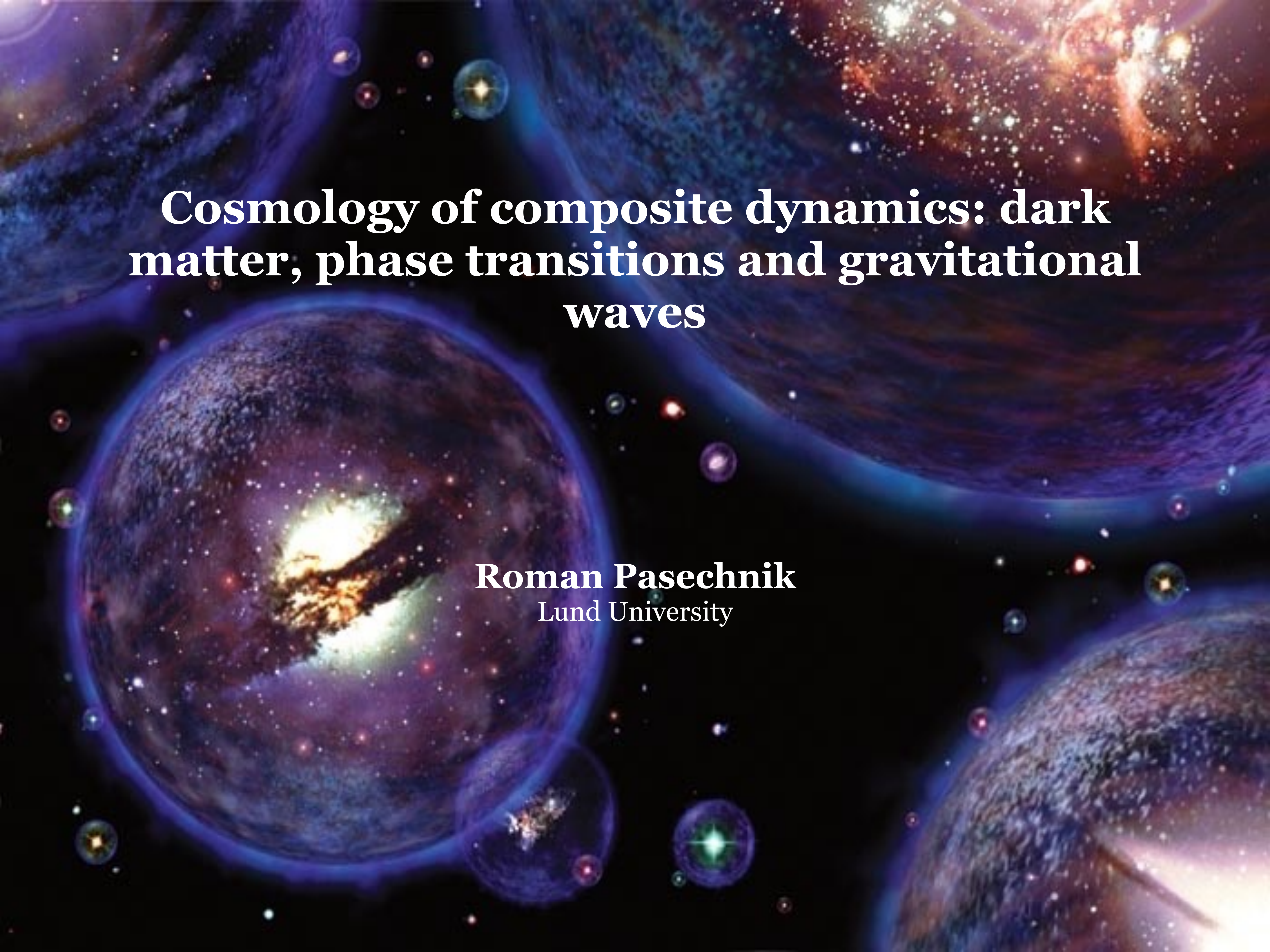


Cosmology of composite dynamics: dark matter, phase transitions and gravitational waves

Roman Pasechnik
Lund University

Strongly coupled dynamics: outlook

- **Important physical examples of gauge fields are realised in Nature (QCD and electroweak interactions)**

- **Non-perturbative QCD phenomena are far from being understood (e.g. quark confinement, mass gap, QCD phase transitions, hot/dense QCD phenomena etc)**

- **Non-abelian gauge (Yang-Mills) fields are present in most of UV completions of the Standard Model (e.g. GUTs, string/EDs compactifications etc)**

- **Confining dark Yang-Mills sectors are often considered as a possible source of Dark Matter in the Universe (e.g. dark glueballs)**

- **Pure gluons**

  $\Rightarrow$ confinement-deconfinement phase transition

- **Gluons + fermions**

  - Fermions in fundamental representation $\Rightarrow$ chiral phase transition
  - Fermions in adjoint rep. $\Rightarrow$ confinement & chiral phase transition
  - Fermions in 2-index symmetric rep. $\Rightarrow$ confinement & chiral phase transition

- **Gluons + fermions + scalars**

  $\Rightarrow$ not explored yet

Hidden confining (pure) gauge sectors

Many works on confining dark $SU(N)$

- **Self-interacting DM**
  E. D. Carlson *et al.*, Astrophys. J. **398** (1992), 43-52
- **Glueball phenomenology**
  A. Soni and Y. Zhang, Phys. Rev. D **93** (2016) no.11, 115025
- **The dark glueball problem**
  J. Halverson *et al.*, Phys. Rev. D **95** (2017) no.4, 043527
- **The nightmare scenario**
  R. Garani *et al.*, JHEP **12** (2021), 139
- **Thermal Squeezeout**
  P. Asadi *et al.*, Phys. Rev. D **104** (2021) no.9, 095013
- **Gravitational waves from confinement**
  W. C. Huang *et al.*, Phys. Rev. D **104** (2021) no.3, 035005

Open questions remain:

Do we need to describe the cosmological evolution of the dark gluon gas?

How do glueball form from dark gluons?

Is there any constraint on glueball self-interactions?

Is there a reliable estimate of the glueball relic density?

How do we describe strongly coupled sectors at finite T?

- **Pure gluons**

  ⇒ Polyakov loop model — Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060; Huang, Reichert, Sannino, Wang, PRD 104 (2021) 035005

  ⇒ Matrix model — Halverson, Long, Maiti, Nelson, Salinas, JHEP 05 (2021) 154

  ⇒ Holographic QCD model — Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD 105 (2022) 066020; PRL 128 (2022) 131101

- **Gluons + fermions**

  ⇒ Polyakov loop improved Nambu-Jona-Lasinio model — Reichert, Sannino, Wang, Zhang, JHEP 01 (2022) 003; Helmboldt, Kubo, Woude, PRD 100 (2019) 055025

  ⇒ Linear sigma model — Helmboldt, Kubo, Woude, PRD 100 (2019) 055025

  ⇒ Polyakov Quark Meson model — RP, Reichert, Sannino, Wang, JHEP 02 (2024) 159

Polyakov Loop Model for pure gluons I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB 72 (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) \, \mathrm{d}\tau \right]$$

- The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for pure gluons II

- Convenient to define the trace of the Polyakov loop as an order parameter for the $Z(N)$ symmetry

$$\ell\left(\vec{x}\right)=\frac{1}{N}\mathrm{Tr}_c[\mathbf{L}]\,,$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell\rightarrow e^{i\phi}\ell,\qquad \phi=\frac{2\pi j}{N},\qquad j=0,1,\cdots,(N-1)$$

- The expectation value of ℓ i.e. $<\ell>$ has the important property:

$$\langle\ell\rangle=0\quad(T<T_c,\,\mathrm{Confined})\,;\qquad \langle\ell\rangle>0\quad(T>T_c,\,\mathrm{Deconfined})$$

- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle\ell\rangle=\exp\left(i\frac{2\pi j}{N}\right)\ell_0,\qquad j=0,1,\cdots,(N-1)$$

where ℓ_0 is defined to be real and $\ell_0\rightarrow 1$ as $T\rightarrow\infty$

Effective PLM potential

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\mathrm{PLM}}^{(\mathrm{poly})} = T^4 \left(-\frac{b_2(T)}{2} |\ell|^2 + b_4 |\ell|^4 + \cdots - b_3 \left(\ell^N + \ell^{*N} \right) \right)$$

$$\text{where} \quad b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 + a_4 \left(\frac{T_0}{T} \right)^4$$

"$\cdots$" represent any required lower dimension operator than ℓ^N i.e. $(\ell\ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

$$V_{\mathrm{PLM}}^{(3\log)} = T^4 \left(-\frac{a(T)}{2} |\ell|^2 + b(T) \ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4 \right) \right)$$

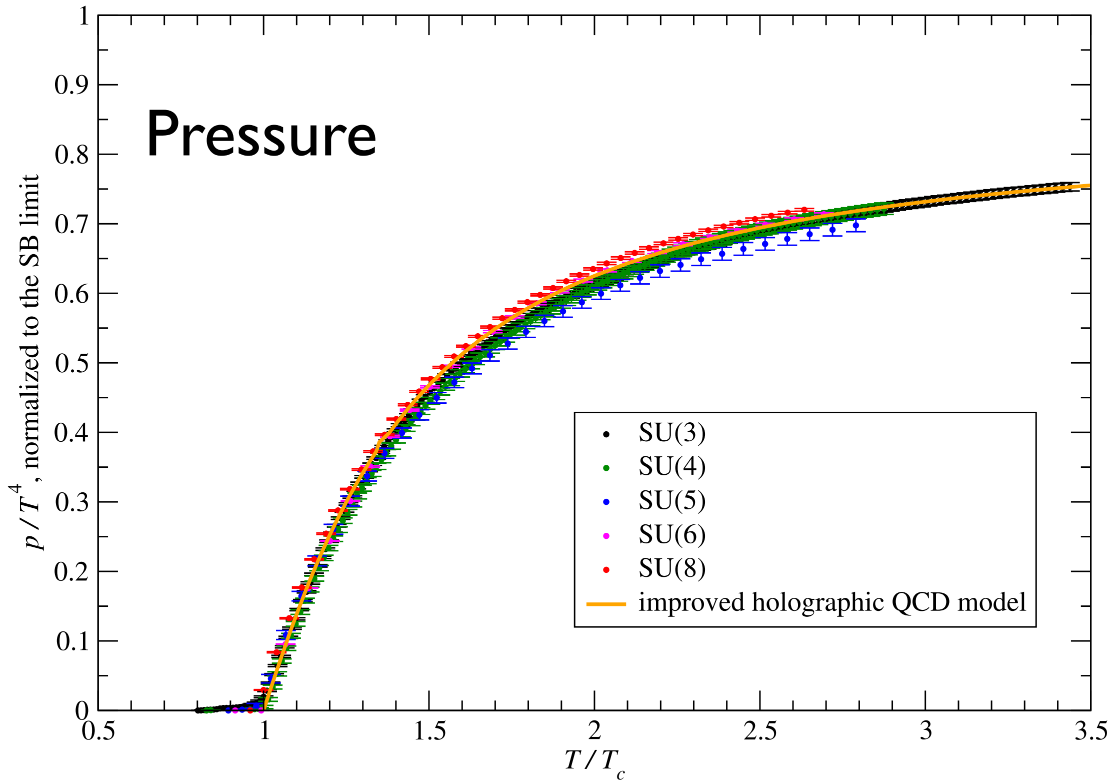
$$a(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3, \quad b(T) = b_3 \left(\frac{T_0}{T} \right)^3$$

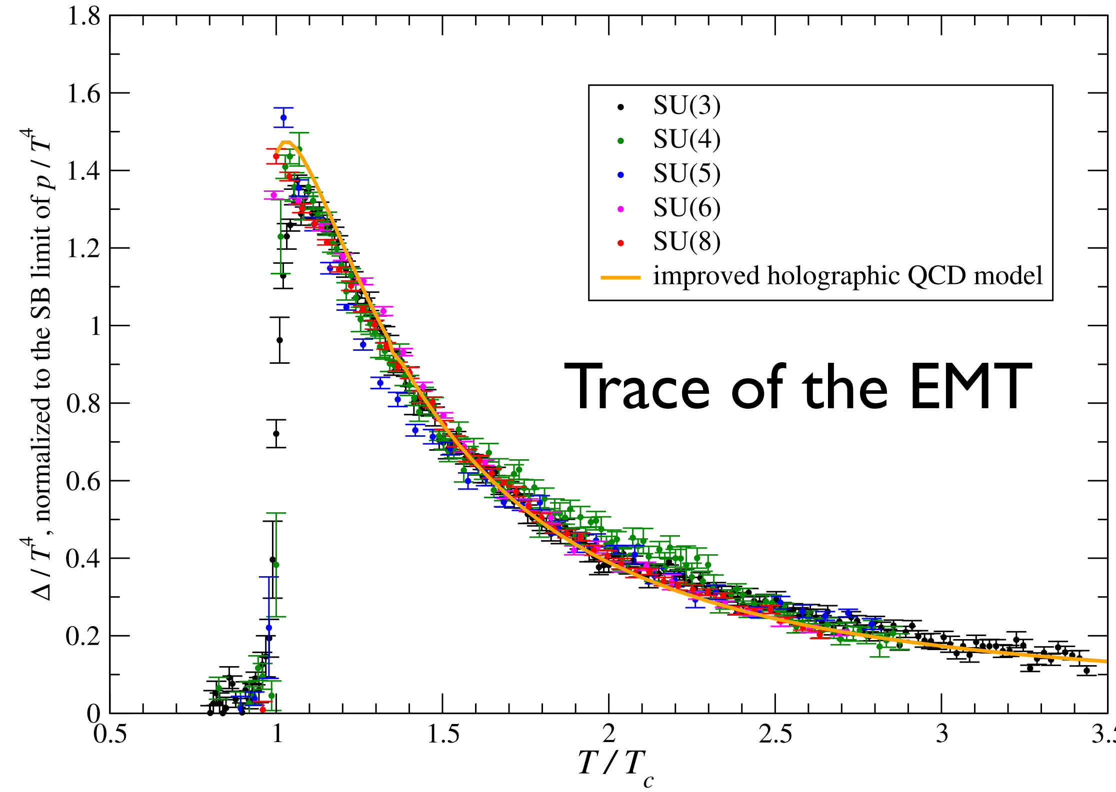
- The a_i, b_i coefficients in $V_{\mathrm{PLM}}^{(\mathrm{poly})}$ and $V_{\mathrm{PLM}}^{(3\log)}$ are determined by fitting the lattice results

Fitting the PLM potential to the lattice data

Lattice data

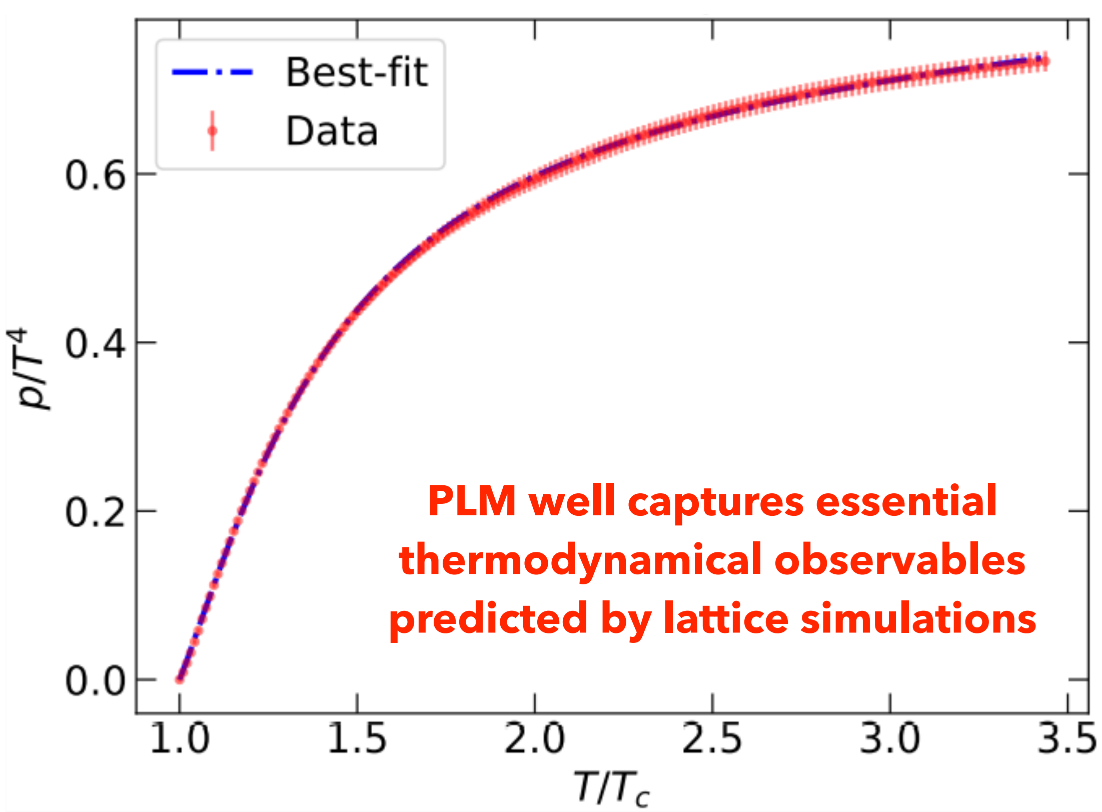
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001

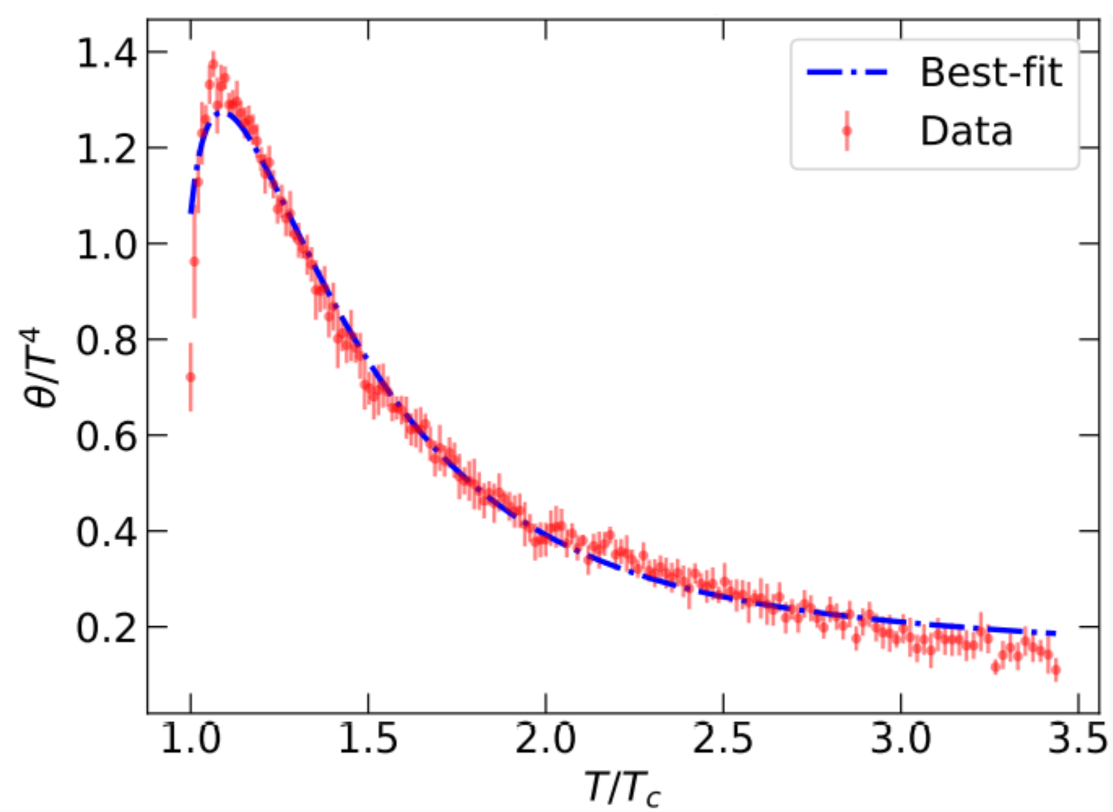




Best fit of the PLM potential

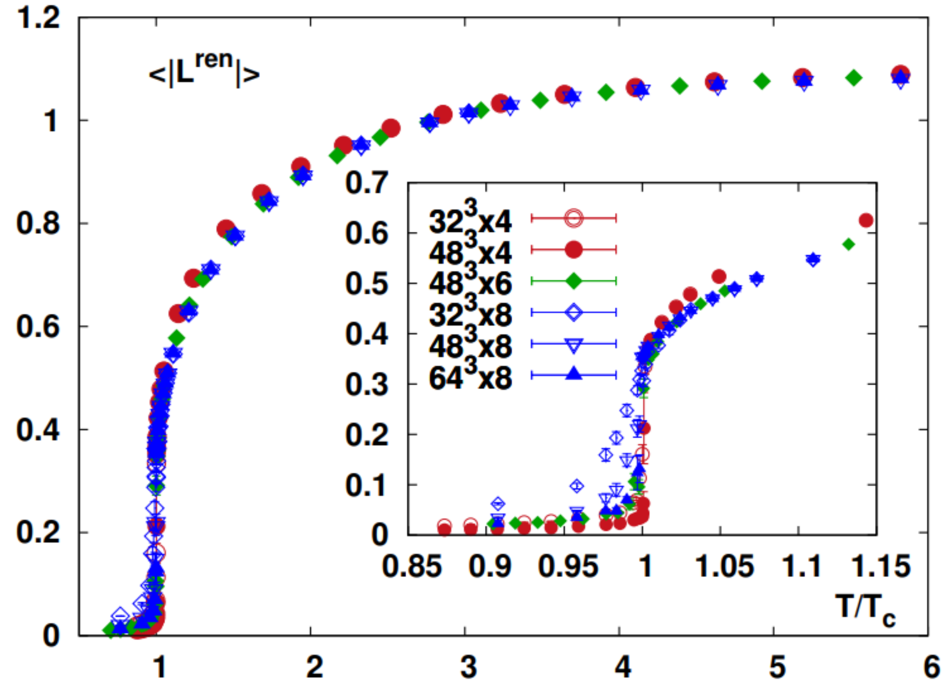
Huang, Reichert, Sannino and Wang, PRD 104 (2021) 035005



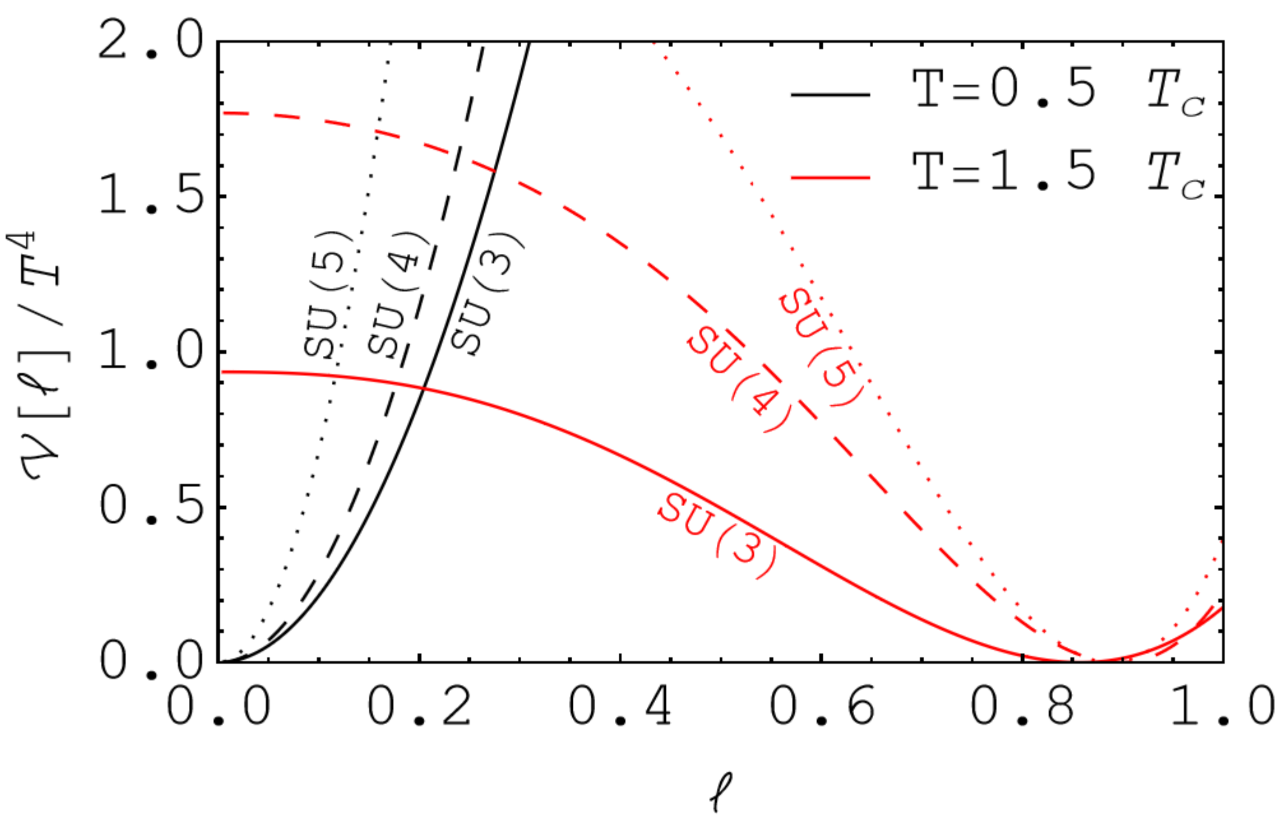


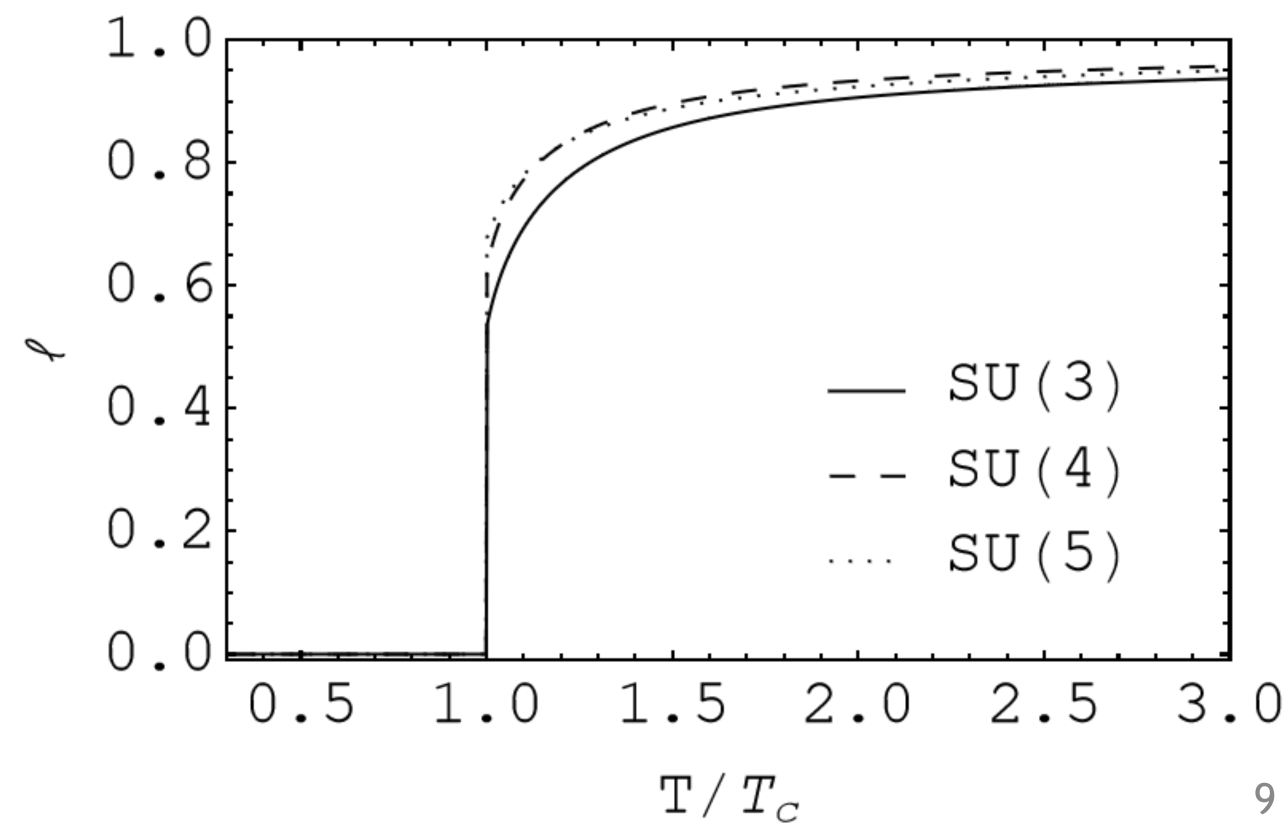
PLM potential and Polyakov loop VEV in SU(N)

P. M. Lo et al., Phys. Rev. D 88 (2013), 074502



Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26
Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12

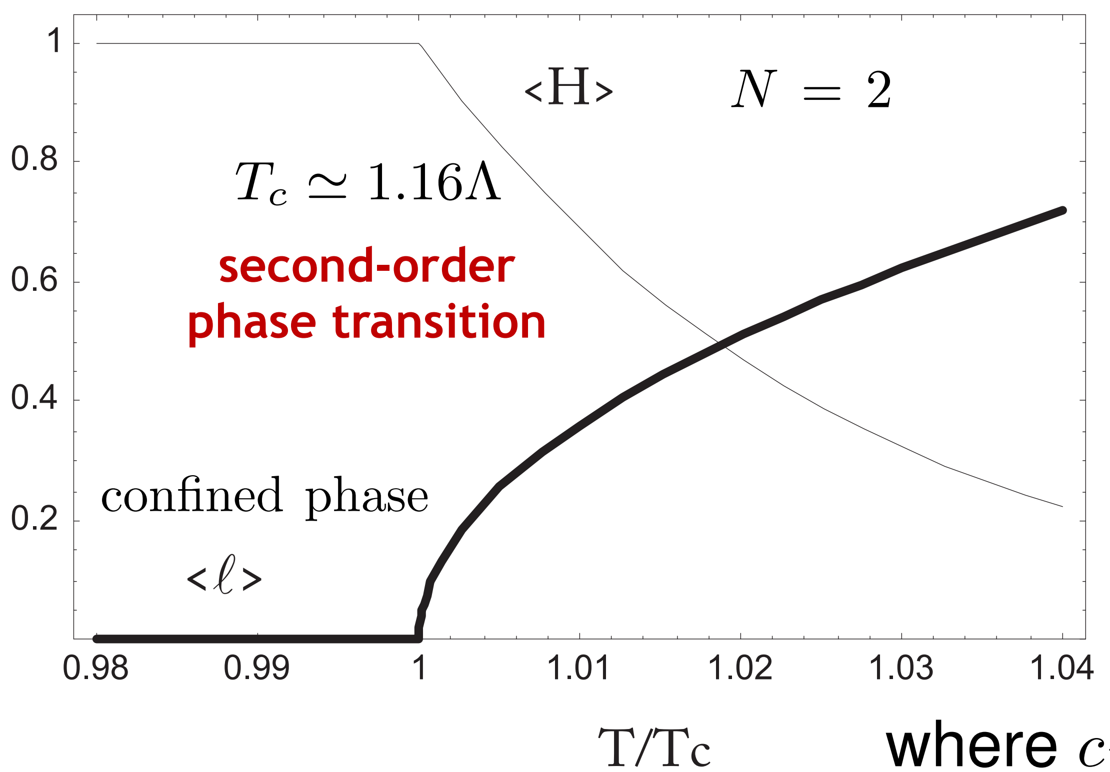




Dark gluon-glueball dynamics

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26

- In the literature, for glueball dark matter production, only ϕ^5 interaction is considered, making the $3 \to 2$ annihilation the only relevant process for DM formation
- However, since glueball is strongly coupled, this naive calculation is not rigorous. A non-perturbative method is required.
- The dark gluon-glueball dynamics can be effectively described by considering the dimension-4 glueball field $\mathcal{H} \propto$ tr$(G^{\mu\nu}G_{\mu\nu})$:



$$\mathcal{L} = \frac{c}{2}\frac{\partial_\mu \mathcal{H} \partial^\mu \mathcal{H}}{\mathcal{H}^{3/2}} - V[\mathcal{H}, \ell] \qquad c = \frac{1}{2\sqrt{e}}\left(\frac{\Lambda}{m_{\rm gb}}\right)^2$$

$$V\left[\mathcal{H}, \ell\right] = \frac{\mathcal{H}}{2}\ln\left[\frac{\mathcal{H}}{\Lambda^4}\right] + T^4 \mathcal{V}\left[\ell\right] + \mathcal{H}\mathcal{P}[\ell] + V_T\left[\mathcal{H}\right]$$

We keep the lowest order in $\mathcal{P}[\ell]$ $\qquad \mathcal{P}[\ell] = c_1 |\ell|^2$

where c_1 is determined by the lattice results (jumping of gluon condensate).

F. Sannino, *Polyakov loops versus hadronic states*, *Phys. Rev. D* **66** (2002) 034013 [hep-ph/0204174].

Thermal evolution of the glueball-dark gluon system

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26

- **Introducing canonically normalised field** $\mathcal{H} = 2^{-8}c^{-2}\phi^4$
  the effective Lagrangian reads:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V[\phi,\ell]\,,$$

$$V[\phi,\ell] = \frac{\phi^4}{2^8c^2}\left[2\ln\left(\frac{\phi}{\Lambda}\right) - 4\ln 2 - \ln c\right] + \\ + \frac{\phi^4}{2^8c^2}\mathcal{P}[\ell] + T^4\mathcal{V}[\ell]\ ,$$

$$\mathcal{P}[\ell] = c_1|\ell|^2\,,$$

$$\mathcal{V}[\ell] = -\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 - b_3(\ell^3 + (\ell^*)^3)\,,$$

$$b_2(T) = \sum_{i=0}^{4} a_i\left(\frac{T_c}{T}\right)^i\ ,$$

- **Fits to lattice results for observables provide:**

a_0	a_1	a_2	a_3	a_4	b_3	b_4
3.72	−5.73	8.49	−9.29	0.27	2.40	4.53

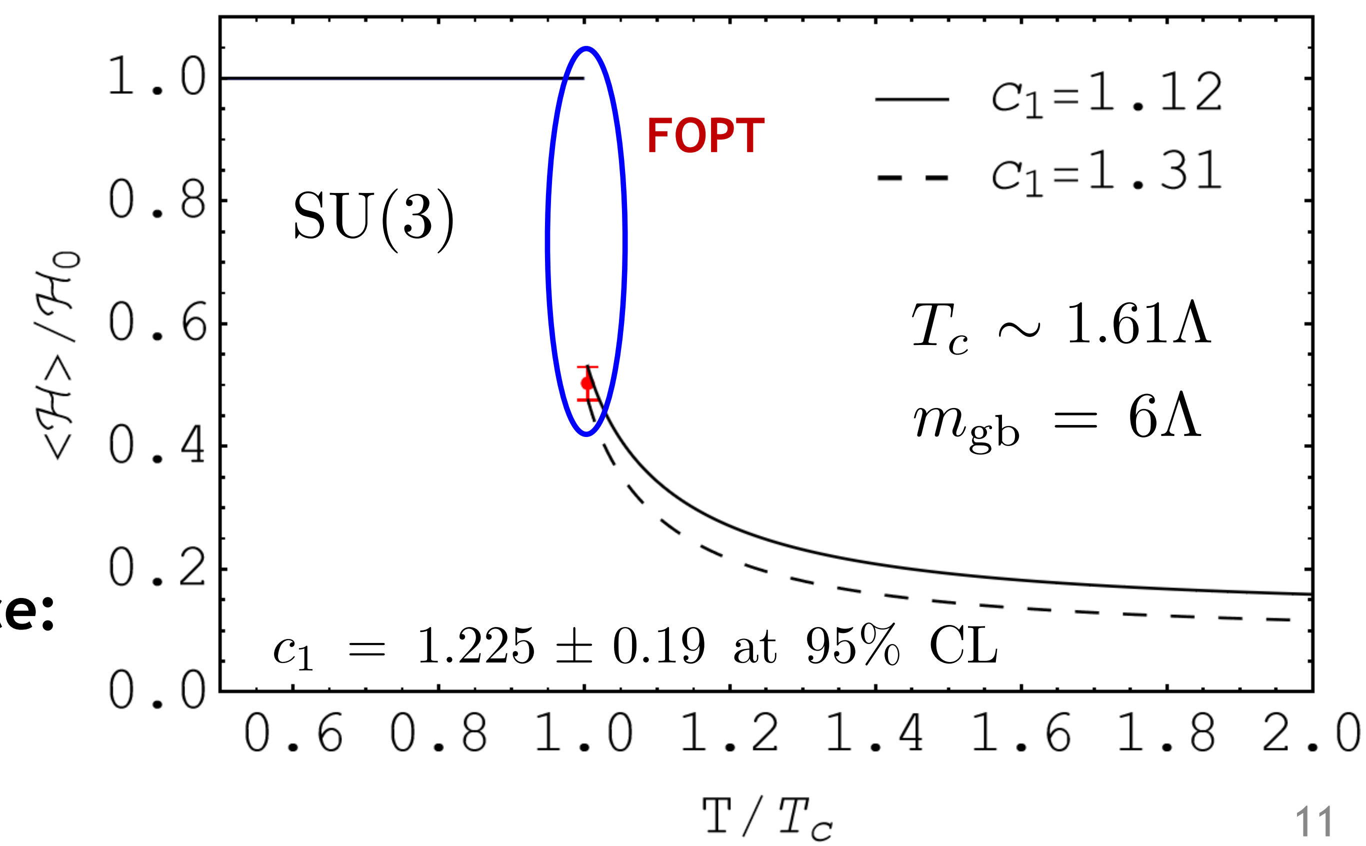
Huang, Reichert, Sannino and Wang,
PRD 104 (2021) 035005

- **Integrating out the Polyakov loop in the high-T phase provides**

$$V[\phi,T] = V[\phi,\ell(\phi,T)]$$

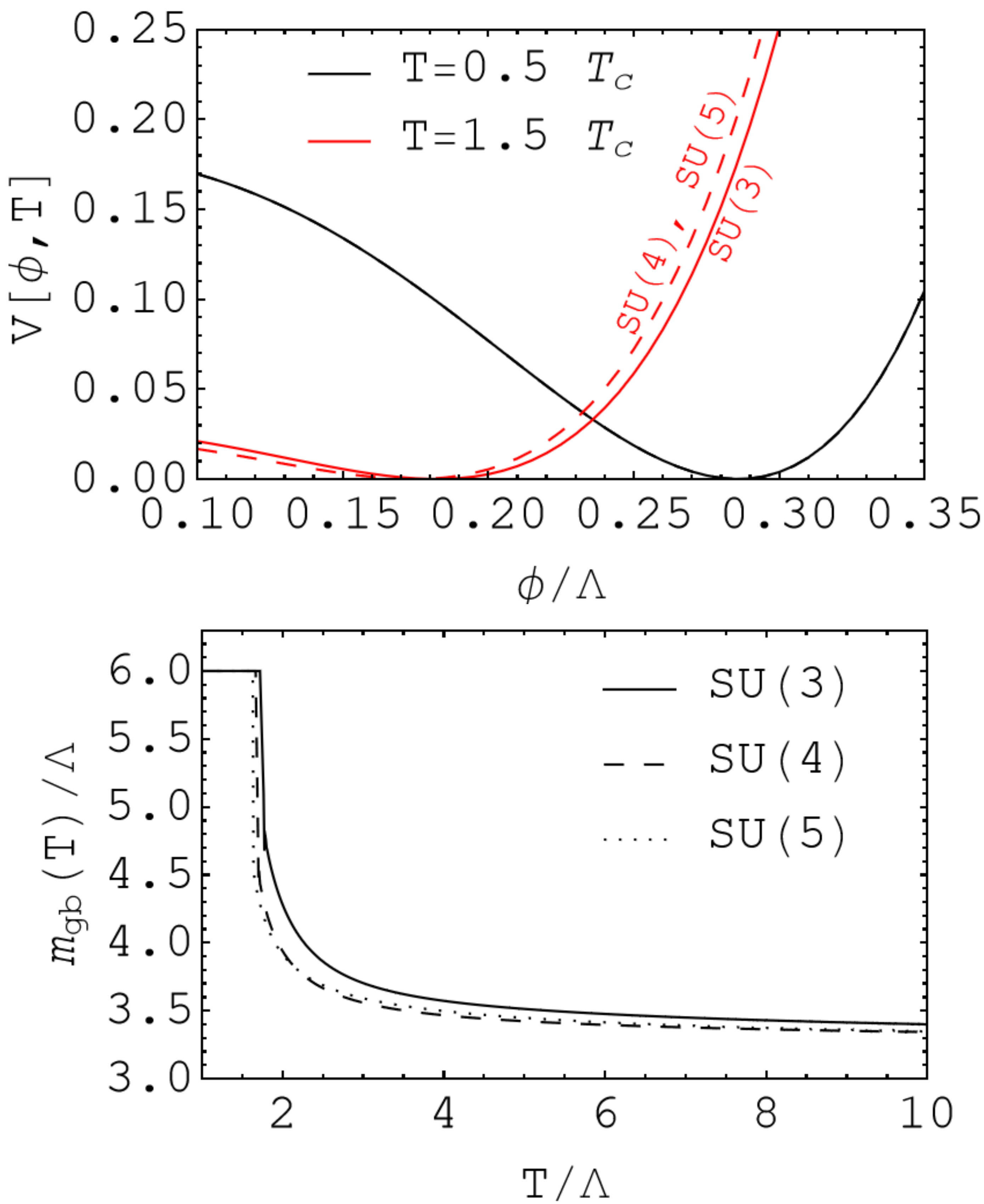
matching the size of discontinuity to lattice:

M. D'Elia, A. Di Giacomo and E. Meggiolaro, *Gauge invariant field strength correlators in pure Yang-Mills and full QCD at finite temperature*, *Phys. Rev. D* **67** (2003) 114504 [hep-lat/0205018].



Thermal potential and glueball mass

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26
Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12



Cosmological evolution of the dark glueball field

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26
Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12

- The glueball field is considered homogeneous and evolves in expanding FLRW universe, with the E.O.M.

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V[\phi, T] = 0\,,$$

- The time variable is found in terms of the photon temperature:

$$t = \frac{1}{2}\sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_\gamma)}}\frac{m_P}{T_\gamma^2}\,, \qquad T_\gamma = \xi_T T$$

  where ξ_T denotes the visible-to-dark sector temperature ratio and $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass and $g_{*,\rho}$ is the number of energy-related degrees of freedom.

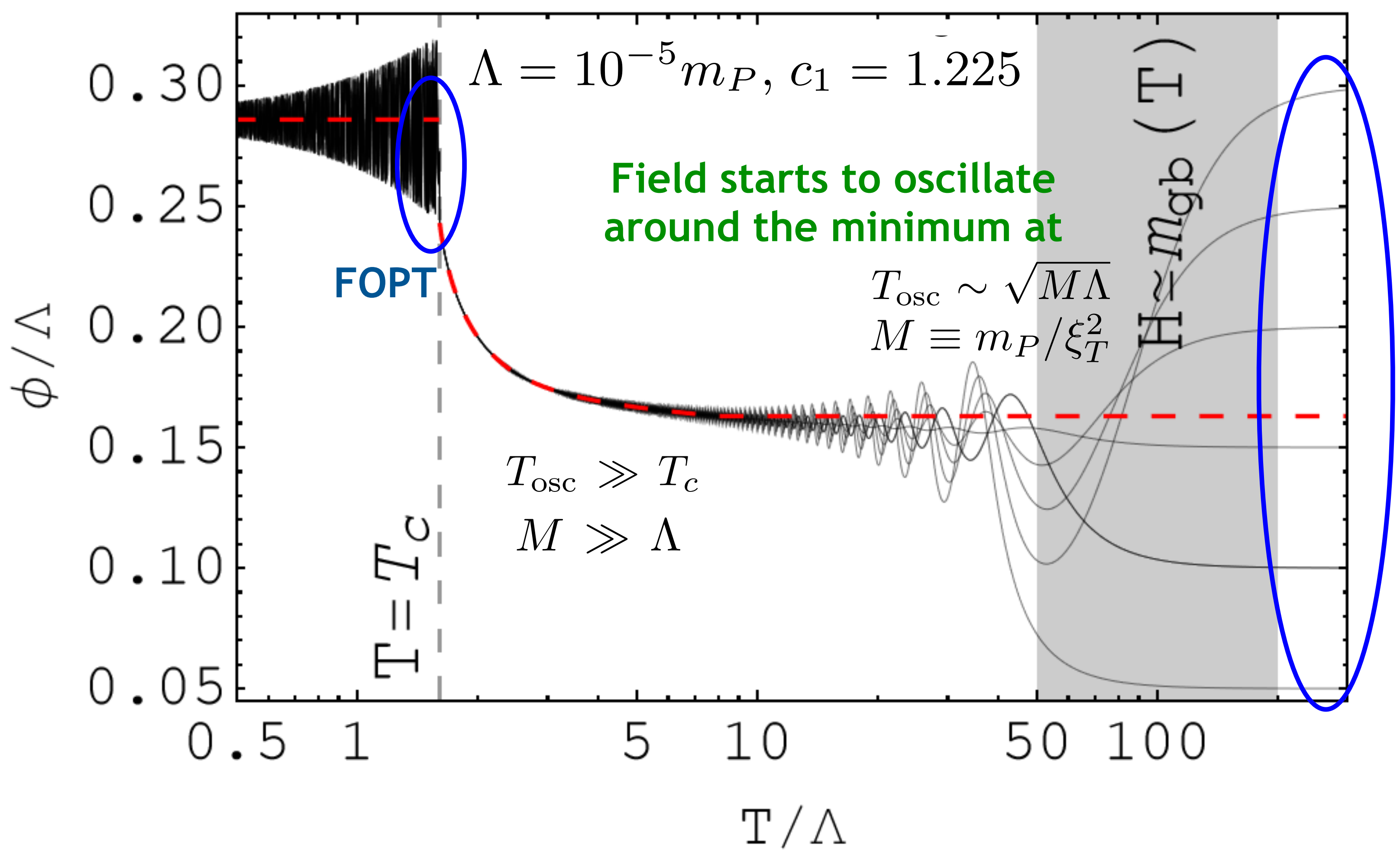
- E.O.M. in terms of the dark sector temperature:

$$\frac{4\pi^3 g_{*,\rho}}{45 m_P^2}\xi_T^4 T^6 \frac{d^2\phi}{dT^2} + \frac{2\pi^3}{45 m_P^2}\frac{dg_{*,\rho}}{dT}\xi_T^4 T^6 \frac{d\phi}{dT} + \partial_\phi V[\phi, T] = 0$$

encodes non-perturbative dynamics of the glueball field at finite T

Cosmological evolution of the dark glueball field

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26
Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12



In the deconfined regime, the field evolution is dominated by Hubble friction (slow evolution)

Oscillations have long time to decay regardless of the initial condition

non-linear interaction terms are important for large amplitudes

- Field starts to oscillate around the minimum of the potential when $H \simeq m_{\rm gb}$ with temperature $T_{\rm OSC} \sim \sqrt{M\Lambda}$
- In early times in deconfined regime, for different initial conditions the field evolution follows the minimum (red dashed line).
- First order phase transition washes out any dependence on initial conditions.

Glueball relic density

Carenza, RP, Salinas, Wang, Phys. Rev. Lett. 129 (2022) no.26, 26
Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12

- Energy stored in these oscillations around $\phi_{\rm min} \approx 0.28\Lambda$ is the relic DM abundance, $\Omega h^2 = \rho/\rho_c$ (critical density $\rho_c = 1.05 \times 10^4\,{\rm eV\,cm^{-3}}$)

$$\rho = \frac{2\pi^3}{45} g_{*,\rho}(T) \frac{T^6}{M^2} \left(\frac{d\phi}{dT}\right)^2 + V[\phi]\,.$$

- Then the relic density today is calculated:

$$\Omega h^2 = \frac{\Lambda}{\rho_c/h^2} \left\langle \frac{\tilde{\rho}}{\tilde{T}^3} \right\rangle_f T_f^3 \left(\frac{T_{\gamma,0}}{\zeta_T T_f}\right)^3 = 0.12 \zeta_T^{-3} \frac{\Lambda}{\Lambda_0}\,,$$

with dilution factor $(T_{\gamma,0}/\zeta_T T_f)^3$ to consider the Universe expansion

- Below freeze-out temperature, the predicted glueball relic density is

$$0.12\zeta_T^{-3} \frac{\Lambda}{137.9\,{\rm eV}} \lesssim \Omega h^2 \lesssim 0.12\zeta_T^{-3} \frac{\Lambda}{82.7\,{\rm eV}}, \quad 1.035 < c_1 < 1.415$$

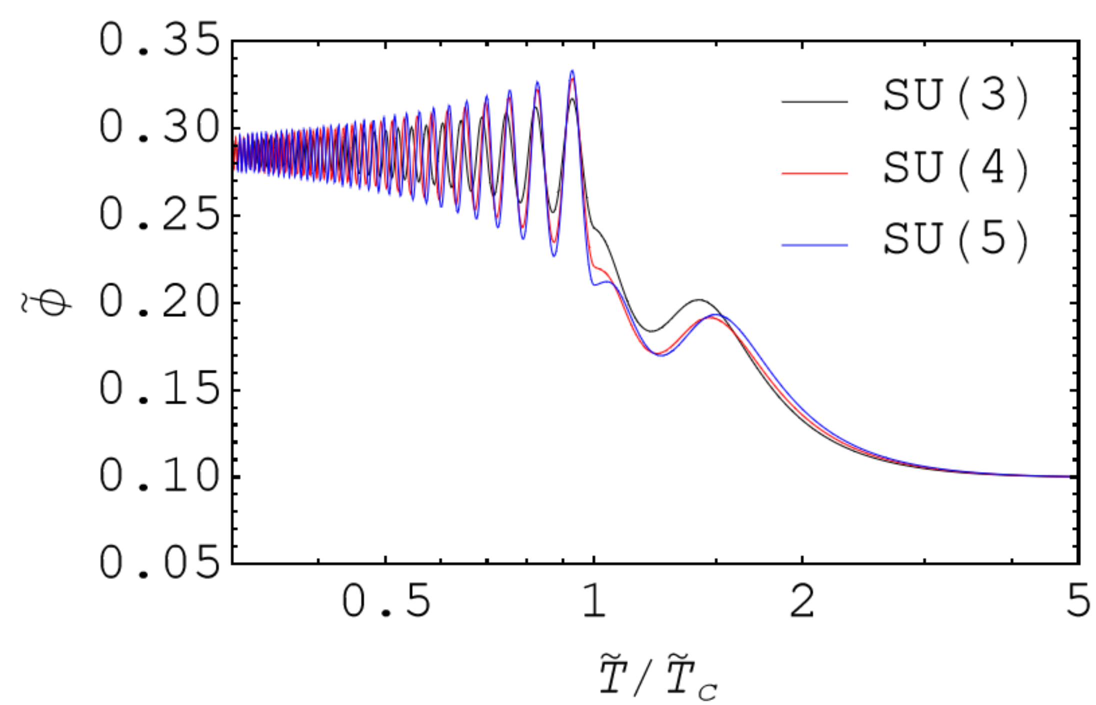
for $\zeta_T^{-1} = 0.1$, the glueball dark matter mass is $\sim 100{\rm MeV}$

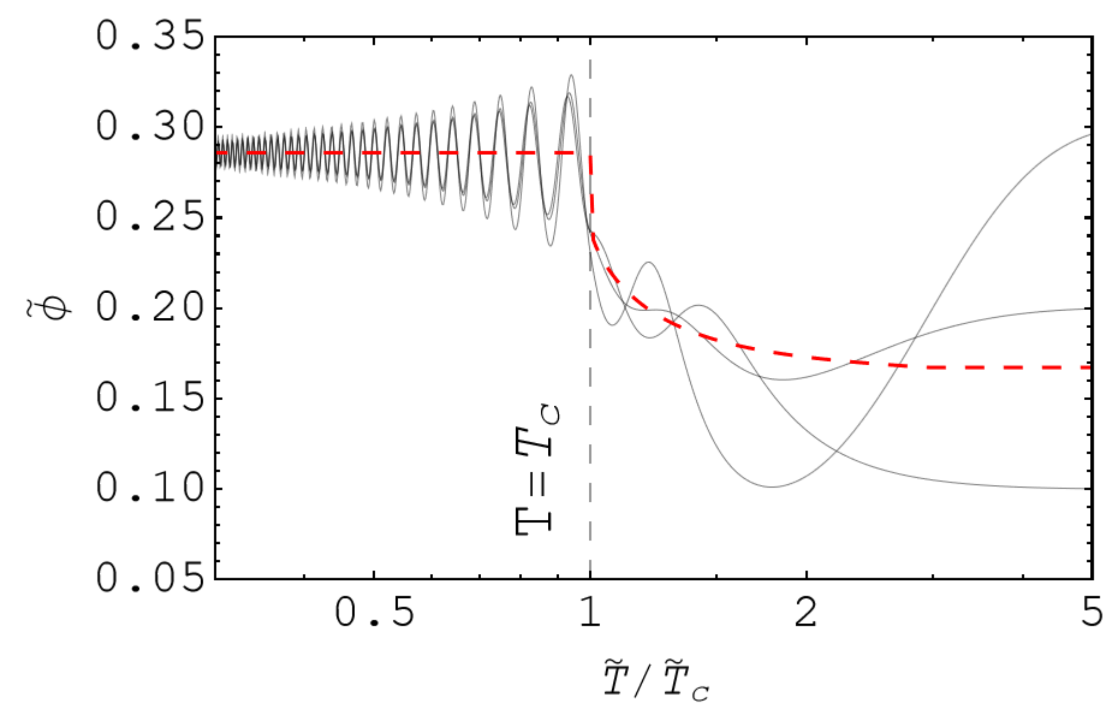
- It is more than a factor of 10 difference compared to the old calculations

$$\Omega h^2 \sim 0.12\, \zeta_T^{-3} \frac{\Lambda}{5.45\,{\rm eV}}$$

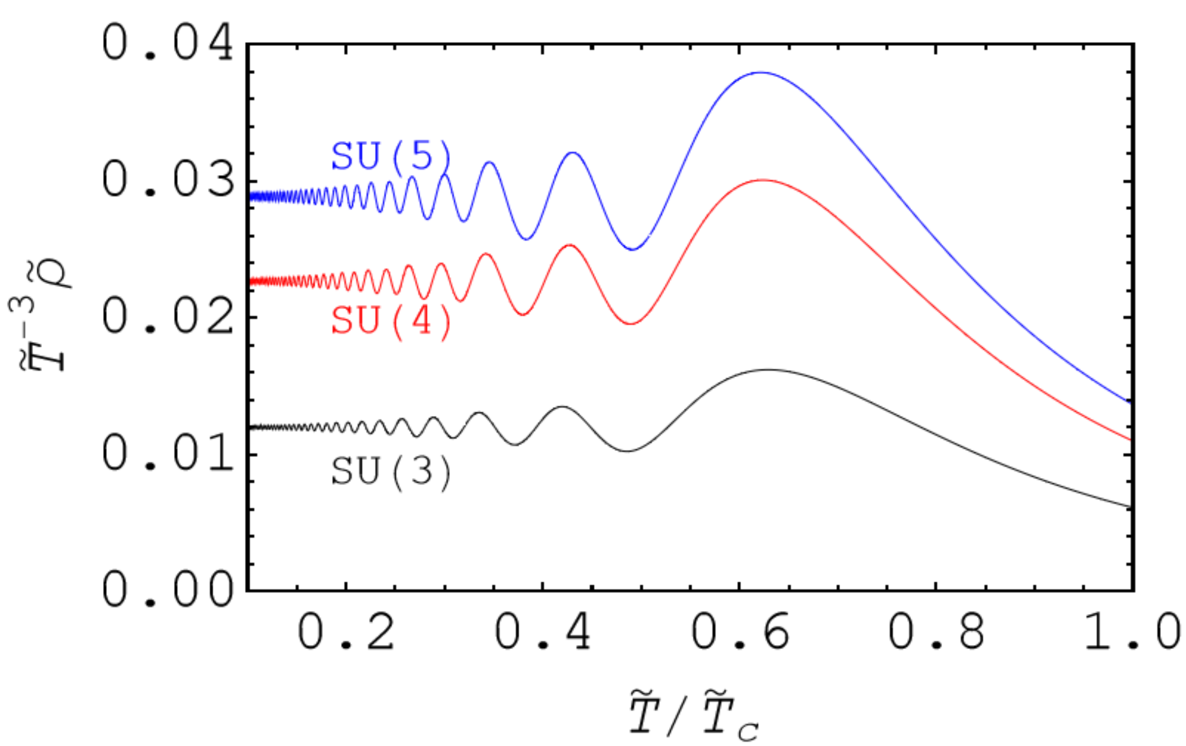
N and initial conditions dependence

Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12





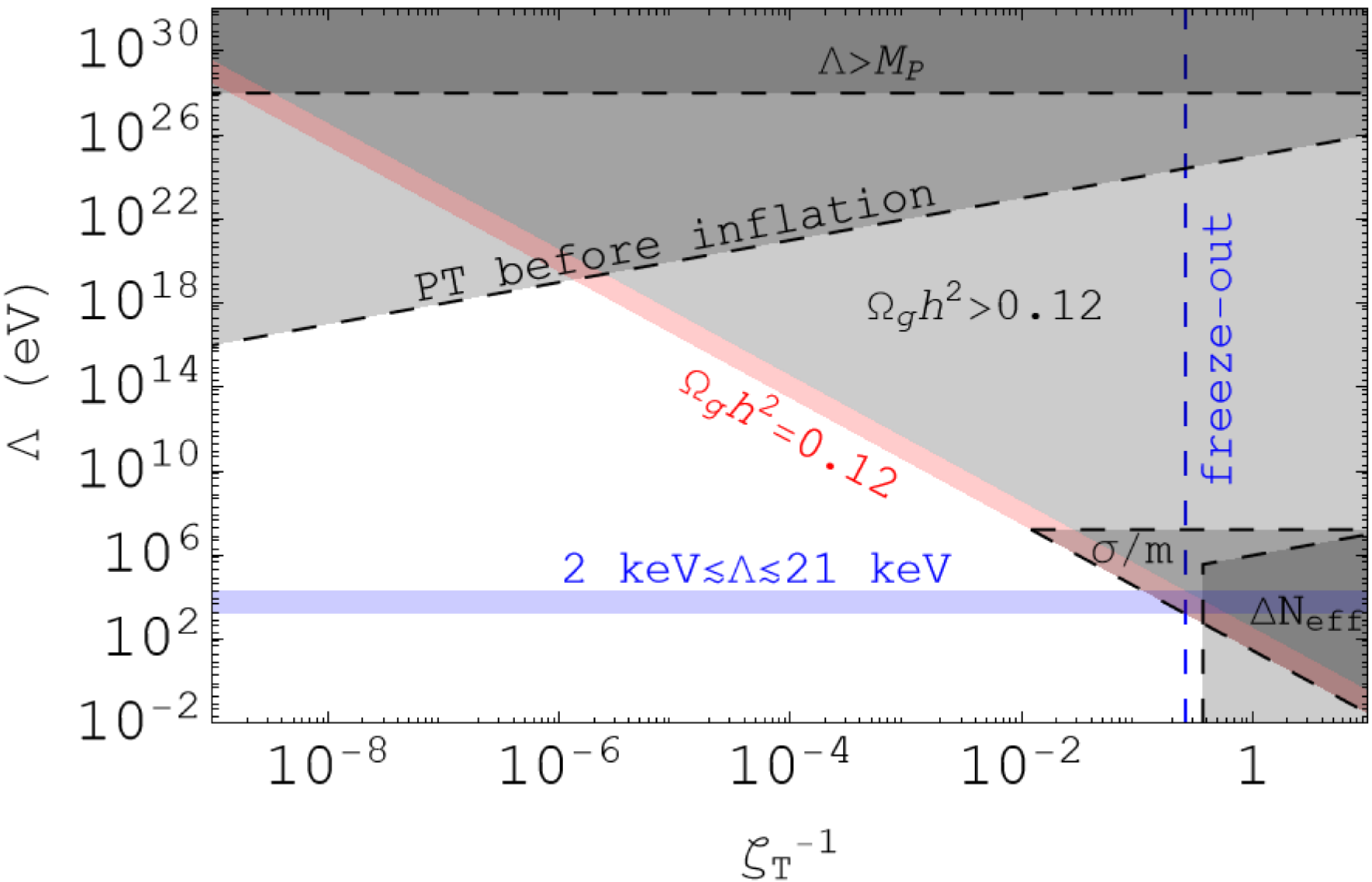
Weak dependence on the gauge group and initial conditions



N	c_1	$100 \times \left\langle \frac{\tilde{\rho}}{\tilde{T}^3} \right\rangle_f$	Λ_0 (eV)
3	1.225 ± 0.19	$0.59^{+0.15}_{-0.14}$	133 ± 32
4	1.225 ± 0.8	$1.1^{+1.0}_{-0.9}$	204 ± 168
5	1.225 ± 0.8	$1.3^{+1.2}_{-1.0}$	139 ± 109

Glueball DM parameter space

Carenza, Ferreira, RP and Wang, Phys. Rev. D 108 (2023) no.12, 12



A large portion of the parameter space is viable

Including fermions: the PQM model

B. Schaefer, J. Pawlowski, J. Wambach PRD 76 (2007) 074023
B. Schaefer, M. Wagner, PPNP 62 (2009) 391
RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- The Polyakov quark meson model (PQM) is widely used as an effective theory to study the first order chiral phase transition
- The Lagrangian of the PLSM where mesons couple to a spatially constant temporal background gauge field reads

$$\mathcal{L} = \bar{q}\left(i\not{D} - g\left(\sigma + i\gamma_5 T^a \pi_a\right)\right) q + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \pi_a\right)^2 - V^{(\mathrm{poly})}_{\mathrm{PLM}} + V_{\mathsf{LSM}} + V_{\mathrm{medium}}\,, \text{where } \not{D} = \gamma_\mu \partial_\mu - i\gamma_0 A_0$$

- V_{LSM} under symmetry $SU(N_f) \times SU(N_f)$ with N_f flavours reads

$$V_{\mathsf{LSM}} = \frac{1}{2}\left(\lambda_\sigma - \lambda_a\right) \mathrm{Tr}\left[\Phi^\dagger \Phi\right]^2 + \frac{N_f}{2} \lambda_a \mathrm{Tr}\left[\Phi^\dagger \Phi \Phi^\dagger \Phi\right] - m^2 \mathrm{Tr}\left[\Phi^\dagger \Phi\right] - 2\left(2N_f\right)^{N_f/2-2} c \left(\det \Phi^\dagger + \det \Phi\right)$$

where the meson field Φ is a $N_f \times N_f$ matrix defined as

$$\Phi = \frac{1}{\sqrt{2N_f}}\left(\sigma + i\eta'\right) I + \left(a_a + i\pi_a\right) T^a\,, I \equiv \text{identity matrix}$$

Thermal corrections: the CJT Method

J. Cornwall, R. Jackiw, E. Tomboulis PRD 10 (1974) 2428
G. Amelino-Camelia, PRD 47 (1993) 2356
RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- Cornwall, Jackiw and Tomboulis (CJT) first proposed a generalized effective action $\Gamma\left(\phi, G\right)$ of composite operators, where the effective action not only depends on $\phi(x)$ but also on the propagator $G(x, y)$
- The effective action becomes the generating functional of the two-particle irreducible (2PI) vacuum graphs rather than the conventional 1PI diagrams
- The CJT method is equivalent to summing up the infinite class of “daisy" and “super daisy" graphs and is thus useful in studying such strongly coupled models beyond mean-field approximation
- The PQM with the CJT method compared to other model computations such as holography and the PNJL model, can bridge perturbative and non-perturbative regimes of the effective theory

The CJT Method: formalism

J. Cornwall, R. Jackiw, E. Tomboulis PRD 10 (1974) 2428
G. Amelino-Camelia, PRD 47 (1993) 2356
RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- In CJT formalism, the finite temperature effective potential with generic scalar field ϕ is given by:

$$V_{\mathrm{CJT}}(\phi, G) = V_0(\phi) + \frac{1}{2}\sum_i \int_\beta \ln G_i^{-1}(\phi; k) + \frac{1}{2}\sum_i \int_\beta \left[D^{-1}(\phi; k) G(\phi; k) - 1\right] + V_2(\phi, G)\,,$$

  $\sum_i$ runs over all meson species; $D^{-1}(\phi; k) \equiv$ tree level propagator
  $V_2(\phi, G) \equiv$ infinite sum of the two-particle irreducible vacuum graphs
- Using the Hartree approximation, $V_2(\phi, G)$ is simplified to a one "double bubble" diagram. In the simplest one-meson case, $V_2 \propto \left[\int_\beta G(\phi; k)\right]^2$.
- We therefore obtain a gap equation by minimizing the above effective potential with respect to the dressed propagator $G_i(\phi; k)$:

$$\frac{1}{2} G_i^{-1}(\phi; k) = \frac{1}{2} D_i^{-1}(\phi; k) + 2\frac{\delta V_2(\phi, G)}{\delta G_i(\phi; k)}$$

The CJT Method: thermal masses and effective potential

RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- Using the gapped equation, the thermal mass is given by ($R_i \equiv M_i/T$):

$$M_\sigma^2 = m_\sigma^2 + \frac{T^2}{4\pi^2}\Bigg[\left(3\lambda_\sigma - \delta_{4,N_f}\frac{3}{2}c\right) I_B(R_\sigma) + \left((N_f^2-1)(\lambda_\sigma + 2\lambda_a) + \delta_{4,N_f}\frac{15}{2}c\right) I_B(R_a) + \left(\lambda_\sigma + \delta_{4,N_f}\frac{3}{2}c\right) I_B(R_\eta) + \left((N_f^2-1)\lambda_\sigma - \delta_{4,N_f}\frac{15}{2}c\right) I_B(R_\pi)\Bigg],$$

- CJT improved finite temperature effective potential:

$$V_{\mathrm{FT}}^{\mathrm{LSM}}(\sigma) = \frac{T^4}{2\pi^2}\sum_i \left[J_B(R_i^2) - \frac{1}{4}\left(R_i^2 - r_i^2\right) I_B(R_i^2)\right],$$

$$I_B(R^2) = 2\frac{\mathrm{d}J_B(R^2)}{\mathrm{d}R^2} = \int_0^\infty \mathrm{d}x \frac{x^2}{\sqrt{x^2+R^2}} \frac{1}{e^{\sqrt{x^2+R^2}}-1},$$

$$J_B(R^2) = \int_0^\infty \mathrm{d}x\, x^2 \ln\left(1 - e^{-\sqrt{x^2+R^2}}\right)$$

First-order phase transitions and bubble's nucleation

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$$

- The generic three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)^2 + V_{\text{eff}}(\rho, T)\right] ,$$

  where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$
- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use percolation temperature T_p: the temperature at which 34% of false vacuum is converted
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*) e^{\beta(t-t_*)}$$

Phase transition characteristics

Huang, Reichert, Sannino, Wang
PRD 104 (2021) 035005

- The inverse duration time then follows as

$$\beta = -\frac{\mathrm{d}}{\mathrm{d}t}\frac{S_3(T)}{T}\bigg|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T\frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T=T_*},$$

where we used that $\mathrm{d}T/\mathrm{d}t = -H(T)T$.

- We define the strength parameter α from the trace of the energy-momentum tensor θ weighted by the enthalpy

$$\alpha = \frac{1}{3}\frac{\Delta\theta}{w_+} = \frac{1}{3}\frac{\Delta e - 3\Delta p}{w_+}, \qquad \Delta X = X^{(+)} - X^{(-)}, \text{ for } X = (\theta, e, p)$$

$(+)$ denotes the meta-stable phase (outside of the bubble) while $(-)$ denotes the stable phase (inside of the bubble).

- The relations between enthalpy w, pressure p, and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}, \qquad e = \frac{\partial p}{\partial \ln T} - p, \qquad p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors

Gravitational wave spectrum: an outlook

- Contributions from bubble collision and turbulence are subleading
  The GW spectrum from sound waves is given by

$$h^2\Omega_{\mathrm{GW}}(f) = h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \left(\frac{f}{f_{\mathrm{peak}}}\right)^3 \left[\frac{4}{7} + \frac{3}{7}\left(\frac{f}{f_{\mathrm{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\mathrm{peak}} \simeq 1.9 \cdot 10^{-5}\,\mathrm{Hz} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \left(\frac{T}{100\,\mathrm{GeV}}\right) \left(\frac{\tilde{\beta}}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \simeq 2.65 \cdot 10^{-6} \left(\frac{v_w}{\tilde{\beta}}\right) \left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \Omega_{\mathrm{dark}}^2 \qquad \Omega_{\mathrm{dark}} = \frac{\rho_{\mathrm{rad,dark}}}{\rho_{\mathrm{rad,tot}}}$$

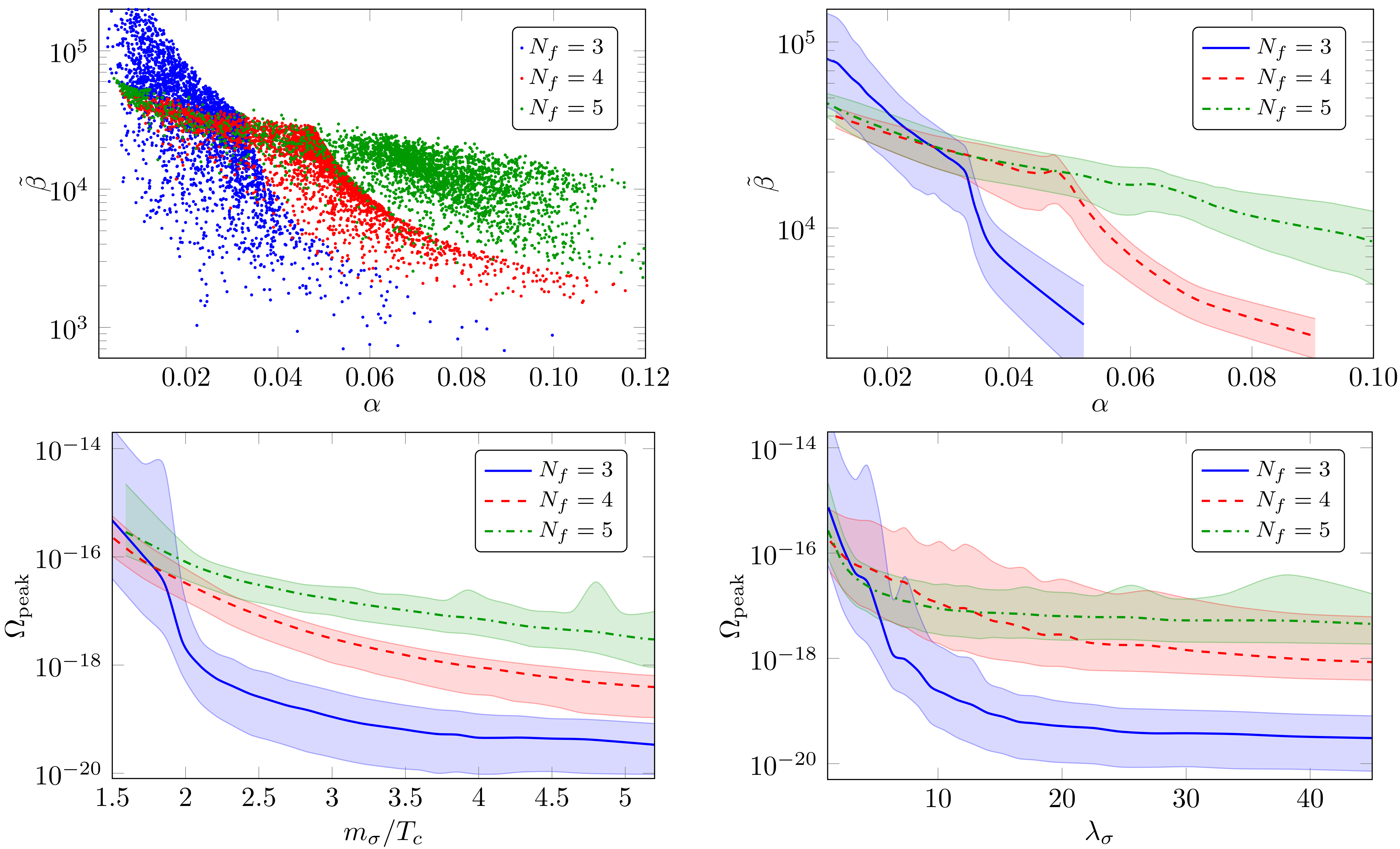
- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.
- The efficiency factor for the sound waves κ_{sw} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\mathrm{sw}} = \sqrt{\tau_{\mathrm{sw}}}\,\kappa_v \qquad \tau_{\mathrm{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \text{ for } \beta >> 1 \qquad \kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98+\alpha}}$$

where $\bar{U}_f$ is the root-mean-square fluid velocity $\qquad \bar{U}_f^2 \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$

Phase diagram and gravitational waves in the PQM model

RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159



The strongest signal we found can almost reach the LISA sensitivity

Summary:

- We developed a new approach based upon the well-established thermal EFT and the existing lattice results to calculate the glueball CDM relic density incorporating confinement effects and non-perturbative self-interactions

- While in the present work we considered only SU(3), due its generality, our approach can be easily applied to different gauge groups

- A dark gauge sector interacting only via gravitational interactions with the SM and a confinement scale at the eV scale might explain the DM abundance without spoiling other cosmological observables

- Our method is suitable for investigations of the glueball formation in modified cosmological histories, requiring only a simple modification of the main evolution equation

- We analysed the phase transitions in the Polyakov-loop extended LSM utilising the CJT method and computed the resulting primordial gravitational wave spectra showcasing an enhancement for weak sigma self-interactions and light sigma meson