Characterization of wakes and impedances in non-ultrarelativistic regime

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Acknowledgements: Giovanni Rumolo, Elena de la Fuente García

16.05.2024 | CEI Section Meeting

Outline

- **Introduction**
- **Simulation technique for non-ultrarelativistic beams**
  - Numerical cancellation of the direct space charge
- **Simulations of a resistive wall chamber with the Wakefield Solver**
  - Longitudinal study
  - Transverse study
- **Simulations of a pillbox cavity with the Eigenmode and Wakefield solvers**
  - Longitudinal impedance
  - Transverse impedance
- **Conclusions**
- **Next steps**

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Beam coupling impedance

- The **beam coupling impedance** describes the **interaction of a** particle **beam with the** surrounding **environment.**
- For a device of length l, the beam coupling impedance is defined as

$$Z_{\parallel} = -\frac{1}{q_0}\int_0^l E_s\, e^{jks}\, ds$$

$$Z_{x,y} = \frac{j}{q_0}\int_0^l [E_{x,y} - \beta Z_0 H_{y,x}]\, e^{jks}\, ds$$

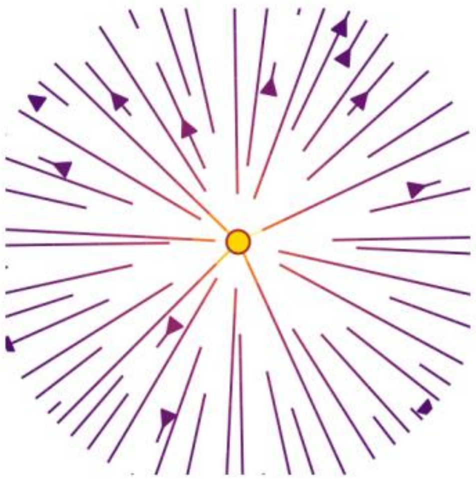
with $E_{s,x,y}$ and $H_{x,y}$ electric and magnetic induced fields in the frequency domain.

- When $\beta < 1$, the induced fields **also** include the **indirect space charge** field, which is related to the interaction of the particles among each other due to the external environment:

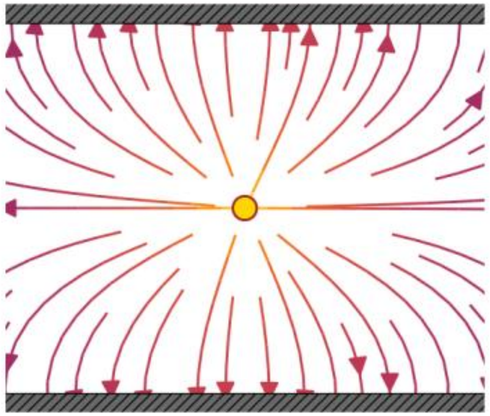
$$Z_{tot}(\beta) = Z(\beta) + Z^{ISC}(\beta)$$

Space charge

- When $\boldsymbol{\beta} < \mathbf{1}$, the **charged particles** of a beam **also create self-fields**, that lead to the direct space charge effect.
- **Direct space charge** is related only to the interaction of the particles among each other in open space.
- While *indirect space charge* is typically directly *taken into account* in the impedance model, the **direct space charge impedance has to be removed**.



Direct space charge in open space.



Indirect space charge with material boundaries.

Kevin Li, Collective effects – an introduction

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Electromagnetic simulations for non-ultrarelativistic beams

- For **ultrarelativistic beams**, the **reliability of CST** electromagnetic simulations has been **extensively proved**.
- But **CST can't discriminate between the fields** induced by the beam, so the simulated beam coupling impedance of a device under test (DUT) is

$$Z_{DUT}^{tot}(\beta) = Z_{DUT}(\beta) + Z_{DUT}^{ISC}(\beta) + Z^{SC}(\beta)$$

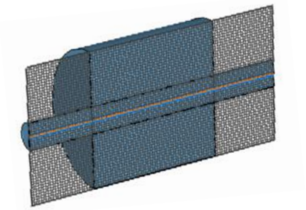
where $Z_{DUT}^{ISC}(\beta)$ is the indirect space charge impedance due to the DUT and $Z^{SC}(\beta)$ is the direct space charge impedance.

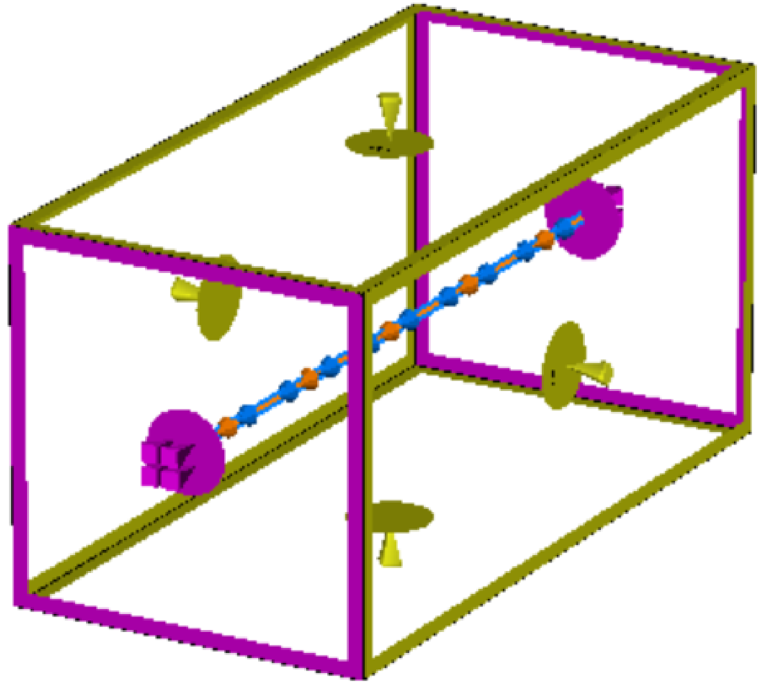
- For $\beta = 1$ it results $Z^{SC}(\beta) = 0$ and $Z_{DUT}^{ISC}(\beta) = 0$.

- For **non-ultrarelativistic beams**, the main **complication** consists in **removing the contribution of the direct space charge** of the source bunch.

Simulations of the bounding box

- CST simulations take place within a delimited domain called ***bounding box***.
  - Since CST is a numerical solver, it discretizes the domain with a mesh grid.





- The **bounding box** (bb) can be **simulated without changing its discretization**, by excluding all the elements of the DUT from the simulation.

- The resulting beam coupling impedance can be written as

$$Z_{bb}^{tot}(\beta) = Z^{SC}(\beta) + Z_{bb}^{ISC}(\beta)$$

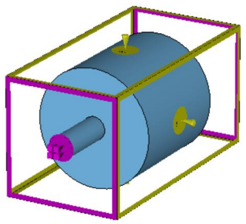
where $Z_{bb}^{ISC}(\beta)$ is the indirect space charge impedance of the bounding box.

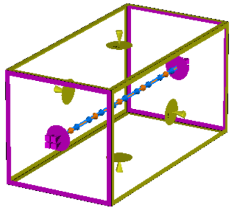
Numerical cancellation of $Z^{SC}(\beta)$ [1]

- Two simulations are run with the same mesh:
  1. Simulation of the device under test: $Z^{tot}_{DUT}(\beta) = Z_{DUT}(\beta) + Z^{ISC}_{DUT}(\beta) + Z^{SC}(\beta)$

     —

  2. Simulation of the bounding box: $Z^{tot}_{bb}(\beta) = Z^{SC}(\beta) + Z^{ISC}_{bb}(\beta)$





to **remove $Z^{SC}(\beta)$ directly from simulations**: =

$$Z^{tot}_{DUT}(\beta) - Z^{tot}_{bb}(\beta) = Z_{DUT}(\beta) + Z^{ISC}_{DUT}(\beta) - Z^{ISC}_{bb}(\beta)$$

- $Z^{ISC}_{bb}(\beta)$ and $Z^{ISC}_{DUT}(\beta)$ can be analytically calculated and removed.
- This **technique** can **also** be applied **directly to the wake potential**.

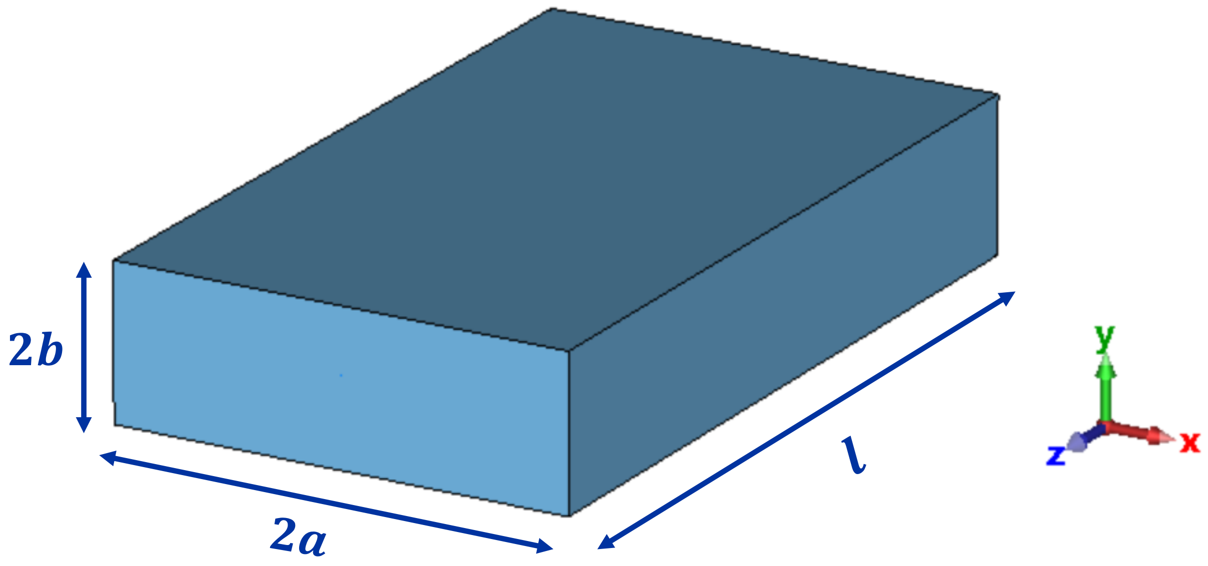
[1] C. Zannini *et al.*, "Electromagnetic simulations for non-ultrarelativistic beams and applications to the CERN low energy machines"

Resistive chamber

- The first device that was considered is a resistive chamber of dimensions

$$a = 30\,mm$$
$$b = 10\,mm$$
$$l = 100\,mm$$

- The infinitely thick walls are simulated directly through the boundary condition “conducting wall”.

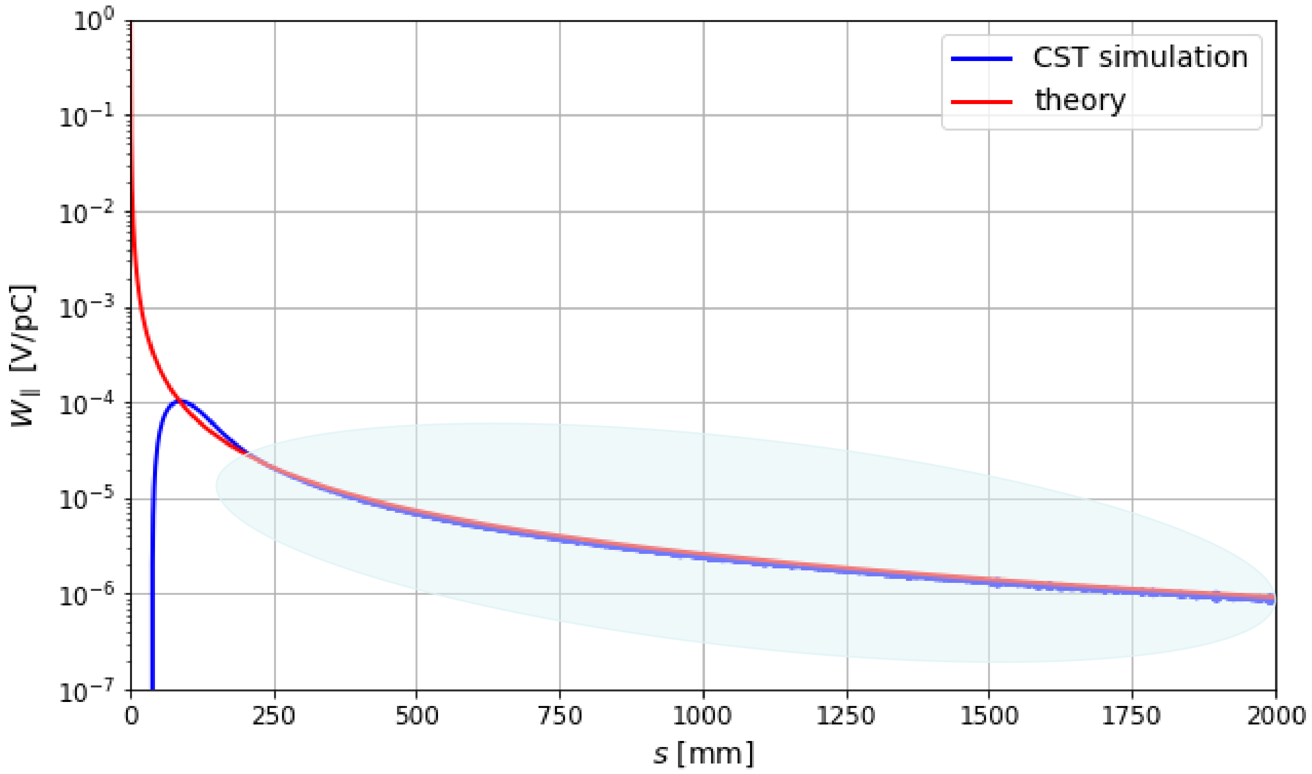


- For the wakefield calculation, **the direct integration method had to be used**, because it is the only one that can also be employed for non-ultrarelativistic beams.

Longitudinal wake potential for $\beta = 1$: comparison between CST simulation and theory

- The *accuracy* of the simulation in the ultrarelativistic case had to be *checked due to* the use of the *direct integration method*.

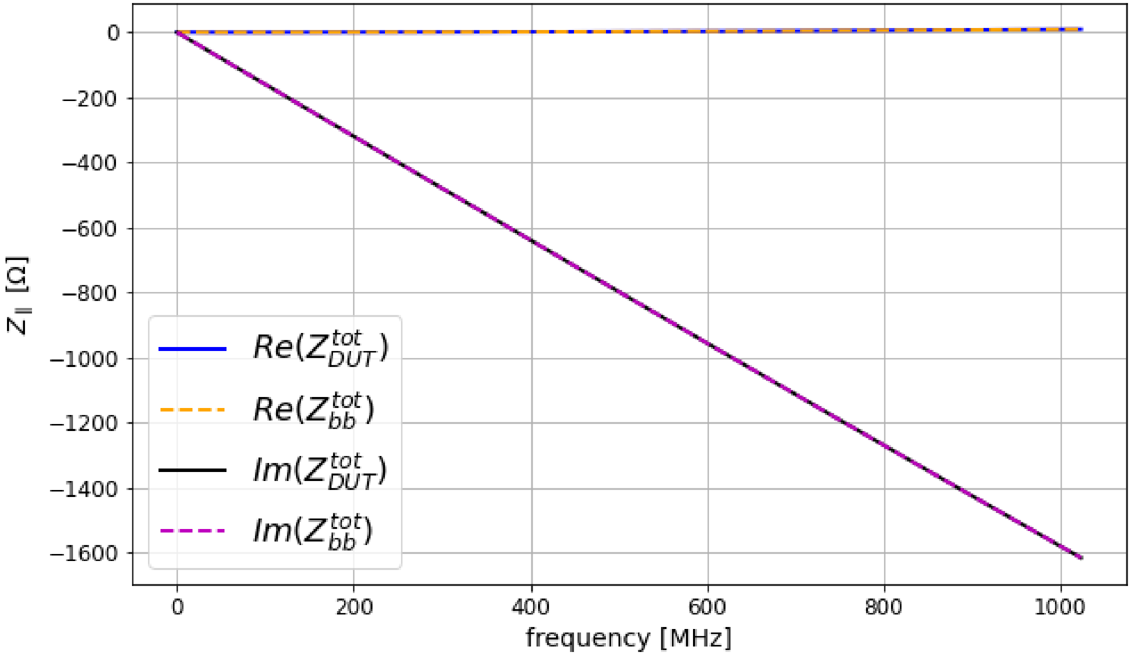
- In the long range there is a **good agreement between the theoretical and simulated** longitudinal wake potentials.



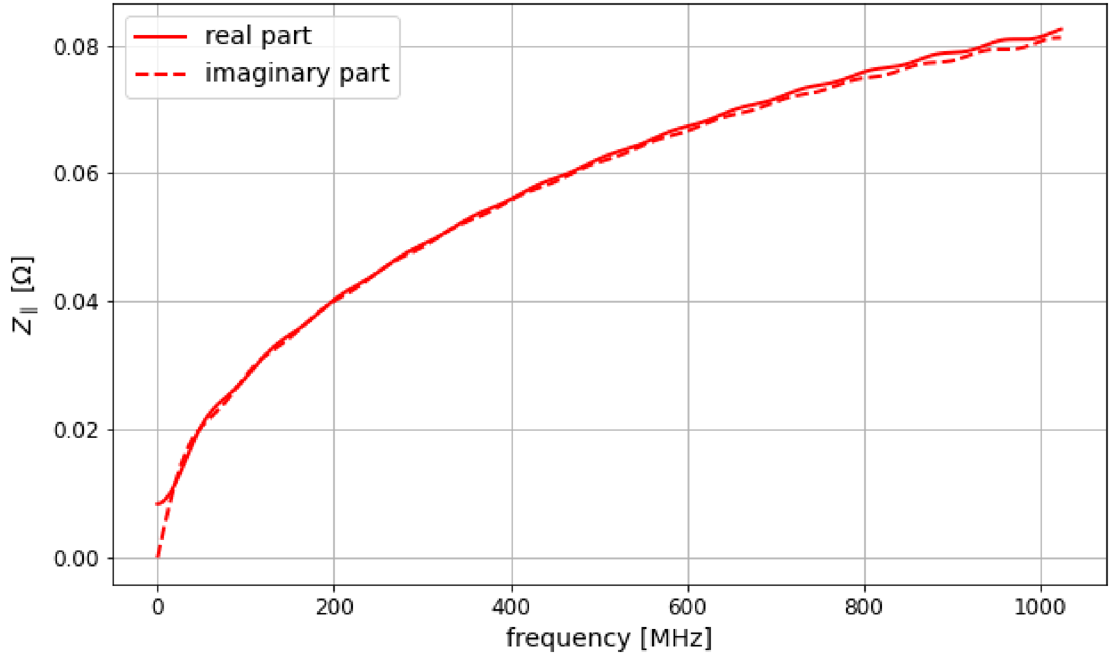
Example of application: longitudinal impedance of a resistive chamber, in the case $\beta = 0.5$

In the case of a resistive chamber with infinitely thick walls, the bounding box is the chamber itself, so $Z_{DUT}^{ISC}(\beta) = Z_{bb}^{ISC}(\beta)$ and we directly obtain $Z_{DUT}^{tot}(\beta) - Z_{bb}^{tot}(\beta) = Z_{DUT}(\beta)$:

Simulations of the resistive chamber ($\boldsymbol{Z_{DUT}^{tot}}$) and bounding box ($\boldsymbol{Z_{bb}^{tot}}$)



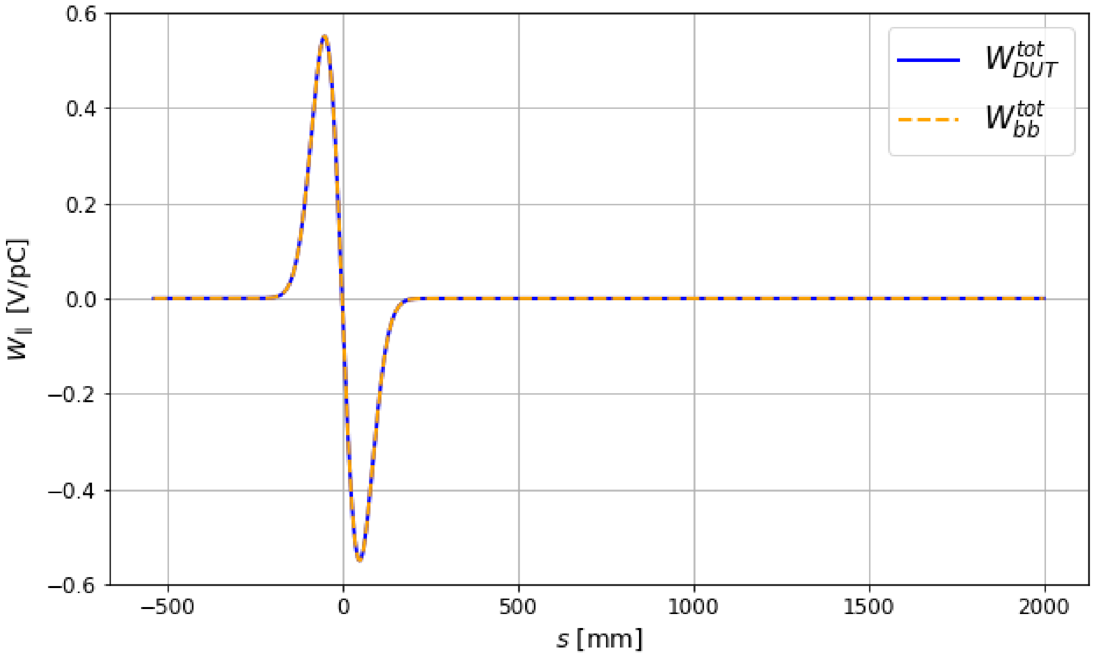
Longitudinal impedance **after numerical cancellation: $\boldsymbol{Z_{DUT}^{tot}(\beta) - Z_{bb}^{tot}(\beta)}$**



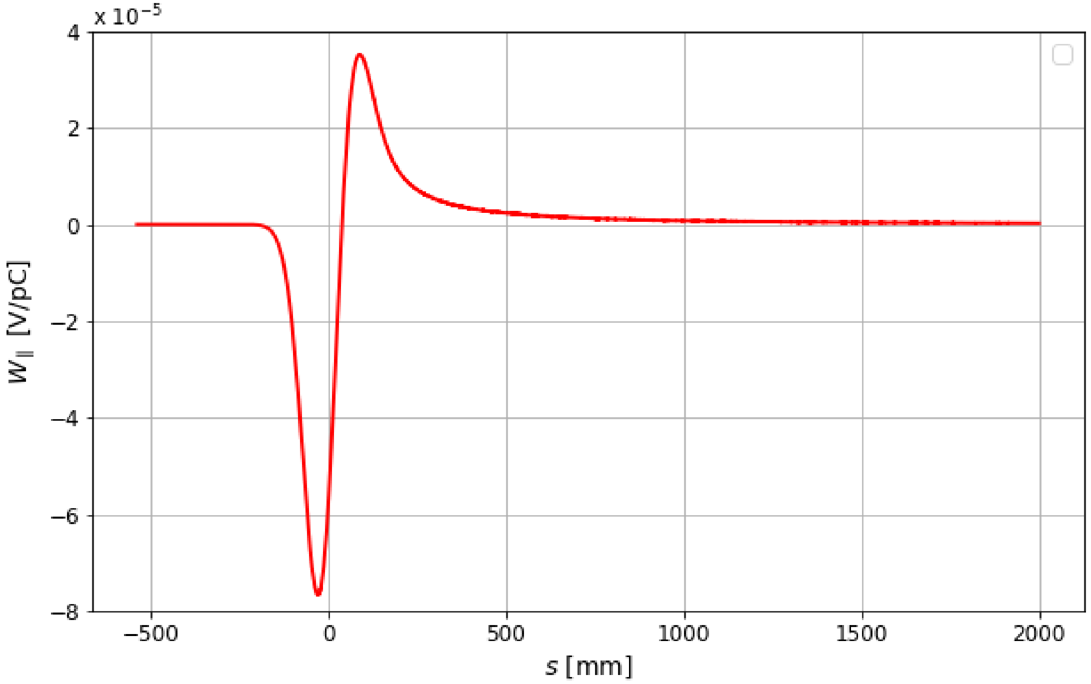
Example of application: longitudinal wake potential of a resistive chamber, in the case $\beta = 0.5$

The **technique** can also be applied **directly to the wake potential**:

Simulations of the resistive chamber (W_{DUT}^{tot}) and bounding box (W_{bb}^{tot})



Longitudinal wake potential **after numerical cancellation: $W_{DUT}^{tot}(\beta) - W_{bb}^{tot}(\beta)$**



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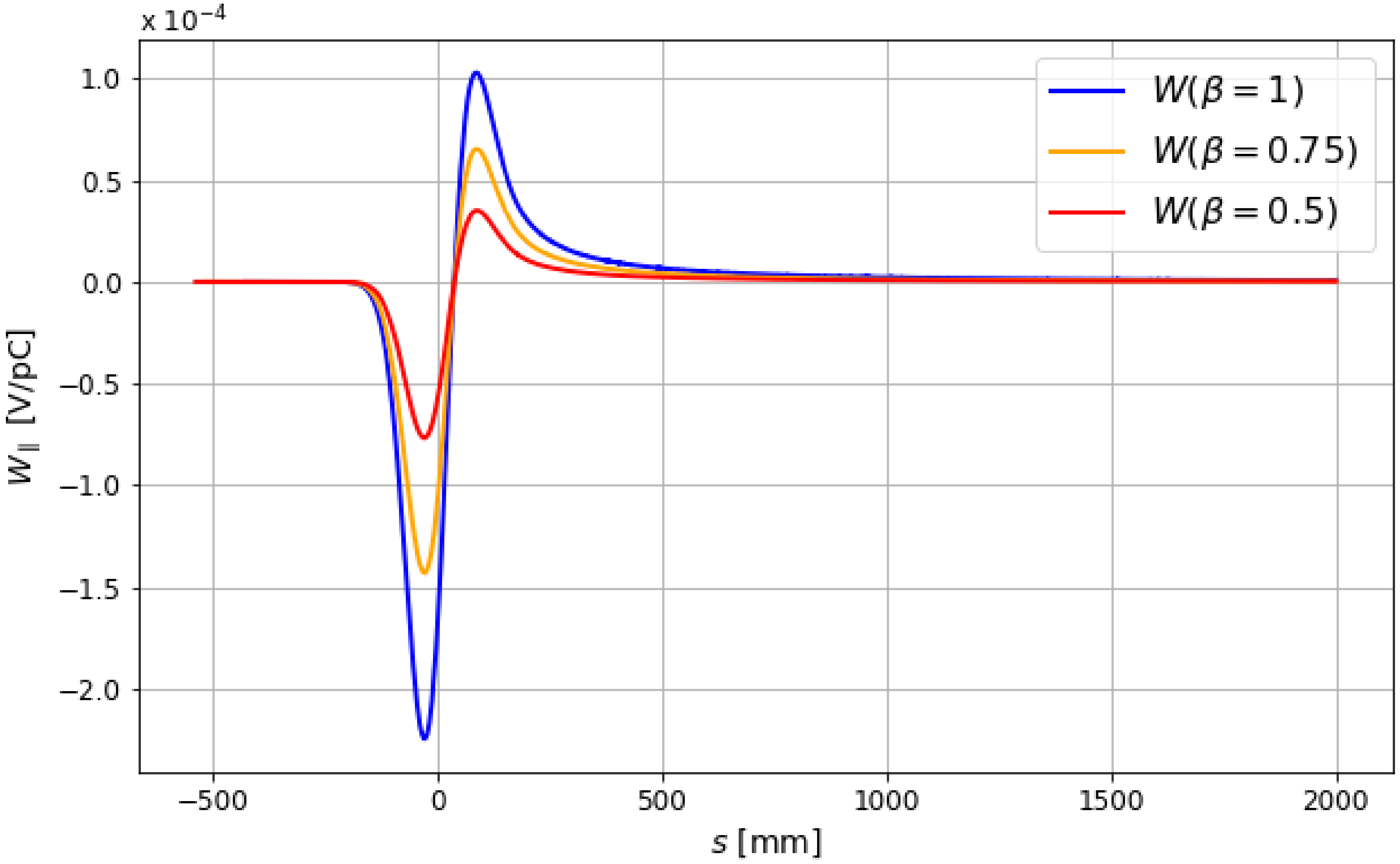
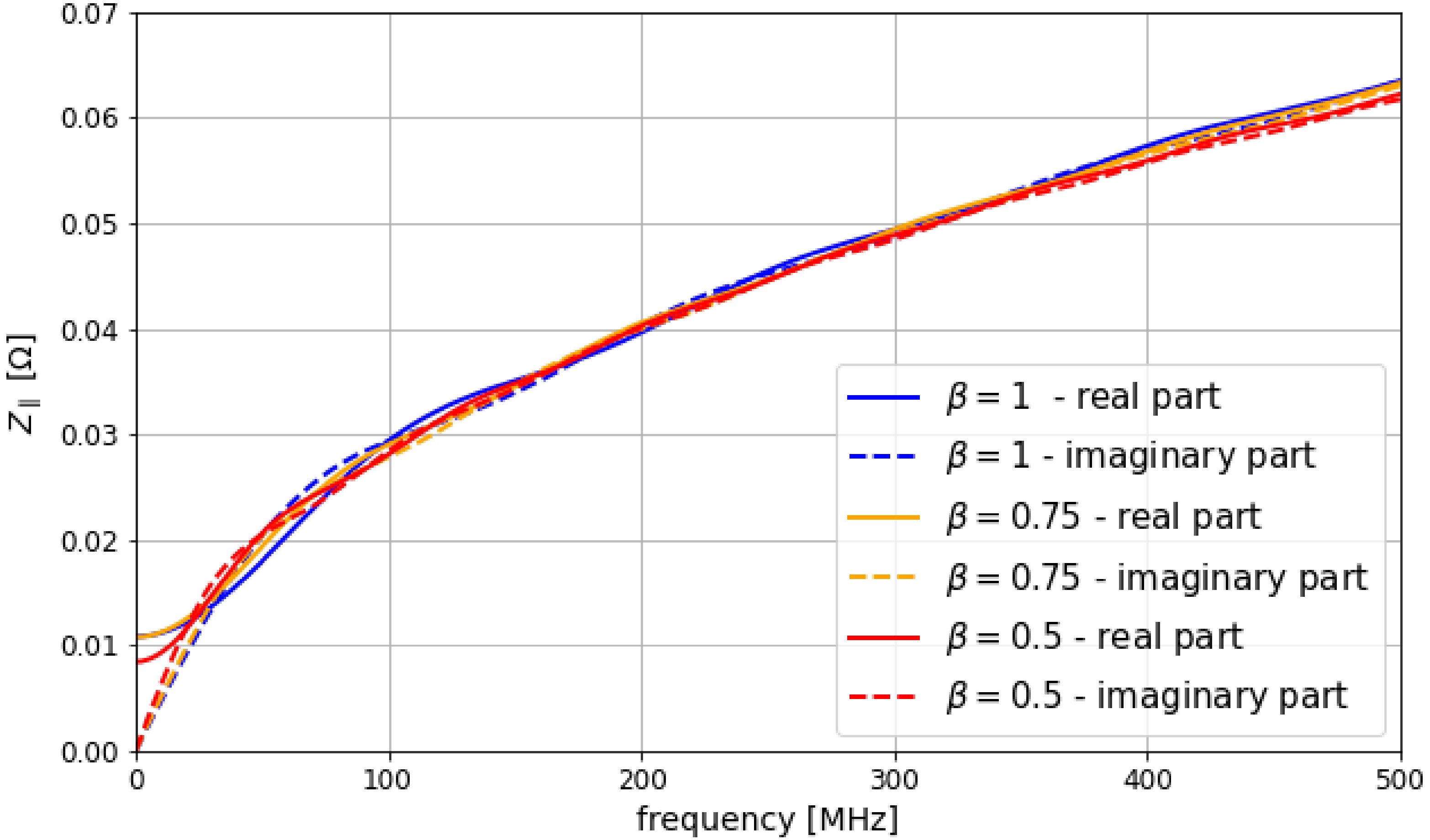
Longitudinal wake potential varying β

It can be observed that the longitudinal wake potential **scales with $\boldsymbol{\beta^{\frac{3}{2}}}$**.



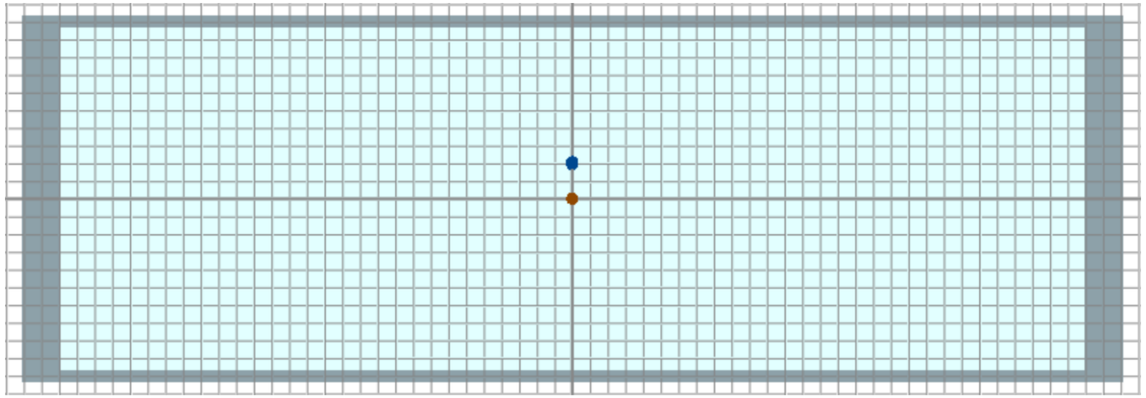
Longitudinal impedance varying β

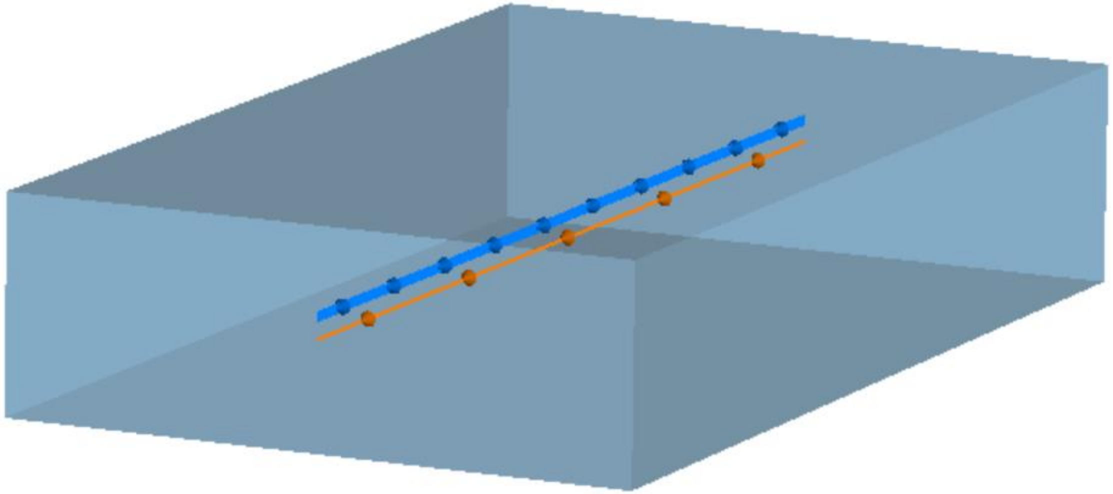
As expected, the longitudinal impedance **doesn’t change with β**.



Settings for transverse simulations

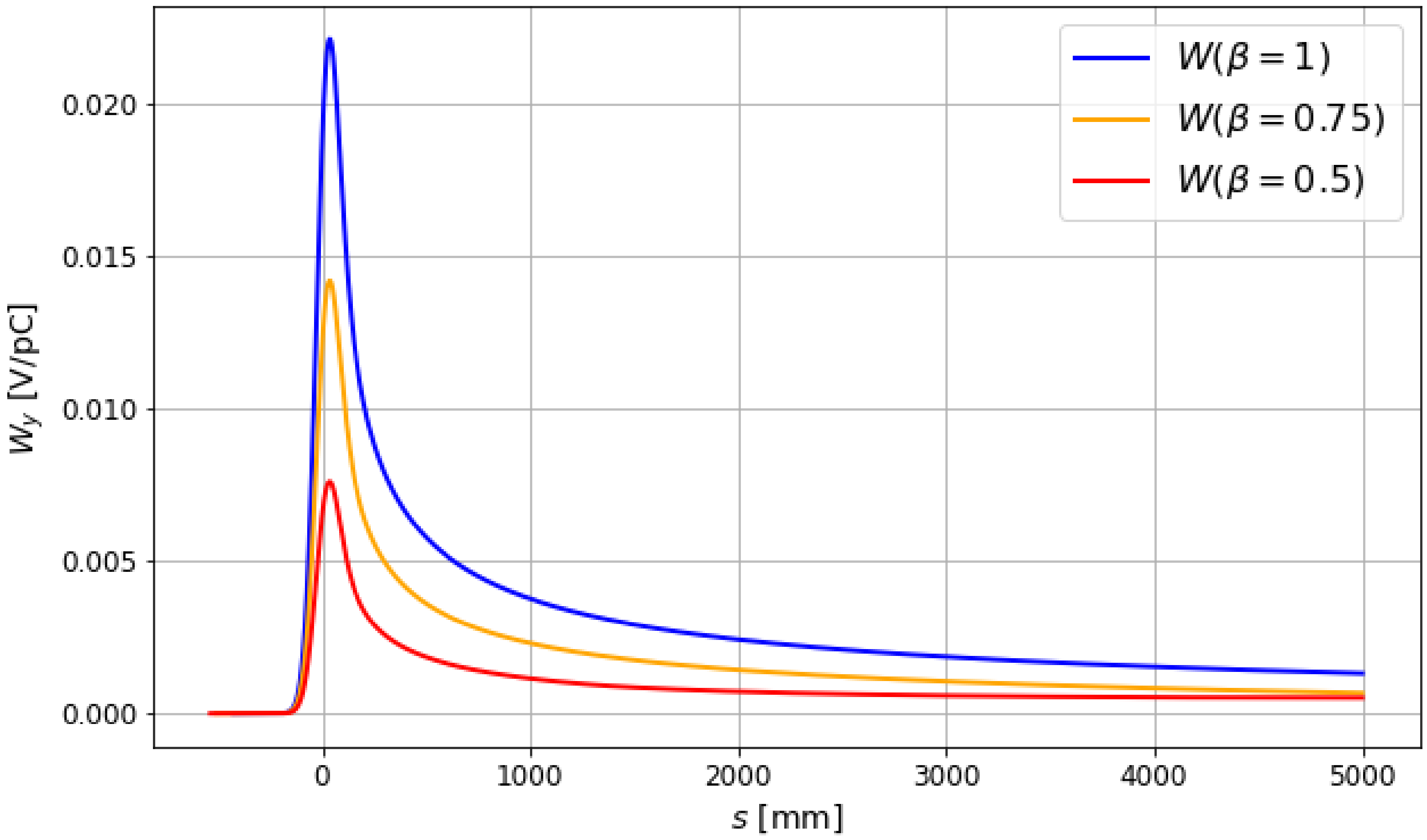
The **dipolar vertical transverse impedance** was simulated: the integration path stays on axis while the beam is displaced vertically with an offset of 20% of the vertical half-aperture.





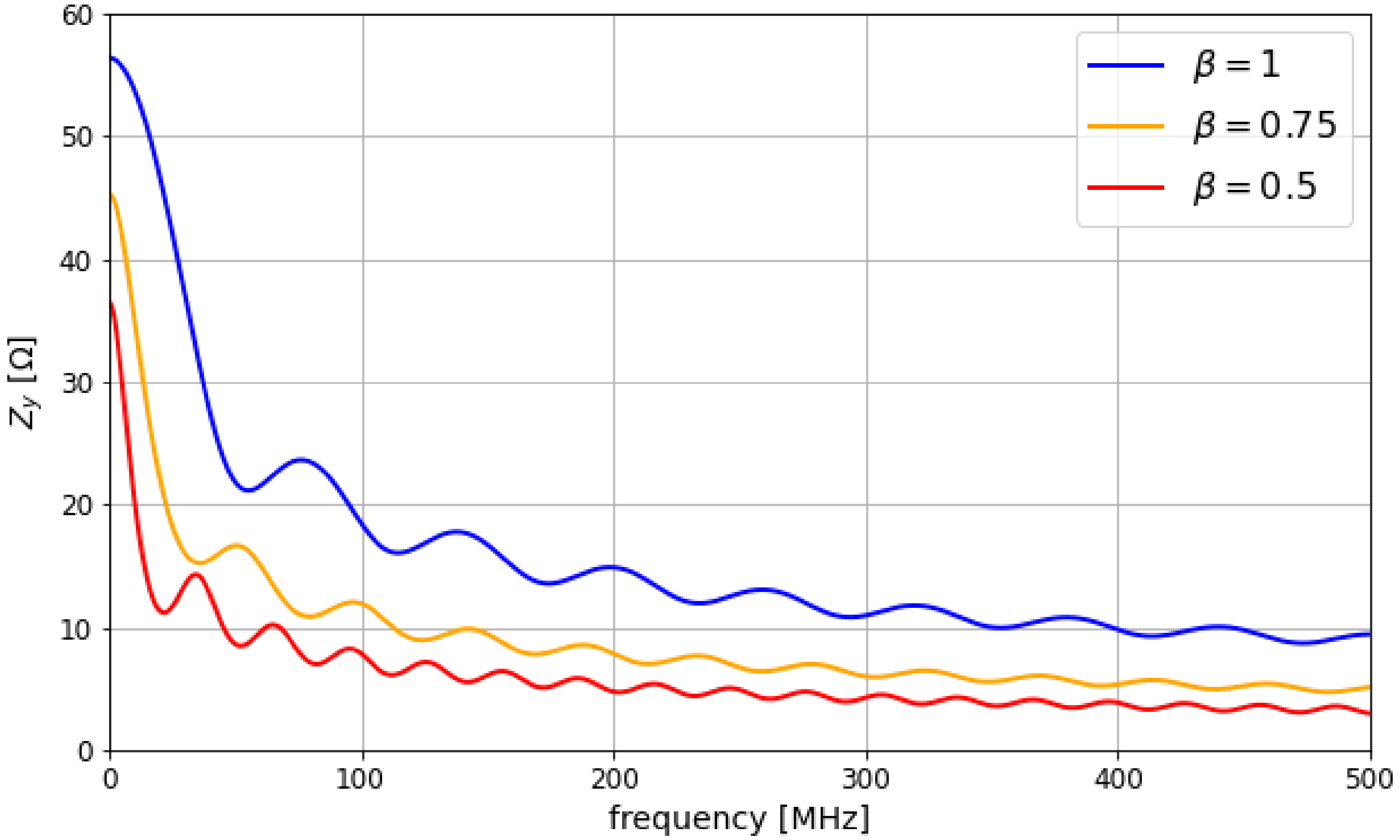
Transverse wake potential varying β

It can be observed that the longitudinal wake potential **scales with** $\boldsymbol{\beta^{\frac{3}{2}}}$.



Transverse impedance varying β

Even though the use of the direct integration method leads to numerical issues, it looks like the transverse impedance **scales with $\boldsymbol{\beta^{-1}}$.**



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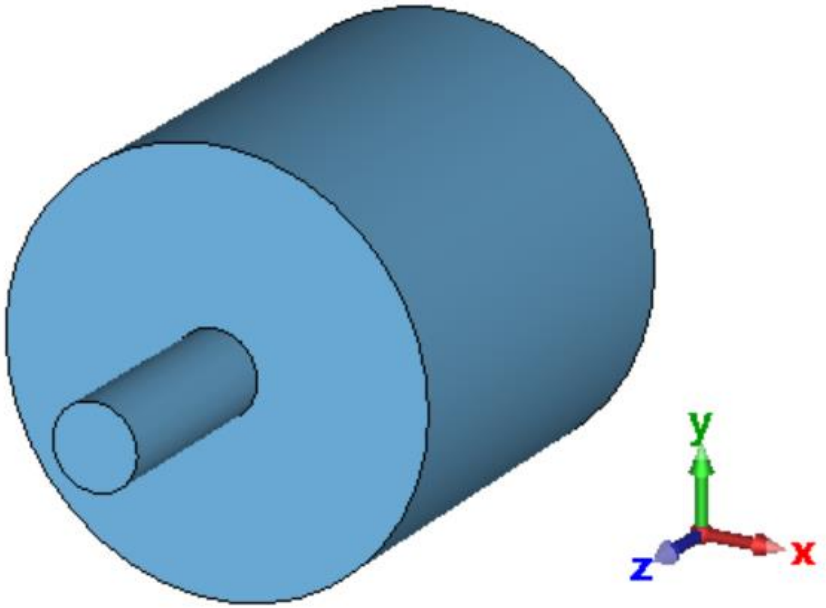
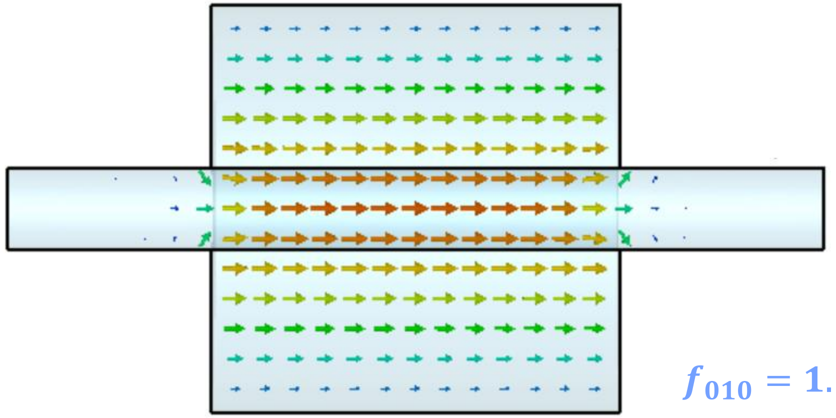
Pillbox cavity



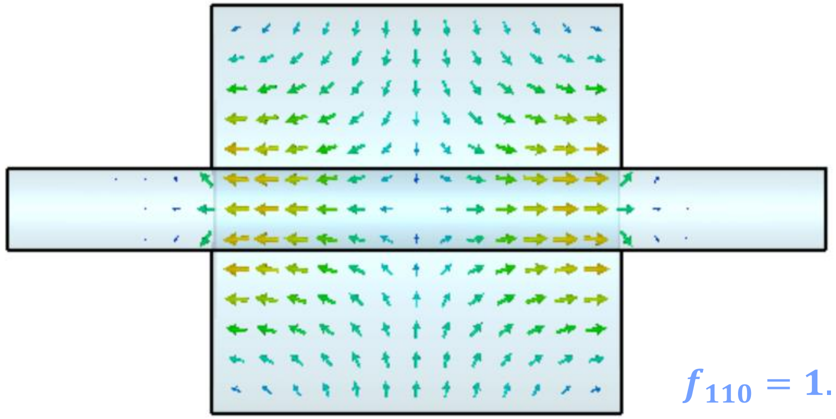
Radius of the pipe	2 cm
Radius of the pillbox	10 cm
Length of the pipe	20 cm
Length of the pillbox	40 cm

Study of the first two resonant modes:



TM010

$f_{010} = 1.150\ GHz$



TM110

$f_{110} = 1.375\ GHz$

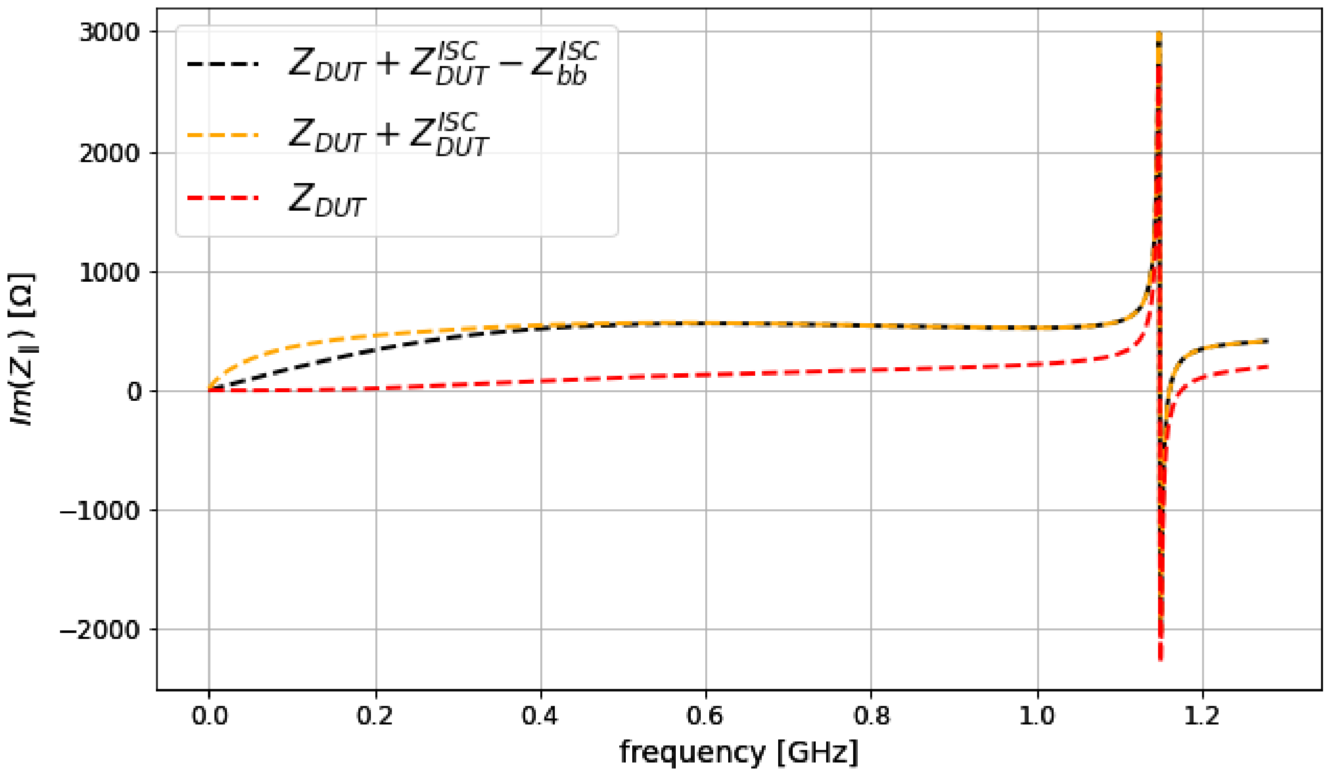
Example of application: longitudinal impedance of a pillbox, in the case $\beta = 0.5$

- $Im(Z)$ is the one affected by space charge.

- For a pillbox we get:

$$Z_{DUT}(\beta) + Z_{DUT}^{ISC}(\beta) - Z_{bb}^{ISC}(\beta)$$

- $Z_{bb}^{ISC}(\beta)$ and $Z_{DUT}^{ISC}(\beta)$ can be analytically calculated and removed.



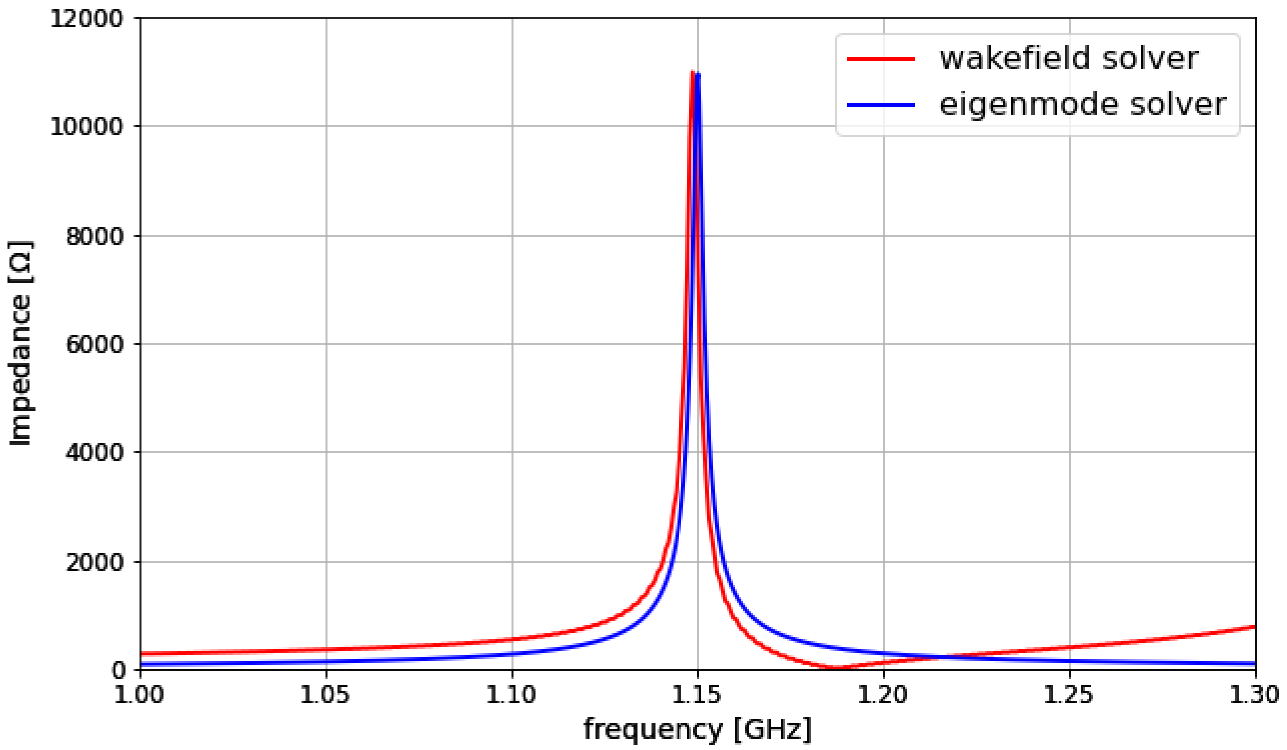
Eigenmode Solver vs Wakefield Solver

- Wakefield Solver (WF): directly provides the **impedance spectrum**.
- Eigenmode Solver (EM): provides three parameters.
  - **impedance spectrum reconstructed** based on the broad-band resonator model.

Eigenmode solver results	
Resonant frequency ω_r	**1.15 GHz**
Quality factor Q	**450**
Shunt impedance Rs	**21954 Ω**

$$Z(\omega) = \frac{R_s}{1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

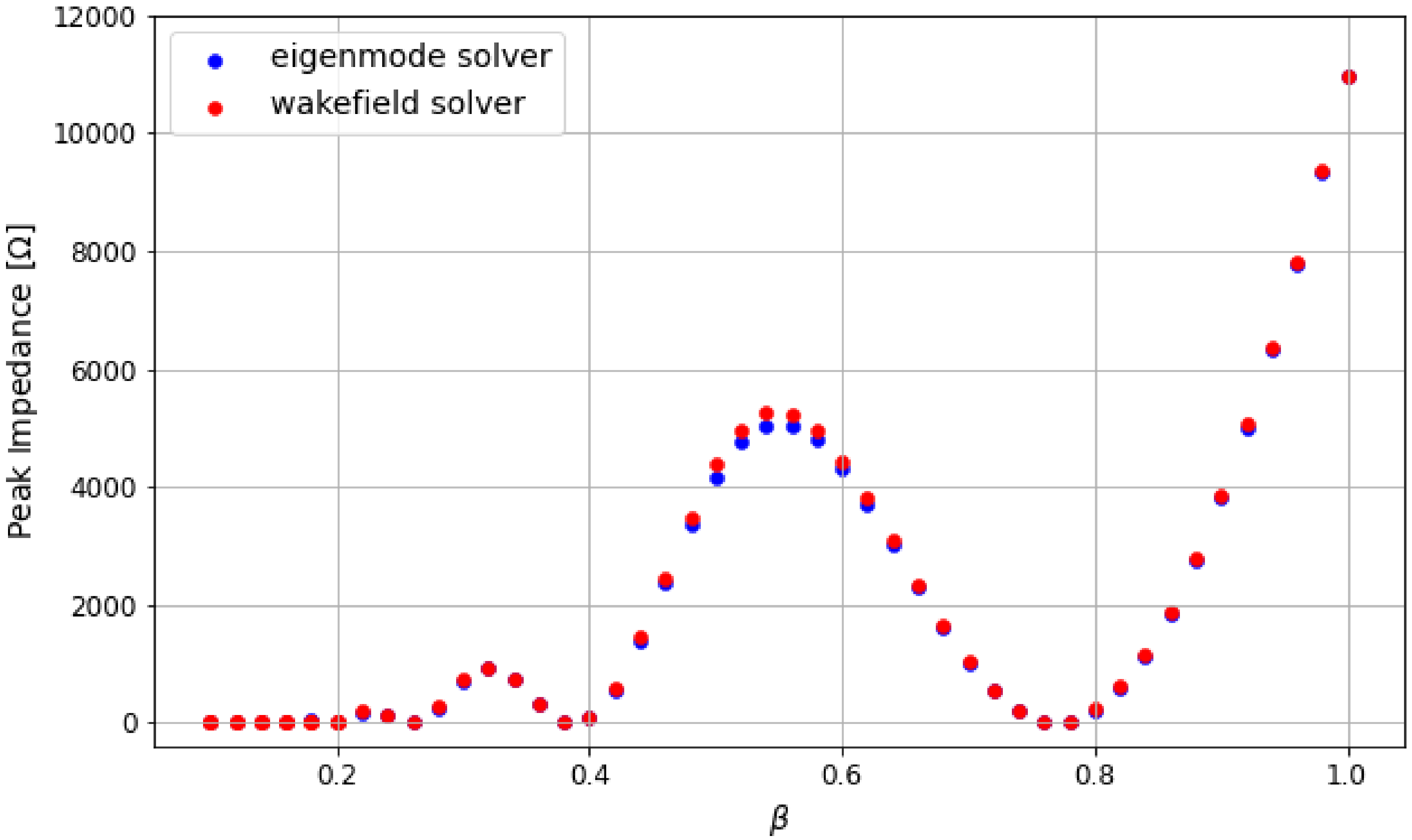
broad-band resonator model



Beam coupling impedance varying β

- Parametric study of the real part of the impedance at f_{010} varying β.

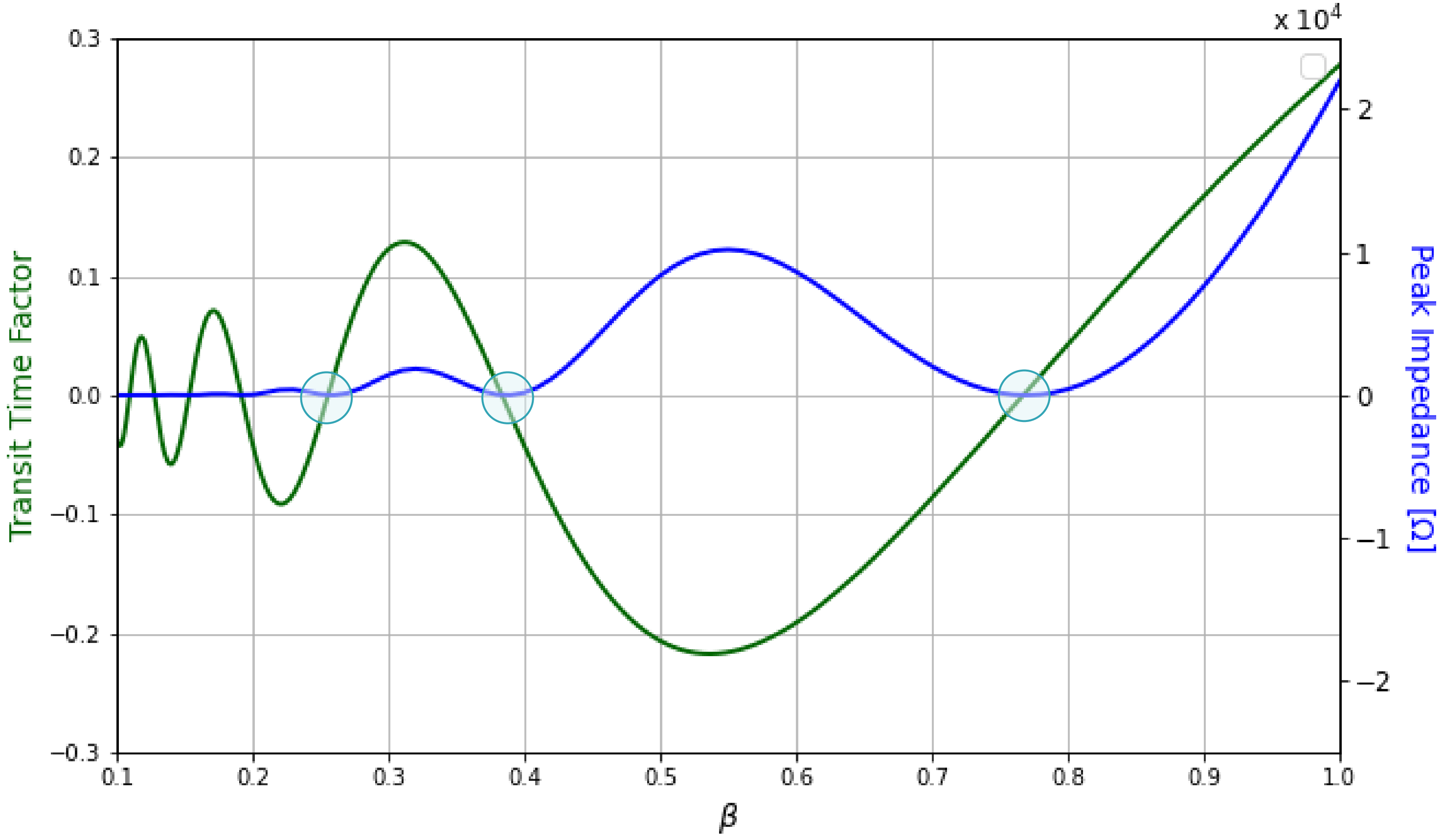
- **Good agreement** between the two solvers:
  - Relative error < 5%
- **This agreement is not obvious bc in EM the particle velocity is taken into account in post processing**



Beam coupling impedance varying β: relationship with the Transit Time Factor

- Study to understand the shape of the curve
  - in particular *values of **β** for which the **peak impedance goes to 0***.
- It can be explained analytically looking at the transit time factor for the fundamental mode

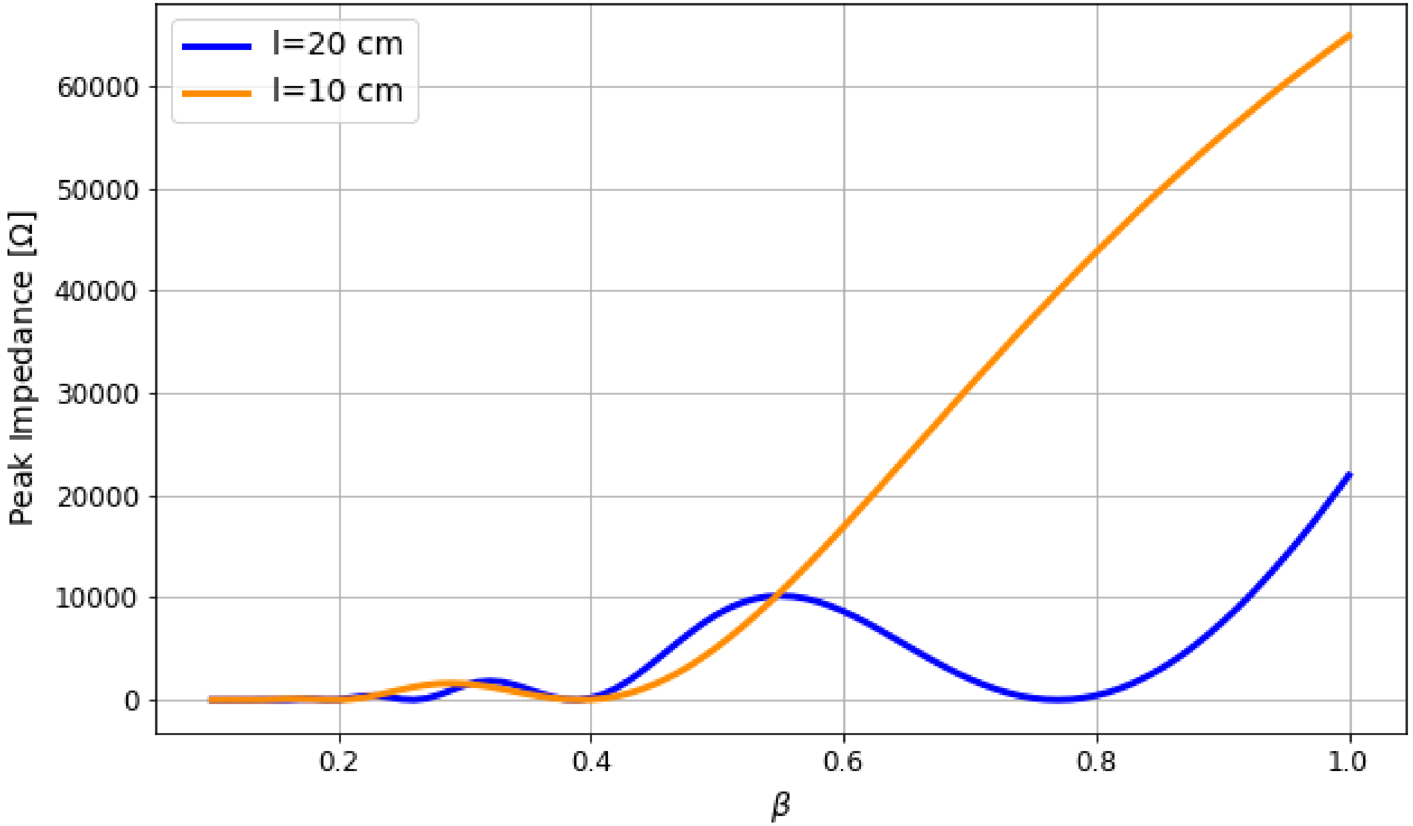
$$T = \frac{\sin\left(\frac{\pi l}{\beta\lambda}\right)}{\frac{\pi l}{\beta\lambda}}$$



Beam impedance varying the pillbox's length

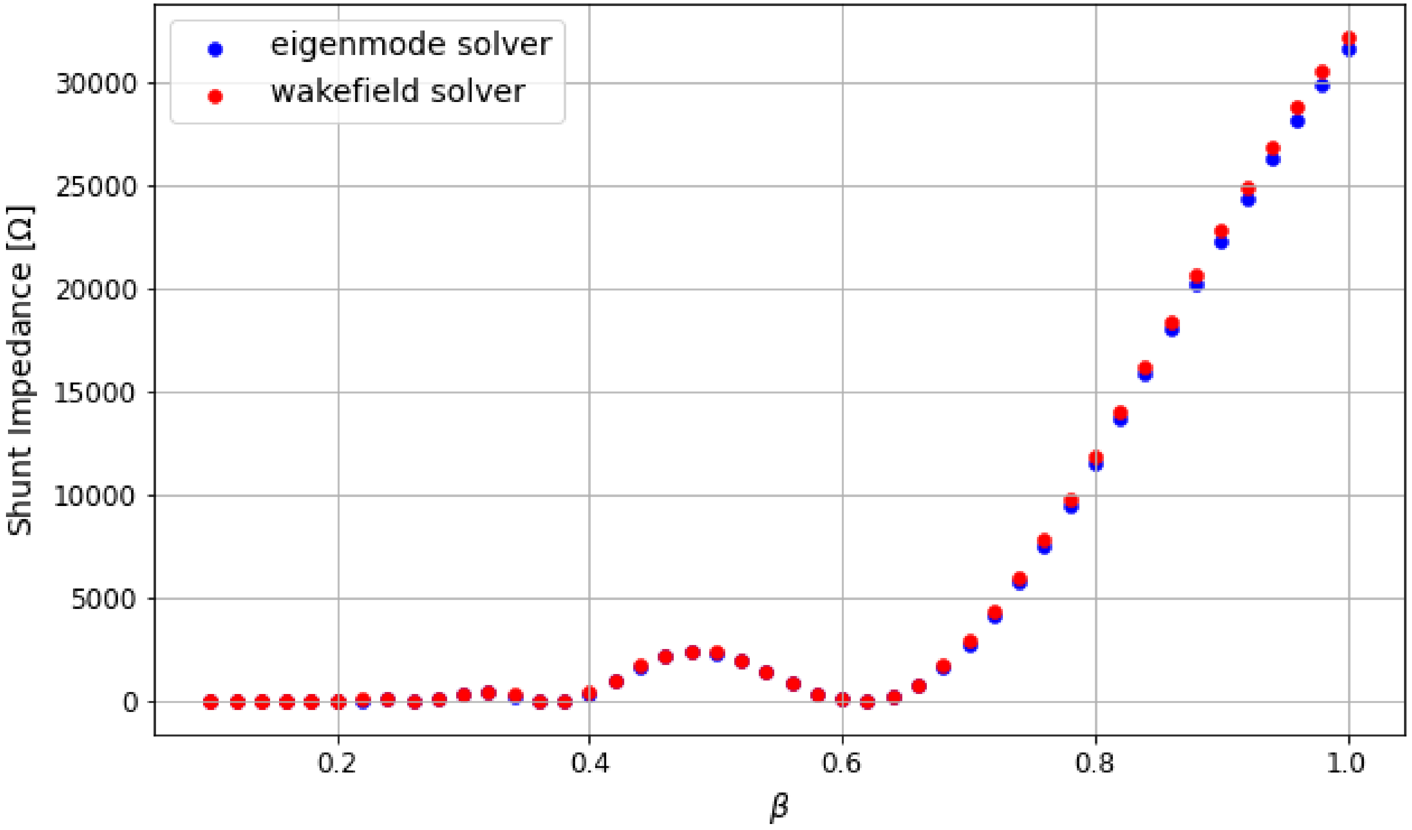
$$T = \frac{\sin\left(\frac{\pi l}{\beta\lambda}\right)}{\frac{\pi l}{\beta\lambda}}$$

When $l \ll \lambda$, we have $T \to 1$, so if we reduce the length of the pillbox the **last minimum** is reached **for a lower β.**



Peak impedance for the second resonant mode

- Also, for the second mode there is **good agreement between the two solvers**.
  - Relative error < 5%

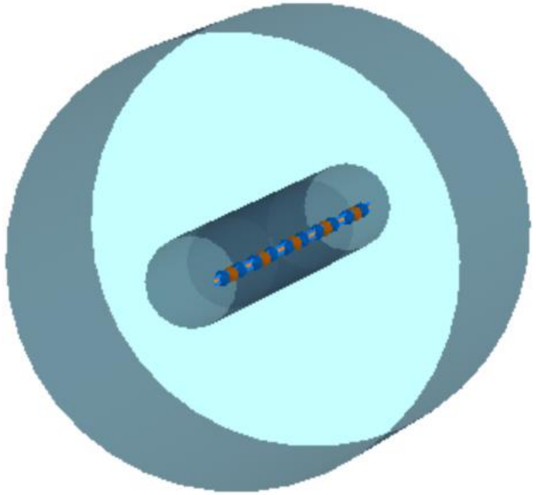


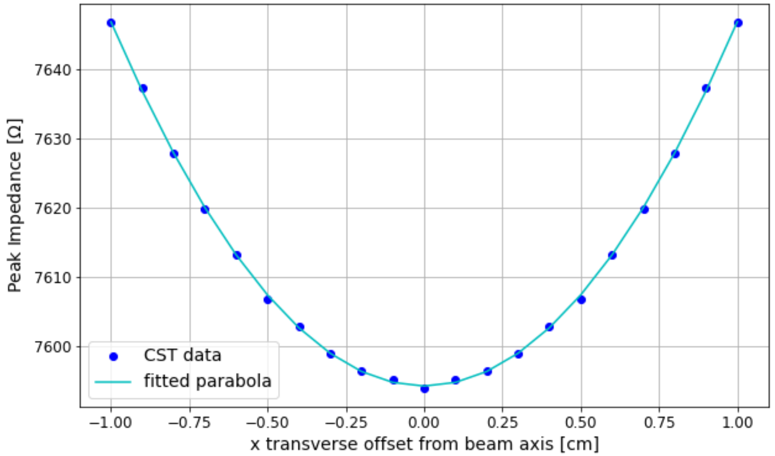
Settings for transverse simulations

The **generalized transverse impedance** was simulated with an offset of 10% of the radius of the pillbox.

- WF Solver: **beam and integration path** are **directly displaced**.
- EM Solver: the longitudinal impedance at f_r is calculated at different transverse offsets, with the expectation of obtaining a **parabola**:

$$Z_{\parallel} = Z_{\parallel,0} + Z_{\parallel,1x} \cdot x_0^2 + Z_{\parallel,1y} \cdot y_0^2$$

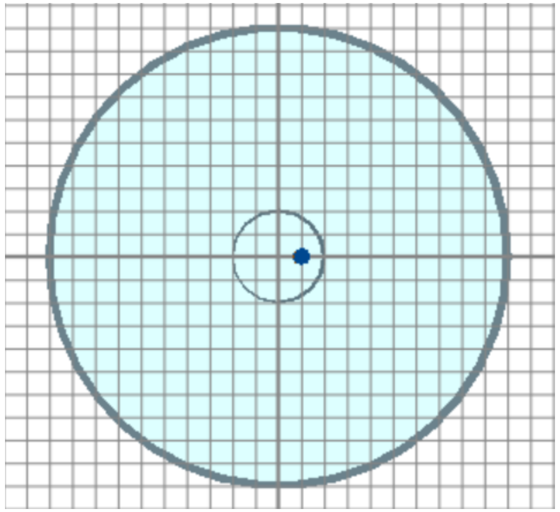




The **transverse impedance** is computed through to the Panofsky-Wenzel theorem:

$$Z_x^{gen} = \frac{Z_{\parallel,1x}(f_r) \cdot c}{2\pi f_r}$$

with $Z_{\parallel,1x}(f_r)$ from the fit.

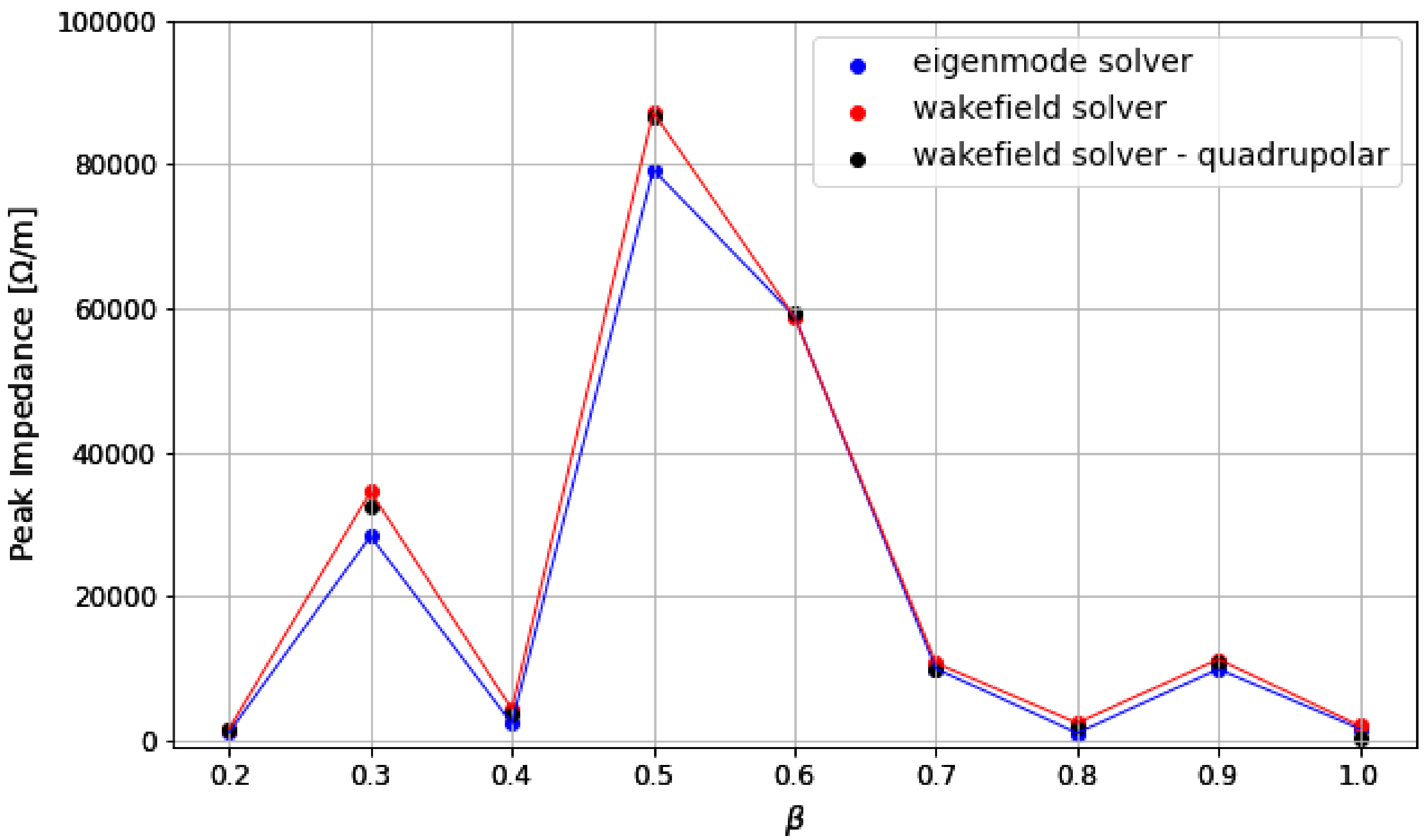


Generalized transverse beam impedance varying β and role of the quadrupolar component

Good agreement between the two solvers.

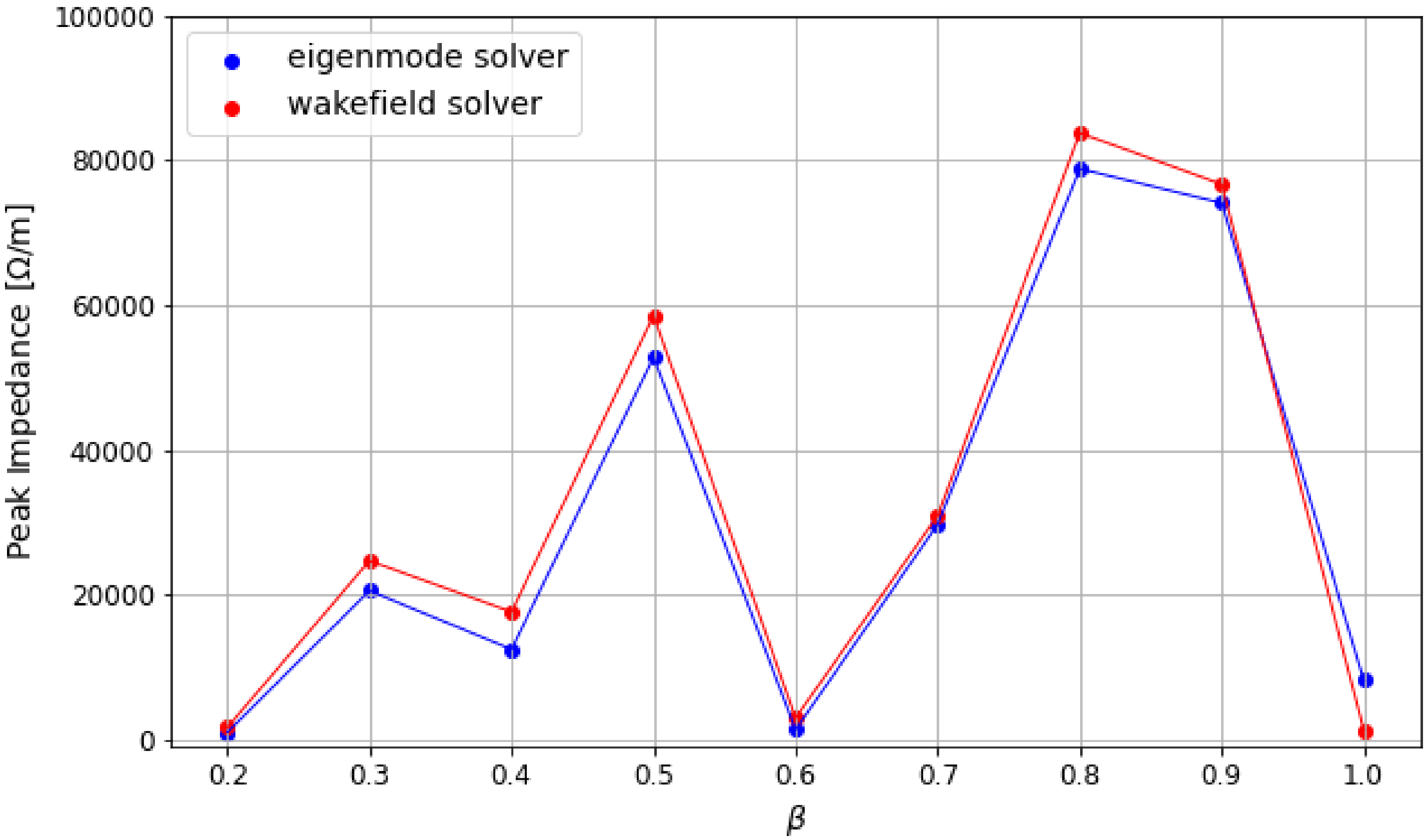
The **mode** is mainly **quadrupolar**:

- $\boldsymbol{\beta = 1}$**: no radial field dependence**
  - $\boldsymbol{Z_{x,y}^{quad} = 0} \rightarrow Z_{x,y}^{gen}$ small
- $\boldsymbol{\beta < 1}$**: radial field dependance**
  - $\boldsymbol{Z_{x,y}^{quad} \neq 0} \rightarrow Z_{x,y}^{gen}$ higher



Generalized transverse beam impedance varying β for the second mode

Also, in the second mode there is **good agreement** between the two solvers.



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Conclusions

- **Low-beta** simulations are **extremely challenging** due to **a series of factors** (mesh convergence, direct integration method, removal of direct space charge).
- The numerical cancellation technique for the **removal** from the simulation **of the direct space charge** contribution was **benchmarked** with a **resistive wall beam chamber**:
  - the **wake potential**, both longitudinal and transverse, **scales with** $\boldsymbol{\beta}^{\frac{3}{2}}$;
  - the **longitudinal impedance doesn’t change with β**, as expected;
  - the **transverse impedance scales with** $\boldsymbol{\beta}$, as expected.
- Simulations of a **pillbox cavity**:
  - Numerical cancellation has been applied successfully.
  - **Good agreement between** the **Eigenmode Solver and** the **Wakefield Solver**
    - The non-ultrarelativistic Wakefield simulations are accurate.
    - The **Eigenmode Solver approximation** of adding particle velocity only in post-processing with the transit time factor **has been found to be accurate**.

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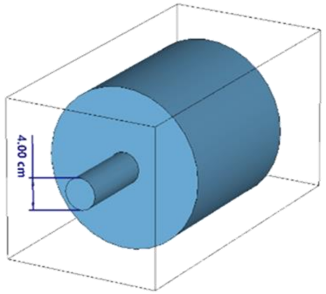
Next steps

- Since the way that CST runs its simulations and the reason behind the numerical issues are not known, using an electromagnetic solver whose implementation is know would be useful: **low-beta simulations** are going to be run **with wakis**
  - First user of wakis
- The presented **study** will be applied to the **PSB FINEMET cavities**, whose impedance model can be improved because it currently doesn’t account for non-ultrarelativistic beams.

Low-beta simulations with wakis

Courtesy of
Elena de la Fuente García

```python
from wakis import GridFIT3D, SolverFIT3D, WakeSolver
import pyvista as pv

# ---------- Domain and Grid setup ---------
# Number of mesh cells
Nx = 57
Ny = 57
Nz = 109
#dt = 5.707829241e-12

# Geometry Import
stl_cavity = 'cavity.stl'
stl_pipe = 'beampipe.stl'
stl_solids = {'cavity': stl_cavity, 'pipe': stl_pipe}

# Materials
stl_materials = {'cavity': 'vacuum', 'pipe':  'vacuum'}
background = [1.0, 1.0, 100] # lossy metal [ε_r, μ_r, σ]

# Domain bounds (from stl)
surf = pv.read(stl_cavity) + pv.read(stl_pipe)
xmin, xmax, ymin, ymax, zmin, zmax = surf.bounds

# Set grid and geometry
grid = GridFIT3D(xmin, xmax, ymin, ymax, zmin, zmax, Nx, Ny, Nz,
                stl_solids=stl_solids,
                stl_materials=stl_materials)
#grid.inspect()
```




```python
# ------------ Beam source ----------------
# Beam parameters and wake obj.
beta = 0.8          # beam relativistic beta
sigmaz = beta*6e-2  # [m] -> multiplied by beta to have f_max cte
q = 1e-9            # [C]
xs = 0.             # x source position [m]
ys = 0.             # y source position [m]
xt = 0.             # x test position [m]
yt = 0.             # y test position [m]
# tinj = 8.53*sigmaz/(beta*c)  # injection time offset [s]

wake = WakeSolver(q=q, sigmaz=sigmaz, beta=beta,
            xsource=xs, ysource=ys, xtest=xt, ytest=yt,
            save=True, logfile=True)
```

GitHub

https://github.com/ImpedanCEI/FITwakis

benchmarks/betacavity/

Simulation of a cylindrical pillbox below cut-off for different relativistic β values.

Next steps

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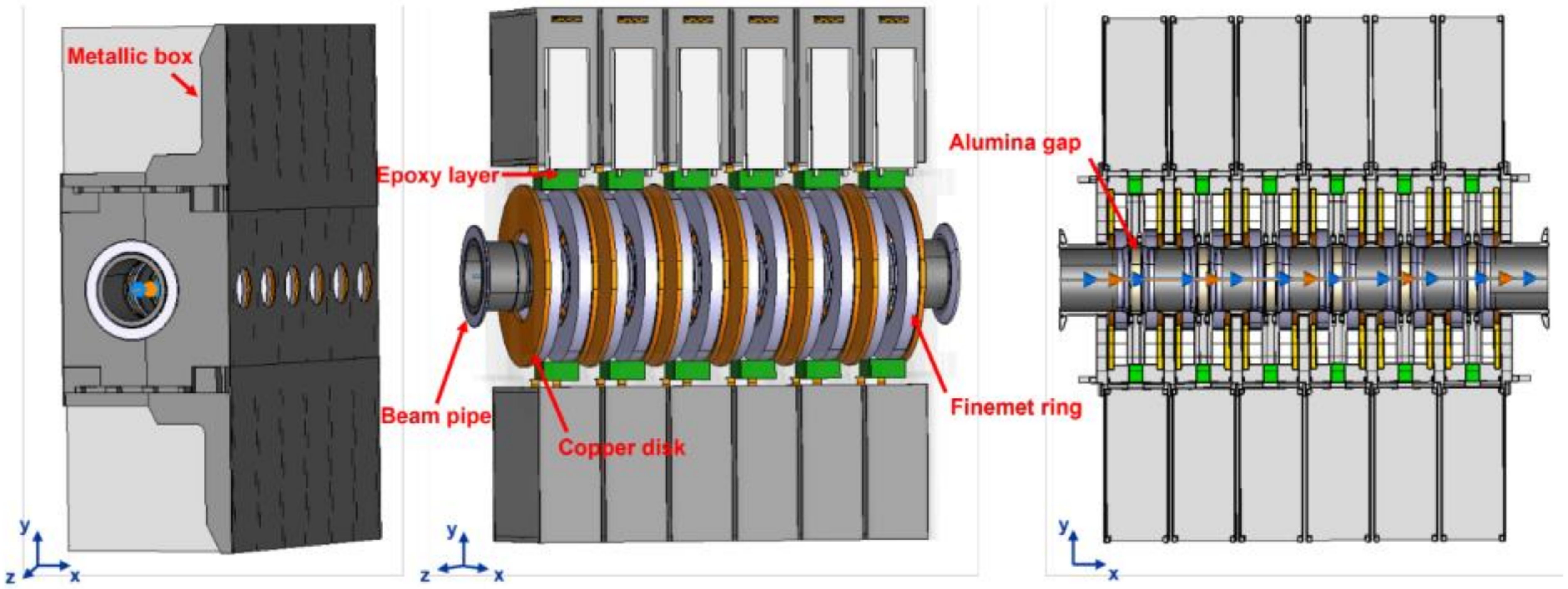
Beam coupling impedance simulations of the PSB FINEMET cavities

Study on the FINEMET cavities' **realistic 3D model**, simplified for electromagnetic simulations:

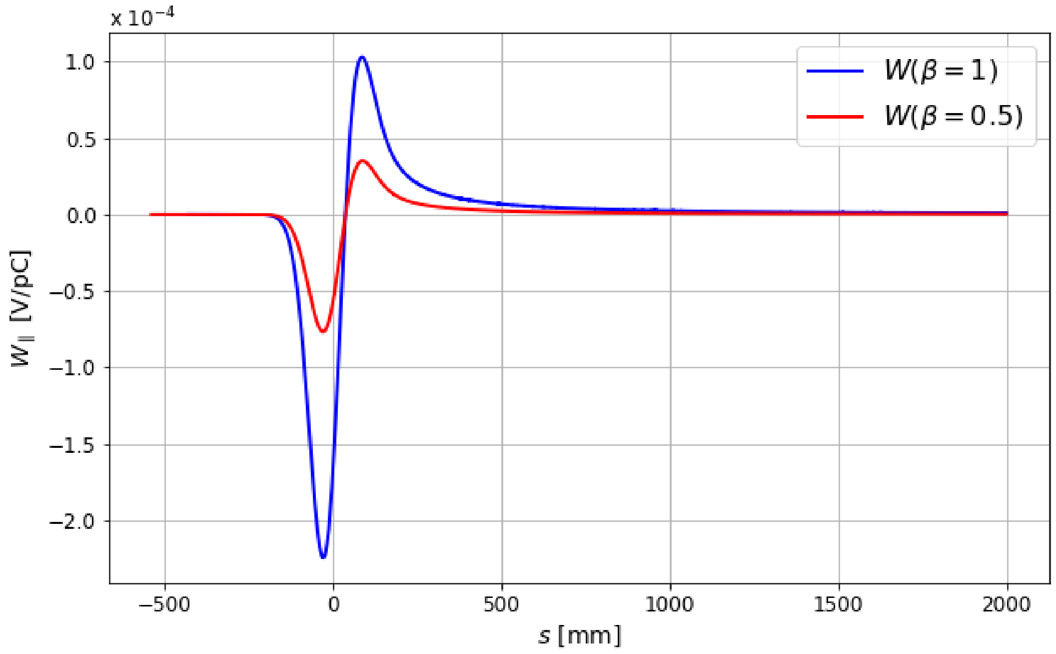


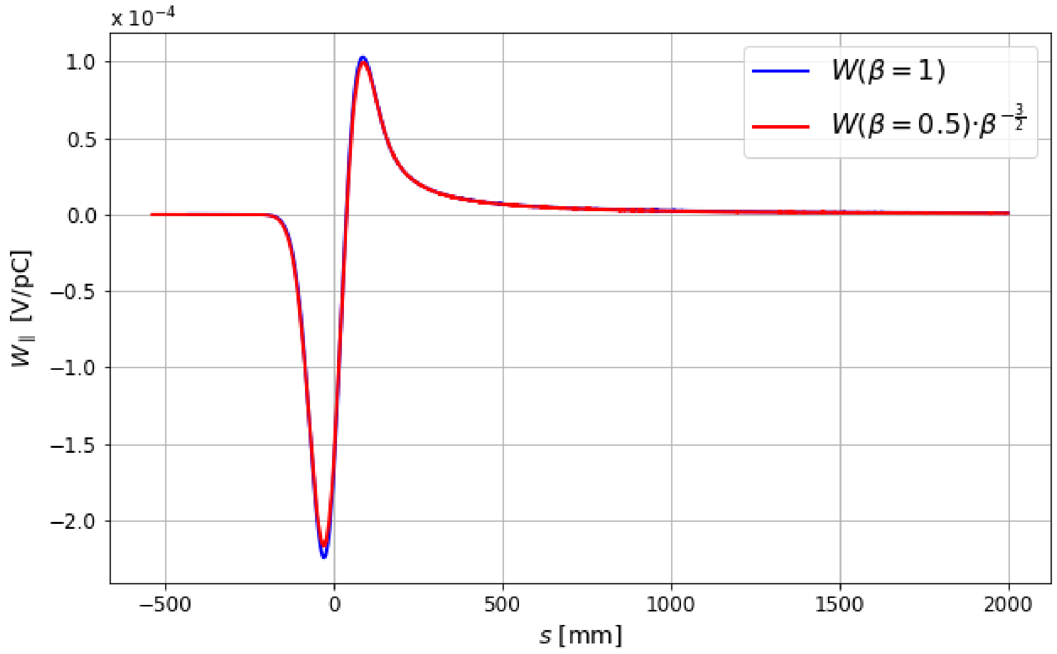
Thank you for your attention

Backup slides

Longitudinal wake potential: comparison between $\beta = 1$ and $\beta = 0.5$

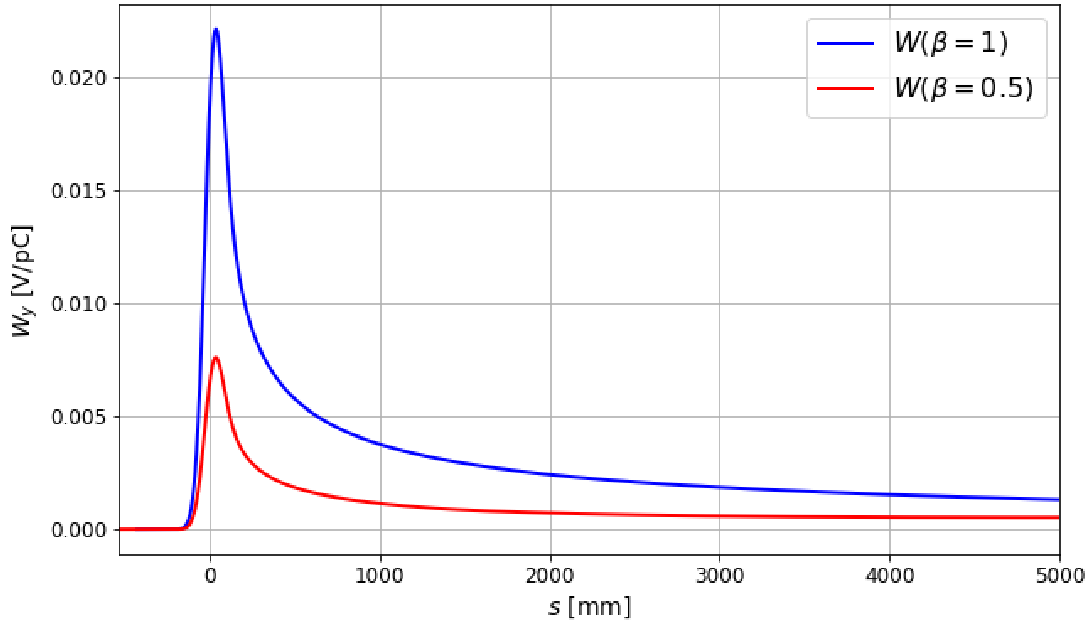
It can be observed that the longitudinal wake potential **scales with $\beta^{-\frac{3}{2}}$**.

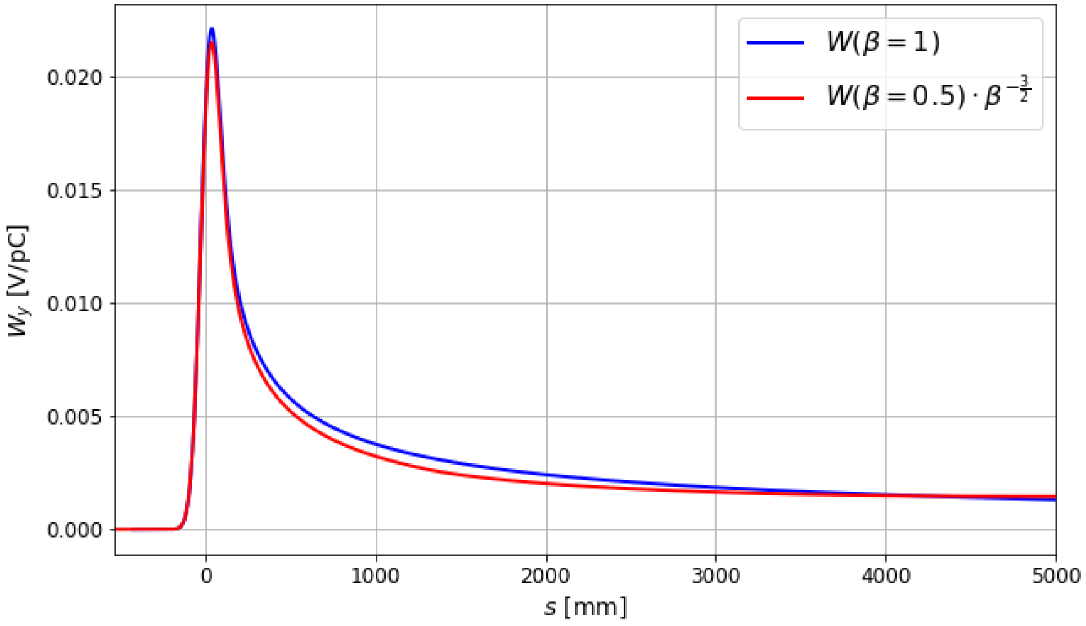




Transverse wake potential: comparison between $\beta = 1$ and $\beta = 0.5$

It can be observed that the longitudinal wake potential **scales with $\beta^{-\frac{3}{2}}$**.



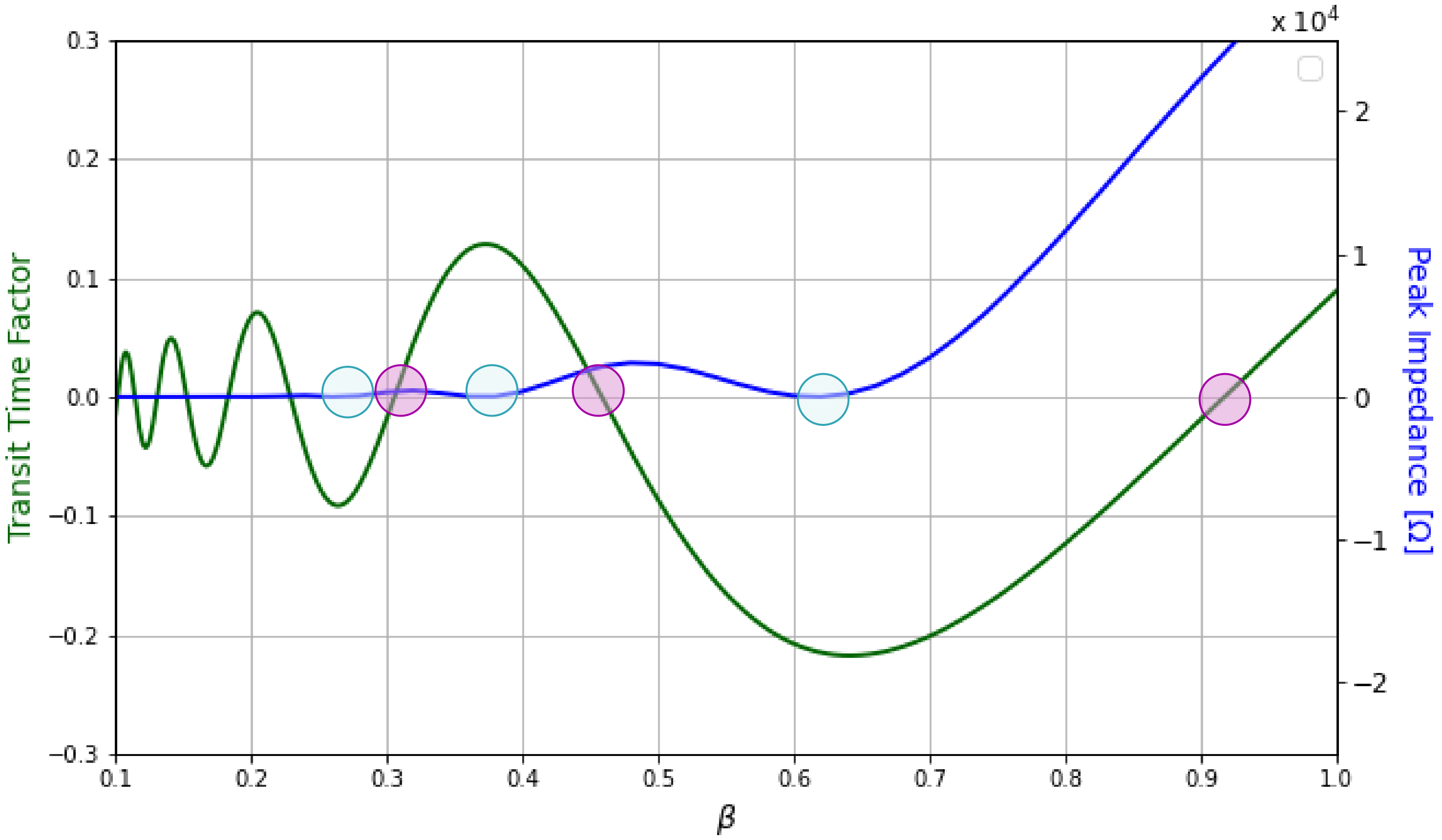


Peak impedance of the second mode varying β: relationship with the Transit Time Factor

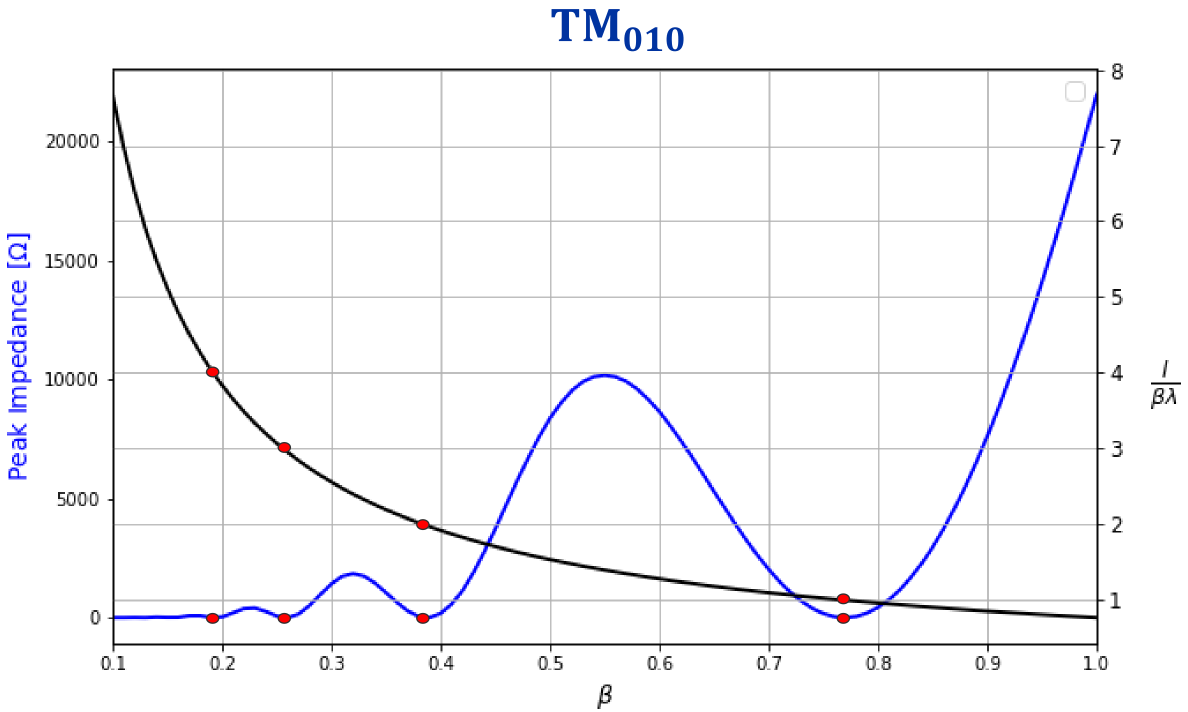
- The impedance in CST is **multiplied by the transit time factor T**, that for the fundamental mode is:

$$T = \frac{\sin\left(\frac{\pi l}{\beta\lambda}\right)}{\frac{\pi l}{\beta\lambda}}$$

- This formula doesn't work for higher order modes.
- Changes in the formula for the other modes are being studied.



Peak impedance and $\frac{l}{\beta\lambda}$ varying β for TM_{010} mode



Peak impedance and $\frac{l}{\beta\lambda}$ varying β for TM_{110} mode