Calculating Power Corrections

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Non Perturbative and Topological Aspects of QCD
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Outline

- Overview on QCD renormalons
- The large-n_f limit
- The transverse momentum distribution of a vector boson in hadronic collision
- Linear power corrections affecting shape observables in e^+e^- collisions
- Fully analytic method and factorisation
- Results for thrust and C-parameter
- Conclusions and outlooks

Introduction

- Entering in a very high precision era for LHC physics (HL-LHC)
- A huge amount of new data at an unprecedented accuracy is expected
- Keep increasing the accuracy of theoretical computations
- Need an input on Non-Perturbative (NP) (hadronisation) corrections, scaling as $\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)^p$
- Absence of a solid theoretical background for estimating NP corrections for generic collider observables which do not admit an OPE

Impact of Power Corrections

- QCD Master formula

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i,p}(x_1) f_{j,p}(x_2) d\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2) \times \left[1 + \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^p\right]$$

- $$d\hat{\sigma} = d\hat{\sigma}_{\mathrm{LO}} + \underbrace{\left(\frac{\alpha_s}{\pi}\right) d\hat{\sigma}_{\mathrm{NLO}}}_{\simeq 10\%} + \underbrace{\left(\frac{\alpha_s}{\pi}\right)^2 d\hat{\sigma}_{\mathrm{NNLO}}}_{\simeq 1\%} + \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^p d\hat{\sigma}_{\mathrm{NP}}$$

- For $Q \simeq 100$ GeV and $p = 1$

$$\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^p \simeq 1 - 10\%$$

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- For $Q \simeq 100$ GeV and $p = 1$

$$\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^p \simeq 1 - 10\%$$

Summary

It is crucial to properly estimate Non-Perturbative corrections

Overview on renormalons

- A generic observable D in a renormalizable QFT $D[\alpha] = \sum c_n \alpha^{n+1}$

This series diverges with factorial growth
$c_n = a^n n!$

Need to truncate the series at its minimum value $n_{\min} = 1/(|a|\alpha)$

$$c_{n_{\min}} \alpha^{n_{\min}+1} \simeq \sqrt{\frac{2\pi}{n_{\min}}} e^{-\frac{1}{|a|\alpha}}$$

- The Borel technique is a useful help

$$B[D](t) = \underbrace{\sum c_n \frac{t^n}{n!}}_{\text{Borel Transform}} \Rightarrow \tilde{D} = \underbrace{\int_0^\infty \mathrm{d}t e^{-t/\alpha} B[D](t)}_{\text{Borel integral}} = \int_0^\infty \mathrm{d}t e^{-t/\alpha} \frac{1}{1-at}$$

For $a > 0$ (fixed sign series) we find a pole along the integration path

$$\tilde{D}_\pm = \int_0^\infty \mathrm{d}t e^{-t/\alpha} \frac{1}{1 - at \pm i\eta}, \text{ the ambiguity is } \tilde{D}_+ - \tilde{D}_- \propto e^{-1/(a\alpha)}$$

What about QCD?

- An $\mathcal{O}(\alpha_s)$ correction to a generic observable

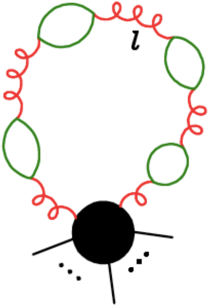
$$\frac{1}{Q^p}\int^Q \mathrm{d}l^p \alpha_s$$

- An all-order computation leads to replace

$$\frac{1}{Q^p}\int_0^Q \mathrm{d}l^p \frac{\alpha_s(Q)}{1-\alpha_s(Q)b_0 \ln\left(Q^2/l^2\right)}$$

$$= \frac{1}{Q^p}\sum_n (-1)^n (2b_0)^n \alpha_s^{n+1}(Q)\int_0^Q \mathrm{d}l l^{p-1}\ln\left(\frac{l}{Q}\right)$$

$$\propto \underbrace{\sum_n \left(\frac{2b_0}{p}\right)^n n!\,\alpha_s^{n+1}(Q)}_{\text{Fixed sign asymptotic series!}} \to n_{\min} = \frac{p}{2b_0\alpha_s(Q)}$$



- The ambiguity takes the form

$$e^{-p/(2b_0\alpha_s(Q))} = \left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^p$$ $\to$ The factorial growth leads to power corrections!

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Summary

The factorial growth for a perturbative series is dubbed a **Renormalon** as it is strictly connected to the renormalization group flow of the QFT

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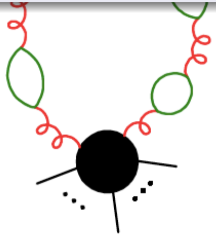

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Case of our Interest

We talk about **Infrared linear ($p=1$) renormalons** arising from low momentum regions

$$\propto \underbrace{\sum_n \left(\frac{2b_0}{p}\right)^n n!\,\alpha_s^{n+1}(Q)}_{\text{Fixed sign asymptotic series!}} \rightarrow n_{\min} = \frac{p}{2b_0\alpha_s(Q)}$$

- The ambiguity takes the form

$$e^{-p/(2b_0\alpha_s(Q))} = \left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^p$$ $\rightarrow$ The factorial growth leads to power corrections!

The Large-n_f method

- Renormalons arise taking all the radiative corrections bulding up the running of α_s
- The running of α_s arises from the fermion bubble insertions along the gluon propagator if one works within the Large-n_f (n_f is the number of flavour) limit
- We take the Abelian limit of QCD ($n_f \to -\infty$), decorating each gluon line with fermionic bubbles

$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2,\epsilon)-\Pi_{\rm ct} = \alpha_s(\mu)\left(\frac{-n_f T_R}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right]$$

- Terms $(\alpha_s n_f)^k$ are fully calculable for each k
- Non-Abelianization at the end of the computation by implementing the Large-b_0 approximation

$$n_f \to -\frac{11 C_A}{4 T_R} + n_l$$

The Large-n_f method

- An all order computation can be handled by taking the $\mathcal{O}(\alpha_s)$ corrections due to a gluon with mass λ
- For a generic IR-safe observable O we take its expectation value (Ferrario Ravasio, Nason, Oleari ('19))

$$\langle O \rangle = \underbrace{\langle O \rangle^{(\mathrm{b})}}_{\text{Born}} - \frac{1}{\alpha_s} \int \mathrm{d}\lambda \frac{\mathrm{d}\langle O \rangle_\lambda^{(1)}}{\mathrm{d}\lambda} \overbrace{\left[\frac{1}{\pi b_0} \arctan \frac{\pi b_0 \alpha_s}{1 + b_0 \alpha_s \log \lambda^2/\mu_C^2} \right]}^{\text{Beneke,'98}}$$

where

$$\langle O \rangle_\lambda^{(1)} = \underbrace{T_V(\lambda) + T_R(\lambda)}_{\text{Virtual and real corrections for a massive gluon}} + \underbrace{T_R^{\Delta}(\lambda)}_{\text{Nason,Seymour('95)}}$$

$$T_R^{\Delta}(\lambda) = \frac{1}{\sigma} \frac{3\pi}{\alpha_s T_F} \lambda^2 \int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\lambda) \delta(\lambda^2 - m_{q\bar{q}}^2) \left[O(\Phi_{q\bar{q}}) - O(\Phi_{(q\bar{q})}) \right]$$

- All the logarithmically divergent terms as $\lambda \to 0$ cancel as O is IR-safe
- A linear term in λ in $\langle O \rangle_\lambda^{(1)}$ $\to$ IR linear renormalon

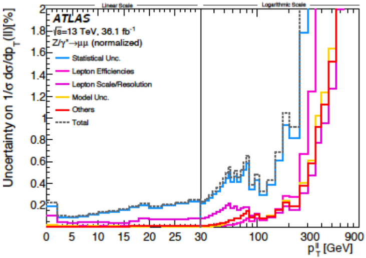
Large-n_f limit in literature

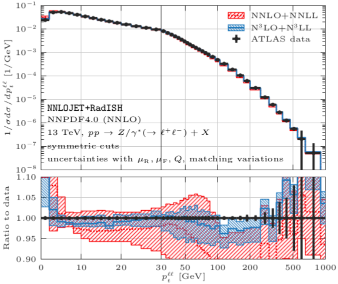
It provides a reliable framework for estimating renormalon corrections

- Beneke, Braun (1995): looking for power corrections in Drell-Yan total cross section, proving that claims about resummation as probe for linear power corrections were unfounded;
- Nason, Seymour (1995): issues about power corrections in shape variables observables;
- Ferrario Ravasio, Nason, Oleari (2019): leptonic observables in top production and decay are affected by IR linear renormalons;
- Ferrario Ravasio, GL, Nason (2020): absence of IR linear renormalons in the p_T distribution of a Z boson in hadronic collisions;
- Caola, Ferrario Ravasio, GL, Melnikov, Nason (2021)+Ozcelik(2022): estimate of leading power corrections affecting Shape Variables in the 3-jet region;
- Nason, Zanderighi (2023): impact of power corrections on α_s fits using shape variables in the three-jet region

The p_T of the Z

- One of the cleanest and best measured LHC observables
- Useful for BSM research and for constraining α_s and PDFs at LHC (Boughezal et al. ('17))
- Sub-percent level precision for normalized distributions measured at LHC (ATLAS and CMS ('15, '19))
- Theoretical uncertainties still at the percent level
- Z+jet @NNLO in QCD (Boughezal, Campbell et al. ('16), Gehrmann-De Ridder, Gehrmann et al. ('16), Gehrmann-De Ridder et al. ('18))
- Current state of the art is N3LO+N^{3}LL (Chen et al. ('22))





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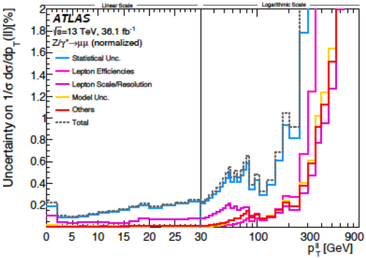
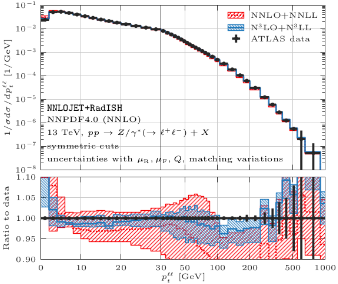
Motivation

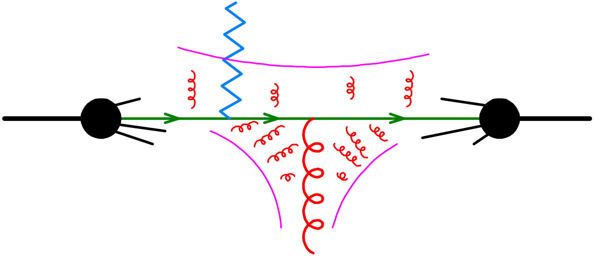
Given the high precision reached for this observable, it is crucial to look for the presence of IR linear renormalons in the moderately large transverse momentum region!

- Current state of the art is N3LO+N^{3}LL (Chen et al. ('22))





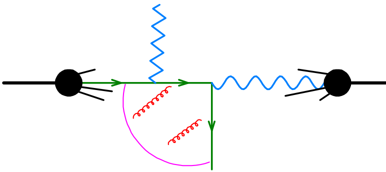
The p_T of the Z: a kinematic argument

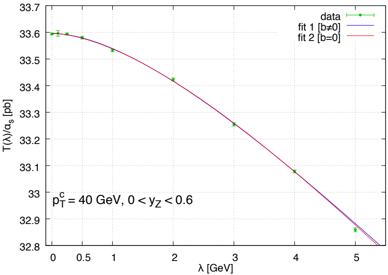


- The soft radiation pattern is not azimuthally symmetric
- A IR linear renormalon is strictly related to soft emissions

If we model a IR linear renormalon as due to the emission of a soft particle with transverse momentum $\sim \Lambda_{\text{QCD}}$, we may assume that it can also affect the p_T^Z by recoil!

- We consider the process $d(p_1)\gamma(p_2) \to Z(p_3)d(p_4)$ to work in the *Large-n_f* limit and to preserve the azimuthal color asymmetry ($E_{CM} = 300$ GeV)





We (Ferrario Ravasio, GL, Nason ('20)) found

$$\langle O \rangle_\lambda^{(1)} \sim \left(\frac{\lambda}{p_T^c}\right)^2 \log\left(\frac{\lambda}{p_T^c}\right)$$

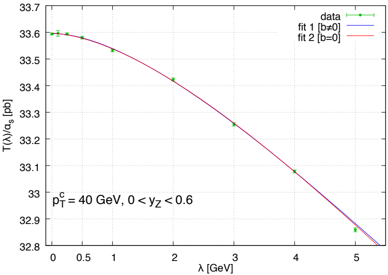
No numeric evidence of a IR linear renormalon for the transverse momentum of the Z boson!

The p_T of the Z: working in the Large-n_f limit

- We consider the process $d(p_1)\gamma(p_2) \to Z(p_3)d(p_4)$ to

Question

Is it possible to provide an analytic argument about the presence (absence) of linear power corrections?



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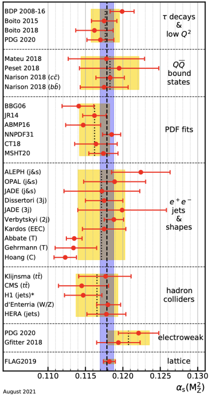
Non Perturbative corrections to Shape Variables

- Shape Variables measure the geometry of a collision and are routinely used for the extraction of α_s from e^+e^- data, from high precision calculations
- It is crucial to consider NP corrections, related to hadronization effects



- From analytic techniques we obtain the values of α_s
  1. $\alpha_s = 0.1135 \pm 0.0010$ [1006.3080] from Thrust
  2. $\alpha_s = 0.1123 \pm 0.0015$ [1501.04111] from C-parameter

- Several standard deviations away from the latest world average value $\alpha_s = 0.1179 \pm 0.0009$ [PDG]

Non Perturbative corrections to Shape Variables

- In the past NP corrections have been considered in the two-jet limit (Webber ('95), Nason and Seymour ('95), Dasgupta and Webber ('96) and many others. . .)
- The extracted value of NP corrections was then extrapolated to the three-jet region for fitting α_s, assuming thery were equal

Is this assumption reliable?

- In a recent work (Luisoni, Monni, Salam ('20)) the authors showed that the NP correction in the three-jet symmetric point is different from the one in the two-jet limit for the C-parameter

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Main Goal

Need a general framework to evaluate NP corrections affecting Shape Variables in a generic three-jet configuration

Linear Power Corrections: an analytic argument

Question

In a theory including a gluon with mass λ, under which hypotheses do $\mathcal{O}(\lambda)$ terms arise/cancel?

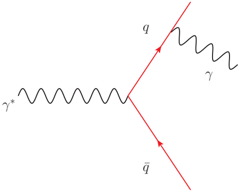
- We (Caola, Ferrario Ravasio, GL, Melnikov, Nason ('21)) observed that
  1. For processes involving massless particles, virtual corrections cannot give rise to linear power corrections
  2. NLO QCD computation taking a gluon with mass λ
     Collinear regions of phase space take the form
     $$\int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2 + \lambda^2} f(\eta, \phi)$$
     For certain kind of observables (C-parameter, Thrust,...) one has
     $$\lim_{\eta \to \pm\infty} f(\eta, \phi) = e^{-|\eta|}$$
  3. For this kind of observables we can rely on the leading soft approximation

Main result

No $\mathcal{O}(\lambda)$ corrections from observables that are inclusive with respect to the soft radiation!

Linear power corrections for Shape Variables

- Linear Power corrections in the three-jet region in the large-n_f limit $\gamma^*(q) \to q(p_1) + \bar{q}(p_2) + \gamma(p_3)$



- Contributions induced by the emission of a soft massive gluon with mass λ, which can further splits $g^*(k) \to q(l)\bar{q}(\bar{l})$

Linear power corrections affecting the cumulative distribution in the Large-n_f limit

$$\Sigma(v) = \sum_F \int \mathrm{d}\sigma_F \theta(V(\Phi_F) - v)$$

Linear Power Corrections to Shape Variables

- $\mathcal{O}(\lambda)$ terms for an observable V can only arise from the splitting contribution to $\Sigma(v;\lambda)$

$$\mathcal{T}_\lambda[\Sigma(v;\lambda)] = \mathcal{T}_\lambda\left[\int \mathrm{d}\Phi_{q\bar{q}} 2\pi\delta(m_{q\bar{q}}^2 - \lambda^2)|M(\{p\},l,\bar{l})|^2\theta(V(\{p\},l,\bar{l}) - v)\right]$$

Expanding in the soft limit we get

$$\mathcal{T}_\lambda[\Sigma(v;\lambda)] = \overbrace{\mathcal{T}_\lambda\left[\int \mathrm{d}\Phi_{q\bar{q}} 2\pi\delta(m_{q\bar{q}}^2 - \lambda^2)|M(\{p\},l,\bar{l})|^2\theta(V(\tilde{p}) - v)\right]}^{\text{No linear terms from this integration!}} +$$

$$\mathcal{T}_\lambda\left[\int \mathrm{d}\Phi_{q\bar{q}} 2\pi\delta(m_{q\bar{q}}^2 - \lambda^2)|M(\{p\},l,\bar{l})|^2\delta(V(\{\tilde{p}\}) - v) \underbrace{[V(\{p\},l,\bar{l}) - V(\{\tilde{p}\})]}_{\text{This term is suppressed in the soft region!}}\right]$$

We can rely on the leading soft approximation for $M(\{p\},l,\bar{l})$

Linear Power Corrections to Shape Variables

- The phase space can be factorized

$$\mathrm{d}\Phi_{q\bar{q}}\delta(\lambda^2-(l+\bar{l})^2) \propto \underbrace{\mathrm{d}\tilde{\Phi}_b}_{\gamma^*\to q\bar{q}\gamma} \times \underbrace{[\mathrm{d}k]}_{\text{gluon phase space}} \times \underbrace{[\mathrm{d}l][\mathrm{d}\bar{l}]}_{g^*\to q\bar{q}}$$

- We use a smooth mapping in k (in the soft limit) to construct the real momenta $\{p_i\}$ from the underlying Born momenta $\{\tilde{p}_i\}$

$$p_i^\mu = \tilde{p}_i^\mu + R^\mu_{i,\nu}(\{\tilde{p}\})k^\nu + \mathcal{O}(k_0^2)$$

- We consider shape observables V (C-parameter and Thrust) such that

$$V(\{p\},l,\bar{l}) - V(\{\tilde{p}\}) = [V(\{\tilde{p}\},l,\bar{l}) - V(\{\tilde{p}\})] + [V(\{p\}) - V(\{\tilde{p}\})] + \mathcal{O}(k_0^2)$$

- We only need

$$V(\{p\},l,\bar{l}) - V(\{\tilde{p}\}) \to V(\{\tilde{p}\},l,\bar{l}) - V(\{\tilde{p}\})$$

Only the emission term matters!

- Linear power corrections can only arise from

$$V(\{\tilde{p}\}, l, \bar{l}) - V(\{\tilde{p}\})$$

After a bit of algebra

$$\mathcal{T}_\lambda[\Sigma(v;\lambda)] = \int \mathrm{d}\sigma^{\mathrm{b}}(\tilde{\Phi}_b)\delta(V(\{\tilde{p}\}) - v) \times \left[\mathcal{N}\mathcal{T}_\lambda[I_V(\{\tilde{p}\},\lambda)]\right]$$

with

$$I_V(\{\tilde{p}\},\lambda) = \int [\mathrm{d}k] \frac{J_\mu J_\nu}{\lambda^2} \theta\left(\omega_{\max} - \frac{(kq)}{\sqrt{q^2}}\right) \int [\mathrm{d}l][\mathrm{d}\bar{l}](2\pi)^4 \delta^{(4)}(k - l - \bar{l})$$
$$\times \underbrace{\mathrm{Tr}\left[\not{l}\gamma^\mu \not{\bar{l}}\gamma^\nu\right]}_{g^* \to q\bar{q}} [V(\{\tilde{p}\}, l, \bar{l}) - V(\{\tilde{p}\})]$$

where

- $J^\mu = \frac{\tilde{p}_1^\mu}{(\tilde{p}_1 k)} - \frac{\tilde{p}_2^\mu}{(\tilde{p}_2 k)}$ is the soft eikonal current for the emission of a massive gluon
- $\omega_{\max}$ is a UV cut-off on the gluon energy in the rest frame of q

Factorized approach: general case

Separation of soft emission and recoil contributions

$$V(\{p\},l,\bar{l}) - V(\{\tilde{p}\}) = V(\{\tilde{p}\},l,\bar{l}) - V(\{\tilde{p}\}) + \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) k^{\nu}$$

$$+ \underbrace{\left\{ \frac{\partial V(\{\tilde{p}\},l,\bar{l})}{\partial \tilde{p}_i^{\mu}} - \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} \right\}}_{\text{Suppressed in the soft region!}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) k^{\nu} + \mathcal{O}(k_0^2)$$

We take observables linear in the soft emissions

$$V(\{p\},l,\bar{l}) - V(\{\tilde{p}\}) \approx V(\{\tilde{p}\},l) - V(\{\tilde{p}\}) + \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) l^{\nu}$$

$$+ V(\{\tilde{p}\},\bar{l}) - V(\{\tilde{p}\}) \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) \bar{l}^{\nu}$$

Factorized approach: general case

Separation of soft emission and recoil contributions

Main point

If the observable is linear in the soft emission we can study its variation due to the emission of a single soft massless parton!

Suppressed in the soft region!

We take observables linear in the soft emissions

$$V(\{p\},l,\bar{l}) - V(\{\tilde{p}\}) \approx V(\{\tilde{p}\},l) - V(\{\tilde{p}\}) + \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) l^{\nu}$$

$$+ V(\{\tilde{p}\},\bar{l}) - V(\{\tilde{p}\}) \frac{\partial V(\{\tilde{p}\})}{\partial \tilde{p}_i^{\mu}} R^{\mu}_{i,\nu}(\{\tilde{p}\}) \bar{l}^{\nu}$$

Factorized approach: general case

We can consider the equation

$$V(\{p\},l) - V(\{\tilde{p}\}) = \frac{l_t}{q} h_V(\eta,\varphi)$$

- The expression vanishes in the soft limit
- No linear power corrections can arise from hard collinear divergences (Caola, Ferrario Ravasio, GL, Melnikov, Nason ('21))
- h_V must be suppressed for large $|\eta|$ and arbitrary φ

$$I_V^{\mathrm{unreg}} = W_V \times \lambda F$$

$$W_V = \int \frac{\mathrm{d}\eta \mathrm{d}\varphi}{2(2\pi)^3} \frac{h_V(\eta,\varphi)}{q}$$

$$F(p_1,p_2,\tilde{l}) = 16\pi \int [\mathrm{d}k] \frac{J^\mu J^\nu}{\lambda^3} \left\{ -2\tilde{l}^\mu \tilde{l}^\nu \frac{\lambda^8}{(2k\tilde{l})^5} - \frac{g^{\mu\nu}\lambda^6}{2(2k\tilde{l})^3} \right\}$$

$$= -\frac{5\pi}{64} \rightarrow \text{Constant value!}$$

Factorized approach: general case

We can consider the equation

Expression to evaluate

$$\mathcal{T}_\lambda[\Sigma(v;\lambda)] = \int \mathrm{d}\sigma^{\mathrm{b}}(\tilde{\Phi}_{\mathrm{b}})\delta(V(\tilde{\Phi}_{\mathrm{b}}) - v)\frac{\lambda}{q}[\mathcal{N}F(qW_V)]$$
$$= \int \mathrm{d}\sigma^{\mathrm{b}}(\tilde{\Phi}_{\mathrm{b}})\delta(V(\tilde{\Phi}_{\mathrm{b}}) - v)\frac{\lambda}{q}\left[-\frac{15\pi}{64}\alpha_s\pi C_F \int \mathrm{d}\eta\frac{\mathrm{d}\varphi}{2\pi}h_V(\eta,\varphi)\right]$$

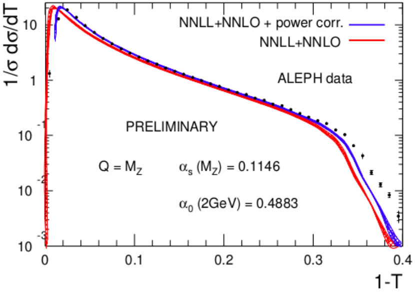
$$I_V^{\mathrm{unreg}} = W_V \times \lambda F$$

$$W_V = \int \frac{\mathrm{d}\eta\mathrm{d}\varphi}{2(2\pi)^3}\frac{h_V(\eta,\varphi)}{q}$$

$$F(p_1,p_2,\tilde{l}) = 16\pi\int[\mathrm{d}k]\frac{J^\mu J^\nu}{\lambda^3}\left\{-2\tilde{l}^\mu\tilde{l}^\nu\frac{\lambda^8}{(2k\tilde{l})^5} - \frac{g^{\mu\nu}\lambda^6}{2(2k\tilde{l})^3}\right\}$$
$$= -\frac{5\pi}{64} \to \text{Constant value!}$$

NP correction as a shift in the shape variable

[Monni '12]



- NP corrections show up as a shift in the Shape Observable

$$\tilde{\Sigma}^{\rm had}(v) = \Sigma(v - \delta_{\rm NP}(v)) \approx \tilde{\Sigma}(v) - \frac{1}{\sigma}\frac{{\rm d}\sigma}{{\rm d}V}\delta_{\rm NP}(v)$$

- As the total cross section is free from linear power corrections we get

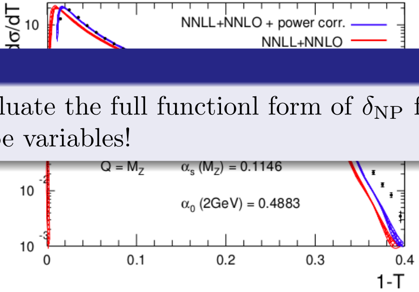
$$\delta_{\rm NP}(v) = \frac{\mathcal{T}_\lambda[\Sigma(v;\lambda)]}{{\rm d}\sigma/{\rm d}V}$$

- Use the parameterisation

$$\delta_{\rm NP}(v) = h\zeta(v), \quad h \equiv \delta_{\rm NP}(0)$$

NP correction as a shift in the shape variable

[Monni '12]



Our result

With our framework we are able to evaluate the full functionl form of δ_{NP} for specific shape variables!

$$\tilde{\Sigma}(v) - \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}V}\delta_{\mathrm{NP}}(v)$$

- As the total cross section is free from linear power corrections we get

$$\delta_{\mathrm{NP}}(v) = \frac{\mathcal{T}_\lambda[\Sigma(v;\lambda)]}{\mathrm{d}\sigma/\mathrm{d}V}$$

- Use the parameterisation

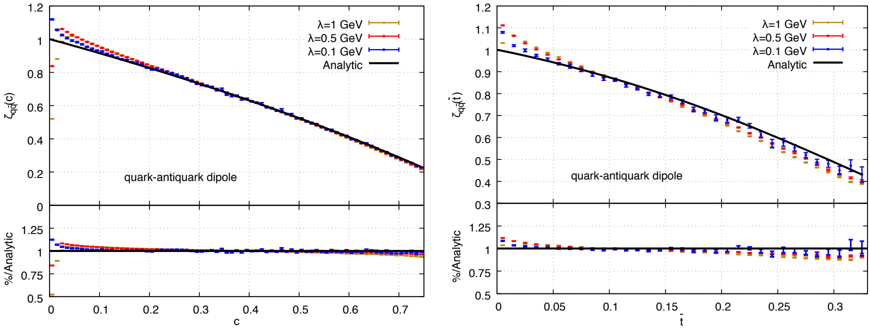
$$\delta_{\mathrm{NP}}(v) = h\zeta(v), \quad h \equiv \delta_{\mathrm{NP}}(0)$$

NP shift in the Shape Variable: $q\bar{q}$ dipole emission

- Start with the process $\gamma^* \to q + \bar{q} + \gamma$
- The only emitting dipole is the $q\bar{q}$

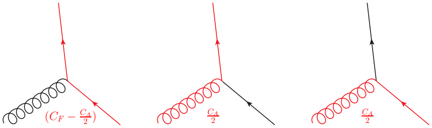
$$\delta_{\rm NP}^{q\bar{q}\gamma} = h\zeta_{q\bar{q}}(v), \quad \zeta_{q\bar{q}}(0) = 1$$

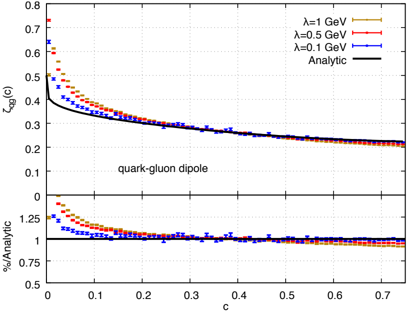
- Comparison of the analytic result with a full large-n_f computation for $\lambda = 0.1,\ 0.5,\ 1$ GeV, $q = 100$ GeV

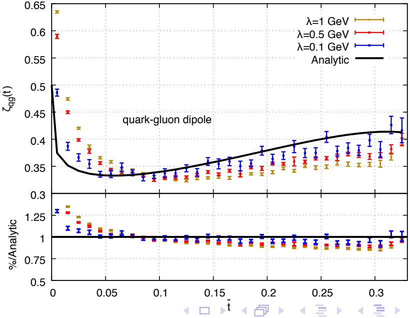


NP shift in the Shape Variable: realistic case

- Linear power corrections come from the soft limit of the emission amplitude
- We extended our considerations to the realistic process $\gamma^* \to q\bar{q}g$
- Three emitting dipoles ($q\bar{q}$, qg, $\bar{q}g$)



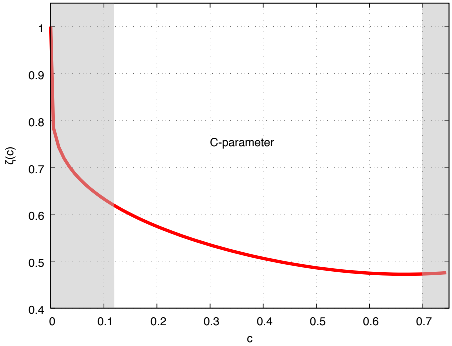


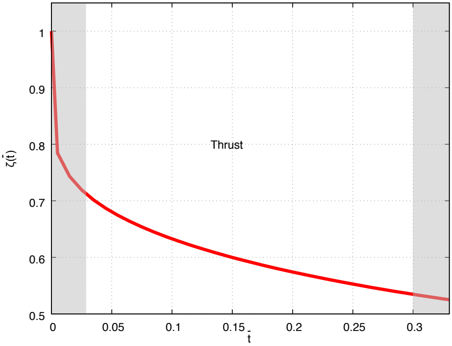


NP shift in the Shape Variable: realistic case

- The contributions from the three dipoles are additive

$$\zeta^{q\bar{q}g}(v) = \zeta_{q\bar{q}}(v)\frac{C_F - C_A/2}{C_F} + \zeta_{qg}(v)\frac{C_A}{C_F}$$





- The shape of NP correction is non trivial in the three-jet region!
- Estimate of the NP correction in the three-jet region for the first time!

Conclusions and Outlooks

- Understanding Non-Perturbative corrections to collider processes is now crucial, given the high precision reached at LHC
- Large-n_f method is a reliable framework to investigate $\mathcal{O}(\Lambda_{\text{QCD}}/Q)^p$ corrections
- No linear terms if integrating inclusively over the soft radiation phase space: analytical proof about the absence of IR linear renormalons in the p_T distribution of the Z boson in hadronic collisions (Ferrario Ravasio, GL, Nason ('20))
- Fully analytic prediction for NP corrections for Thrust and C-parameter away from the two-jet region (Caola, Ferrario Ravasio, GL, Melnikov, Nason, Ozcelik ('22))
- Impact of these findings for α_s fits (Nason, Zanderighi ('23))
- Linear Power Corrections to the electroweak single top production at LHC (Makarov, Melnikov, Nason, Ozcelik ('23))
- Linear Power Corrections to top quark pair production (Makarov, Melnikov, Nason, Ozcelik ('23))
- Next directions: extensions to other observables, study of resummation effects and phenomenological applications

THANKS FOR THE ATTENTION!!!

BACKUP

Renormalons structure

For $\left.\frac{\mathrm{d}\langle O\rangle_\lambda^{(1)}}{\mathrm{d}\lambda}\right|_{\lambda=0} = A$ (constant), we take the $\lambda < \mu_C$ $(\mu_C = \mu e^{5/6})$ region

$$-\frac{1}{b_0\alpha_s}\left.\frac{\mathrm{d}\langle O\rangle_\lambda^{(1)}}{\mathrm{d}\lambda}\right|_{\lambda=0}\int_0^{\mu_C}\frac{\mathrm{d}\lambda}{\pi}\arctan\frac{\pi b_0\alpha_s}{1+b_0\alpha_s\ln\frac{\lambda^2}{\mu_C^2}}$$

Taking $a = b_0\alpha_s$, $\lambda/\mu_C = l$

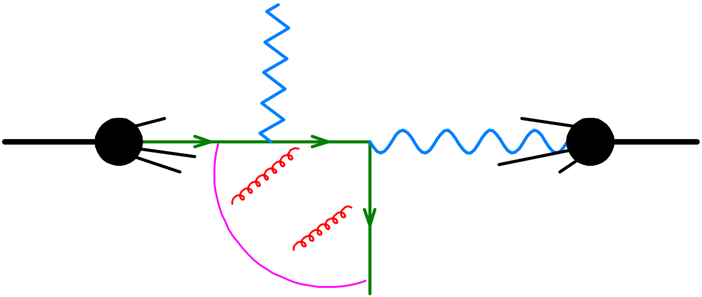
$$\int_0^1\frac{\mathrm{d}l}{\pi a}\arctan\frac{\pi a}{1+a\ln l^2} = \overbrace{\frac{1}{\pi a}\arctan(\pi a)+\int_0^1\mathrm{d}z\frac{\pi a\cos(\pi z/2)-\sin(\pi z/2)}{1+(z\pi a)^2}}^{\text{Analytic}}$$
$$+\underbrace{\frac{1}{\pi a}\mathrm{P}\int_0^\infty\mathrm{d}t\frac{\exp(-\frac{t}{2a})}{1-t}}_{\text{Borel integral}}-\underbrace{\frac{1}{a}\exp\left(-\frac{1}{2a}\right)}_{\text{Ambiguity}}$$

Replacing $a = b_0\alpha_s = 1/\ln(\mu_C^2/\Lambda^2)$

$$\exp\left(-\frac{1}{2a}\right) = \frac{\Lambda}{\mu_C}$$

The p_T of the Z in the large-n_f limit

- No existing large-n_f computation for a process with a Born level gluon
- $d(p_1)\gamma(p_2) \rightarrow Z(p_3)d(p_4)$ as proxy for the real QCD process



- The azimuthal asymmetry of the soft emission pattern is preserved

An all-order large-n_f computation is possible

The p_T of the Z in the large-n_f limit

Evaluation of the cross section

$$\sigma = \sigma_{\mathrm{B}} - \frac{1}{b_0 \alpha_s(\mu)} \int_0^\infty \frac{\mathrm{d}\lambda}{\pi} \frac{\mathrm{d}T(\lambda)}{\mathrm{d}\lambda} \arctan \frac{\pi b_0 \alpha_s}{1 + b_0 \alpha_s \ln^2 \frac{\lambda^2}{\mu_C^2}}$$

Computation of $T(\lambda)$ for different λ values

$$T(\lambda) = T_V(\lambda) + T_R(\lambda) + T_\oplus(\lambda) + T_\ominus(\lambda) + T_\ominus^\Delta(\lambda) + T_R^\Delta(\lambda)$$

- The Δ terms can be neglected as we are inclusive with respect to the splitting of the gluon in a $q\bar{q}$ pair
- DIS scheme for subtracting the initial state collinear singularity for the gluon ($T_\oplus(\lambda)$)

The p_T of the Z: $T(\lambda)$

- The integration diverges as $p_{\mathrm{T},Z}$ approaches 0

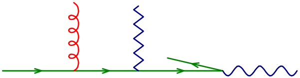
$$F_{\mathrm{supp}} = \frac{p_{\mathrm{T},Z}^4}{p_{\mathrm{T},Z}^4 + p_{\mathrm{T,cut}}^4}$$

- $T_V(\lambda)$:
  1. UV divergences extracted in CDR ($d = 4 - 2\epsilon$) and canceled in the total
  2. IR divergences regulated by the gluon mass λ
- $T_R(\lambda)$ evaluated in 4 dimensions:
  1. IR divergences when g gets soft or collinear to either the initial or final d quark (arising as $\ln\lambda$, $\ln^2\lambda$ as $\lambda \to 0$)
  2. IR singularity associated with the splitting of the initial state photon

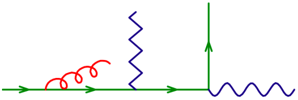
$$T_R(\lambda) = \int \mathrm{d}\mathbf{\Phi}_{g^*} R_{g^*}(\mathbf{\Phi}_{g^*})\Theta(\mathbf{\Phi}_{g^*})$$

The real squared amplitude is divided in three regions

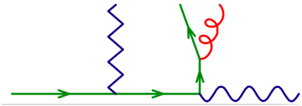
$$R^{(1)} = \frac{\frac{1}{p^2_{\mathrm{T},d}}}{\frac{1}{p^2_{\mathrm{T},d}} + \frac{1}{m^2_{\mathrm{T},g}} + \frac{(E_d+E_g)^2}{E_d E_g m^2_{d,g}}} R$$



$$R^{(2)} = \frac{\frac{1}{m^2_{\mathrm{T},g}}}{\frac{1}{p^2_{\mathrm{T},d}} + \frac{1}{m^2_{\mathrm{T},g}} + \frac{(E_d+E_g)^2}{E_d E_g m^2_{d,g}}} R$$



$$R^{(3)} = \frac{\frac{(E_d+E_g)^2}{E_d E_g m^2_{d,g}}}{\frac{1}{p^2_{\mathrm{T},d}} + \frac{1}{m^2_{\mathrm{T},g}} + \frac{(E_d+E_g)^2}{E_d E_g m^2_{d,g}}} R$$



The p_T of the Z: results

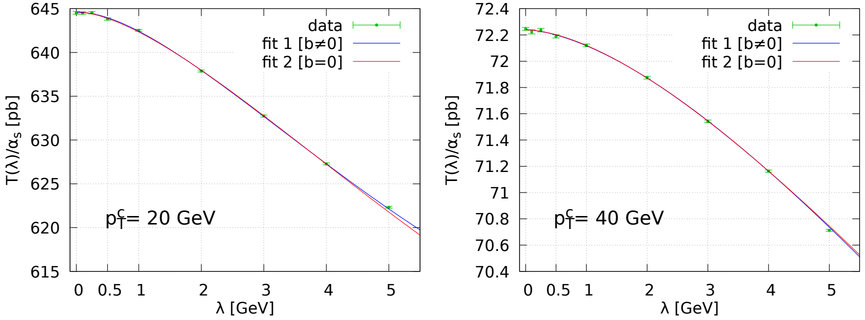
Our setup:

- $f_d^{(1)}(x = x_\oplus) = f_\gamma^{(2)}(x = x_\ominus) = \frac{(1-x)^3}{x}$
- $M_Z = 91.188$ GeV
- $E_{\rm CM} = 300$ GeV
- $\sigma(p_T^Z > p_{\rm T}^c)$ $(p_{\rm T}^c = 20, 40$ GeV$)$
- $\sigma(p_T^Z > p_{\rm T}^c, 0 < y < y_{\rm cut} = 0.6)$
- $\mu_{\rm F} = M_Z$

We fit $T(\lambda)$ with the fitting function

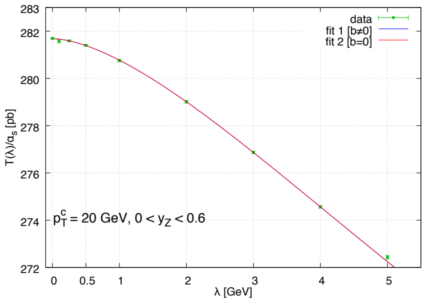
$$f(\lambda) = a\left[1 + b\left(\frac{\lambda}{p_{\rm T}^c}\right) + c\left(\frac{\lambda}{p_{\rm T}^c}\right)^2 \ln^2\left(\frac{\lambda}{p_{\rm T}^c}\right) + d\left(\frac{\lambda}{p_{\rm T}^c}\right)^2 \ln\left(\frac{\lambda}{p_{\rm T}^c}\right)\right]$$

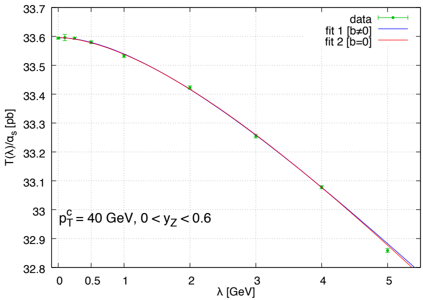
The p_T of the Z:results



- $\lambda = 5$ GeV excluded from the fit
- b first included and then set to 0 to study its impact on $T(\lambda)$
- $b = 0.009 \pm 0.004$ for $p_T^c = 20$ GeV and $b = 0.024 \pm 0.017$ for $p_T^c = 40$ GeV

The p_T of the Z: results for a more exclusive analysis



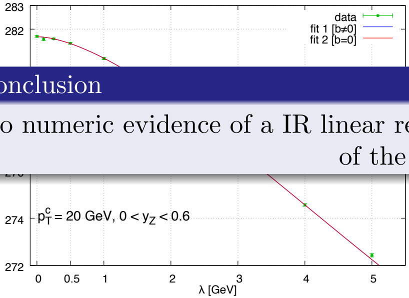


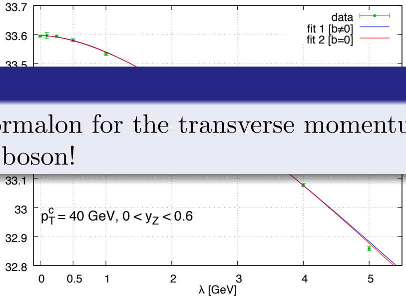
- $b = -0.001 \pm 0.009$ for $p_{\mathrm{T}}^{c} = 20$ GeV and $b = 0.015 \pm 0.025$ for $p_{\mathrm{T}}^{c} = 40$ GeV

We (Ferrario Ravasio, GL, Nason ('20)) found

$$\langle O \rangle_{\lambda}^{(1)} \sim \left(\frac{\lambda}{p_{\mathrm{T}}^{c}}\right)^{2} \ln\left(\frac{\lambda}{p_{\mathrm{T}}^{c}}\right)$$

The p_T of the Z: results for a more exclusive analysis





Conclusion

No numeric evidence of a IR linear renormalon for the transverse momentum of the Z boson!

- $b = -0.001 \pm 0.009$ for $p_{\mathrm{T}}^c = 20$ GeV and $b = 0.015 \pm 0.025$ for $p_{\mathrm{T}}^c = 40$ GeV

We (Ferrario Ravasio, GL, Nason ('20)) found

$$\langle O \rangle_\lambda^{(1)} \sim \left(\frac{\lambda}{p_{\mathrm{T}}^c}\right)^2 \ln\left(\frac{\lambda}{p_{\mathrm{T}}^c}\right)$$

The p_T of the Z: results for a more exclusive analysis

Conclusion

No numeric evidence of a IR linear renormalon for the transverse momentum of the Z boson!

Question

Is it possible to provide an analytic argument about the presence (absence) of linear power corrections?

- $b = -0.001 \pm 0.009$ for $p_T^c = 20$ GeV and $b = 0.015 \pm 0.025$ for $p_T^c = 40$ GeV

We (Ferrario Ravasio, GL, Nason ('20)) found

$$\langle O \rangle_\lambda^{(1)} \sim \left(\frac{\lambda}{p_T^c}\right)^2 \ln\left(\frac{\lambda}{p_T^c}\right)$$

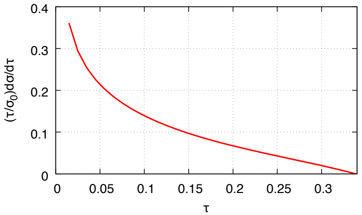
Shape Variables: Details of the Computation

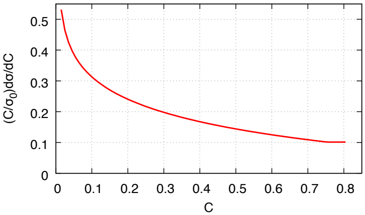
- Two shape variables O such that $\frac{d\sigma}{dO} = \sigma_0 \delta(O)$ at LO
  1. Thrust $(2/3 < T < 1)$

$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i E_i} \rightarrow \begin{cases} \tau = 0 & \text{2-jet region} \\ \tau = 1/3 & \text{3-jet symmetric point} \end{cases}$$

  2. C-parameter $(0 < C < 1)$

$$3 - \frac{3}{2Q^2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{E_i E_j} \rightarrow \begin{cases} C = 0 & \text{2-jet region} \\ C = 3/4 & \text{3-jet symmetric point} \end{cases}$$





- The C-parameter has a *Sudakov shoulder* within the physical range $(C = 3/4)$

Direct analytic integration: the case of the C-parameter

We take $V = C$ and decompose

$$I_C(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda) = -\frac{3\lambda}{4\pi^3 q} \sum_{i=1}^{5} I_i(x, y, \lambda)$$

where

$$I_i(x, y, \lambda) = \int_0^{\beta_{\max}} \mathrm{d}\beta G_i(\beta, x, y)$$

with

- $q = \sqrt{q^2}$
- β is the velocity of the massive gluon in the q rest frame ($\beta_{\max} = \sqrt{1 - \lambda^2/\omega^2_{\max}}$)
- x and y parameterise the three-jet kinematics
- G_5 is the most interesting one

Direct analytic integration: the case of the C-parameter

- G_5 integrable at $\beta = 1$ but not at $\beta = 0$

$$I_5^{\text{reg}} = \int_{\beta_{\min}}^{1} \mathrm{d}\beta \; G_5(x, y, \lambda)$$

- The result can be expressed in terms of the two complete elliptic integrals

$$G_5 = \frac{\sqrt{1-\beta^2}\ln\left(\frac{1+\beta}{1-\beta}\right)\ln\left(\frac{\sqrt{1-\beta^2 c_{12}^2}+\beta s_{12}}{\sqrt{1-\beta^2 c_{12}^2}-\beta s_{12}}\right)}{64\beta^8 s_{12} x(x(y-1)+1)(xy-1)\sqrt{1-\beta^2 c_{12}^2}}$$
$$\times\Big(\beta^6 x\left[x^2(y-1)y + x\left(-4y^2+4y-5\right)+5\right] + \beta^4\left[x^2\left(54y^2-54y-\right.\right.$$
$$\left.-\,21x^3(y-1)y+55x-38\right] + 5\beta^2\left[x^2\left(-24y^2+24y+5\right)\right.$$
$$\left.+\,11x^3(y-1)y-17x+12\right] - 35(x-2)\left(x^2(y-1)y+x-1\right)\Big).$$

$$K(z) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-zt^2)}},$$
$$E(z) = \int_0^1 \frac{\mathrm{d}t\,\sqrt{1-zt^2}}{\sqrt{1-t^2}}$$

$$\mathcal{T}_\lambda[\Sigma(c,\lambda)] = \int \mathrm{d}\sigma^{\mathrm{b}}\delta(C(\Phi_{\mathrm{b}})-c)$$
$$\times\, \alpha_s C_F \frac{45\pi}{16}\frac{s_{12}^3}{1-z_3}\left[\frac{(1+z_3)}{2}K(c_{12}^2) - (1-z_1 z_2)E(c_{12}^2)\right]\left(\frac{\lambda}{q}\right)$$

Direct analytic integration: the case of the C-parameter

- G_5 integrable at $\beta = 1$ but not at $\beta = 0$

$$I_5^{\rm reg} = \int^1 \mathrm{d}\beta\, G_5(x,y,\lambda)$$

Check

- In the two-jet limit ($c_{12} = 0,\ z_2 \to 0,\ z_1 + z_3 \to 1$):

$$\frac{\mathcal{T}_\lambda[\Sigma(0,\lambda)]}{\mathrm{d}\sigma/\mathrm{d}C|_{c=0}} = -\frac{15}{16}\pi^2\left(\frac{\lambda}{q}\right)\alpha_s$$

- In the three-jet symmetric point ($c_{12} = 0, z_i = 1/3$):

$$\frac{\mathcal{T}_\lambda[\Sigma(3/4,\lambda)]}{\mathrm{d}\sigma/\mathrm{d}C|_{c=3/4}} = \frac{15}{32}\sqrt{3}\pi[3K(1/4) - 4E(1/4)]\left(\frac{\lambda}{q}\right)\alpha_s$$

$-54y -$

$).$

$$E(z) = \int_0^1 \frac{\mathrm{d}t\ \sqrt{1-zt^2}}{\sqrt{1-t^2}}$$

$$\mathcal{T}_\lambda[\Sigma(c,\lambda)] = \int \mathrm{d}\sigma^{\rm b}\delta(C(\Phi_{\rm b}) - c)$$
$$\times\, \alpha_s C_F \frac{45\pi}{16}\frac{s_{12}^3}{1-z_3}\left[\frac{(1+z_3)}{2}K(c_{12}^2) - (1 - z_1 z_2)E(c_{12}^2)\right]\left(\frac{\lambda}{q}\right)$$

Shape Variables: Computation in the Large-n_f limit

- Computation of
$$\langle O\rangle_\lambda^{(1)} = T_V(\lambda) + T_R(\lambda) + T_R^\Delta(\lambda)$$
with
$$T_R^\Delta(\lambda) = \frac{1}{\sigma_0}\frac{3\pi}{\alpha_s T_F}\lambda^2 \int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\lambda)\delta(\lambda^2 - m_{q\bar{q}}^2)\Big[O(\Phi_{q\bar{q}}) - O(\Phi_{(q\bar{q})})\Big]$$
- The integration diverges in the two-jet limit
$$F_{\text{supp}} = C^2$$
- $T_V(\lambda)$:
  1. IR divergences regulated by the gluon mass λ
  2. UV divergences regulated in CDR ($d = 4 - 2\epsilon$) and canceled in the total
- $T_R(\lambda)$ evaluated in 4 dimensions:
  1. IR divergences arising as γ gets soft or collinear to either d or $\bar{d}$
  2. IR divergences when g gets collinear to either d or $\bar{d}$ (arising as $\log\lambda, \log^2\lambda$ singularities as $\lambda \to 0$)

Shape Variables: Computation in the Large-n_f limit

$$T_R(\lambda) = \frac{1}{\sigma_0}\int \mathrm{d}\Phi_{3+1} R^{(\lambda)}_{g^*}(\Phi_{3+1}) O_{3+1}$$

- The real squared amplitude is divided in three regions

$$R = R^{(1)} + R^{(2)} + R^{(3)}$$

$$R^{(1)} = \frac{f^2_{d\gamma} + f^2_{\bar{d}\gamma}}{f^2_{d\gamma} + f^2_{\bar{d}\gamma} + f^2_{dg} + f^2_{\bar{d}g}} R \quad (\gamma \parallel d(\bar{d}), \gamma \text{ soft})$$

$$R^{(2)} = \frac{f^2_{dg}}{f^2_{d\gamma} + f^2_{\bar{d}\gamma} + f^2_{dg} + f^2_{\bar{d}g}} R \quad (g \parallel d)$$

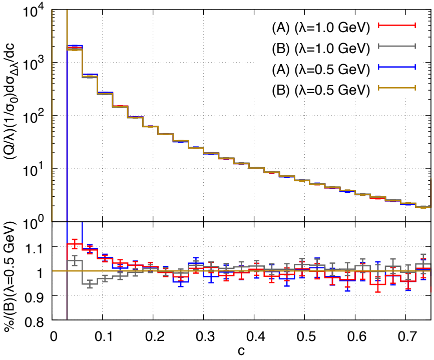
$$R^{(3)} = \frac{f^2_{\bar{d}g}}{f^2_{d\gamma} + f^2_{\bar{d}\gamma} + f^2_{dg} + f^2_{\bar{d}g}} R \quad (g \parallel \bar{d})$$

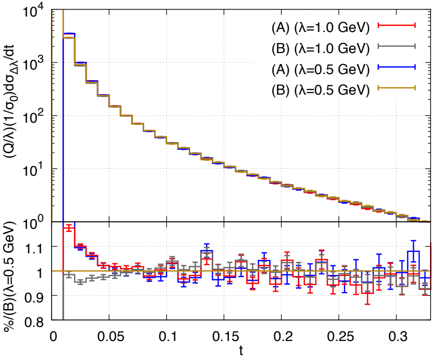
$$f_{ij} = \frac{E_i + E_j}{(k_i + k_j)^2} \quad (i, j = d, \bar{d}, \gamma, g)$$

- $R^{(1)}$ integrated within the `POWHEG-BOX`, $R^{(2)}, R^{(3)}$ with a separated Fortran code
- $\gamma^* \to d\bar{d}\gamma q\bar{q} \Rightarrow$ IR finite as $\lambda \to 0$, QED singularity from γ (`POWHEG-BOX`)

Shape Variables: Results for Kinematical Distributions

- $\langle O\rangle_\lambda^{(1)} - \langle O\rangle_0^{(1)}$, with $O = \delta(z - z(\Phi))$, for t $= 1 -$ Thrust and C-parameter
- Computation for $\lambda = 0.5, 1$ GeV, for $Q = 100$ GeV
- Comparison between analytical approach (A) and Large-n_f limit (B)



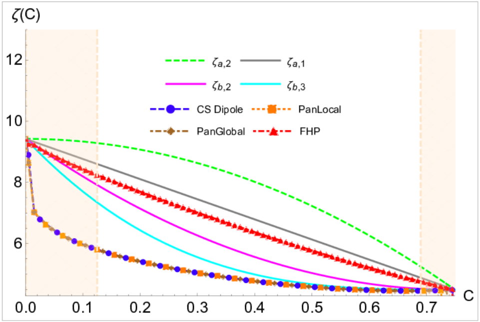


- Behaviour in λ is nearly linear
- Excellent agreement between the two methods
- $\mathcal{O}(\lambda^2)$ entering for $C \lesssim 0.15$ and t $\lesssim 0.07$

Comparison with literature

- Ambiguous prediction in the bulk of the three-jet region
- Different results depending on the way of handling the recoil due to the emission of a soft massless gluon
- Perfect agreement with our results if using a mapping which satisfies our requirements (Catani-Seymour, PanLocal, PanGlobal)

[Luisoni, Monni, Salam ('20)]



- All the recoil schemes give the same prediction at the endpoints $c = 0$, $c = 3/4$, where the recoil effect are strongly suppressed

Impact on α_s fits

- Impact of NP corrections on α_s fits (Nason, Zanderighi '23)