Signal Induction

DRD1 Gaseous Detector School

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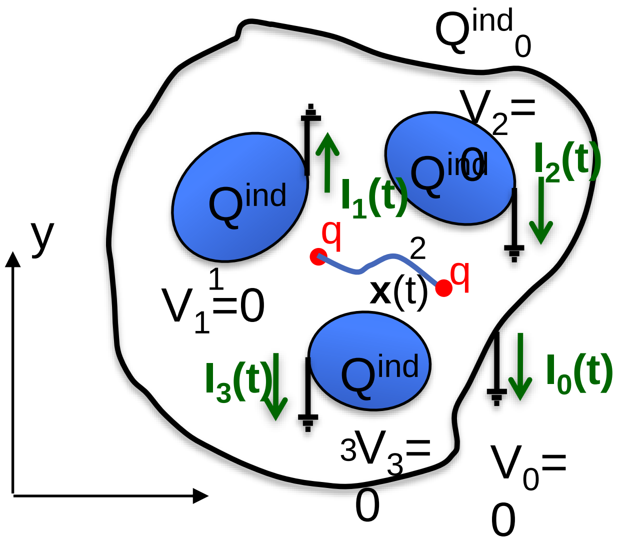
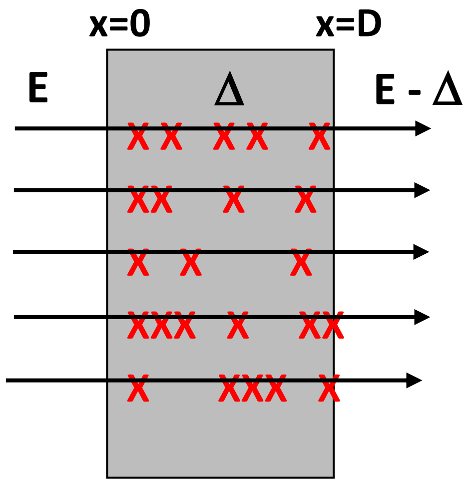
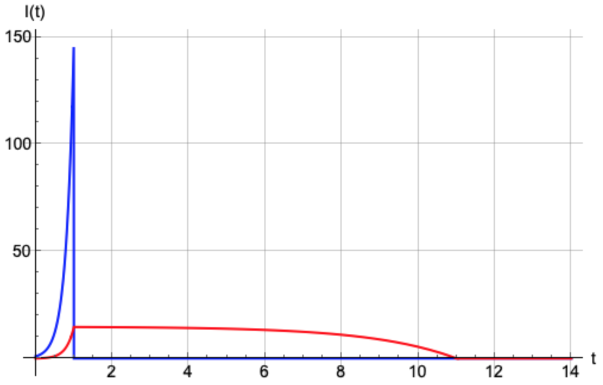
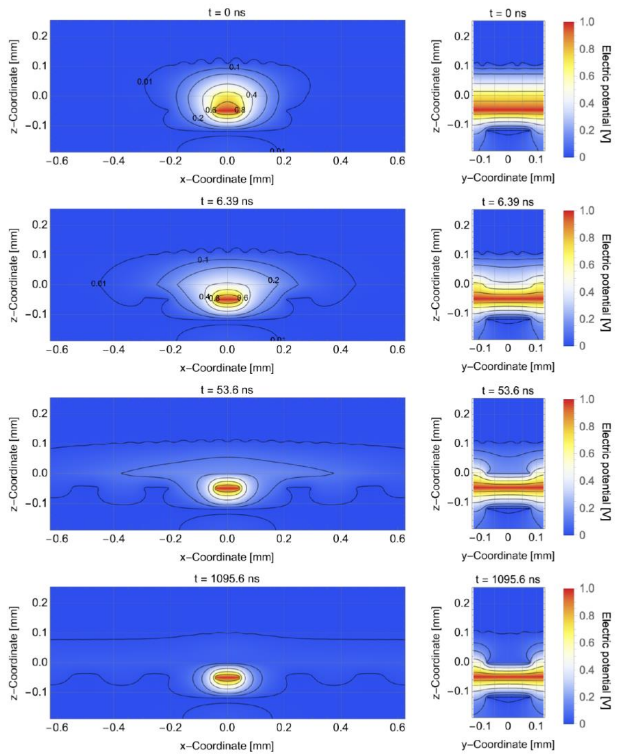
Nov. 29 2024

Outline









Signal induction principles and theorems

Energy Loss, charge deposit, transport properties

Examples: Wire chambers, GEMs, Micromegas

Theorems and examples for detectors with resistive elements

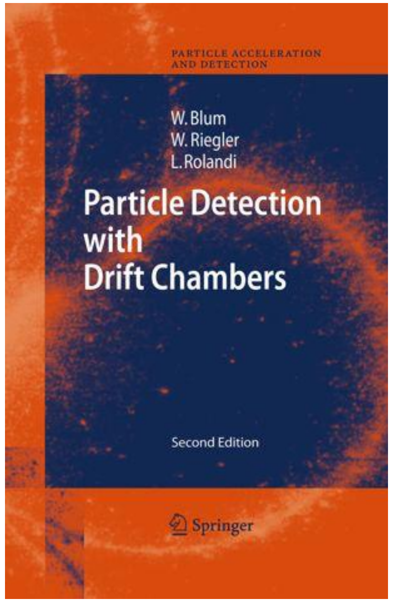
Lecture Series

Signals in particle detectors,
CERN Academic Training Lectures 2019

https://indico.cern.ch/event/843083/

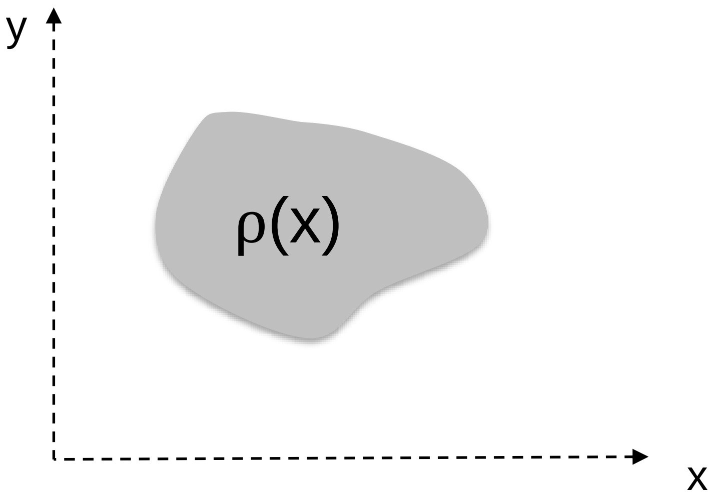
Signals, Noise and Signal processing in Particle Detectors,
CERN Academic Training Lectures 2024

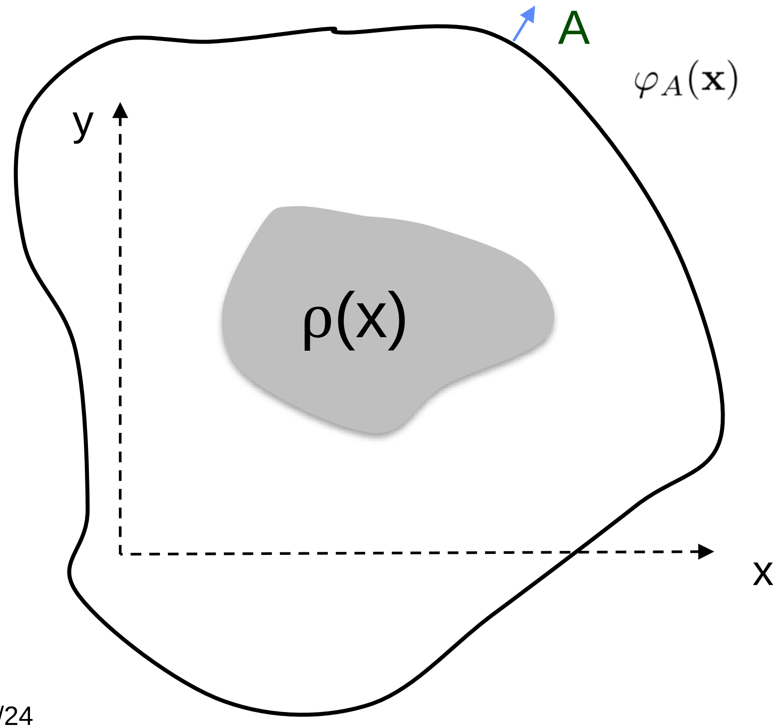
https://indico.cern.ch/event/1458895/



Particle Detection with Drift Chambers,
Springer, 2008 2nd Edition

Electrostatics





Potential for a given charge distribution

$$\varphi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x' \qquad \mathbf{E}(\mathbf{x}) = -\nabla\varphi(\mathbf{x})$$

Charge distribution with boundary condition, Poisson equation:

$$\Delta\varphi(\mathbf{x}) = -\frac{\rho(\mathbf{x})}{\varepsilon_0} \qquad \varphi(\mathbf{x})|_{\mathbf{x}=\mathbf{A}} = \varphi_A(\mathbf{x})$$

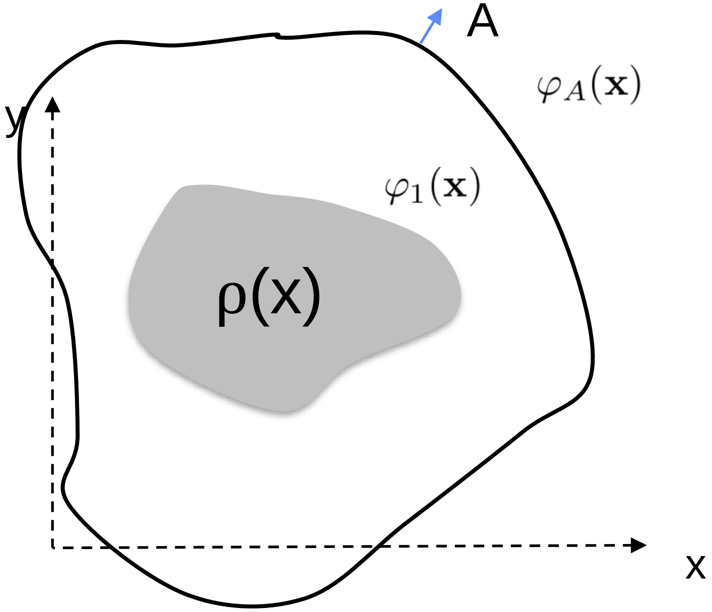
In regions of space without charge and given boundary, Laplace equation:

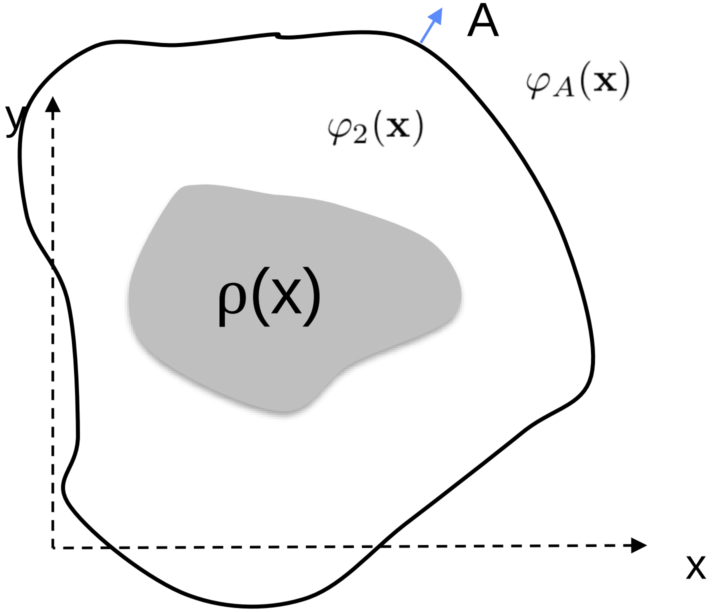
$$\Delta\varphi(\mathbf{x}) = 0 \qquad \varphi(\mathbf{x})|_{\mathbf{x}=\mathbf{A}} = \varphi_A(\mathbf{x})$$

Gauss Law:

$$\oint_A \mathbf{E}(\mathbf{x})d\mathbf{A} = \frac{1}{\varepsilon_0}\oint_V \rho(\mathbf{x})dV$$

Uniqueness





Let's assume we have two solutions of the same Poisson equation

$$\Delta\varphi_1(\mathbf{x}) = -\rho(\mathbf{x})/\varepsilon_0 \qquad \Delta\varphi_2(\mathbf{x}) = -\rho(\mathbf{x})/\varepsilon_0$$

$$\varphi_1(\mathbf{x})|_{\mathbf{x}=A} = \varphi_2(\mathbf{x})|_{\mathbf{x}=A} = \varphi_A(\mathbf{x})$$

The difference between these two solutions must satisfy the Laplace equation

$$\varphi(\mathbf{x}) = \varphi_1(\mathbf{x}) - \varphi_2(\mathbf{x}) \qquad \Delta\varphi(\mathbf{x}) = 0$$

In general it holds that

$$\nabla\left(\varphi(\mathbf{x})\nabla\varphi(\mathbf{x})\right) = \left(\nabla\varphi(\mathbf{x})\right)^2 + \varphi(\mathbf{x})\Delta\varphi(\mathbf{x})$$

Using the fact that $\Delta\varphi(\mathbf{x}) = 0$ and applying Gauss' theorem we have

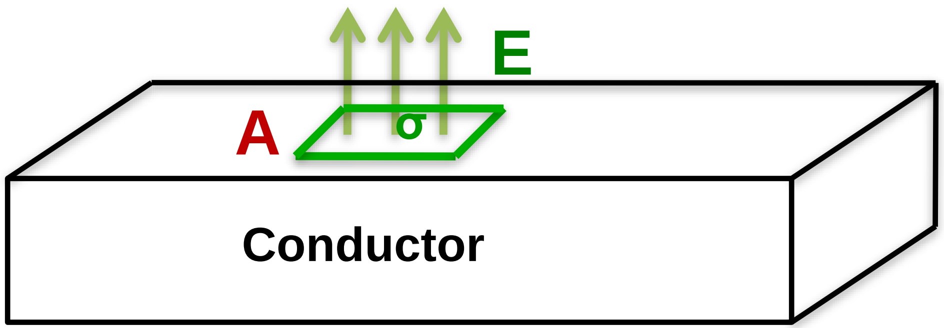
$$\oint_A \varphi(\mathbf{x})\nabla\varphi(\mathbf{x})d\mathbf{A} = \int_V \left(\nabla\varphi(\mathbf{x})\right)^2 d^3x$$

In case $\varphi_1 = \varphi_2$ on the surface we have $\varphi = 0$ on the surface and the left side vanishes. That means that the right hand side must also vanish and we have

$$\nabla\varphi(\mathbf{x}) = 0 \quad \rightarrow \quad \nabla\varphi_1(\mathbf{x}) = \nabla\varphi_2(\mathbf{x})$$

Defining the potential $\varphi_1(\mathbf{x})$ on the entire surface therefore uniquely defines the electric field in the volume. The same holds in case we define $\nabla\varphi_1 d\mathbf{A}$ on the entire surface. Evidently the uniqueness theorem also holds in case we define $\varphi_1(\mathbf{x})$ on a fraction of the surface and $\nabla\varphi_1 d\mathbf{A}$ on the rest of the surface.

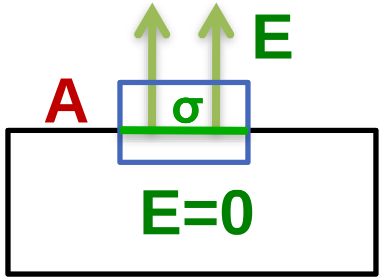
Conductors



In conducting materials, charge will move as long as there is an electric field present.

A static situation is only achieved once the electric field inside the conductor is zero and therefore also the charge density inside the conductor is zero. All the charge is sitting on the surface of the conductor.

The electric field on the surface of the conductor must be perpendicular to the conductor, since a component parallel to the conductor would again move the charge.



$$\oint_S \mathbf{E}(\mathbf{x})\, d\mathbf{A} = \frac{1}{\varepsilon_0} \oint_V \rho(\mathbf{x})\, d^3x$$

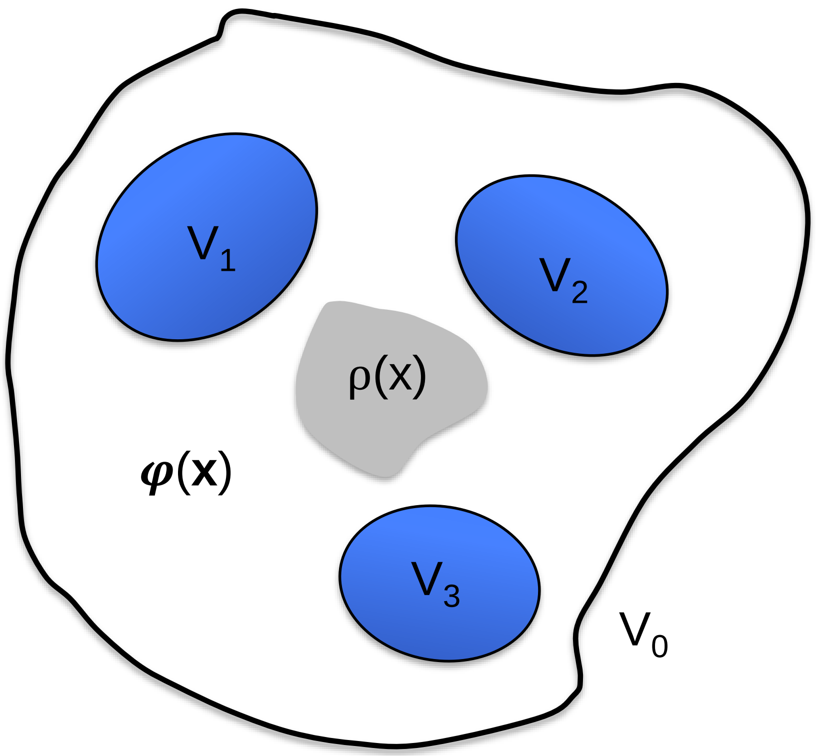
Moving a test charge across the surface of the conductor one crosses the field lines perpendicular and no work is performed. A conductor surface is therefore and equipotential surface.

Since the electric field inside the conductor is zero and the field-lines are perpendicular to the surface, we can use Gauss law to relate the surface charge density to the field on the surface:

$$EA = \frac{1}{\varepsilon_0}\sigma A \qquad \rightarrow \qquad \sigma = \varepsilon_0 E$$

→ The charge density (C/m^2) at a given point on the conductor surface is equal to ε_0 times the electric field in this point.

Induced charge on metal electrodes



Poisson equation:

$$\Delta\varphi(\mathbf{x}) = -\rho(\mathbf{x})/\varepsilon_0 \qquad \varphi(\mathbf{x})|_{\mathbf{x}=\mathbf{A}_n} = V_n$$

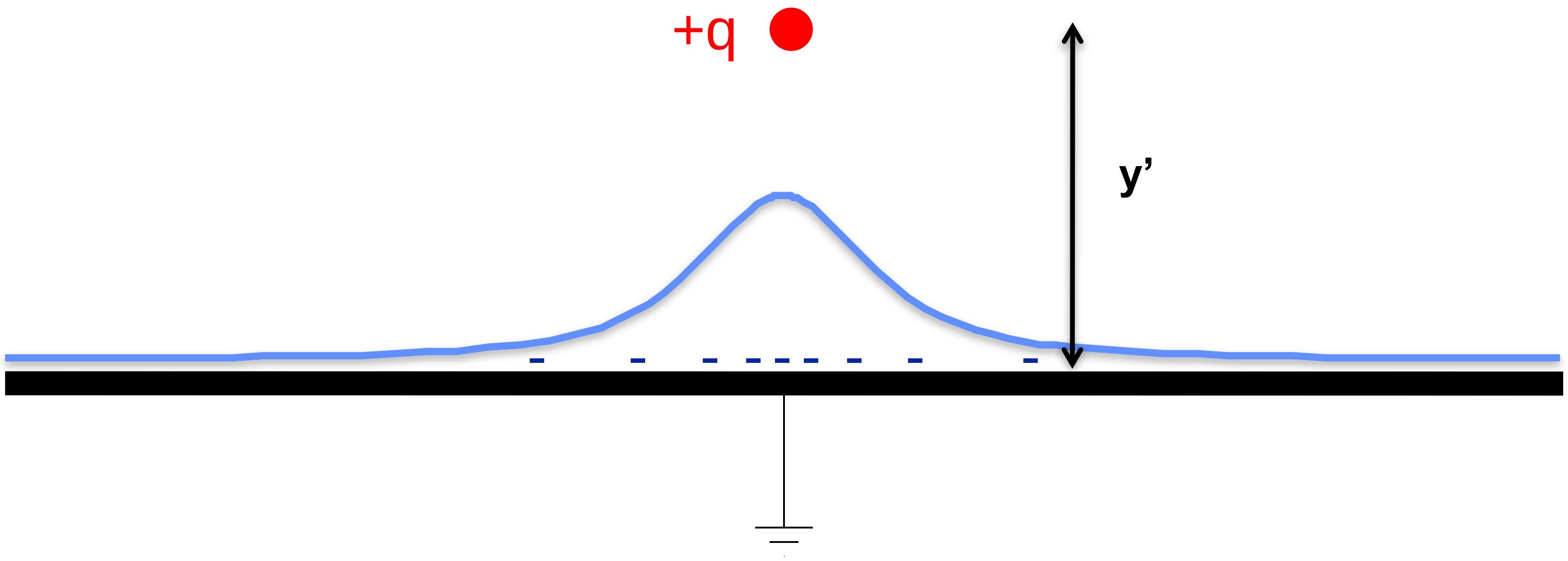
The charge on the electrode is given by the integral of the electric field over the surface of the electrode

$$Q_n = -\varepsilon_0 \oint_{\mathbf{A}_n} \nabla\varphi(\mathbf{x}) d\mathbf{A}$$

The Principle of Signal Induction on Metal Electrodes by Moving Charges

Induced charge on an infinite plane

A point charge q at a distance y' above a grounded metal plate 'induces' a surface charge.



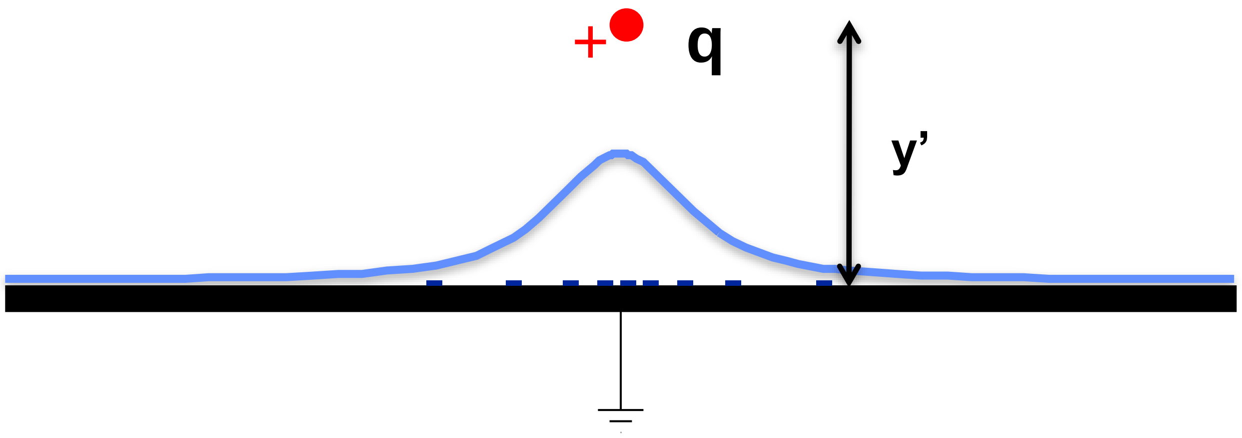
Induced charge on an infinite plane

In order to find the charge induced on an electrode we therefore have to

a) Solve the Poisson equation with boundary condition that φ=0 on the conductor surface.

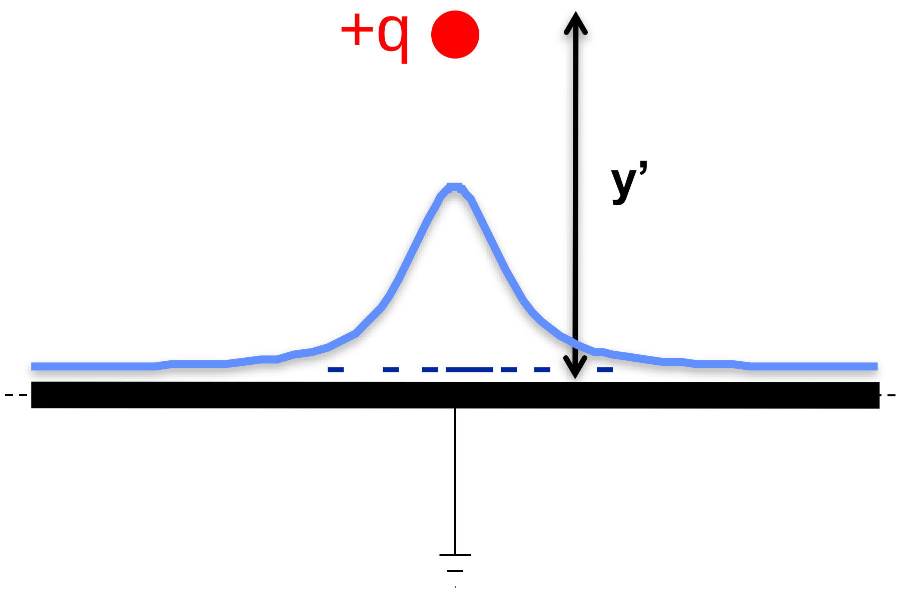
b) Calculate the electric field E on the surface of the conductor

c) Integrate $\sigma = e_0 E$ over the electrode surface



Induced charge on an infinite plane

The solution for the field of a point charge in front of a metal plate is equal to the solution of the charge together with a (negative) mirror charge at y=-y'.



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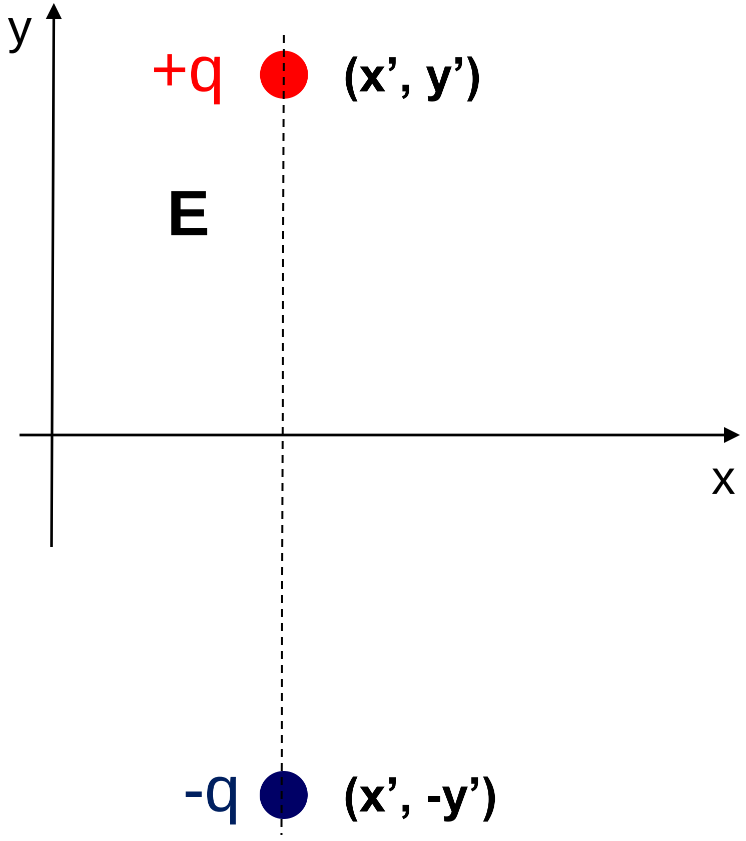


The potential for the above problem is therefore:

$$\varphi(x,y,z)=\frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}-\frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x-x')^2+(y+y')^2+(z-z')^2}}$$

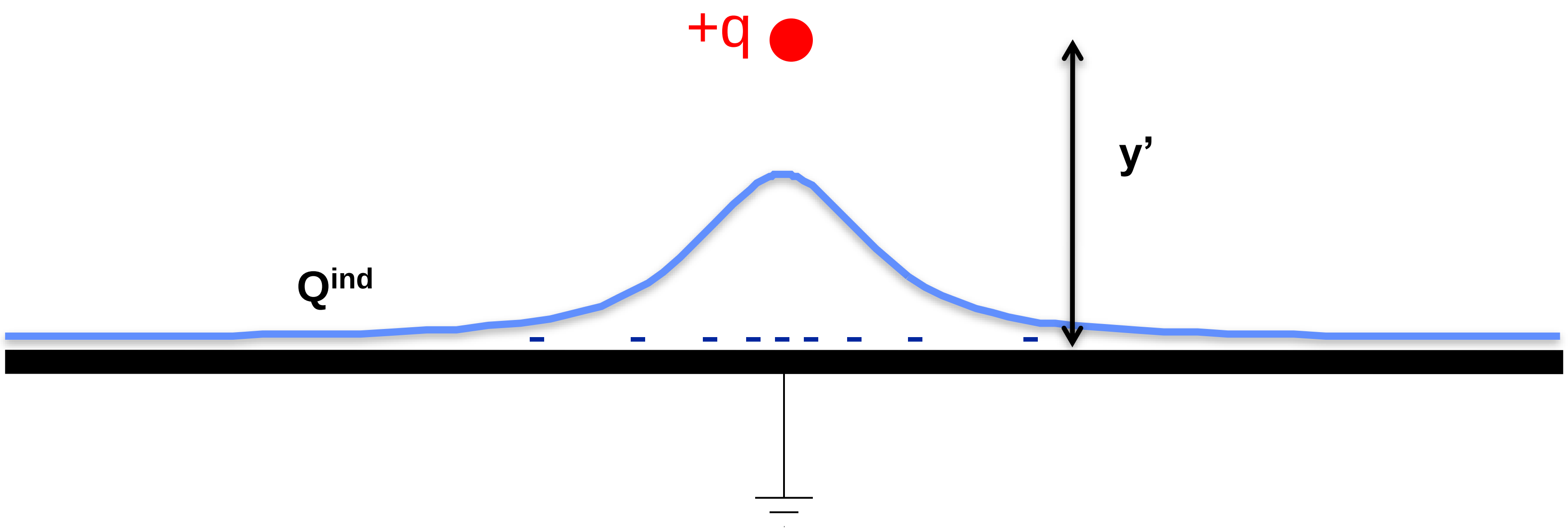
Induced charge on an infinite plane

We therefore find a surface charge density of

$$\sigma(x,y) = -\varepsilon_0 \frac{\partial \varphi}{dy}|_{y=0} = -\frac{q}{2\pi} \frac{y'}{\left((x-x')^2 + y'^2 + (z-z')^2\right)^{3/2}}$$

And therefore a total induced charge of

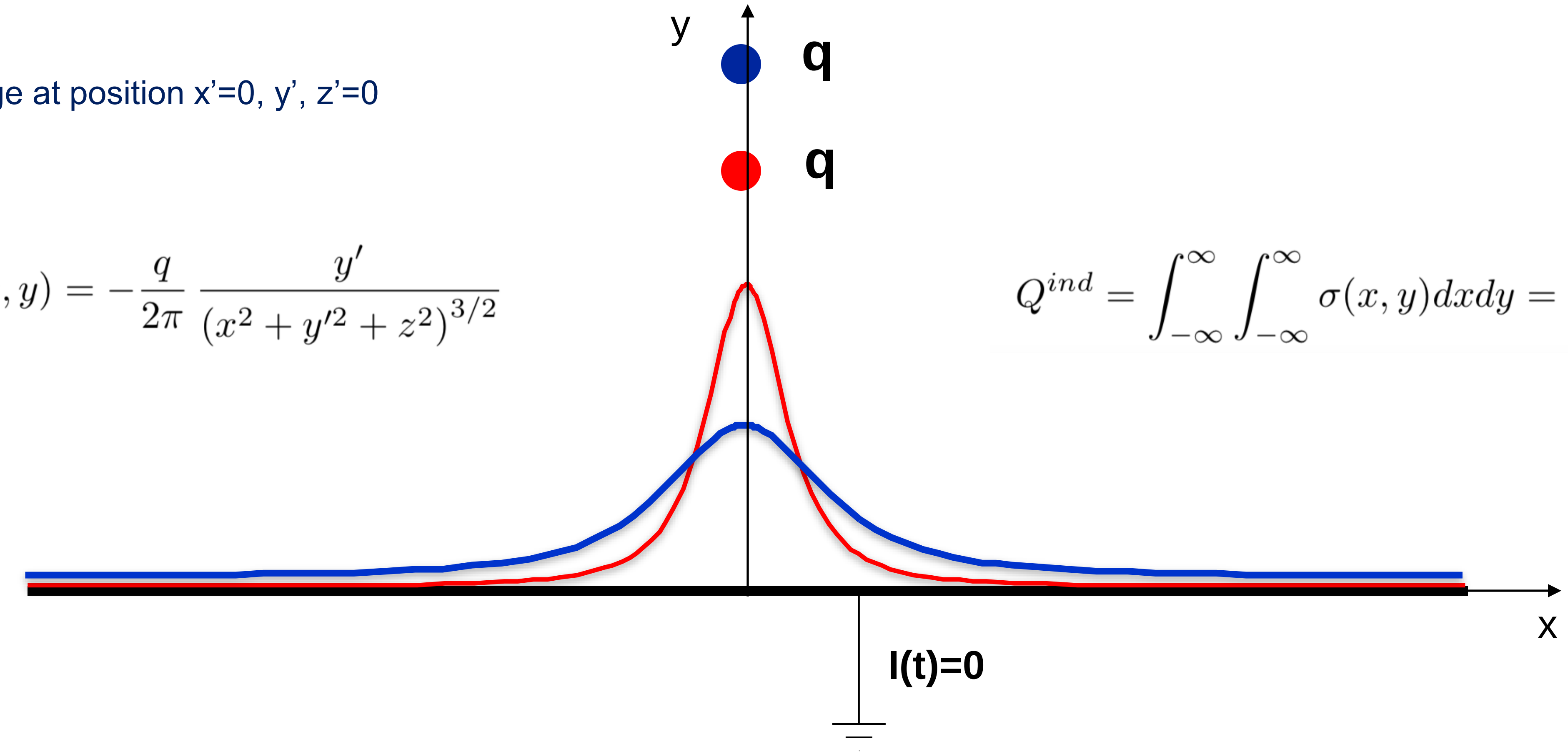
$$Q^{ind} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma(x,y) dx dy = -q$$



Induced charge on an infinite plane

- Moving the point charge closer to the metal plate, the surface charge distribution becomes more peaked
- The total induced charge is however always equal to –q,
- The charge is just rearranged on the surface and no current is flowing between the plate and ground..

Charge at position x'=0, y', z'=0



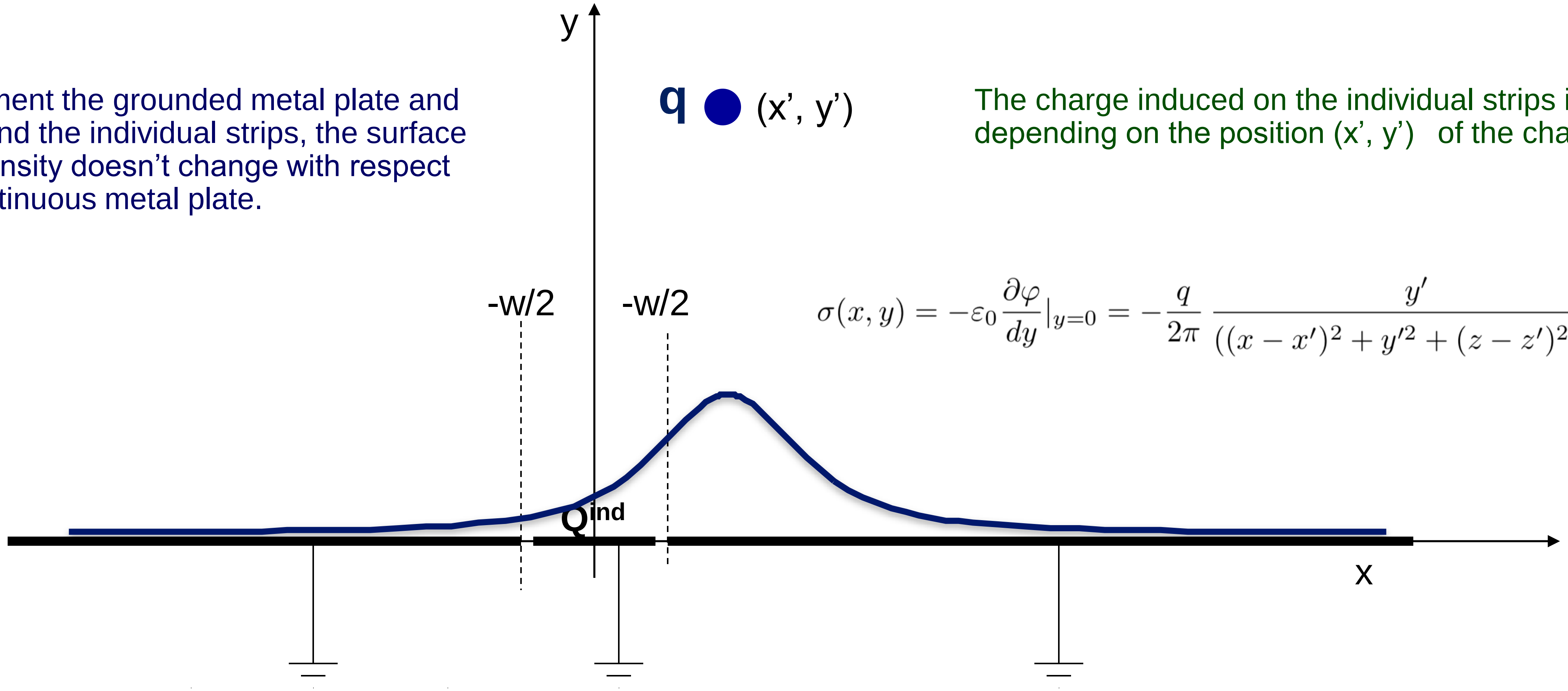
$$\sigma(x,y) = -\frac{q}{2\pi}\frac{y'}{(x^2+y'^2+z^2)^{3/2}}$$

$$Q^{ind} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sigma(x,y)dxdy = -q$$

Induced charge on strip electrode

If we segment the grounded metal plate and if we ground the individual strips, the surface charge density doesn't change with respect to the continuous metal plate.

The charge induced on the individual strips is now depending on the position (x', y') of the charge.



$$\sigma(x,y) = -\varepsilon_0 \frac{\partial \varphi}{dy}|_{y=0} = -\frac{q}{2\pi}\,\frac{y'}{((x-x')^2 + y'^2 + (z-z')^2)^{3/2}}$$

$$Q^{ind}(x',y') = \int_{-\infty}^{\infty}\int_{-w/2}^{w/2} \sigma(x,y)dxdy = -\frac{q}{\pi}\left(\arctan\frac{w-2x'}{2y'} - \arctan\frac{w+2x'}{2y'}\right)$$

Induced current on strip electrode

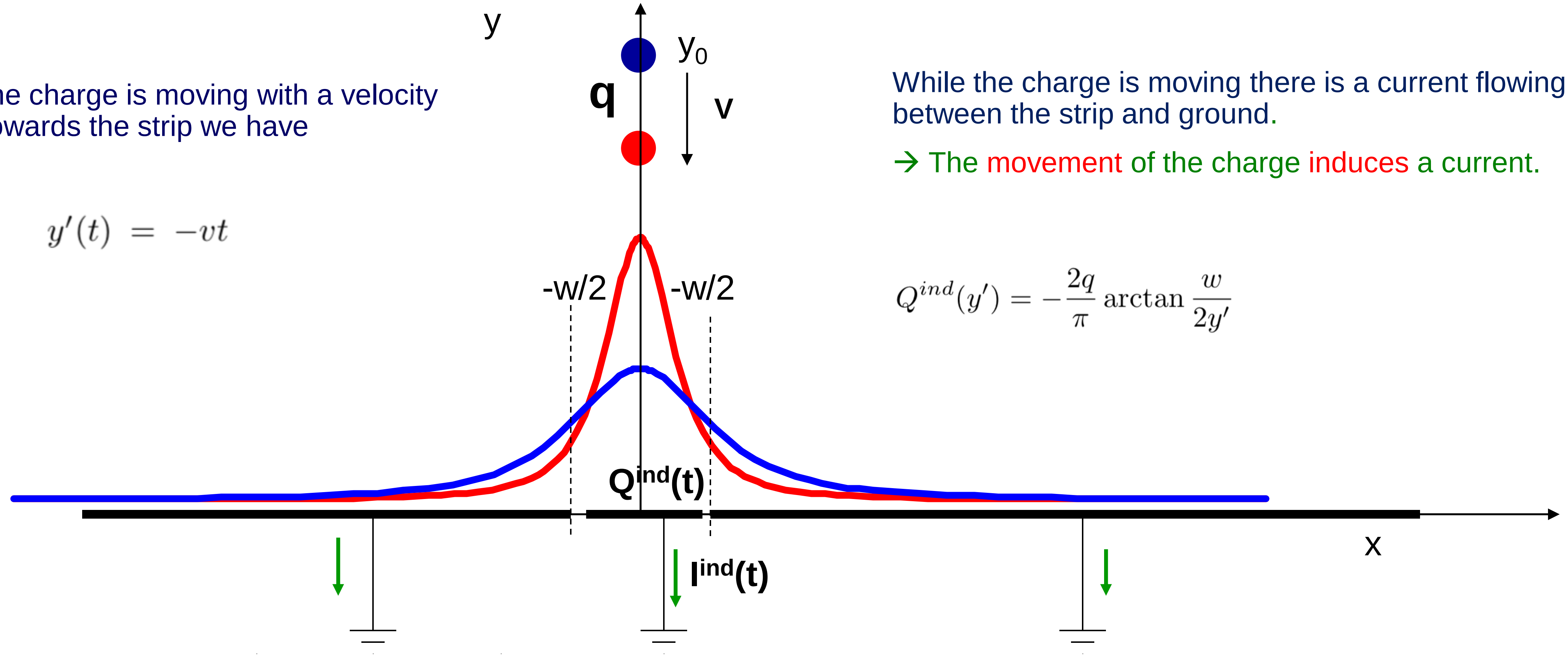
If the charge is moving with a velocity v towards the strip we have

$$y'(t) = -vt$$

While the charge is moving there is a current flowing between the strip and ground.

→ The movement of the charge induces a current.

$$Q^{ind}(y') = -\frac{2q}{\pi}\arctan\frac{w}{2y'}$$

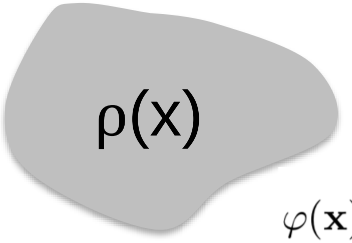


$$Q^{ind}(t) = q\frac{2}{\pi}\arctan\frac{w}{2vt} \qquad t < 0$$

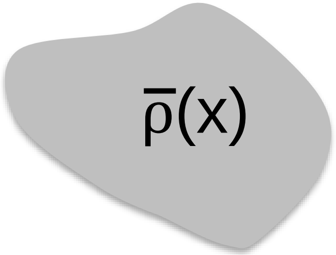
$$I^{ind}(t) = -\frac{dQ^{ind}(t)}{dt} = q\frac{4w}{\pi\left(w^2 + 4v^2t^2\right)}v \qquad t < 0$$

Reciprocity theorem

Two arbitrary charge distributions $\rho(\mathbf{x})$ and $\overline{\rho}(\mathbf{x})$



$$\varphi(\mathbf{x}) = \int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$



$$\overline{\varphi}(\mathbf{x}) = \int \frac{\overline{\rho}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$

$$W = \int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x}) d^3x = \int\int \frac{\overline{\rho}(\mathbf{x})\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x d^3x' = \int \rho(\mathbf{x}')\overline{\varphi}(\mathbf{x}') d^3x'$$

$$\int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x}) d^3x = \int \rho(\mathbf{x})\overline{\varphi}(\mathbf{x}) d^3x$$

This is simply a result of the fact that the Coulomb force depends only on the relative distance of the charges and not on absolute position in space.

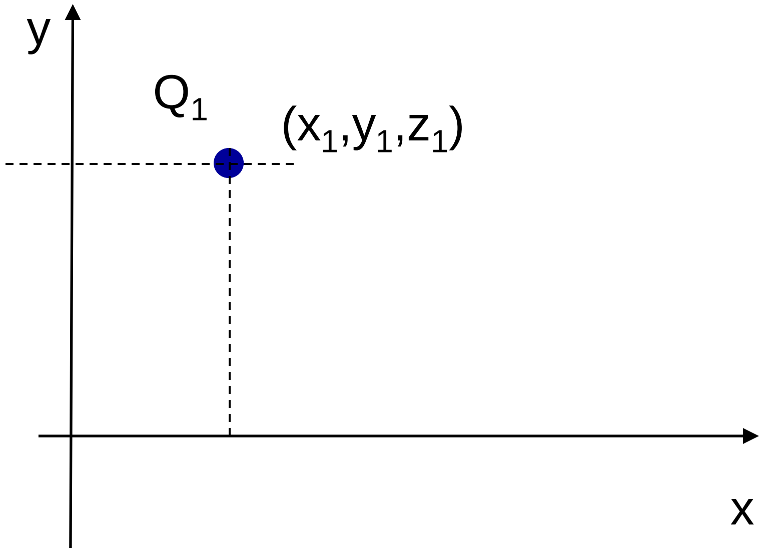
The expressions can be interpreted as the work needed to move one charge distribution in the electric field of the other charge distribution, actio = reactio

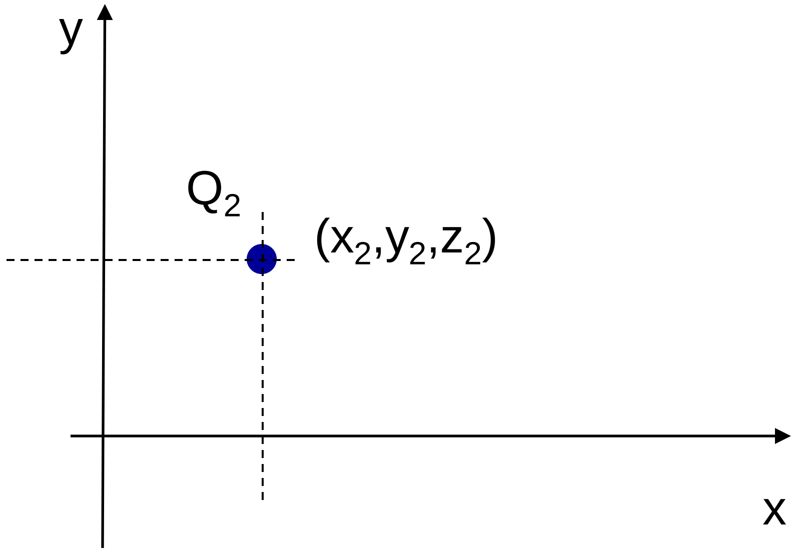
Sounds like a trivial statement, but has very practical consequences.

Reciprocity theorem

$$\int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x})d^3x = \int \rho(\mathbf{x})\overline{\varphi}(\mathbf{x})d^3x$$

Example: two point charges





$$\rho(\mathbf{x}) = Q_1\delta(\mathbf{x}-\mathbf{x}_1) \quad \varphi(\mathbf{x}) = \frac{Q_1}{4\pi\varepsilon_0|\mathbf{x}-\mathbf{x}_1|}$$

$$\overline{\rho}(\mathbf{x}) = Q_2\delta(\mathbf{x}-\mathbf{x}_2) \quad \overline{\varphi}(\mathbf{x}) = \frac{Q_2}{4\pi\varepsilon_0|\mathbf{x}-\mathbf{x}_2|}$$

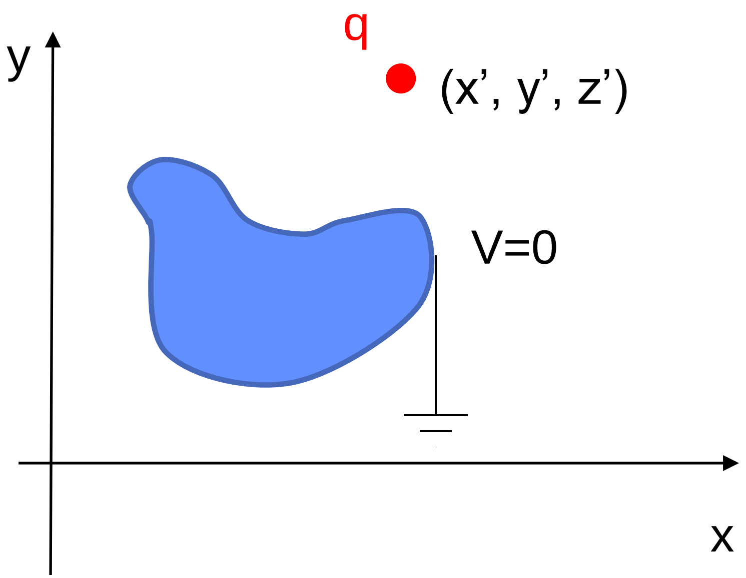
$$\frac{Q_1Q_2}{4\pi\varepsilon_0|\mathbf{x}_1-\mathbf{x}_2|} = \frac{Q_1Q_2}{4\pi\varepsilon_0|\mathbf{x}_2-\mathbf{x}_1|}$$

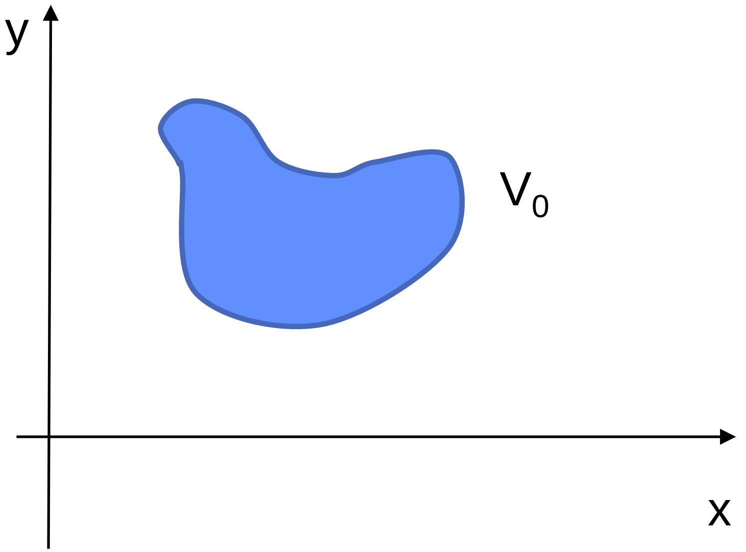
→ Correct but not really useful

Reciprocity

$$\int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x})d^3x = \int \rho(\mathbf{x})\overline{\varphi}(\mathbf{x})d^3x$$

Grounded metal electrode of arbitrary shape
Point charge q at (x', y', z')





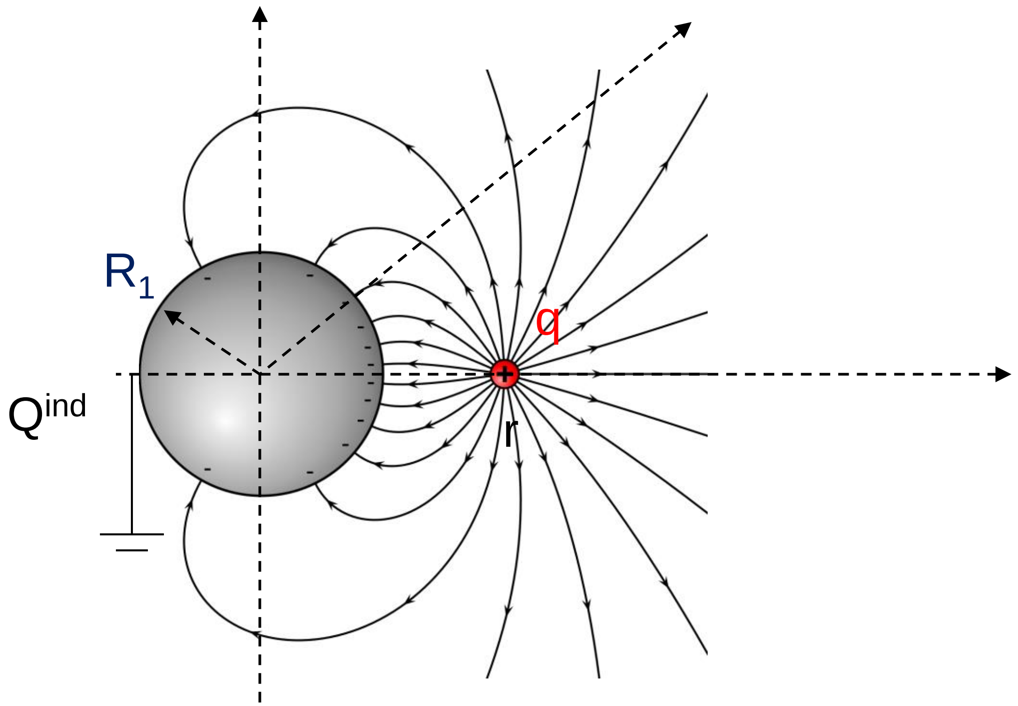
$$\rho(\mathbf{x}) = q\delta(\mathbf{x}-\mathbf{x}') + \rho_{surf}(\mathbf{x}) \qquad \varphi(\mathbf{x})|_{surf} = 0$$

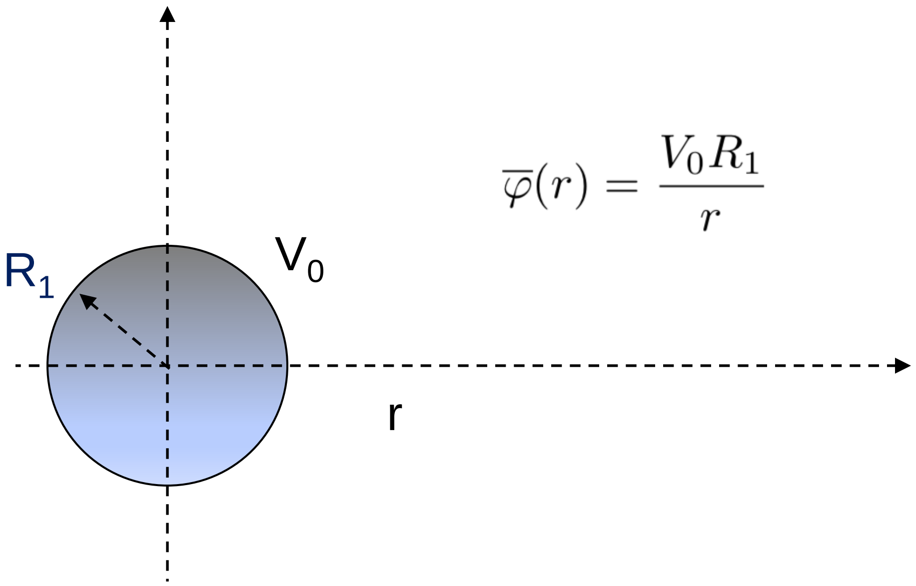
$$\overline{\rho}(\mathbf{x}) = \overline{\rho}_{surf}(\mathbf{x}) \qquad \overline{\varphi}(\mathbf{x})|_{surf} = V_0$$

$$\begin{aligned}
\int \overline{\rho}_{surf}(\mathbf{x})\varphi(\mathbf{x})d^3x &= \int [q\delta(\mathbf{x}-\mathbf{x}') + \rho_{surf}(\mathbf{x})]\overline{\varphi}(\mathbf{x})d^3x \\
0 &= q\overline{\varphi}(\mathbf{x}') + V_0\int \rho_{surf}(\mathbf{x})d^3x \\
0 &= q\overline{\varphi}(\mathbf{x}') + V_0 Q^{ind} \\
Q^{ind} &= -\frac{q}{V_0}\overline{\varphi}(\mathbf{x}')
\end{aligned}$$

→ Knowing the potential for the electrode at voltage V_0 in absence of any external charges we can directly calculate the charge induced on the grounded electrode in presence of a point charge !!

Induced charge on a grounded sphere

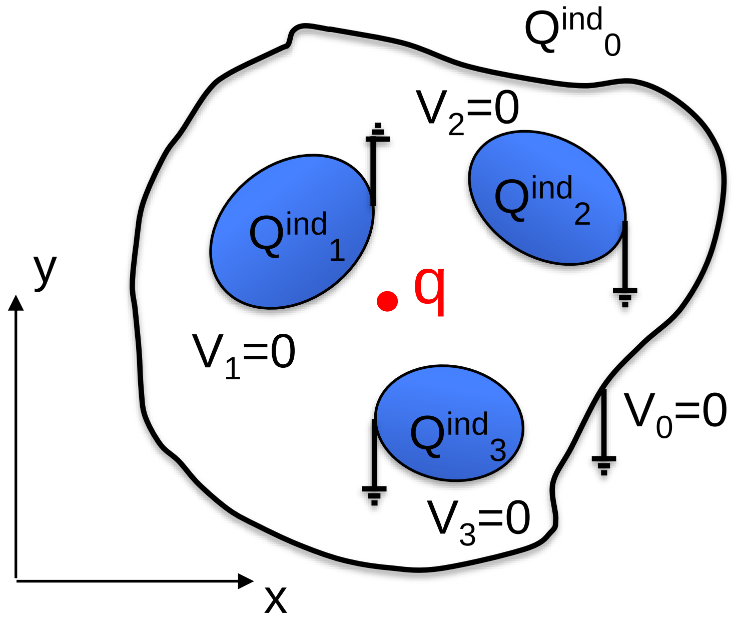




$$\overline{\varphi}(r) = \frac{V_0 R_1}{r}$$

$$Q^{ind}(r) \;=\; -\frac{q}{V_0}\overline{\varphi}(\mathbf{x}') \;= -q\frac{R_1}{r}$$

Theorem, induced charge

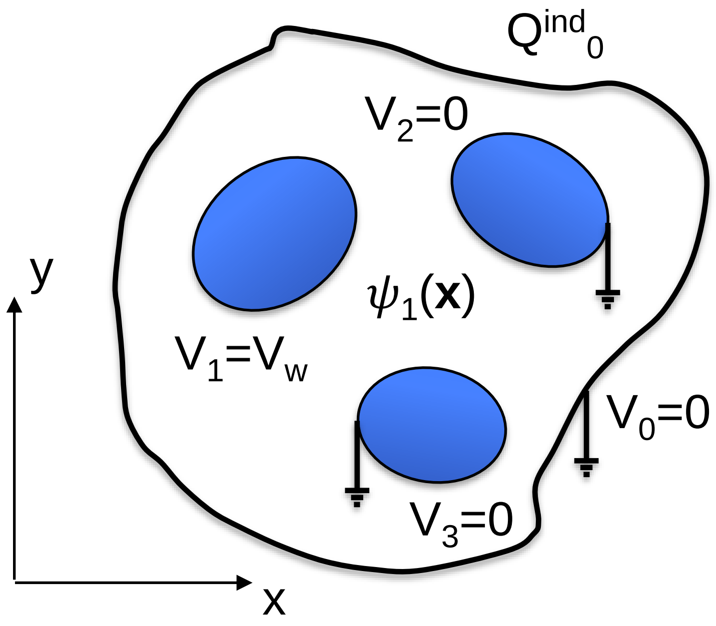


The charge induced on a grounded conducting electrode by a point charge q at position **x** can be calculated the following way:

Remove the point charge, put the electrode in question to potential V_w while keeping all other electrodes at ground potential.

This defines the weighting potential $\psi_n(\mathbf{x})$ of this electrode and the induced charge is

$$Q_n^{ind} = -\frac{q}{V_w}\psi_n(\mathbf{x})$$

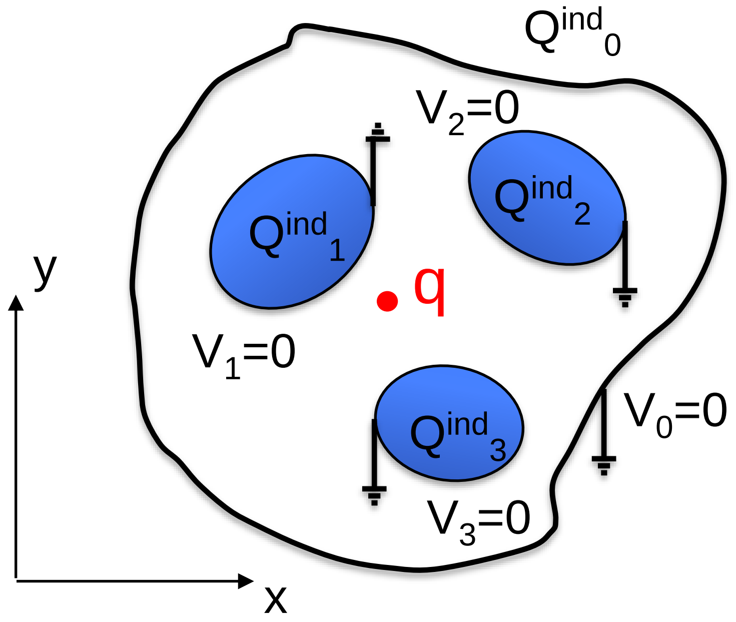


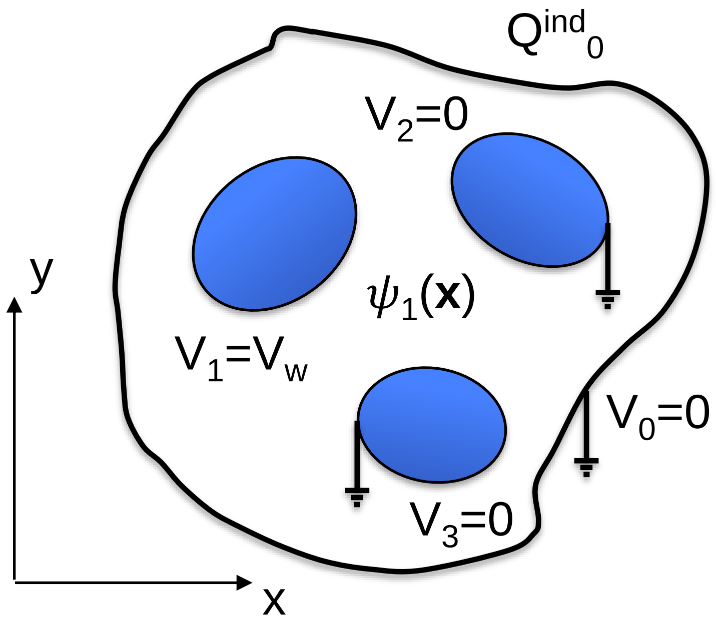
We therefore do not have to solve the Poisson equation for a point charge but we just have to solve the Laplace equation for the given boundary conditions on the electrodes.

For detectors with long electrode, like wire chambers, RPCs, silicon strip detectors, we only have to solve the 2D Laplace equation instead of the 3D Poisson equation.

Specifically for numeric field calculations this is much easier and numerically stable.

Theorem, sum of induced charges





The sum of all induced charges is defined by the sum of all weighting potentials

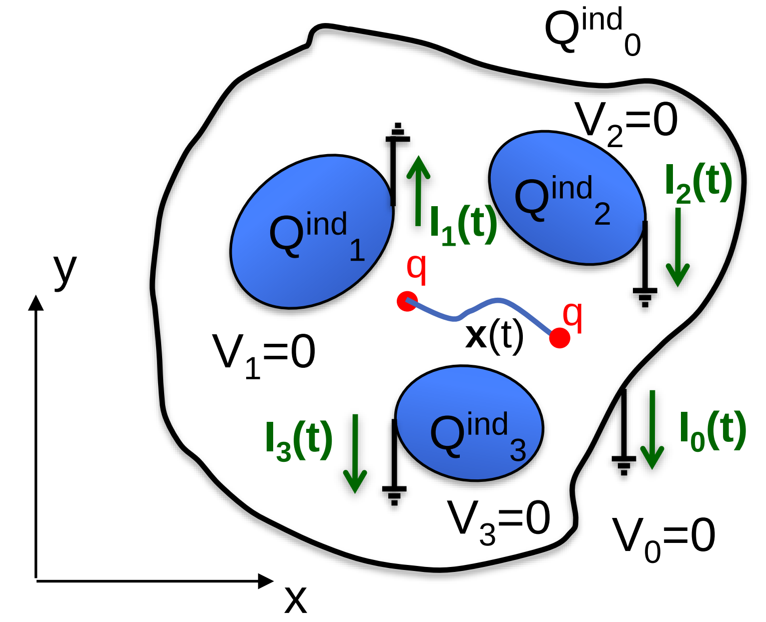
$$\sum_{n=0}^{N} Q_n^{ind} = -\frac{q}{V_w} \sum_{n=0}^{N} \psi_n(\mathbf{x})$$

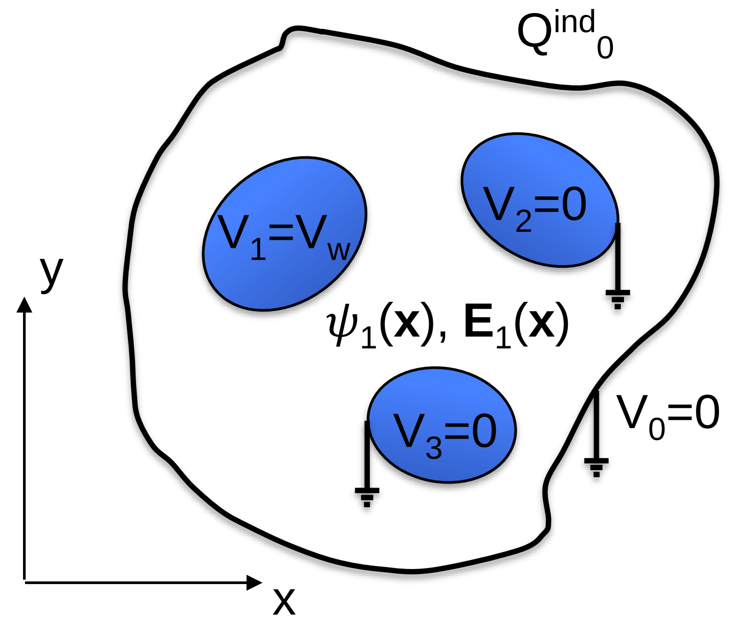
This sum is related to the situation where all electrodes are put to voltage V_w. This will however result in a constant potential V_w in the entire volume between the electrodes, so we have

$$\sum_{n=0}^{N} \psi_n(\mathbf{x}) = V_w \qquad \sum_{n=0}^{N} Q_n^{ind} = -q$$

The sum of all charges induced by a charge q is equal to –q, in case the geometry is such that one electrode encloses all others.

Theorem, induced current





The current induced on a grounded conducting electrode by a point charge q moving along a trajectory **x(t)** can be calculated the following way:

$$I_n^{ind}(t) = -\frac{Q_n^{ind}(\mathbf{x}(t))}{dt} = \frac{q}{V_w}\nabla\psi_n(\mathbf{x}(t))\dot{\mathbf{x}}(t) = -\frac{q}{V_w}\mathbf{E}_n(\mathbf{x}(t))\dot{\mathbf{x}}(t)$$

This weighting field $\mathbf{E}_n(\mathbf{x})$ is given by

$$\mathbf{E}_n(\mathbf{x}) = -\nabla\psi_n(\mathbf{x})$$

→ Ramo-Shockley theorem

Since the sum of all induced charges is constant and equal to –q at any time, the sum of all induced currents is zero at any time, in case there is one electrode enclosing all the others.

$$\sum_{n=0}^{N} I_n^{ind}(t) = 0$$

Ramo Shockley theorem (Reciprocity theorem)

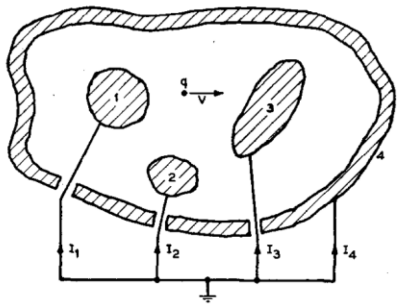


FIG. 1. Schematic representation of conductors and currents.

Currents to Conductors Induced by a Moving Point Charge

W. SHOCKLEY
Bell Telephone Laboratories, Inc., New York, N. Y.
(Received May 14, 1938)

General expressions are derived for the currents which flow in the external circuit connecting a system of conductors when a point charge is moving among the conductors. The results are applied to obtain explicit expressions for several cases of practical interest.

584 *Proceedings of the I.R.E.* *September, 1939*

Currents Induced by Electron Motion*

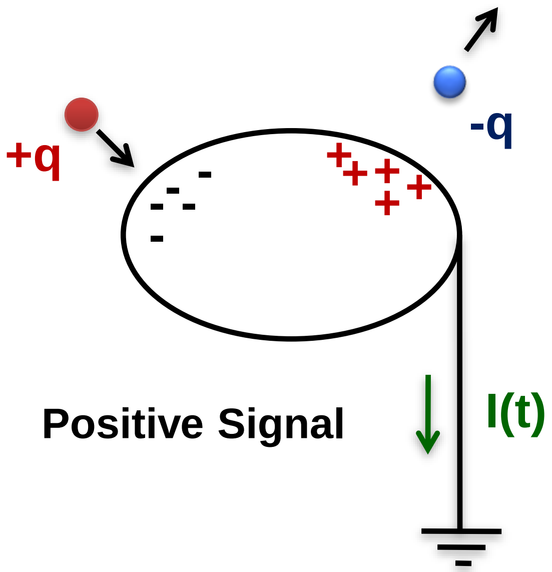
SIMON RAMO†, ASSOCIATE MEMBER, I.R.E.

Summary—*A method is given for computing the instantaneous current induced in neighboring conductors by a given specified motion of electrons. The method is based on the repeated use of a simple equation giving the current due to a single electron's movement and is believed to be simpler than methods previously described.*

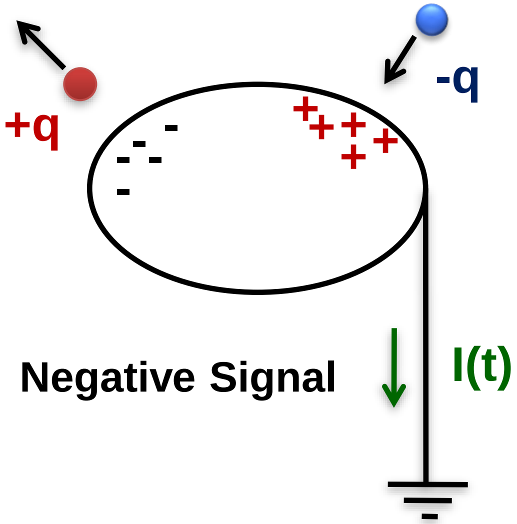
METHOD OF COMPUTATION

The method is based on the following equation, whose derivation is given later:

Signal polarity



Positive Signal



Negative Signal

The definition of the induced current

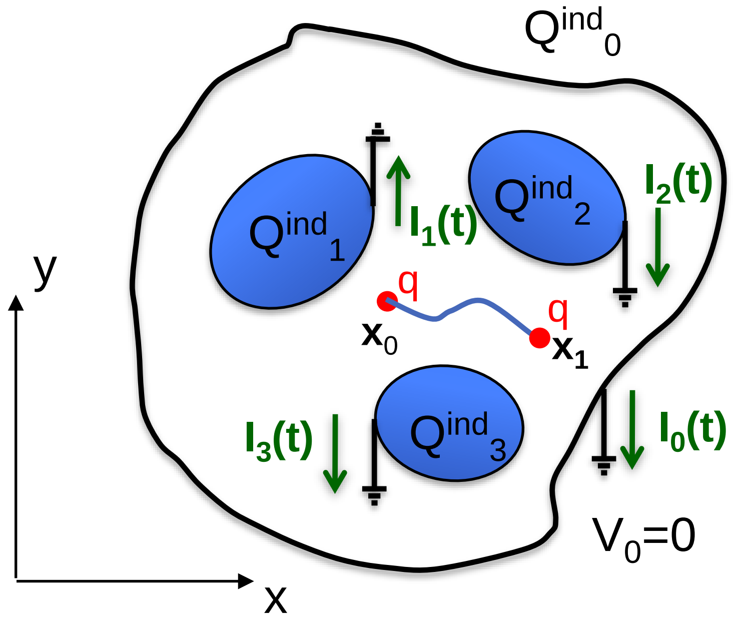
$$I^{ind}(t) = -\frac{dQ^{ind}(t)}{dt}$$

defines that the current arrow points away from the electrode.
A positive current refers to a reduction of charge on the electrode.

The signal is positive if:
Positive charge is moving from electrode to ground or
Negative charge is moving from ground to the electrode

The signal is negative if:
Negative charge is moving from electrode to ground or
Positive charge is moving from ground to the electrode

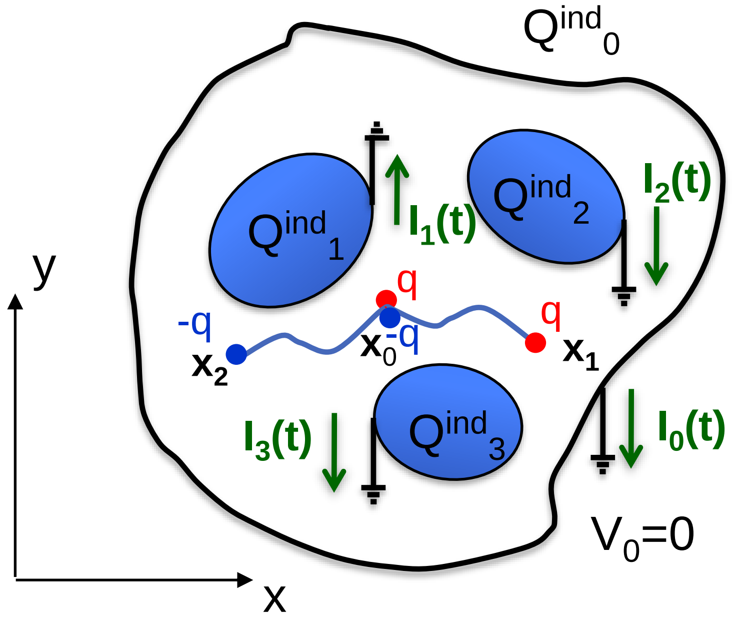
Theorem, total induced charge



The total charge flowing between the electrode and ground when the charge q is moving from point $\mathbf{x}_0=\mathbf{x}(t_0)$ to $\mathbf{x}_1=\mathbf{x}(t_1)$

$$Q_n = \int_{t_0}^{t_1} I_n^{ind}(t)dt = -\frac{q}{V_w}\int_{t_0}^{t_1} E_n(\mathbf{x}(t))\dot{\mathbf{x}}(t)dt = \frac{q}{V_w}\left[\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_0)\right]$$

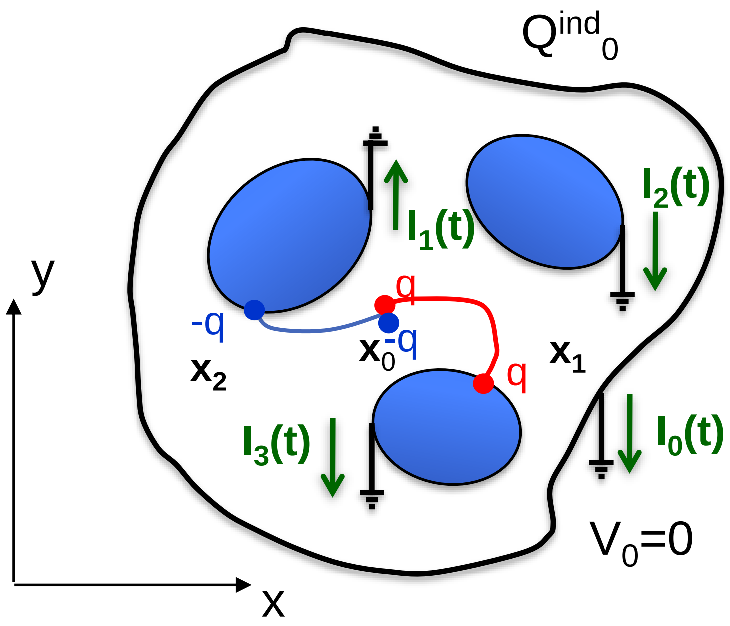
This charge is independent of the path between $\mathbf{x}_1$ and $\mathbf{x}_2$.



If a pair of charges q, -q is produced at position $\mathbf{x}_0$ and q is moving to $\mathbf{x}_1$ while –q is moving to $\mathbf{x}_2$, the charge flowing between electrode n and ground is

$$\begin{aligned} Q_n &= \int_{t_0}^{t_1} I_n^{ind}(t)dt \\ &= \frac{q}{V_w}\left[\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_0)\right] - \frac{q}{V_w}\left[\psi_n(\mathbf{x}_2) - \psi_n(\mathbf{x}_0)\right] \\ &= \frac{q}{V_w}\left[\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_2)\right] \end{aligned}$$

Theorem, total induced charge



$$\int_0^\infty I_1(t)dt = -q$$
$$\int_0^\infty I_2(t)dt = 0$$
$$\int_0^\infty I_3(t)dt = q$$

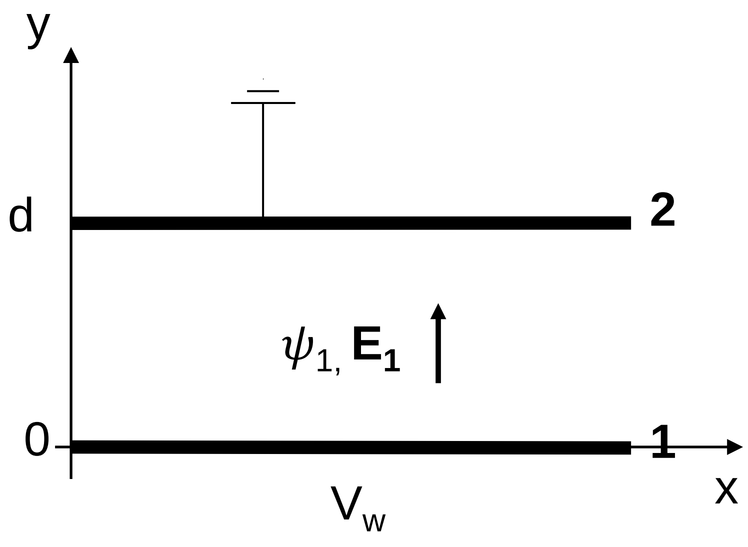
If a pair of charges q, -q is produced at position $\mathbf{x}_0$ and q is moving to $\mathbf{x}_1$ while –q is moving to $\mathbf{x}_2$, the charge flowing between electrode n and ground is

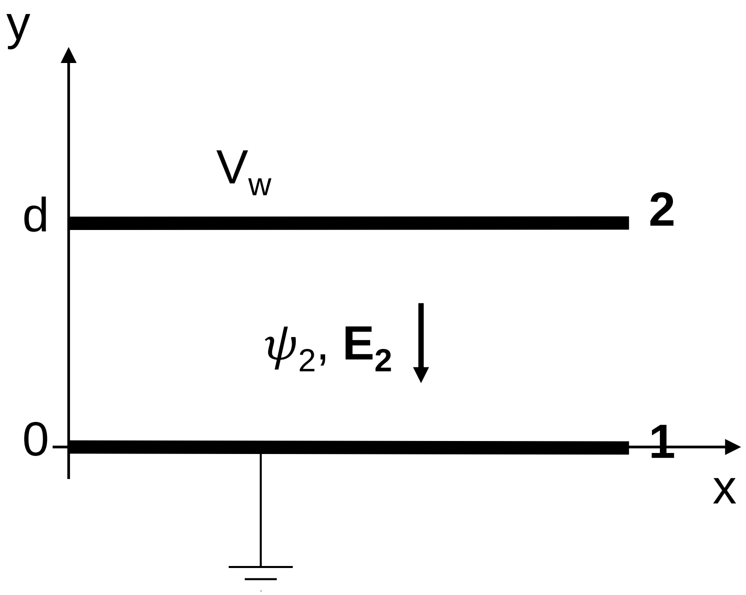
$$\begin{aligned} Q_n &= \int_{t_0}^{t_1} I_n^{ind}(t)dt \\ &= \frac{q}{V_w}\left[\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_0)\right] - \frac{q}{V_w}\left[\psi_n(\mathbf{x}_2) - \psi_n(\mathbf{x}_0)\right] \\ &= \frac{q}{V_w}\left[\psi_n(\mathbf{x}_1) - \psi_n(\mathbf{x}_2)\right] \end{aligned}$$

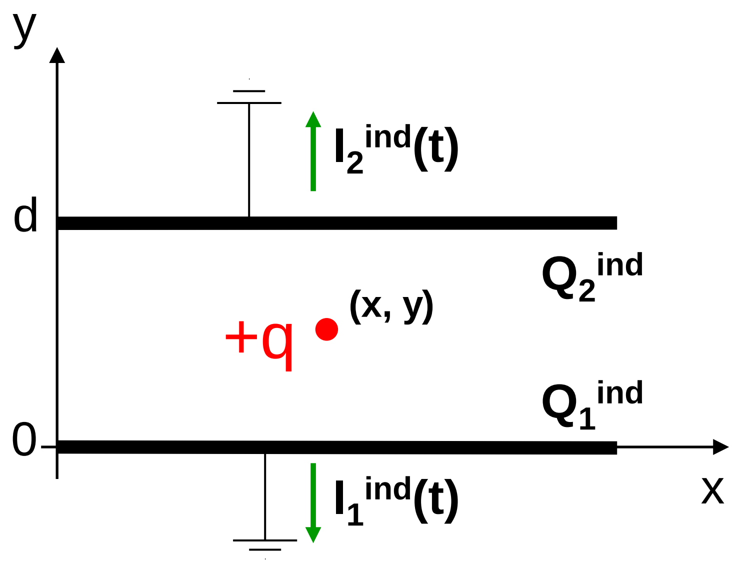
In case q moves to the surface of electrode n (where $\psi_n(\mathbf{x}) = V_w$) and –q to the surface of any other electrode (where $\psi_n(\mathbf{x})$ is 0), to total charge that has been flowing between the electrode and ground is equal to q.

If charges are created in pairs it holds that the total charge flowing between an electrode and ground is equal to the total charge that has arrived at this electrode, once ALL charges have arrived at the electrodes.

Parallel plate geometry







Weighting potential, electrode1

$$\psi_1(y) = V_w\left(1 - \frac{y}{d}\right)$$

$$E_1(y) = \frac{V_w}{d}$$

Weighting potential, electrode 2

$$\psi_2(y) = V_w\frac{y}{d}$$

$$E_2(y) = -\frac{V_w}{d}$$

Induced charge

$$Q_1^{ind}(y) = -\frac{q}{V_w}\psi_1(y) = -q\left(1 - \frac{y}{d}\right)$$

$$Q_2^{ind}(y) = -\frac{q}{V_w}\psi_2(y) = -q\frac{y}{d}$$

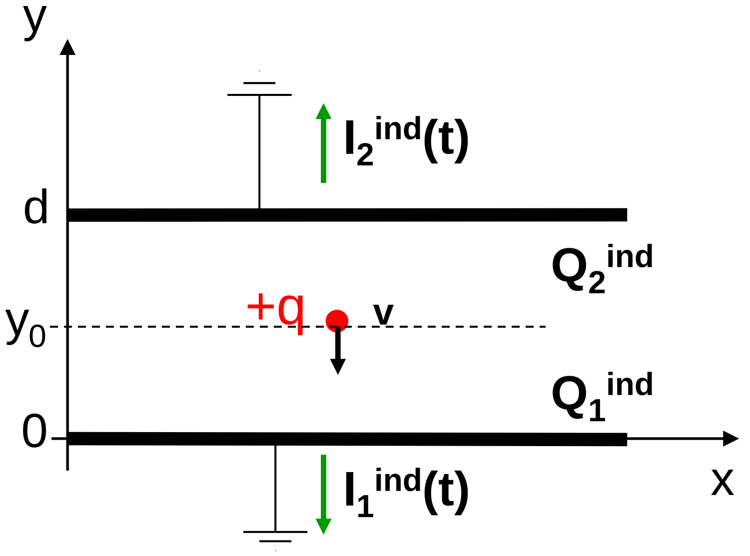
Sum of the weighting potentials

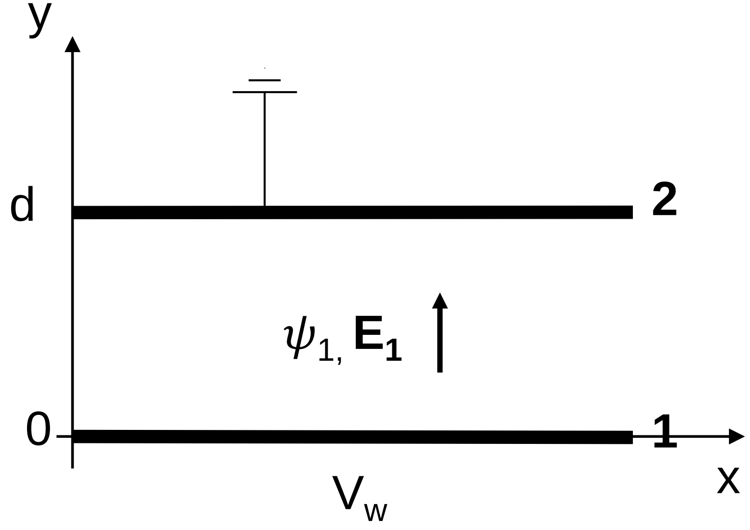
$$\psi_1(y) + \psi_2(y) = V_w$$

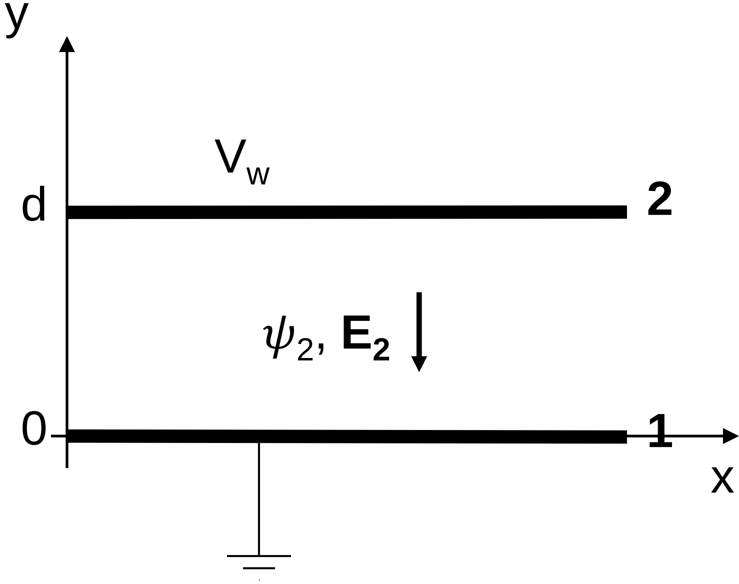
Sum of induced charges

$$Q_1^{ind} + Q_2^{ind} = -q$$

Parallel plate geometry







$$y(t) = y_0 - vt \qquad \frac{dy(t)}{dt} = -v \qquad 0 < t < \frac{y_0}{v}$$

$$I_1(t) = -\frac{q}{V_w} E_1(y(t)) \frac{dy}{dt} = q\frac{v}{d}$$

$$I_2(t) = -\frac{q}{V_w} E_2(y(t)) \frac{dy}{dt} = -q\frac{v}{d}$$

$$I_1(t) + I_2(t) = 0$$

Total induced charge on electrode 1

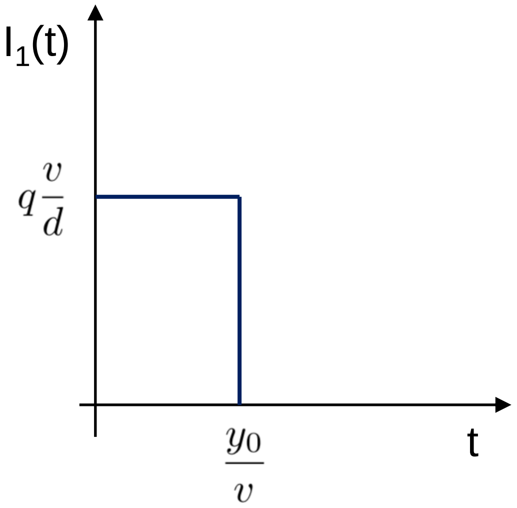
$$Q_{1tot}^{ind} = \int_0^{y_0/v} I_1(t)dt = \frac{q}{V_w}\left[\psi_1(0) - \psi_1(y_0)\right] = q\frac{y_0}{d}$$

Weighting field, electrode1

$$E_1(y) = \frac{V_w}{d}$$

Weighting field, electrode 2

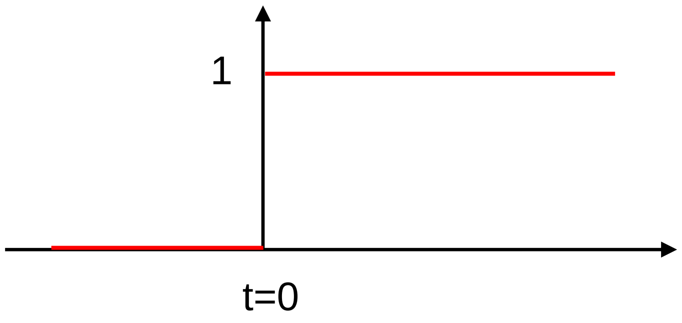
$$E_2(y) = -\frac{V_w}{d}$$



Heaviside step function

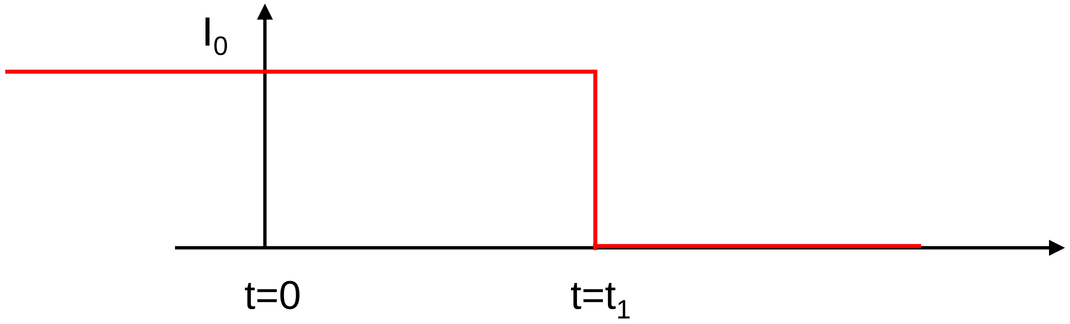
The Heaviside step function is defined as

$$\Theta(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$



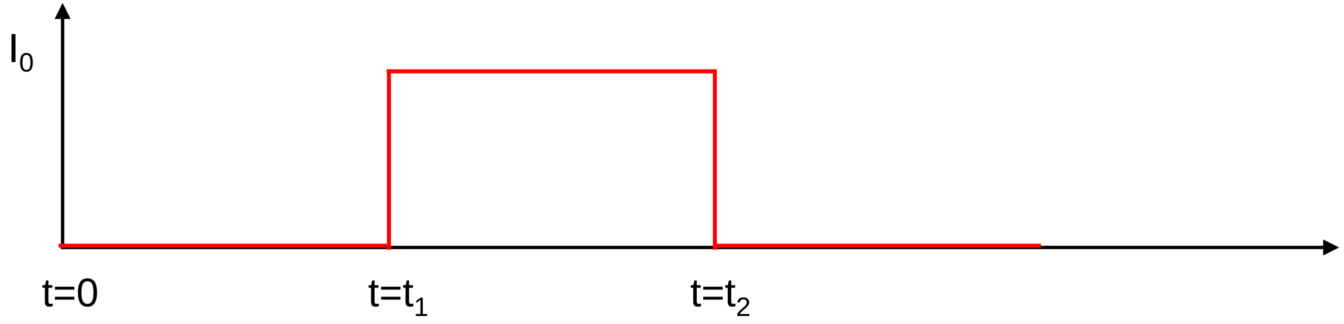
In case we have a constant current signal with amplitude I_0 starting at t=0 and lasting until $t=t_1$ we have

$$I(t) = I_0 \Theta(t_1 - t)$$

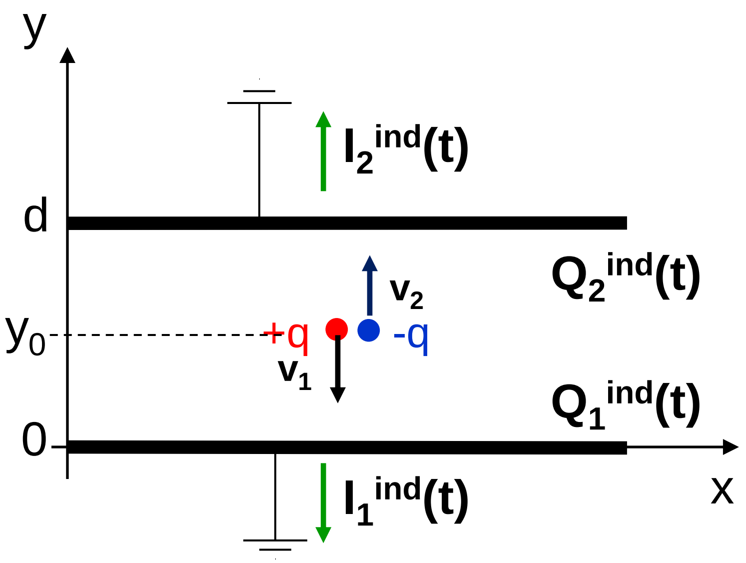


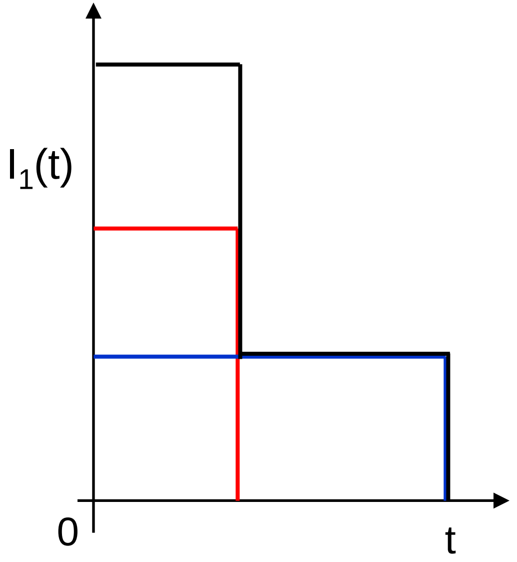
In case we have a constant current signal with amplitude I_0 starting at $t=t_1$ and lasting until $t=t_2$ we have

$$I(t) = I_0 \left[\Theta(t_2 - t) - \Theta(t_1 - t)\right]$$



Parallel plate geometry





Two charges +q, -q moving from y_0 to the electrodes with velocities v_1 and v_2, arriving at the electrodes at times t_1 and t_2

$$t_1 = \frac{y_0}{v_1} \qquad t_2 = \frac{d - y_0}{v_2}$$

Induced currents

$$I_1(t) = q\frac{v_1}{d}\Theta(t_1 - t) + q\frac{v_2}{d}\Theta(t_2 - t) \qquad I_2(t) = -I_1(t)$$

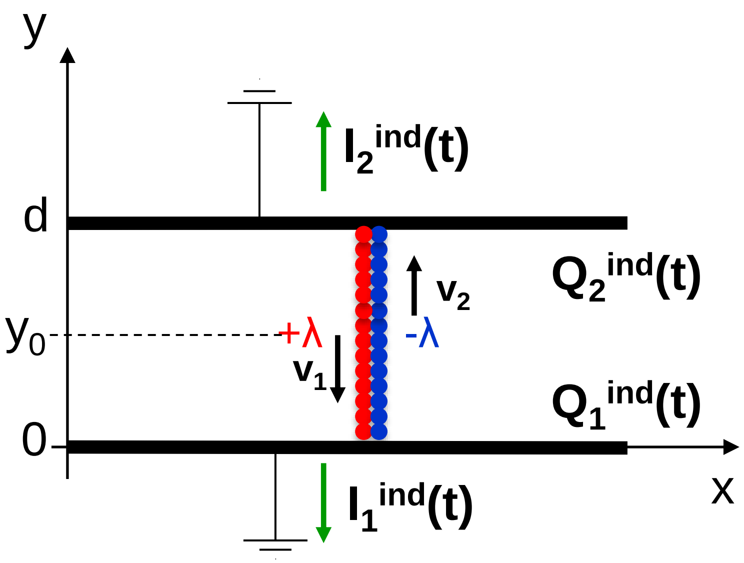
$$I_2(t) = -I_1(t)$$

Total induced charges

$$\begin{aligned} Q_{1tot}^{ind} &= \int_0^\infty I_1(t)dt \\ &= \frac{q}{V_w}\left[\psi_1(0) - \psi_1(y_0)\right] - \frac{q}{V_w}\left[\psi_1(d) - \psi_1(y_0)\right] \\ &= q \end{aligned}$$

In all physics processes, pairs of charges with opposite sign are produced at the same position, which results in the fact that the total induced charge is equal to the charge that has arrived at the electrode, once ALL charges have arrived at the electrodes.

Parallel plate geometry

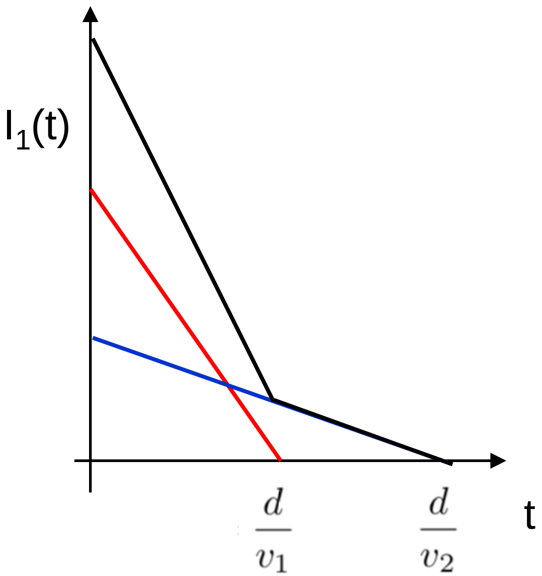


Charge particles passing sensors leave a trail of positive and negative charges (electrons/Ions, electrons/holes). Assuming a uniform distribution along the track we have two 'line charges' + λ, - λ (C/cm), moving with velocities v_1 and v_2, with the last charges arriving at the electrode at t_1 and t_2

$$t_1 = \frac{d}{v_1} \qquad t_2 = \frac{d}{v_2}$$

The induced current due to the movement of these charges is the sum of two 'triangles', Q= λd

$$\begin{aligned} I_1(t) &= \lambda v_1 \left(1 - \frac{t}{t_1}\right) \Theta(t - t_1) + \lambda v_2 \left(1 - \frac{t}{t_2}\right) \Theta(t - t_2) \\ &= \frac{Q}{t_1} \left(1 - \frac{t}{t_1}\right) \Theta(t - t_1) + \frac{Q}{t_2} \left(1 - \frac{t}{t_2}\right) \Theta(t - t_2) \end{aligned}$$



The total induced charge on electrode 1 is

$$\begin{aligned} Q_{1tot}^{ind} &= \int_0^\infty I_1(t)dt \\ &= \frac{\lambda d}{2} + \frac{\lambda d}{2} \\ &= \lambda d \\ &= Q \end{aligned}$$

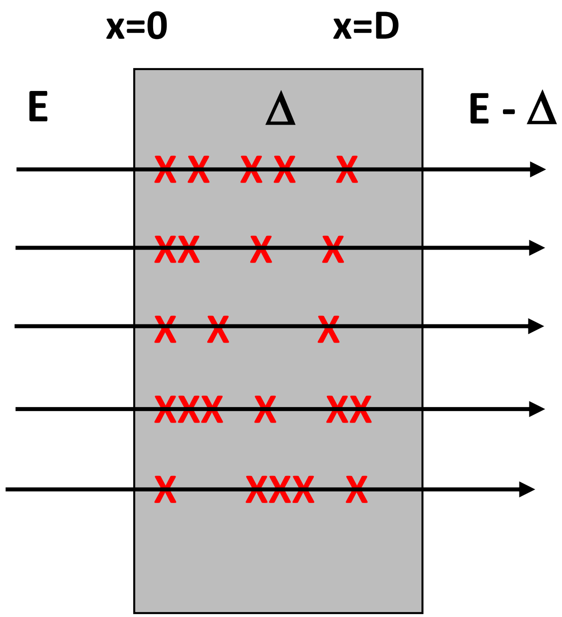
Charge Deposit

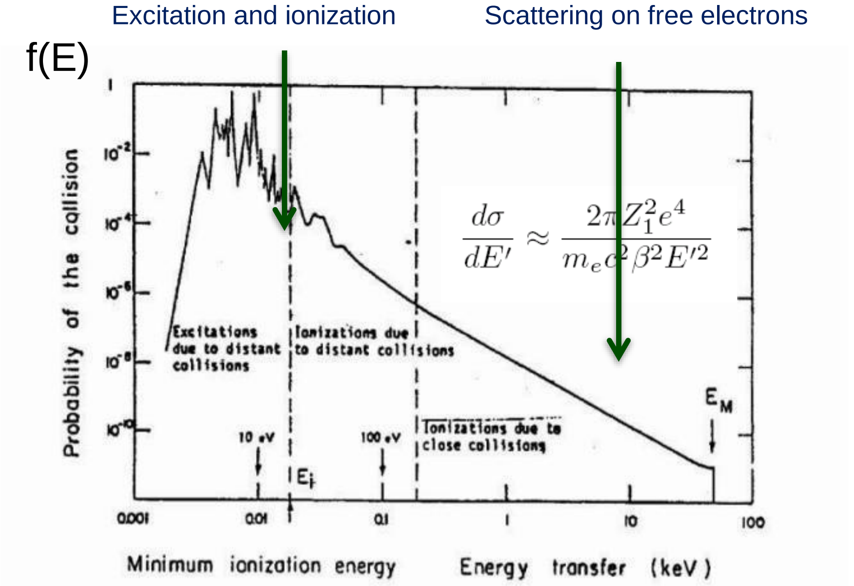
Fluctuations of the Energy Loss

A charged particle passing through a piece of material will produce a trail of ionization.

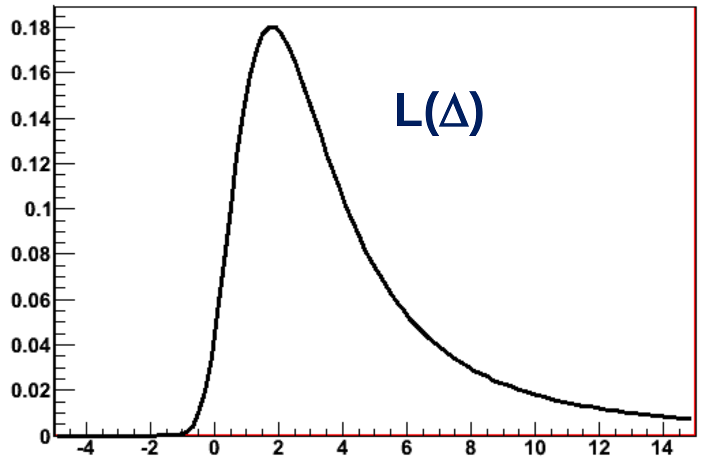
Since the individual interactions with the atomic electrons are independent, the number of primary interactions follows a Poisson distribution.

The probability f(E) for transferring an energy E to the atomic electron in an interaction is given by the Rutherford crossection at large energy transfers and by the atomic atomic shell structure at low energy transfers.





Convolution



Landau distribution $L(x)$ according to

$$L(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} \exp\left[sx + s\ln s\right] ds \quad (A.1)$$

$$= \frac{1}{\pi}\int_0^{\infty} \exp(-\pi/2\, t)\cos(tx + t\ln t)dt \quad (A.2)$$

$$= \frac{1}{\pi}\int_0^{\infty} \exp\left[-tx - t\ln t\right]\sin(\pi t)dt \quad (A.3)$$

Expression (A.2) is well suited for evaluation for $x < 0$, while eq. (A.3) is well suited for evaluation for $x > 0$. For large values of x the Landau distribution approximates to

$$L(x) \approx \frac{1}{x^2} \quad (A.4)$$

$$x = \frac{E}{\overline{n}\epsilon} + C_\gamma - 1 - \ln\overline{n} \qquad \overline{n} = \frac{N_A \rho Z_2 k D}{A\epsilon}$$

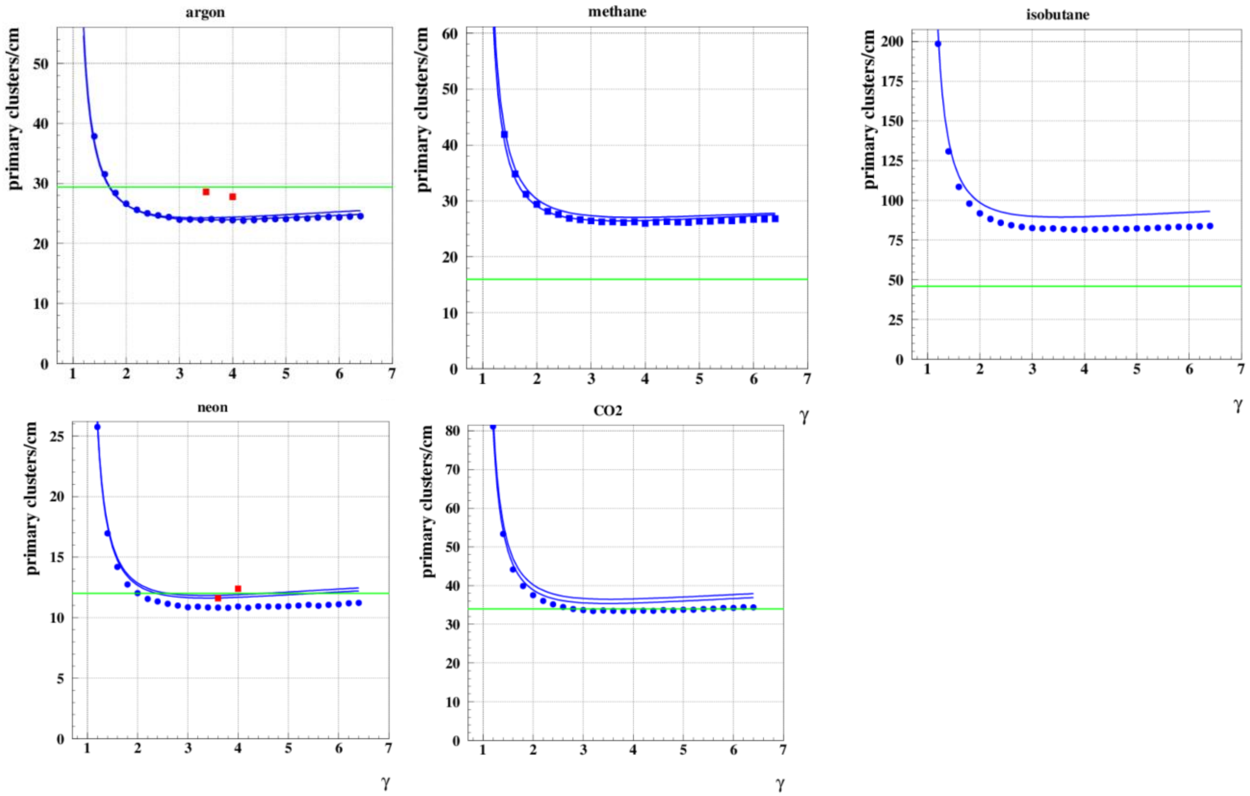
$$\ln\epsilon = \ln\frac{I^2}{E_{max}} + 2\beta^2$$

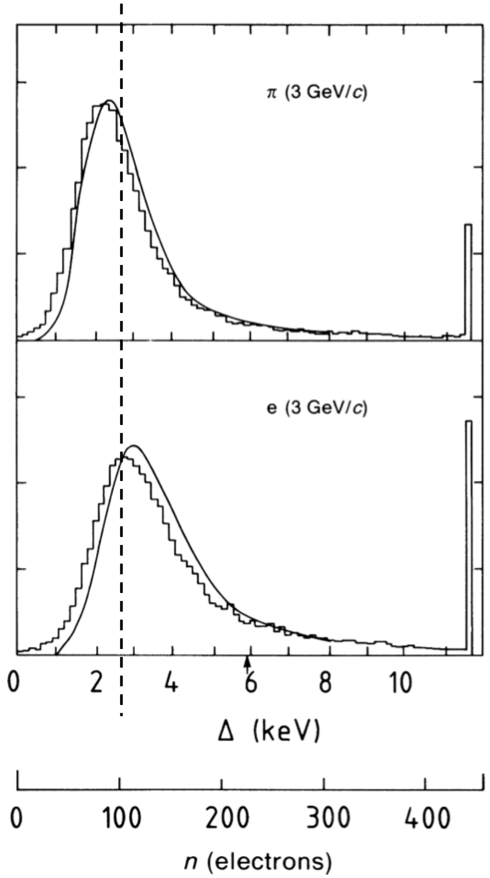
$$E_{MP} = \overline{n}\epsilon(x_0 + 1 - C_\gamma + \ln\overline{n}) \approx \overline{n}\epsilon(0.2 + \ln\overline{n})$$

$$\Delta E_{FWHM} = 4.02\,\overline{n}\epsilon = \frac{4.02\, N_A \rho Z_2 k D}{A}$$

$$\frac{\Delta E_{FWHM}}{E_{MP}} = \frac{4.02}{0.2 + \ln\overline{n}}$$

Charge deposit in gases





The solid lines represent measurements from

[27] F. Rieke, W. Prepejchal, Phys. Rev. A 6 (1972) 1507.

The points represent results from a PAI model

1.5cm Argon MP approx. 67/cm

Average distance between clusters

Argon	0.41mm
Neon	0.83mm
Isobutane	0.12mm

For single particle detection, internal gain is used in gas detectors

Transport of electrons and ions in gases

Transport of Electrons in Gases: Drift-velocity

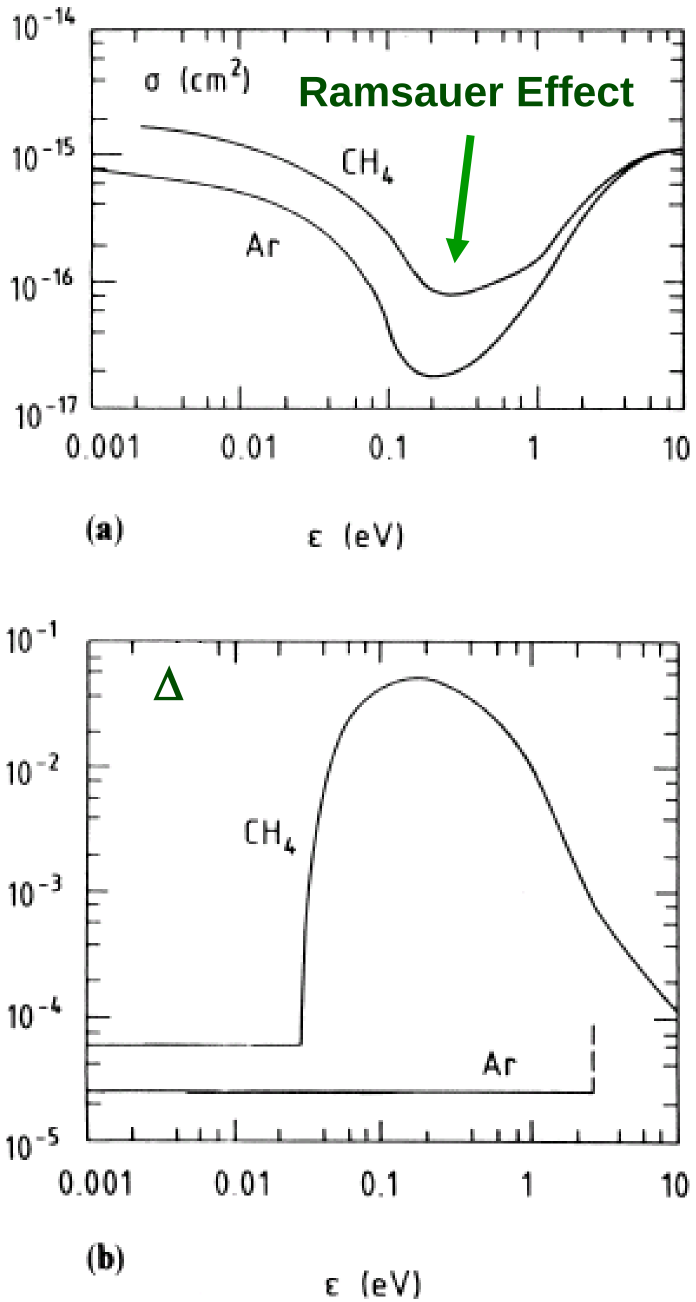
Electrons are completely ‘randomized’ in each collision. The actual drift velocity v along the electric field is quite different from the average velocity u of the electrons i.e. →about 100 times smaller.

The velocities v and u are determined by the atomic crossection $\sigma(\varepsilon)$ and the fractional energy loss $\Delta(\varepsilon)$ per collision (N is the gas density i.e. number of gas atoms/m^3, m is the electron mass.):

$$v = \sqrt{\frac{eE}{mN\sigma}\sqrt{\frac{\Delta}{2}}} \qquad u = \sqrt{\frac{eE}{mN\sigma}\sqrt{\frac{2}{\Delta}}} \qquad \frac{u}{v} = \sqrt{\frac{2}{\Delta}}$$

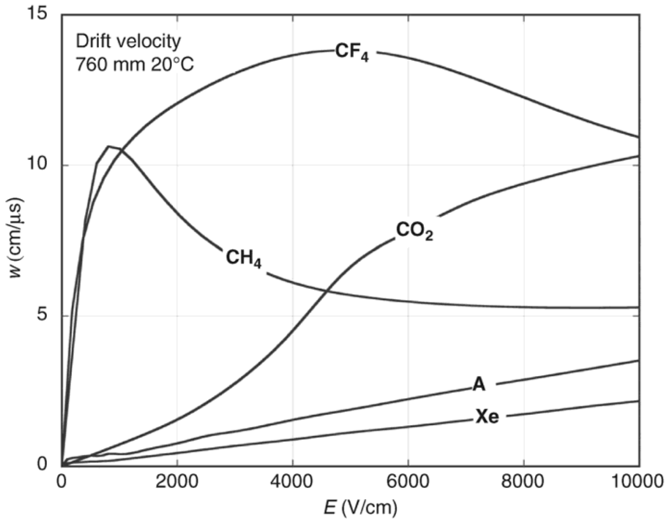
Because $\sigma(\varepsilon)$und $\Delta(\varepsilon)$ show a strong dependence on the electron energy in the typical electric fields, the electron drift velocity v shows a strong and complex variation with the applied electric field.

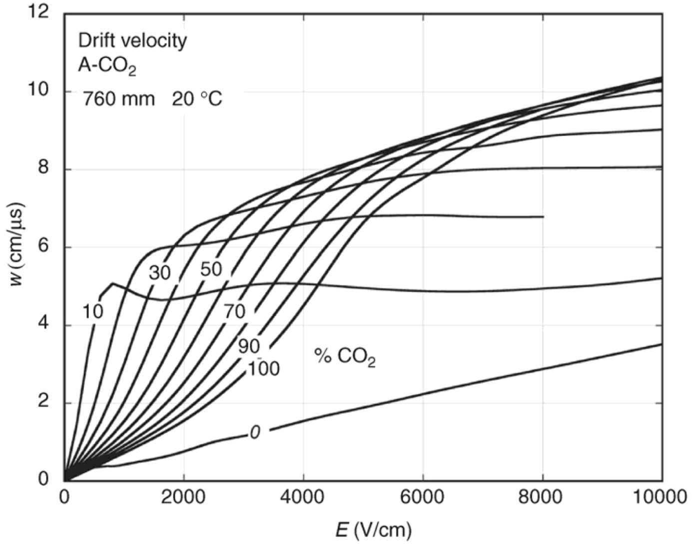
v is depending on E/N: doubling the electric field and doubling the gas pressure at the same time results in the same electric field.

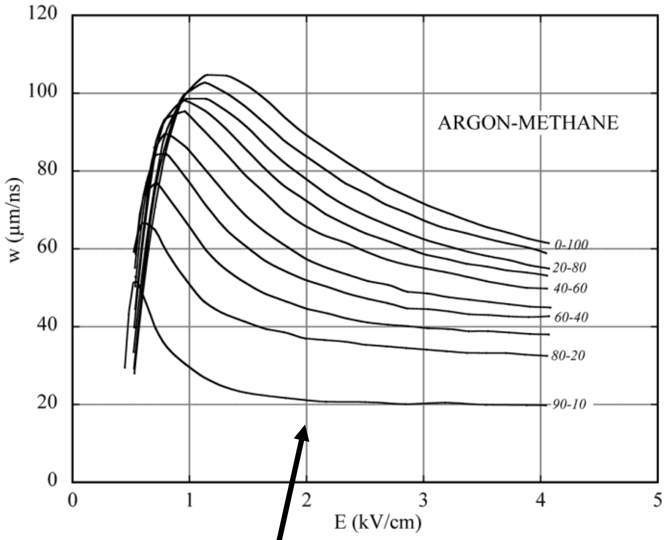


Transport of Electrons and Ions in Gases

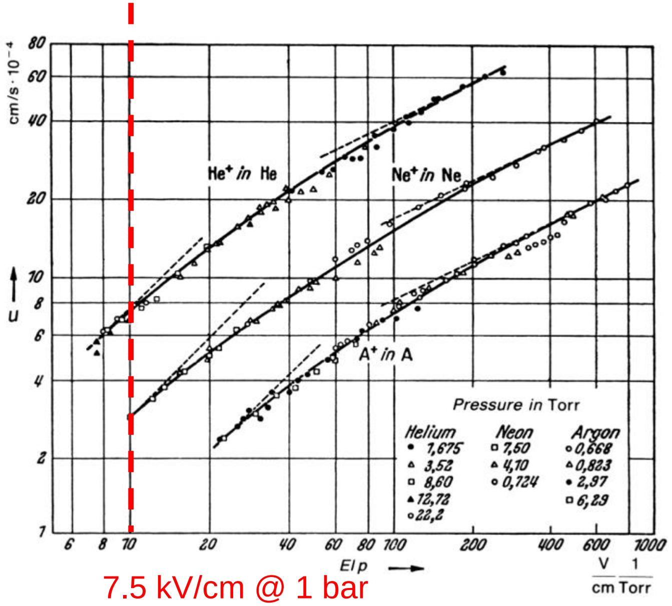
Typical electron drift velocities are v=20-140um/ns (20 000-140 000m/s). The microscopic velocity u is about ca. 100mal larger.







Typical ion drift velocities are around 1000 times smaller than electron drift velocities.



7.5 kV/cm @ 1 bar

Gas	Ion	Mobility ($cm^2 V^{-1} s^{-1}$)
He	He^+	10.40 ± 0.10
Ne	Ne^+	4.14 ± 0.2[a]
Ar	Ar^+	1.535 ± 0.007
Kr	Kr^+	0.96 ± 0.09
Xe	Xe^+	0.57 ± 0.05

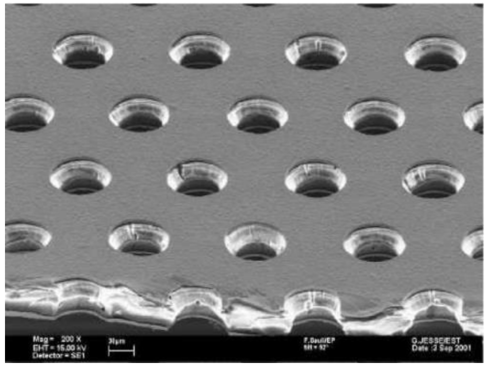
[a] Average over several measurements

at low fields ($u \propto E$) and at high fields ($u \propto \sqrt{E}$)

Argon Ions at 2kV/cm → 0.03um/ns
1000x slower than electrons

GEM Detector

Gas Electron Multiplier GEM



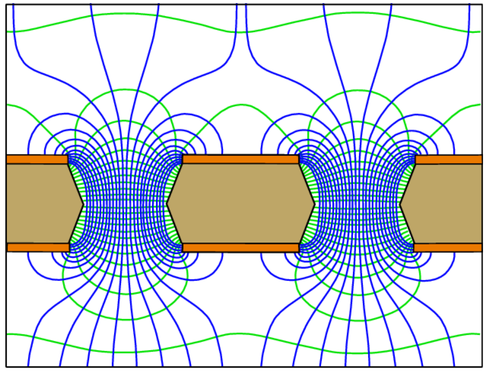
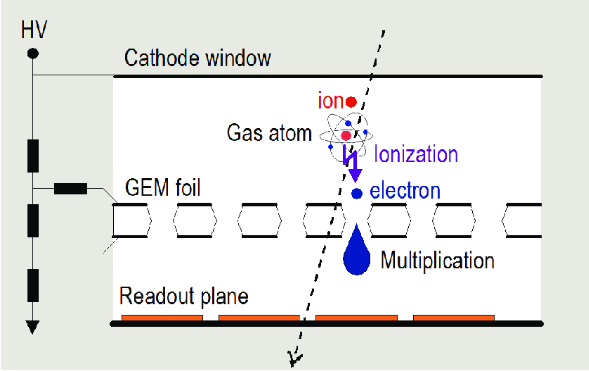
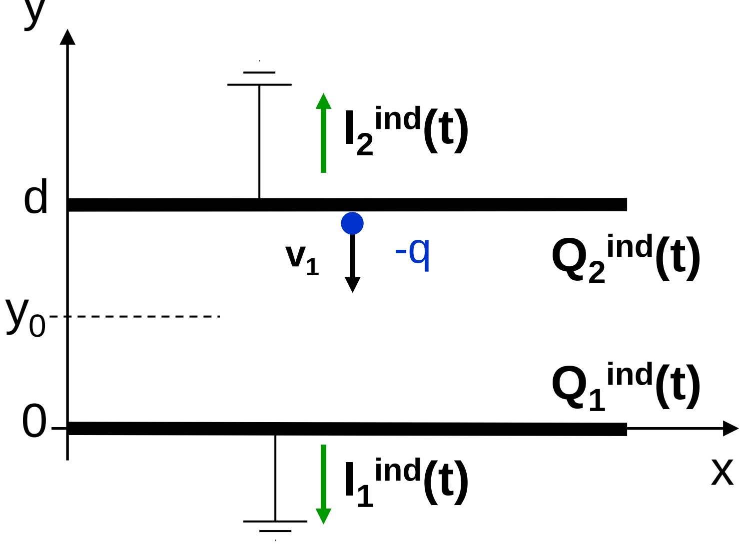
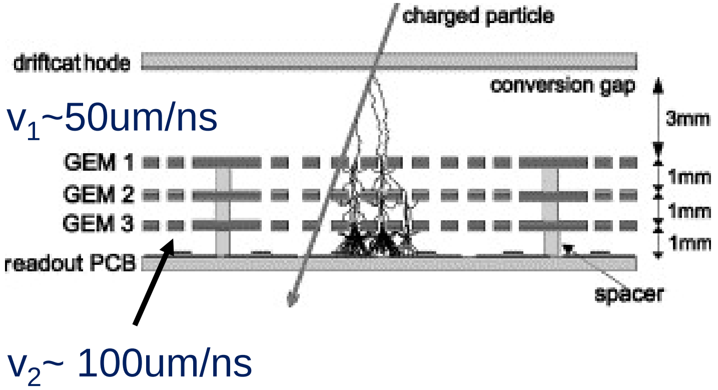
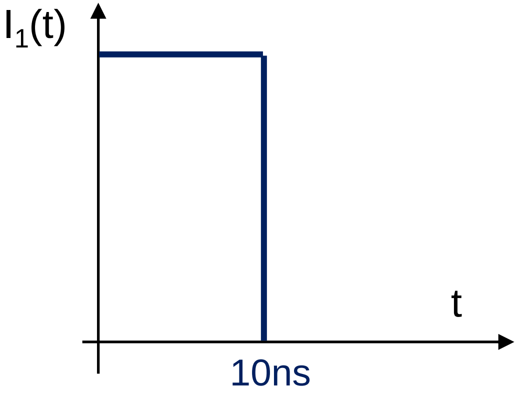


Fig. 2: Electric field lines in a GEM detector









Single electrons moving through a GEM hole are multiplied in the strong electric field.

Ions are moving to the top side of the GEM or back into the transfer gap.

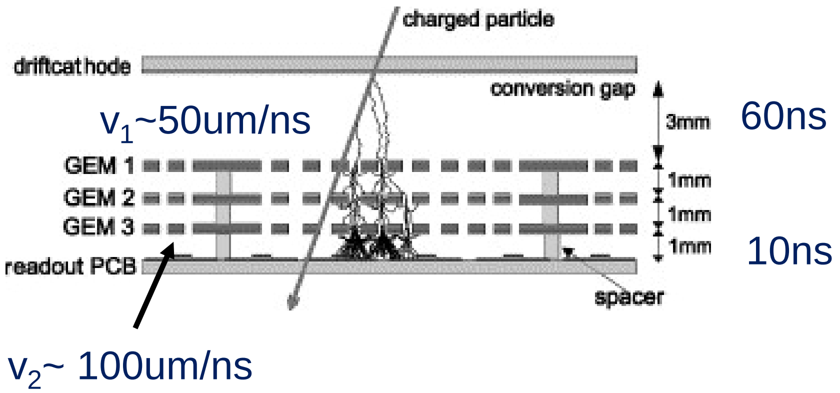
Electrons are exiting the holes and move to the next amplification stage.

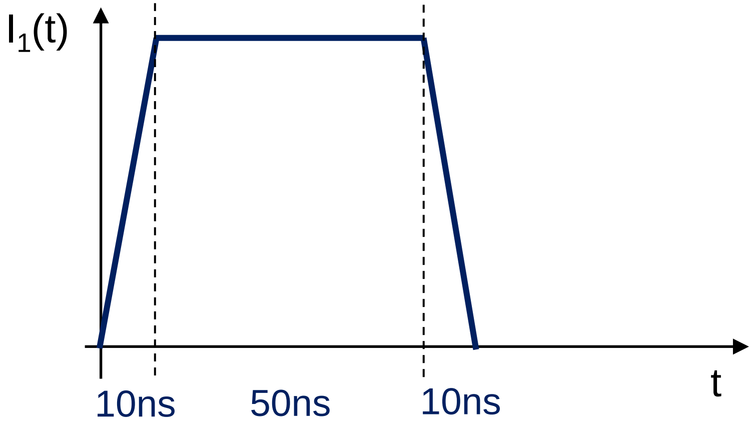
In the last gap the ‘Induction gap’ the electrons are moving to the readout electrodes and induce the signal.

The geometry is equivalent to a parallel plate chamber with only electrons moving through the entire gap.

The signal from a single electron starting in the conversion gap is a ‘box’ with a duration equivalent to the electron transit time in the induction gap approx. 10ns.

Gas Electron Multiplier GEM





The conversion gap above the top GEM is the volume where charge particles deposit e+e- pairs.

Typical thickness is 3-15mm (2.5m for the ALICE TPC !)

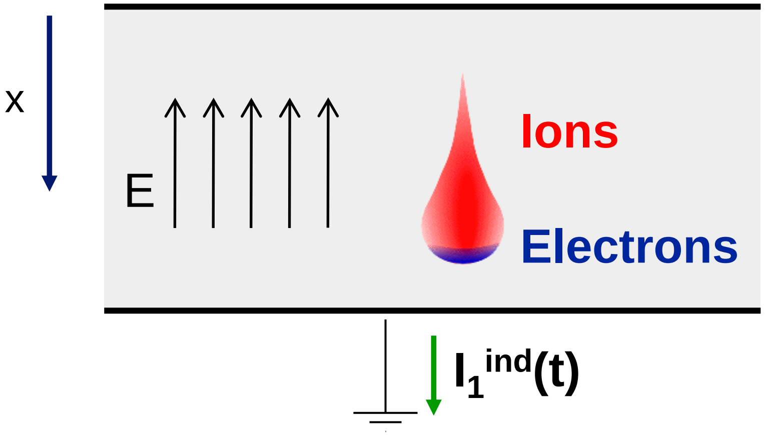
For a continuous charge deposit the signal has trapezoid shape where the length is dominated by the drift time in the conversion gap.

Parallel field avalanches

Gas avalanche multiplication

At sufficiently high electric fields (100kV/cm) the electrons gain energy in excess of the ionization energy → secondary ionzation etc. etc. → exponential increase → avalanche → Townsend coefficient α

$$N_e(x) = e^{\alpha x} \qquad N_e(t) = e^{\alpha v t}\Theta\left(d/v_e - t\right)$$



The current induced by these moving electrons is

$$I_e^{ind}(t) = -\frac{e_0 v_e}{d} N_e(t)$$

The ions move from the point of creation in opposite direction from the point of creation and the induced current is

$$n_I(x)dx = \alpha e^{\alpha x} dx \qquad dI_I^{ind}(x,t)dx = -\frac{e_0 v_I}{d}\alpha e^{\alpha x}[\Theta(x/v_I + x/v_e - t) - \Theta(x/v_e - t)]dx$$

Gain 10^3: ad=7
Gain 10^4: ad=9.2

$$I_I^{ind}(t) = \int_0^d dI_I^{ind}(x,t)dx = -\frac{e_0 v_I}{d}\left[(e^{\alpha d} - e^{\alpha v_e v_I t/(v_e+v_I)})\Theta(d/v_e + d/v_I - t) - (e^{\alpha d} - e^{\alpha v_e t})\Theta(d/v_e - t)\right]$$

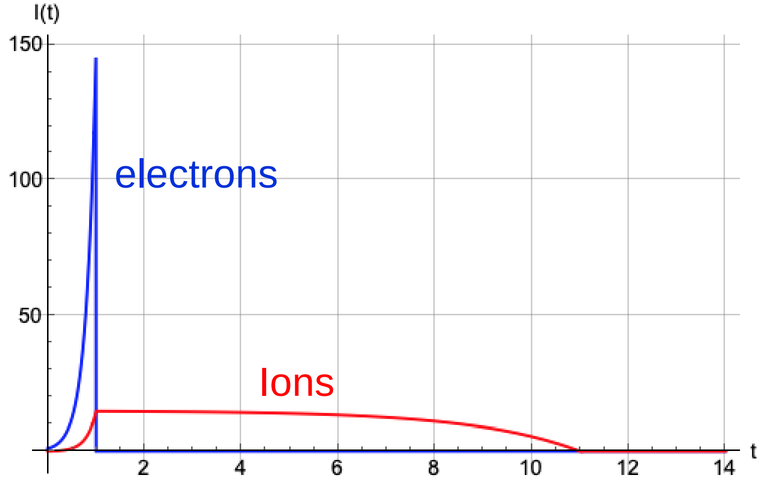
The total charge induced by the electrons and the ions is

$$Q_e^{ind} = \int_0^{d/v_e} I_e^{ind}(t)dt = -e_0\frac{e^{\alpha d} - 1}{\alpha d}$$

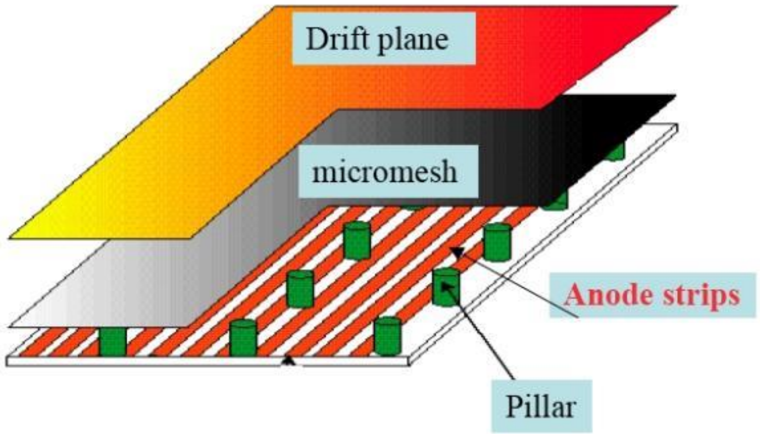
$$Q_I^{ind} = \int_0^{d/v_I} I_e^{ind}(t)dt = -e_0\frac{e^{\alpha d}(\alpha d - 1) + 1}{\alpha d}$$

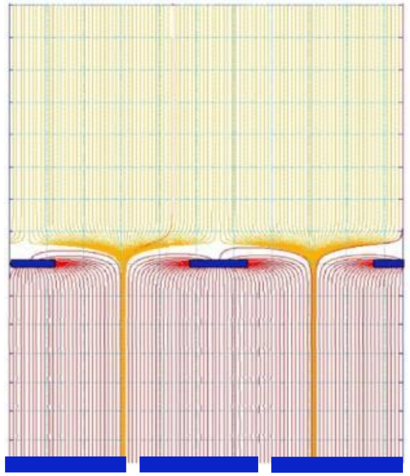
$$Q_e^{ind} + Q_I^{ind} = -e_0 e^{\alpha d}$$

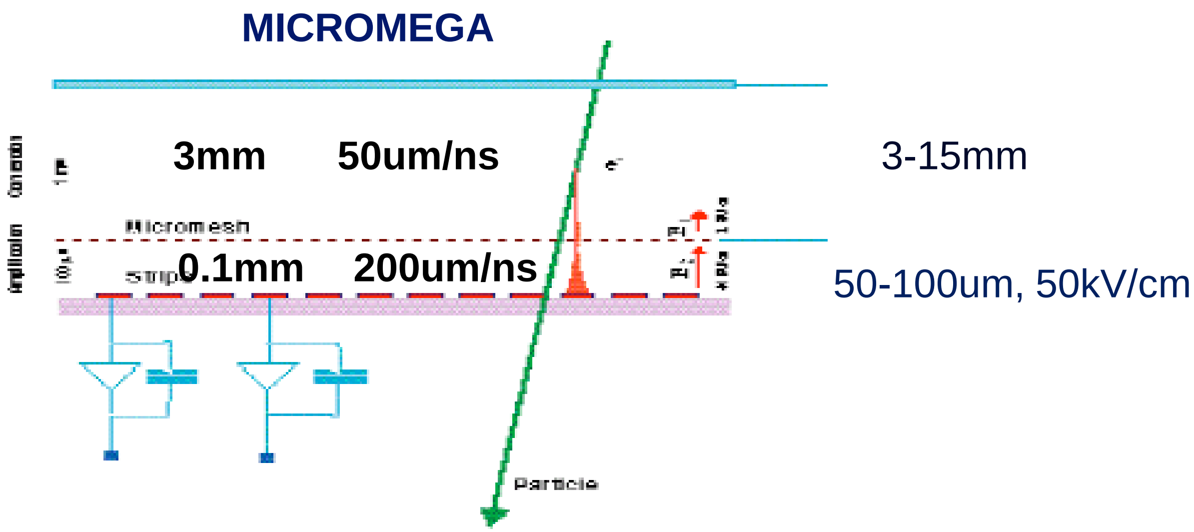
$$\frac{Q_e}{Q_I} = \frac{e^{\alpha d} - 1}{(\alpha d - 1)e^{\alpha d} + 1} \approx \frac{1}{\alpha d - 1} \qquad \frac{Q_e}{Q_e + Q_I} \approx \frac{1}{\alpha d} \quad \text{for} \quad e^{\alpha d} \gg 1$$



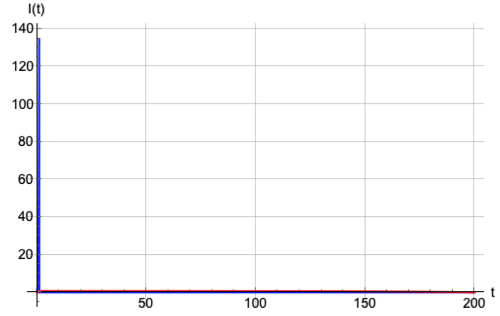
MicroMeshGAS detectors



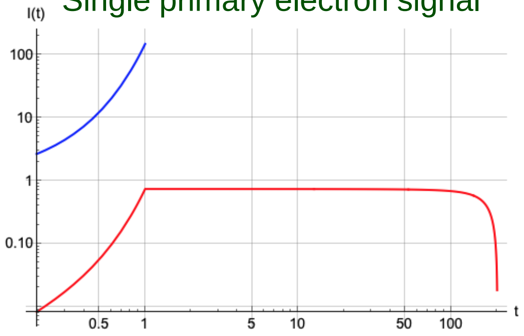




Single primary electron signal



Single primary electron signal



Electrons movement in the induction gap takes about 0.1mm/v_1=0.5ns.

Collecting all electrons from the drift gap takes e.g. 3mm/v_1=60ns.

The MICROMEGA electron signal has a length of about 60ns.

Ion movement – e.g. Argon Ions take 130ns for 50kV/cm and 100um gap, so the total length of the ions component is around 180ns.

When using ‘fast’ electronics one does not integrate the full charge → ballistic deficit.

Wire Chambers

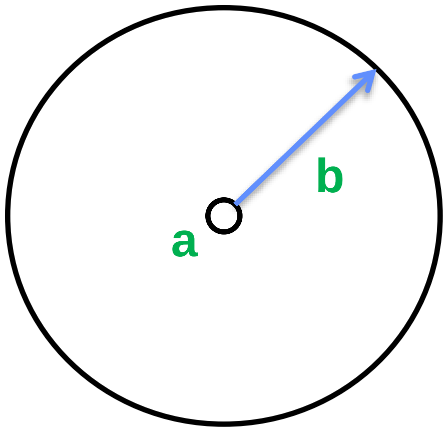
Wire Chamber Signals

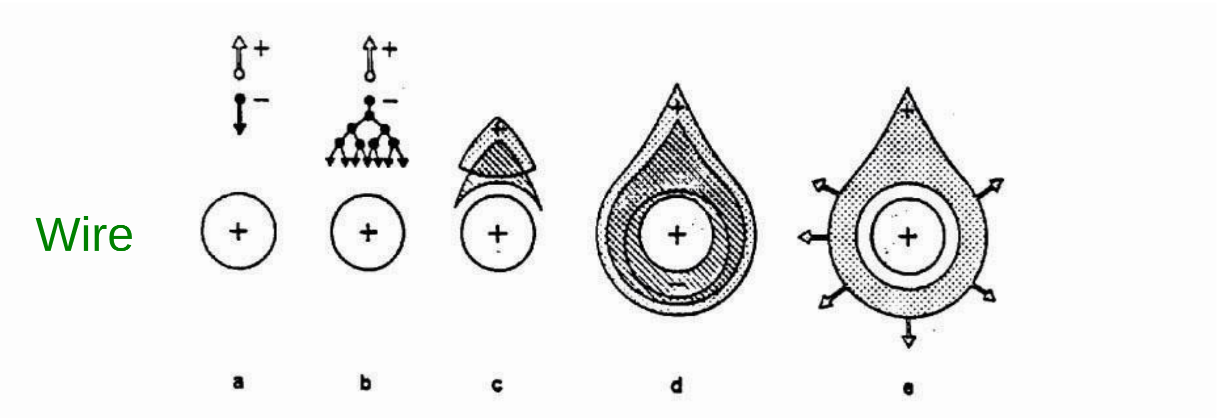
Wire with radius (10-25μm) in a tube of radius b (1-3cm):

$$\varphi(r) = \frac{V}{\ln(a/b)} \ln(r/b) \qquad E_r(r) = \frac{V}{\ln(b/a)} \frac{1}{r}$$

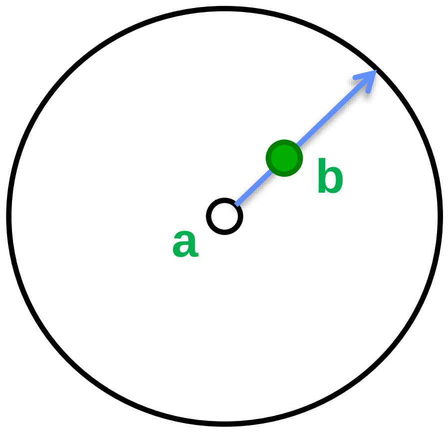
Electric field close to a thin wire (100-300kV/cm). E.g. V_0=1000V, a=10μm, b=10mm, E(a)=150kV/cm

Electric field is sufficient to accelerate electrons to energies which are sufficient to produce secondary ionization → electron avalanche → signal.





Wire Chamber Signals



The electrons are produced very close to the wire, so as a first approximation we can assume that the signal is only due to N_{tot} ions moving from the wire surface to the tube wall:

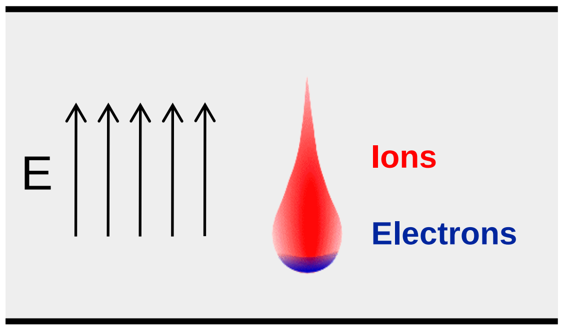
$$\varphi(r) = \frac{V}{\ln(a/b)} \ln(r/b) \qquad E_r(r) = \frac{V}{\ln(b/a)} \frac{1}{r}$$

Ions move with a velocity proportional to the electric field. $v(r) = \mu E(r)$

$$\frac{dr(t)}{dt} = \mu \frac{V}{\ln(b/a)} \frac{1}{r(t)} \quad \rightarrow \quad r(t) = a\sqrt{1 + t/t_0} \qquad 0 < t < t_{max}$$

$$t_0 = \frac{a^2 \ln(b/a)}{2\mu V} \qquad t_{max} = t_0 \left(\frac{b^2}{a^2} - 1 \right)$$

Weighting field of the wire: Remove charge and set wire to V_w while grounding the tube wall.



$$\psi_1(r) = -\frac{V_w \; ln(r/b)}{\ln(b/a)} \qquad E_1(r) = \frac{V_w}{r \ln(b/a)}$$

The induced current is therefore

$$I_1^{ind}(t) = -\frac{N_{tot} e_0}{V_w} E_1 \left[r(t) \right] \frac{dr(t)}{dt} = -\frac{N_{tot} e_0}{2 \ln(b/a)} \frac{1}{t + t_0}$$

Wire Chamber Signals

$$I_1^{ind}(t) = -\frac{N_{tot}e_0}{V_w} E_1\left[r(t)\right] \frac{dr(t)}{dt} = -\frac{N_{tot}e_0}{2\ln(b/a)} \frac{1}{t+t_0}$$

$$Q_1^{ind}(t) = \int_0^t I_1^{ind}(t')dt' = -\frac{N_{tot}e_0}{2\ln(b/a)} \ln\left(1+\frac{t}{t_0}\right)$$

$$\frac{Q_1^{ind}(t)}{Q_{tot}} = \frac{Q_1^{ind}(t)}{-N_{tot}e_0} = \frac{1}{2\ln(b/a)} \ln\left(1+\frac{t}{t_0}\right)$$

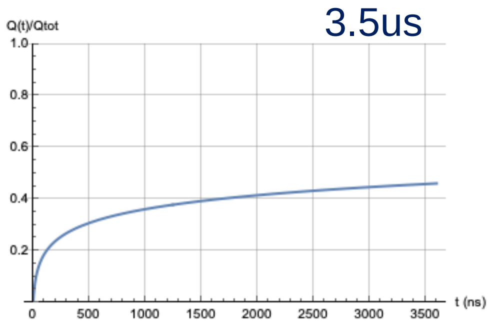
ATLAS muon drift tubes

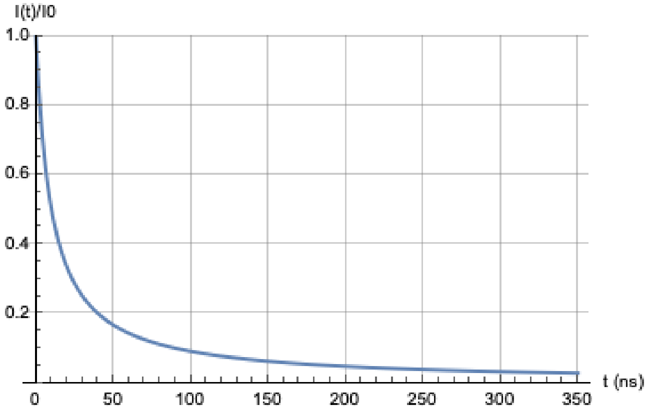
V_0...voltage on the wire $\approx 3500V$

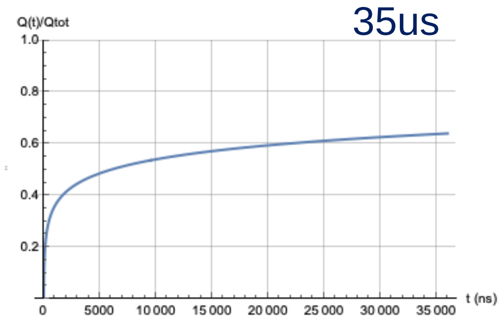
a...wire radius $= 25\mu m$

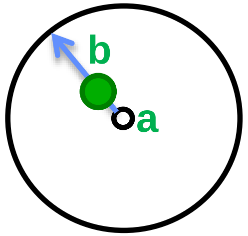
b...tube radius $= 1.46cm$

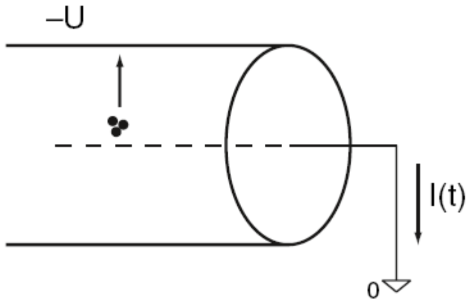
$t_0 = 11ns \qquad t_{max} = 3.73msec$







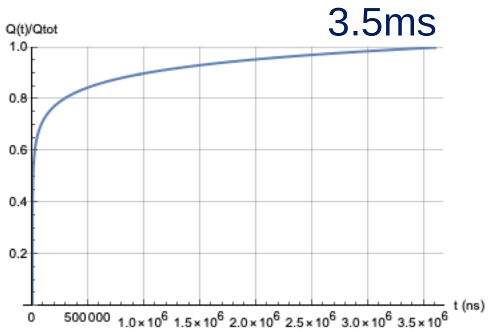




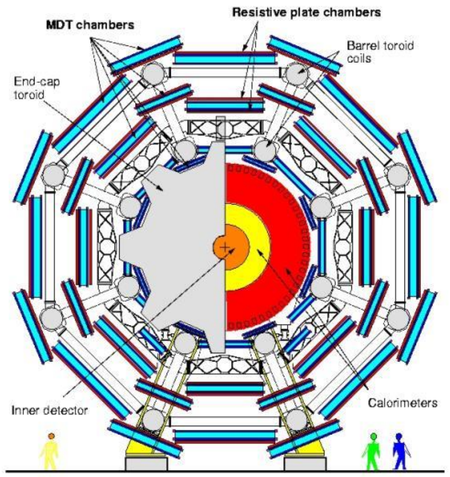
The signal from a single primary electron has $1/(t+t_0)$ shape with a very long tail …

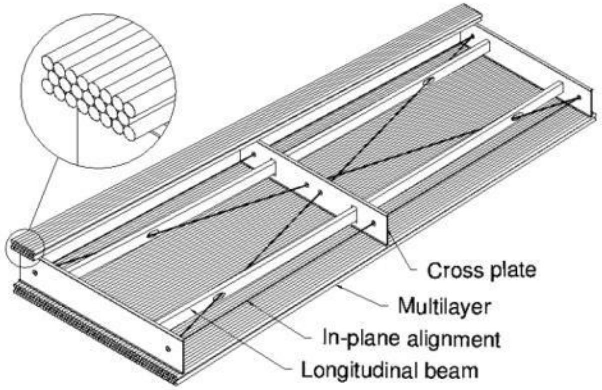
Typically only a small fraction (e.g. 10%) of the total avalanche charge is induced during the electronics integration time.

Ballistic deficit. In Micropattern detectors one can integrate all the charge → 10 times lower gas gain for the same signal.



ATLAS muon system drift tubes





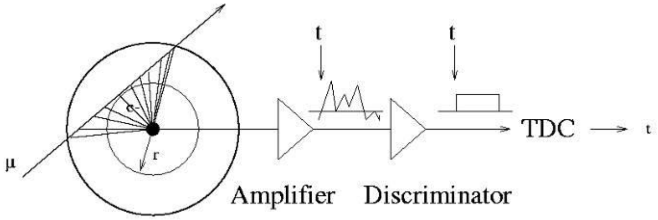
Tubes of 3cm diameter are assembled into chambers.

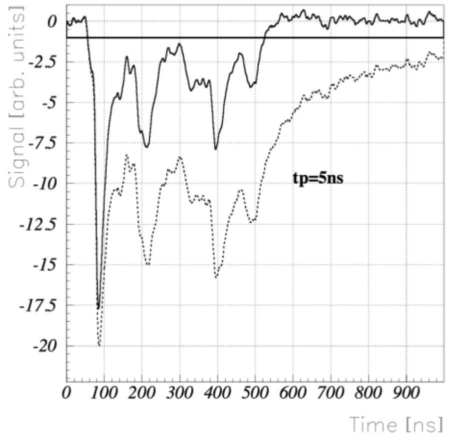
The first electrons arriving at the wire determine the distance of the track from the wire → drift tube.

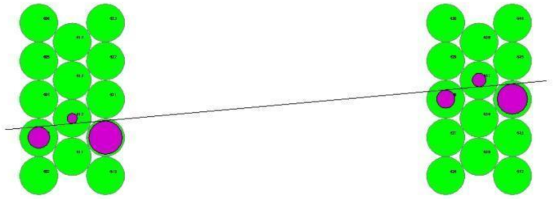
The last arriving electrons are originating from r=1.5cm, so the last electrons always arrive at the same time.

The long signal tail needs dedicated electronics filtering to ensure limited deadtime.

80um position resolution over a few thousand m^2 area with only 330 000 channels !







Detector with Resistive Elements

Quasi-static approximation of Maxwell's equations

Assuming a conductivity sigma of the material we have a current according to

$$\mathbf{j}(\mathbf{x},t) = \sigma(\mathbf{x})\mathbf{E}(\mathbf{x},t)$$

Maxwell's equations for this situation

$$\begin{aligned}
\nabla\mathbf{D}(\mathbf{x},t) &= \rho(\mathbf{x},t) \qquad \mathbf{D}(\mathbf{x},t) = \varepsilon(\mathbf{x})\mathbf{E}(\mathbf{x},t) \\
\nabla\mathbf{B}(\mathbf{x},t) &= 0 \qquad \mathbf{B}(\mathbf{x},t) = \mu(\mathbf{x})\mathbf{H}(\mathbf{x},t) \\
\nabla\times\mathbf{E}(\mathbf{x},t) &= -\frac{\partial\mathbf{B}(\mathbf{x},t)}{\partial t} \\
\nabla\times\mathbf{H}(\mathbf{x},t) &= \frac{\partial\mathbf{D}(\mathbf{x},t)}{\partial t} + \mathbf{j}_e(\mathbf{x},t) + \sigma(\mathbf{x})\mathbf{E}(\mathbf{x},t)
\end{aligned}$$

The current j_e(x, t) is an 'externally impressed' current, which is related to the 'externally impressed' charge density ρ_e by

$$\nabla\mathbf{j}_e(\mathbf{x},t) = -\frac{\partial\rho_e(\mathbf{x},t)}{\partial t}$$

If we assume that this impressed current is only changing slowly we can neglect Faraday's law and approximate

$$\nabla\times\mathbf{E}(\mathbf{x},t)\approx 0 \qquad \mathbf{E}(\mathbf{x},t) = -\nabla\varphi(\mathbf{x},t)$$

and we can then write the electric field as the gradient of a potential, an by taking the divergence of the last equation …

$$\nabla(\nabla\times\mathbf{H}(\mathbf{x},t)) = \frac{\partial\nabla\mathbf{D}(\mathbf{x},t)}{\partial t} + \nabla\mathbf{j}_e(\mathbf{x},t) + \nabla[\sigma(\mathbf{x})\mathbf{E}(\mathbf{x},t)] = 0$$

$$\nabla\left[\varepsilon(\mathbf{x})\nabla\frac{\partial\varphi(\mathbf{x},t)}{\partial t} + \sigma(\mathbf{x})\nabla\varphi(\mathbf{x},t)\right] = -\frac{\partial\rho_e(\mathbf{x},t)}{\partial t}$$

Quasi-static approximation of Maxwell's equations

Performing the Fourier Transform of the quasi-static equation

$$\nabla\left[\varepsilon(\mathbf{x})\nabla\frac{\partial\varphi(\mathbf{x},t)}{\partial t}+\sigma(\mathbf{x})\nabla\varphi(\mathbf{x},t)\right]=-\frac{\partial\rho_e(\mathbf{x},t)}{\partial t}$$

we find

$$\nabla\left[\varepsilon(x)\nabla i\omega\varphi(\mathbf{x},\omega)+\sigma(\mathbf{x})\nabla\varphi(\mathbf{x},\omega)\right]=-i\omega\rho_e(\mathbf{x},\omega)$$

$$\nabla\left[\left(\varepsilon(x)+\sigma(\mathbf{x})/i\omega\right)\nabla\varphi(\mathbf{x},\omega)\right]=-\rho_e(\mathbf{x},\omega)$$

[1] H.A. Haus, J.R. Melcher, Electromagentic Fields and Energy, Prentice-Hall, Englewood Cliffs, NJ, 1989.

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The quasi-static electromagnetic approximation for weakly conducting media☆

Th. Heubrandtner, B. Schnizer*

Institut für Theoretische Physik, Technische Universität Graz, Petersgrasse 16, 8010 Graz, Austria

So we can write this equation as

$$\nabla\left[\varepsilon_{\text{eff}}(\mathbf{x})\nabla\varphi(\mathbf{x},\omega)\right]=-\rho_e(\mathbf{x},\omega)\qquad\varepsilon_{\text{eff}}(\mathbf{x})=\varepsilon(x)+\sigma(\mathbf{x})/i\omega\qquad\rho(\mathbf{x},\omega)=-\nabla\left[\varepsilon(\mathbf{x})\nabla\varphi(\mathbf{x},\omega)\right]\qquad\rho_e(\mathbf{x},\omega)=-\nabla\left[\varepsilon_{\text{eff}}(\mathbf{x})\nabla\varphi(\mathbf{x},\omega)\right]$$

This is the Poisson equation with an effective permittivity !

→ We can therefore find the time dependent solutions for a medium with a given conductivity by solving the electrostatic Poisson equation in the Fourier domain !

→ Knowing the electrostatic solution for a given permittivity ε(x) we just have to replace ε(x) by ε(x)+σ(x)/iω and perform the inverse Fourier transform !

Extension of the Ramo Shockley theorem

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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

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Extended theorems for signal induction in particle detectors VCI 2004

W. Riegler*

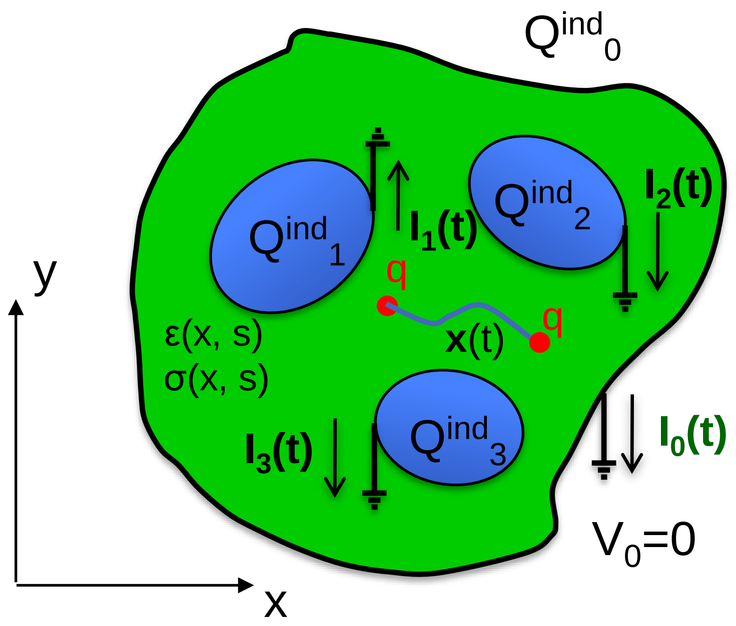
CERN, PH Division, Rt. De Meyrin, Geneva 23CH-1211, Switzerland

Available online 13 August 2004

Abstract

Most particle detectors are based on the principle that charged particles leave a trail of ionization in the detector and that the movement of these charges in an electric field induces signals on the detector electrodes. Assuming detector elements that are insulating and electrodes with infinite conductivity one can calculate the signals with an electrostatic approximation using the so-called ‘Ramo theorem’. This is the standard way for the calculation of signals e.g. in wire chambers and silicon detectors. In case the detectors contain resistive elements, which is, e.g. the case in resistive plate chambers or underdepleted silicon detectors, the time dependence of the signals is not only given by the movement of the charges but also by the time-dependent reaction of the detector materials. Using the quasistatic approximation of Maxwell’s equations we present an extended formalism that allows the calculation of induced signals for detectors with general materials by time dependent weighting fields. As examples, we will discuss the signals in resistive plate chambers and underdepleted silicon detectors.

Theorem, induced current



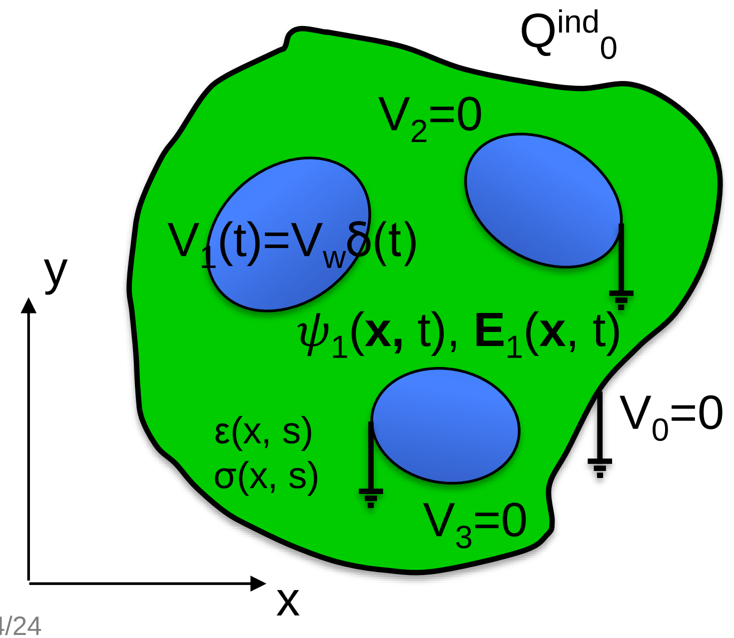
Applying the delta voltage pulse to the electrode in question we find the potential ψ_n(x, t) and the field E_n(x, t) from which the induced current can be calculated the following way:

$$I_n^{ext}(s) = -sQ_n^{ext}(s) = \frac{s}{V_w}\int_V \psi_n(\mathbf{x}, s)\rho_e(\mathbf{x}, s)d^3x$$

$$\rho_e(\mathbf{x}, t) = q\delta(\mathbf{x} - \mathbf{x}_1(t))$$

$$I_n^{ext}(t) = -\frac{q}{V_w}\int_0^t \mathbf{E}_n(\mathbf{x}_1(t'), t - t')\dot{\mathbf{x}}_1(t')dt'$$

→ Ramo-Shockley theorem extension for conducting media

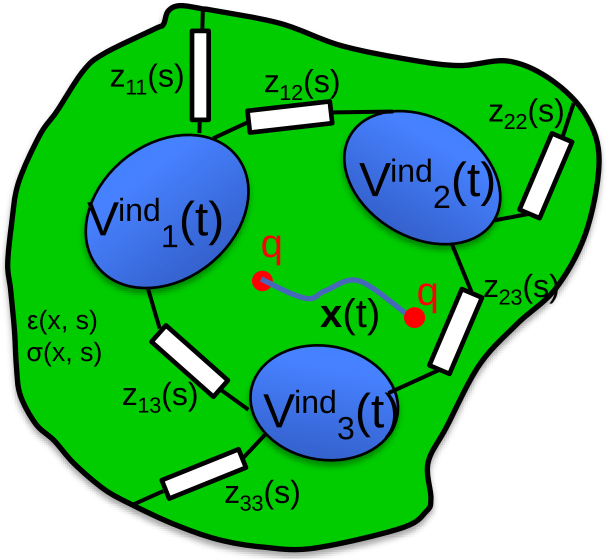


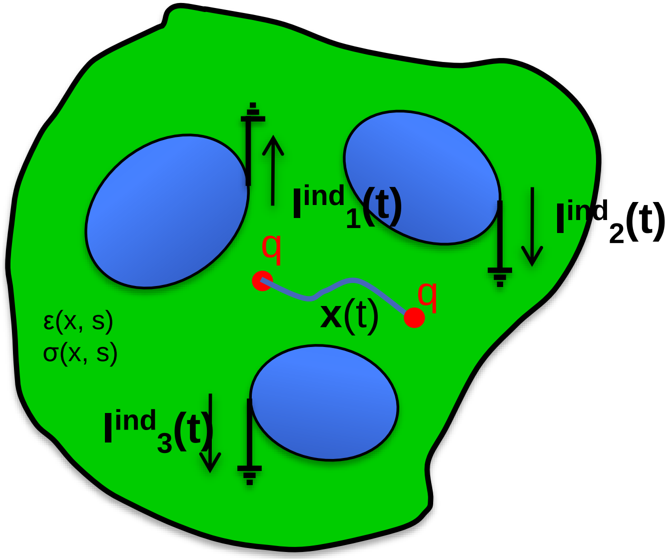
Note that E_n is not physical potential, since the delta function gives it a dimension of V/cm s.

In case the material is an insulator there is no time dependence of the weighting field and we recuperate Ramo's theorem.

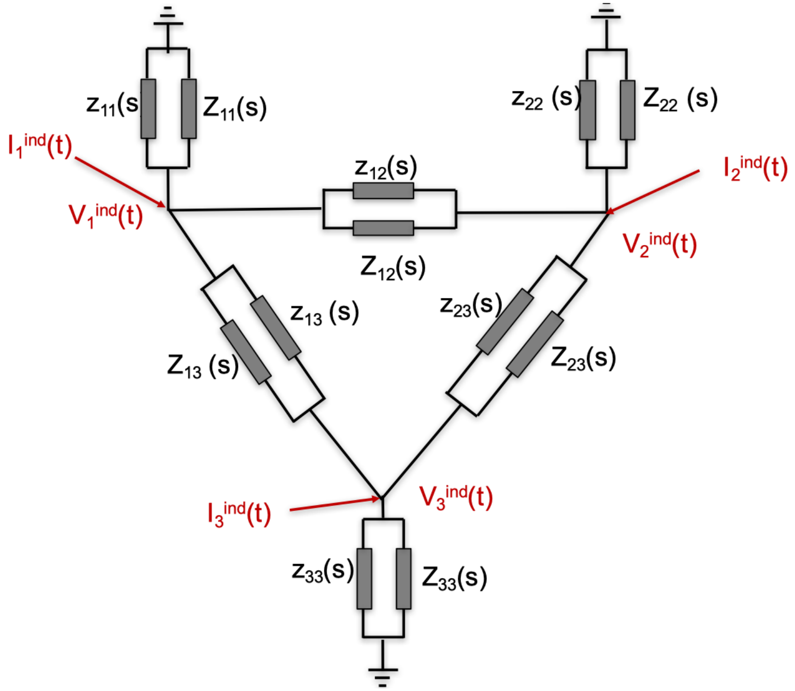
$$\mathbf{E}_n(\mathbf{x}, t) = \mathbf{E}_{n0}(\mathbf{x})\delta(t - t') \qquad I_n^{ext}(t) = -\frac{q}{V_w}\mathbf{E}_{n0}(\mathbf{x}_1(t))\dot{\mathbf{x}}_1(t)dt$$

Equivalent circuit, Impedance elements





In case the electrodes are not insulated but connected with discrete linear impedance components we can consider them as part of the medium and we therefore just have to add these elements in the equivalent circuit.



$$Z_{nm}(s) = -\frac{1}{y_{nm}(s)} \quad n \neq m \qquad Z_{nn}(s) = \frac{1}{\sum_{m=1}^{N} y_{nm}(s)} = -\frac{1}{y_{0n}} \quad n = m$$

$$y_{mn}(s) = \frac{s}{V_w} \oint_{\mathbf{A}_n} \varepsilon_{eff}(\mathbf{x}, s) \nabla \psi_m(\mathbf{x}, s) d\mathbf{A}$$

VRIJE UNIVERSITEIT BRUSSEL

Thesis submitted in fulfilment of the requirements for the award of the degree of Doctor of Sciences

RESISTIVE ELECTRODES AND PARTICLE DETECTORS

Modeling and Measurements of Novel Detector Structures

Djunes Janssens

February 2024

Promotor: Prof. Dr. J. D'Hondth
Co-promotors: Dr. H. Schindler

Sciences and Bio-Engineering Sciences
Physics department

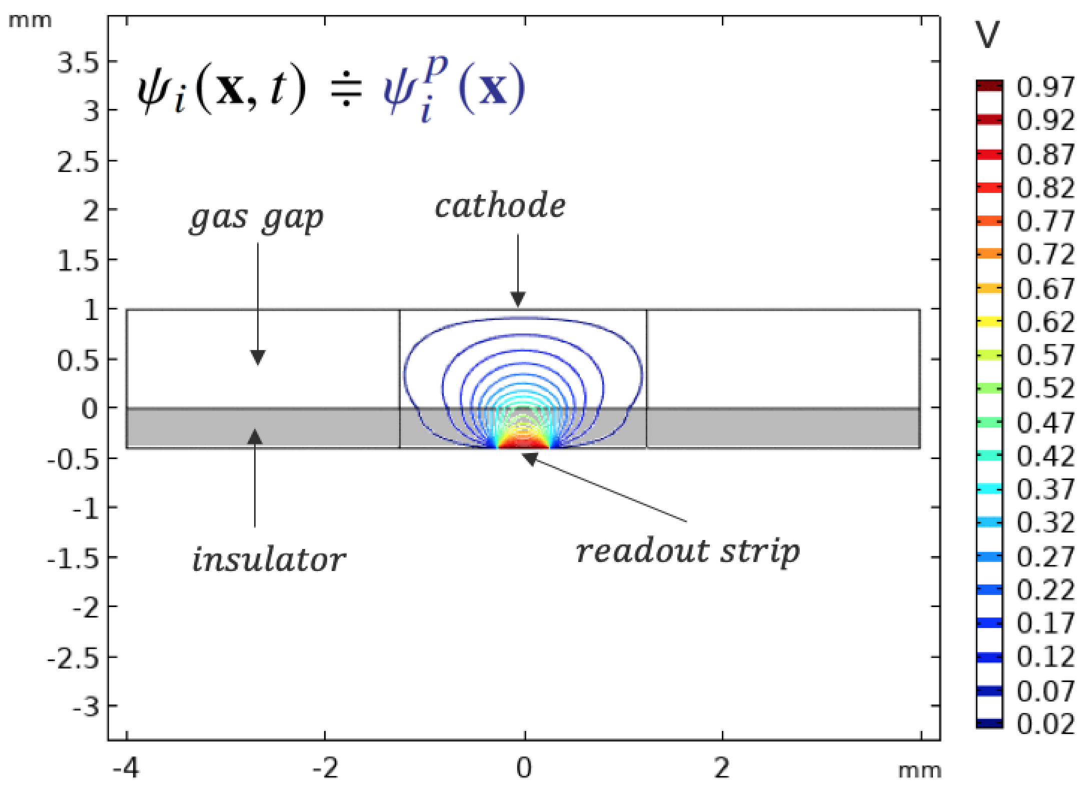
Djunes Janssens

https://cds.cern.ch/record/2890572

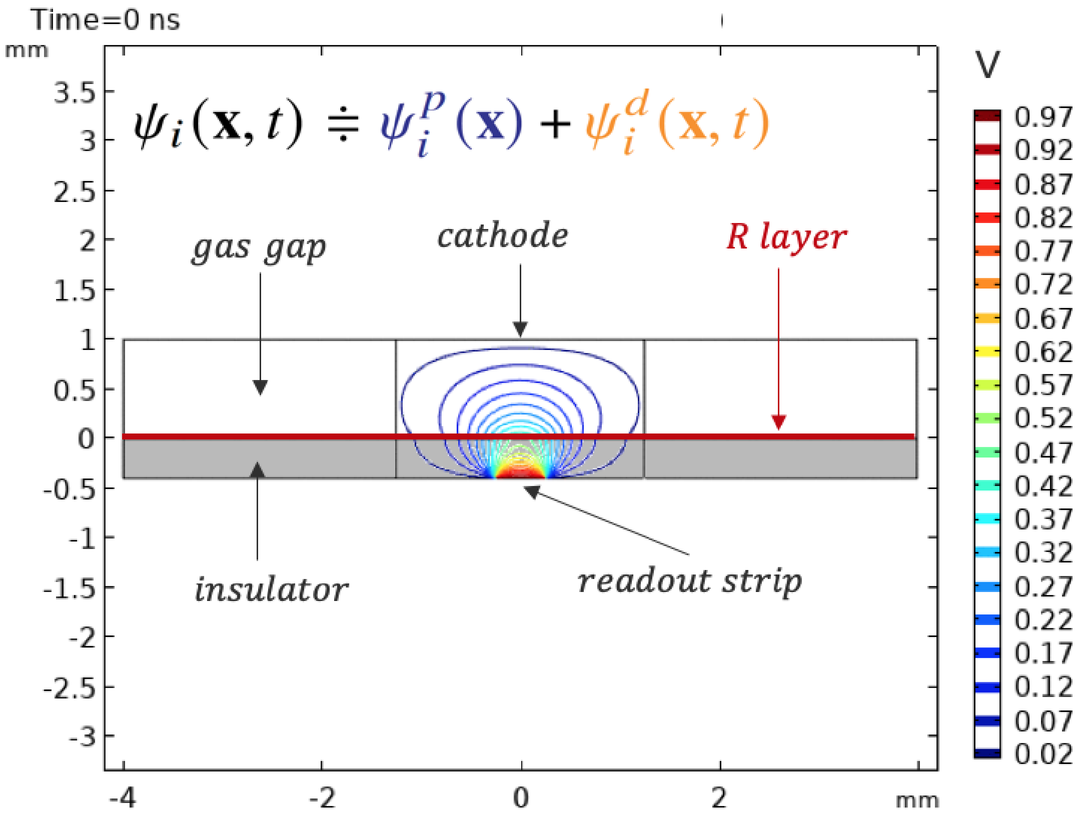
Micromegas toy-model example

The time-dependent weighting potential $\psi_i(\mathbf{x}, t)$ is comprised of a static prompt and a dynamic delayed component:

Static weighting potential of a readout strip



Dynamic weighting potential of a readout strip with resistive layer



Micromegas toy-model example

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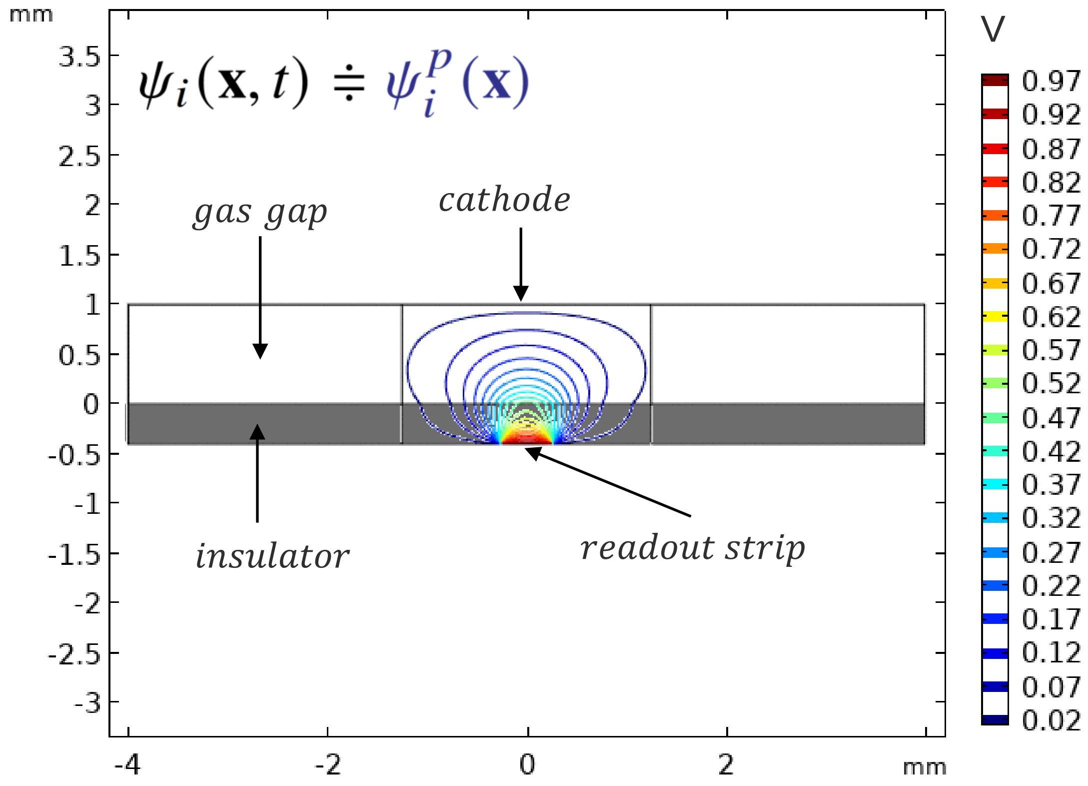
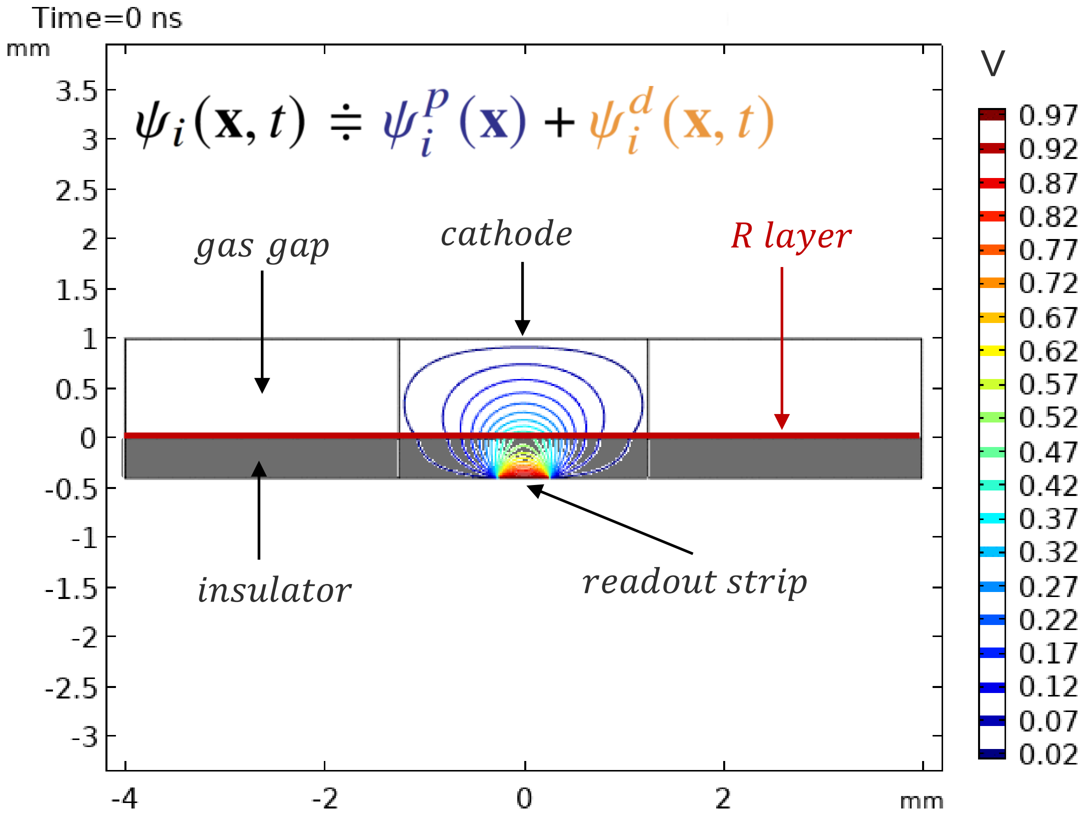
Static weighting potential of a readout strip



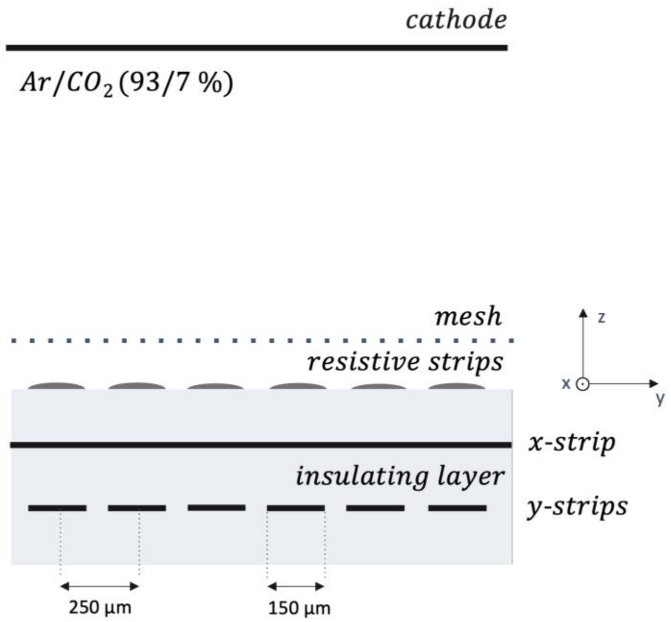
Dynamic weighting potential of a readout strip with resistive layer

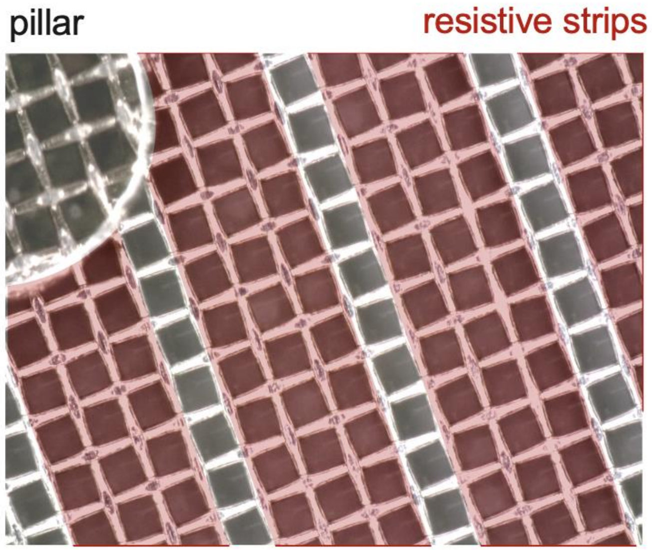


Resistive strip bulk Micromegas

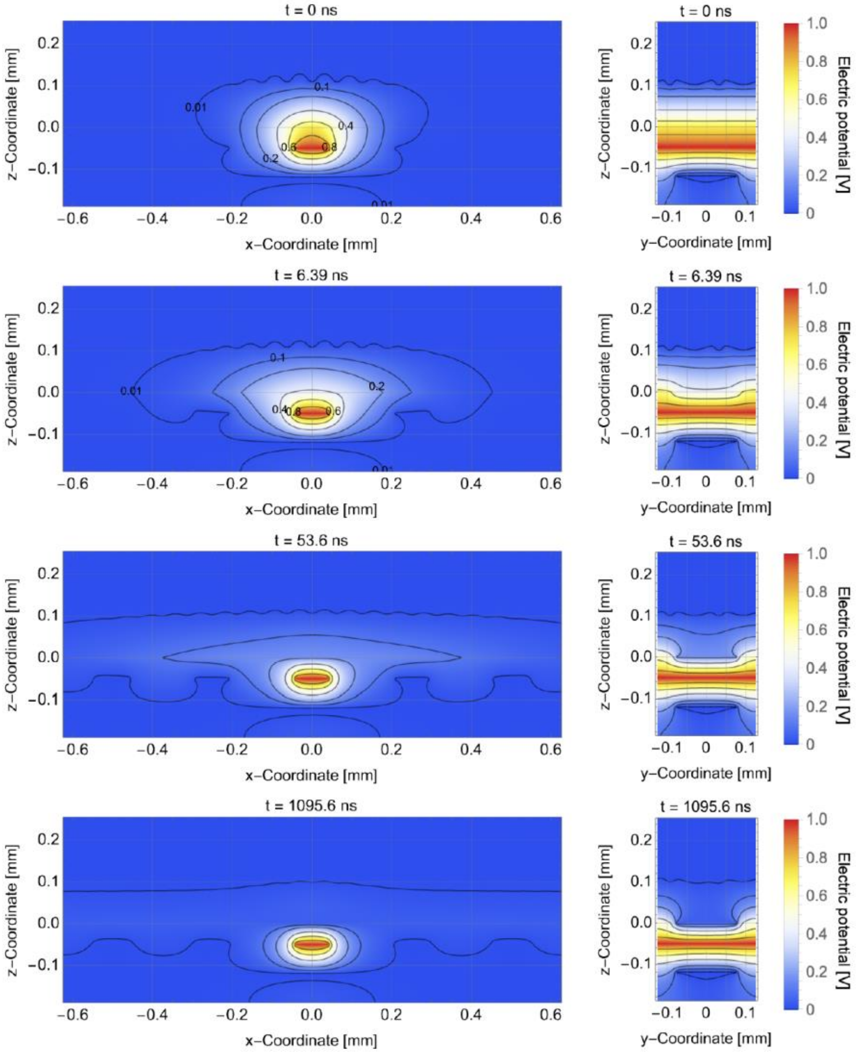
While the previous slide the embedded readout electrodes were beneath a thin resistive layer, the ATLAS-type MM, used in the NSW upgrade, instead features perpendicular resistive strips over a dielectric foil.

Here we use a 2D strip readout to capture the delayed component coming from the resistive strips.





Calculated dynamic weighting potential



T. Alexopoulos, et al., Nucl. Instrum. Meth. A 640 (2011) 110.
M. Byszewski and J. Wotschack, JINST 7 C02060 (2011).

Djunes Janssens

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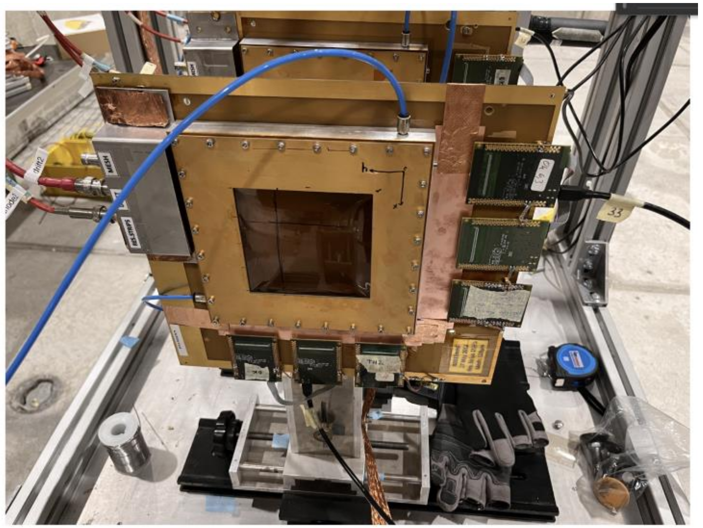
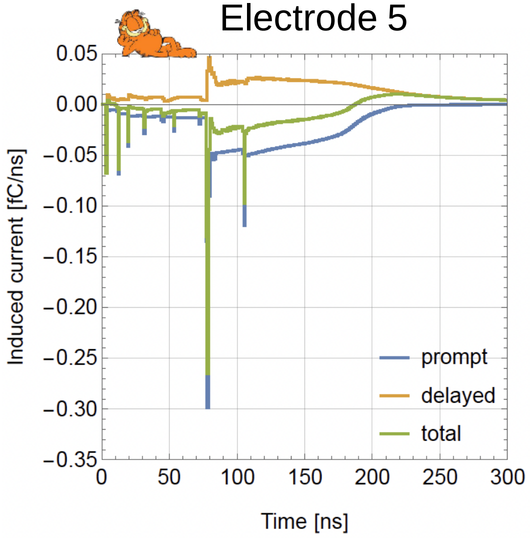
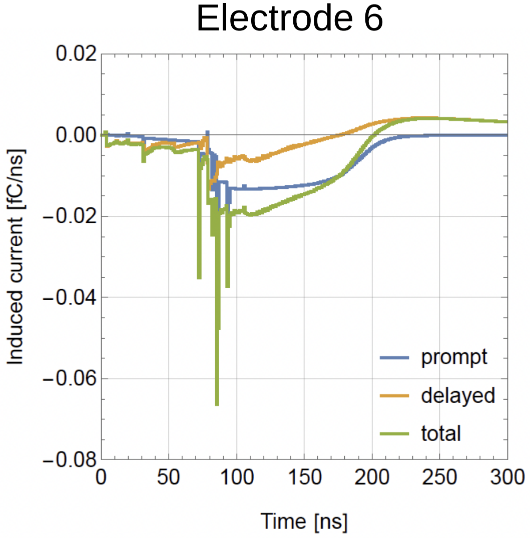
EP R&D
RD51 Collaboration

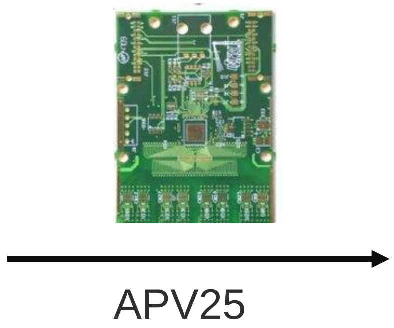
Resistive strip bulk Micromegas

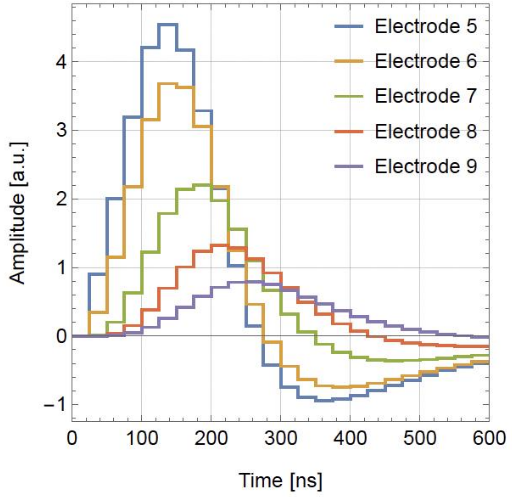
After having calculated the signals induced on the strip electrodes, the electronics with which the detector is read out needs to be taken into account.





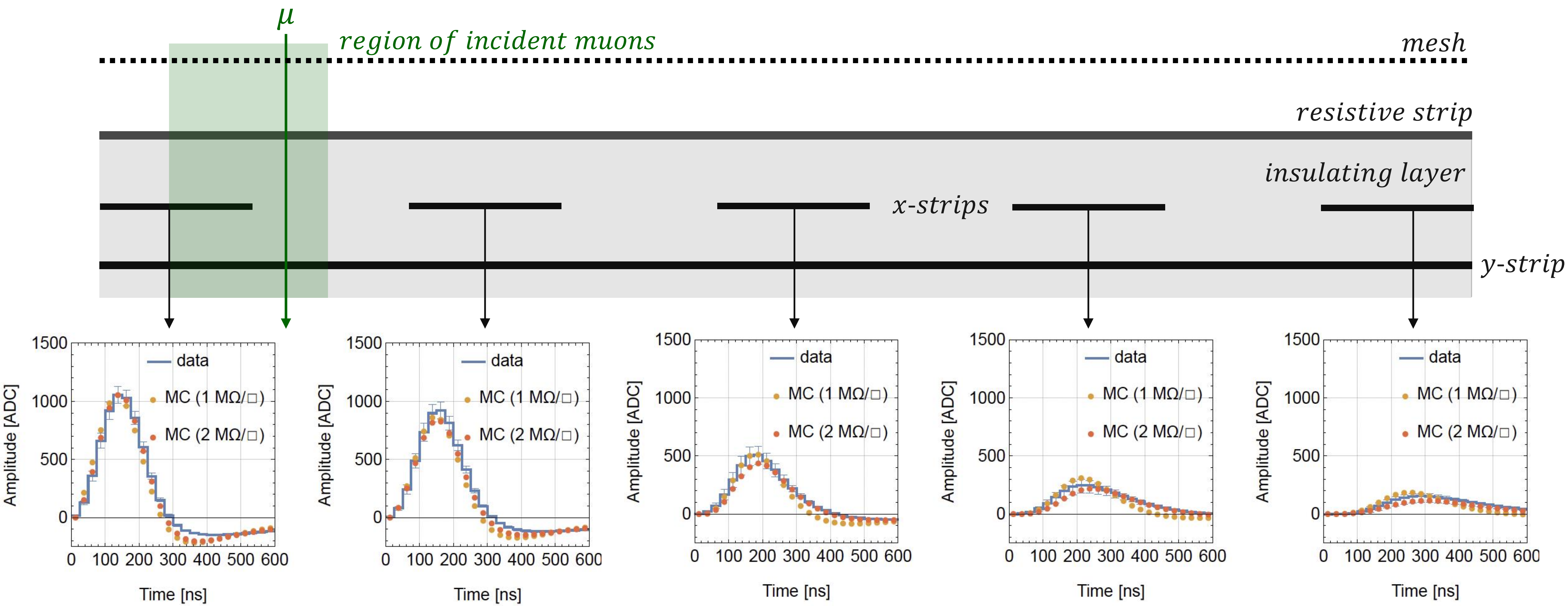






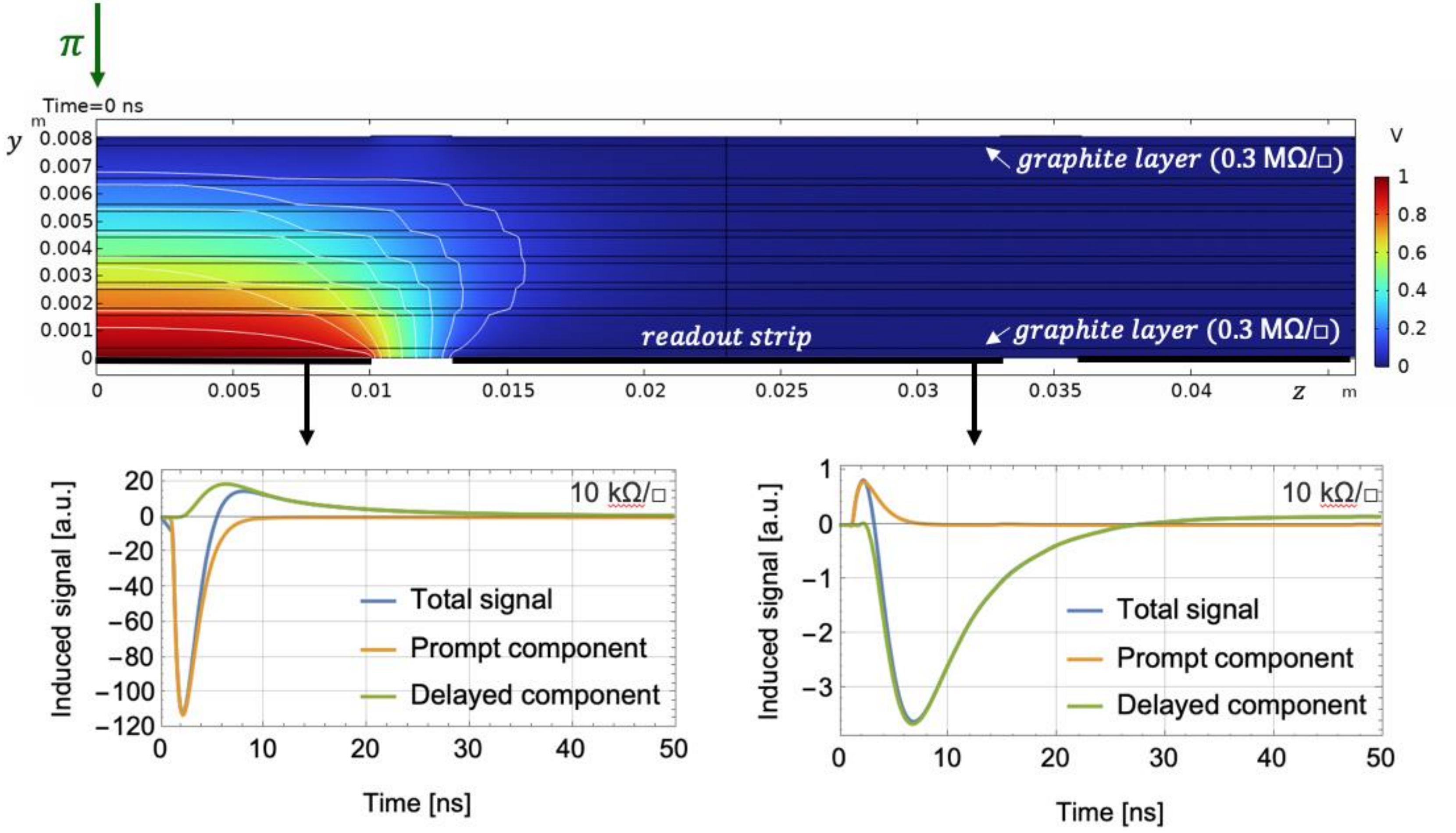
Resistive strip bulk Micromegas

- For the comparison we look at the average induced current response of neighboring strips. This averaging is performed over muon events positioned between the leading and the next-to-leading strip.



6-gap MRPC

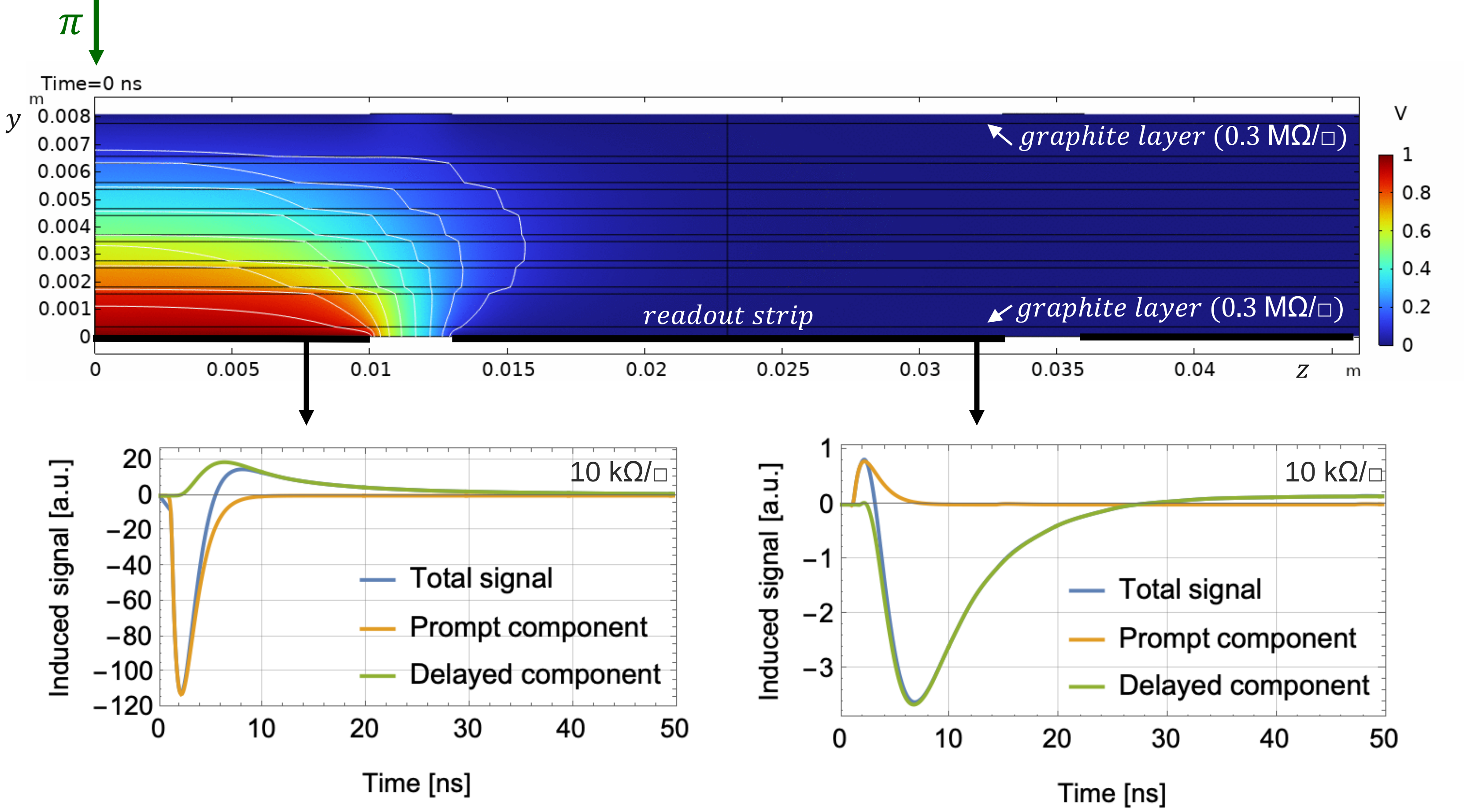
The dynamic weighting potential was calculated using COMSOL and then imported into Garfield++ for the induced signal calculations. Given a graphite layers with O(100 kΩ/□), the signal induced by electrons remains unaffected.



Consistent with G. Battistoni et al., NIM in Physics Research 202 (1982) 459.

6-gap MRPC

The dynamic weighting potential was calculated using COMSOL and then imported into Garfield++ for the induced signal calculations. Given a graphite layers with O(100 kΩ/□), the signal induced by electrons remains unaffected.



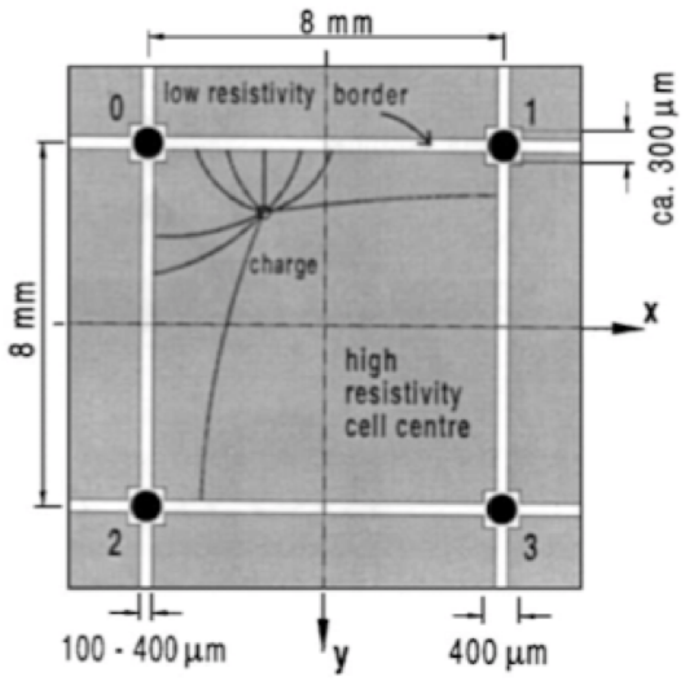
Djunes Janssens

Consistent with G. Battistoni et al., NIM in Physics Research 202 (1982) 459.

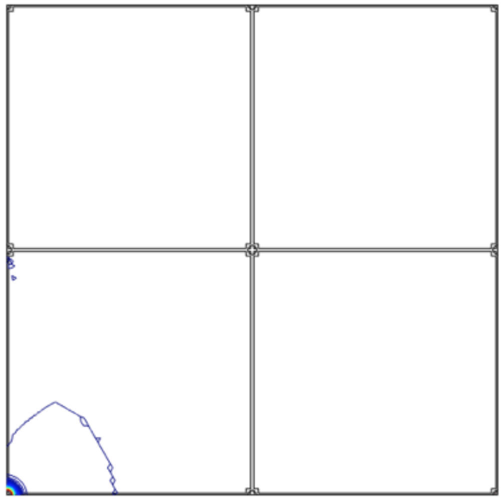
Signal formation in a MicroCAT detector

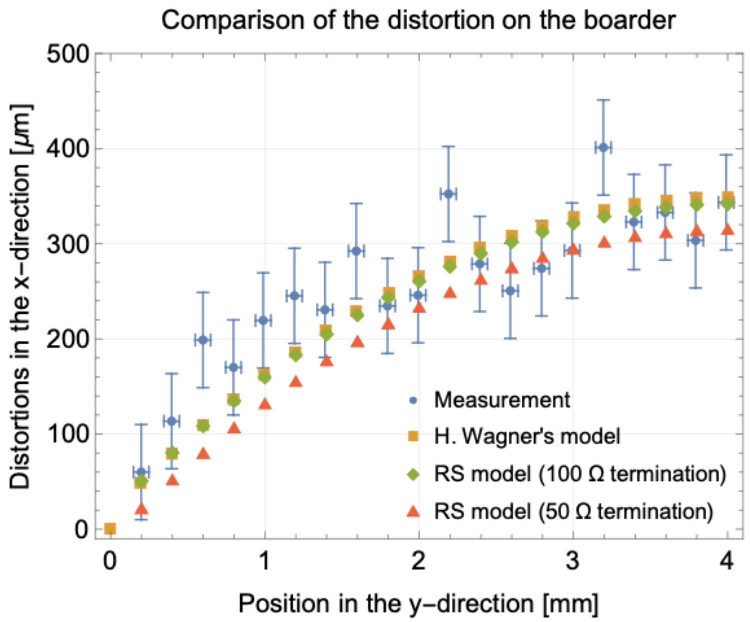
The MicroCAT's two-dimensional interpolating readout structure allows for a reduced number of electronic readout channels without loss of spatial resolution.

This resistive readout concept has recently enjoyed renewed interest with the development of a DC-Coupled LGAD device: *arXiv:2204.07226 [physics.ins-det]*.



Weighting potential map for one readout node





RD51 Collaboration

F. Bartol et al. J. Phys. III France, 6 (1996), p. 337
A. Sarvestani et al., Nucl. Instrum. Meth. A 410 (1998) 238–258
H. Wagner et al. Nucl. Instrum. Meth. A 482 (2002) 334–346

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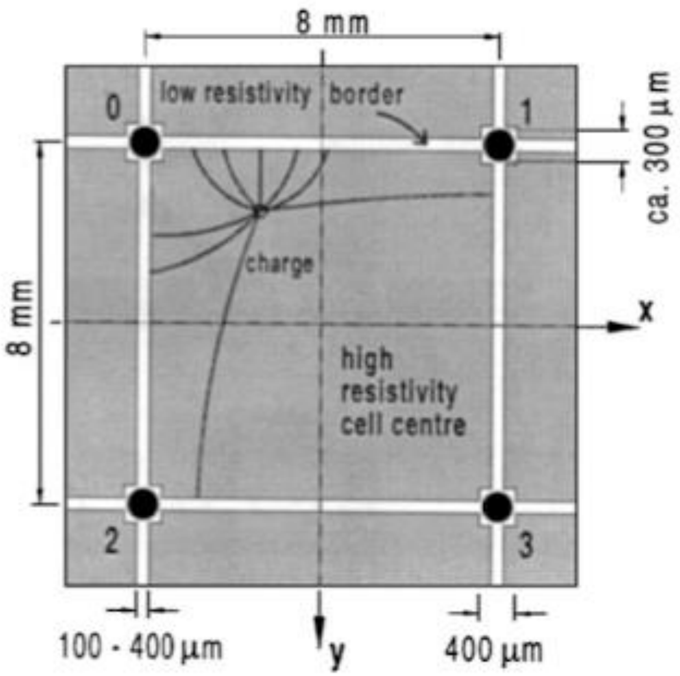

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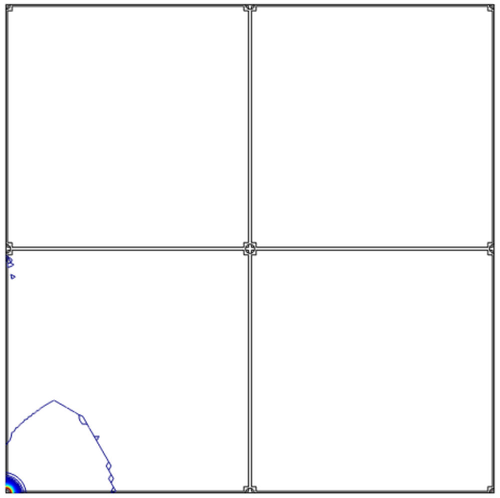
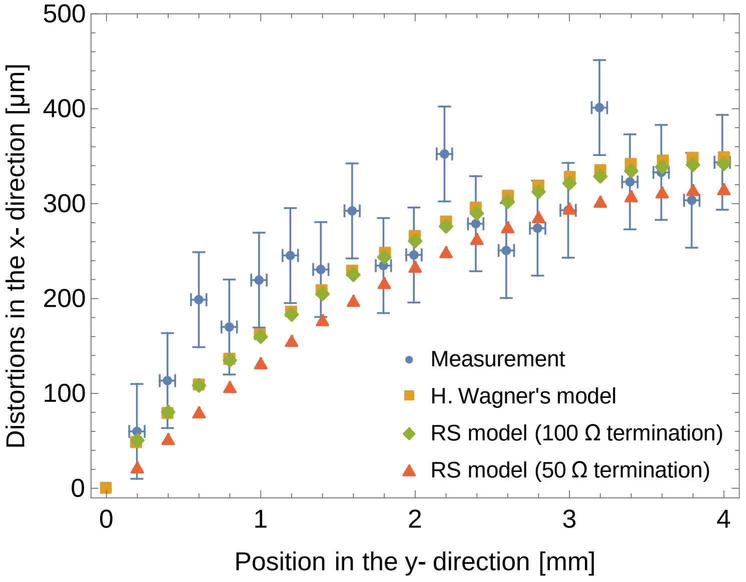


Comparison of the distortion on the boarder



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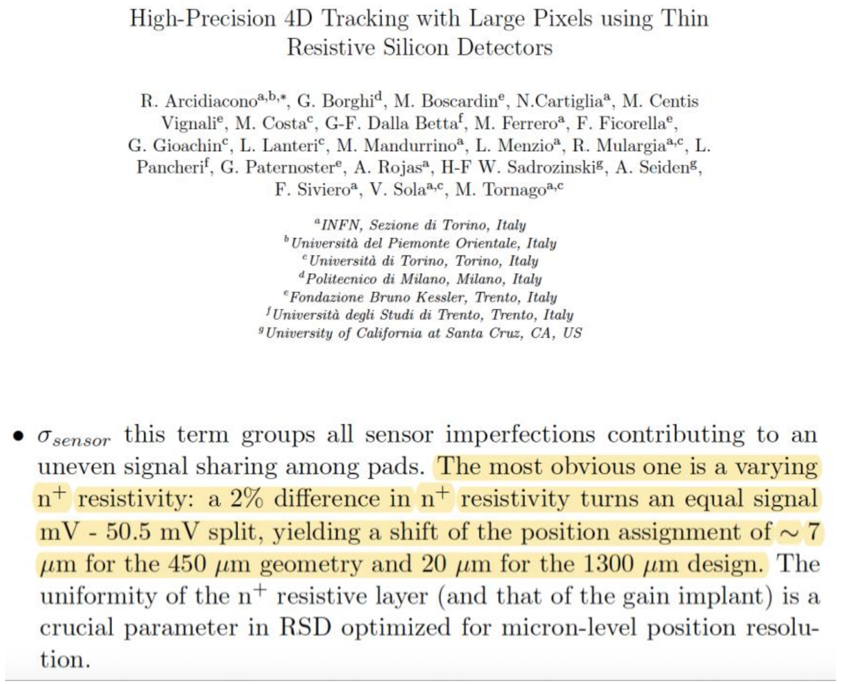
MicroCAT resistive position interpolation readout

During production, the resistivity can fluctuate on the surface of the resistive layer. This could make the timing response or reconstruction capability of your detector non-uniform over the active area.



High-Precision 4D Tracking with Large Pixels using Thin Resistive Silicon Detectors

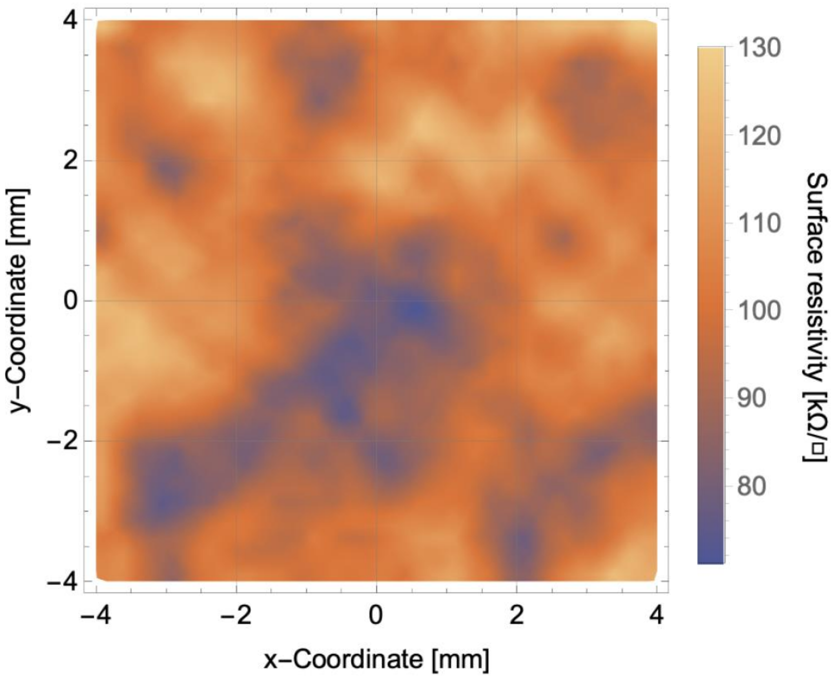
R. Arcidiacono[a,b,*], G. Borghi[d], M. Boscardin[e], N.Cartiglia[a], M. Centis Vignali[e], M. Costa[c], G-F. Dalla Betta[f], M. Ferrero[a], F. Ficorella[e], G. Gioachin[c], L. Lanteri[c], M. Mandurrino[a], L. Menzio[a], R. Mulargia[a,c], L. Pancheri[f], G. Paternoster[e], A. Rojas[a], H-F W. Sadrozinski[g], A. Seiden[g], F. Siviero[a], V. Sola[a,c], M. Tornago[a,c]

[a] *INFN, Sezione di Torino, Italy*
[b] *Università del Piemonte Orientale, Italy*
[c] *Università di Torino, Torino, Italy*
[d] *Politecnico di Milano, Milano, Italy*
[e] *Fondazione Bruno Kessler, Trento, Italy*
[f] *Università degli Studi di Trento, Trento, Italy*
[g] *University of California at Santa Cruz, CA, US*

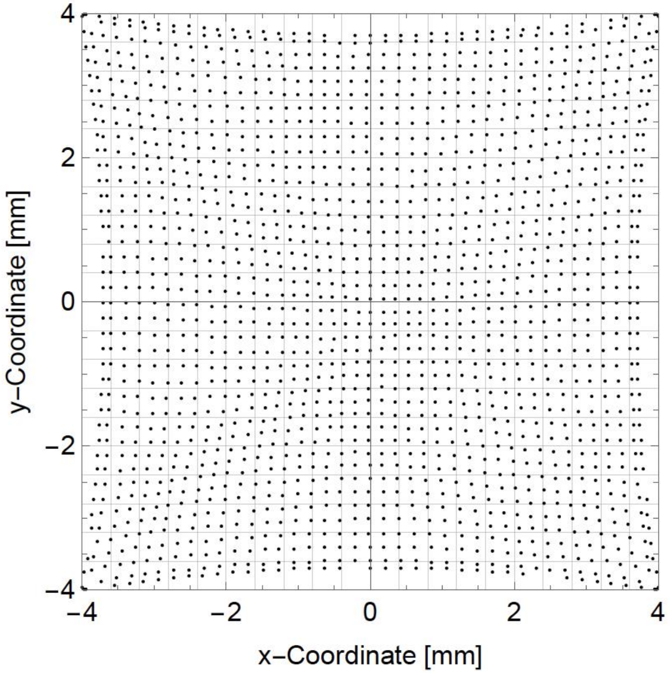
- σ_{sensor} this term groups all sensor imperfections contributing to an uneven signal sharing among pads. The most obvious one is a varying n^+ resistivity: a 2% difference in n^+ resistivity turns an equal signal mV - 50.5 mV split, yielding a shift of the position assignment of ~ 7 μm for the 450 μm geometry and 20 μm for the 1300 μm design. The uniformity of the n^+ resistive layer (and that of the gain implant) is a crucial parameter in RSD optimized for micron-level position resolution.

R. Arcidiacono et al., arXiv:2211.13809v1 [physics.ins-det]

Map of non-uniform resistivity in one readout cell



Correction map



Summary

The movement of charges induces signals on metal electrodes.

The signal shapes are therefore determined by the dynamics of the charges inside the detector i.e. by the movement of electrons and Ions in gaseous detectors.

These signals can be calculated by means of weighting potentials and weighting fields that are solutions of the Laplace equation.

The formalism can be extended to detectors containing resistive elements by use of time dependent weighting fields.

Signals in GEM detectors are mainly due to the movement of the electrons in the induction gap.

Signals in Micromegas detectors are characterized by a short electron ‘spike’ and a longer ‘flat’ ion tail.

Signals in wire chambers are mainly due to the movement of ions and are characterize by a long hyperbolic $1/(t+t_0)$ tail.