LARGE BLUE ISOCURVATURE FROM A CLASSICAL CONFORMAL LIMIT

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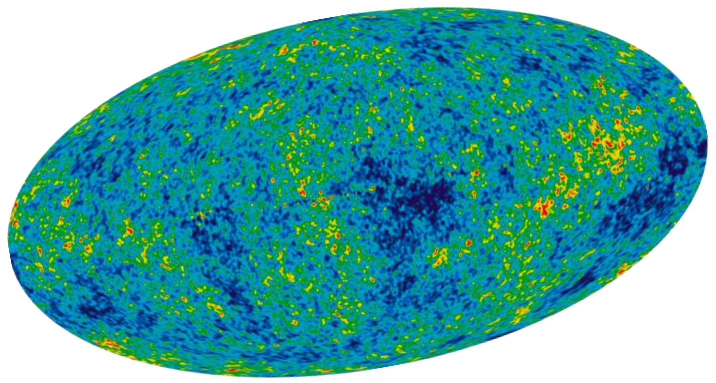
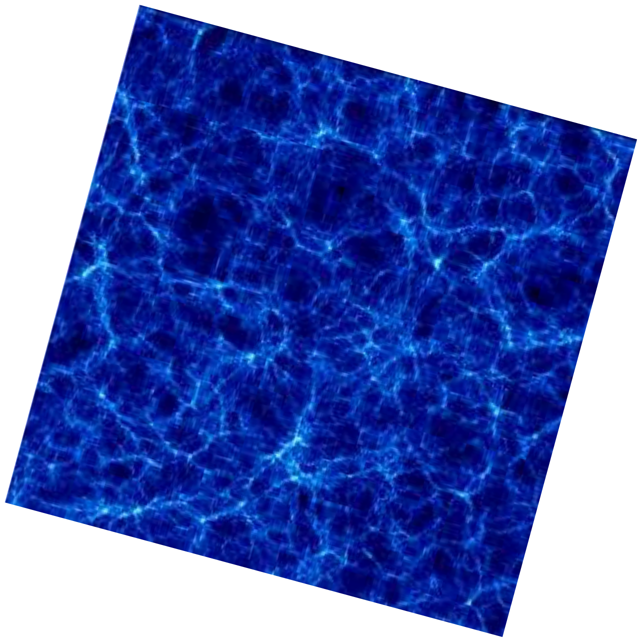
May 04, 2024

Isocurvature Perturbations [2110.02272, 2309.17010]

$$S_{ij} = 3\left(\zeta_i - \zeta_j\right)$$



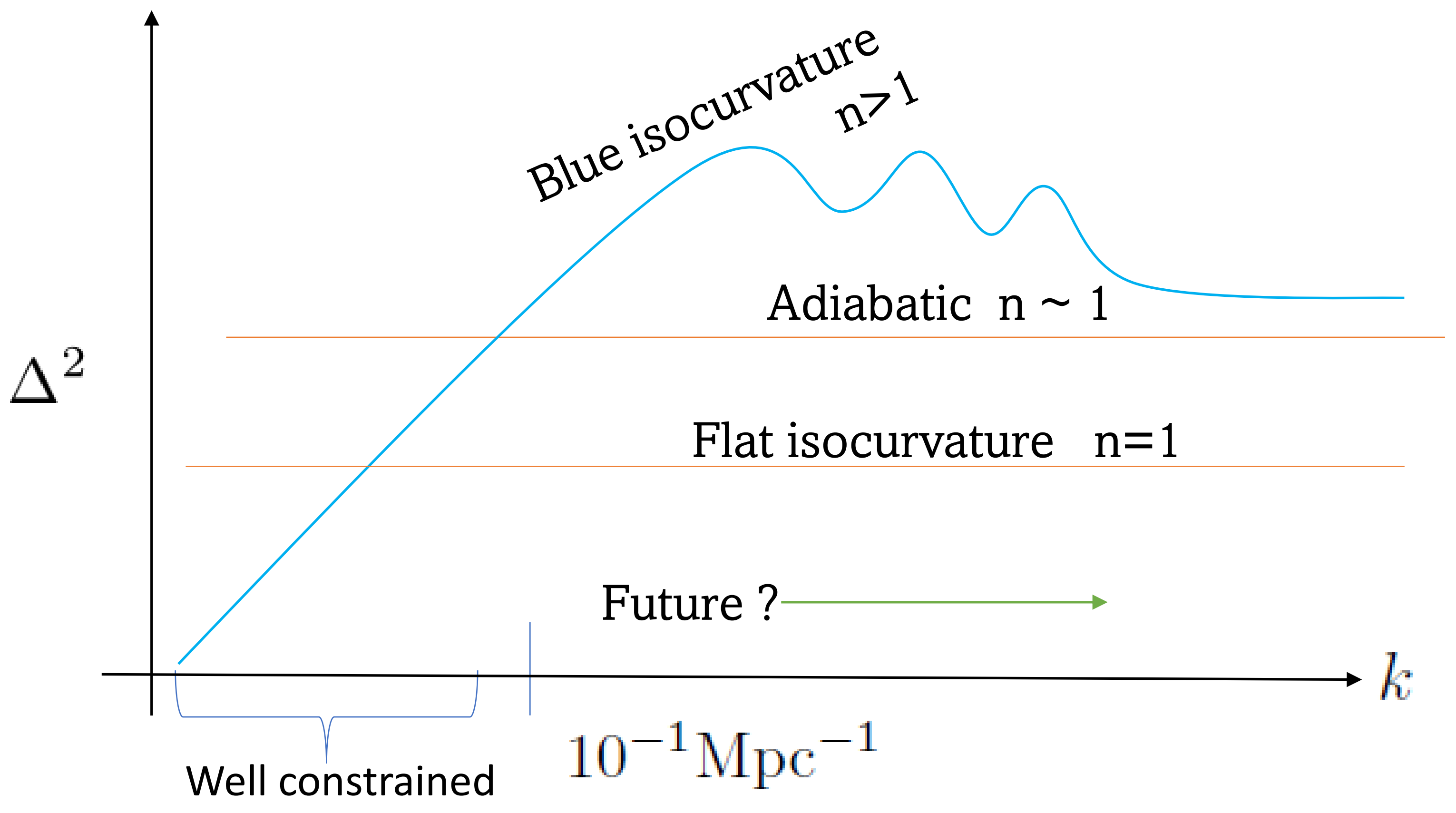


Adiabatic	Isocurvature
single dynamical mode	Multiple independent dynamical modes

$$\delta_c^{\rm ad} = \delta_b^{\rm ad} = \frac{3}{4}\delta_\gamma^{\rm ad} = \frac{3}{4}\delta_\nu^{\rm ad}$$

$$\delta_c^{\rm iso} = \delta_c - \delta_c^{\rm ad} = \delta_c - \frac{3}{4}\delta_\gamma^{\rm ad}$$

Currently, scale invariant isocurvature perturbations are observationally constrained to be less than 2% on large (CMB) scales at k=0.05/Mpc. [**1807.06211**]



$$\Delta^2(k) \sim k^{n-1}$$

Blue n > 1
Red n < 1

2-sigma hint found in **1711.06736**, **1707.09354** from combined Planck+BOSS-DR12 data analysis.

(not statistically significant yet)

Why study blue isocurvature

- Isocurvature can offer valuable insights into inflation and the presence of spectator DM fields and their mass scales.

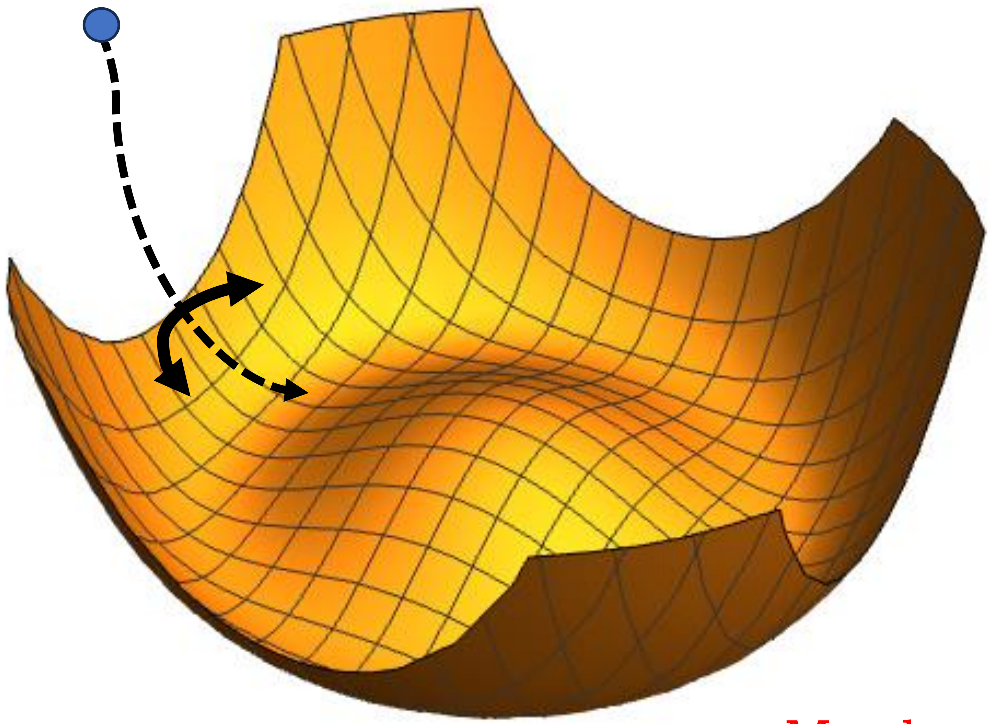
- Blue isocurvatures with nI > 2.4, uniquely hint towards spectator fields with <u>time-dependent mass</u> during inflation. [**1509.0585**]

- Blue isocurvature can relax the constraint on H-f parametric region

$$O\left(\frac{H_{\text{inf}}}{f_{\text{PQ}}\theta_i}\right)$$

Generate blue-tilted spectra via non-equilibrium radial field

S. Kasuya, M. Kawasaki [0904.3800]



$$\Phi = R e^{i\frac{a}{R}}$$

$$\sqrt{\Delta^2_{\frac{\delta a}{a}}} \approx \frac{H/2\pi}{R\theta_i}$$

$R : O(M_{\rm pl}) \to O(f_{\rm PQ})$

Dynamical non-equilb mass

Massless axion

$$\Box \delta a = 0$$

1501.05618

$$\left(\Box - \frac{\Box R}{R}\right)\delta a = 0$$

Questions:

1. Mass of spectator field to obtain large blue-tilt (niso > 2)?
   - > H

2. How do you switch off the mass at the end of the rolling to ensure that the axion density isn't diluted away by inflation?

3. How do we get a large initial displacement and not worry about quartic interaction term?

All conditions satisfied by Kasuya-Kwasaki's SUSY axion model.
Q1 via SUGRA and Q2&3 via "SUSY flat-directions".

Kasuya-Kawasaki model relies on having a SUSY flat-direction (and two dynamical chiral PQ fields) and no quartic self interaction.

Is there a generic way to generate large blue isocurvature without flat-direction, for single PQ field and with quartic interaction term?

U(1) PQ field

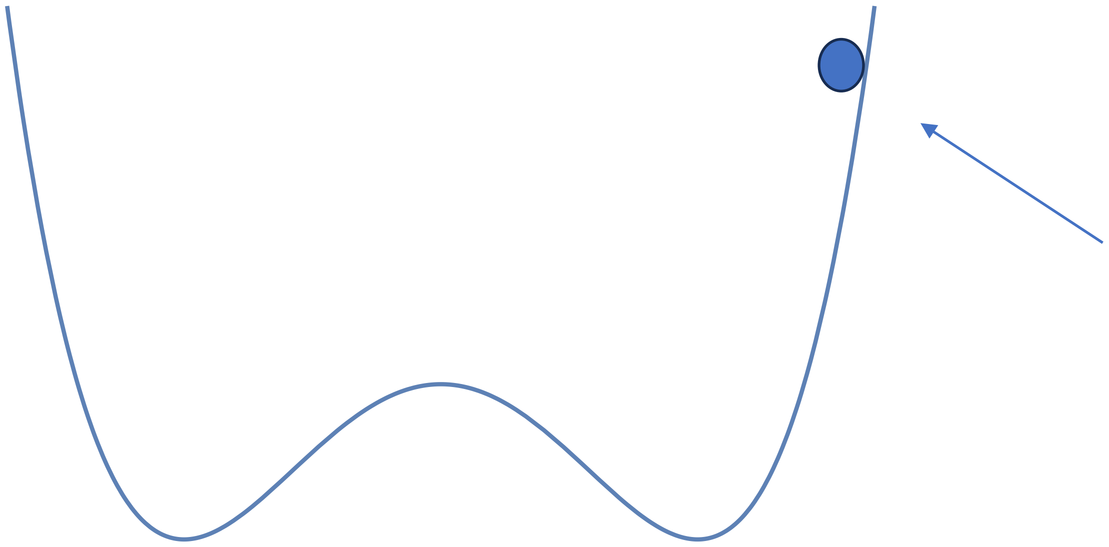
$$\Phi = \frac{1}{\sqrt{2}}\Gamma e^{i\frac{\Sigma}{\Gamma}}$$

Γ	Radial field
Σ	Axial field

PQ symmetry breaking potential

$$V = -M^2\Gamma^2 + \frac{\lambda}{4}\Gamma^4$$

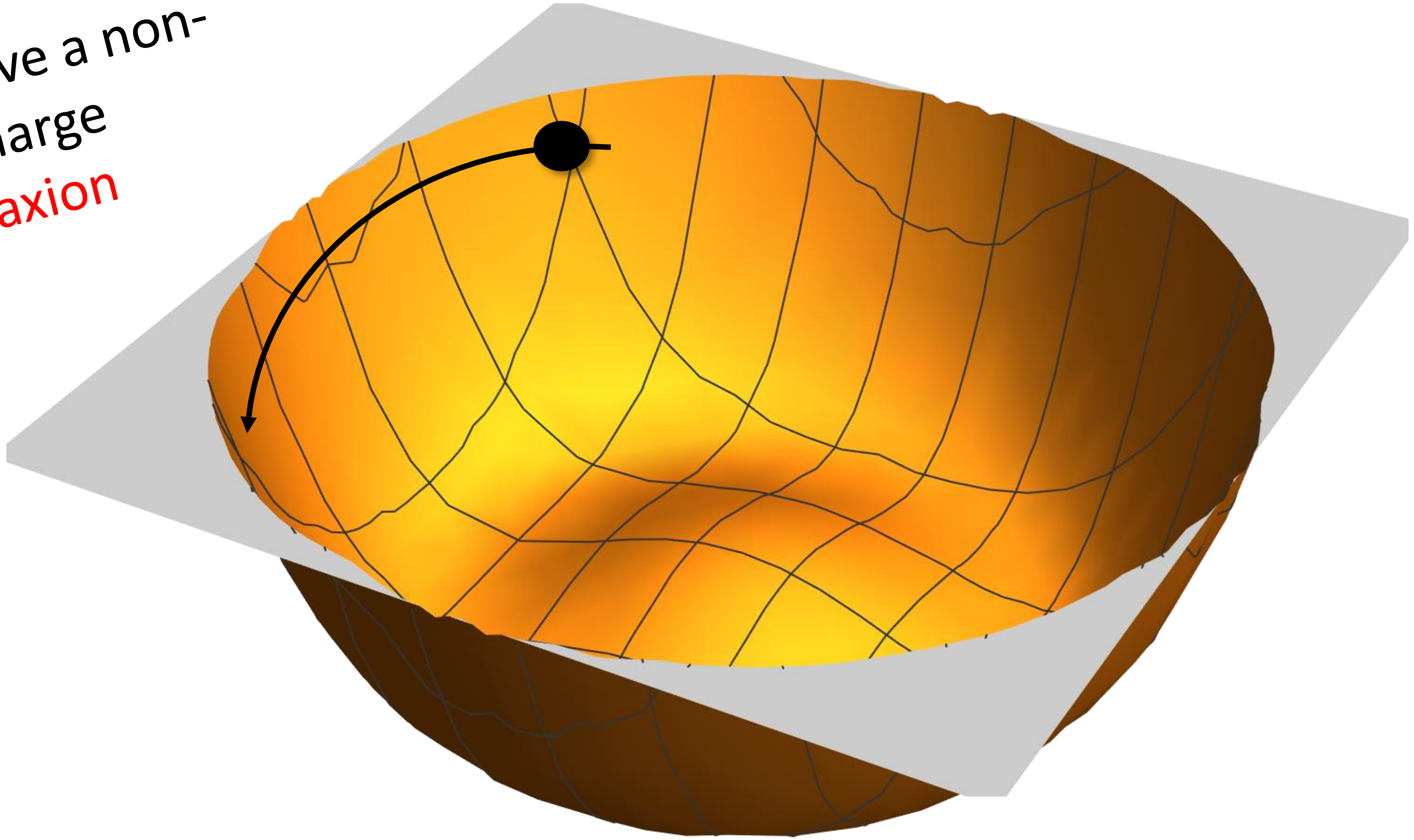
Has the usual quartic self-interaction term



Displaced away from minimum vev

Impart initial angular momentum, or, equivalently have a non-zero U(1)PQ charge

a.k.a rotating axion



$$n_I = 3$$

$$\Delta_s^2 \propto k^2$$

Generates blue isocurvature spectrum

Classical background conformal solution

$$Y = a\Gamma$$

$$S = \int d\eta d^3x \left[\frac{1}{2} \left[(\partial_0 Y)^2 - 2\frac{\partial_0 a}{a} Y \partial_0 Y + \left(\frac{\partial_0 a}{a} \right)^2 Y^2 \right] + \frac{1}{2} Y^2 (\partial_0 \theta)^2 - \left(-M^2 a^2 Y^2 + \frac{\lambda}{4} Y^4 \right) \right].$$

Classical background conformal solution

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Y field is conformal and doesn't see the curved spacetime until the end of 'spiral' rolling

Two ways to get a $Y_0 =$ constant solution:

1. $\partial_0 \theta =$ constant and $\sqrt{\lambda} Y^2 \gg \max\left(M^2 a^2, 2H^2 a^2\right)$, or
2. $\partial_0 \theta = 0$ and $\lambda = 0$ and $M^2 = -2H^2$ (this is not useful to us as the minimum is at 0)

$$Q^{(0)} = Y^2 \partial_0 \theta$$

Conserved U(1)PQ charge

The axial rotations cause a strong mixing between the radial and axial fluctuations.

$$a^2\left[\partial_\eta \delta a, \partial_\eta \delta\Gamma\right] \approx -2i\partial_\eta \theta_0 \delta^{(3)}(\vec{x} - \vec{y})$$

In such a scenario, it is not obvious how to consistently quantize the two **strongly coupled** fields, define proper vacuum, identify the Goldstone mode and get correct correlation functions.

Explicit quantization

$$\delta\psi^n = (a\delta\Gamma, a\Gamma_0\delta\theta)^n$$

$$\begin{aligned} [\delta\psi^n(\eta,\vec{x}), \delta\psi^m(\eta,\vec{x})] &= 0, \\ [\pi^n(\eta,\vec{x}), \pi^m(\eta,\vec{x})] &= 0, \\ [\delta\psi^n(\eta,\vec{x}), \pi^m(\eta,\vec{x})] &= i\delta^{nm}\delta^{(3)}(\vec{x}-\vec{y}) \end{aligned}$$

$\longrightarrow$

$$\begin{aligned} [\delta\psi^n, \delta\psi^m] &= 0, \\ [\delta\psi^n, \partial_\eta\delta\psi^m] &= i\delta^{nm}\delta^{(3)}(\vec{x}-\vec{y}) \\ [\partial_\eta\delta\psi^n, \partial_\eta\delta\psi^m] &= i\delta^{(3)}(\vec{x}-\vec{y}) \begin{bmatrix} 0 & 2\partial_\eta\theta_0 \\ -2\partial_\eta\theta_0 & 0 \end{bmatrix}. \end{aligned}$$

$$\delta\psi^n = \int \frac{d^3p}{(2\pi)^{3/2}} \left[a_{\vec{p}}^{++} c_{++} V_{++}^n e^{-i\omega_{++}\eta} + a_{\vec{p}}^{+-} c_{+-} V_{+-}^n e^{-i\omega_{+-}\eta} + h.c.\right] e^{i\vec{p}\cdot\vec{x}}$$

2 annihilating ladder operators with ++ and +- belonging to different eigen-solutions

Post quantization and identifying proper vacuum

Goldston mode

$$\lim_{k \ll \sqrt{\lambda} Y_c} \omega_{+-}^2 \approx \frac{k^2}{3}$$

$$\lim_{k \gg \sqrt{\lambda} Y_c} \omega_{+-}^2 \approx k^2$$

Behaves like a **radiation-matter fluid**

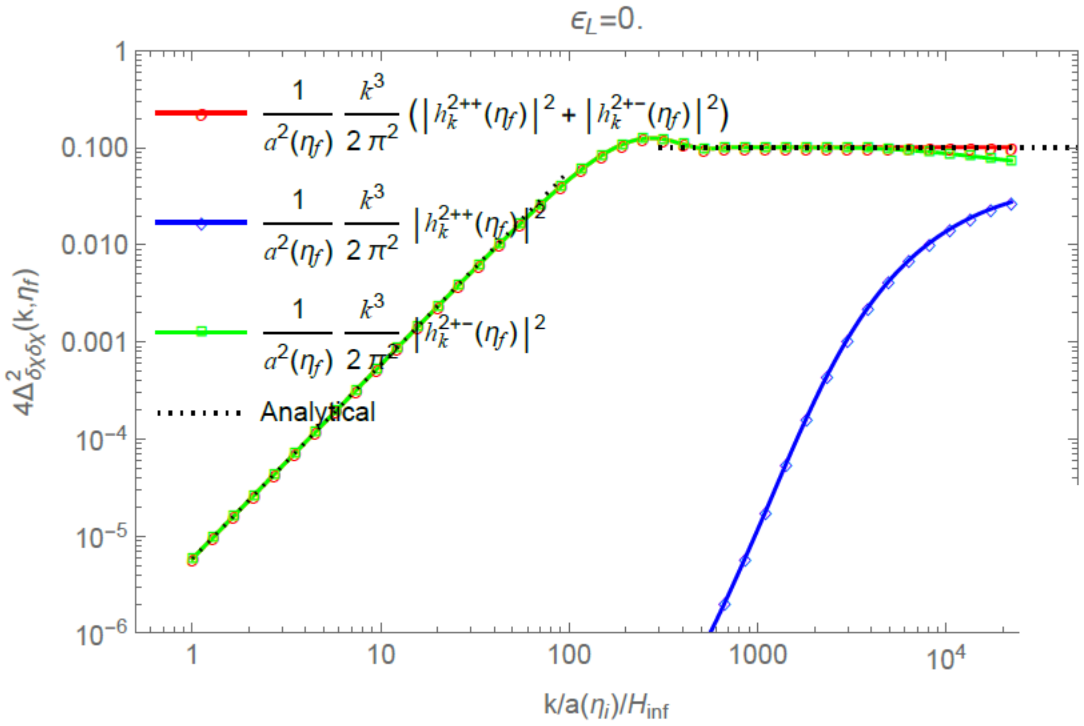
$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3}$$

The theory behaves like a CFT that flows from k²/3 → k²

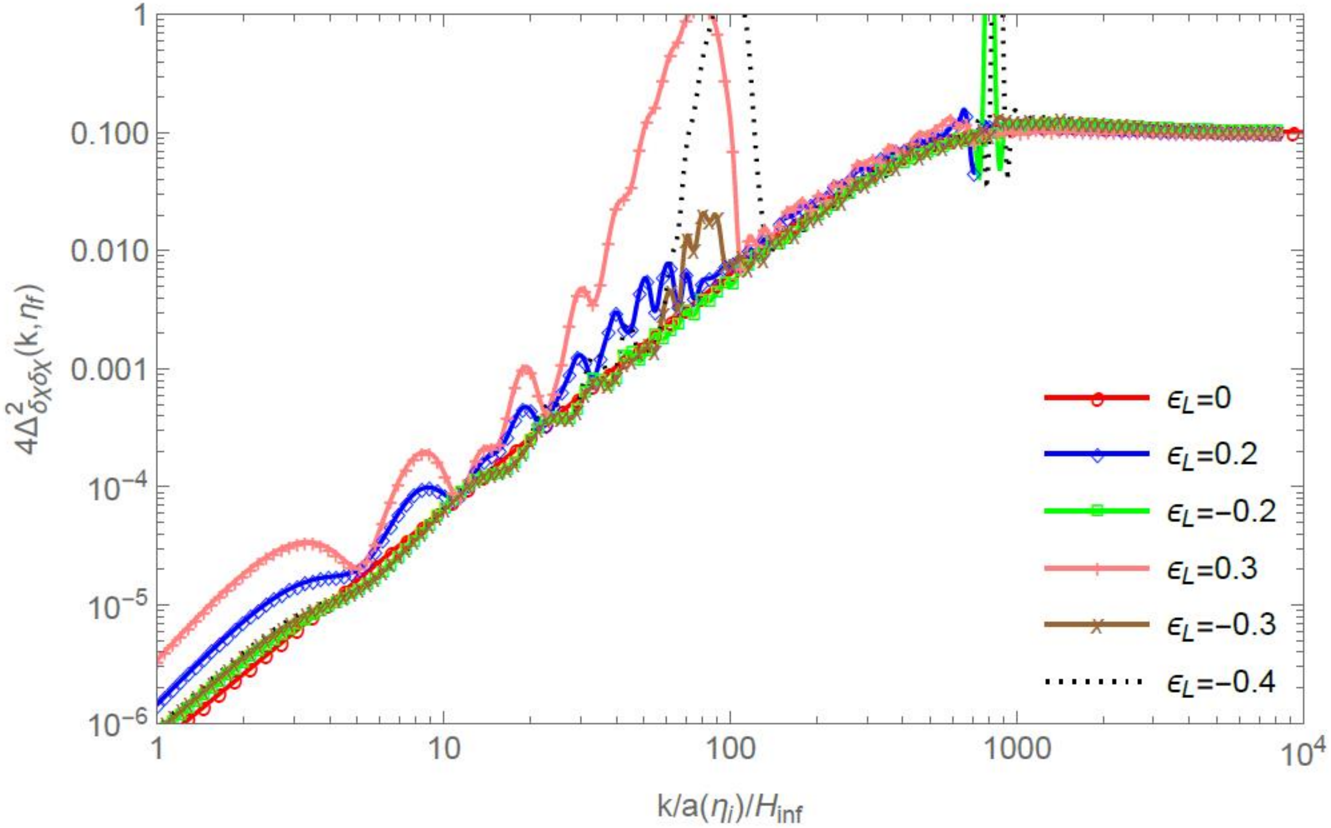
$\partial_\eta \theta_0$-induced mixing is generating an axion pressure-supported acoustic wave in the mixture of axions and radial fields.

“Correlation remains Minkowskian even after modes have $k < aH$”

until rotations seize



Plots of axion blue-tilted isocurvature



Rotating axion during inflation has cool properties

- resembles a CFT that flows from $k^2/3 \rightarrow k^2$
- Isocurvature quantity frozen even during subhorizon evolution
- Goldston mode behaving like a radiation-matter fluid during rotation
- Kinetic correlation even though no field correlation.
- Mimics a flat direction without having flat direction. (Y=constant)

$$\langle \partial_\eta \delta\Gamma \partial_\eta \delta\chi \rangle \neq 0 \text{ even though } \langle \delta\Gamma \delta\chi \rangle = 0$$

Thanks…

Talking points

1. Initiate discussion into how blue-tilted isocurve can evade constraints.
    1. State current and previous analysis in support of this hypothesis.
2. Sketch caricature argument to generate blue spectrum and how the duration of rolling is controlled.
3. Discuss KK model and how flat-direction helps stabilize the system to ensure larger rolling period through a Hubble-induced mass term.
4. Discuss new axion model with quartic potential
5. Classical conformal limit to avoid radial field going to zero.
6. Non-zero U(1)PQ charge
7. Quantization of the strongly coupled fields.
    1. Goldston mode behaving like a radiation fluid (what is the intuitive reason?)
8. How the correlation remains Minkowskian even when $k/aH<1$.
9. Show some plots and present some results for deviation from conformal limit.

SUSY axion model

S. Kasuya, M. Kawasaki [0904.3800]

Renormalizable superpotential: $W_{\rm PQ} = h\left(\Phi_+\Phi_- - F_a^2\right)\Phi_0$

subscripts on Φ indicate U(1) PQ charges.

$$\Phi_+\Phi_- - F_a^2 = 0 \qquad \Phi_0 = 0 \qquad \text{flat-direction}$$

$$V = \frac{1}{2}c_+ H^2 \left|\Phi_+\right|^2 + \frac{1}{2}c_- H^2 \left|\Phi_-\right|^2 + \frac{1}{2}\left|\Phi_+\Phi_- - F_a^2\right|^2$$

Kaehler induced mass terms

F-term