Fixed Points of 4d QFTs

…and what they can do for us

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critical points

thermal or quantum phase transitions

continuous phase transition
Landau-Ginsburg-type universality
conformal / topological phase transitions
deconfined criticality, ``emergence”

symmetry breaking, generation of mass

conformal field theory

scale invariance vs conformal invariance
conformal bootstrap, CFT data

critical points in 4d

UV critical points

- fundamental definition of QFT — Wilson '71
- asymptotic freedom — Gross, Wilzcek '73 , Politzer '73
- asymptotic near-freedom — Bailin, Love '74
- asymptotic safety — Weinberg '79

IR critical points

- Banks-Zaks conformal window — Caswell '74, Banks, Zaks, '82
- weak-strong dualities — Seiberg '95

today:

Interacting fixed points in …

4d QFTs at weak coupling

4d QFTs at strong coupling

4d quantum gravity

renormalisation group

quantum fluctuations modify interactions
couplings depend on energy

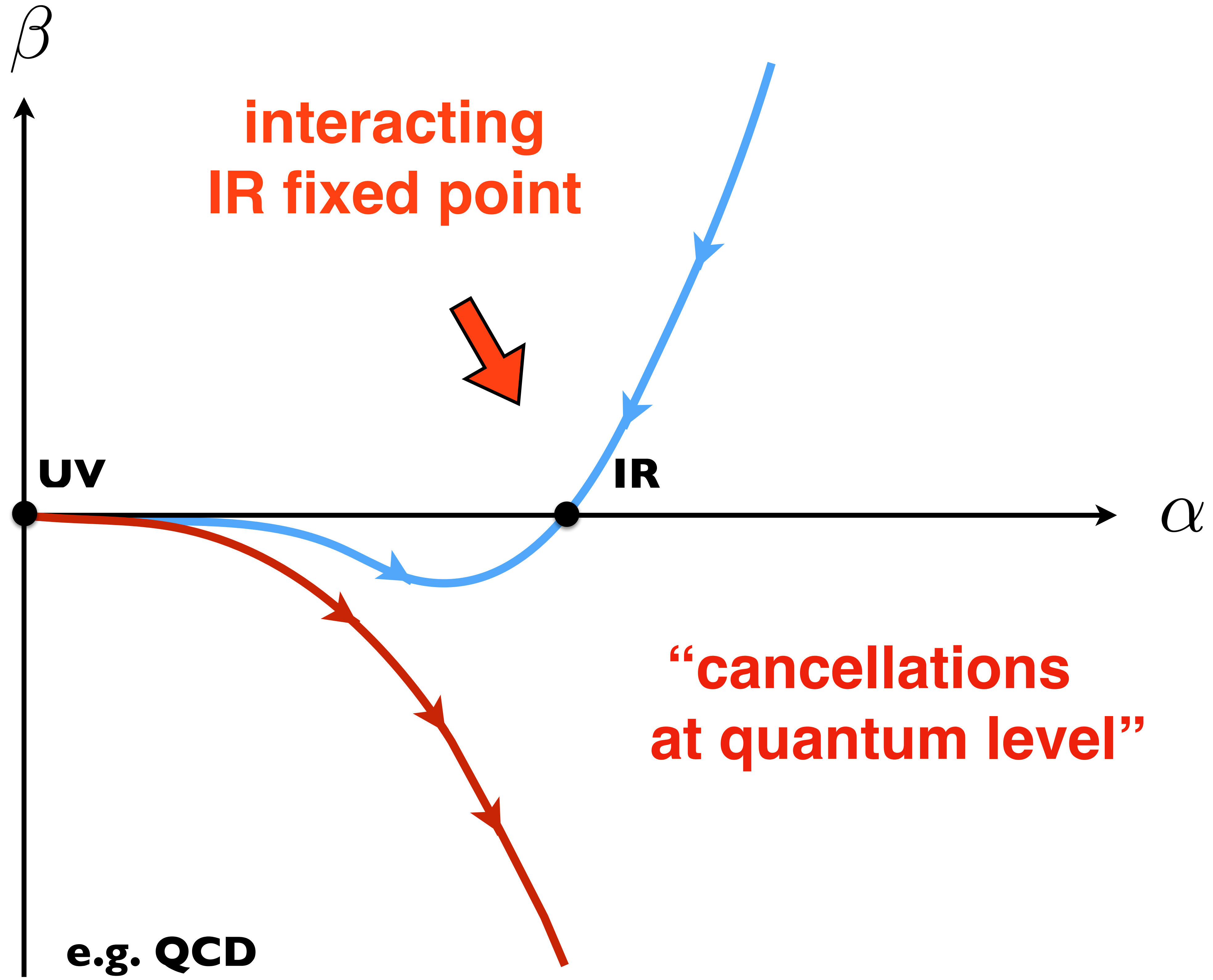
$$\mu \frac{d\alpha}{d\mu} = \beta(\alpha)$$

fluctuations	$\hbar$
energy scale	μ
couplings	$\alpha(\mu)$

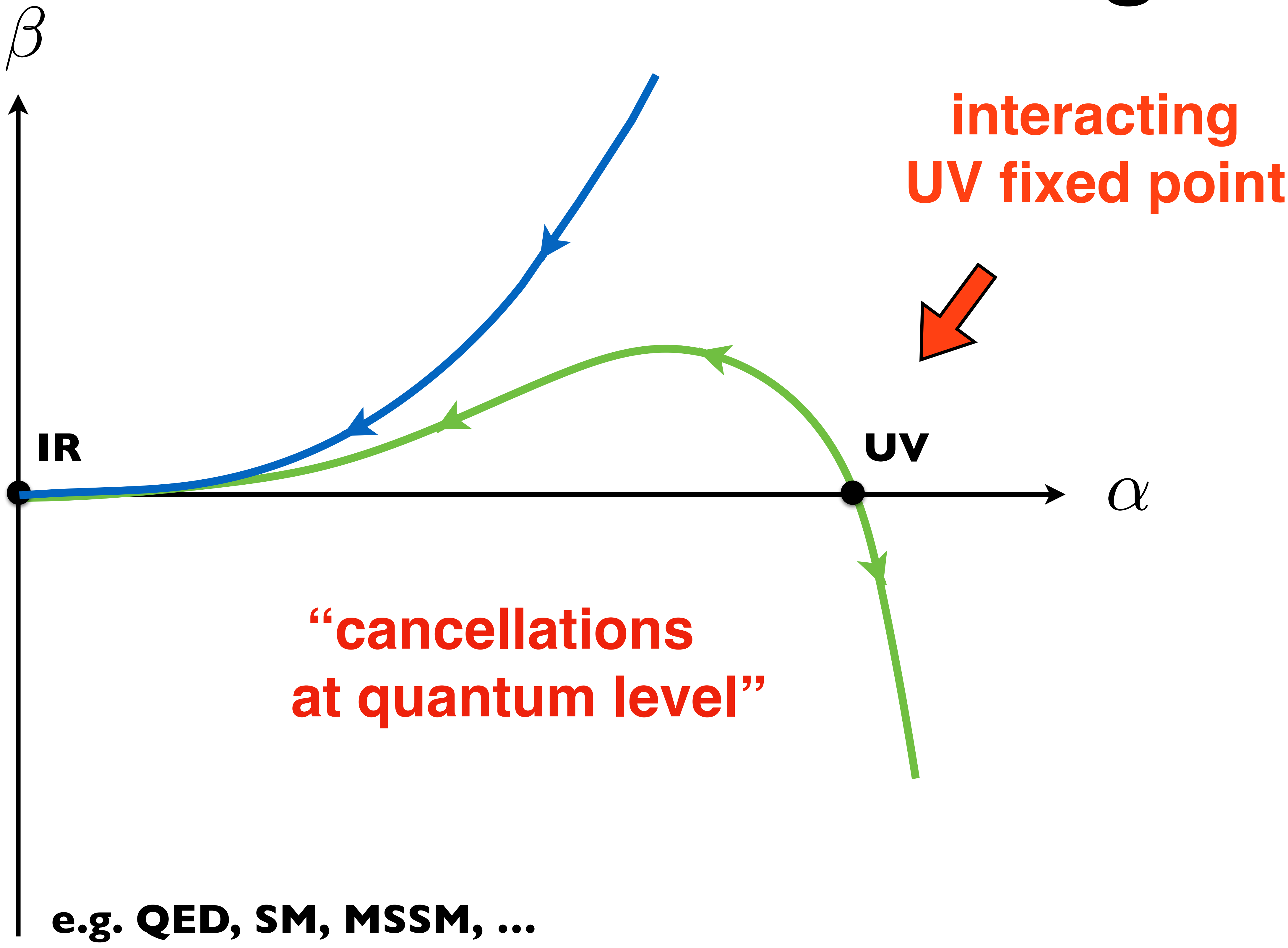
QFT provides us with

predictions into regions where we cannot (yet) make measurements

renormalisation group



renormalisation group



weakly coupled fixed points

4d QFTs

fields **vectors** A_μ^a , **fermions** ψ_I , **scalars** ϕ^A

path integral $$Z[J] = \exp -i \int d^4x (L + L_{\rm gf} + L_{\rm gh} + J^i \, \Phi_i)$$

action $$L = \frac{1}{4g_a^2} \mathrm{Tr}\, F^a_{\mu\nu} F_a^{\mu\nu} + i\psi_I \not{D} \psi_I + \frac{1}{2}(D_\mu \phi^A)^2$$
$$+\frac{1}{2} Y^A{}_{IJ} \phi^A \, \psi_I \, \xi \, \psi_J + \frac{1}{4!} \lambda_{ABCD} \phi^A \phi^B \phi^C \phi^D$$

Machacek, Vaughn '85

4d QFTs

fields — **vectors** A^a_μ , **fermions** ψ_I , **scalars** ϕ^A

path integral

$$Z[J] = \exp -i \int d^4x (L + L_{\rm gf} + L_{\rm gh} + J^i\,\Phi_i)$$

action

$$L = \frac{1}{4g_a^2}\,{\rm Tr}\,F^a_{\mu\nu}F_a^{\mu\nu} + i\psi_I \not{D}\psi_I + \frac{1}{2}(D_\mu\phi^A)^2$$
$$+\frac{1}{2}Y^A{}_{IJ}\phi^A\,\psi_I\,\xi\,\psi_J + \frac{1}{4!}\lambda_{ABCD}\phi^A\phi^B\phi^C\phi^D$$

gauge → g_a

Yukawa → $Y^A{}_{IJ}$

quartics → λ_{ABCD}

Machacek, Vaughn '85

Perturbativity guarantees that …

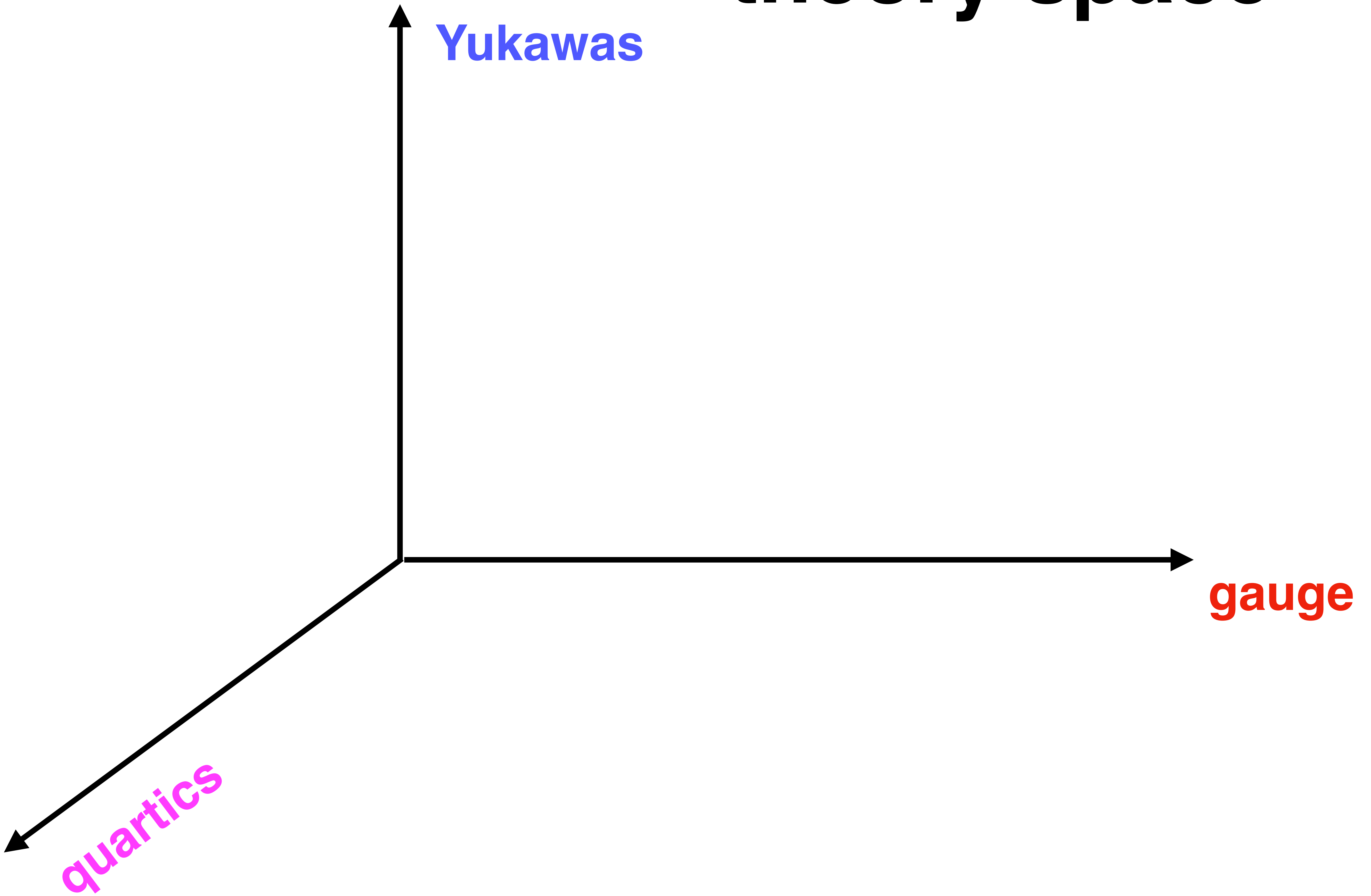
- masses = relevant
- higher dimensional interactions = irrelevant
- classically marginal couplings = key

  general RGEs are known

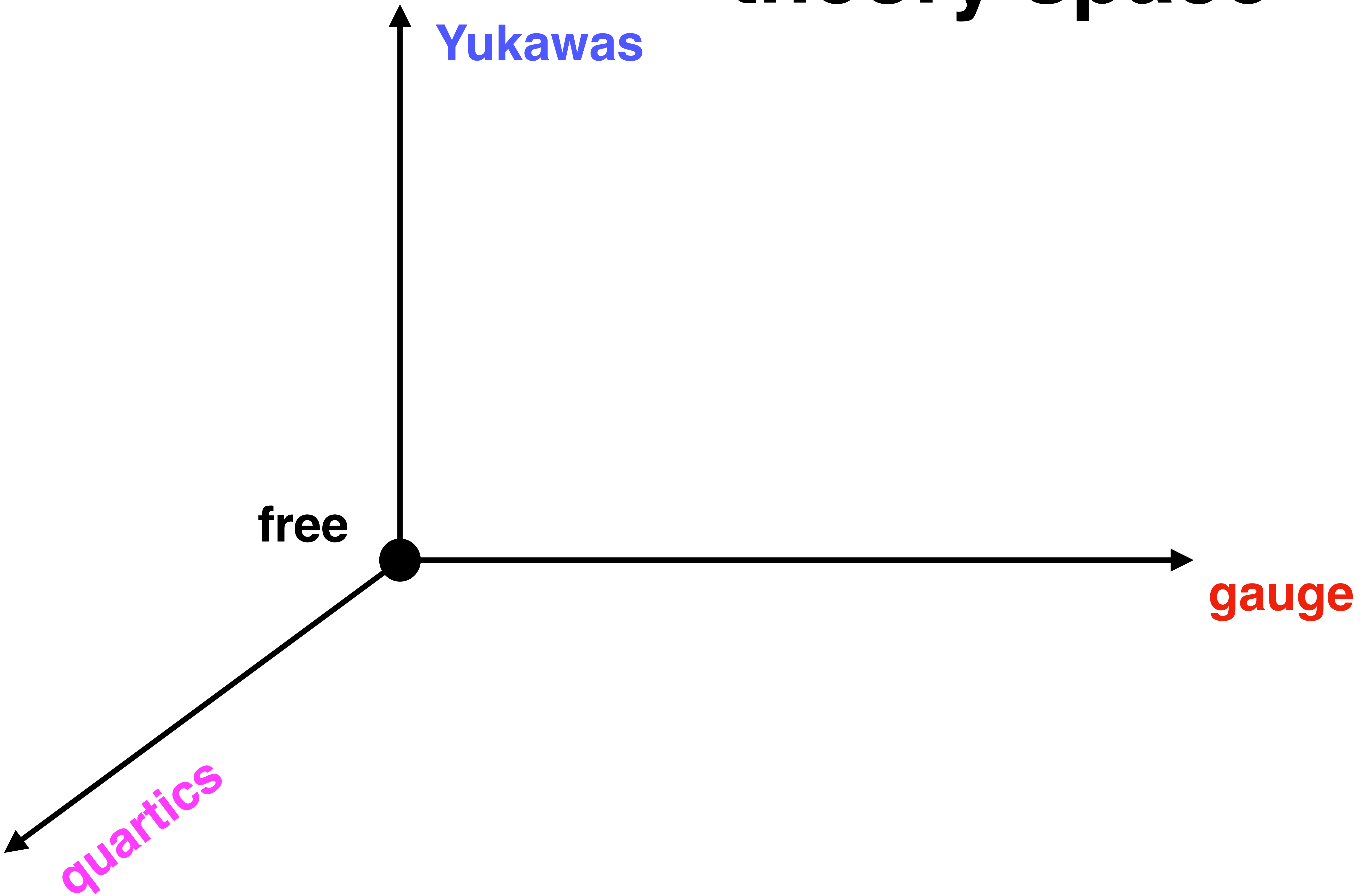
Machacek, Vaughn ’85

Davies, Herren, Thomsen ’21
Bednyakov, Pilkener ‘21

“theory space”

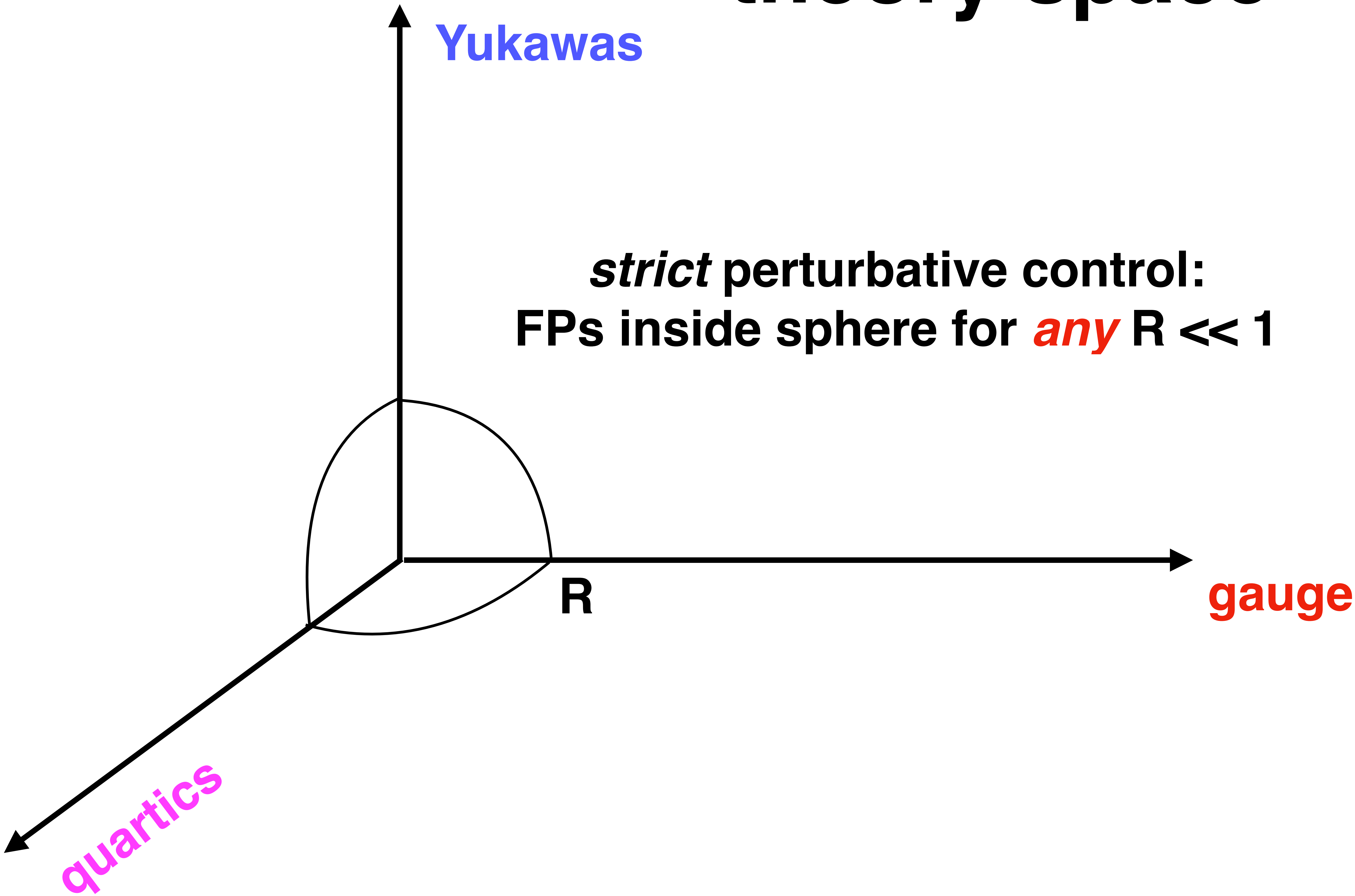


“theory space”



“theory space”

strict perturbative control:
FPs inside sphere for *any* R << 1



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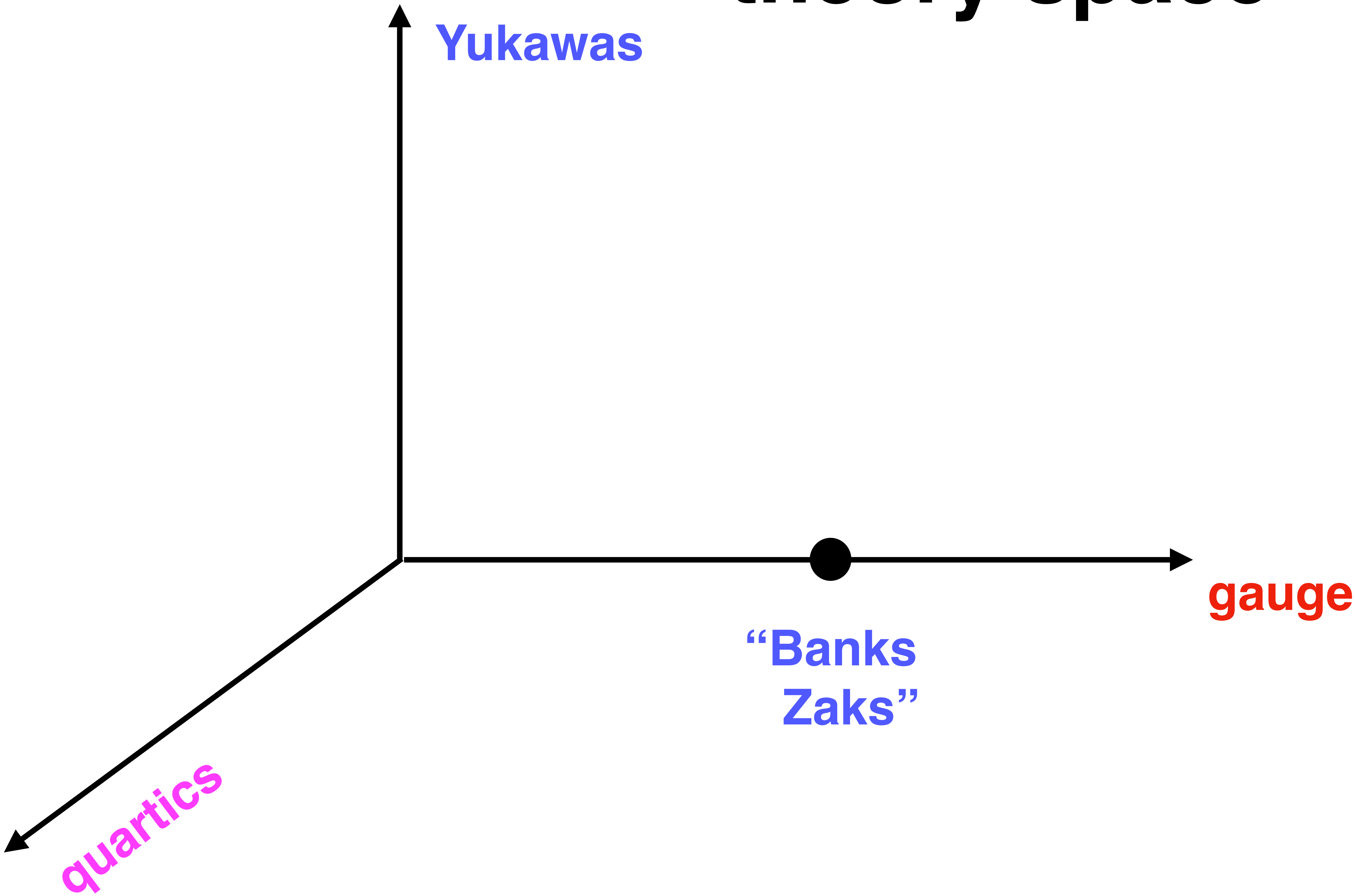

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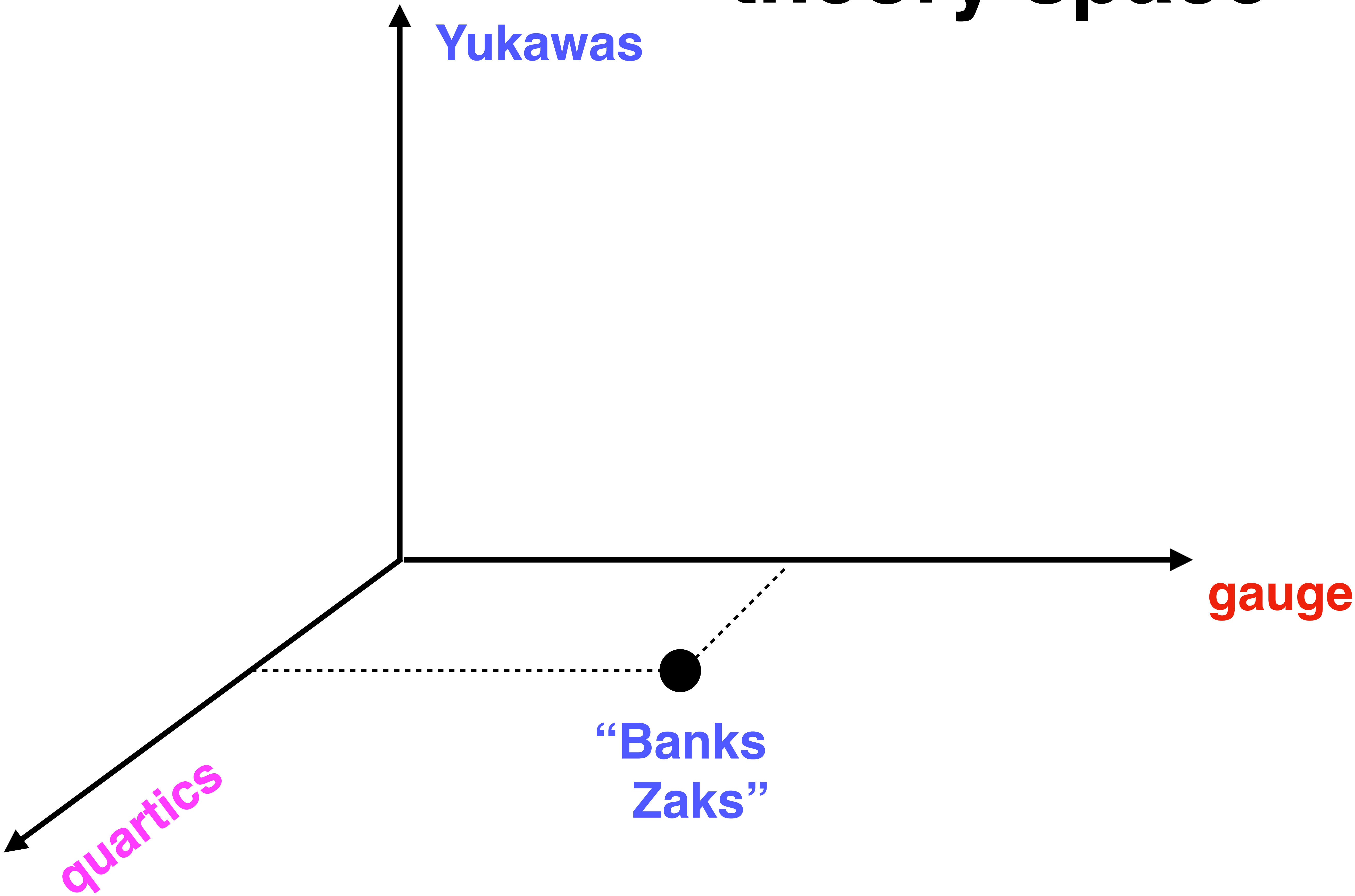

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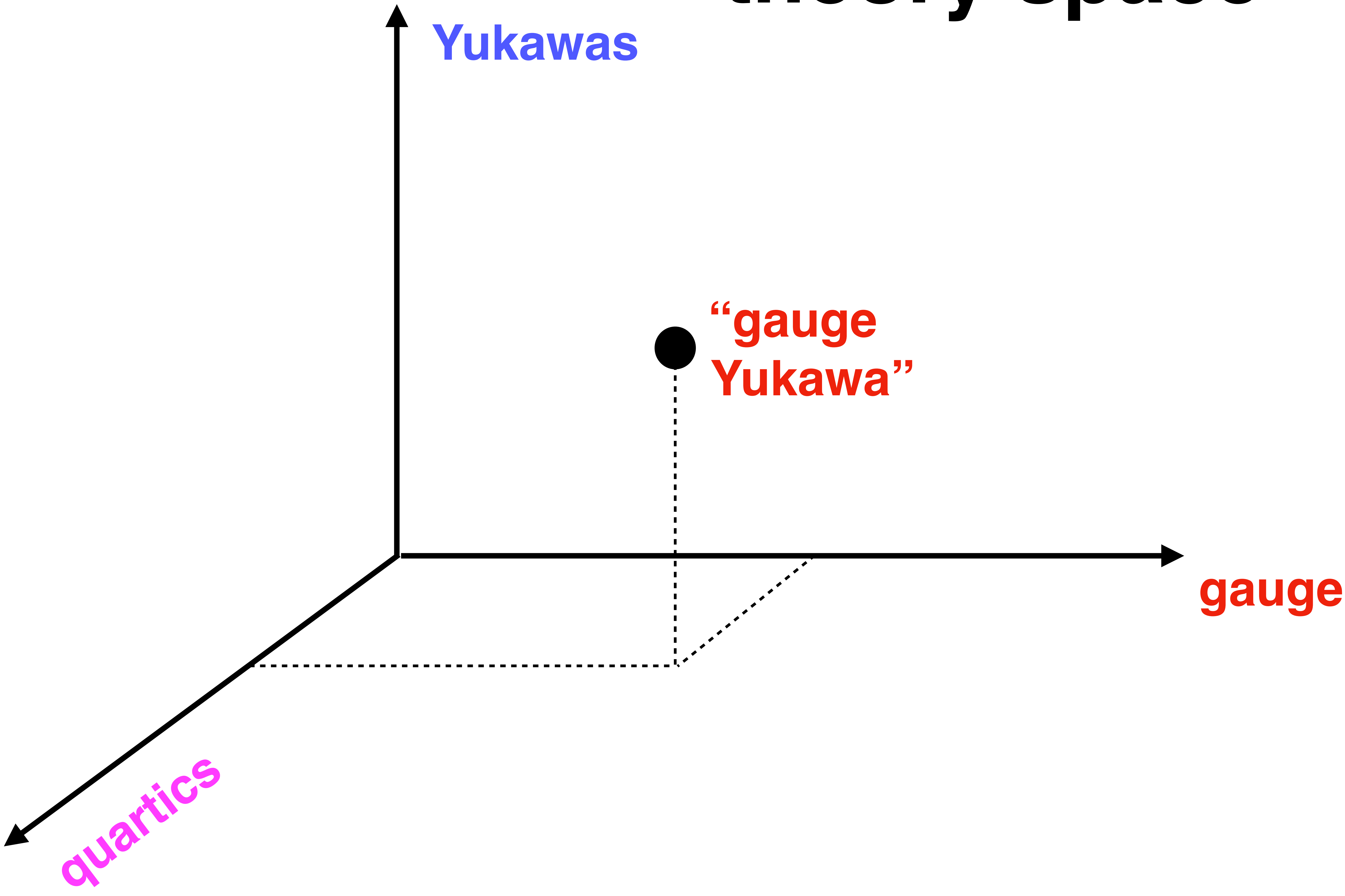

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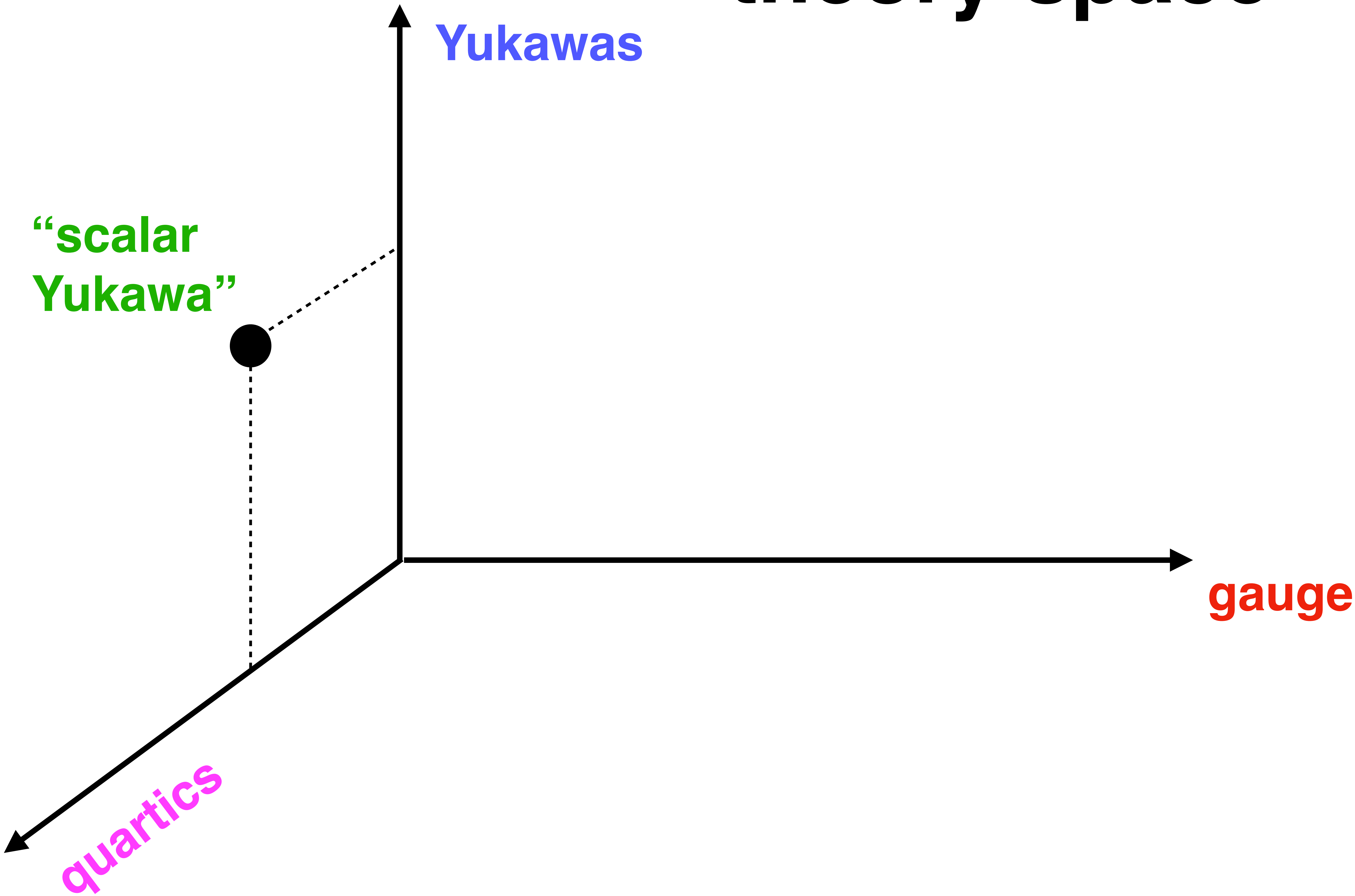

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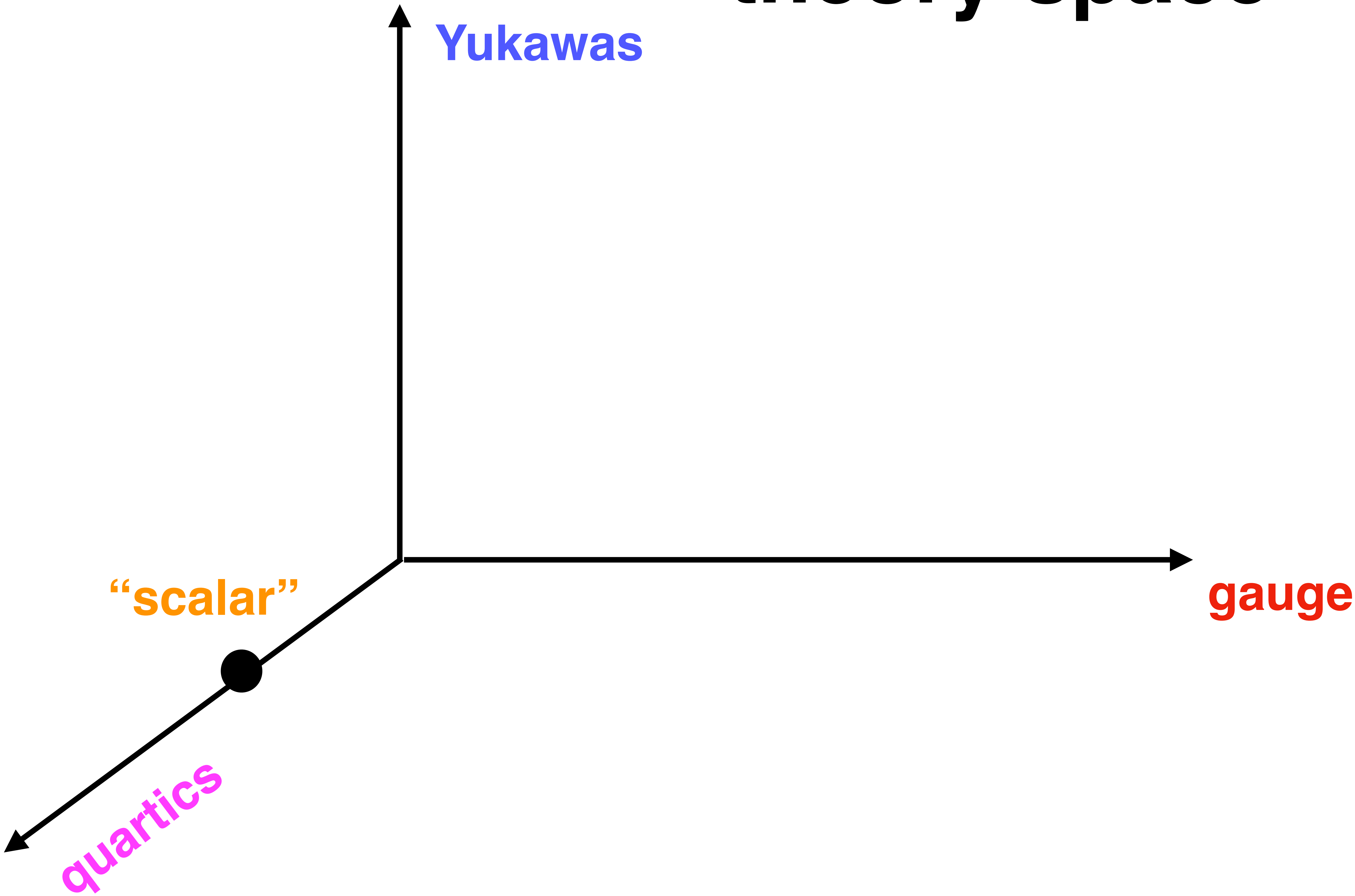


“theory space”

couplings	interacting fixed points				
gauge	Y	Y	Y	N	N
Yukawas	N	N	Y	N	Y
quartics	N	Y	Y	Y	Y
	“Banks Zaks”	“Banks Zaks”	“gauge Yukawa”	“scalar”	“scalar Yukawa”

result

couplings	YES non-abelian			NO abelian			NO	
gauge	Y	Y	Y	Y	Y	Y	N	N
Yukawas	N	N	Y	N	N	Y	N	Y
quartics	N	Y	Y	N	Y	Y	Y	Y
weak FPs	IR	IR	IR UV	no	no	no	no	no

Banks Zaks

gauge Yukawa

Banks Zaks

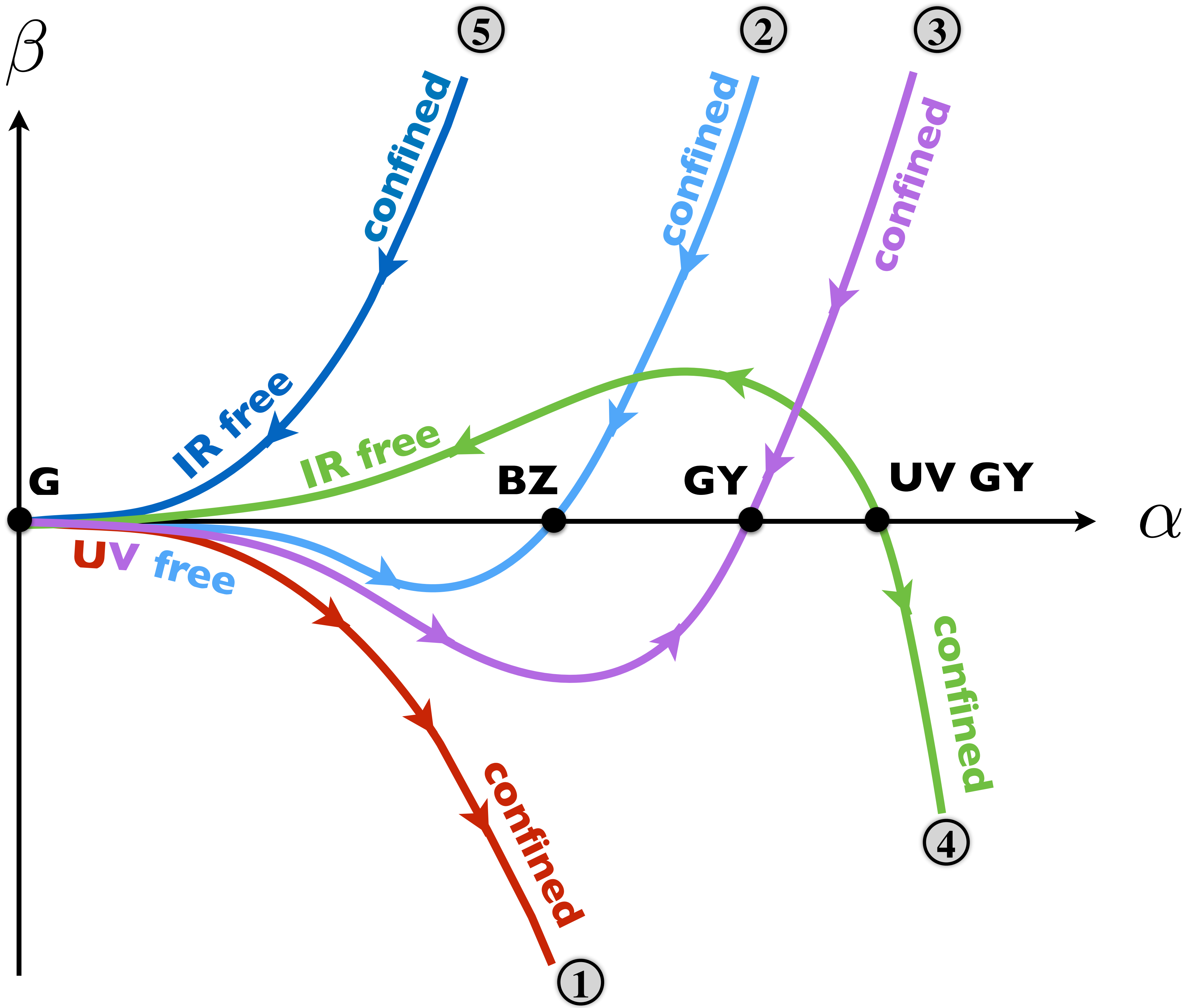
gauge Yukawa

scalar

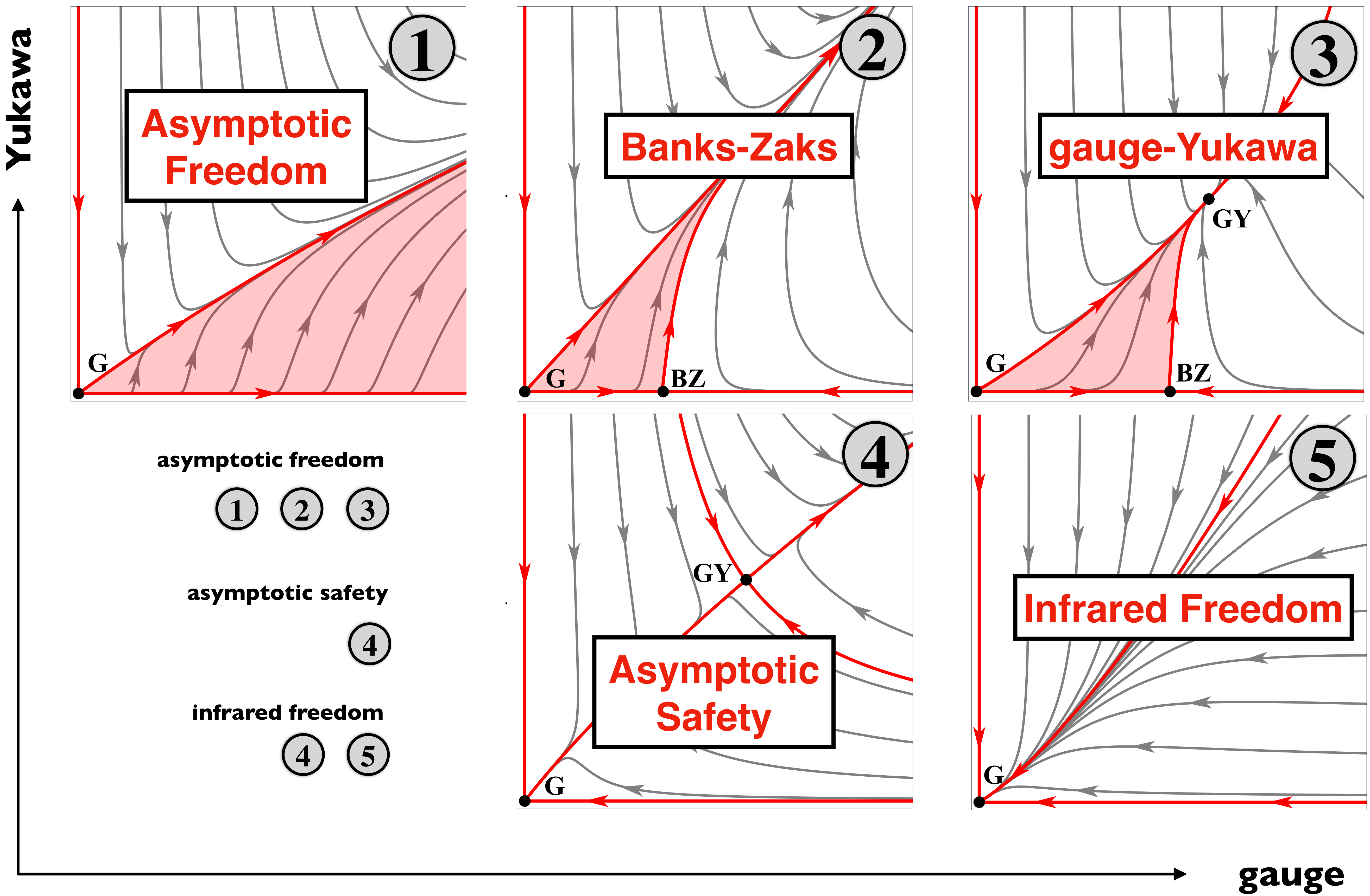
scalar Yukawa

AD Bond, DF Litim, **Theorems for Asymptotic Safety of Gauge Theories,** 1608.00519 (EPJC)
AD Bond, DF Litim, **Price of Asymptotic Safety,** 1801.08527 (PRL)

renormalisation group



“phase diagrams”



result

	YES			NO			NO	
couplings	non-abelian			abelian				
gauge	Y	Y	Y	Y	Y	Y	N	N
Yukawas	N	N	Y	N	N	Y	N	Y
quartics	N	Y	Y	N	Y	Y	Y	Y
weak FPs	IR	IR	IR UV	no	no	no	no	no
	Banks Zaks	Banks Zaks	gauge Yukawa	Banks Zaks	Banks Zaks	gauge Yukawa	scalar	scalar Yukawa

AD Bond, DF Litim, **Theorems for Asymptotic Safety of Gauge Theories,** 1608.00519 (EPJC)
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Banks Zaks

gauge Yukawa

Banks Zaks

gauge Yukawa

scalar

scalar Yukawa

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result

	YES			NO			NO	
couplings	non-abelian			abelian				
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Yukawas	N	N	Y	N	N	Y	N	Y
quartics	N	Y	Y	N	Y	Y	Y	Y
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Banks Zaks

gauge Yukawa

Banks Zaks

gauge Yukawa

scalar

scalar Yukawa

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AD Bond, DF Litim, **Price of Asymptotic Safety,** 1801.08527 (PRL)

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Banks Zaks

gauge Yukawa

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scalar Yukawa

AD Bond, DF Litim, **Theorems for Asymptotic Safety of Gauge Theories,** 1608.00519 (EPJC)
AD Bond, DF Litim, **Price of Asymptotic Safety,** 1801.08527 (PRL)

why no UV BZ?

gauge coupling

$$\alpha = \frac{g^2}{(4\pi)^2} \qquad \beta = -B\,\alpha^2 + C\,\alpha^3 + \mathcal{O}(\alpha^4)$$

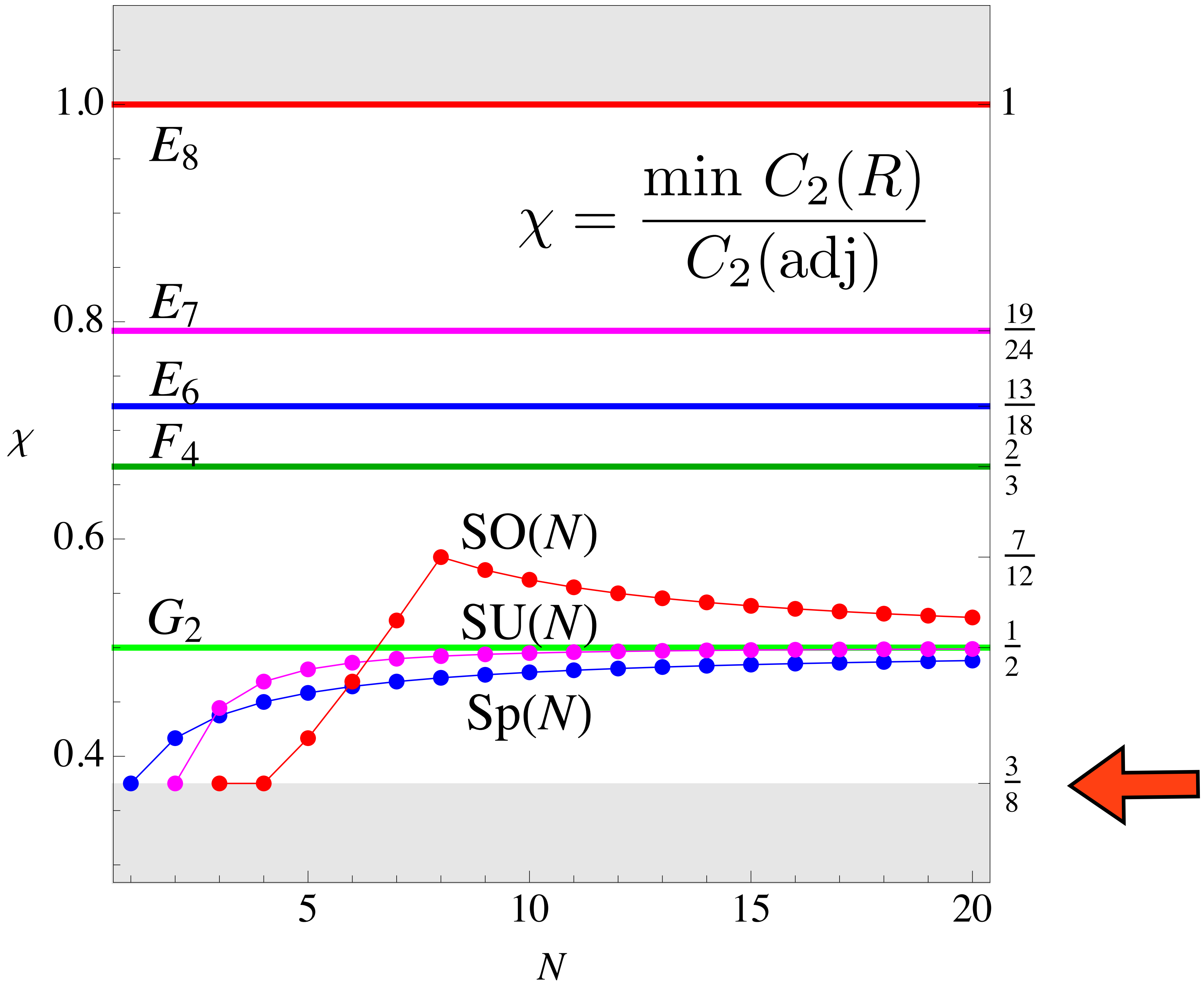
weakly coupled fixed point $\quad 0 < \alpha^* = B/C \ll 1$

competition between matter and gauge fields

UV BZ requires $\quad \boxed{B < 0, C < 0}$

necessary condition $\quad \chi = \frac{\min\, C_2(R)}{C_2(\mathrm{adj})} < \frac{1}{11}$ **impossible!**

here's why.



Bond, Litim, 1608.00519/EPJC

why Yukawas?

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

one loop   gauge   Yukawa

here's why.

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

one loop gauge Yukawa

Yukawas *always* slow down the running of gauge couplings

template UV theories

Lagrangian	Fields	Interactions
$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu}$	SU(N) + Nf Diracs	gauge
$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$	or	
$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$	SO(N) + Nf Majoranas	
$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$	or	Yukawas
$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$	Sp(N) + Nf Majoranas	
$L_V = -v\,({\rm Tr}\, H^\dagger H)^2\,.$	and	quartics
	Scalars H_ij	

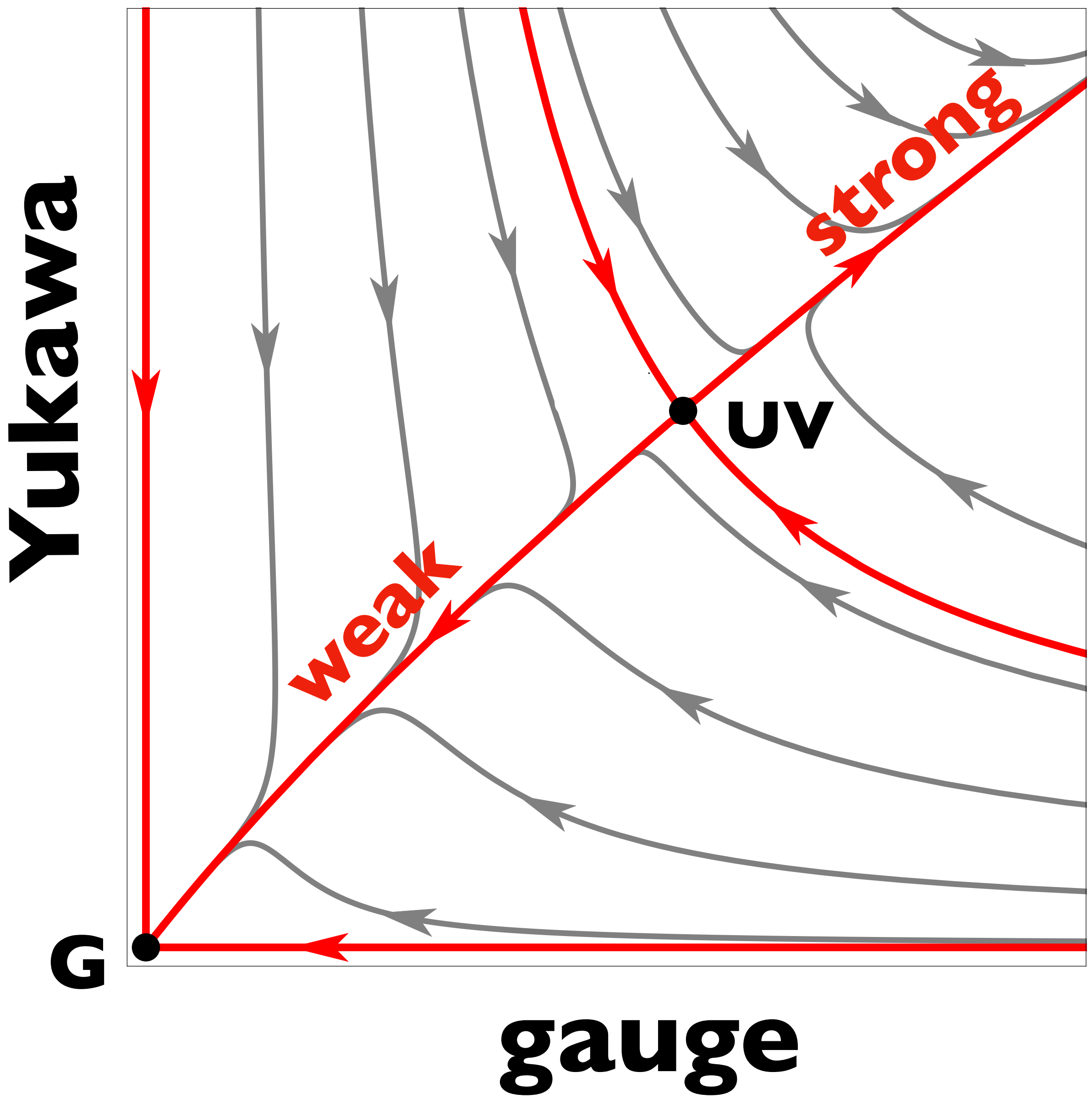
Parameter: N, Nf, masses

DF Litim, F Sannino, **Asymptotic safety guaranteed,** 1406.2337

AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615

AD Bond, DF Litim, T Steudtner, **Asymptotic safety with Majorana fermions and new large N equivalences** 1911.11168

template UV



SU(N) + Diracs
+ mesons

SO(N) + Majoranas
+ mesons

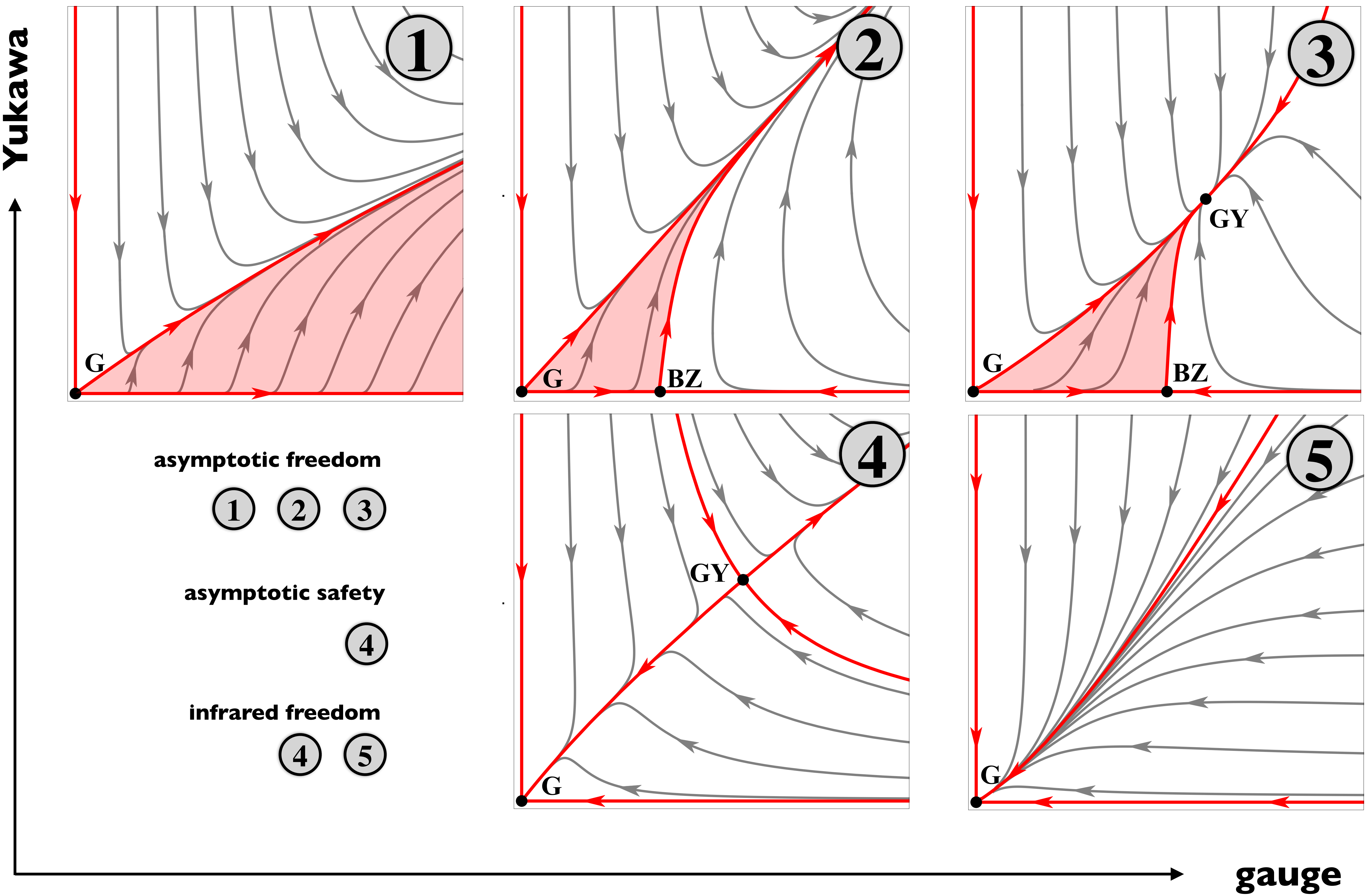
Sp(N) + Majoranas
+ mesons

DF Litim, F Sannino, **Asymptotic safety guaranteed,** 1406.2337
AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615
AD Bond, DF Litim, T Steudtner, **Asymptotic safety with Majorana fermions and new large N equivalences** 1911.11168

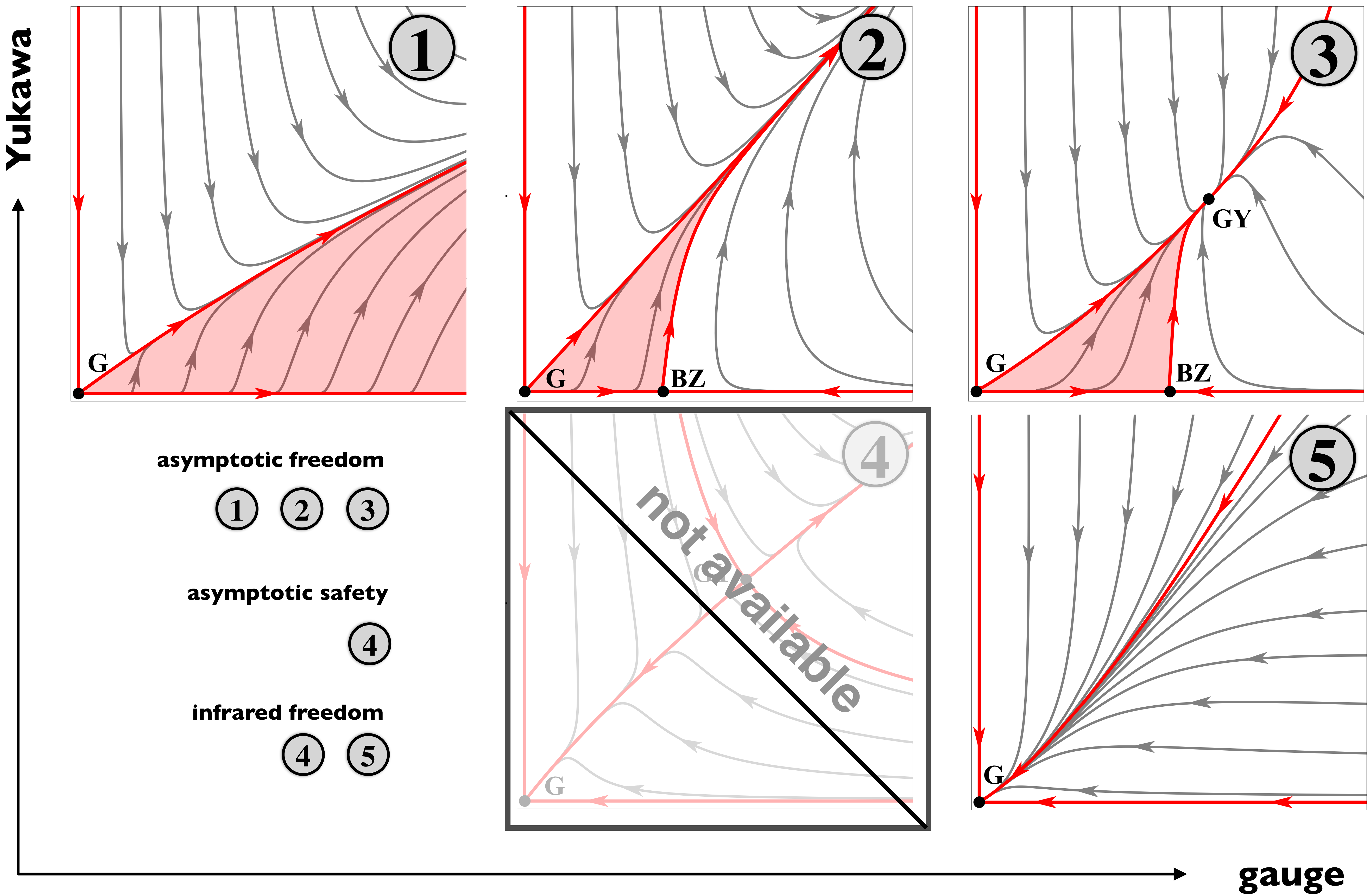
what about supersymmetry?

AD Bond, DF Litim, **Asymptotic Safety Guaranteed in Supersymmetry,** 1709.06953 (PRL)
DF Litim, G Hiller, K Moch, **Fixed Points in Supersymmetric Extensions of the SM,** 2202.01264 (EPJC)
AD Bond, DF Litim, **Asymptotic Safety Guaranteed in Strongly Coupled Gauge Theories,** 2202.08223 (PRD)

N=0 supersymmetry

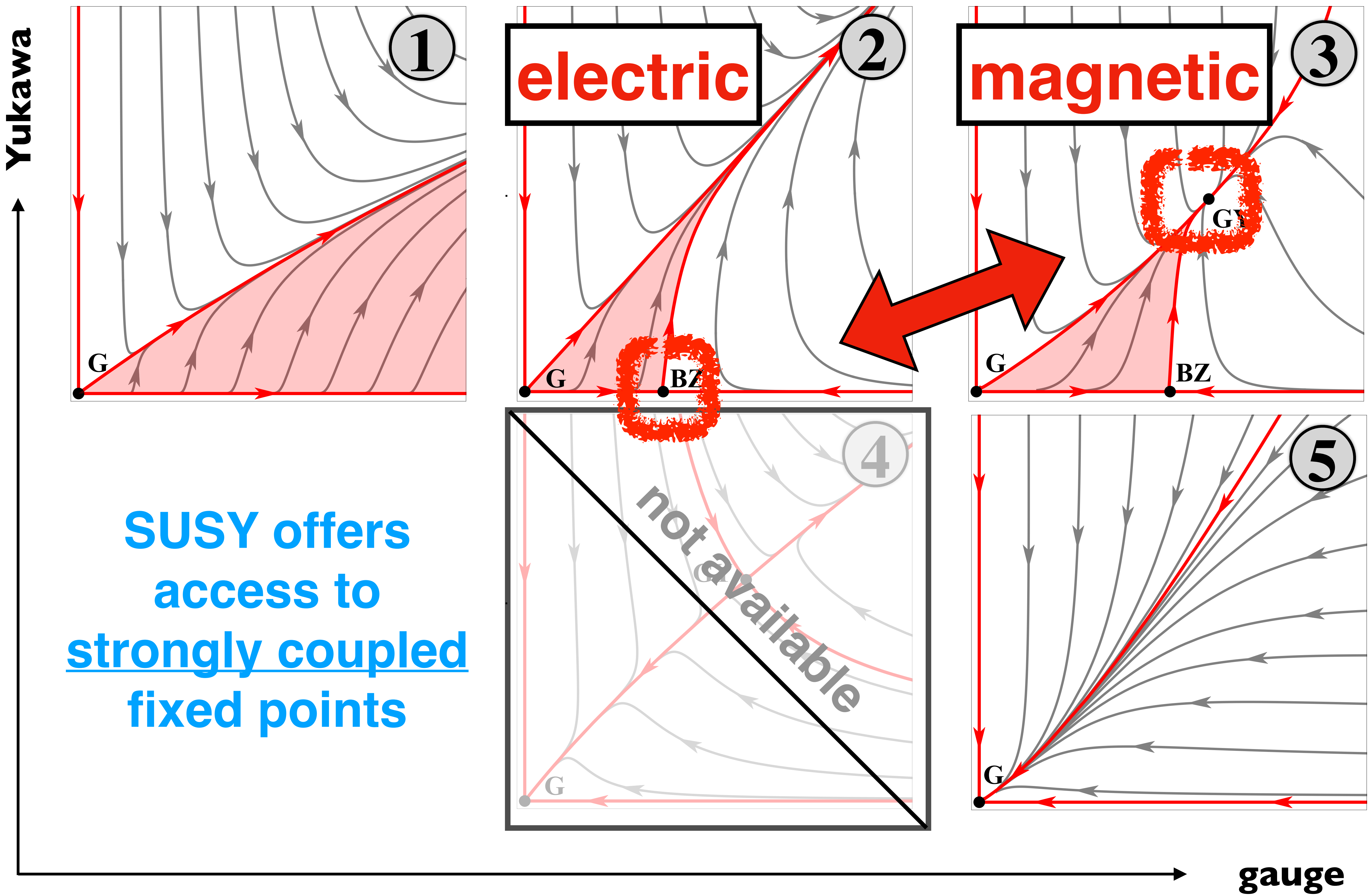


N=1 supersymmetry



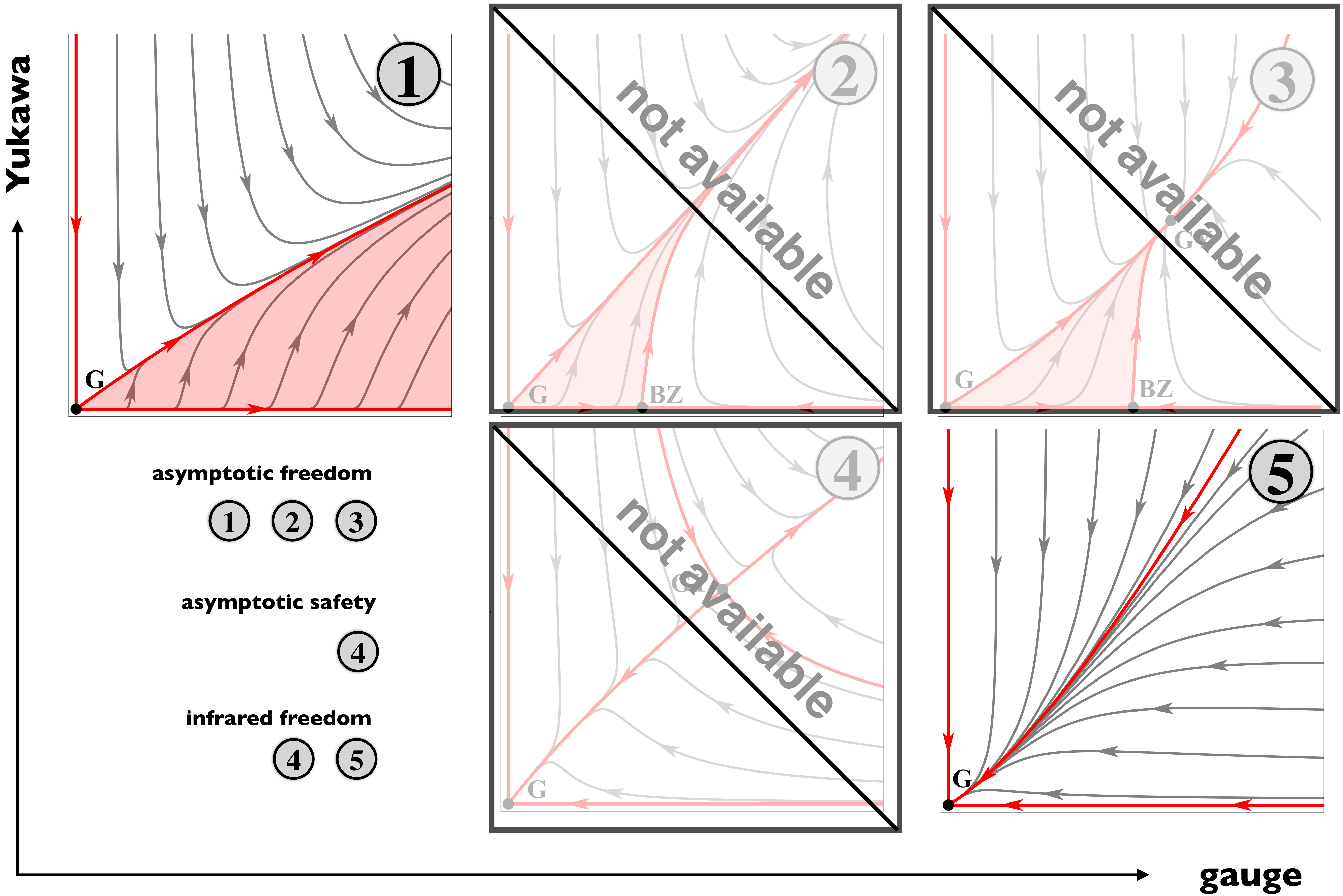
Seiberg duality

Seiberg '95

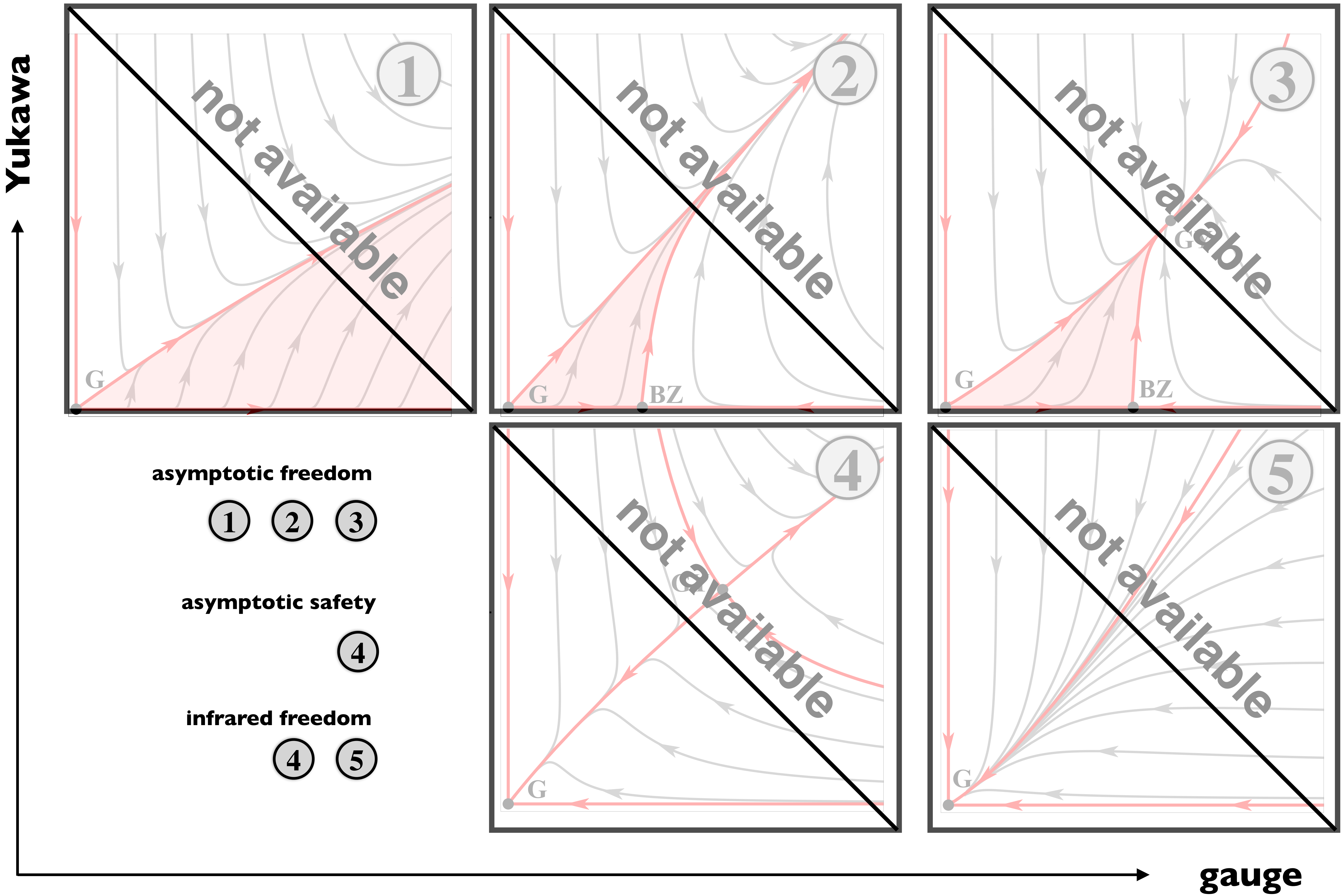


SUSY offers access to <u>strongly coupled</u> fixed points

N=2 supersymmetry



N=4 supersymmetry



why no Susy UV fixed point

superfield anomalous dimension

$$2\,\frac{d_R}{d_G}\,|\gamma_R|^2 = B\,\alpha_* + \mathcal{O}(B\,\alpha_*^2, \alpha_*^3)$$

S Martin, J Wells, hep-ph/0011382/PRD

asymptotic freedom requires B > 0

primary mechanism does not work

why ~~no~~ Susy UV fixed point

superfield anomalous dimension

$$2\,\frac{d_R}{d_G}\,|\gamma_R|^2 = B_i\,\alpha_*^i + \mathcal{O}(B\,\alpha_*^2, \alpha_*^3)$$

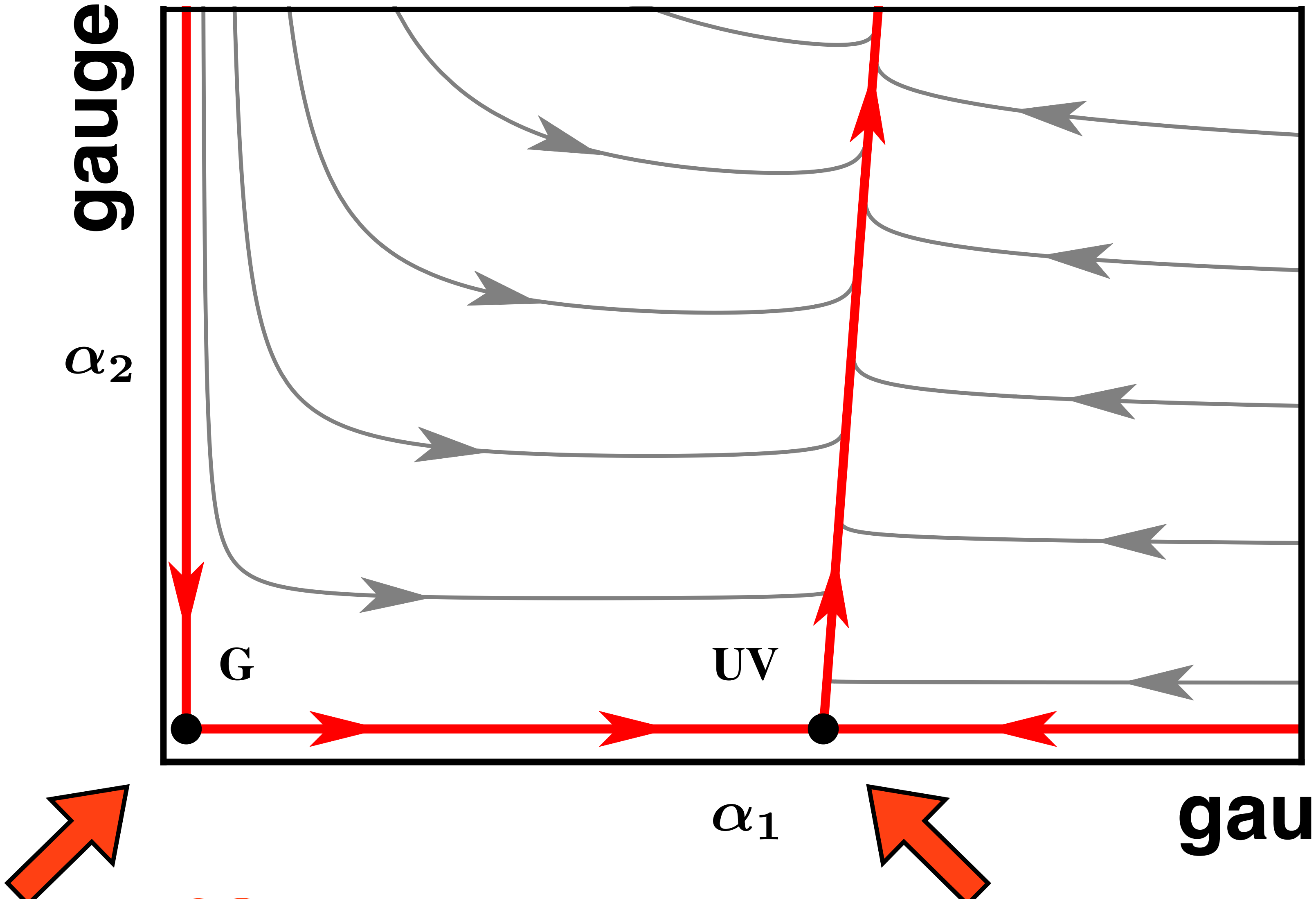
S Martin, J Wells, hep-ph/0011382/PRD

remedy: **semi-simple** susy gauge theories

some $B_i < 0$ a possibility

AD Bond, DF Litim, 1709.06953/PRL

Susy UV fixed points



SU(N) x SU(M)
+ chiral superfields
+ superpotential

no asymptotic freedom

e.g. MSSM

interacting UV fixed point

AD Bond, DF Litim, **Asymptotic Safety Guaranteed in Supersymmetry,** 1709.06953 (PRL)
Asymptotic Safety Guaranteed for Strongly Coupled Gauge Theories, 2202.08223 (PRD)

MSSM extensions

MSSM + new quark singlets + new leptons + superpotential

$$W_1 = Y^{ijk}\overline{d}_i Q_j L_k + \overline{Y}^{ijk}\overline{u}_i Q_j \overline{L}_k + x_b\, y_b \overline{d}_3 Q_3 H_d + x_t\, y_t \overline{u}_3 Q_3 H_u \ ,$$

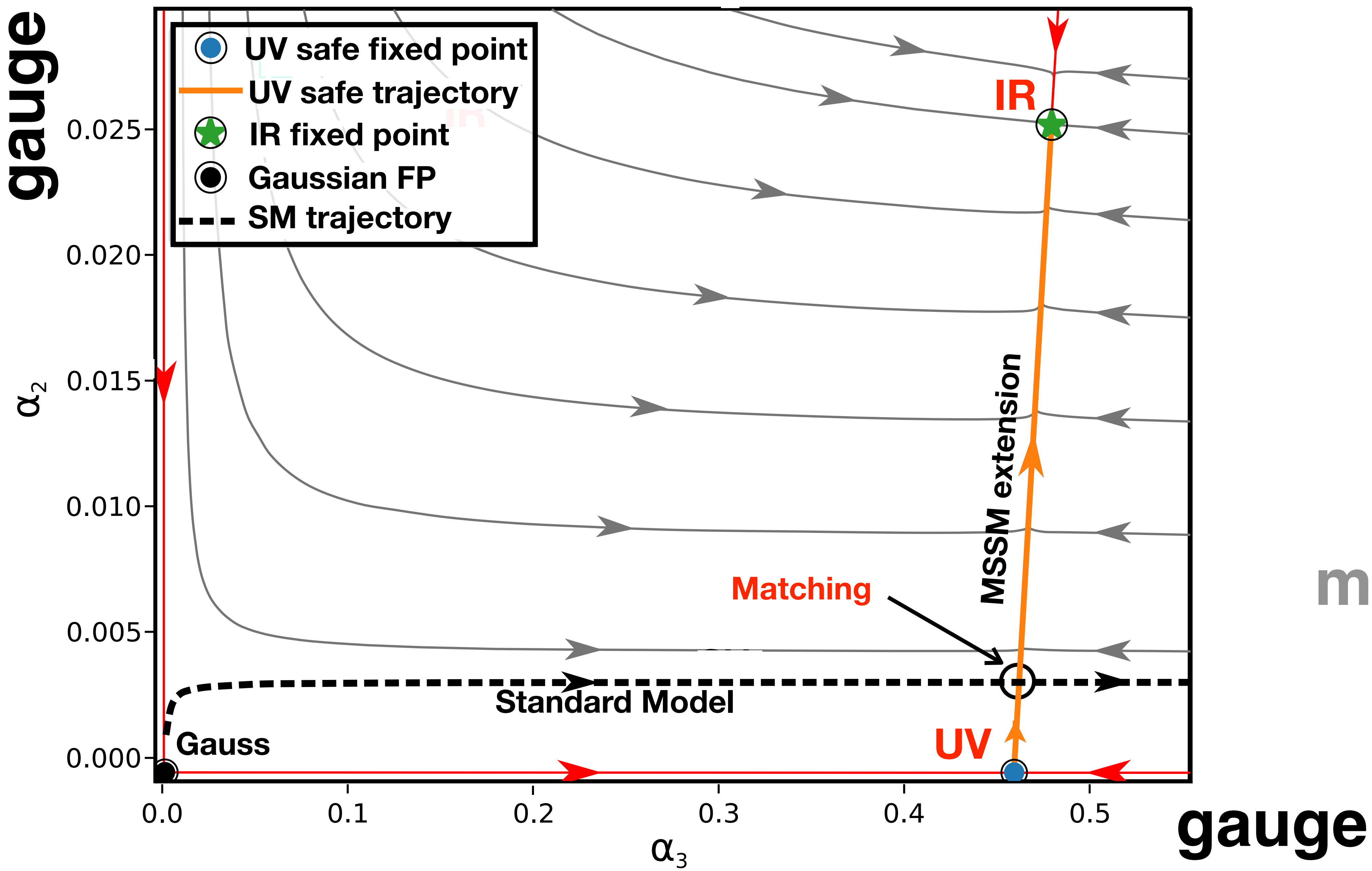
2 Loop RGEs

scan over
3.5M models

G Hiller, D Litim, S Moch, **FPs in supersymmetric extensions of the Standard Model** 2202.01264 (EPJC)

MSSM extensions

MSSM + new quark singlets
+ new leptons + superpotential



however: matching scale too low

G Hiller, D Litim, S Moch, **FPs in supersymmetric extensions of the Standard Model** 2202.01264 (EPJC)

a) 4d critical points

Case	Condition	Fixed Point
$i)$	$g_i = \mathbf{Y}^A_{JK} = \lambda_{ABCD} = 0$	Gaussian
$ii)$	some $g_i \neq 0$, all $\mathbf{Y}^A_{JK} = 0$	Banks-Zaks
$iii)$	some $g_i \neq 0$, some $\mathbf{Y}^A_{JK} \neq 0$	gauge-Yukawa

weakly-coupled fixed points necessitate
non-abelian gauge fields

systematic classification of 4d fixed points
“universality classes”

AD Bond, DF Litim, **Price of Asymptotic Safety,** PRL 122 (2019) 211601 [1801.08527]

b) conformal field theory

a-theorem Komargodski, Schwimmer '11

"any Lorentz-invariant and unitary 4d QFT which is under perturbative control in the deep UV or IR asymptotes to a CFT"

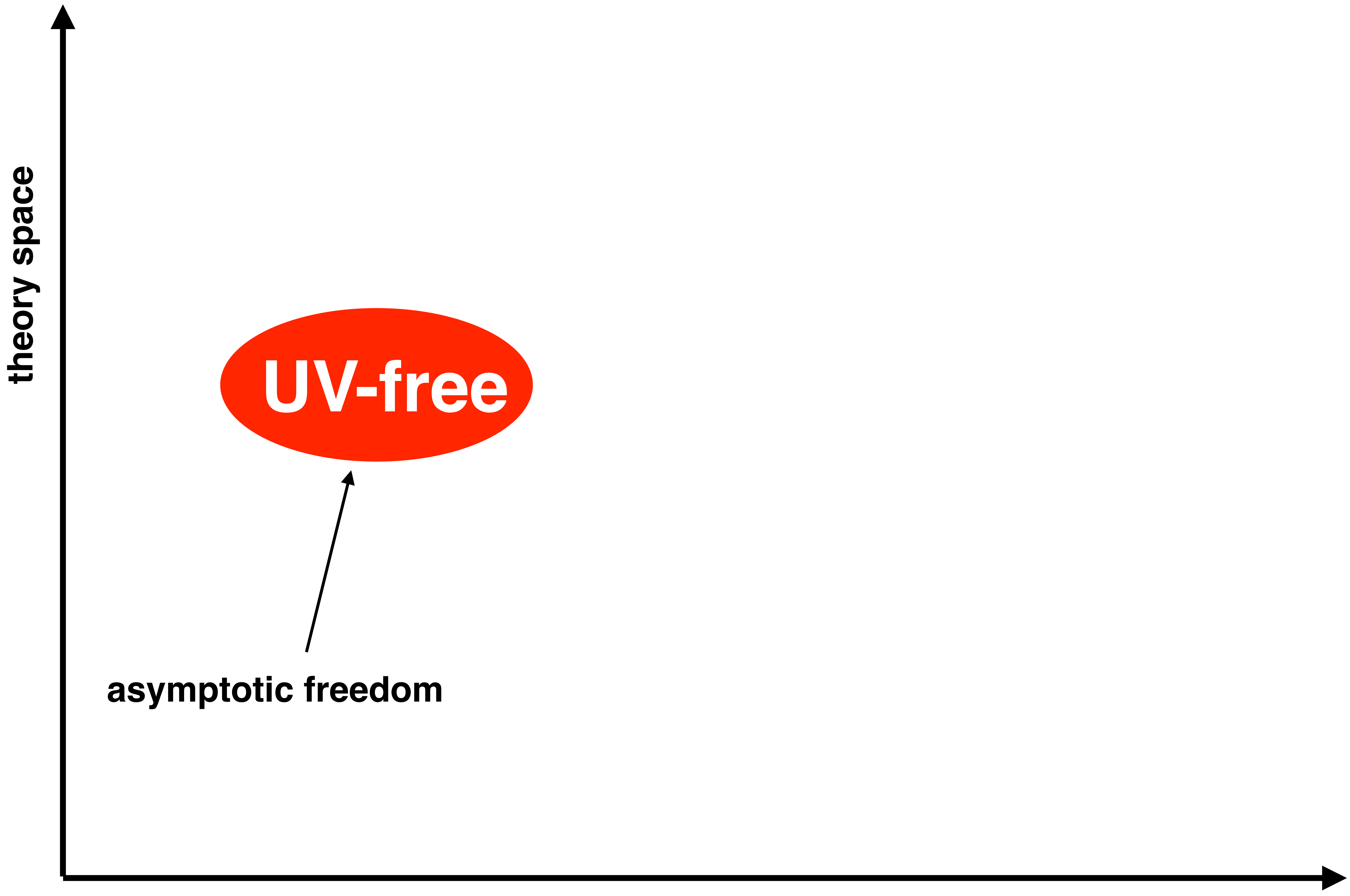
Polchinski '88
Luty, Polchinski, Rattazzi '12

→ **any weakly-coupled 4d CFT must contain non-abelian gauge fields**

→ **4d space-time is distinguished**

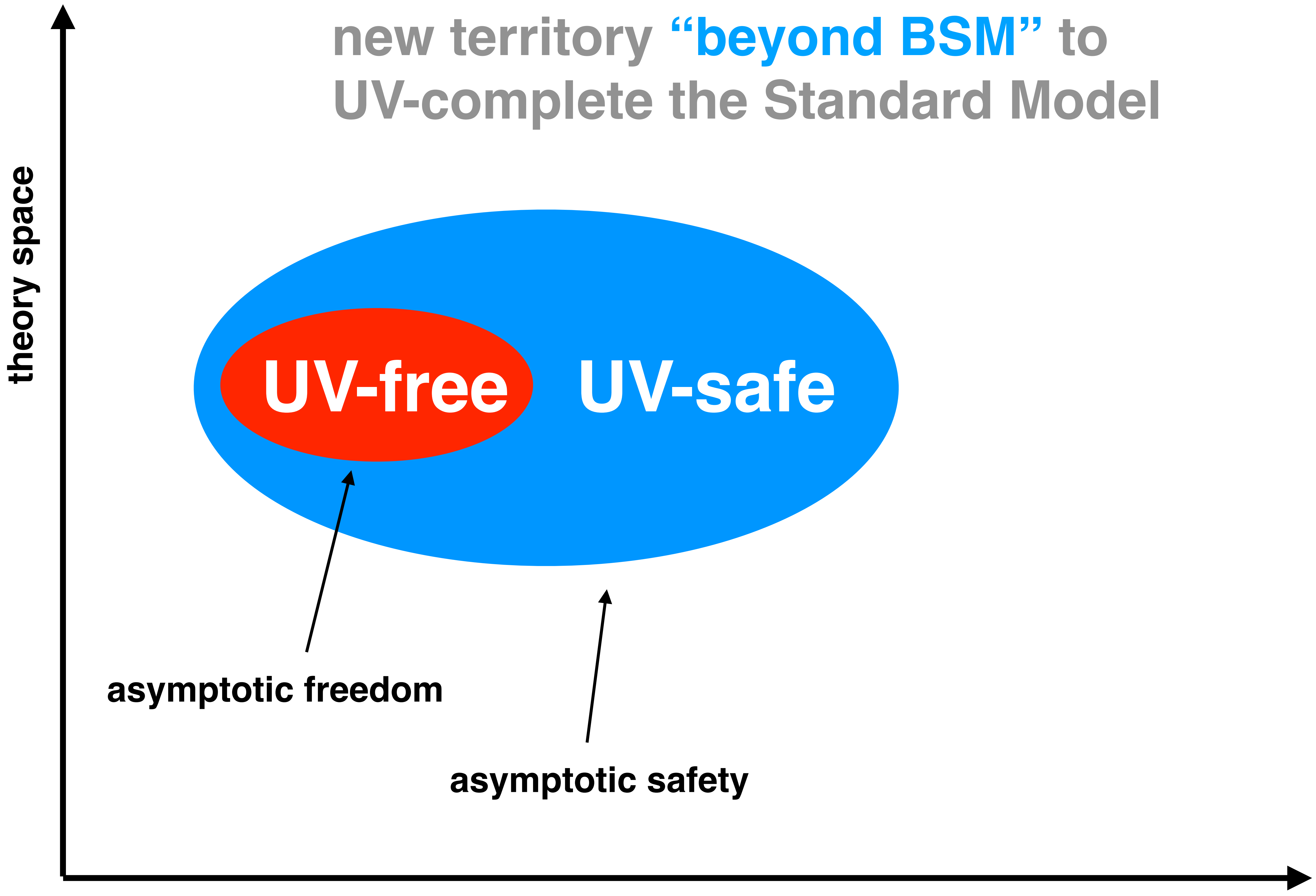
→ **QFT + RG offer access to CFT data complementary to bootstrap**

c) model building

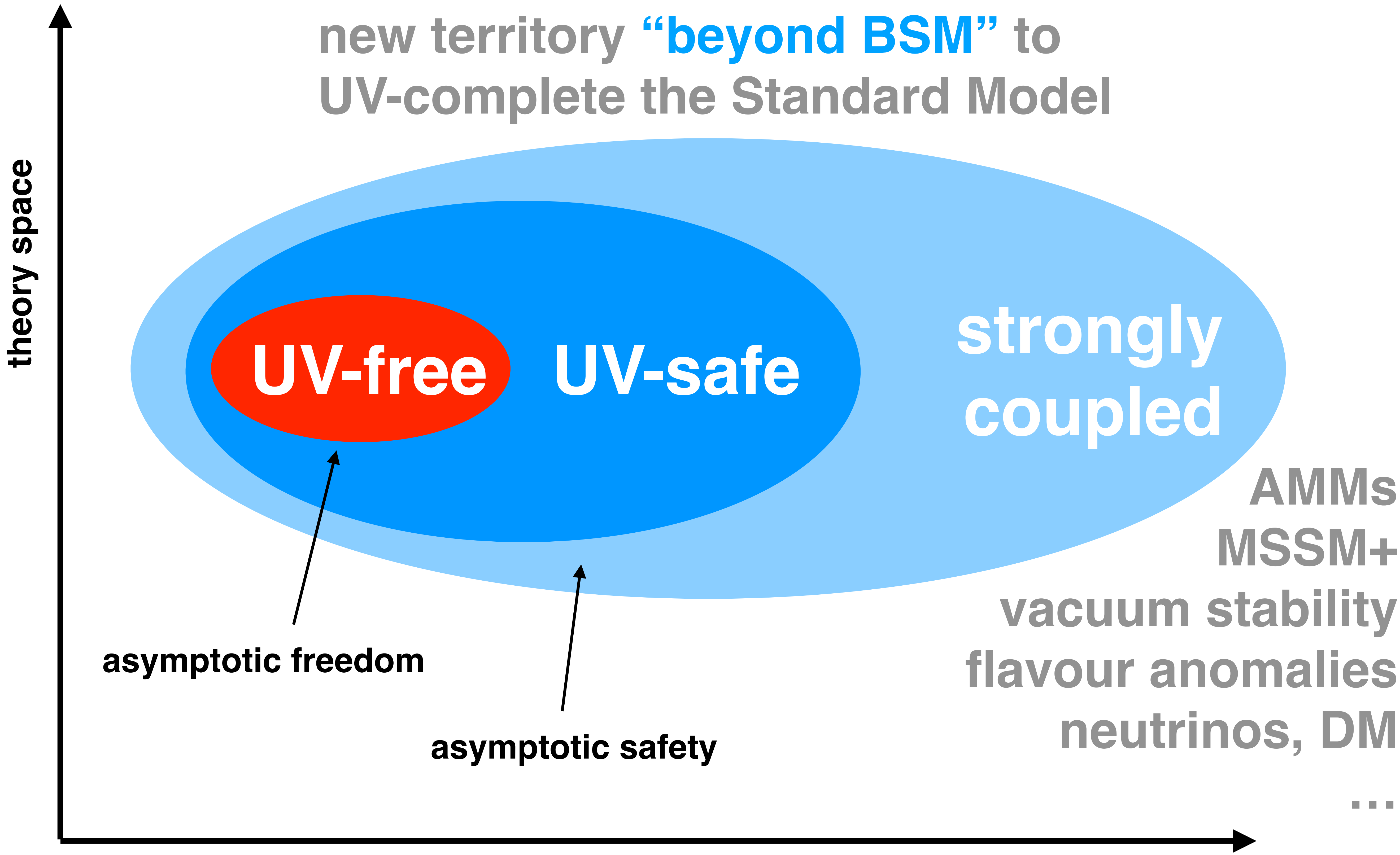


c) model building

new territory “beyond BSM” to
UV-complete the Standard Model



c) model building



strongly coupled fixed points

strongly coupled FPs

w/ supersymmetry: YES

non-renormalisation theorems
infinite order RGEs Novikov, Shifman, Vainstein, Zakharov '83 '86
Seiberg duality Seiberg '95
***a* - maximisation** Intrilligator, Wecht '03

AD Bond, DF Litim, **Asymptotic Safety Guaranteed in Strongly Coupled Gauge Theories,** 2202.08223 (PRD)

w/o supersymmetry: challenges

lattice
conformal bootstrap
Schwinger-Dyson
functional RG
perturbation theory

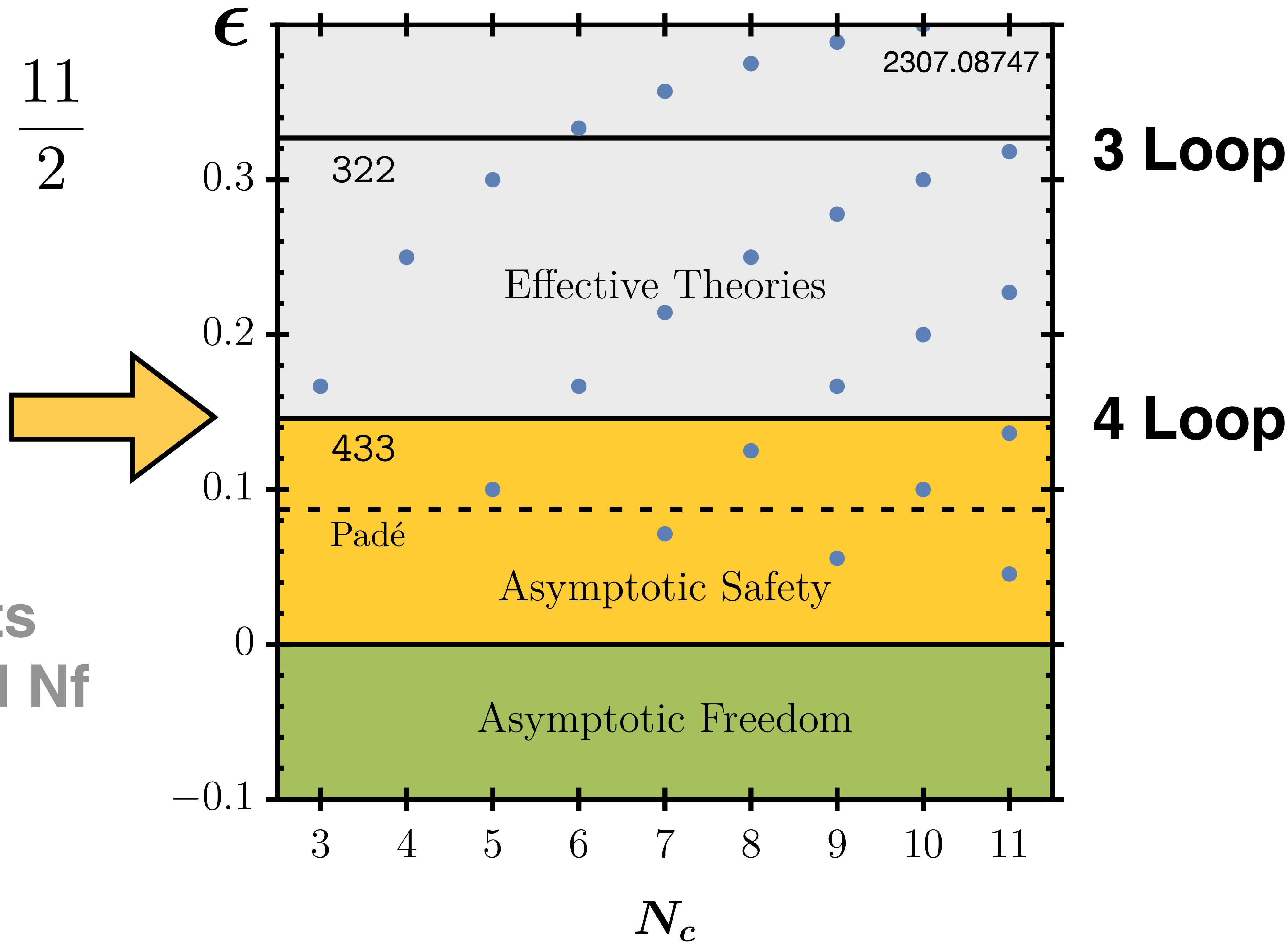
Di Pietro and M. Serone, **Looking through the QCD Conformal Window with Perturbation Theory** 2003.01742

UV conformal window from PT

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

size of the conformal window

constraints for Nc and Nf



AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615 (PRD)
AD Bond, DF Litim, G Medina Vazquez, **Conformal windows beyond asymptotic freedom**, 2107.13020 (PRD)
DF Litim, N Riyaz, E Stamou, T Steudtner, **Asymptotic safety guaranteed at four loop**, 2307.08747 (PRD)

4d quantum gravity

gravitation

physics of classical gravity

Einstein's theory of general relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda\, g_{\mu\nu} + 8\pi G_N\, T_{\mu\nu}$$

Newton's coupling $G_N = 6.7 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg}\, s^3}$

cosmological constant $\Lambda \approx 10^{-35} s^{-2}$

what's new with gravity?

degrees of freedom: **spin 2**

perturbatively non-renormalisable

Newton's coupling is **dimensionful** $[G_N] = 2 - D < 0$

UV fixed point?

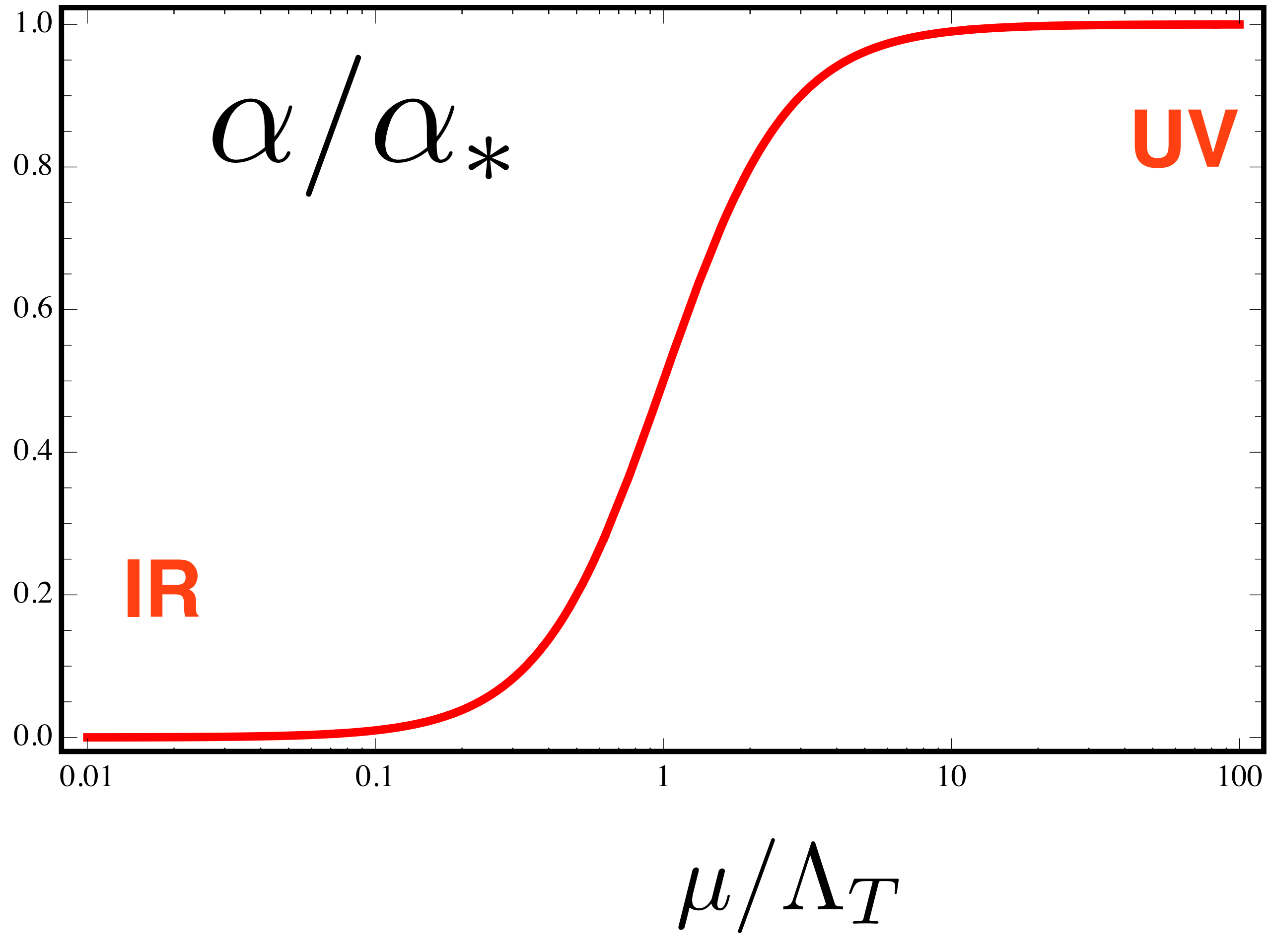
requires **large** anomalous dimensions

non-perturbative tools mandatory

renormalisation group

	dimension	coupling
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$

Gastmans et al '78
Christensen, Duff '78
Weinberg '79
Kawai et al '90



$$G(\mu) \approx G_N$$

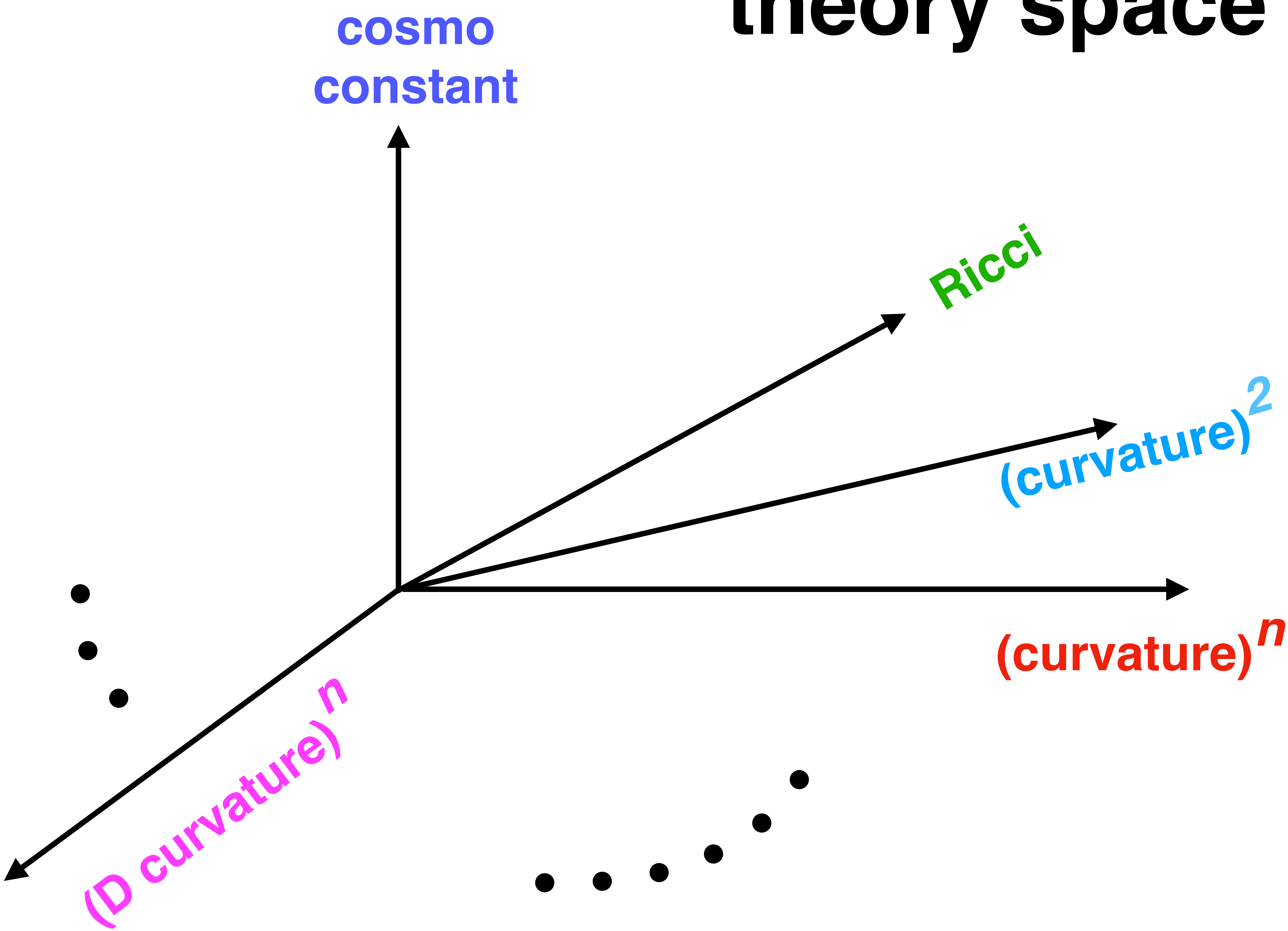
classical GR

$$G(\mu) \approx \frac{\alpha_*}{\mu^{D-2}}$$

quantum GR

UV fixed point implies weakened gravity

“theory space”



renormalisation group

functional RG

Reuter ’96

evidence for FP in 4d

Reuter ’96, Percacci ’02, Litim ’03
Codello, Rahmende, Percacci ’08, Niedermaier ‘08
Benedetti, Machado, Saueressig ’09
Eichhorn, Gies ‘10 , Folkerts, Litim, Pawlowski ’12
Falls, Litim, Nikolakopoulos, Rahmede ’13, ’14, ’16
Falls, Litim, Schroeder ’18, Eichhorn, Schiffer ’22
Kluth, Litim ’20, 22, ‘23
Fehre, Litim, Pawlowski, Reichert ‘22
Pawlowski, Reichert ’23
...

convergence of

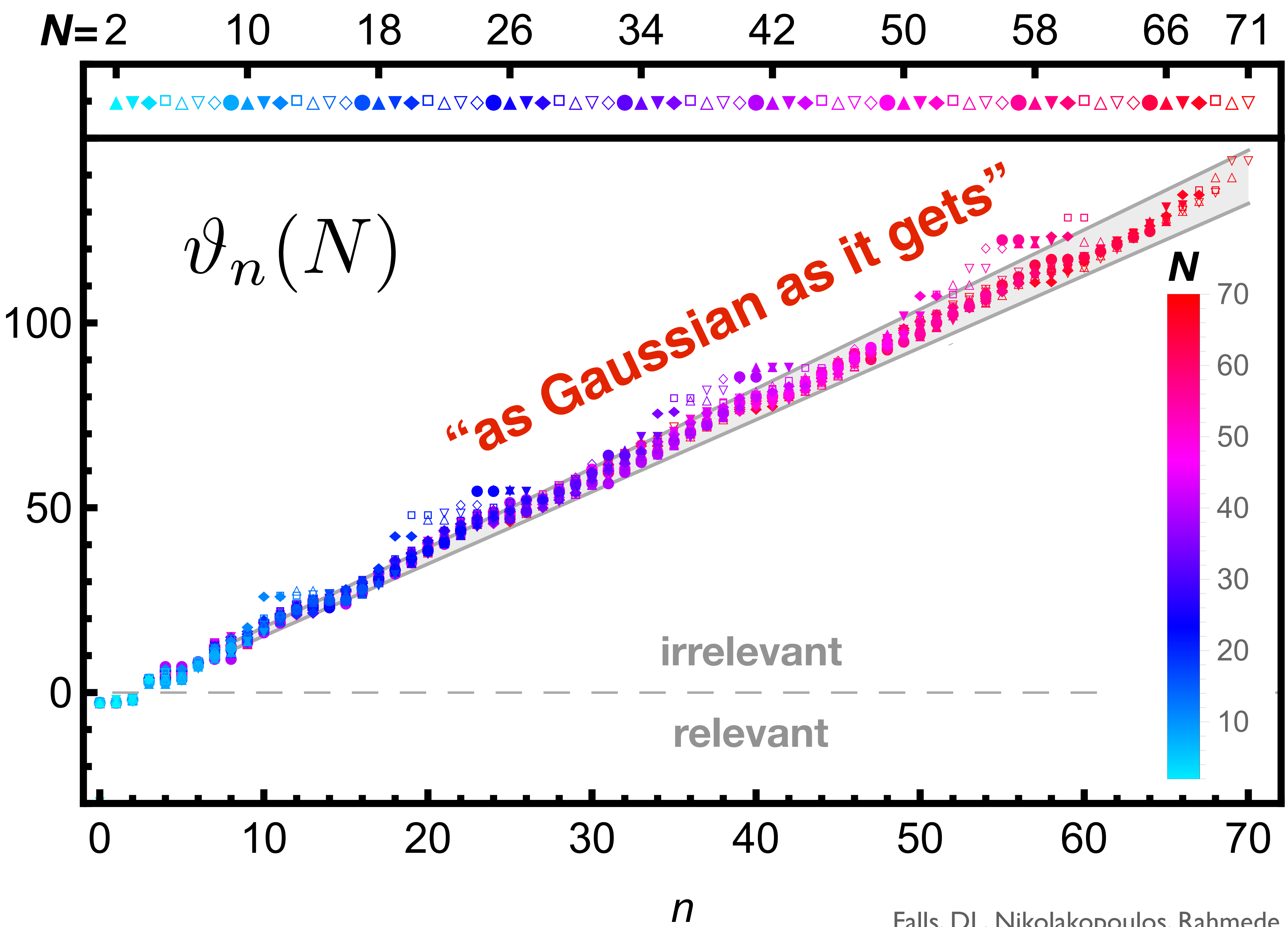
higher curvature interactions

Falls, Litim, Nikolakopoulos, Rahmede ’13, ’14, ’16
Falls, Litim, Schroeder ’18
Kluth, Litim ’20, 22, ’23
Baldazzi, Falls, Kluth, Knorr ’23
...

gravitational correlation functions

Christiansen, Meibohm, Pawlowski, Reichert ’15
Fehre, Litim, Pawlowski, Reichert ‘22
Pawlowski, Reichert ’23
...

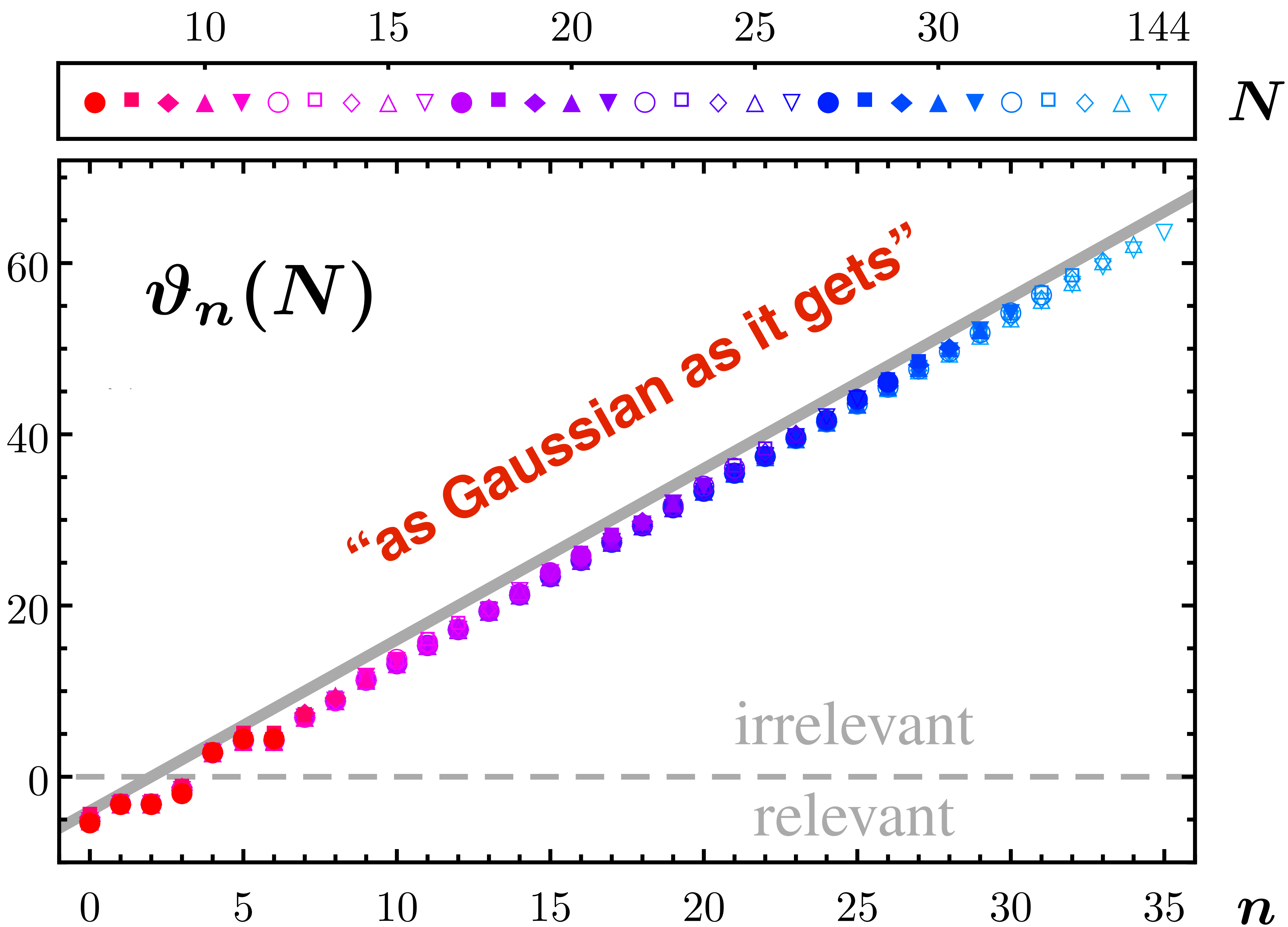
Ricci scalars



Falls, DL, Nikolakopoulos, Rahmede '13, '14, '16
Falls, DL, Schroeder, '18

$$\Gamma_k = \int d^d x \sqrt{g}[F_k(\text{Riem}^2) + R \cdot Z_k(\text{Riem}^2)]$$

Riemann



Yannick Kluth, DF Litim, **Fixed points of quantum gravity and the dimensionality of the UV critical surface,** 2008.09181

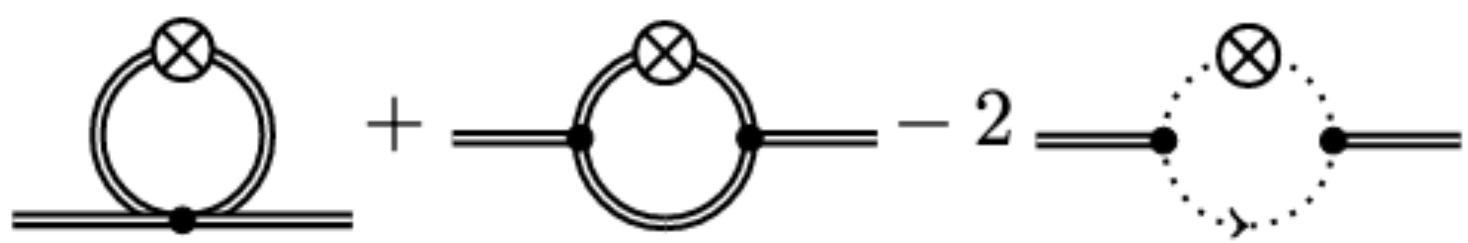
Lorentzian quantum gravity

Einstein Hilbert

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^4 x \, |\det g_{\mu\nu}|^{\frac{1}{2}} \left(\mathcal{R} - 2\Lambda\right)$$

use **Callan Symanzik flow** $\quad R_k = Z_\phi \, k^2$

$$\partial_t \Gamma_k^{(hh)} = -\frac{1}{2}$$



$$- \partial_t S^{(hh)}_{\mathrm{ct},k}$$

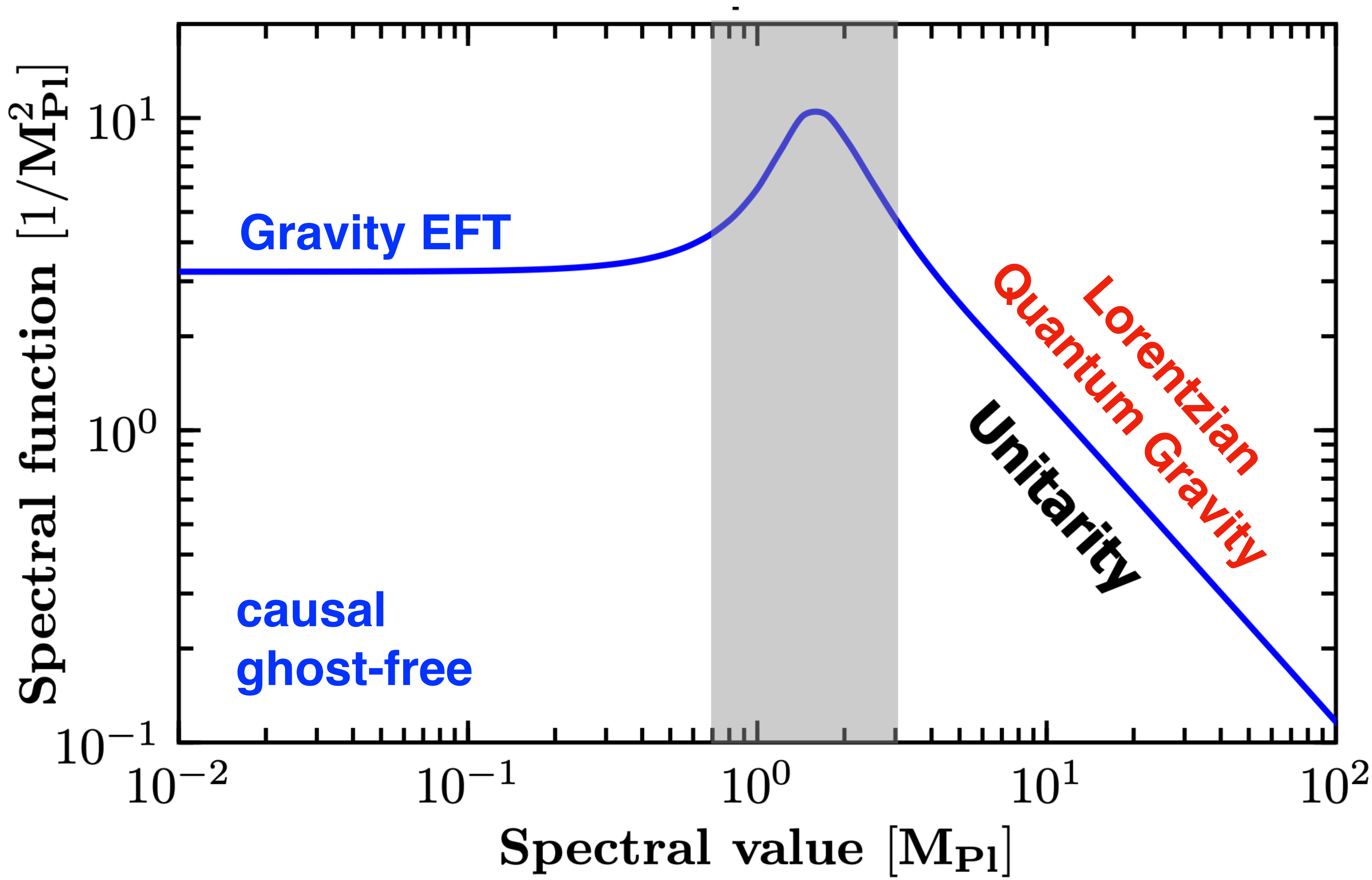
to find **CL spectral representation** for the graviton

$$\mathcal{G}_{hh}(p_0, |\vec{p}\,|) = \int_0^\infty \frac{\mathrm{d}\lambda}{\pi} \frac{\lambda \, \rho_h(\lambda, |\vec{p}\,|)}{\lambda^2 + p_0^2}$$

Fehre, Litim, Reichert, Pawlowski, **Lorentzian Quantum Gravity and the Graviton Spectral Function**, 2111.13232 (PRL)

Lorentzian quantum gravity

result: UV fixed point in Lorentzian signature



Fehre, Litim, Reichert, Pawlowski, **Lorentzian Quantum Gravity and the Graviton Spectral Function**, 2111.13232 (PRL)

conclusions

rigorous fixed points in 4d QFTs

weak coupling: theorems, templates, classification,
non-Abelian gauge fields key, links with CFTs

new directions for BSM

challenges: strong coupling

evidence for UV fixed points in 4d quantum gravity

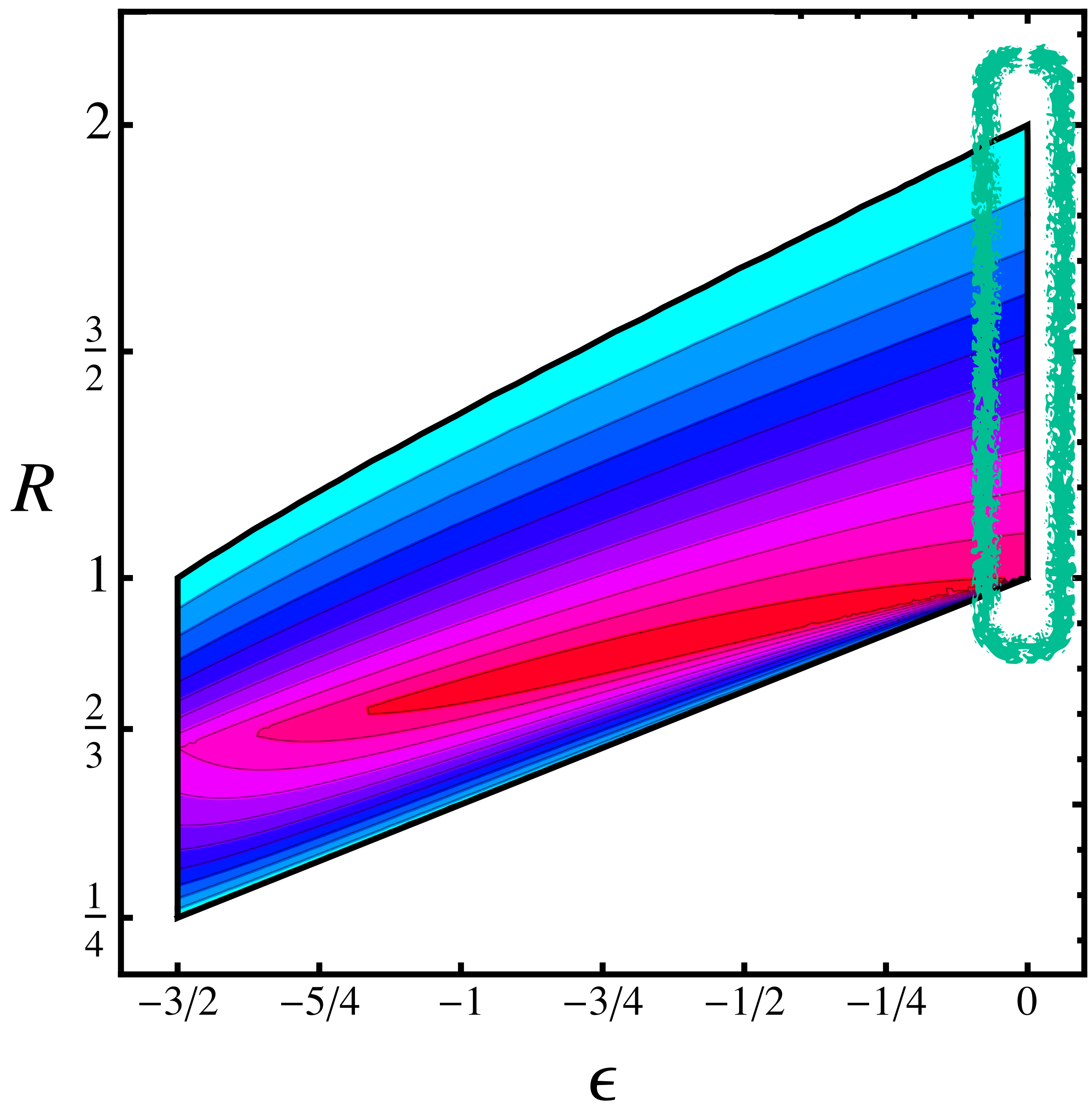
convergence, near-Gaussianity
Lorentzian signature, graviton spectral function

...stay tuned

extra bits and pieces

Non-Perturbative UV Conformal Window

$$R = \frac{N_2}{N_1}$$



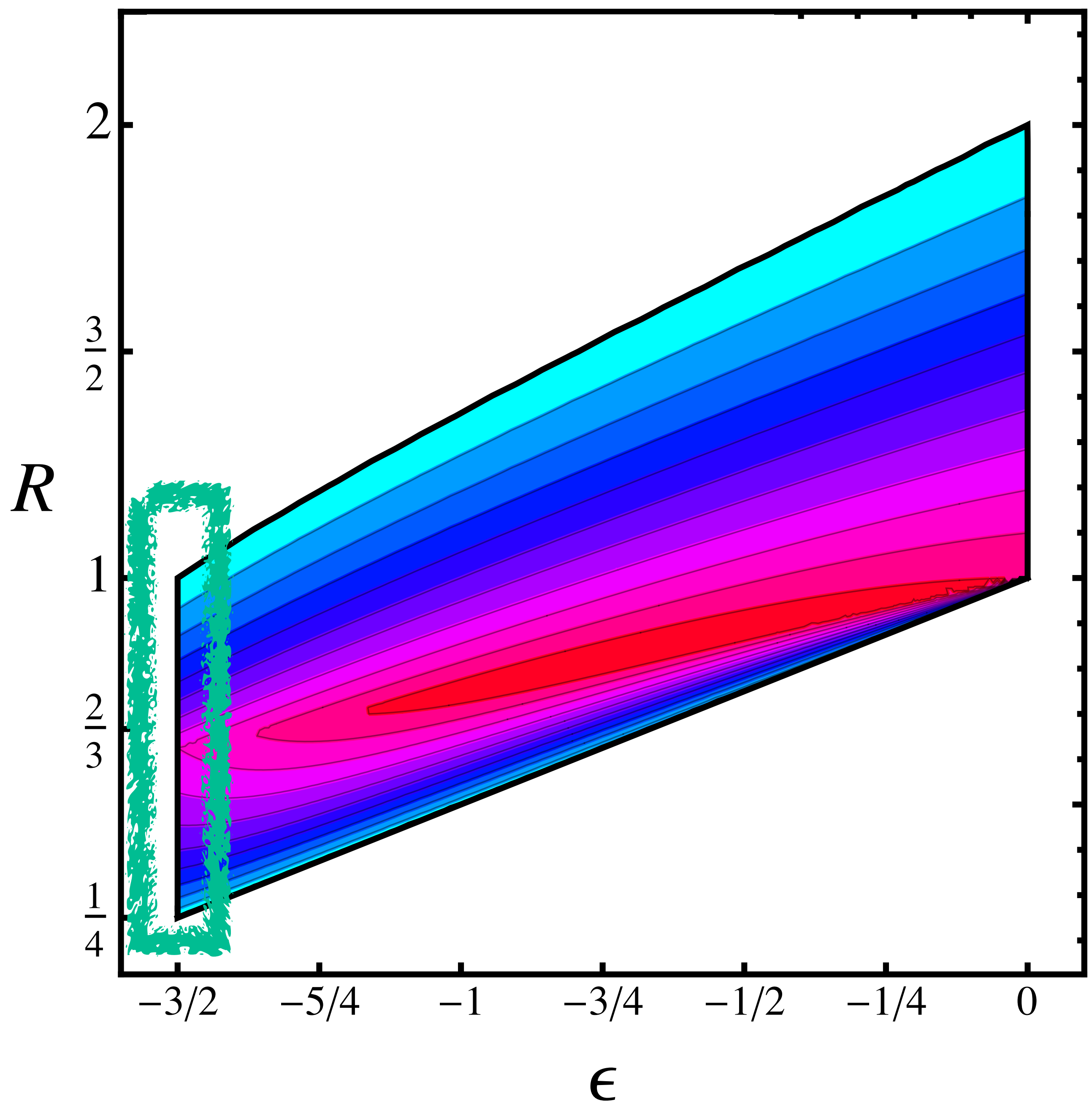
weak coupling

$$\epsilon = \frac{N_F + N_2}{N_1} - 3$$

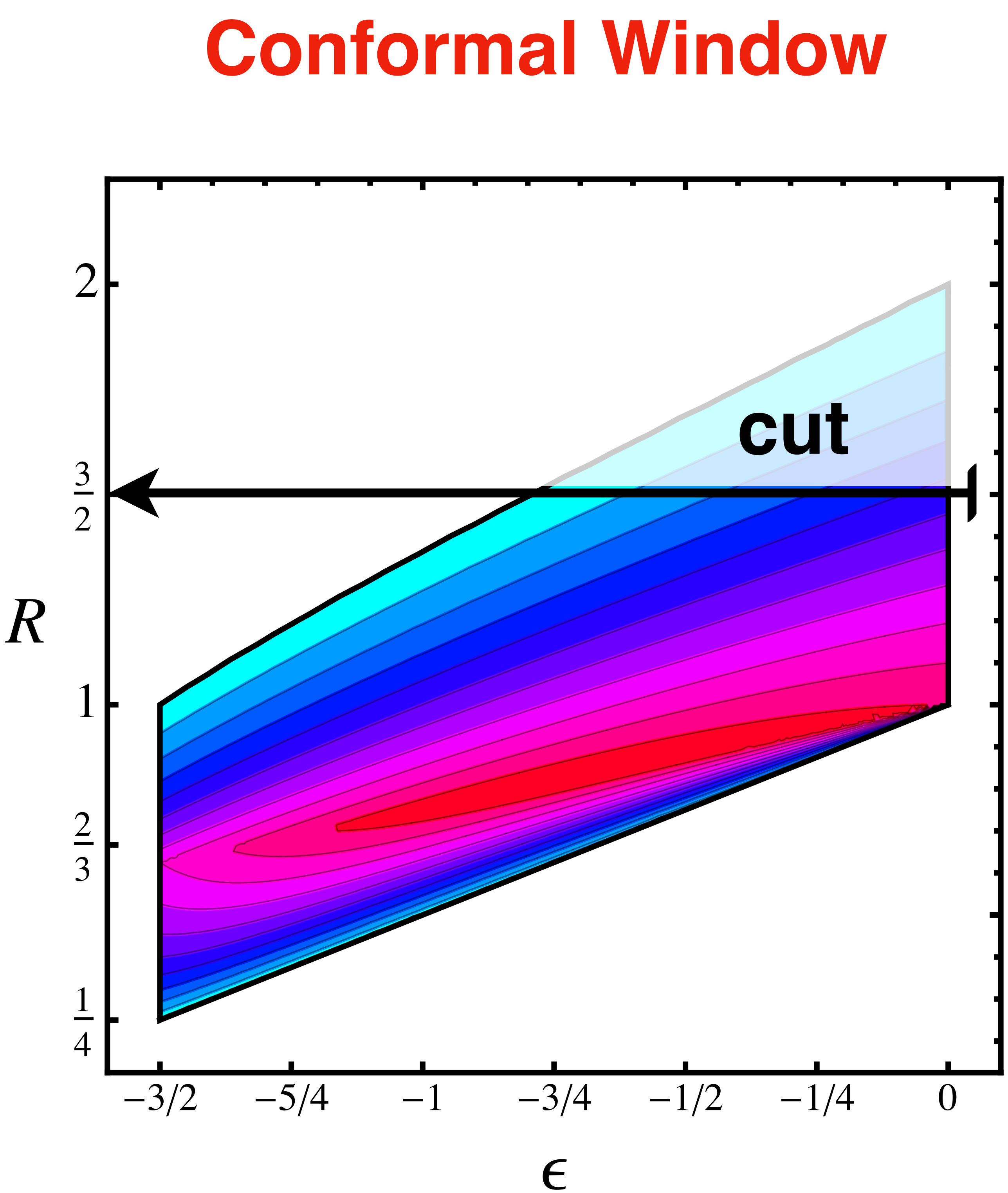
Non-Perturbative UV Conformal Window

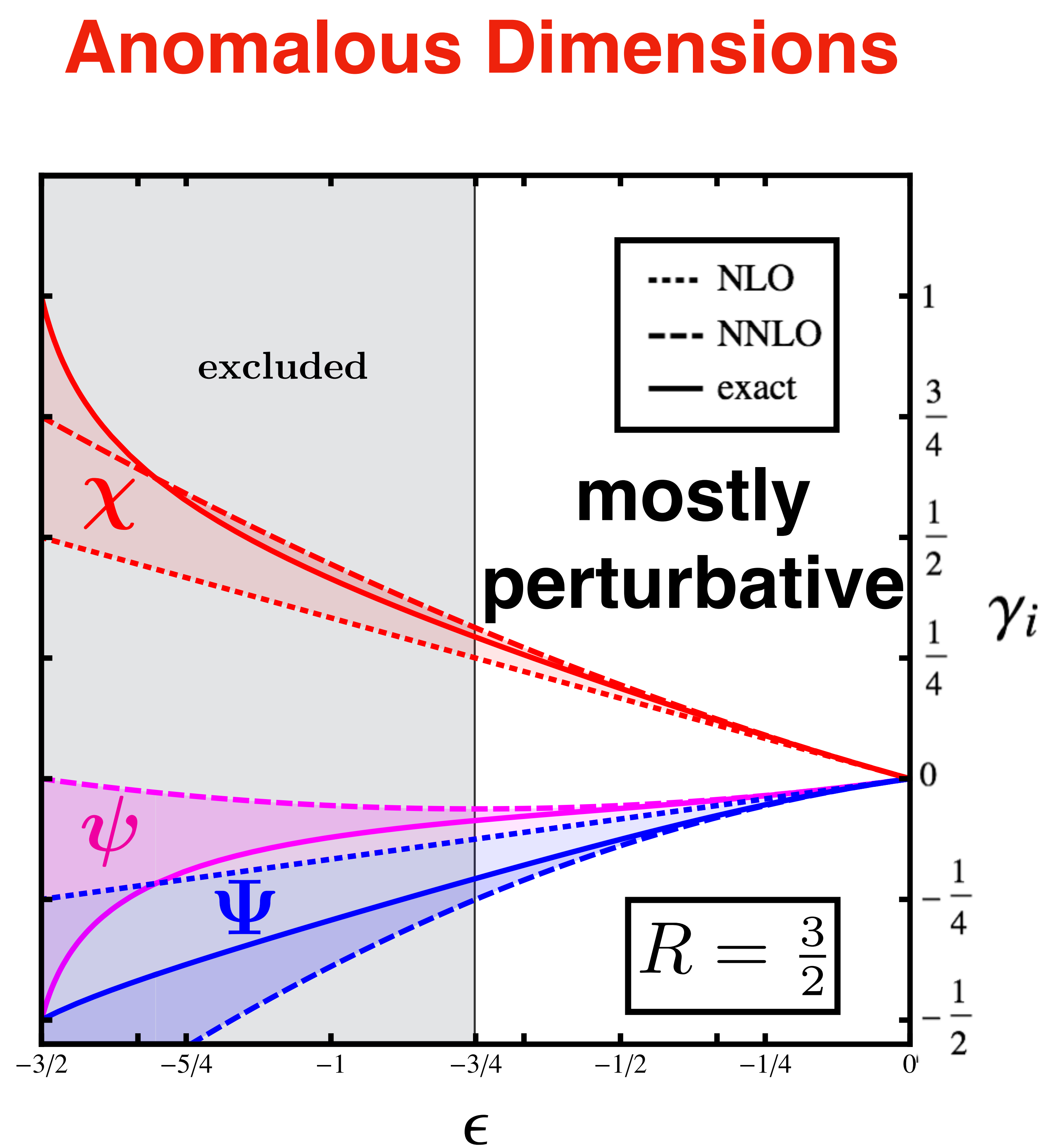
$$R = \frac{N_2}{N_1}$$

unitarity



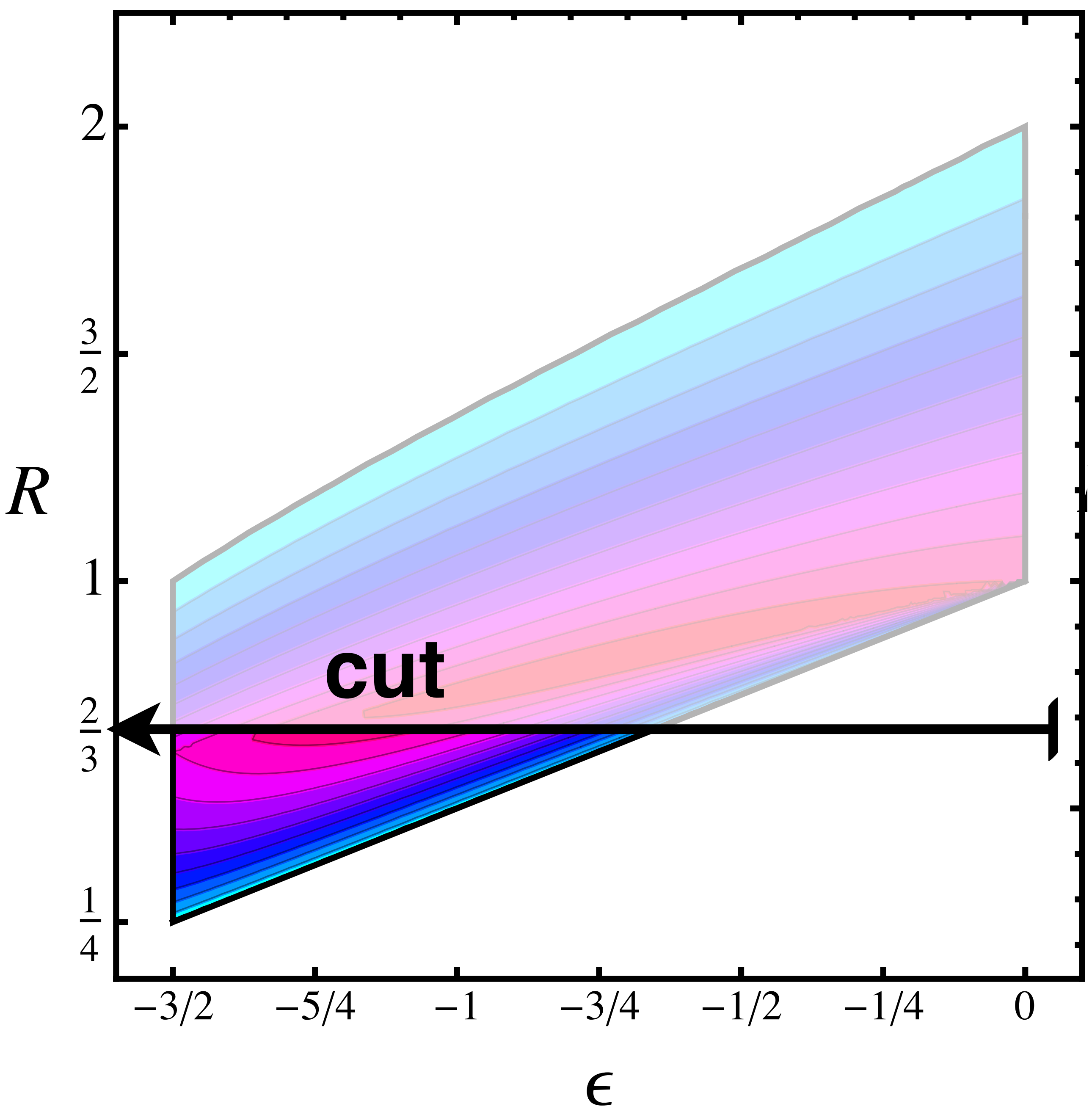
$$\epsilon = \frac{N_F + N_2}{N_1} - 3$$





NLO = 2-loop
NNLO = 3-loop

Conformal Window



Anomalous Dimensions