Generalized Global Symmetries and Nonperturbative Quantum Flavodynamics

Seth Koren

University of Notre Dame

Leptons: 2211.07639 with Clay Córdova, Sungwoo Hong, Kantaro Ohmori

Quarks: 2402.12453 with Clay & Sungwoo

Related ideas in my

SM proton stability: 2204.01741

(B-L) BF theory for the lithium problem: 2204.01750

SM flavor 2-group: 2212.13193 with Clay

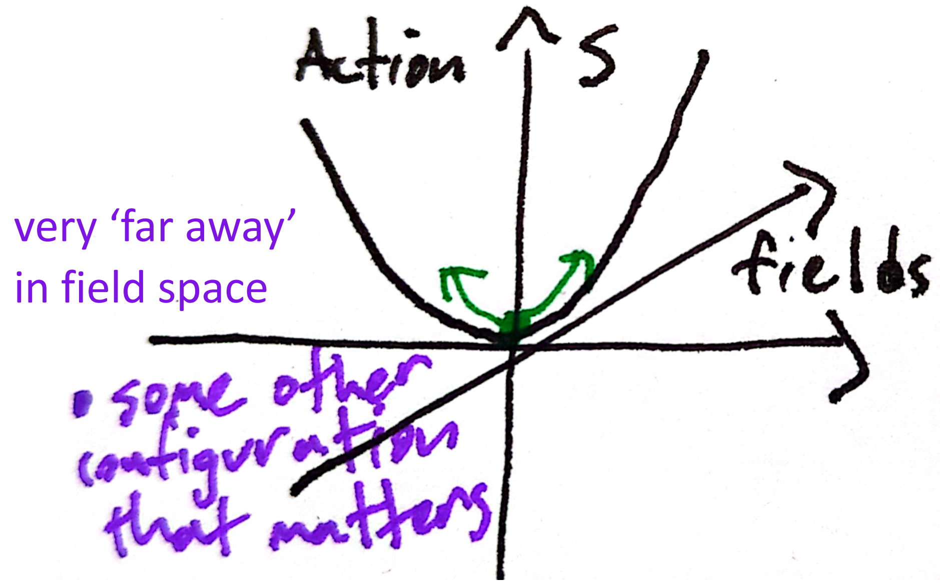
Fractional charges: 2405.X with Adam Martin, 240X with Sam Homiller, and more on the way

Nonperturbative QFT effects

Most of our time spent understanding *perturbative* QFT effects, sensibly

Expand around vacuum, calculate e.g. $\langle \psi_{\text{out}} | S | \psi_{\text{in}} \rangle$

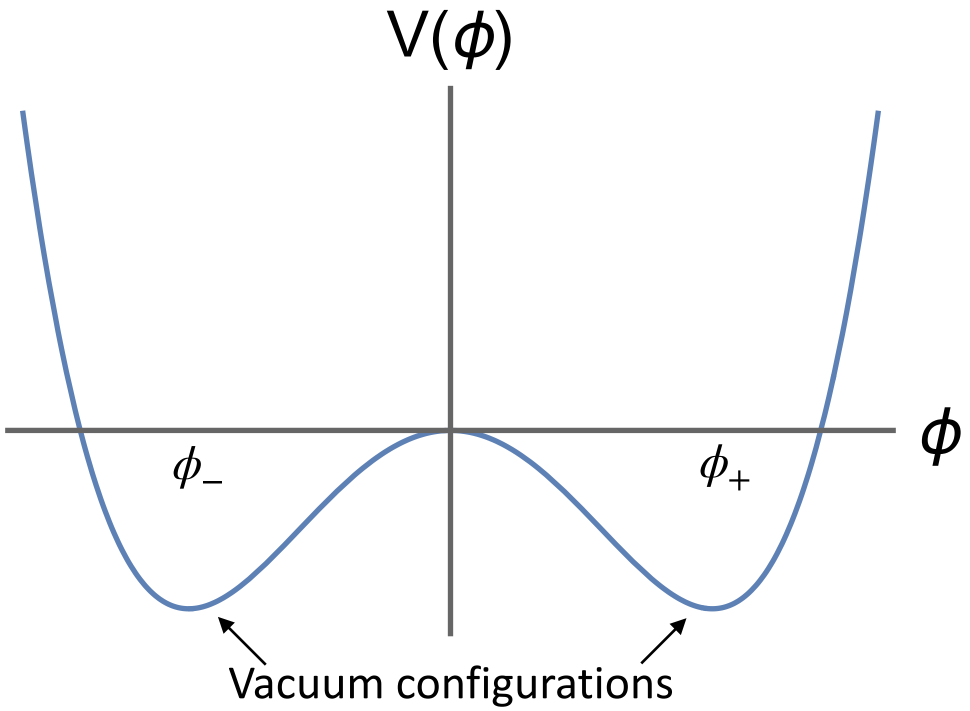
All great. But quantum field theory
is richer than perturbation theory!



Topology in field theory

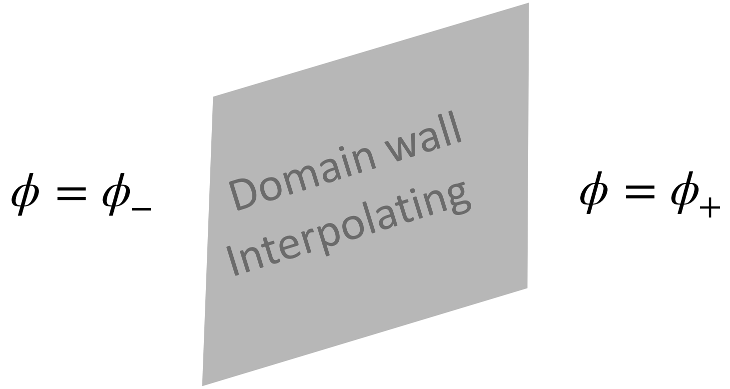
Often there are ‘topological quantum numbers’ that classify field space

A $\mathbb{Z}_2$-symmetric scalar breaking $\mathbb{Z}_2 \to \varnothing$, distinct vacua $\pi_0(\mathscr{M}_{\text{vac}}) = \mathbb{Z}_2$

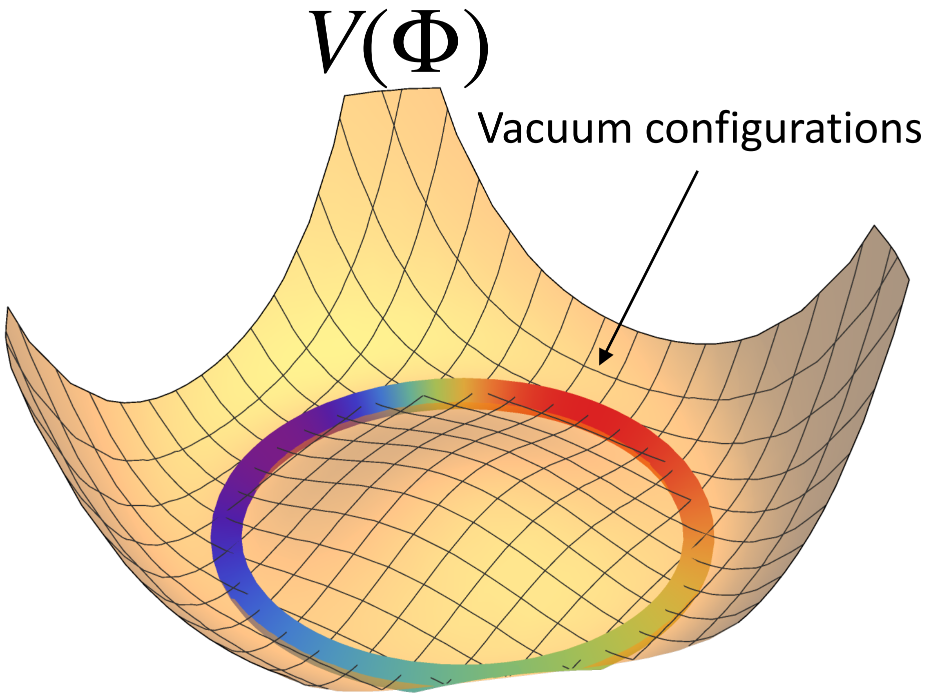


Local vacuum solutions

$\phi(x) : \mathbb{R}^4 \to \mathscr{M}_{\text{vac}}$ can have defects; domain walls separate different regions

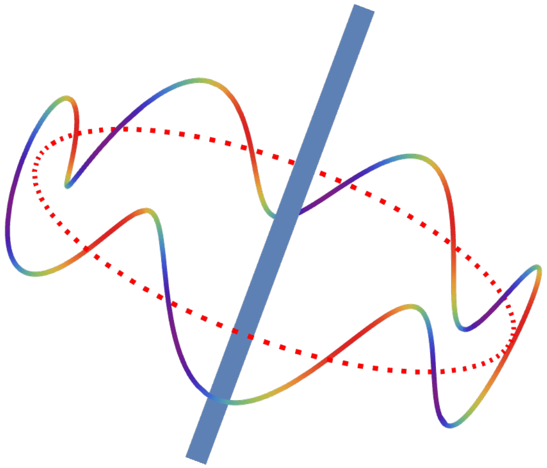


Higher-dimensional topology



A $U(1)$-symmetric scalar breaking $U(1) \to \varnothing$,
$\pi_0(\mathscr{M}_{\text{vac}}) = 1$, $\pi_1(\mathscr{M}_{\text{vac}}) = U(1)$

$\Phi(x) : \mathbb{R}^4 \to \mathscr{M}_{\text{vac}}$ can have winding number which leads to cosmic strings



B-L strings for the lithium problem 2204.01750

How can we tell what sorts of physics effects these objects can lead to?

Generalized Global Symmetries

Symmetries are important!

Usually look at Lagrangian data and consider transforming local operators

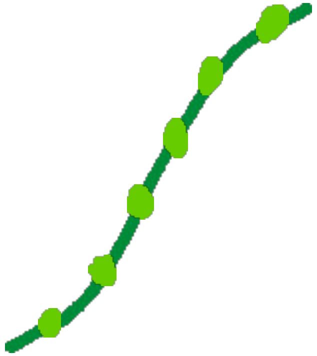
$$\psi^a(x) \to R^a{}_b \psi^b(x)$$

But what about these extended operators associated to this nonperturbative, topological data in our theory?

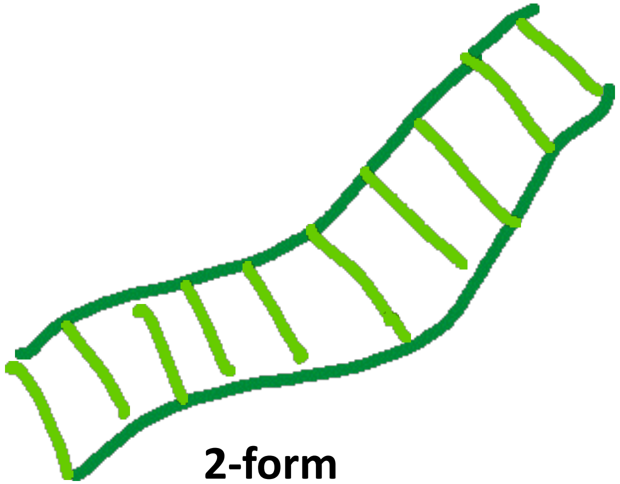
GGS Framework
Gaiotto, Kapustin, Seiberg, Willett
1412.5148

Higher-form symmetries

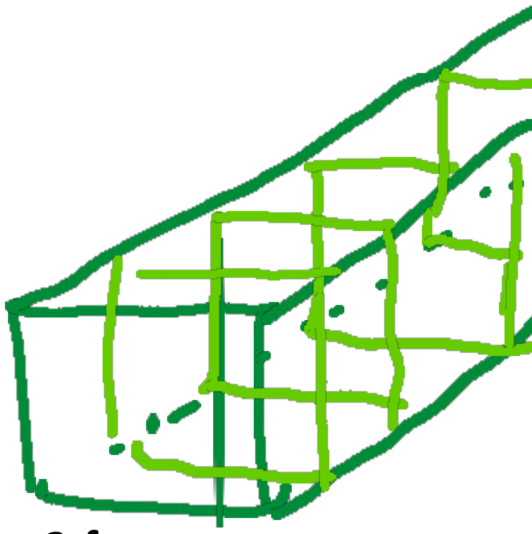
0-form symmetry
charged local
operators
e.g. particles



1-form
line operators
e.g. Wilson line



2-form
surface operators
e.g. cosmic string



3-form
volume operators
e.g. domain wall

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^{\mu\nu} = 0$$

Generally $\partial_\mu J^{\mu_1 \mu_2 \cdots \mu_{p+1}} = 0$ antisymmetric

Higher-form symmetries

0-form symmetry
charged local
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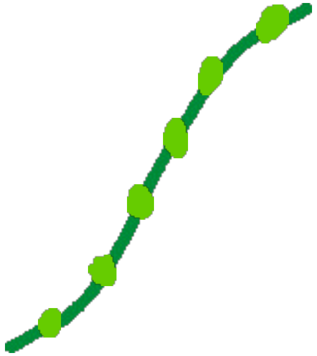
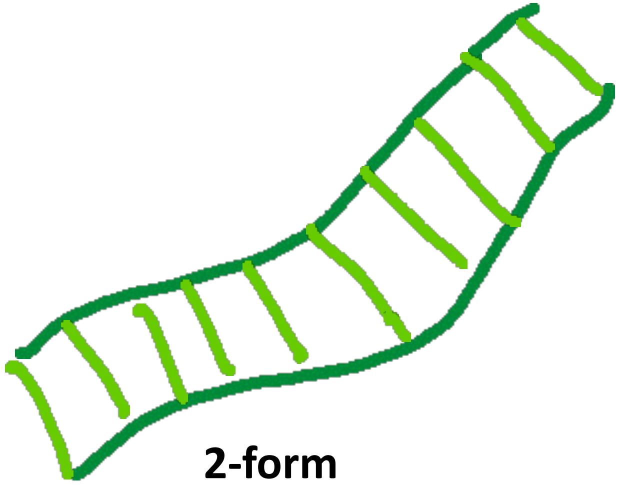
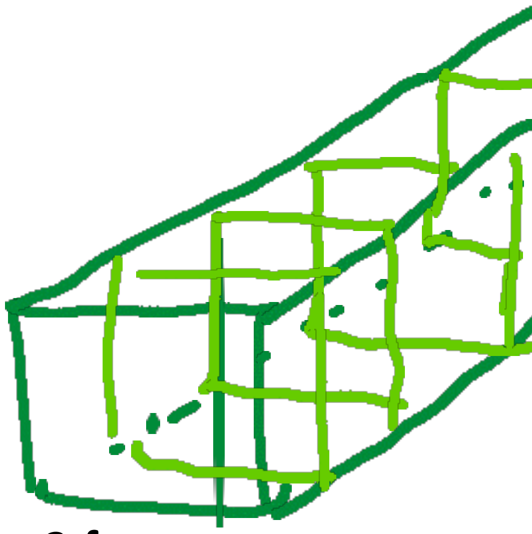


1-form
line operators
e.g. Wilson line



2-form
surface operators
e.g. cosmic string



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Generally $\partial_\mu J^{\mu_1 \mu_2 \cdots \mu_{p+1}} = 0$ antisymmetric

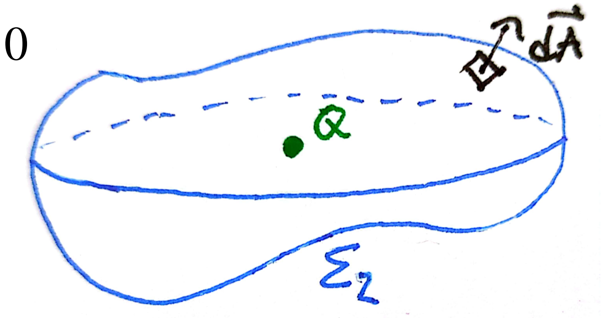
Break by adding charged operator to Lagrangian e.g. $\delta\mathscr{L} = MNN$

Break only with the appearance of new dynamical degrees of freedom!

Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.

$t = 0$



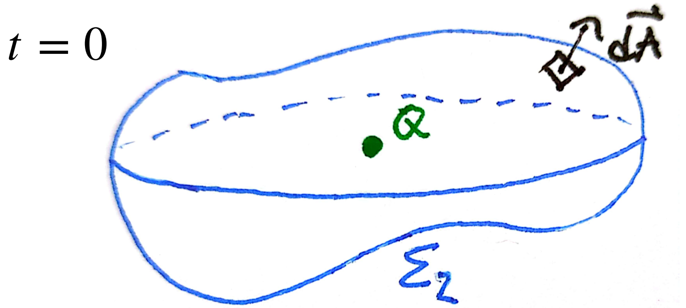
$$Q_{\text{enclosed}} = \int_{\Sigma_2} \vec{E} \cdot d\vec{A}$$

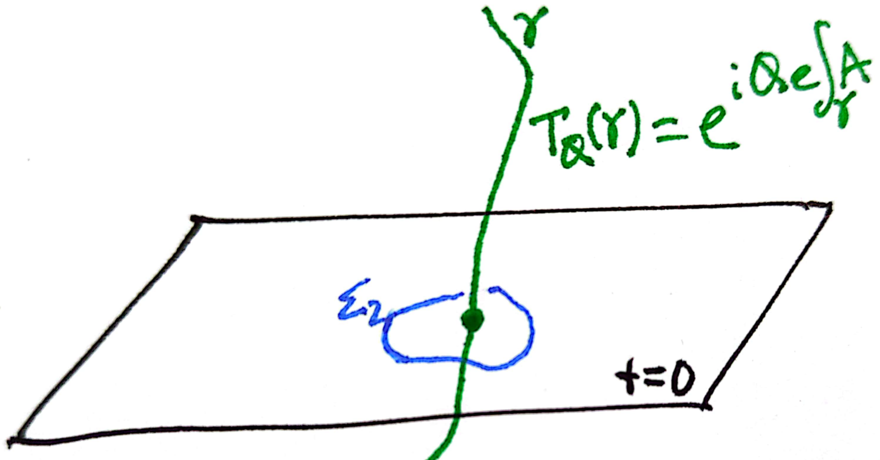
Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.

In pure electromagnetism, the photon field strength is conserved $J_E^{\mu\nu} \sim \frac{1}{e^2} F^{\mu\nu}, \quad \partial_\mu J_E^{\mu\nu} = 0$

Gauss' law computes a Noether charge for an electric 1-form symmetry!





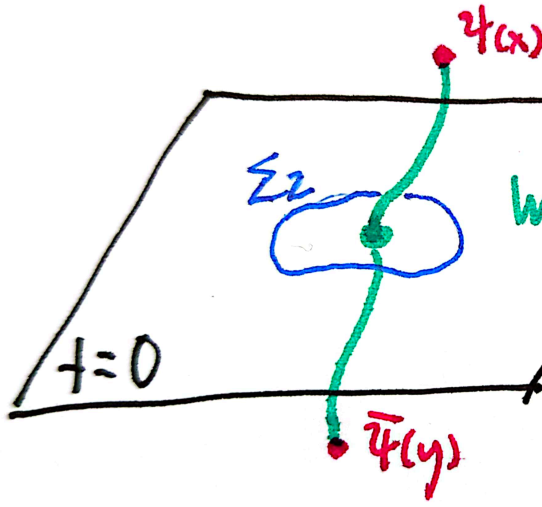
$$Q_{\text{enclosed}} = \int_{\Sigma_2} \vec{E} \cdot d\vec{A}$$

$$Q_{\text{enclosed}} = \int_{\Sigma_2} F^{\mu\nu} dS^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$$

Emergent 1-form symmetry

The 1-form symmetry is emergent in the low-energy, long-distance theory $E \ll m_e$.

Once we see the dynamical electron, then Wilson lines can 'end'.



$W_q = e^{iq\int_x^y A}$

That is, Gauss' law really breaks for $E > m_e$ because the Gaussian surface is no longer topological.

Emergent 1-form symmetry

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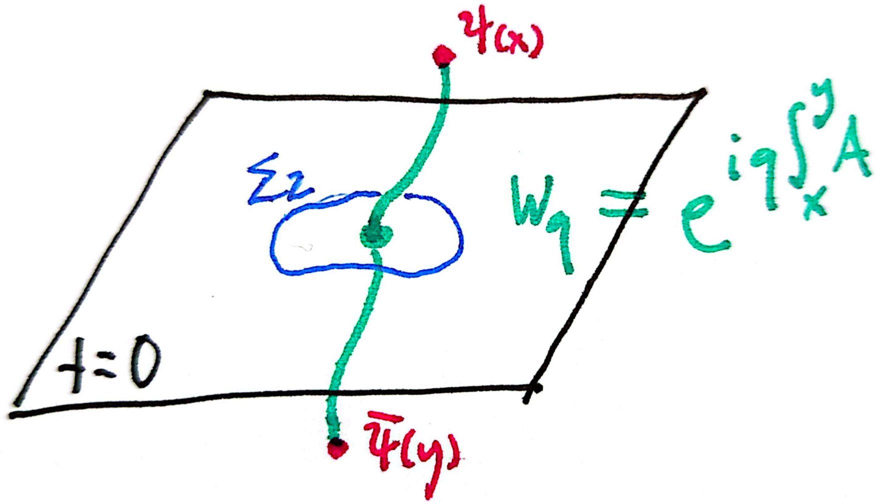
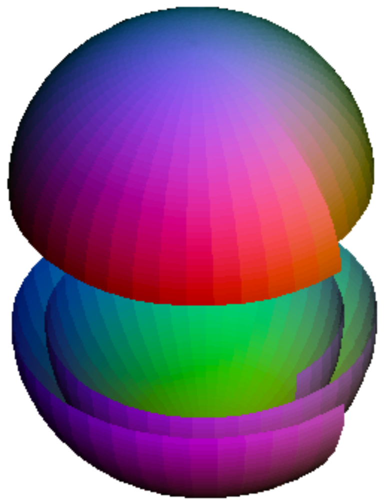


That is, Gauss' law really breaks for $E > m_e$ because the Gaussian surface is no longer topological.

Mutatis mutandis a magnetic one-form symmetry for a theory H with 't Hooft lines classified by $\pi_1(H)$

Instantons and Anomalies



Yang-Mills field configurations can carry topological quantum numbers $\pi_3(SU(N)) = \mathbb{Z}$.

Sometimes a global symmetry, say $U(1)_X$, can be good classically but quantum-mechanically be anomalous

$$\partial_\mu J^\mu_X = 0 \quad \longrightarrow \quad \partial_\mu J^\mu_X = \frac{\mathscr{A}}{8\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$$

It's the instantons which bring this symmetry violation to life

$\int_{\mathbb{R}^4} F^{\mu\nu}\tilde{F}_{\mu\nu} \propto \int_{\partial\mathbb{R}^4 \simeq S^3} \hat{n}_\mu J^\mu_{\mathrm{CS}}$ = number of times A_μ 'winds' around infinity

Unsaturated Anomalies - Missing Instantons

We said instantons are the field configurations which can saturate the anomaly

$$\partial_\mu J^\mu_X = \frac{\mathscr{A}}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

But what about when they don't?

E.g. famously $\pi_3\left(U(1)\right) = 1$ and *there are no Abelian instantons in* $\mathbb{R}^4$, so $\int_{\mathbb{R}^4} F\tilde{F} = 0$

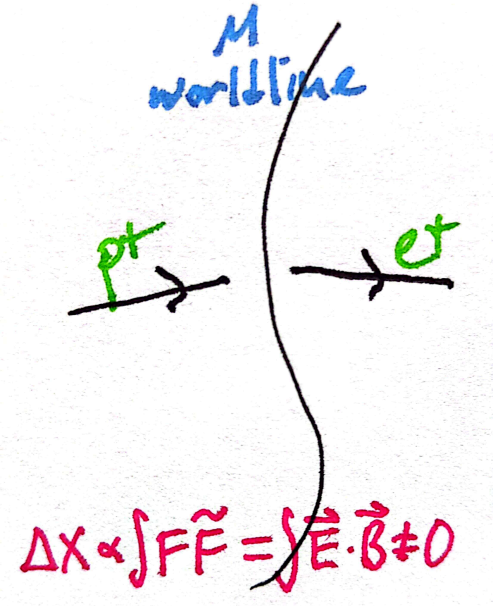
Old lesson: X is anomalous but S-matrix preserves X anyway

EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous there must remain a subtle sort of symmetry that demands the S-matrix preserves X

A hint: X *can* be violated around magnetic monopoles

c.f. Callan-Rubakov



A confused effective field theorist

Dirac '31
Callan, Rubakov '80s
Ongoing…

There's a subtler notion of symmetry!

X not fully broken, but converted to a non-invertible symmetry! This must act both on local fields and on 't Hooft lines.

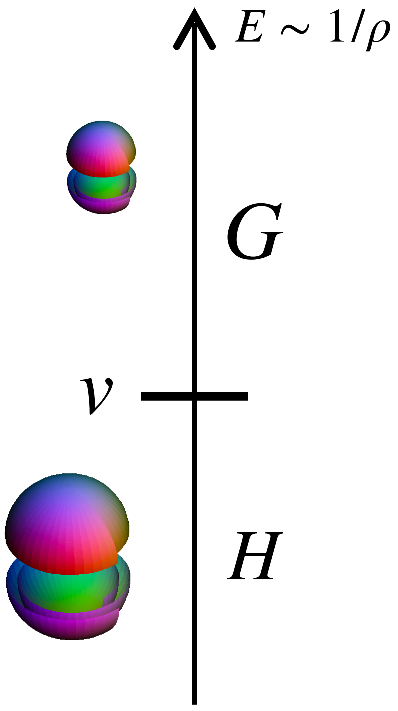
Choi, Lam, Shao
2205.05086
Córdova, Ohmori
2205.06243

$$\psi(x) \to \psi(x) e^{i\alpha} \qquad e^{i\oint_\gamma A_m} \to e^{i\oint_\gamma A_m + i\alpha \oint_\gamma A}$$

Non-invertible symmetry must break when there are dynamical monopoles

Another victory for naturalness

Small instanton model building



When can ultraviolet instantons have interesting effects?

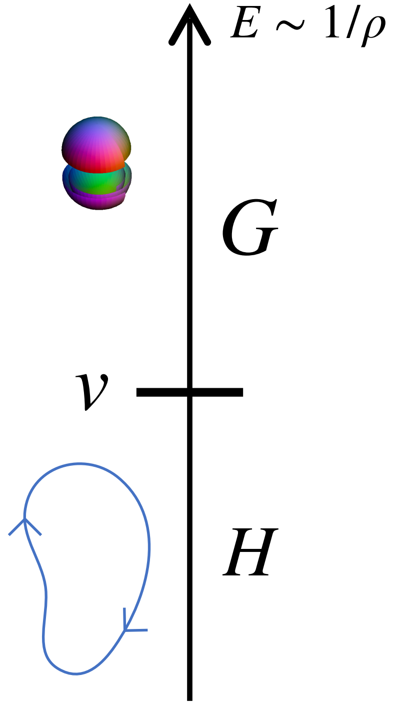
$$A_\mu^{(1-inst)}(x) \propto \frac{\eta_{a\mu\nu}(x-x_0)^\nu J^a}{(x-x_0)^2 + \rho^2}$$

G-instanton effects suppressed below Higgsing at v, and H-instantons (if any) may not have the same effects

What can we tell about small instantons at low energies? Normally, nothing. Need $E \gtrsim v$.

But if the low-energy theory allows H-magnetic representations $\pi_2(G/H) \simeq \pi_1(H) \neq 1$, then this information can subtly be preserved

Model-building logic



A classical global symmetry X protects some operator $\mathcal{O}$ and has an H anomaly

$$\partial_\mu J^\mu_X = \frac{\mathscr{A}}{8\pi^2} H^{\mu\nu}\tilde{H}_{\mu\nu}$$

But some values of $\int_{\mathscr{M}} H\tilde{H}$ not realized for $\mathscr{M} = \mathbb{R}^4$, so X is

not violated in S-matrix of the IR theory and $\mathcal{O}$ still protected

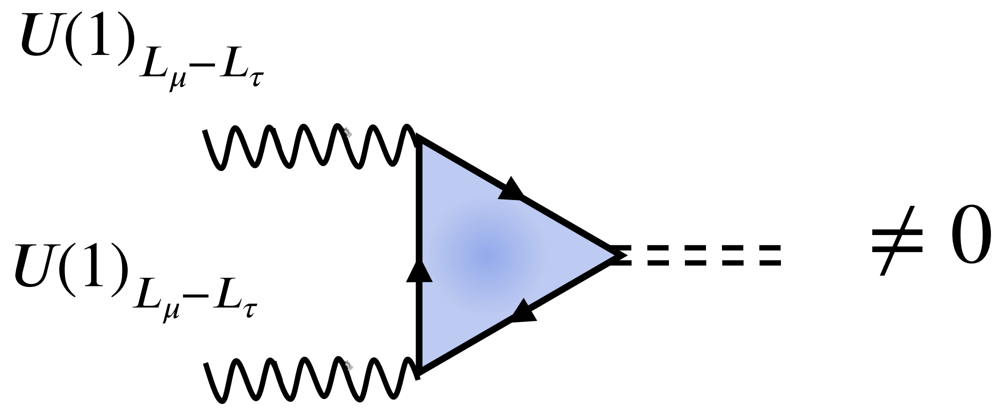
Non-invertible X symmetry tells us $\mathcal{O}$ could be generated *only* by instantons in the theory $G \supset H$ which has G/H-monopoles

Nonperturbative Quantum Lepton Flavodynamics

Neutrino Masses from Generalized Symmetry Breaking

arXiv:2211.07639, Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

Beyond with $Z'_{L_\mu - L_\tau}$ and N!



Non-invertible symmetry protects neutrino masses
either with or without right-handed neutrinos

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $\left(\tilde{H}L\right)^2$

	L_i	$\bar{e}_i$	N_i
$\mathbb{Z}_3^{\tilde{L}+N}$	$+1$	-1	$+1$

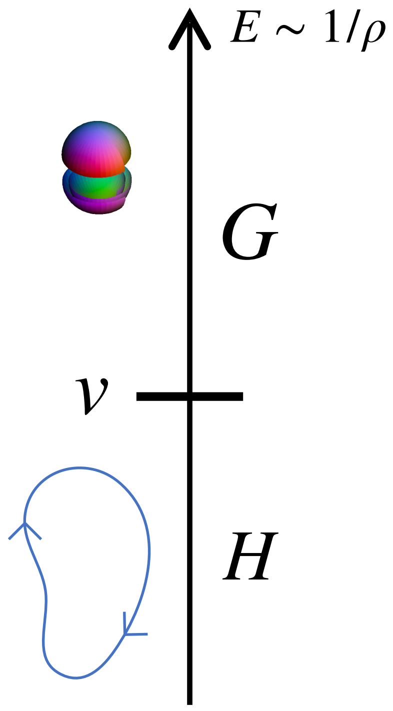
Disallows HLN

Model-building logic

A classical global symmetry $X = \mathbb{Z}_3^L$ protects the operators $\mathcal{O}_{ij} = (\tilde{H}L_i)(\tilde{H}L_j)$ and has an $H = U(1)_{L_\mu - L_\tau}$ anomaly

But while $\int_{\mathcal{M}} H\tilde{H} \in \mathbb{Z}$ generally, $\int_{\mathbb{R}^4} H\tilde{H} = 0$

X is a non-invertible symmetry! In a theory $G \supset H$ with lepton flavor monopoles, $\mathcal{O}_{ij}$ could be classically absent and generated only by G-instantons

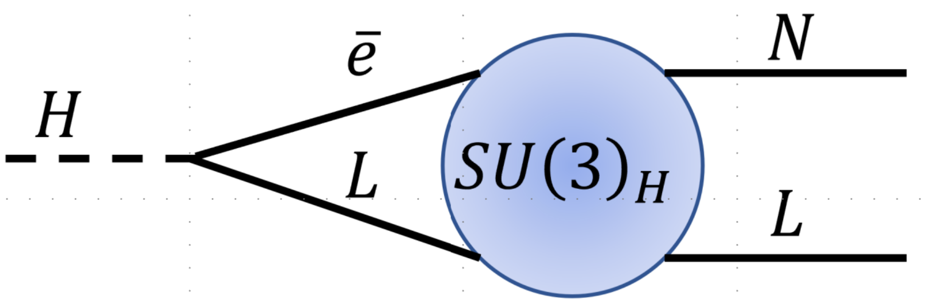


Dirac masses:

Write down charged lepton mass $\mathscr{L} \sim y_\tau H \mathbf{L}\bar{\mathbf{e}}$

	$SU(3)_H$	$U(1)_{\mu-\tau}$	$U(1)_L$	$U(1)_N$
$\mathbf{L}$	3	$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$+1$	0
$\bar{\mathbf{e}}$	$\bar{3}$	$\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	0
$\mathbf{N}$	$\bar{3}$	$\begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	$+1$

Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{L}\mathbf{N}$



$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{L}\mathbf{N}$$

Nonperturbative Quantum Quark Flavodynamics

Non-Invertible Peccei-Quinn Symmetry and the Massless Quark Solution to the Strong CP Problem

arXiv:2402.12453, Clay Córdova, Sungwoo Hong, SK

Strong CP Brief Version

The 'strong CP angle' $\bar{\theta} = \arg e^{-i\theta} \det\left(y_u y_d\right)$ is constrained to $\bar{\theta} \lesssim 10^{-10}$!

SM 'massless up quark solution': UV PQ symmetry sets $y_u = 0$, and observed up quark mass is totally generated by QCD instantons

Georgi-McArthur '81
Kaplan-Manohar '86
Choi, Kim, Sze '88

Beautiful idea but not realized in nature
QCD instanton effects not large enough

Flavour Lattice Averaging Group 2019

Could the quark sector tell us about a model where UV instantons revive this solution?

Quark Horizontal Symmetry

In the quark sector we can gauge flavor in a slightly more subtle way because $N_c = 3 = N_g$!

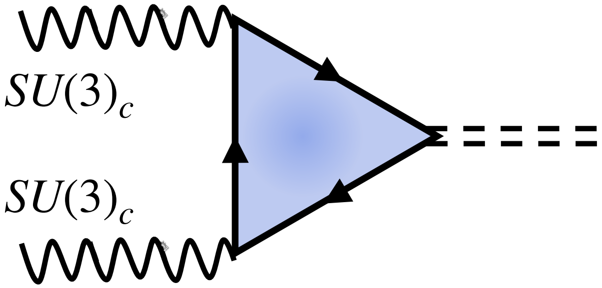
We have the right matter to realize $\left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$

These non-trivial possibilities modify the topological data in a crucial way

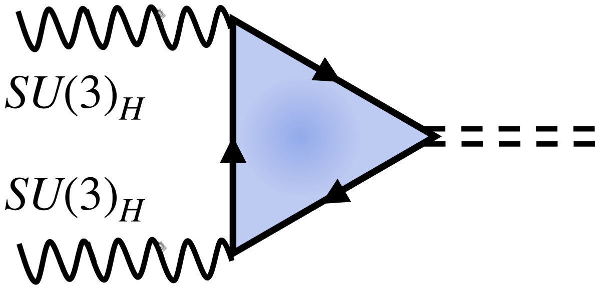
	$SU(3)_c$	$SU(3)_H$
$\mathbf{Q}$	3	3
$\bar{\mathbf{u}}$	$\bar{3}$	$\bar{3}$
$\bar{\mathbf{d}}$	$\bar{3}$	$\bar{3}$

Non-invertible symmetry

When the global structure is non-trivial, there is an interplay between the color and flavor anomalies that gives a non-invertible symmetry!



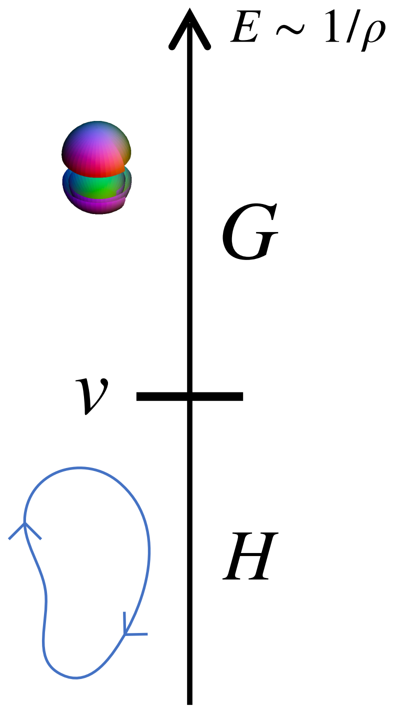
Certain 'fractional instantons' appear as a mixture of the two, but not on $\mathbb{R}^4$!



	Q_i	$\overline{u}_i$	$\overline{d}_i$
$\mathbb{Z}_3^{\tilde{B}+d}$	$+1$	-1	$+1$

A spurion analysis reveals non-invertible symmetries can protect $y_d \to 0$

Model-building logic



A classical global symmetry $X = \mathbb{Z}_3^{\tilde{B}+d}$ protects the operators $\mathcal{O}_{ij} = HQ_i\bar{d}_j$ and has an $H = \left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ anomaly

But while $\int_{\mathcal{M}} H\tilde{H} \in \mathbb{Z}/3$ generally, $\int_{\mathbb{R}^4} H\tilde{H} \in \mathbb{Z}$

X is a non-invertible symmetry! In a theory $G \supset H$ with quark color-flavor monopoles, $\mathcal{O}_{ij}$ could be classically absent and generated only by G-instantons

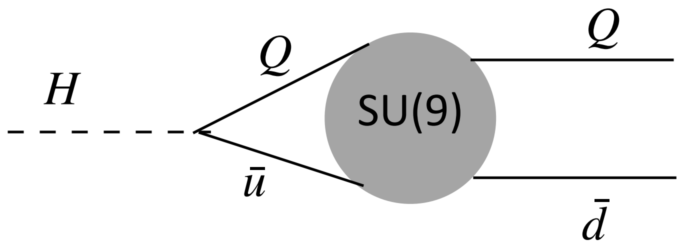
Color-flavor unification!

This all points to a beautiful $SU(9)$ unified theory in which the colors and flavors of the quarks are placed together into the fundamental

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

$$\mathscr{L}_0 = y_t \tilde{H} \mathbf{Q} \bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

Again start with good $U(1)_{\text{PQ}}$ and no strong CP violation, then

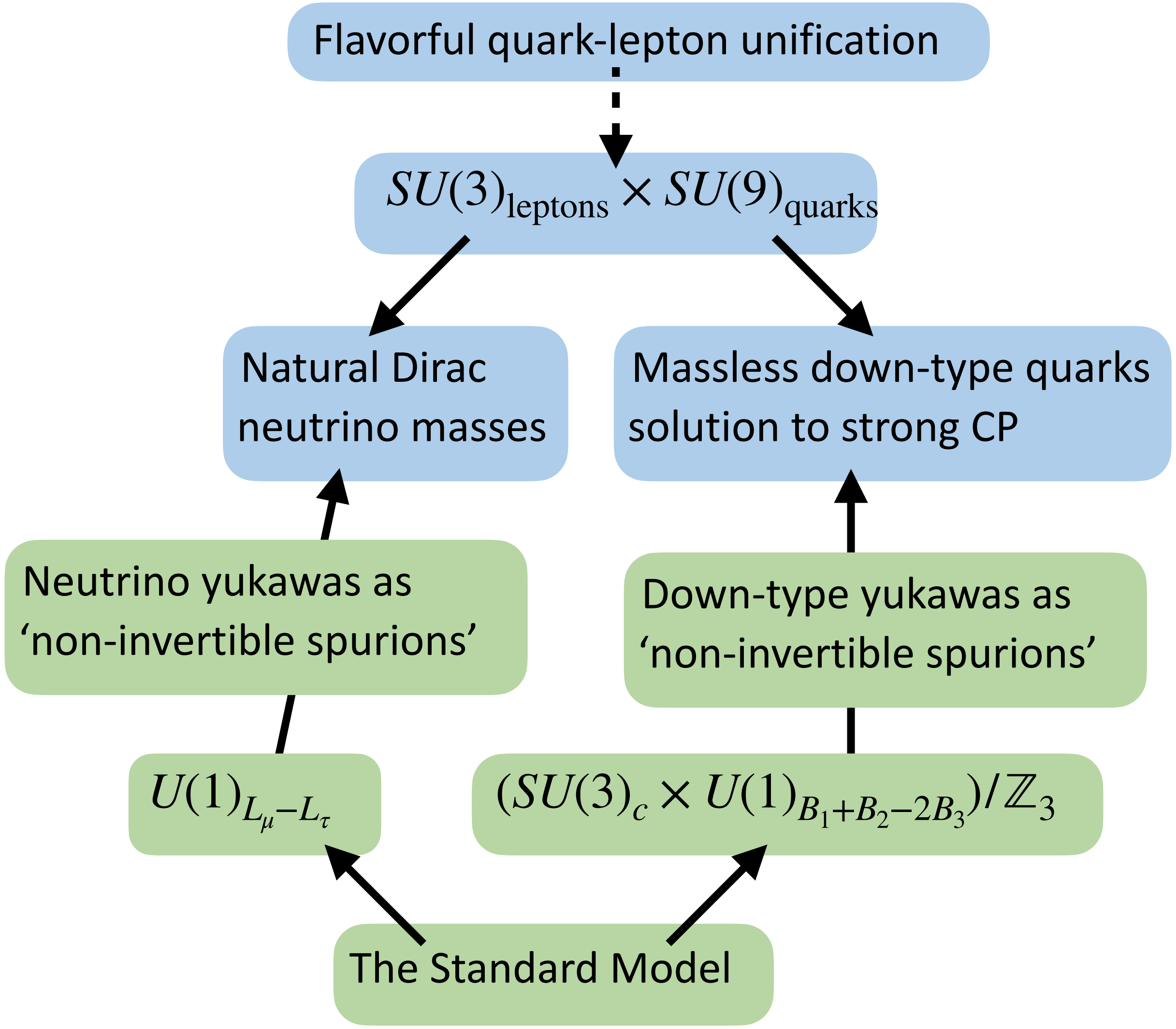


$$\mathscr{L}(\Lambda) \sim y_t H \mathbf{Q} \bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda)}} H \mathbf{Q} \bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2} F\tilde{F}$$

Non-invertible symmetry model building

- Top-down: Theories of quantum flavodynamics have previously-unnoticed nonperturbative effects with super-cool pheno!

- Bottom-up: We uncovered these using powerful new ideas from generalized global symmetries.



Backup slides

Rants and other things I didn’t have time for

Wrong conclusion

- Incorrect takeaway: “They used these fancy new symmetry ideas but in the end the UV model could be explained in terms of instantons. We’ve known about that stuff since the 80s. So who cares about generalized symmetries?”

- Correct takeaway: “These intriguing instanton effects have been sitting this close to the SM for decades and nobody saw it?! What can generalized symmetries tell me about my favorite BSM model??

Massless quark wins on quality

Both axion and massless quark solutions rely on good quality Peccei-Quinn symmetries, but only the former has a quality 'problem' because its required quality is ridiculously unnatural

Worse issue for the axion because

- With PQ-charged scalar ϕ can have all sorts of PQ-violating ops e.g. $\mathscr{L} \supset c_n M_{\rm Pl}^{4-n} \phi^n$
- We have strong astrophysical bounds on $\langle \phi \rangle = f_a \gtrsim 10^8$ GeV
- The *potential* $V_{\rm grav} \sim f_a^4 \left(f_a / M_{\rm pl} \right)^{n-4}$ cannot overpower $V_{\rm inst} \sim \Lambda_{\rm QCD}^4$

Whereas we can sustain some extra additive contribution to M as long as its magnitude is small

$\mathscr{L} \supset c_\Sigma \tilde{H} Q \Sigma \bar{d} / M_{\rm Pl}$ can have some random phase and $O(1)$ coupling as long as $\langle \Sigma \rangle / M_{\rm pl} \lesssim \bar{\theta}$. Quark flavor physics is not too far away!

Quark Weak CP and Strong CP Violation

The 'strong CP angle' $\bar{\theta} = \arg e^{-i\theta} \det\left(y_u y_d\right)$ is constrained to $\bar{\theta} \lesssim 10^{-10}$!

Even worse, we also have the 'weak CP angle' $\tilde{J} = \mathrm{Im}\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right)$

oft parameterized by m_i, θ_{ij}, and the phase $\delta_{\mathrm{CKM}} \sim 1.14$

A small value of $\bar{\theta}$ is not technically natural ⇒ the strong CP problem.

Upon RG evolution, $\delta\bar{\theta} \propto c\delta_{\mathrm{CKM}}$

Ellis & Gaillard '79

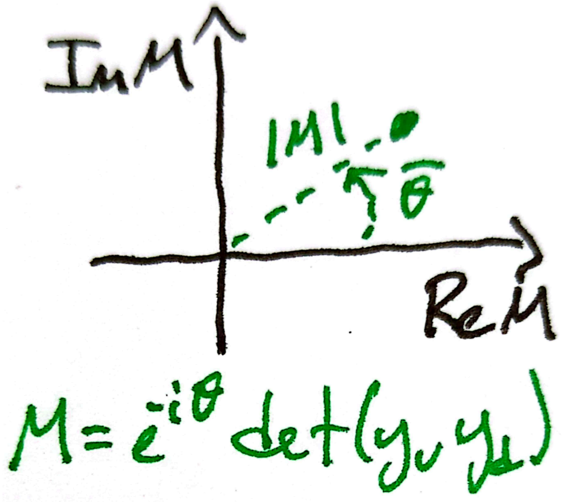
Peccei-Quinn for Strong CP

Now consider a Peccei-Quinn symmetry protecting the up quark mass

$$U(1)_{\mathrm{PQ}}: \quad \bar{u} \to \bar{u}e^{i\alpha} \quad \Rightarrow \quad \tilde{H}Q\bar{u} \text{ charged so } y_u = 0$$

If the PQ symmetry is good, $y_u \to 0$, and so $\det y_u \to 0$ and there's no strong CP violation

Easier to parameterize in 'Cartesian coordinates' for complex parameter $M \in \mathbb{C}$



Def $M = e^{-i\theta} \det\left(y_u y_d\right)$, so $\bar{\theta} = \arg M$

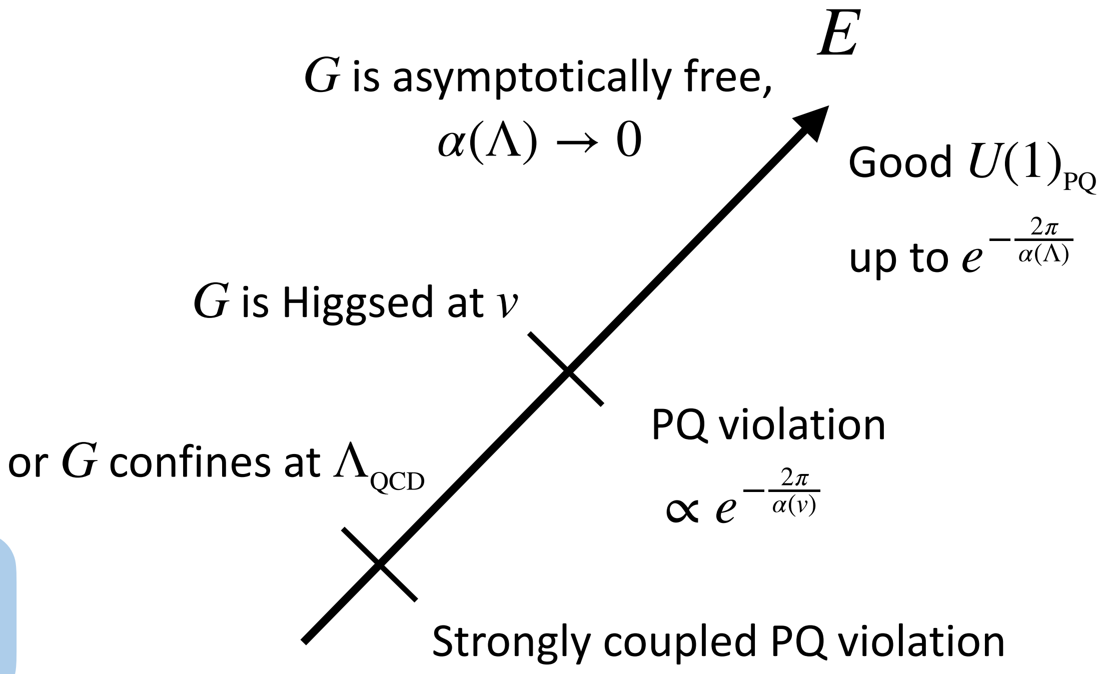
Transforms as CP : $\mathrm{Im}(M) \to -\mathrm{Im}(M)$

Peccei-Quinn Violation

Massless up quark?! Not in the IR.
A PQ symmetry which begins good is violated by instantons at low energies

UV $y_u = 0$ is then violated by QCD instantons to generate mass, automatically $M \in \mathbb{R}_+$.

Georgi-McArthur '81
Kaplan-Manohar '86
Choi, Kim, Sze '88

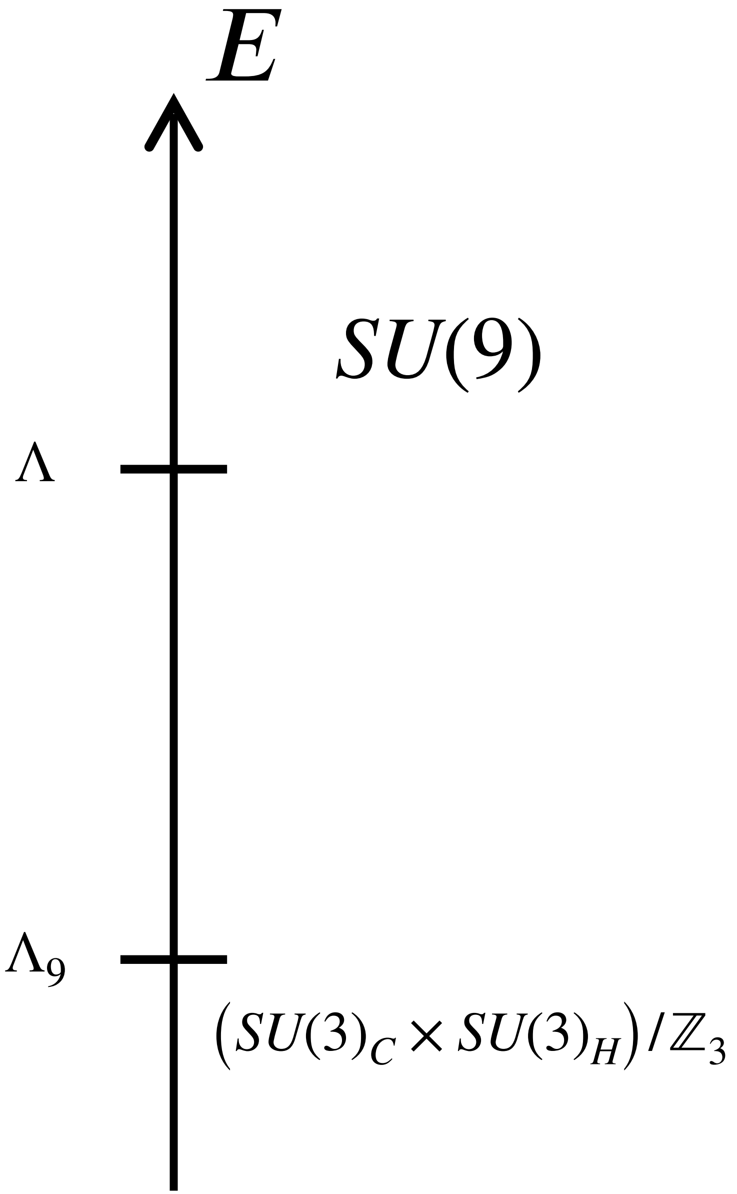


Heroic efforts by lattice physicists tell us the SM does not bear out the massless up quark solution

Flavour Lattice Averaging Group 2019

Could there be any UV model where instantons revive this solution?

Strong CP in more detail



We begin in the far UV with a good $U(1)_{\mathrm{PQ}}$

$$\mathscr{L}_0 = y_t \tilde{H}\mathbf{Q}\bar{\mathbf{u}} + \text{h.c.} + \frac{i\theta_9}{32\pi^2}F\tilde{F}$$

And so of course $M = e^{-i\theta}\det\left(y_u y_d\right) = 0$

We flow down in energies and begin to generate

$$\mathscr{L}(\Lambda) \sim y_t H\mathbf{Q}\bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{\alpha_9(\Lambda)}} H\mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i\theta_9}{32\pi^2}F\tilde{F}$$

With exactly the right phase to ensure

$$\bar{\theta} = \arg e^{-i\theta_9}\det y_u y_d = -\theta_9 + \arg |y_t|^2 e^{i\theta_9} = 0$$

Further at the matching scale

$$\mathscr{L}(\Lambda_9) \sim y_t H\mathbf{Q}\bar{\mathbf{u}} + y_t^{\star} e^{i\theta_9} e^{-\frac{2\pi}{3\alpha_s(\Lambda_9)}} H\mathbf{Q}\bar{\mathbf{d}} + \text{ h.c.} + \frac{i3\theta_9}{32\pi^2}\left(G\tilde{G} + K\tilde{K}\right)$$

And the matching accounts for the yukawas now being 3x3 matrices

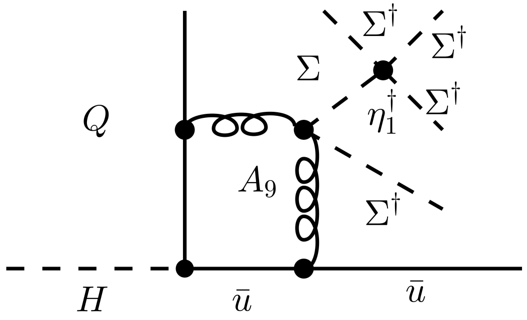
$$\bar{\theta} = -3\theta_9 + \arg\det |y_t|^2 e^{i\theta_9} = 0$$

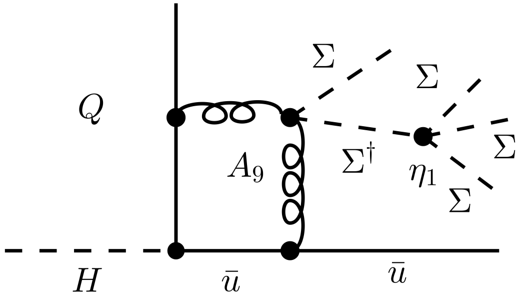
Generating CKM

Idea: Communicating flavor-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate *hermitian* yukawas

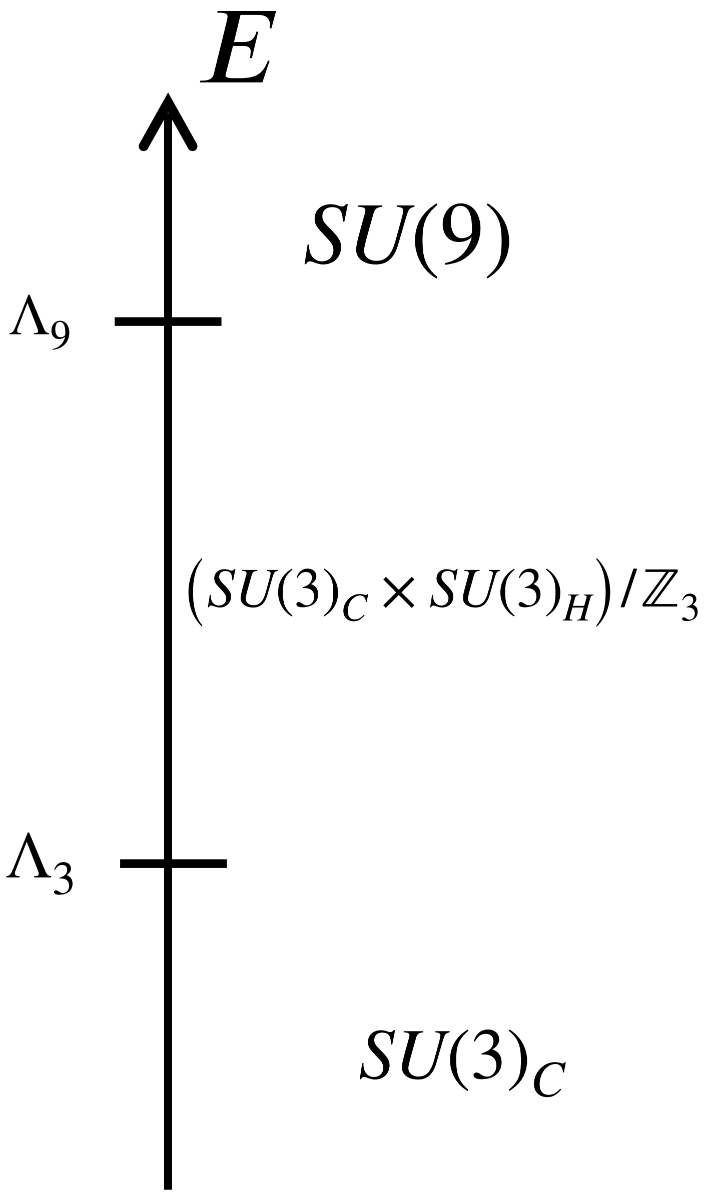
$\bar{\theta} = \arg\det e^{-i\theta} y_u y_d$ automatically zero

$$V_{\mathbb{Z}_4}(\Sigma) = \eta_1 \text{Tr}\left(\Sigma^4\right) + \eta_2 \text{Tr}\left(\Sigma^2\right)^2 + \text{ h.c.}$$





$$(y_u)^a{}_b \sim y_t \left(\mathbb{1}^a{}_b + \frac{\alpha_9}{(4\pi)} \frac{\eta_1^\dagger (\Sigma^{\dagger 4})^a{}_b + \eta_2^\dagger \text{Tr}(\Sigma^{\dagger 2})(\Sigma^{\dagger 2})^a{}_b}{\Lambda_9^4} + \frac{\alpha_9}{(4\pi)} \frac{\eta_1 (\Sigma^4)^a{}_b + \eta_2 \text{Tr}(\Sigma^2)(\Sigma^2)^a{}_b}{\Lambda_9^4} + \dots \right)$$



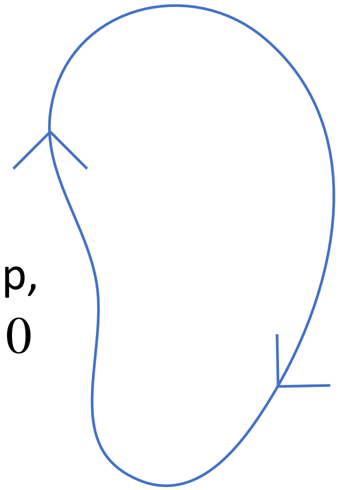
Violation of non-invertible symmetries

The IR generalized symmetry picture is loops of dynamical monopoles which break magnetic one-form symmetry so violate non-invertible symmetry

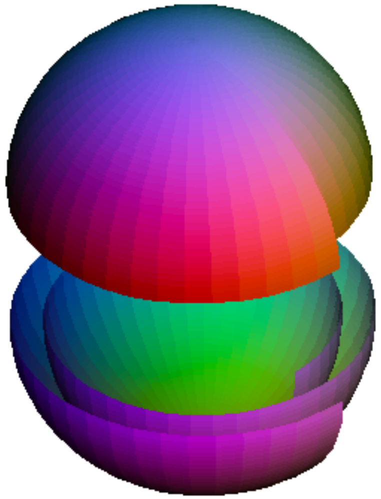
The connection between monopole loops and small instantons is not yet well explored

Fan, Fraser, Reece, Stout 2105.09950

Dyon loop, $q_e, q_m \neq 0$



$$\int E \cdot B \neq 0 \neq \int F\tilde{F}$$



$A^{(n-\mathrm{inst})}$

Some deep relation to Callan-Rubakov