Tortured Phenomenology Department

or, why we need quantum computers

Hank Lamm

May 4, 2024

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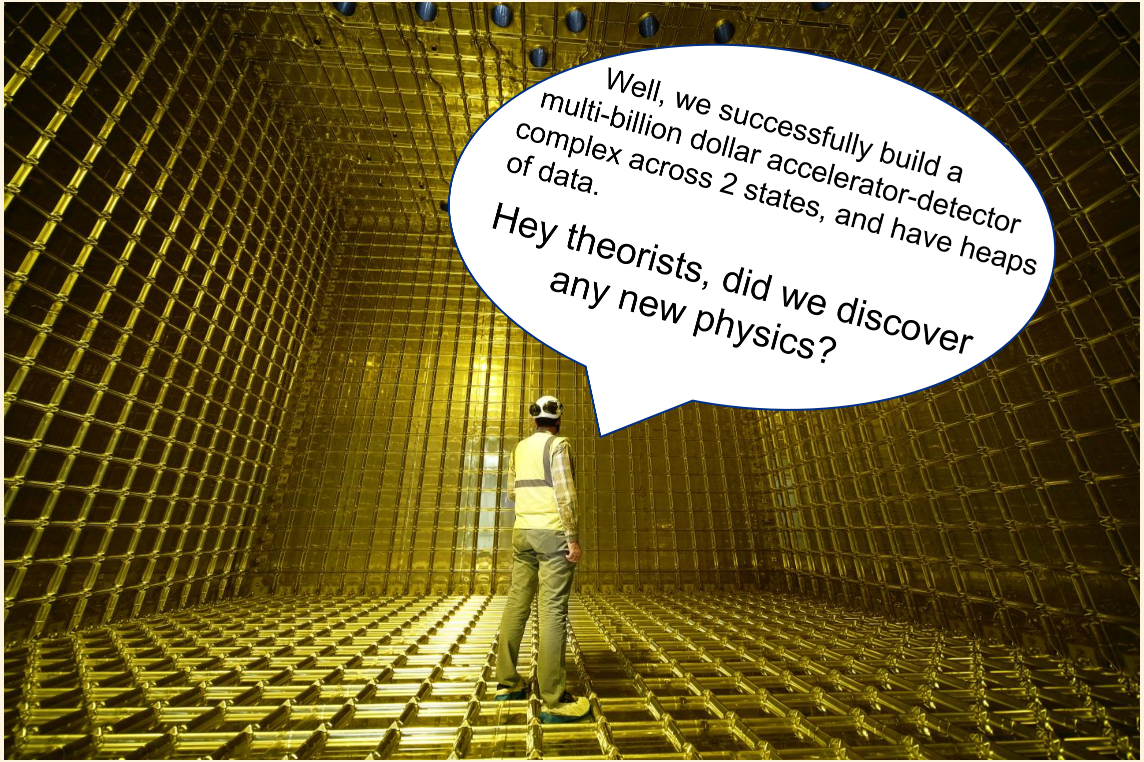
Stated succinctly....



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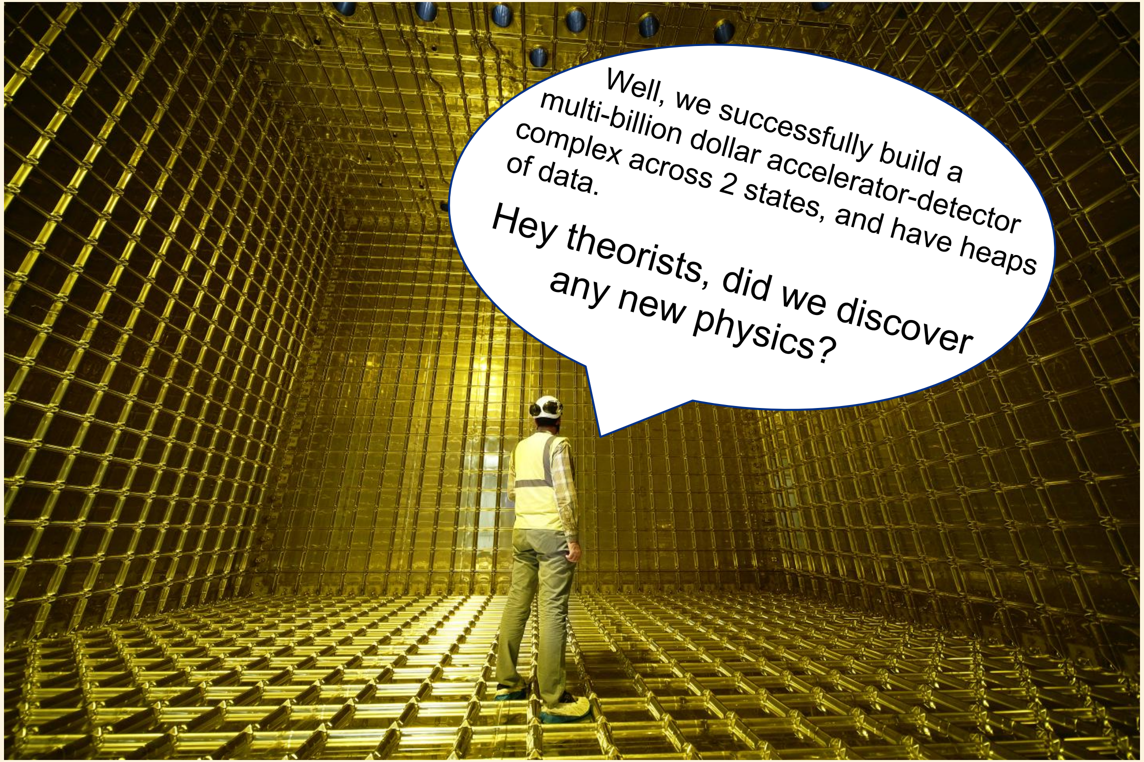

Choose your fighter: the workflow of a theorist

Your brain and pencil and paper

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Throw it in Mathematica

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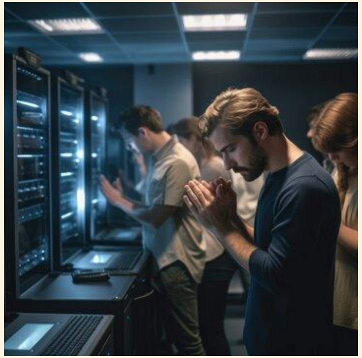
Pray to a supercomputer



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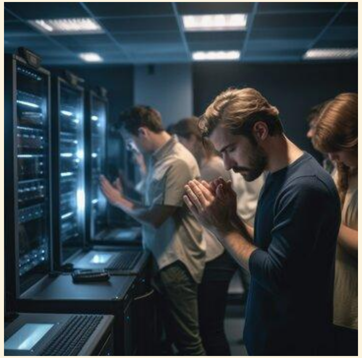
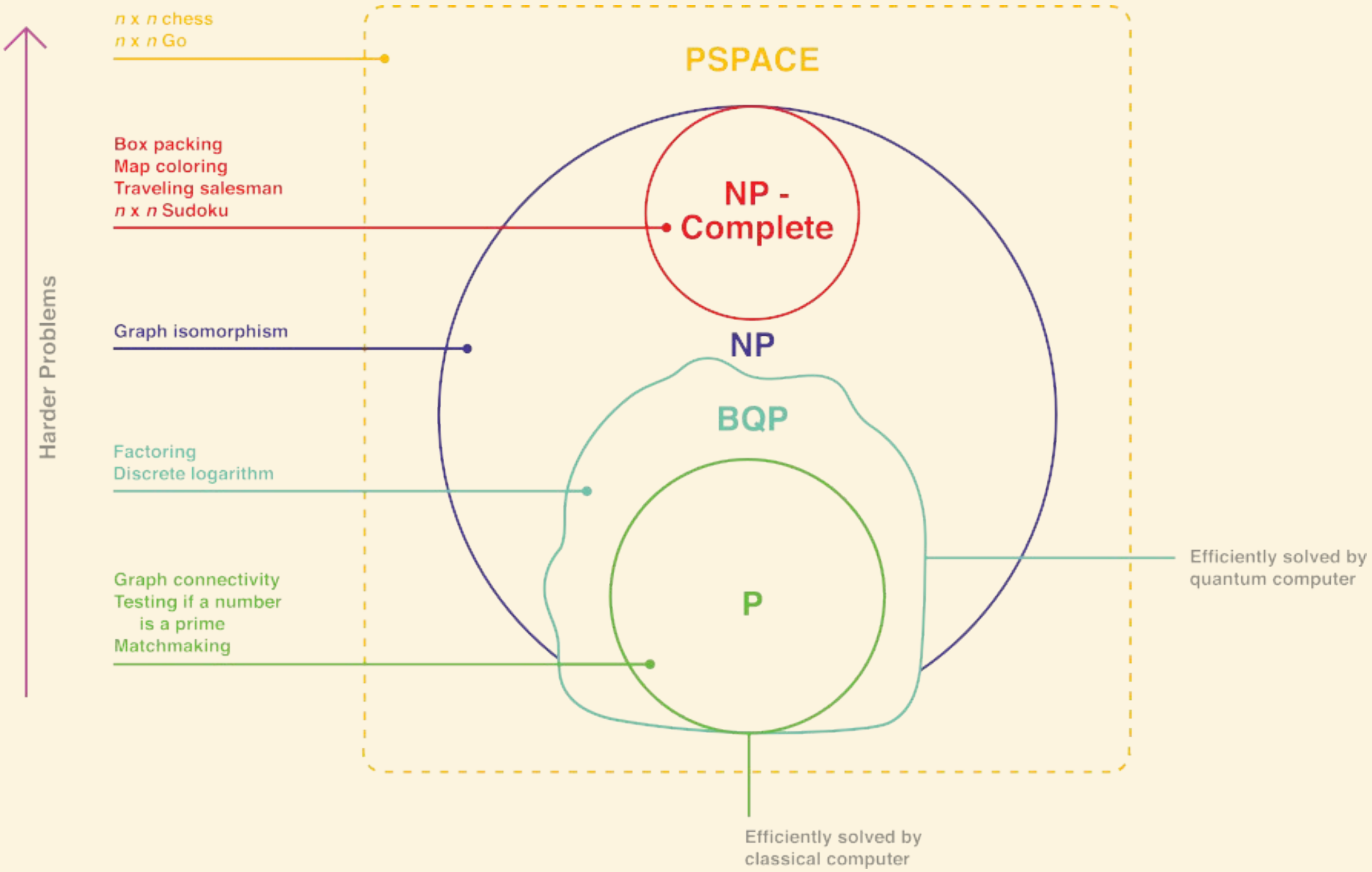


But what if your prayers go unanswered?

Computational complexity and Utility



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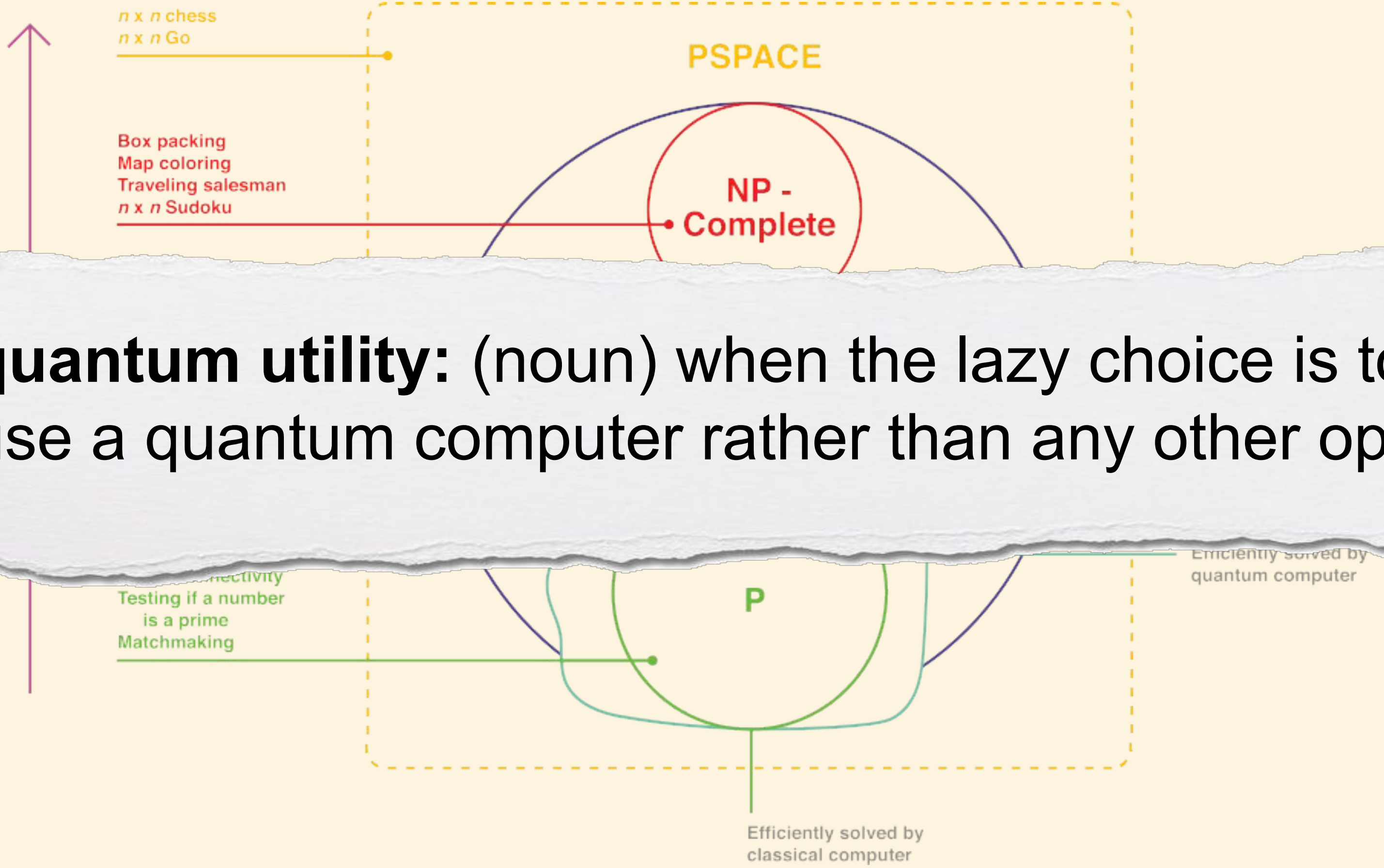
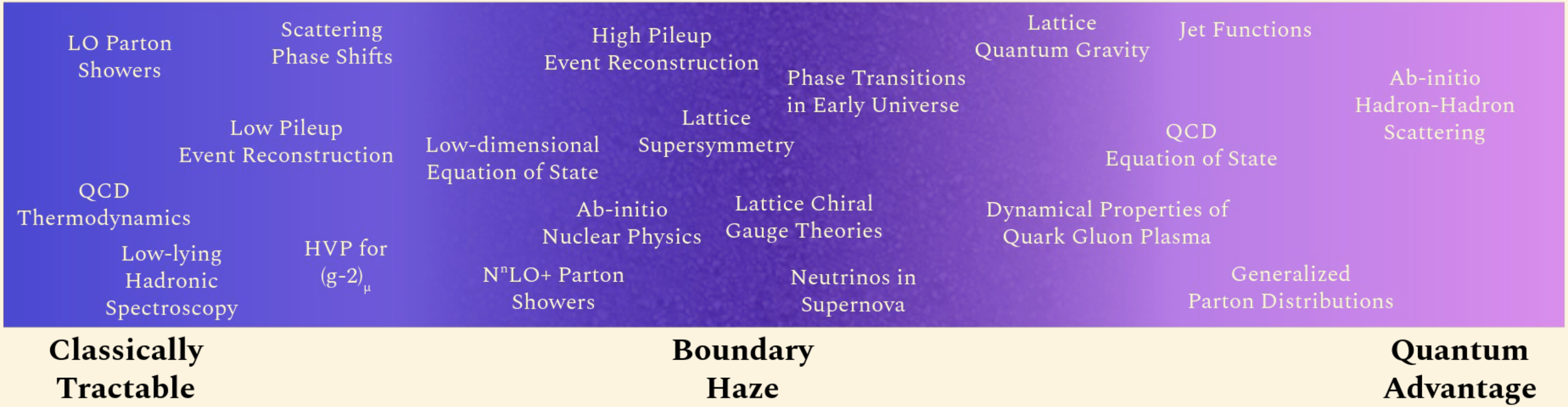


quantum utility: (noun) when the lazy choice is to use a quantum computer rather than any other option

Quantum Computing for Particle Physics, it's a need

- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise **inaccessible**
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE



While broad, these topics often are formulated as **lattice field theories**

Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field

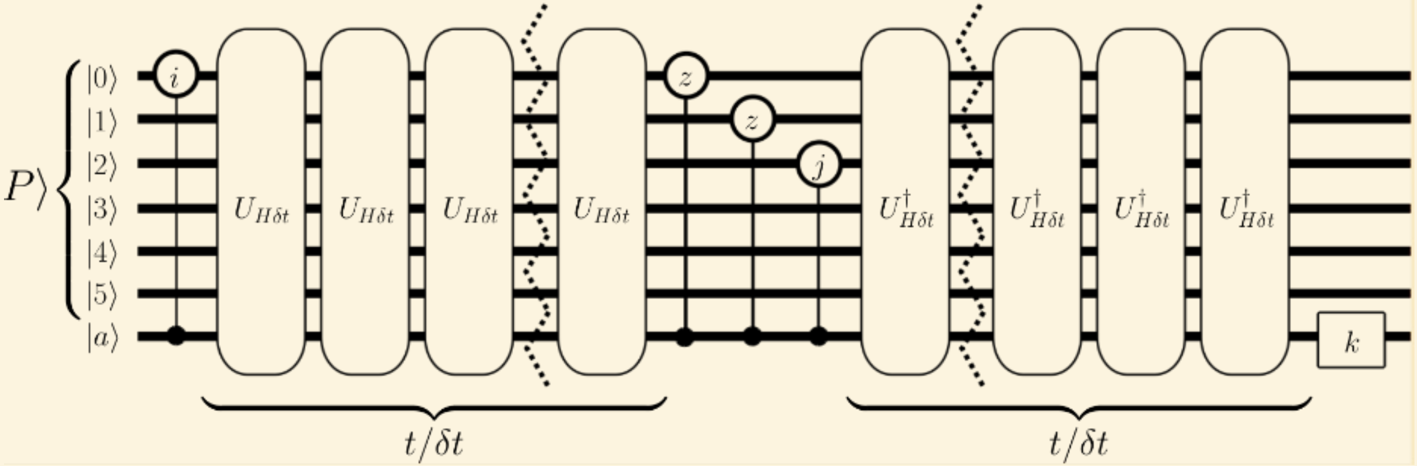
N-point correlators and Quantum Utility

- **Nearly all** HEP quantum utility is time-evolution + *n* Hermitian insertions

$$\langle \prod_i \mathcal{O}_i(t_i) \rangle = \int_{\psi(0)}^{\psi(T)} \mathcal{D}\psi \prod_i \mathcal{O}_i(t_i) e^{-iS} = \langle \psi(T) | \prod_i \mathcal{O}_i(t_i) | \psi(0) \rangle$$

- Example: Hadronic Tensor

$$\langle P | \chi^\dagger(tn^\mu)\chi(0) | P \rangle = \sum_{i,j,k=\{x,y\}} \frac{c_{ij}}{4} \langle P, a | U_{i,j,k} | P, a \rangle$$



Parton Physics on Quantum Computers
Lamm, Lawrence, Yamauchi - *Phys.Rev.Res* 2 (2020) 1, 013272
Formulation HEP problem with potential quantum utility

Where did all the matter come from (aka baryogenesis)?

As a closer target, consider the viscosity of QCD

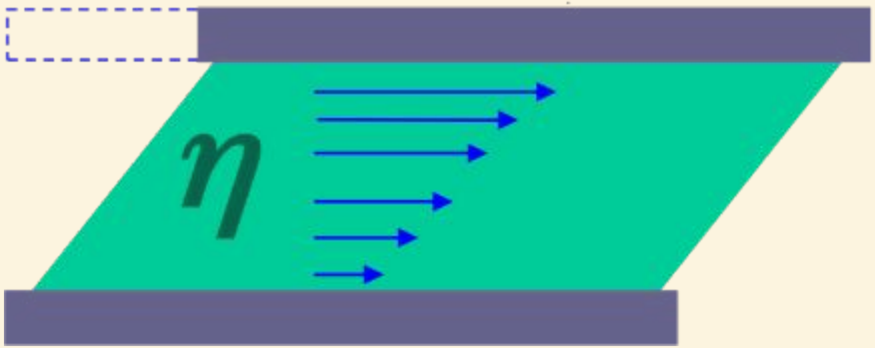
- $\eta = \frac{V}{T} \int_0^\infty \langle T_{12}(t) T_{12}(0) \rangle$

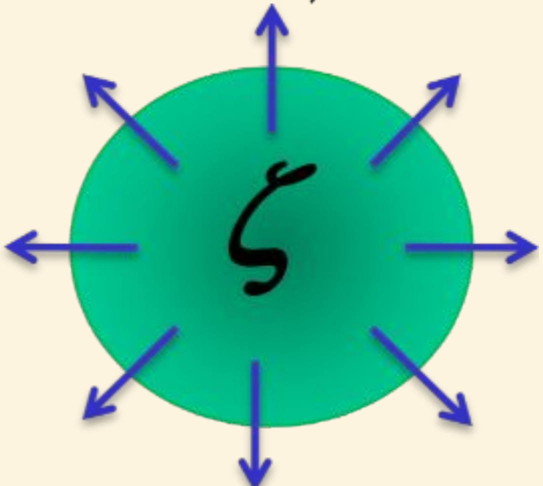
Quantum algorithms for transport coefficients in gauge theories
NuQS Collaboration - *Phys.Rev.D 104 (2021) 9, 094514*
Formulates lattice operators and propose correlators

- I believe its a **"near-term"** goal and allows for focus…
- …while introducing **all** the necessary pieces

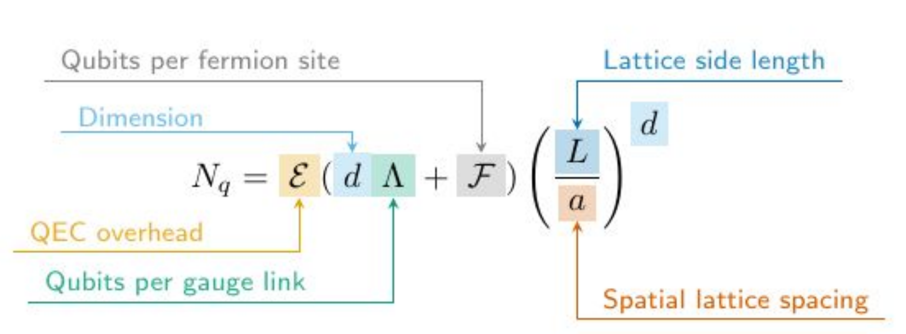
Viscosity of pure-glue QCD from the lattice
Altenkort *et al.* - *2211.08230 [hep-lat]*
State of the art lattice results, but massive uncertainties persist

$$\eta/s = 0.15 - 0.48, T = 1.5T_c$$
$$\zeta/s = 0.017 - 0.059, T = 1.5T_c$$





It's one calculation, Hank. What could it cost?



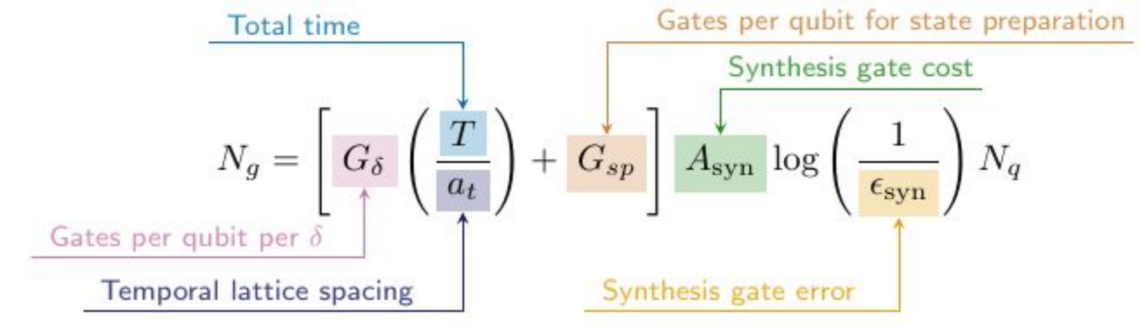
Viscosity of SU(3) on 10^3 Lattice:

2021: O(10^6) lq and **O(10^{55})** T-gates

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer
Kan and Nam - 2107.12769 [quant-ph]
Rough, conservative, model- and algorithm-dependent estimates for viscosity and heavy-ion collisions

2024: O(10^4) lq & **O(10^{12})** T-gates

Primitive Quantum Gates for an SU(3) Discrete Subgroup: S(108)
Gustafson, Ji, Lamm, Murairi, Zhu - *pending on arxiv*



physical qubit: physical system susceptible to noise

logical qubit: collection of physical qubits that encode single qubit of information and allow for error correction

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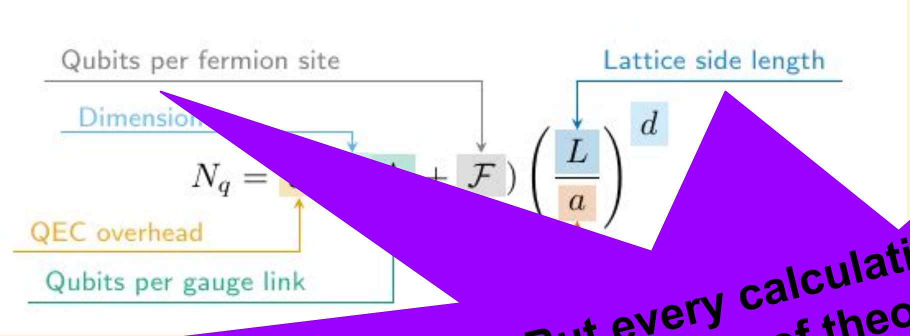


But every calculation has an immense number of theoretical errors that also have to be dealt with, e.g. lattice spacing, finite volume, truncation, trotterization….and this is where theorist are crucial in accelerating timeline

…cosity of SU(3) on 10^3 Lattice:

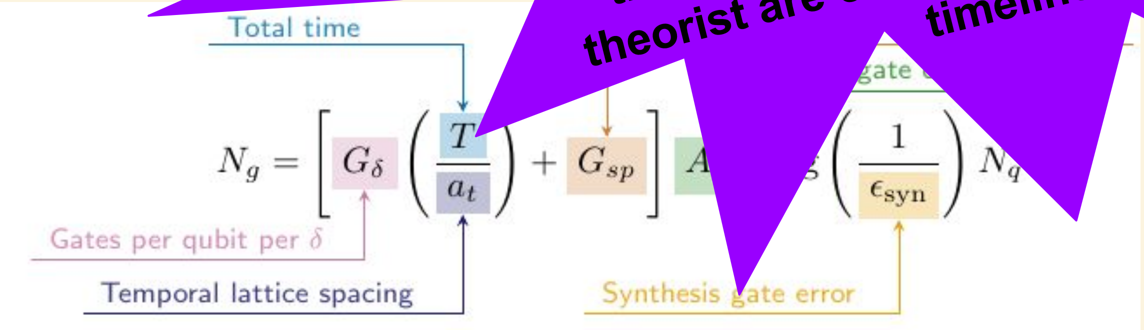
…nd **O(10^{55})** T-gates

…Electrodynamics on a Universal Quantum Computer

…for viscosity and heavy-ion collisions

…(10^{12}) T-gates

…Subgroup: S(108)



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We need phenomenological models

What is **minimal model** for learning new phenomenology?

- Strongly-coupled theories
- Non-equilibrium behavior

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What can we learn from **lower** dimensions?

- Are there models in the same universality classes of interests

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What can we learn from fewer degrees of freedom, **EFTs**?

- SCET? HQET? χpt?

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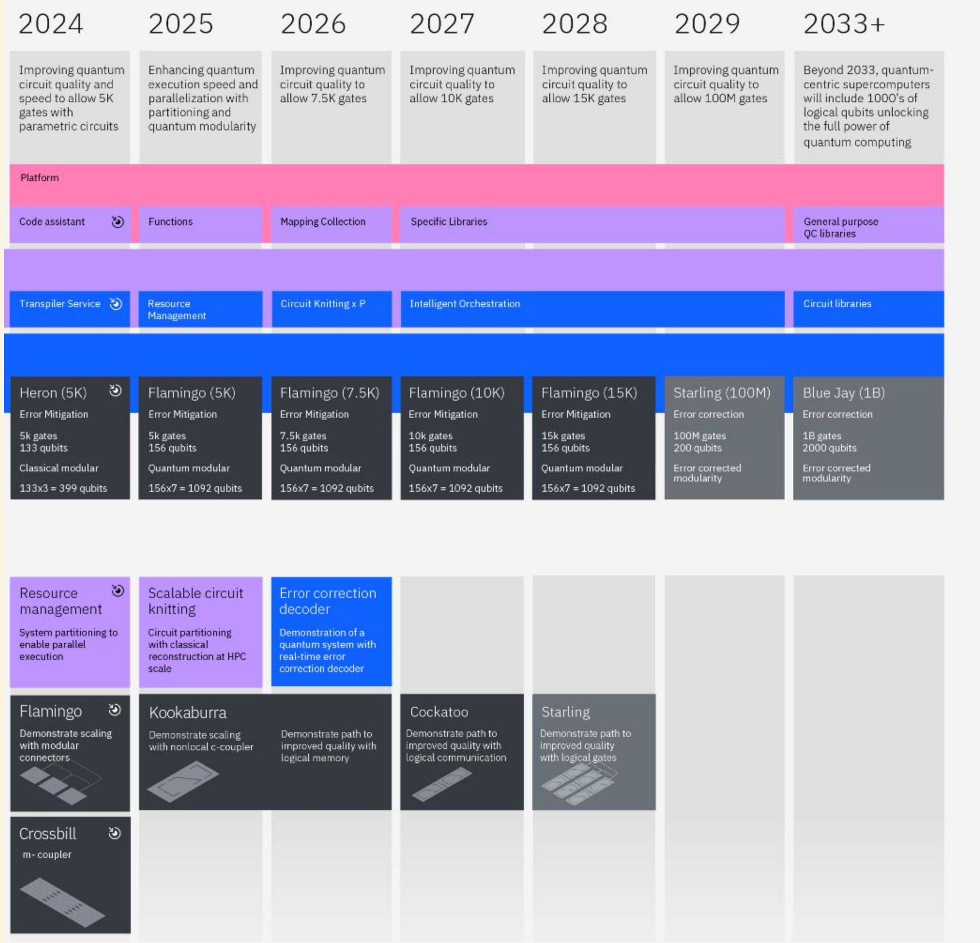
What **observables** have really large uncertainties?

- Consider things with large, model-dependent systematics

Where did we come from, where will we go?

Today, in 2024: Hundreds of physical qubits, thousands of noisy gates



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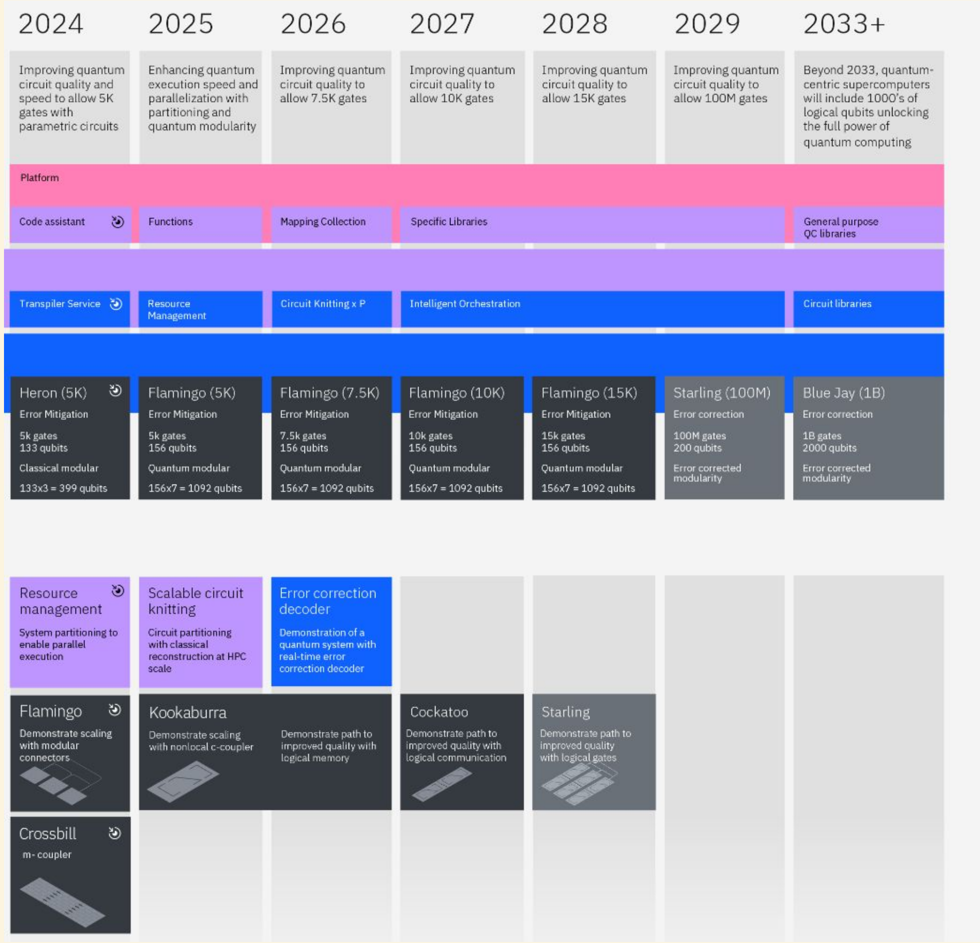
…squinting at the roadmaps…

2027-2029: Tens of logical qubits with few logical gates



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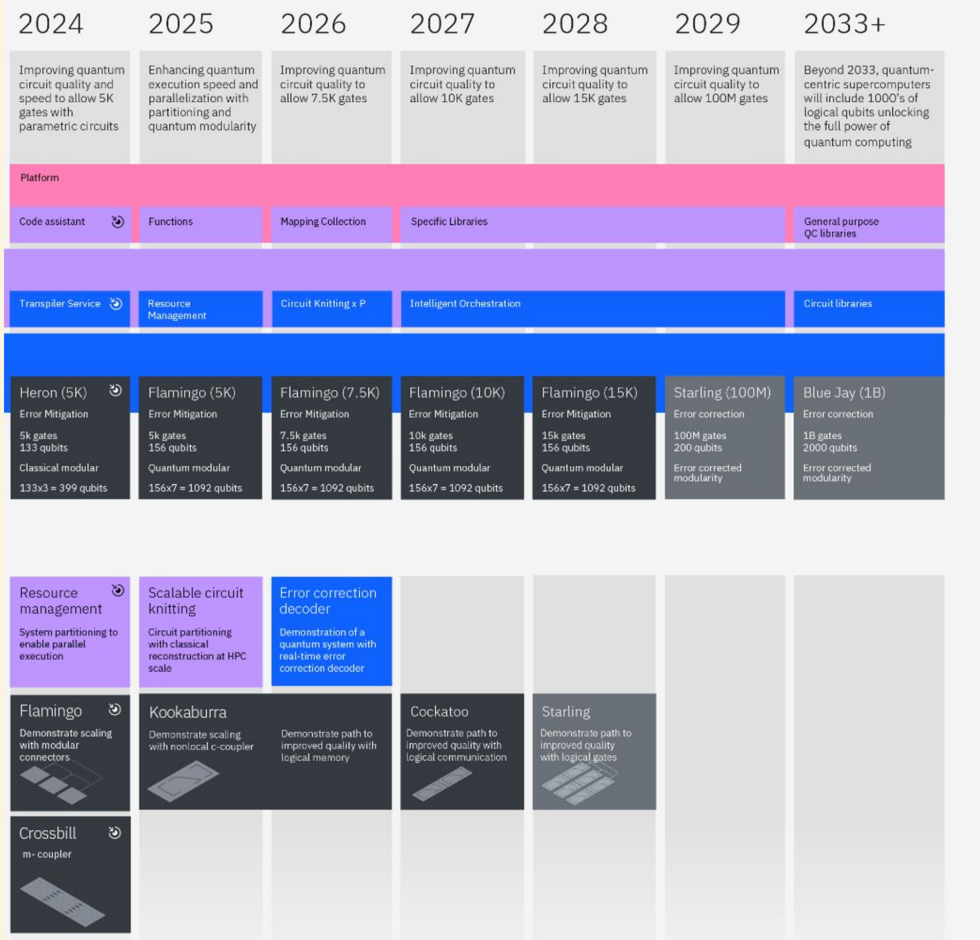
…squinting at the roadmaps…

2027-2029: Tens of logical qubits with few logical gates

…balancing on broken branches…

2030-2040: Fault-tolerant computers with ~1000 qubits



All I need is…(the industrial workforce of a small country)

$$\langle \psi_0 | e^{-iHt} \mathcal{O} e^{iHt} | \psi_0 \rangle$$

- **Formulate the theory**

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- **Prepare a state**

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- Prepare a state
- Time evolve the state

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$$\langle\psi_0|e^{-iHt}\mathcal{O}e^{iHt}|\psi_0\rangle$$

- Formulate the theory
- Prepare a state
- Time evolve the state
- Perform a measurement

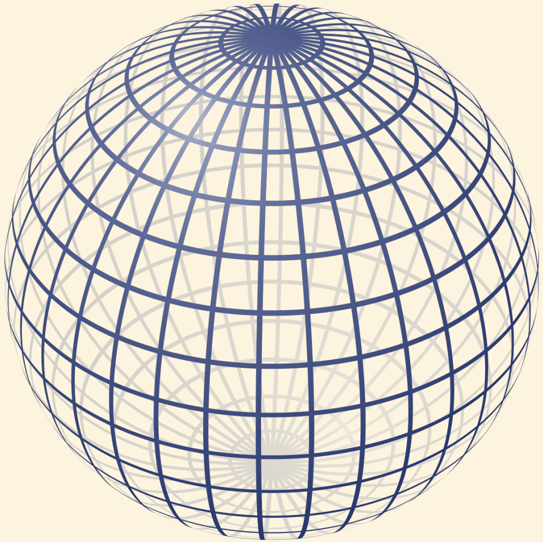
What is a gluon?

or, digitize infinite Hilbert spaces

Mapping infinite bosonic fields to finite quantum register

- Y'all are spoiled by classical computers

$$U(1) \rightarrow e^{i\alpha} \qquad SU(2) \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad SU(3) \rightarrow \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix}$$



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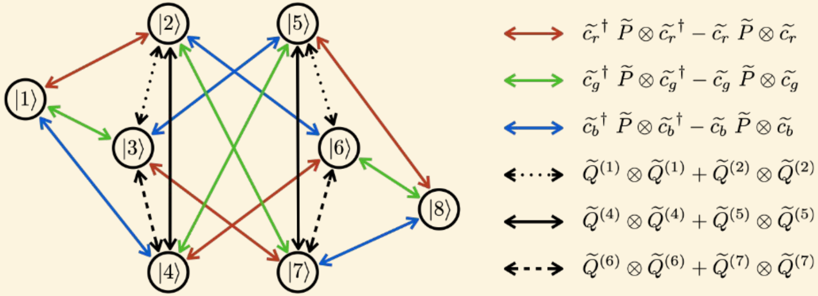
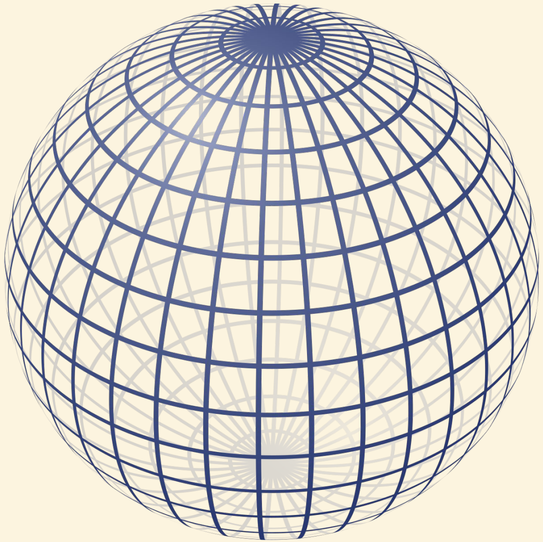
- On classical computers, these correspond to expensive **floating-point registers**
- Typically high connectivity between states e.g





Qu8its for Quantum Simulations of Lattice Quantum Chromodynamics
Illa, Robin, Savage - 2403.14537
Explore the utility of quoctit for quantum simulations of the dynamics of 1+1D LQCD

How to choose a side?

$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \text{ReTr}U_1U_2U_3^\dagger U_4^\dagger$$

E (irreducible representation) basis

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis
Ciavarella, Klco, Savage *Phys.Rev.D 103 (2021) 9, 094501*
Qubit implementation of SU(3) with irrep truncations

Mixed basis

A new basis for Hamiltonian SU(2) simulations
Bauer, D'Andrea, Freytsis, Grabowska - 2307.11829
Formulated an alternative basis that contains parts of E & B basis

B (group element) basis

Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT
Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501
Qubit and Qudit gates for approximating SU(2) with subgroups

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arbitrary precision, gauge fixing, quantum noise, error correction, gate costs, classical simulatability

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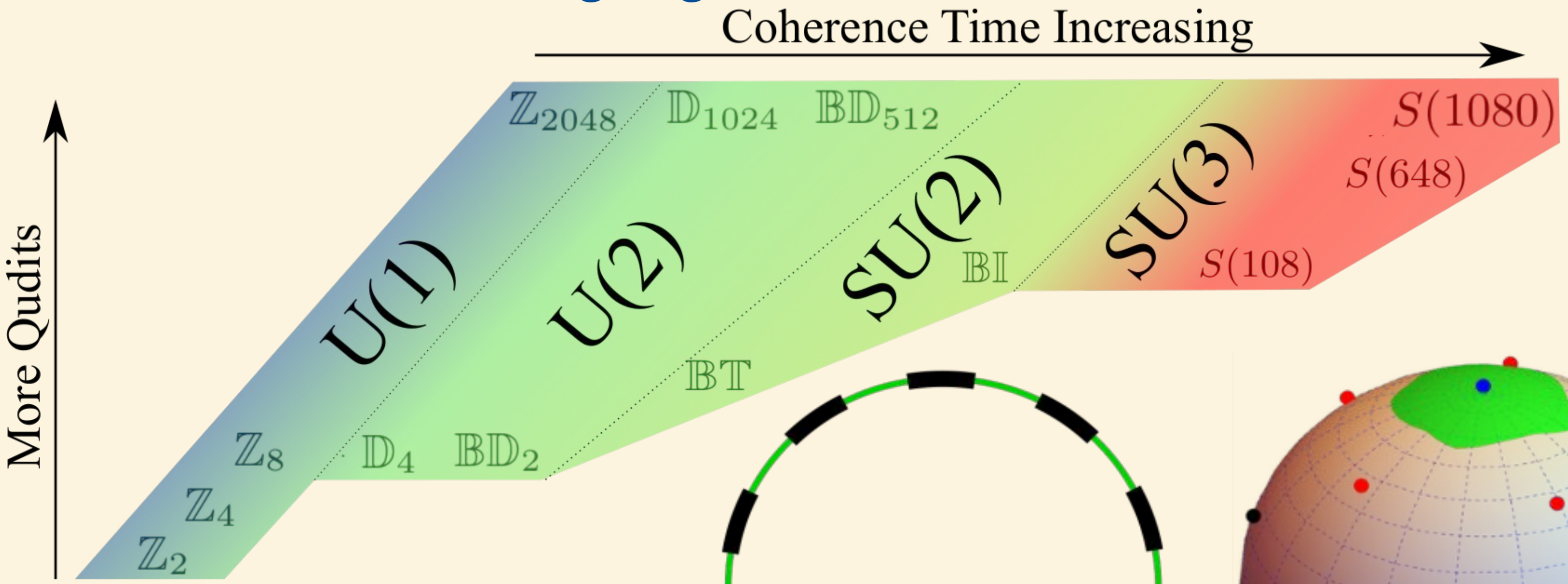
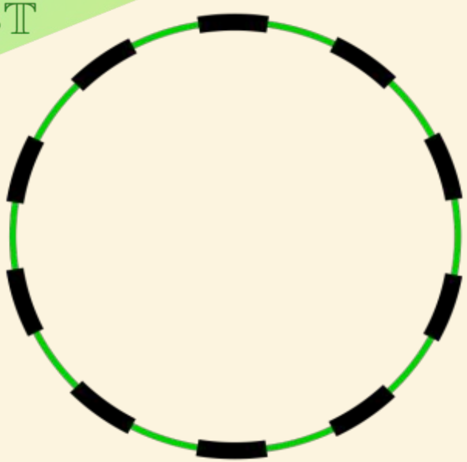
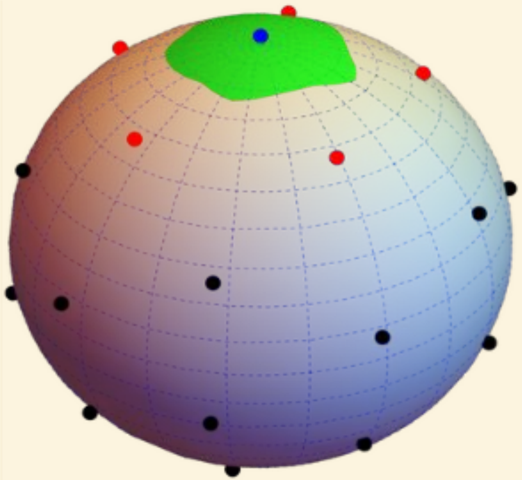
This is not a trivial decision, it breaks symmetries!

The ladder of discrete gauge theories in HEP calculations







Gluon Field Digitization for Quantum Computers
NuQS collaboration - *Phys.Rev.D 100 (2019) 11, 114501*
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

Digitising SU(2) gauge fields and the freezing transition
Hartung *et al. - Eur.Phys.J.C 82 (2022) 3, 237*
Understanding the scaling of freezing transitions with approximations

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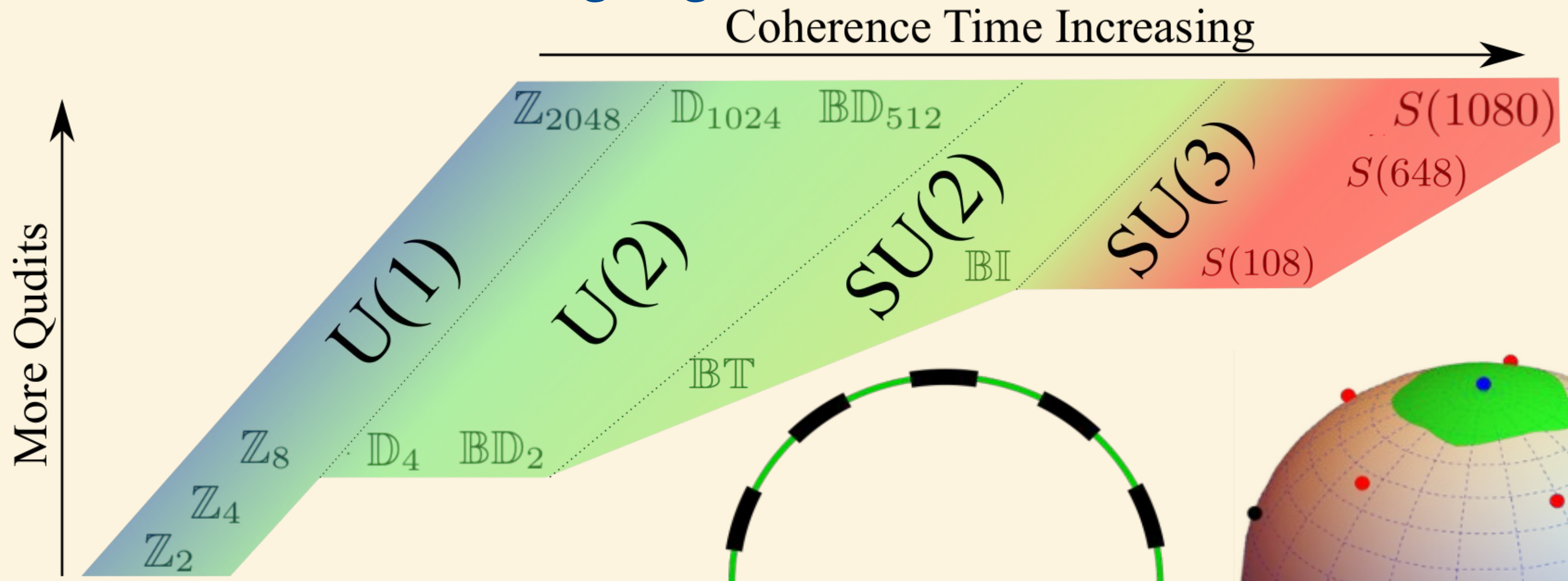

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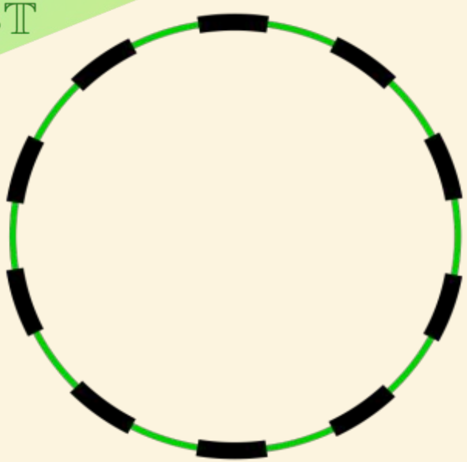
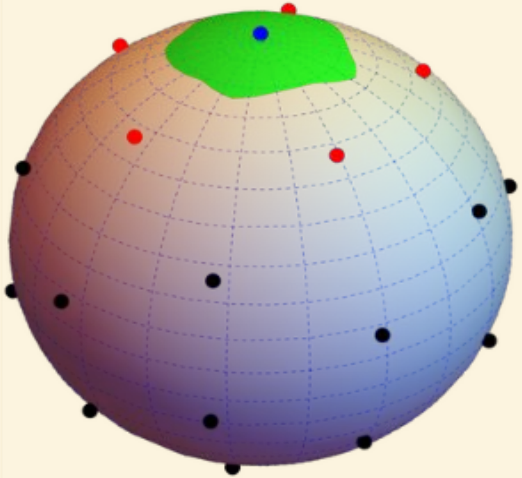


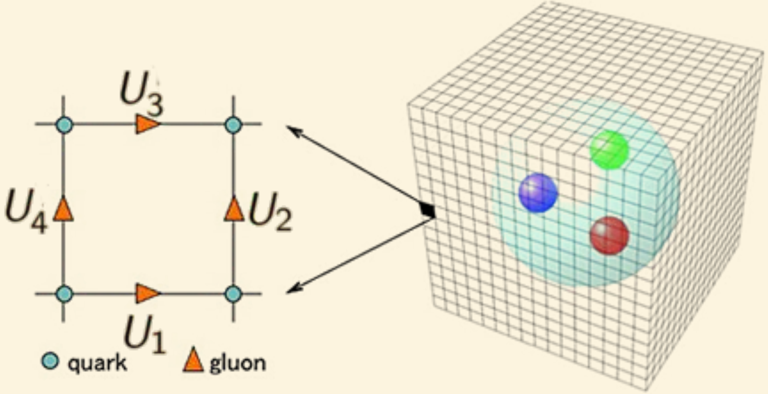


Secretly, these are continuous groups coupled to Higgs fields

How do quarks and gluons interact?

or, picking a lattice Hamiltonian

O(a²) Kogut-Susskind Hamiltonian



$$H_{KS} = \sum_n m_n \psi_n^\dagger \psi + \sum_{n,k} [\psi_n^\dagger U_{nk} \psi_{n+k} + h.c.]$$

Fermionic mass term Fermionic kinetic or *hopping* term

$$+ \sum_n E_n^2 + \sum_{n,k} \mathrm{ReTr}\, U_p$$

Gauge **E** field Gauge **B** or *plaquette* term

Hamiltonian Formulation of Wilson's Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D 11 (1975) 395-408*
Formulated O(a²) lattice Hamiltonian for LGT with staggered matter

But….shouldn't use Kogut-Susskind Hamiltonian

$$K = \frac{g^2}{2a}\mathrm{Tr}[WE_i(\mathbf{x})E_i(\mathbf{x}) + ZE_i(\mathbf{x})U_i(\mathbf{x})E_i(\mathbf{x}+a\hat{i})U_i^\dagger(\mathbf{x})]$$

$$V = \frac{2N}{ag^2}[XP_{ij}(\mathbf{x}) + \frac{Y}{2}(R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x}))]$$

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Formulated $O(a^2)$ lattice Hamiltonian for LGT with staggered matter

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Improved Hamiltonians for Quantum Simulation of Gauge Theories
Carena, Lamm, Li, Liu *PRL 129 (2022) 5*
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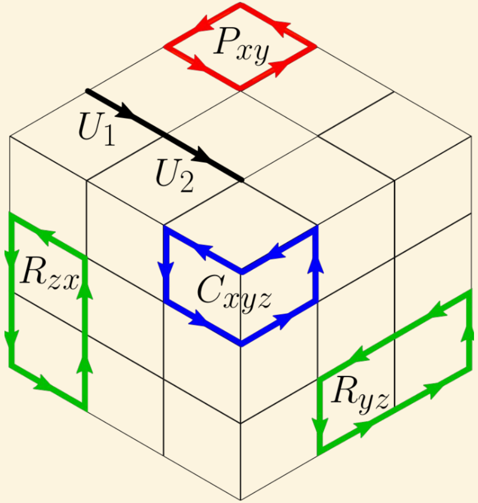

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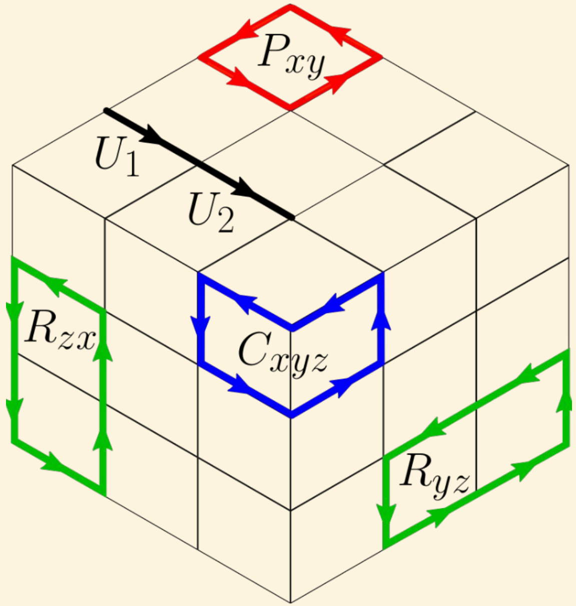

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Comparing to the commonly used H_{KS}, H_I should allow quantum simulations with **>2^d fewer qubits**… [with a gate count] comparable or less than that of H_{KS} for theories with d ≥ 2…

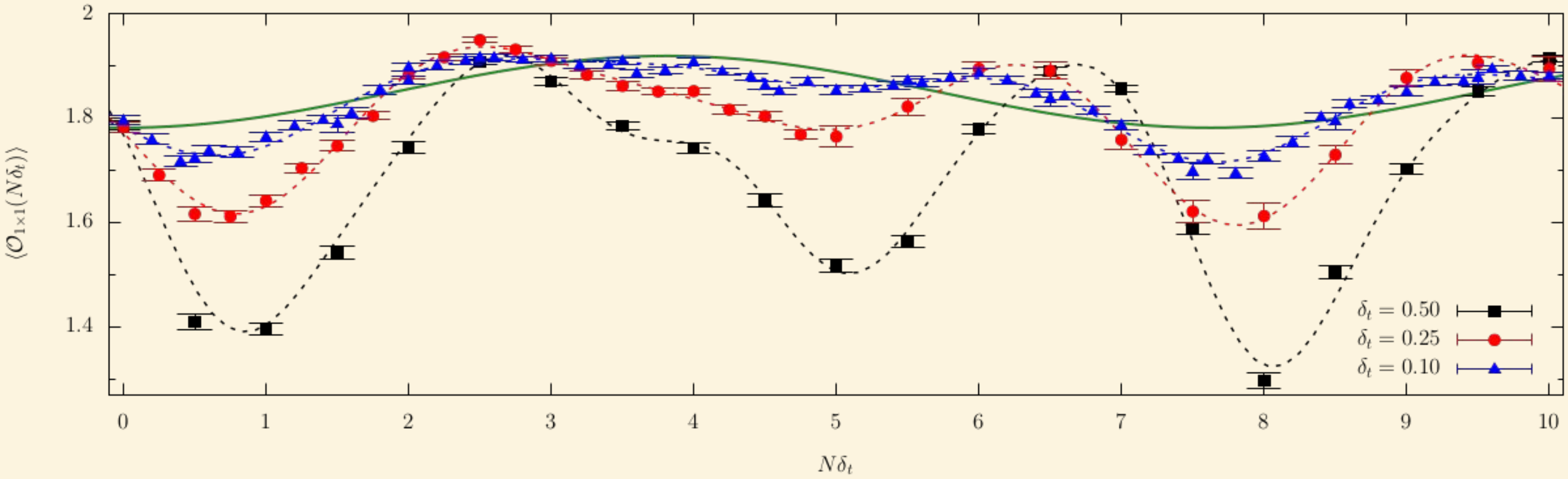
How do I evolve a quantum state?

or, how to approximate e^{iHt}

What is trotterization?

$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t \frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t \frac{H_V}{2}}\right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{-it\left(H_K + H_V + \frac{\delta t^2}{24}(2[H_K,[H_K,H_V]] - [H_V,[H_V,H_K]])\right)\right\}$$



Hey! This is an EFT!

How to estimate Trotter errors

- Loose error bounds obtained from

$$||U(t) - U_{trott}(t))|| \leq (\delta t)^n \sum_{i,j,\cdots}[[H_i, H_j], \cdots]$$

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

$$
\begin{aligned}
\|\mathcal{O}_3\| &= \left\|\left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r)]\right]\right\| &&\leq 4x^3 \quad (k > j),\\
\|\mathcal{O}_5\| &= \left\|\left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r+1)]\right]\right\| &&\leq 4x^3,\\
\|\mathcal{O}_{13}\| &= \left\|\left[H_I^{(l)}(r), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^3 \quad (k > j, l > j),\\
\|\mathcal{O}_{14}\| &= \left\|\left[H_M(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^2\mu \quad (k > j),\\
\|\mathcal{O}_{15}\| &= \left\|\left[H_I^{(l)}(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^3 \quad (k > j),\\
\|\mathcal{O}_{19}\| &= \left\|\left[H_I^{(l)}(r), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^3 \quad (l > j),\\
\|\mathcal{O}_{20}\| &= \left\|\left[H_M(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^2\mu,\\
\|\mathcal{O}_{22}\| &= \left\|\left[H_I^{(l)}(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^3,\\
\|\mathcal{O}_{24}\| &= \left\|\left[H_I^{(l)}(r+2), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| &&\leq 4x^3.
\end{aligned}
$$

- **Overly** conservative: cutoff states are largest EV
  - Empirically, we find MUCH smaller

State-dependent error bound for digital quantum simulation of driven systems
Hatomura - PRA 105, L050601 (2022)
Compares trotter errors for given initial state to norm-based estimates

- Have you heard of **decoupling theorem?**

- Can we use **classical Euclidean calculations** to compute?

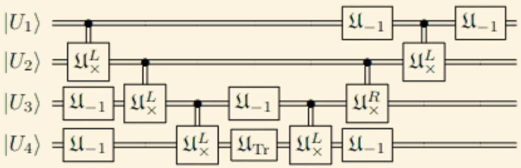
Primitive gates as subroutines

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

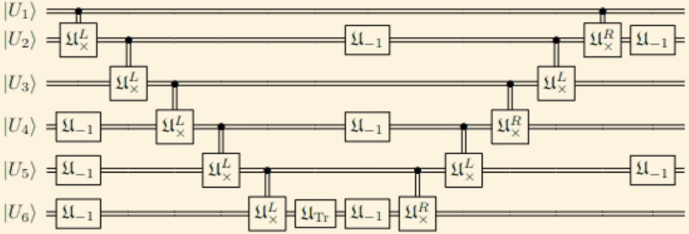
LGT require group operations on registers

– Think **native gates** for gauge theories

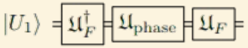
- Inversion gate: $\mathfrak{U}_{-1}\,|g\rangle = |g^{-1}\rangle$
- Multiplication gate: $\mathfrak{U}_{\times}\,|g\rangle\,|h\rangle = |g\rangle\,|gh\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)\,|g\rangle = e^{i\theta\,\mathrm{Re\,Tr}\,g}\,|g\rangle$
- Fourier Transform gate: $\mathfrak{U}_F \sum_{g\in G} f(g)\,|g\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij}\,|\rho, i, j\rangle$



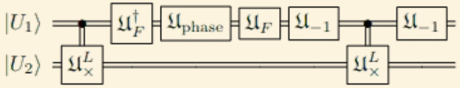
(a) $\mathcal{U}_{V_{KS}}$ assuming linear register connectivity.



(b) $\mathcal{U}_{V_{\mathrm{rect}}}$ assuming linear register connectivity.



(c) $\mathcal{U}_{K_{KS}}$.

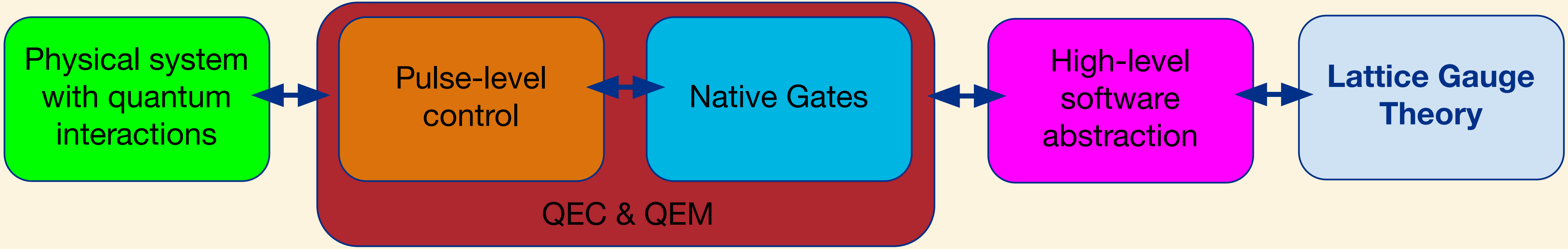


(d) $\mathcal{U}_{K_{2L}}$.

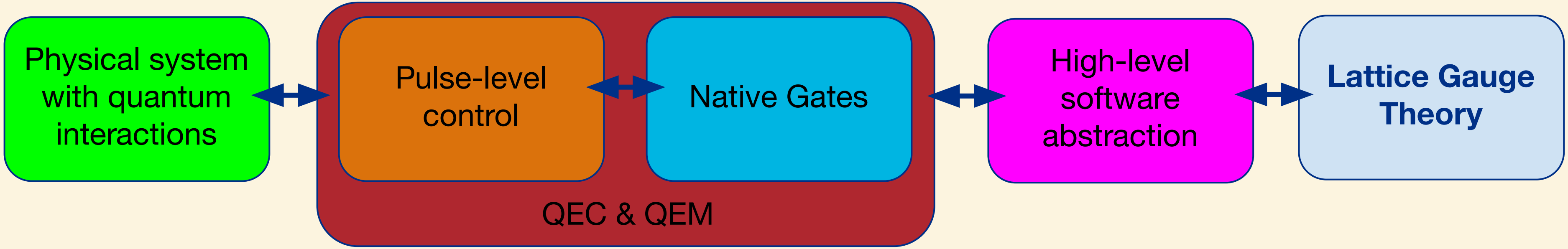
How can we annoy experimentalists?

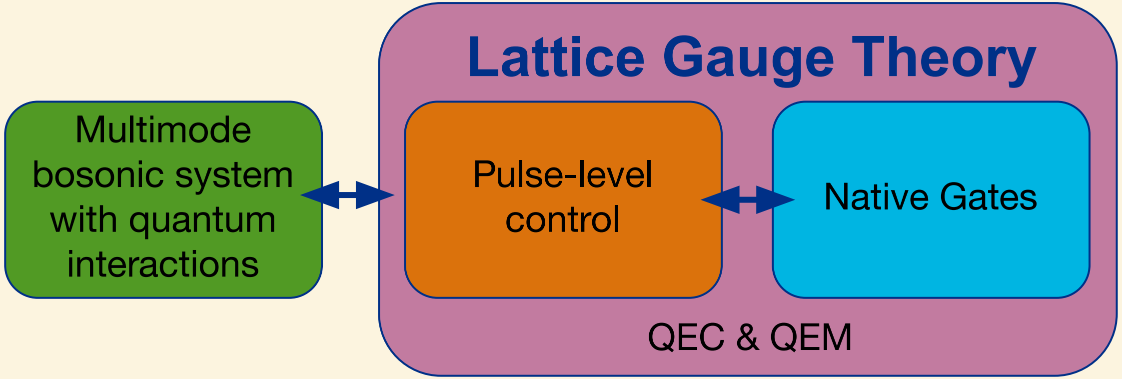
or, HEP for quantum computing

What does the stack look like?



What does the stack look like?





How do bosonic quantum computers work?

$$H_s = \hbar\omega_s\left(b^\dagger b + \frac{1}{2}\right) - \hbar(\omega_q + \chi_s b^\dagger b)\frac{\sigma_z}{2},$$

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

- Instead of defining qudits by Fock states, define them in coherent basis

- α is related to canonical p,q of the oscillator

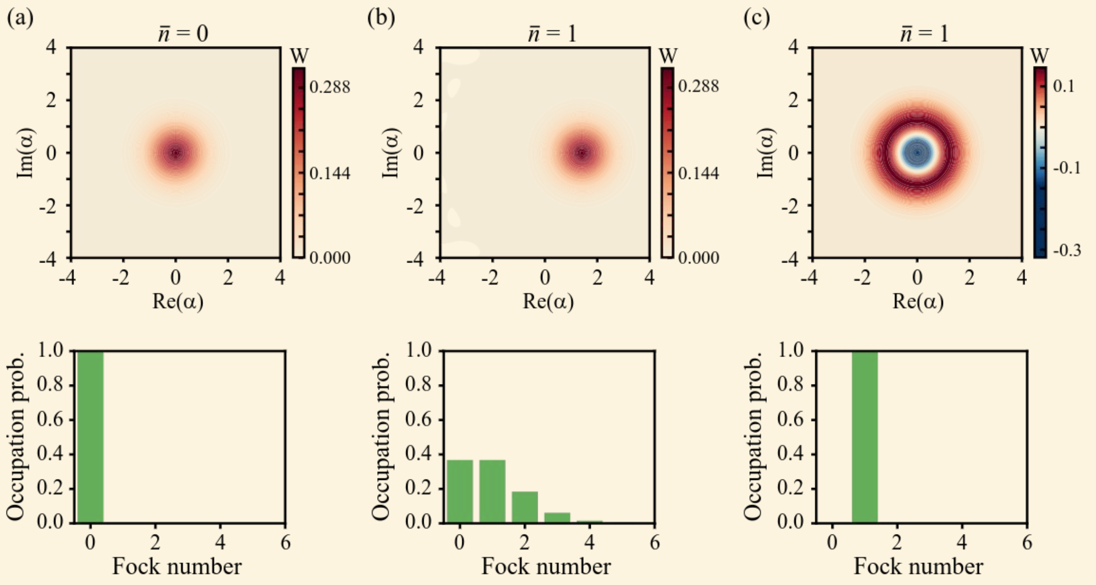
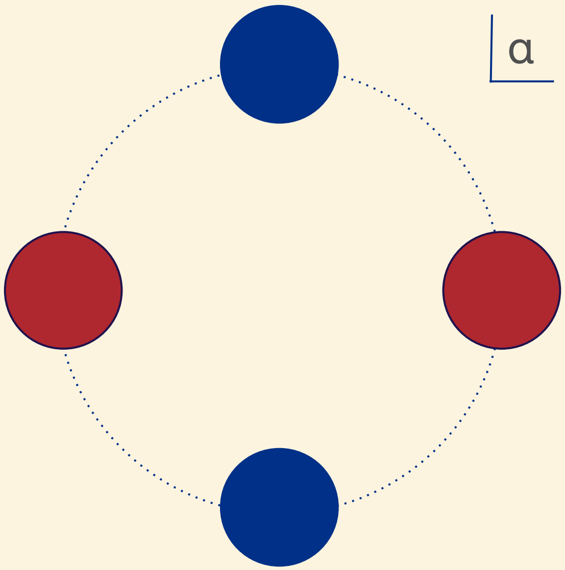


Figure 5: Ideal Wigner tomography (top panel) and corresponding Fock state occupation probability (bottom panel) for (a) the ground state $|0\rangle$, (b) a coherent state ($\alpha = 1$) with an average photon number $\bar{n} = 1$, and (c) the Fock state $|1\rangle$. Note that the populations are distributed across multiple Fock states for the coherent state.

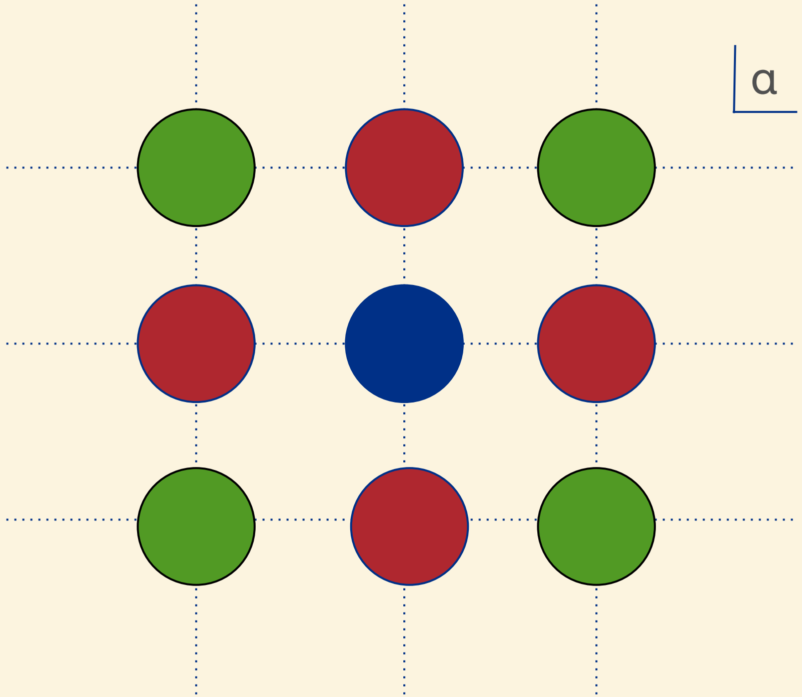
Qudit-based quantum computing with SRF cavities at Fermilab
Roy, Kim, Romanenko, Grassellino - PoS LATTICE2023 (2024) 127
Gentle review of specific hardware and state-of-the-art results

Drawing stars around scars

- Idea is to construct a constellation of states in some "group", and then take a subgroup, or superposition of states to be the codespace



Hardware-Efficient Autonomous Quantum Memory Protection
Leghtas et al. - Phys. Rev. Lett. 111, 120501
Develops psk cat codes and their gates

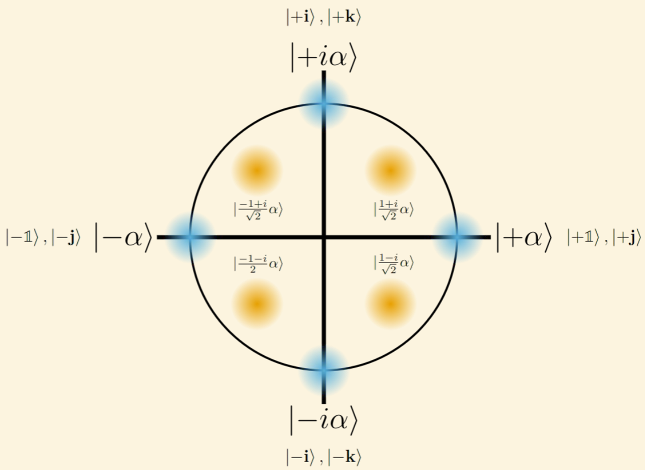


Encoding a qubit in an oscillator
Gottesman, Kitaev, and Preskill - Phys. Rev. A 64, 012310
Develops a lattice-based code

2T-Qudits from two modes

The 2T-qutrit, a two-mode bosonic qutrit
Denys, Leverrier - Quantum 7, 1032 (2023)
Develops a Z_3 qutrit from larger, nonabelian group

$$2T = Q \rtimes \mathbb{Z}_3,$$



The 2T-quocit: a two-mode bosonic qudit for high energy physics
Kurkcuoglu, Lamm, Ogunkoya, Pierattell - in prep
Develops a Q quoctit from larger, nonabelian group

2T-Qudits from two modes

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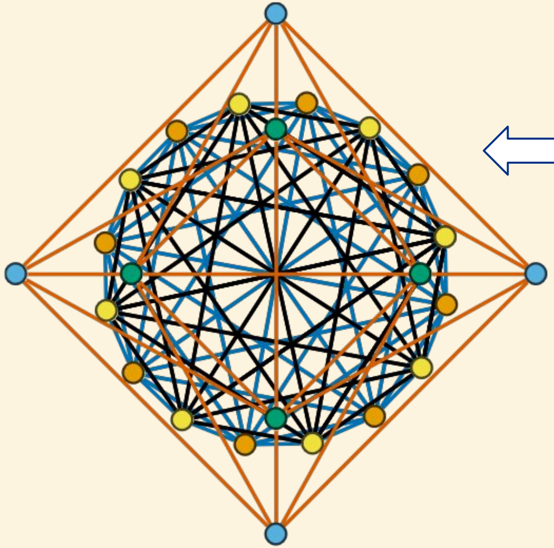

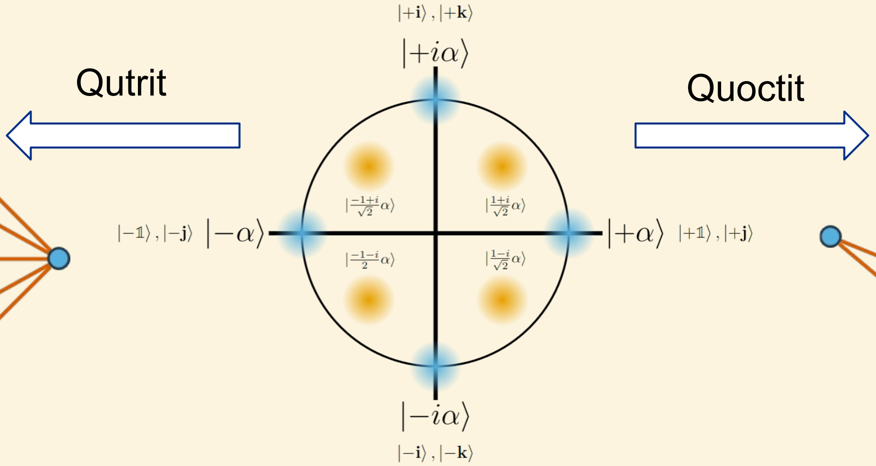

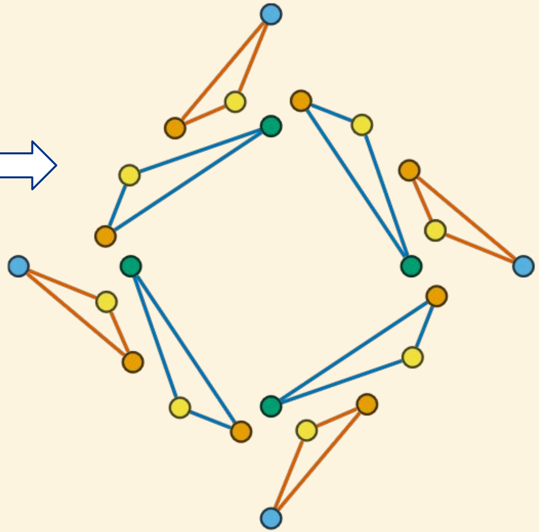

$$Q = \{\pm\mathbf{1}, \pm\mathbf{i}, \pm\mathbf{j}, \pm\mathbf{k}\},$$

$$\omega Q = \left\{\pm\frac{1}{2}(1+\mathbf{i}+\mathbf{j}+\mathbf{k}), \pm\frac{1}{2}(1+\mathbf{i}-\mathbf{j}-\mathbf{k}), \pm\frac{1}{2}(1-\mathbf{i}-\mathbf{j}+\mathbf{k}), \pm\frac{1}{2}(1-\mathbf{i}+\mathbf{j}-\mathbf{k})\right\},$$

$$\omega^2 Q = \left\{\pm\frac{1}{2}(-1+\mathbf{i}+\mathbf{j}+\mathbf{k}), \pm\frac{1}{2}(1-\mathbf{i}+\mathbf{j}+\mathbf{k}), \pm\frac{1}{2}(1+\mathbf{i}-\mathbf{j}+\mathbf{k}), \pm\frac{1}{2}(1+\mathbf{i}+\mathbf{j}-\mathbf{k})\right\},$$

$$|\phi_q\rangle = |q\rangle + |q\omega\rangle + |q\omega^2\rangle, \quad \forall q \in Q,$$

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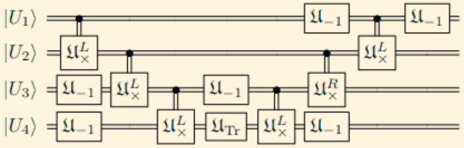
Benefits of Nonabelian Qudits for Primitive Gates

- Inversion gate is virtual (just relabel the states)
- Multiplication gate is a controlled permutations
- Typically "$X^{a,b}$" native gates

TABLE IV. Number of physical T gates and clean ancilla required to implement logical gates for primitive gates for BT.

Gate	T gates	Clean ancilla
$\mathcal{U}_{-1}$	28	0
$\mathcal{U}_{\times}$	154	1
$\mathcal{U}_{Tr}$	$12.65 \log_2(1/\epsilon)$	0
$\mathcal{U}_{QFT}$	$48 \log_2(1/\epsilon) + 96$	0



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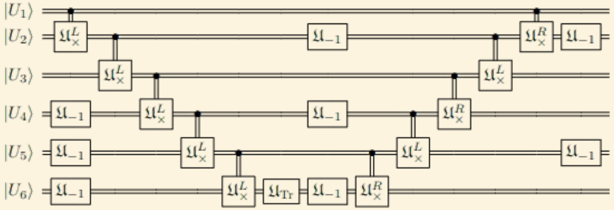

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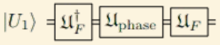

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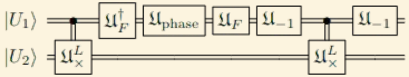

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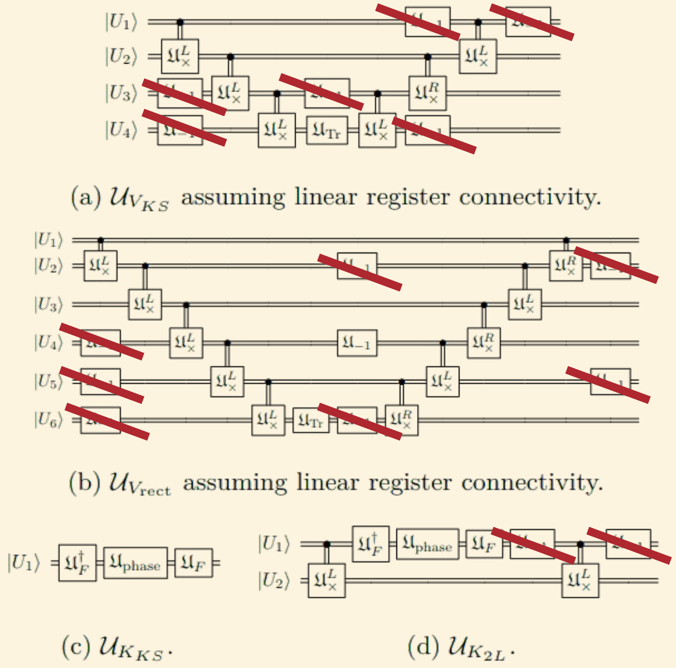

(a) $\mathcal{U}_{V_{KS}}$ assuming linear register connectivity.

(b) $\mathcal{U}_{V_{\text{rect}}}$ assuming linear register connectivity.

(c) $\mathcal{U}_{K_{KS}}$. (d) $\mathcal{U}_{K_{2L}}$.

Could decrease total fault tolerant gate cost by factor of 2

Big unsolved problem in fault tolerant qudits

Problem: Given a **SU(d) element**, find the best approximation **in a subset**

- For qubits, we know efficient algorithms for gate synthesis

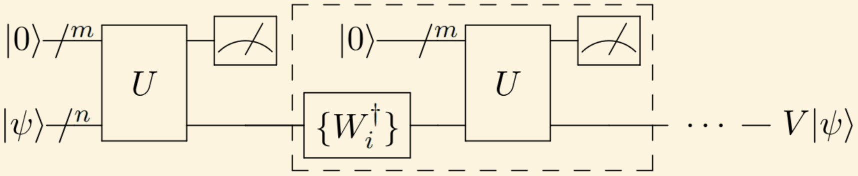

Figure 1: RUS design circuit to implement unitary V.

Efficient Synthesis of Universal Repeat-Until-Success Circuits
Bocharov, Roetteler, Svore - Phys. Rev. Lett. 114, 080502 (2015)
Develop algorithm for single-qubit gate synthesis using number theory

$$N_{T,avg}=1.15 \log_2 1/\varepsilon+9.2$$

- For qudits, shrug…we have an estimated, nonconstructive bound

$$n \gtrsim \frac{\ln\left(\frac{B}{A}\right) + (p^2-1)\ln(1/\epsilon)}{\ln(p(p-1))}$$

A Normal Form for Single-Qudit Clifford+T Operators
Jain, Kalra, Prakash - arXiv:2011.07970
Propose a normal form for odd prime qudits

Sounds alot like finding the digitization of a gauge theory to me…

Endgame

- The road to **quantum utility in HEP** will be **long** and **winding**
- It is coming into focus that the answer to the questions

  Do we need it? Are the benefits to HEP specialized hardware? Should theorists be engaged now?

  are all **YES!**
- Many **theoretical issues** remain unresolved, and could reduce costs
- **Error Correction** and **Gate Synthesis** may benefit from HEP insight
- **Qudit devices** including bosonic computers have **novelties** worth leveraging