Orbifold stability of asymptotic GUTs

ANCA PREDA

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A. Preda, Z. Wang [in preparation]

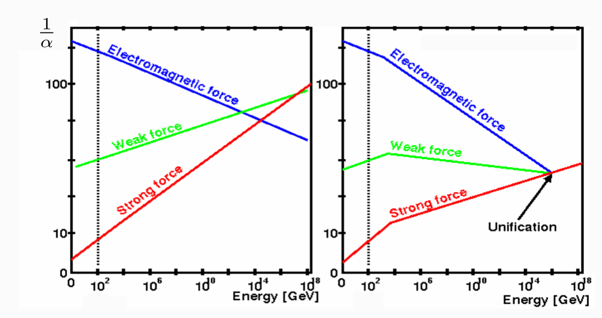
LUND UNIVERSITY

BSM^2 - Beyond the Standard Model
BrainStorming Meeting

Outline

Grand Unification

Standard picture:



SM gauge couplings meet at some high scale

$\updownarrow$

Physics described by a **unified gauge group** $\mathcal{G} \supset \mathcal{G}_{\rm SM}$
e.g. $SU(5),\ SO(10),\ E_6 \ldots$

Asymptotic Grand Unified Theories (aGUTs)

- Grand Unified Theories (GUTs) formulated in 5 or more space-time dimensions. [1]

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defined on $\mathbb{R}^4 \times K$, where $\mathbb{R}^4$ is the usual 4-dimensional Minkowski space and K defines δ *compact* extra dimensions.

- Gauge symmetry is broken using boundary conditions which violate the GUT symmetry

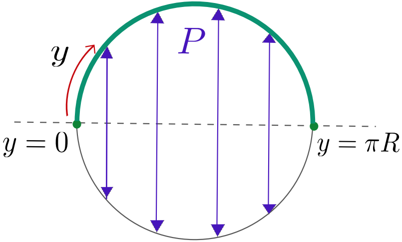
  $\Rightarrow$ different from the usual Higgs mechanism

- **Motivation:** solution to hierarchy problem, lower GUT scale, smaller representations...

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

Example: 5D case

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:



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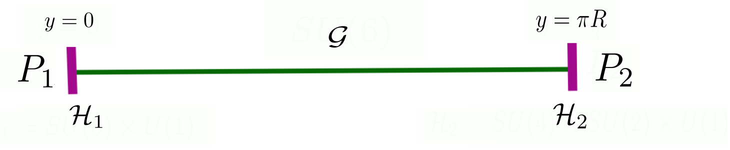
- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:

- The inverse radius R^{-1} sets the scale of compactification.

Example: 5D case

- Each intrinsic $\mathbb{Z}_2$ transformation is specified by a parity matrix P acting on the fields

$$\Phi(x^\mu, -y) = P\Phi(x^\mu, y) = \pm\Phi(x^\mu, y).$$



- Each P_i will break $\mathcal{G} \to \mathcal{H}_i$ on one boundary, such that

$$\mathcal{G}_{4\mathrm{D}} \equiv \mathcal{H}_i \cap \mathcal{H}_j$$

- Viable model must contain the Standard Model

$$\mathcal{G}_{4\mathrm{D}} \supset \mathcal{G}_{\mathrm{SM}}$$

Example: 5D case

A model can be fully defined in terms of [2]

Gauge group $\mathcal{G}$

Parity P

Parity assignments

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Example: 5D case

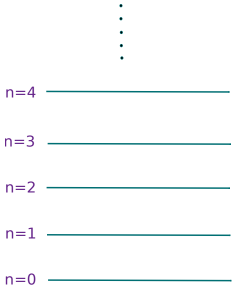
For a field $\Phi\left(x^{\mu}, y\right)$ we can do a **Kaluza-Klein (KK) decomposition**

⇩

Decomposition

$$\Phi\left(x^{\mu}, y\right) = \underbrace{\sum_{n=0}^{\infty} \phi_{+}^{(n)}(x^{\mu}) \cos\left(\frac{ny}{R}\right)}_{\text{parity-even}} + \underbrace{\sum_{n=1}^{\infty} \phi_{-}^{(n)}(x^{\mu}) \sin\left(\frac{ny}{R}\right)}_{\text{parity-odd}}$$

Example: 5D case



- The 4D fields $\phi_{\pm}^{(n)} \equiv$ KK modes with mass of n/R.
- The Standard Model fields are the massless zero modes of ϕ_{+}.
- For $E \ll 1/R$, the heavy Kaluza-Klein towers are integrated out.

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4D effective field theory

Gauge-Higgs Unification[3] [4]

Assume a 5D gauge theory and A_M $(M = 1, \ldots, 5)$ a gauge field

⇓

$$\underbrace{A_\mu(\mu = 1 \ldots 4)}_{\text{4D}} \quad (+) \qquad \text{and} \qquad \underbrace{A_5}_{\text{extra dimension}} \quad (-)$$

⇓

A_5 behaves as a scalar field in 4D

$\equiv$ **Higgs field**

[3] Y. Hosotani, Phys. Lett. B 126 (1983)

[4] R. Contino,et al, Nucl. Phys. B 671 (2003)

Gauge-Higgs Unification

$A_5 \equiv$ **gauge-scalar** embedded in the gauge fields

- Gauge-Higgs scalar A_5 will generate masses for bulk fields when getting a VEV $\langle \mathbf{A_5} \rangle = \alpha$:

$$-\frac{1}{2} F_{5\mu} F^{5\mu}$$

gauge

$$(D_M \Phi)^\dagger \left(D^M \Phi\right)$$

scalar

$$\bar{\Psi} i D_M \Gamma^M \Psi$$

fermions

Gauge-Higgs Unification

There will be a scalar potential for A_5!

...but gauge symmetry forbids the potential at tree level

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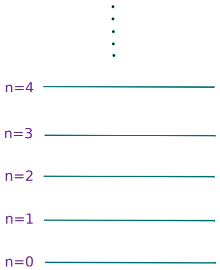
one loop effective potential

(dictates symmetry breaking, mass of the scalars etc.)

One loop effective potential

Summing over KK modes and doing some algebra we find [5]



$$V_{\text{eff}}(\alpha) = \frac{\mp 1}{32\pi^2} \frac{1}{(\pi R)^4} \mathcal{F}(c\alpha)$$

where

$$\mathcal{F}(\alpha) = \frac{3}{2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\alpha)}{n^5}.$$

[5] I. Antoniadis, et al, New Journal of Physics 3 (2001)

One loop effective potential

$$V_{\text{eff}}^{\text{total}}(\alpha) = V_{\text{eff}}^{\text{gauge}}(\alpha) + V_{\text{eff}}^{\text{scalar}}(\alpha) + V_{\text{eff}}^{\text{fermionic}}(\alpha)$$

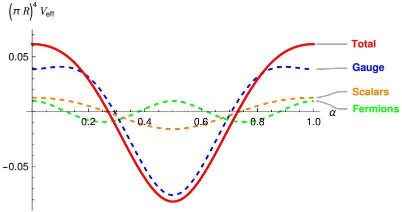


Figure: Potential of an $SU(6)$ model.

- gauge dominated
- minimum at $\alpha = 1/2$

One loop effective potential

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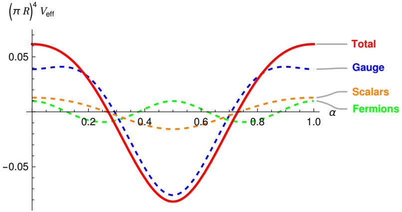

Figure: Potential of an $SU(6)$ model.

- gauge dominated
- minimum at $\alpha = 1/2$

Not a viable model

One loop effective potential

Minimum of $V_{\text{eff}}^{\text{gauge}}(\alpha)$ must be at $\alpha = 0$.

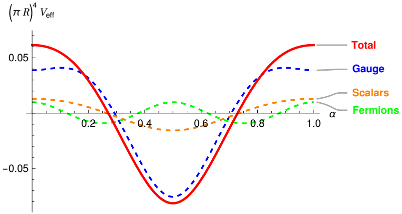

Figure: Potential of an $SU(6)$ model.

Gauge transformation to remove the VEV

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change in the parity on one boundary

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different breaking pattern

One loop effective potential

Minimum of $V_{\text{eff}}^{\text{gauge}}(\alpha)$ must be at $\alpha = 0$.

⇓

criteria of **orbifold stability**

One loop effective potential

Minimum of $V_{\text{eff}}^{\text{gauge}}(\alpha)$ must be at $\alpha = 0$.

$\Downarrow$

criteria of **orbifold stability**

$\Rightarrow$ Derive conditions that models need to satisfy in order to be phenomenologically relevant

Orbifold stability: $SU(N)$

- aGUT in 5D based on the $SU(N)$ gauge group[6]
- Extra dimension compactified on the orbifold $\mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$
- Most general parities

$$P_1 = (+1\cdots,+1,+1,\cdots,+1,-1,\cdots,-1,-1\cdots,-1),$$
$$P_2 = (\underbrace{+1,\cdots,+1}_{p},\underbrace{-1,\cdots,-1}_{q},\underbrace{+1,\cdots,+1}_{r},\underbrace{-1,\cdots,-1}_{s}).$$

[6] N. Haba, T. Yamashita, JHEP 2004 (2004).

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$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

Breaking pattern

$$SU(N) \to SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3\,,$$

where $p + q + r + s = N$.

6 N. Haba, T. Yamashita, JHEP 2004 (2004).

Orbifold stability: $SU(N)$

- Fields will have parity assignments under (P_1, P_2), which can be

$$(+,+) \quad (+,-) \quad (-,+) \quad (-,-)$$

- Parity assignments on the fields are

$$A_\mu \to \begin{matrix} p \\ q \\ r \\ s \end{matrix} \begin{pmatrix} (+,+) & (+,-) & (-,+) & (-,-) \\ (+,-) & (+,+) & (-,-) & (-,+) \\ (-,+) & (-,-) & (+,+) & (+,-) \\ (-,-) & (-,+) & (+,-) & (+,+) \end{pmatrix}.$$

- Gauge-scalar zero modes present in the (p, s) and (q, r) blocks

- Scalars in the bi-fundamental representation of $SU(p) \times SU(s)$ and $SU(q) \times SU(r)$

Orbifold stability: $SU(N)$

$$P_1 = (+1\cdots,+1,+1,\cdots,+1,-1,\cdots,-1,-1\cdots,-1),$$
$$P_2 = (\underbrace{+1,\cdots,+1}_{p},\underbrace{-1,\cdots,-1}_{q},\underbrace{+1,\cdots,+1}_{r},\underbrace{-1,\cdots,-1}_{s}).$$

Depending on the values of (p, q, r, s), we can have [7]

- **Two-block** case: two of (p, q, r, s) non-zero, others zero
- **Three-block** case: three of (p, q, r, s) non-zero, others zero
- **Four-block** case: all four (p, q, r, s) non-zero

[7] G. Cacciapaglia et al. [in preparation]

Orbifold stability: $SU(N)$

Approach: Compute the potential, evaluate it at the two minima ($\alpha = 0,\ 1/2$) and check which is the *global* one:

$$\Delta V_{\text{eff}} = V_{\text{eff}}(1/2) - V_{\text{eff}}(0)$$

Two-block case

- Breaking pattern is

$$SU(N) \to SU(p) \times SU(N-p) \times U(1)$$

- Always **stable**: minimum of $V_{\text{gauge}}^{\text{eff}}$ at $\alpha = 0$

Orbifold stability: $SU(N)$

Three-block case

- Breaking pattern is

$$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2$$

- **Stable** only for $\mathbf{p \geq N/2}$

Four-blocks

- Breaking pattern is

$$SU(N) \to SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3$$

- **Never stable**: always decays into 3-blocks

Orbifold stability: $SU(N)$ results

Examples:

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$
$$SU(6) \to SU(3) \times SU(2) \times U(1)^2 \qquad \text{satisfy } p \geq N/2 \quad ✅$$
$$SU(8) \to SU(4) \times SU(2) \times SU(2)$$

whereas

$$SU(7) \to SU(3) \times SU(3) \times U(1)^2 \qquad \text{has } p \leq N/2 \quad ❌$$

Orbifold stability: $Sp(2N)$, $SO(N)$

- Analysis can be extended to other groups as well: $Sp(2N)$, $SO(N)$
- Same strategy as before for identifying stable configurations
- **New parity definitions** are needed $\leftrightarrow$ group theory

examples:

$$Sp(10) \to SU(3) \times SU(2) \times U(1) \times U(1)$$

$$SO(10)^8 \to SU(3) \times SU(2) \times U(1) \times U(1)$$

$$SO(11) \to SO(3) \times SO(2) \times SO(6) \sim SU(2) \times U(1) \times SU(4)$$

$$SO(12) \to SU(4) \times SU(2) \times U(1) \times U(1)$$

[8] M. Khojali, et al, PACP2022 (2022)

Orbifold stability: final classification

All possible working models for a $d = 5$ aGUT:

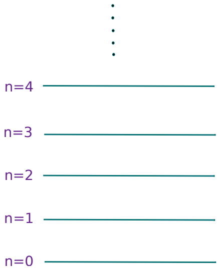
Model	Breaking pattern	Stability criteria
$\mathrm{SU}(N)$	$\mathrm{SU}(N) \to \mathrm{SU}(a) \times \mathrm{SU}(N-a) \times \mathrm{U}(1)$	stable $\forall$ a
	$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(s) \times \mathrm{U}(1)^2$	$p \geq N/2$
$\mathrm{Sp}(2N)$	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2a) \times \mathrm{Sp}(2(N-a))$	stable $\forall$ a
	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2p) \times \mathrm{Sp}(2q) \times \mathrm{Sp}(2s)$	$p \geq N/2$
	$\mathrm{Sp}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N)$	$\mathrm{SO}(2N) \to \mathrm{SO}(2a) \times \mathrm{SO}(2(N-a))$	stable $\forall$ a
	$\mathrm{SO}(2N) \to \mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$p \geq N/2$
	$\mathrm{SO}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N+1)$	$\mathrm{SO}(2N+1) \to \mathrm{SU}(2a+1) \times \mathrm{SU}(2(N-a))$	stable $\forall$ a
	$\mathrm{SO}(2N+1) \to \mathrm{SO}(2p+1) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$2p+1 \geq (2N+1)/2$

Conclusions

- aGUTs as an alternative to standard GUTs
- Viable models have to pass certain criteria ⇒ **orbifold stability**
- For $SU(N)$: two-blocks and three-blocks with $p \geq N/2$ are stable, while four-blocks are not
- The criteria of **orbifold stability** helps identify potentially interesting models
- **Systematic classification** that discards phenomenologically unrealistic scenarios

Back-up slides

Example: 5D case



Each 5D field $\equiv$ **infinite tower** of 4D fields

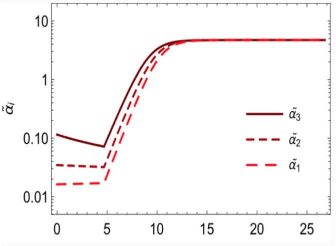
“ground state” (n=0) are the SM states

RGEs for the couplings get modified:

logarithmic $\rightarrow$ power-law dependence

$\Downarrow$

couplings will flow **asymptotically** towards a UV fixed point



One loop effective potential

The effective potential for a scalar field in 4D is given by[5]

$$V_{\text{eff}} = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{d^4 p}{(2\pi)^4} \log[p^2 + m^2]\,, \;\; F_I = 0,\, 1$$
$$= -\frac{1}{32\pi^2}\sum_I (-1)^{F_I} \int_0^\infty dl\; l\; e^{-m^2/l}$$

States appear as towers of KK modes with mass

$$m_n^2 = \frac{(n + c\,\alpha)^2}{R^2}, \qquad c \text{ depends on the representation}$$

The field $\alpha \sim \langle A_5 \rangle$.

[5] I. Antoniadis, et al, New Journal of Physics 3 (2001)

One loop effective potential

Assume a model with gauge group $\mathcal{G}$ and parities (P_1, P_2)

⇓

Gauge transformation Ω : $\quad A'_M \rightarrow \Omega^\dagger A_M \Omega - \frac{i}{g}\Omega^\dagger \partial_M \Omega$ **adjoint**

Boundary conditions imposed on the adjoint become [6]

$$A'_\mu = P'_i \cdot A'_\mu \cdot P'_i - \frac{i}{g} P'_i \cdot \partial_\mu P'_i$$

$$A'_5 = -P'_i \cdot A'_5 \cdot P'_i + \frac{i}{g} P'_i \cdot \partial_5 P'_i$$

where $P'_i = \Omega^\dagger P_i \Omega$.

Freedom to choose the gauge $\Rightarrow \partial_M P'_i = 0$.

⇓

$(P'_1, P'_2) \sim (P_1, P_2)$ **equivalent!**

[6] N. Haba, et al, Prog.Theor.Phys., 111(2004)

Removing the VEV

The $SU(6)$ model with

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{+1, \cdots, +1}_{r=1})$$

has a global minimum at $\alpha = 1/2$.

$\Rightarrow$ remove the value of α using a gauge transformation

$$\Omega(\alpha) = \exp\left(i\frac{g}{R}y\, A_5\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\frac{\alpha y}{R} & \sin\frac{\alpha y}{R} \\ 0 & 0 & 0 & 0 & \sin\frac{\alpha y}{R} & \cos\frac{\alpha y}{R} \end{pmatrix}$$

Removing the VEV

For $y = \pi R$, the gauge transformation flips the last two signs of P_2', such that

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)$$
$$P_2' = (\underbrace{+1, \cdots, +1}_{p=4}, \underbrace{-1, \cdots, -1}_{q=1}, \underbrace{-1, \cdots, -1}_{s=1})$$

The 4D unbroken group will then be $SU(4) \times U(1) \times U(1)$.

Equivalence classes

$$P_1 = (+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1 \cdots, -1),$$
$$P_2 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}).$$

Certain parity configurations are equivalent:

$$\begin{aligned} P_1 \to -P_1 &\quad\Rightarrow\quad p \leftrightarrow r,\quad q \leftrightarrow s \\ P_2 \to -P_2 &\quad\Rightarrow\quad p \leftrightarrow q,\quad r \leftrightarrow s \\ P_i \to -P_i &\quad\Rightarrow\quad p \leftrightarrow s,\quad q \leftrightarrow r \\ P_1 \leftrightarrow P_2 &\quad\Rightarrow\quad q \leftrightarrow r \end{aligned}$$

Equivalence classes given by gauge transformations [6]

$$[p, q, r, s] \sim [p+1, q-1, r-1, s+1] \sim [p-1, q+1, r+1, s-1]$$

[6] N. Haba, et al, Prog.Theor.Phys., 111(2004)

Stability of $Sp(2N)$ and $SO(N)$

- For $Sp(2N)$ groups the Cartan generators are given by

$$X_i = \underbrace{\text{diag}(0, \dots 0, 1, 0, \dots 0)}_{SU(N) \text{ Cartan generator}} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Cartan subalgebra

$$\Omega(\theta_i) = \prod \exp(i\theta_i X_i^C) = \text{diag}(e^{i\theta_1}, \dots e^{i\theta_N}, e^{-i\theta_1}, \dots e^{-i\theta_N})$$

Possible parities $\Rightarrow$

$$P^I_{Sp(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \theta_i = 0, \pi$$

$$P^{II} Sp(2N) = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \theta_i = \pm\pi/2$$

Stability of $Sp(2N)$ and $SO(N)$

- For $SO(2N)$ groups the Cartan generators are given by

$$X_i = \underbrace{\text{diag}(0,\ldots 0,1,0\ldots 0)}_{SU(N)\ \text{Cartan generator}} \otimes \sigma_2$$

⇩

Cartan subalgebra

$$\Omega(\theta_i) = \prod \exp(i\theta_i X_i^C) = \sum_i \text{diag}(0,\ldots 0,1,0\ldots 0) \otimes e^{i\theta_i\sigma_2}$$

Possible parities ⇒

$$P^{I}_{SO(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \theta_i = 0, \pi$$

$$P^{II}_{SO(2N)} = P_{SU(N)} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \theta_i = \pm\pi/2$$