Fusing physics principles and machine learning:

inferring dark matter densities of galaxies using stellar catalogs with incomplete kinematic information

Sung Hak Lim
CTPU-PTC, IBS

Institute for Basic Science

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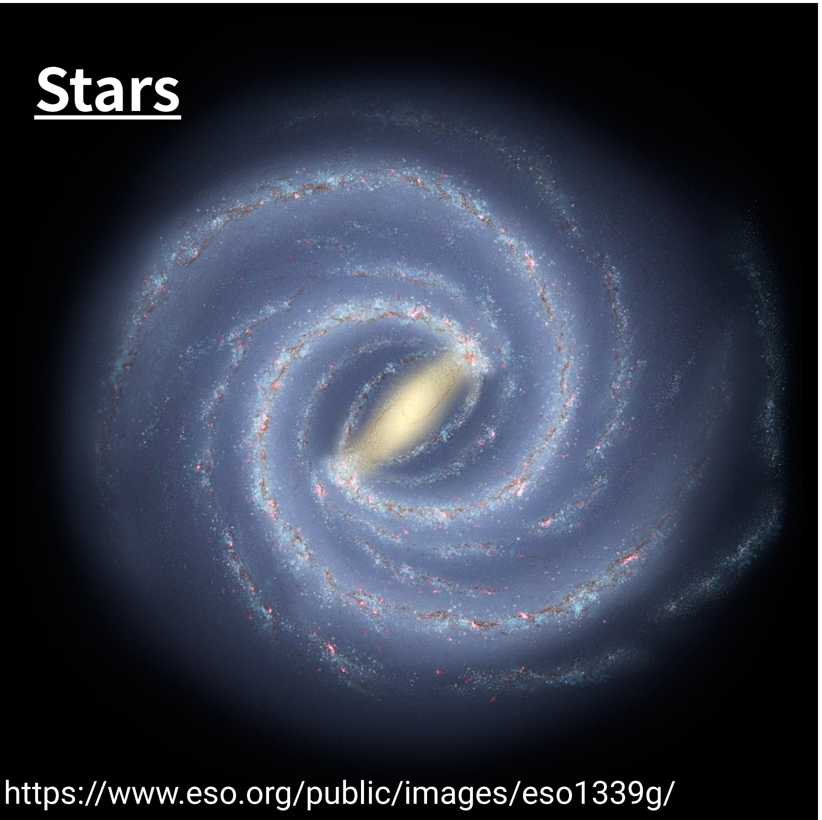
In **galactic dynamics** for studying **dark matter**,
one important and interesting task is…

Q: How to use **star catalogs**
of a galaxy to understand
its galactic dark matter density?

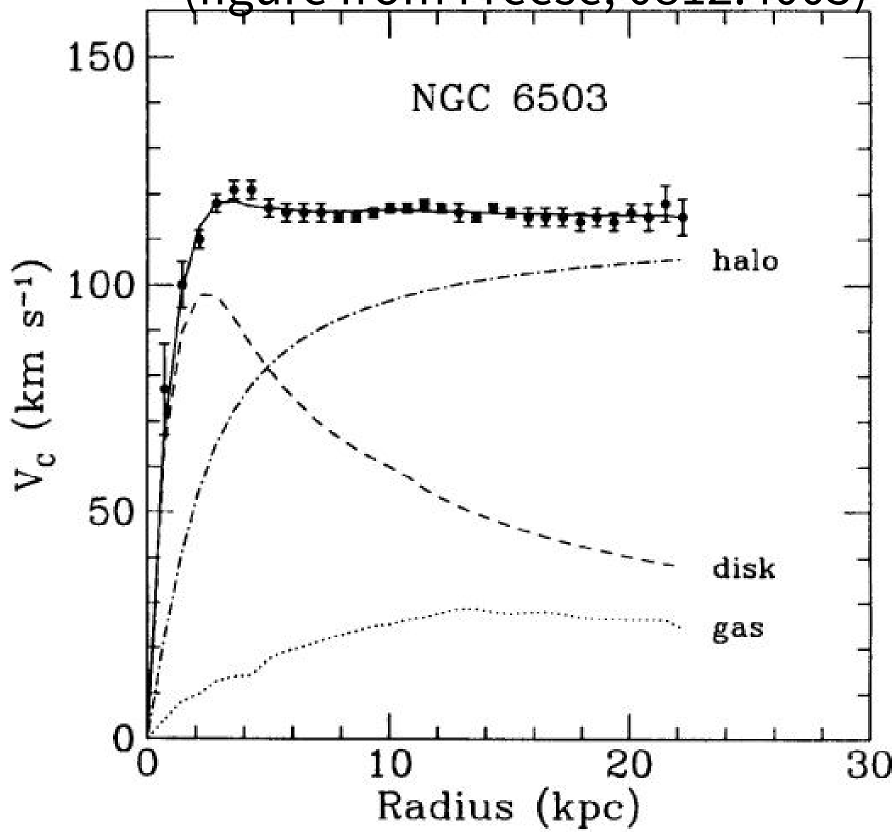


Dark Matter Halo

?

Old school example:
Galaxy rotation curve

(figure from Freese, 0812.4005)

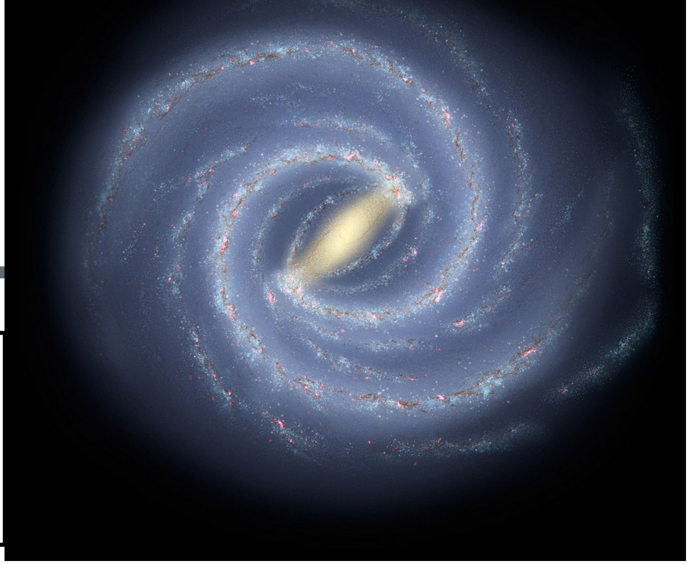


NGC 6503 from NASA Hubble telescope

$$v_{\rm circ}(R) = \sqrt{\frac{GM(R)}{R}}$$

Obtain mass density from enclosed mass M(R)

Outline of Strategy: Fluid Mechanics



Star catalog

$\{(\vec{x}, \vec{v})\}$

Galaxy: hydrodynamic system

Phase space density

$f(\vec{x}, \vec{v})$

Neural Networks for Density Estimation: Normalizing Flows

$$\vec{u}_0 \to \vec{u}_1 \to \cdots \to \vec{u}_n = (\vec{x}, \vec{v})$$

Gravitational accel.

$\vec{a}(\vec{x})$

Solving fluid EOM (Boltzmann Equation)

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}}\right] f(\vec{x}, \vec{v}) = 0$$

Mass density

$\rho(\vec{x})$

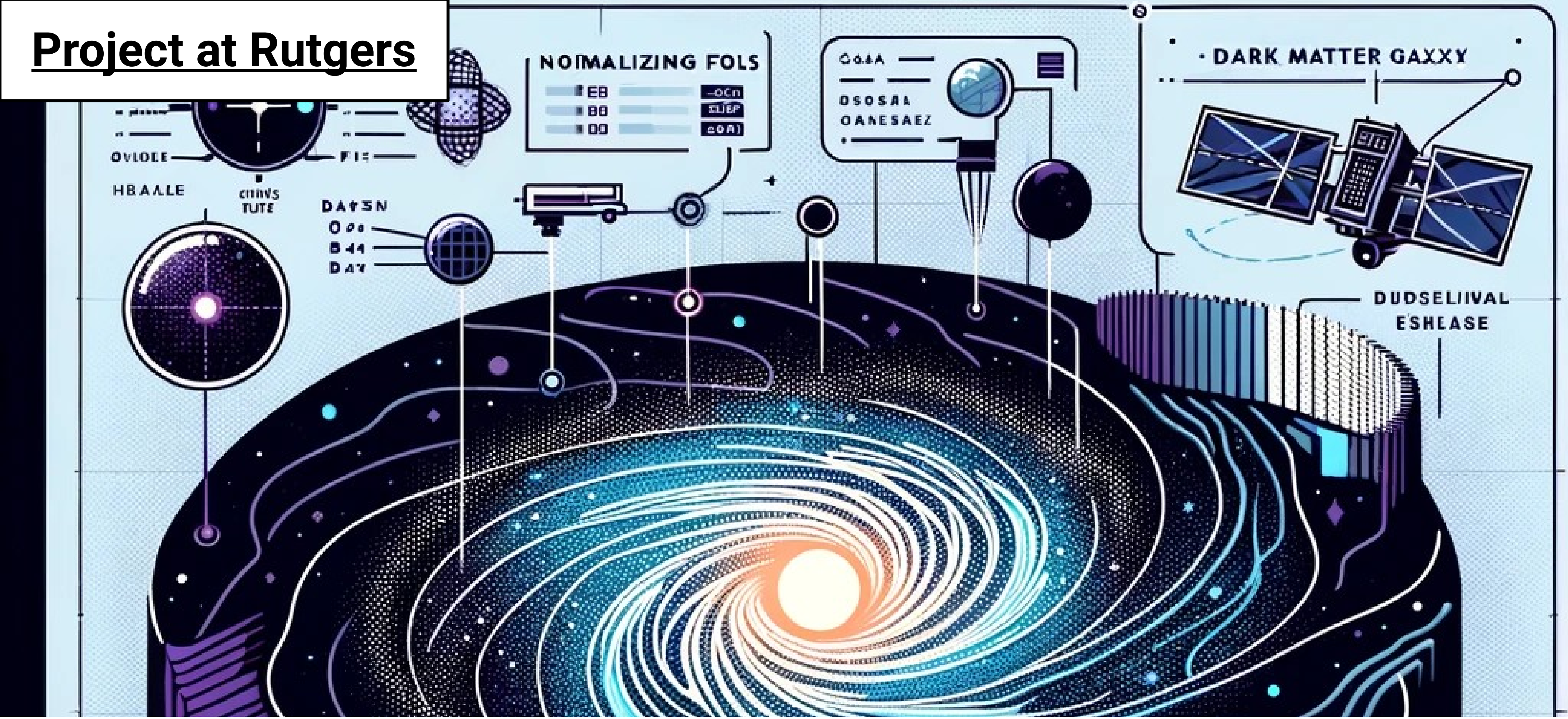
Solving Gauss's Equation

$$-4\pi G\rho = \nabla \cdot \vec{a}$$

arXiv: 2205.01129, 2305.13358

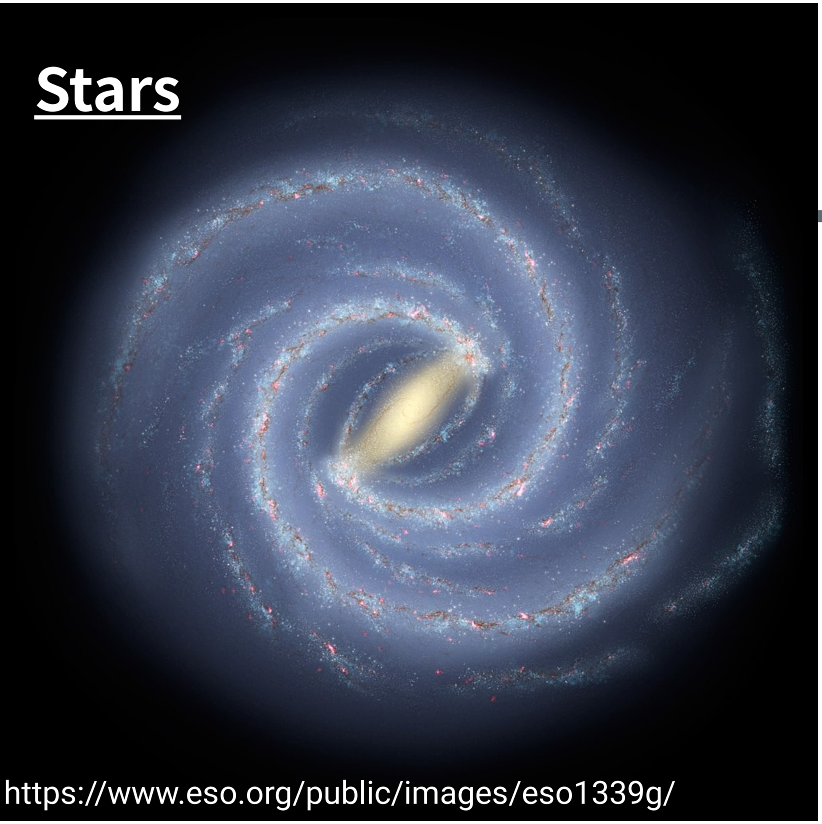
See also Green et. al. arXiv:2011.04673, arXiv:2205.02244

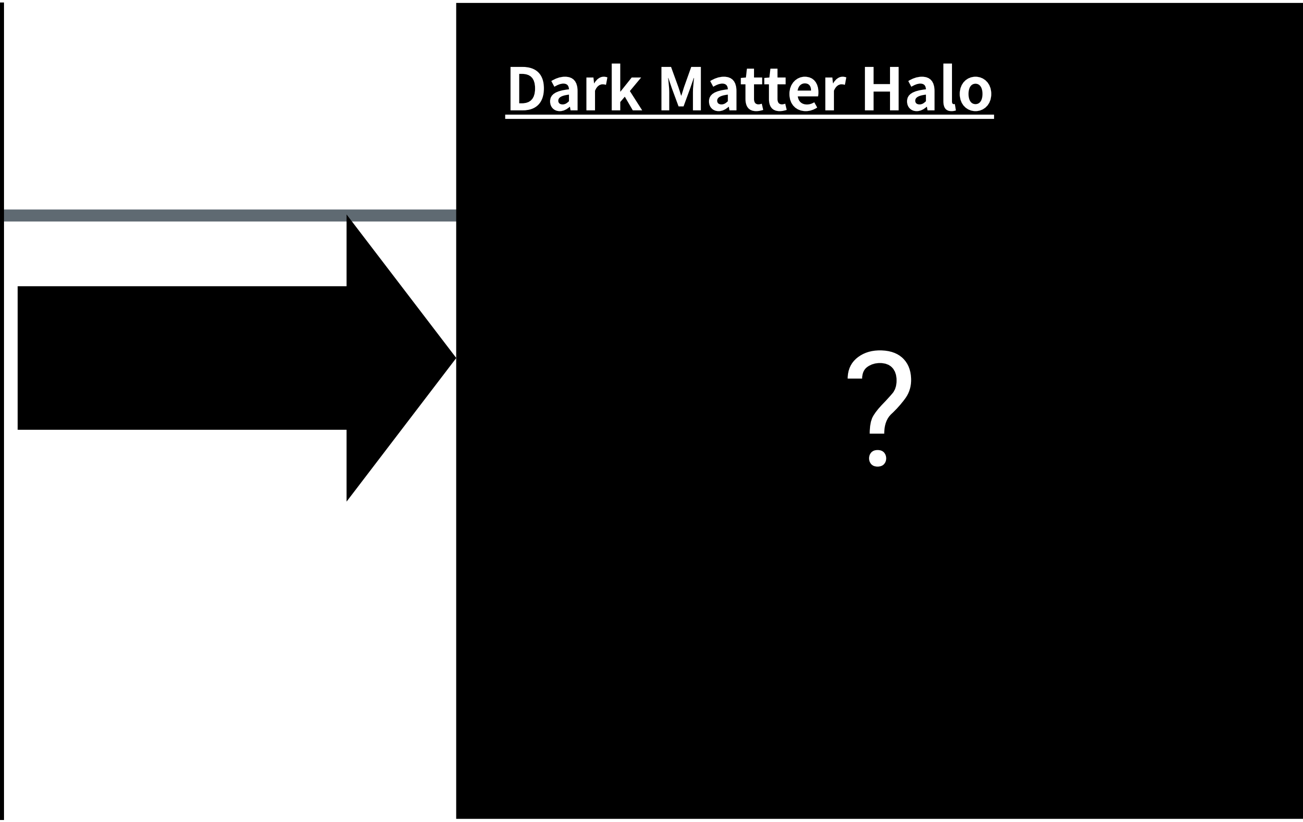
Project at Rutgers



Mapping Dark Matter in the Milky Way using Normalizing Flows and Gaia DR3

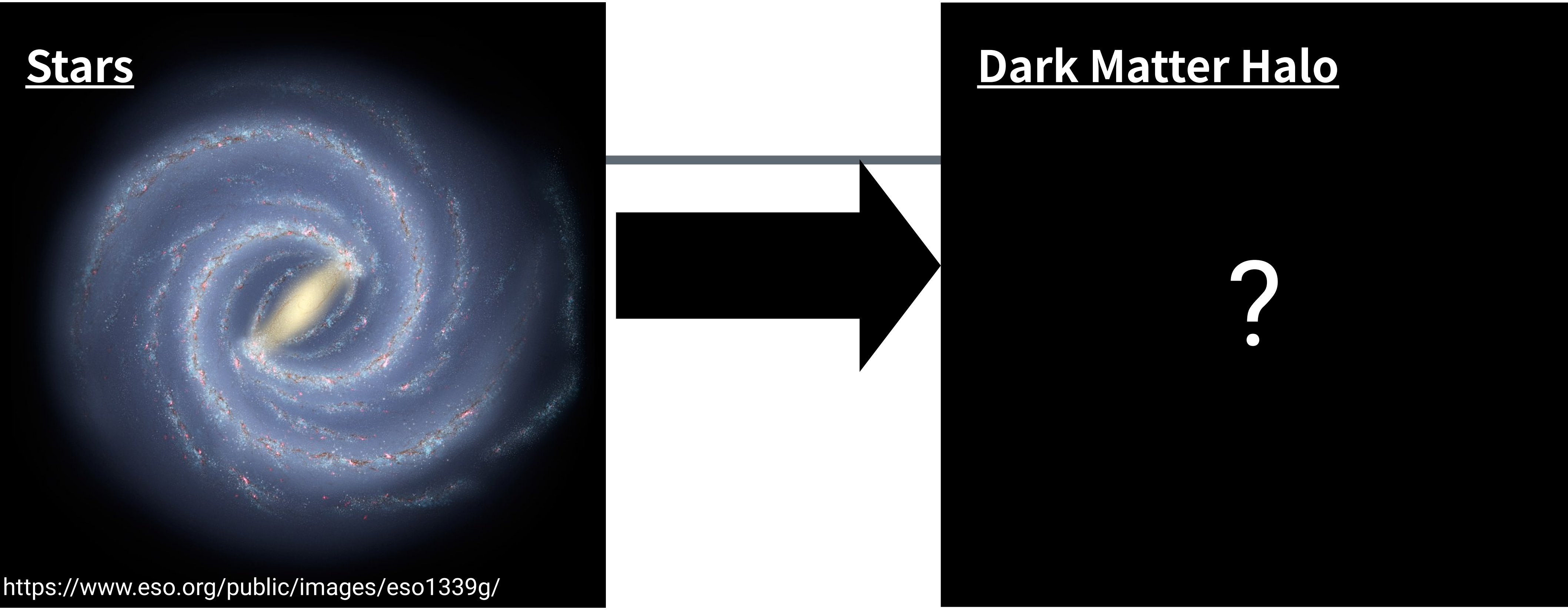
M. R. Buckley, **SHL**, E. Putney, and D. Shih, arXiv:2205.01129, published in MNRAS
SHL, E. Putney, M. R. Buckley, and D. Shih, arXiv:2305.13358





We have an unsupervised ML method to estimate **dark matter density** given **stellar distribution of a galaxy**.

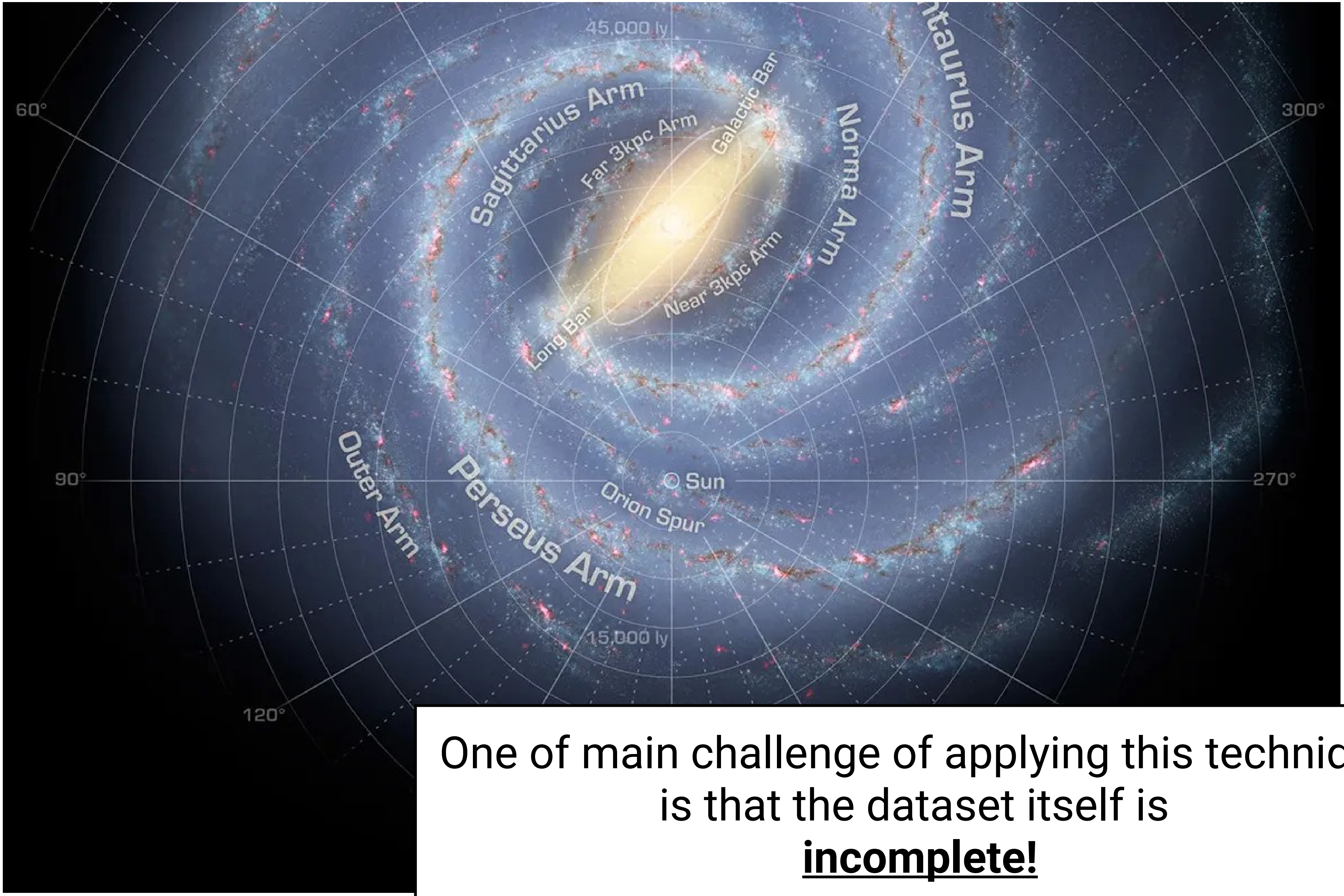
END of story?



We have an ML method to estimate **dark matter density** given **stellar distribution of a galaxy**.

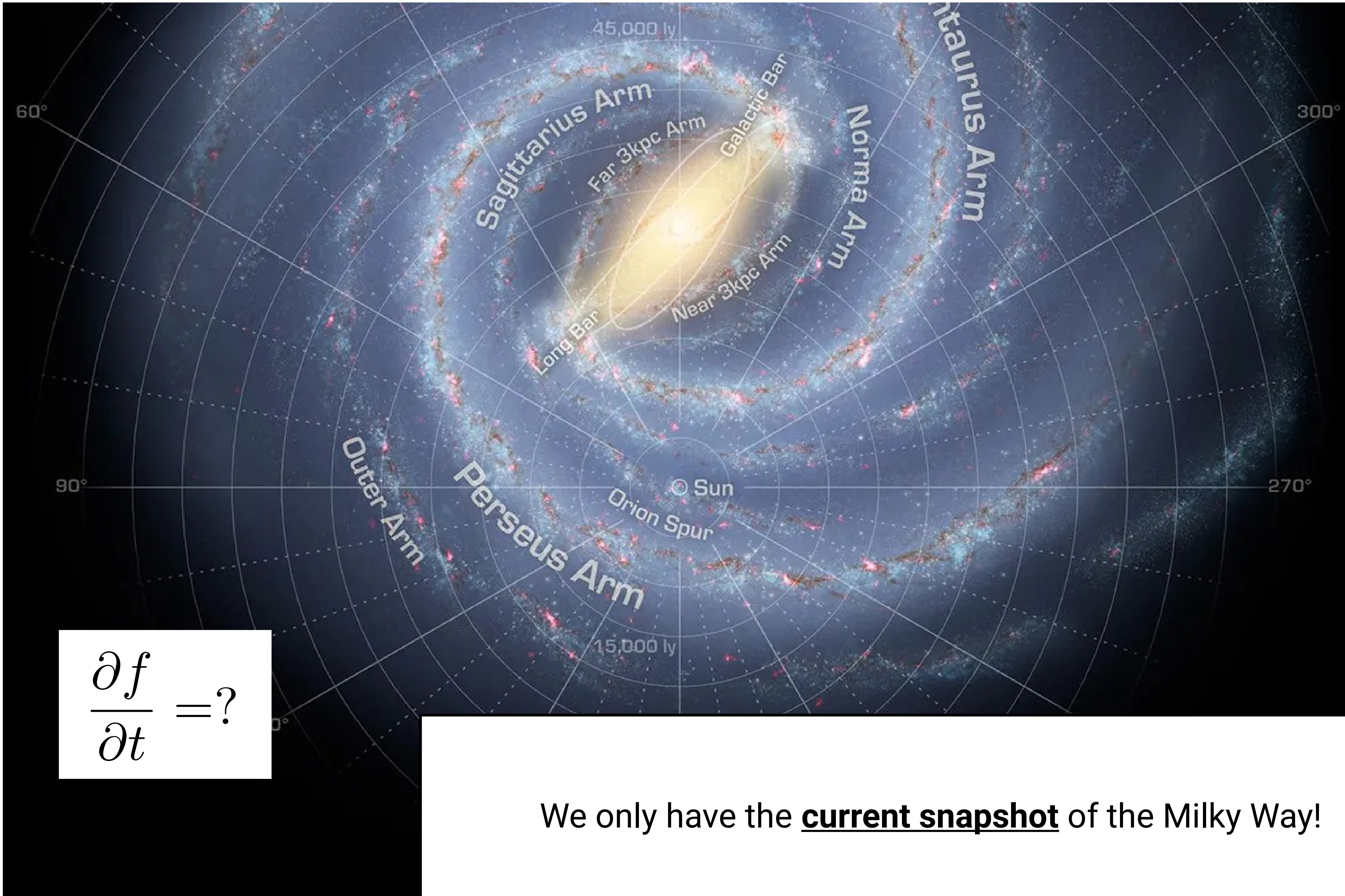
END? → Of course not!

Galactic Dynamics and Incomplete Datasets



One of main challenge of applying this technique is that the dataset itself is **incomplete!**

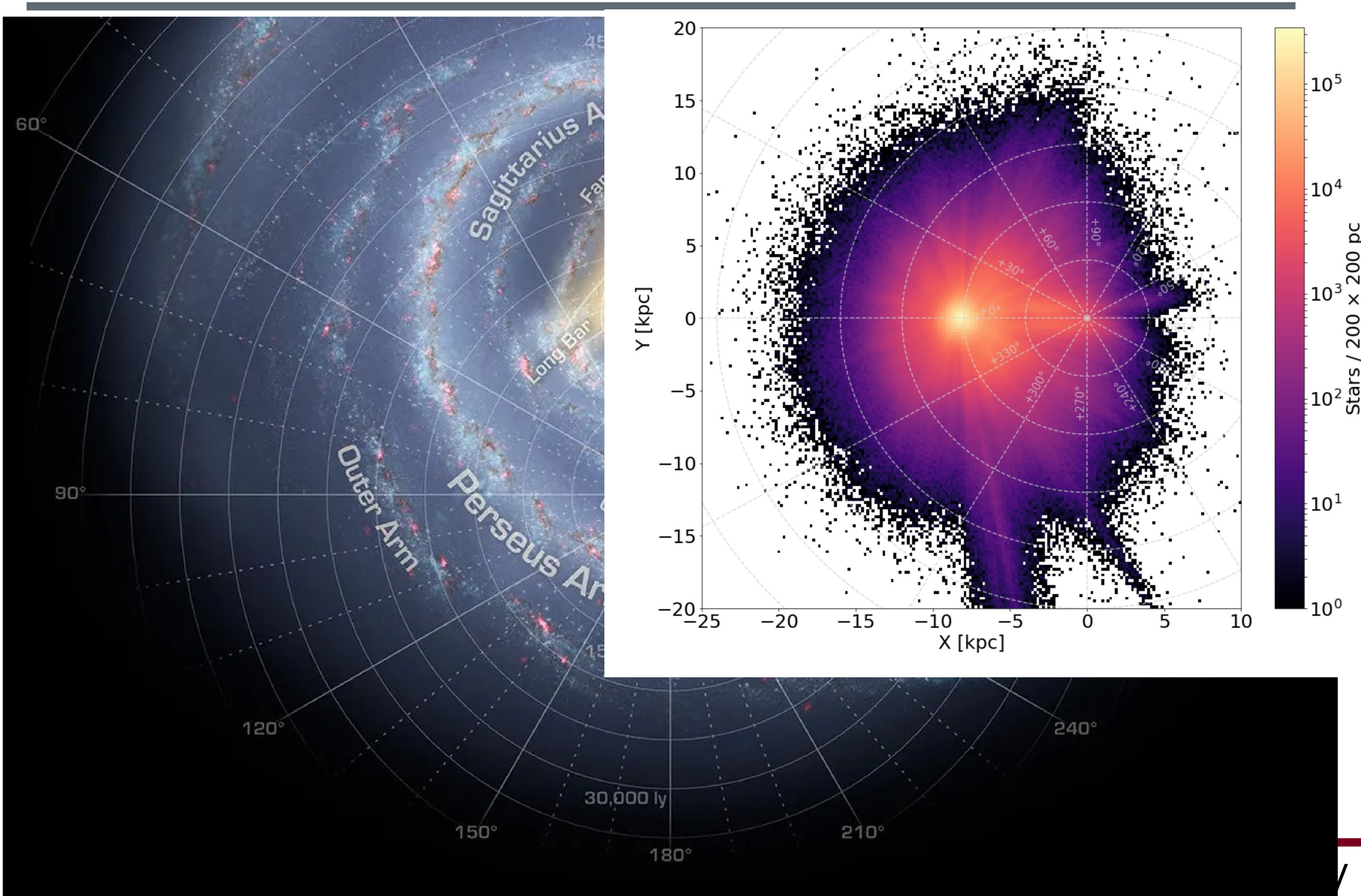
No time derivative information



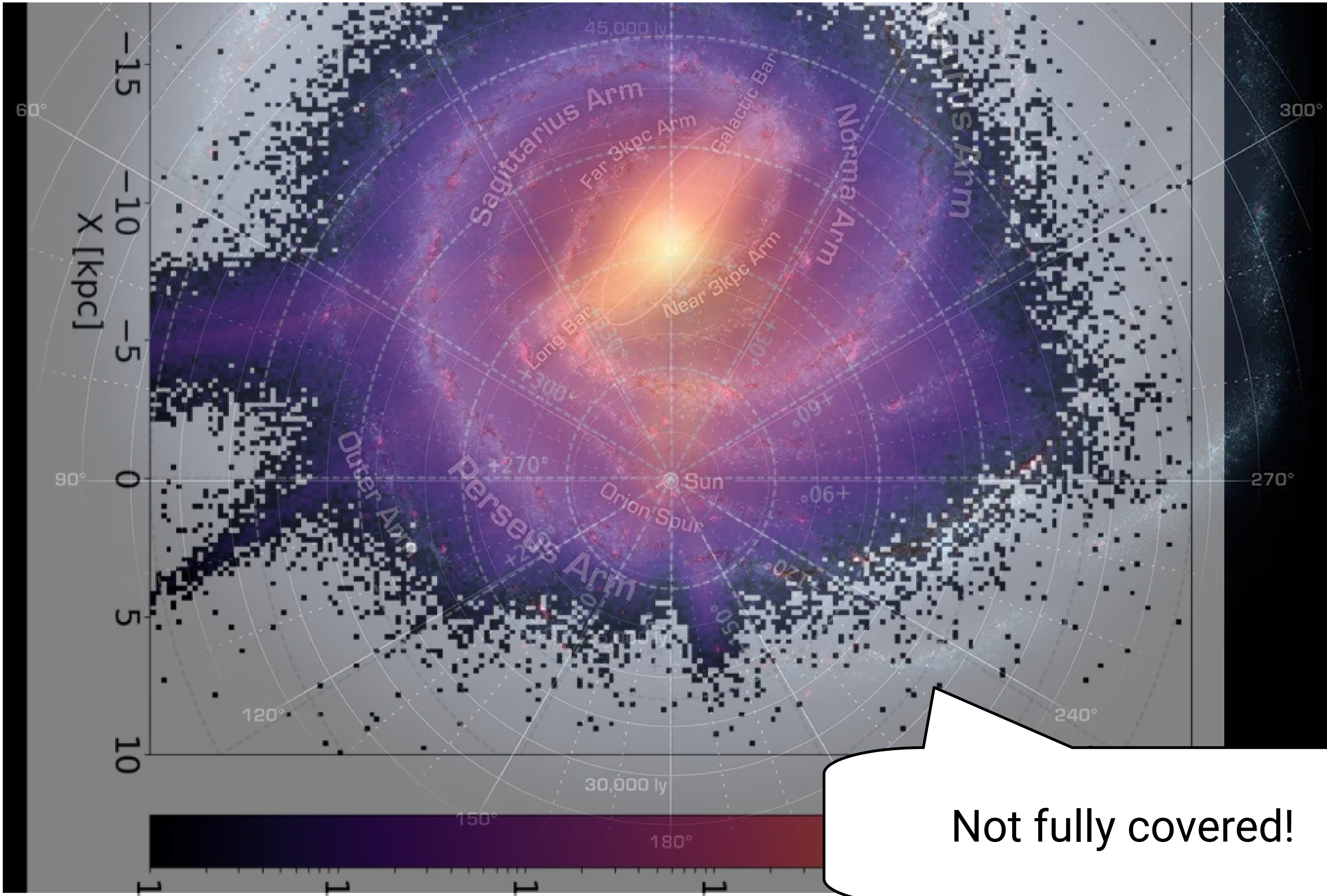
$$\frac{\partial f}{\partial t} = ?$$

We only have the **current snapshot** of the Milky Way!

Radial Velocity Distribution of Gaia DR3



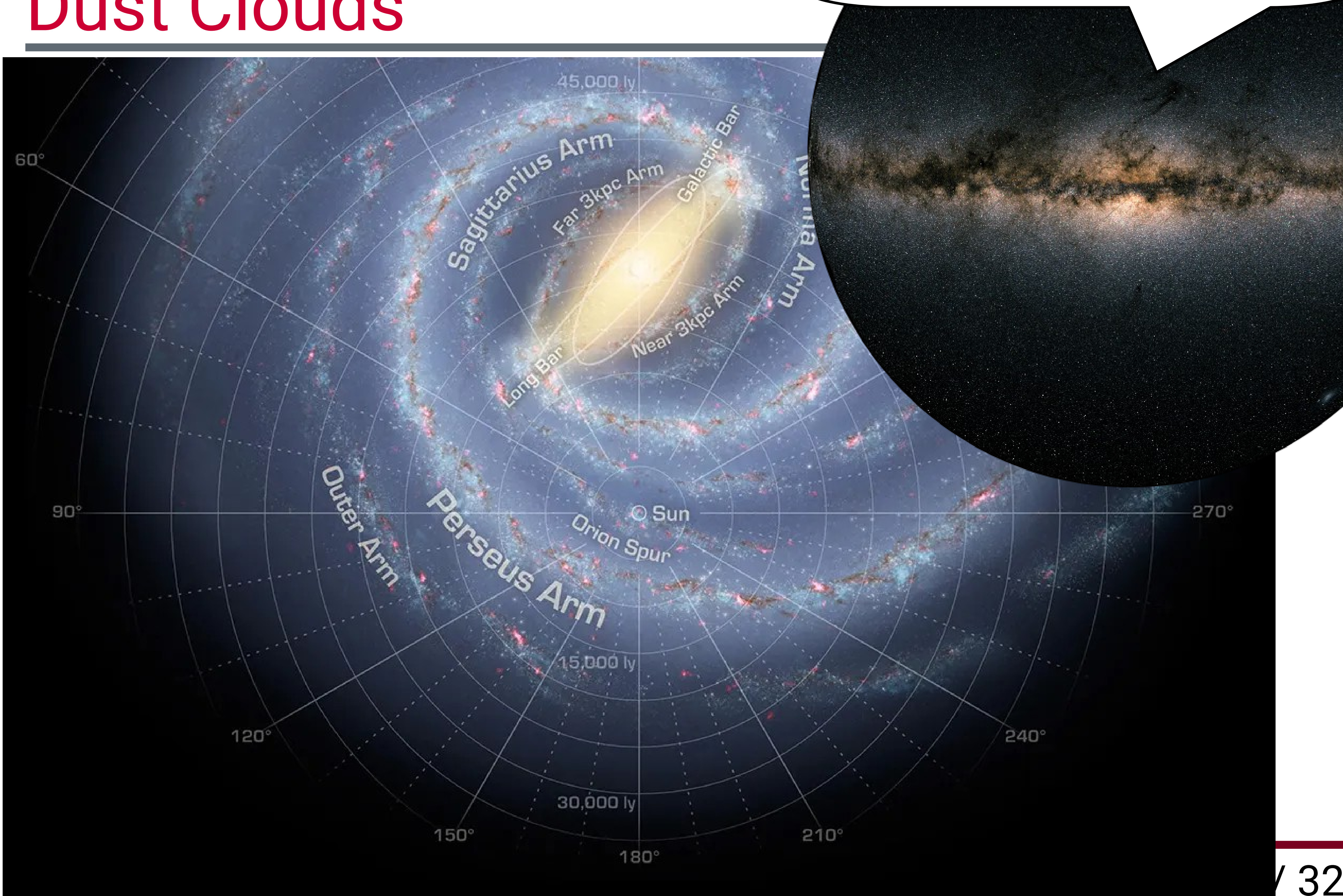
Incompleteness in Spacial Coverage



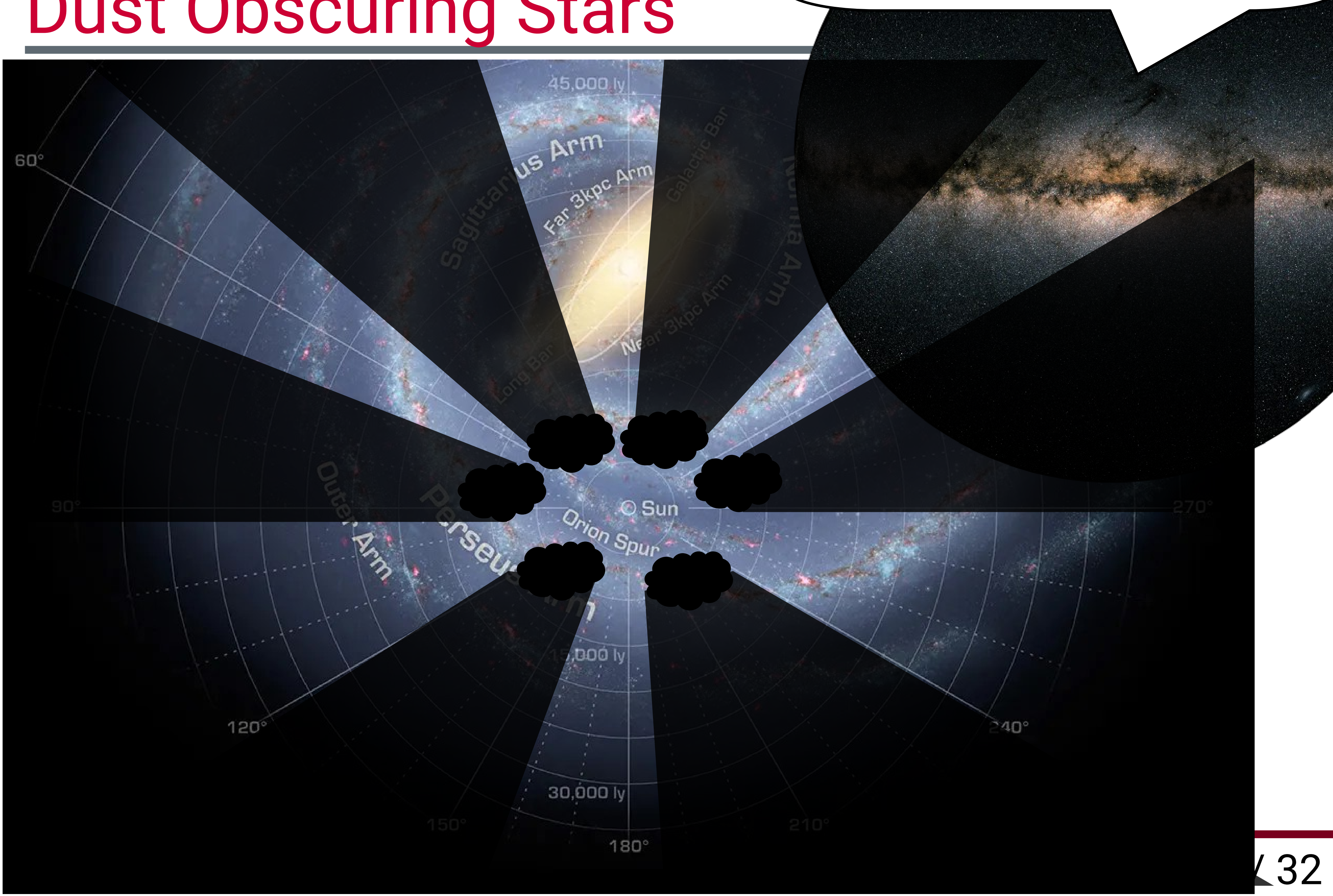
Not fully covered!

Dust Clouds

Intergalactic dust cloud obscuring light from stars!

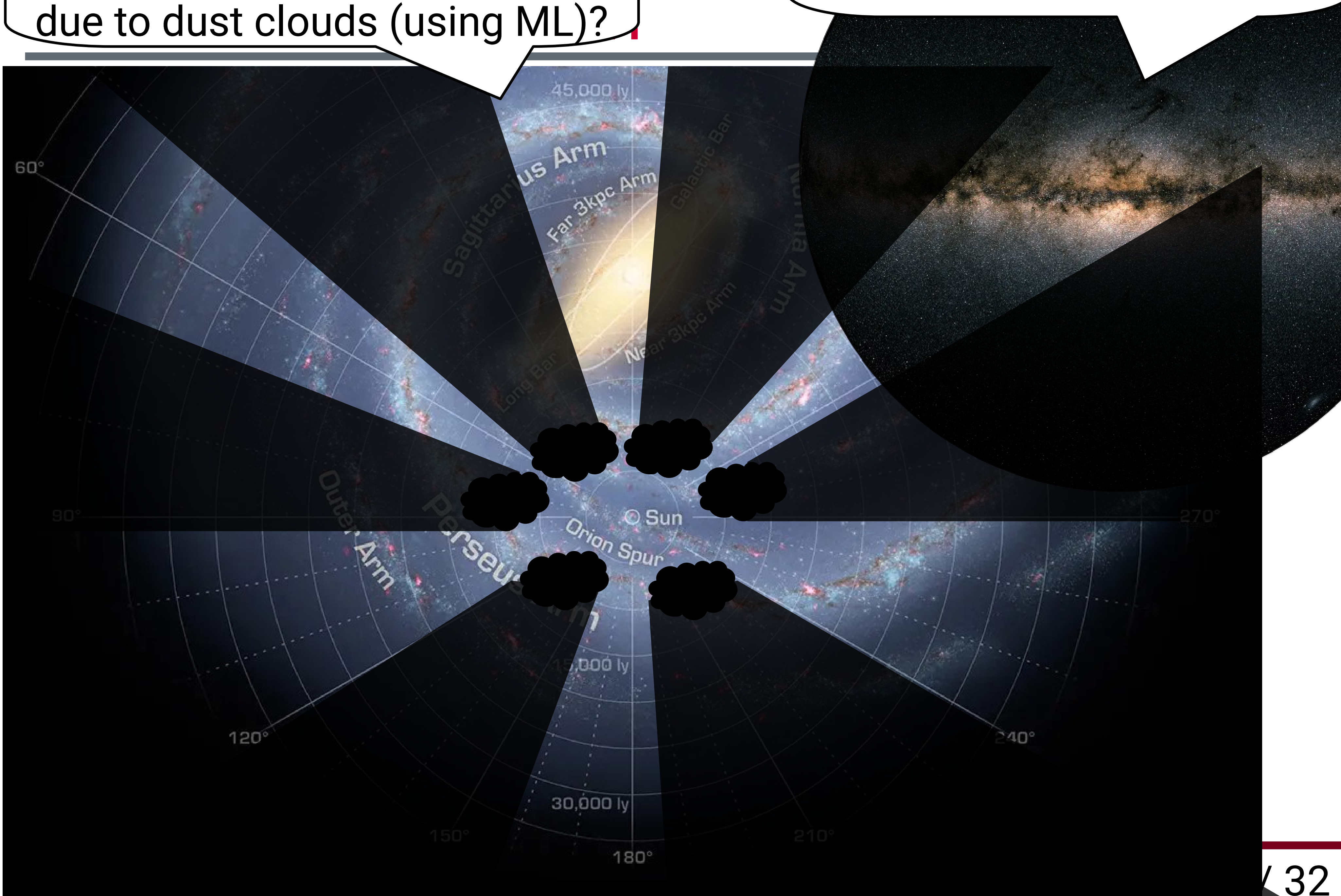


Dust Obscuring Stars



How could we overcome this data **incompleteness** due to dust clouds (using ML)?

Intergalactic dust cloud obscuring light from stars!



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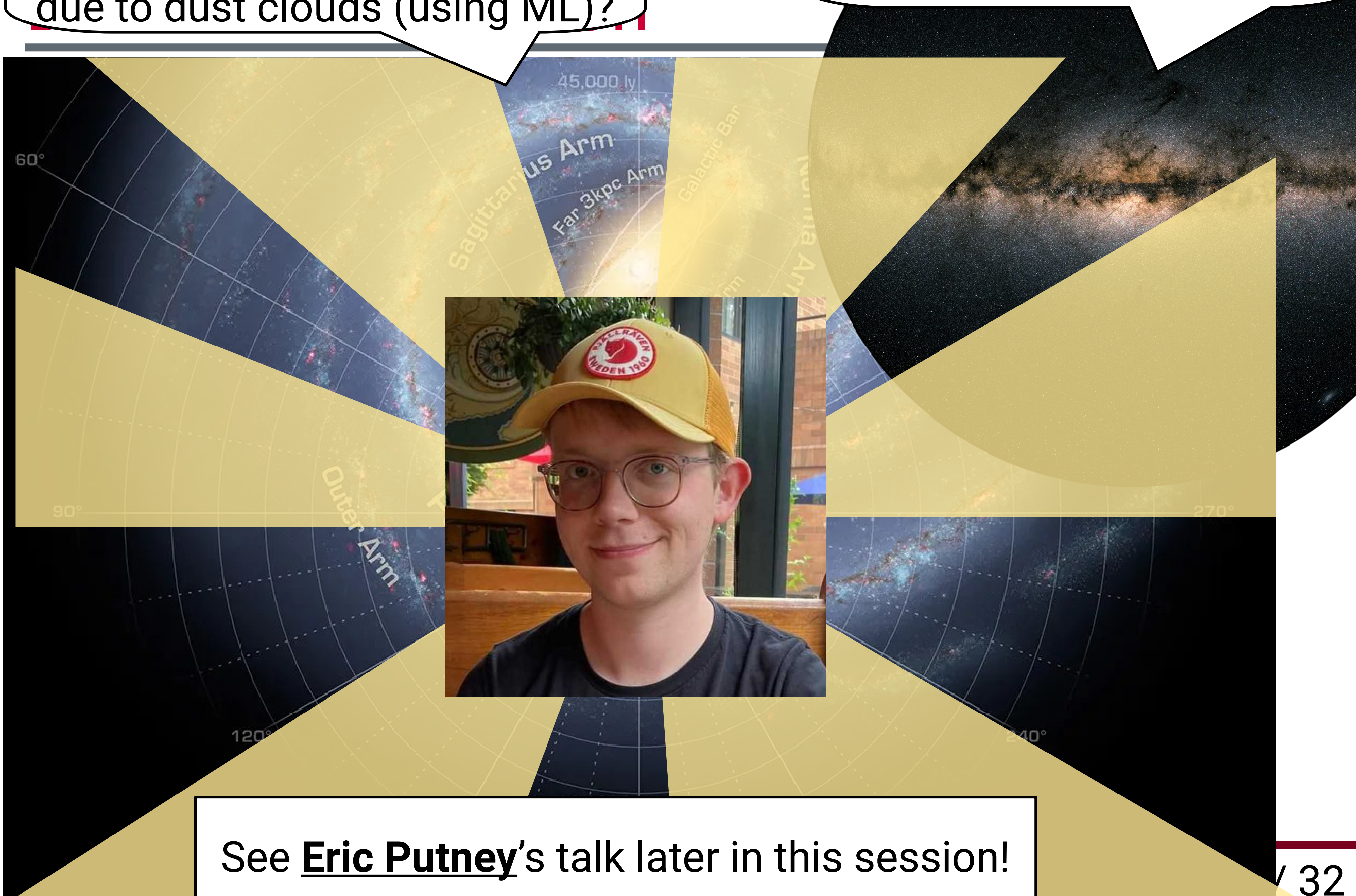
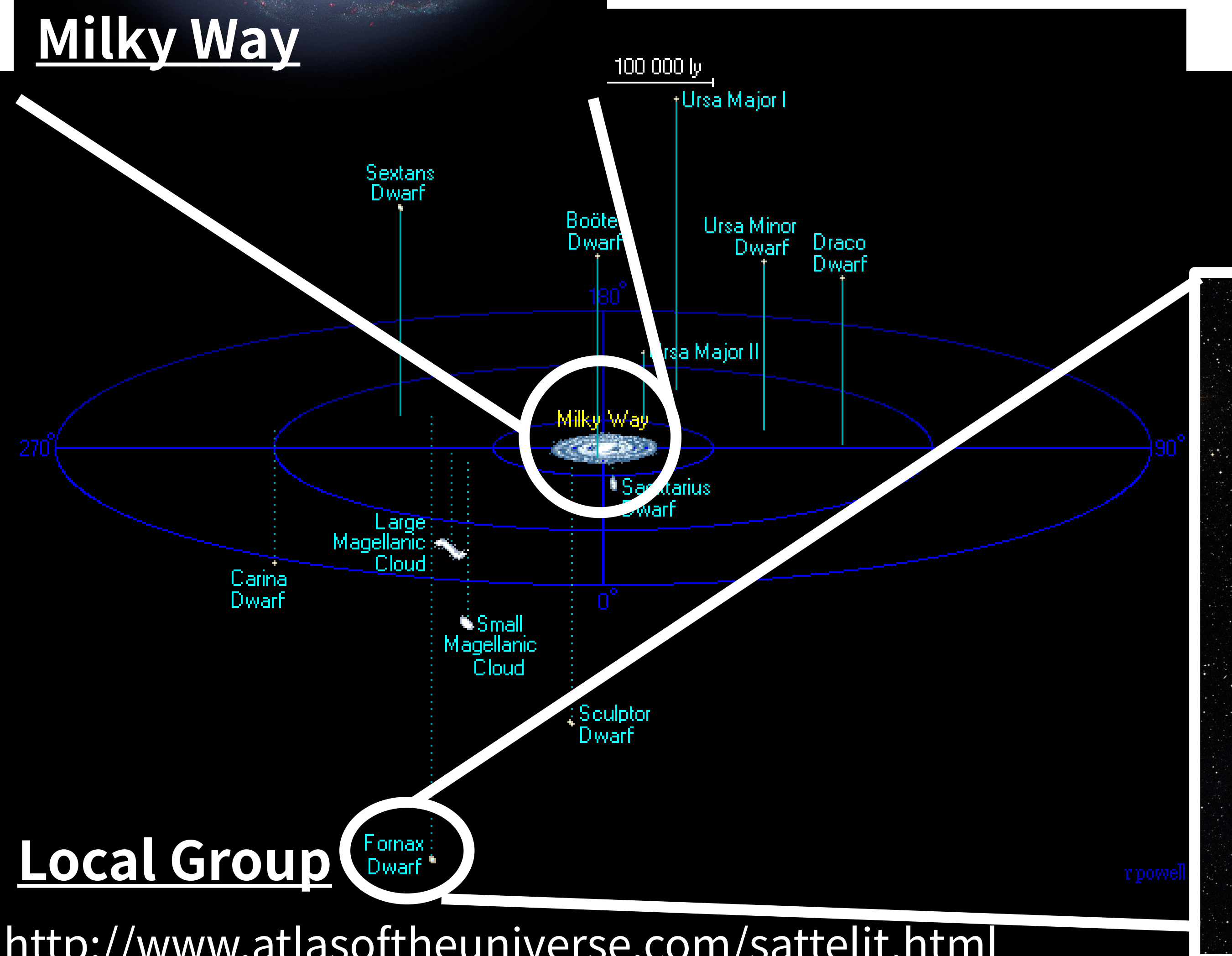


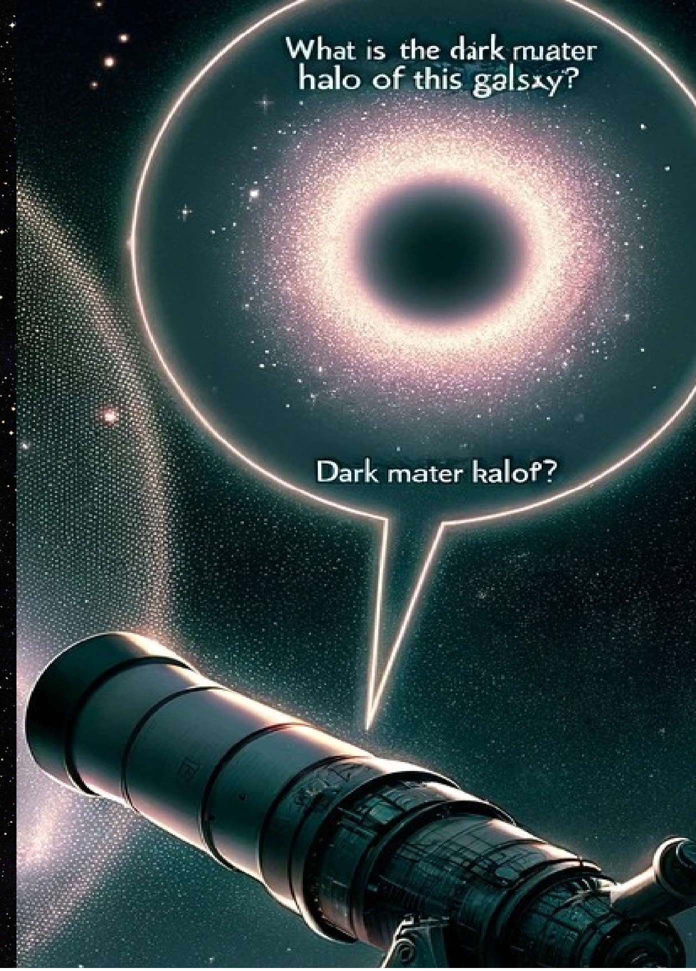
See **Eric Putney**'s talk later in this session!

Is the technique easily applicable to any of **distant dust-free galaxies, like dwarf spheroidal galaxy?**

Answer: both yes and no



http://www.atlasoftheuniverse.com/sattelit.html



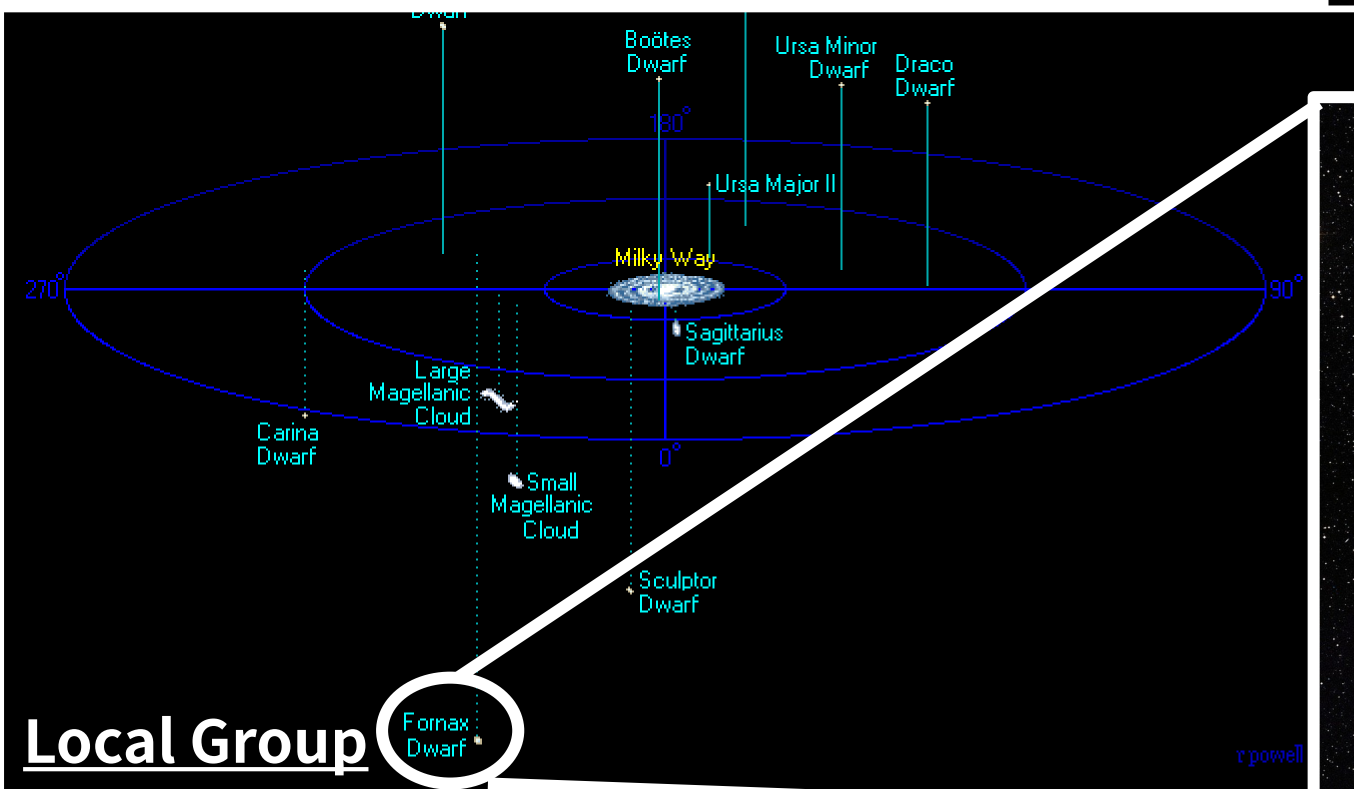
Model-Independent Spherical Jeans Analysis using Equivariant Continuous Normalizing Flows

Collaboration with
K. Hayashi (Ichinoseki College), S. Horigome (IPMU),
M. M. Nojiri (KEK), S. Matsumoto (IPMU),

Dwarf Spheroidal Galaxy?

- A round and faint satellite galaxy, orbiting the Milky Way.
- Almost no gas and dust obscuring stars. Whole galaxy is clearly visible.

Milky Way from Ground



Local Group

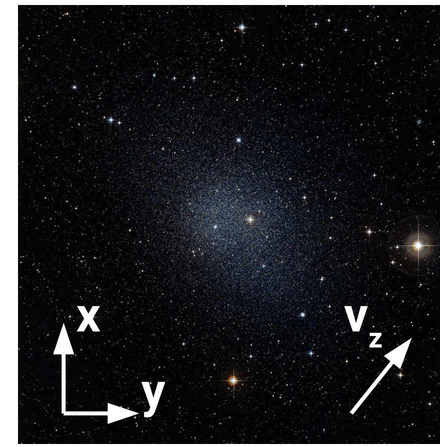
Fornax Dwarf

http://www.atlasoftheuniverse.com/sattelit.html

Challenges in Analyzing dSphs

- Faint galaxy
  → less number of observed stars O[100] ~ O[1000]

- Available kinematic information is **limited**!
  - Position of stars on the sky (x, y) (phot.)
  - ~~Distance to the stars (z)~~
  - ~~Proper motion of stars on the sky (v_x, v_y)~~
  - Radial velocity (v_z) (spec.)



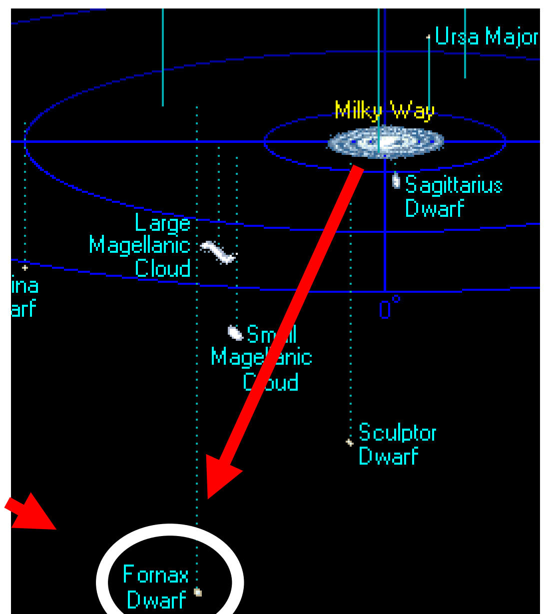
- Phase space density of stars are not accessible, and hence we cannot solve the equation of motion yet.. (Jeans equation)

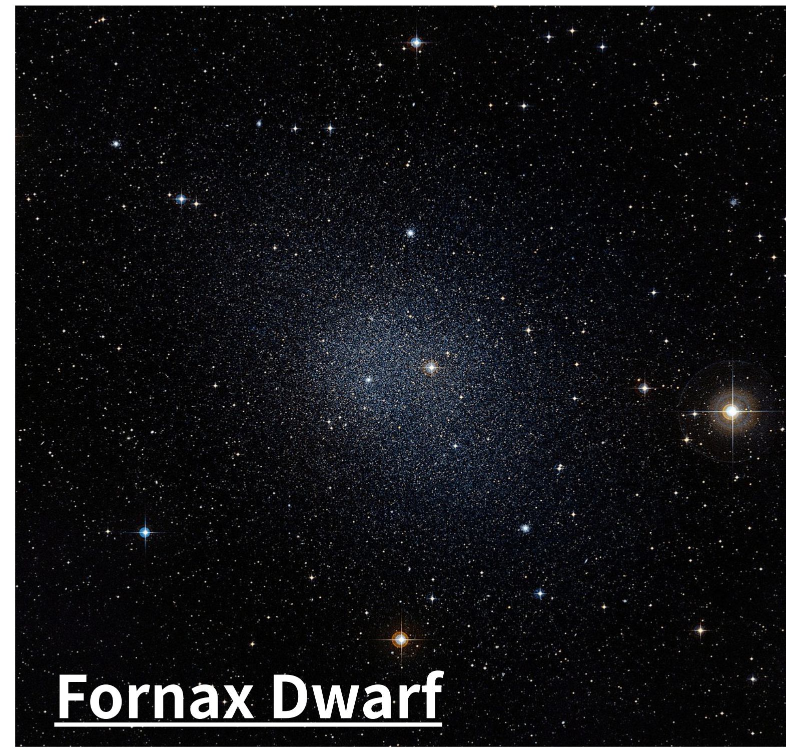
$$\frac{\partial n\langle v_j\rangle}{\partial t} + n\frac{\partial \Phi}{\partial x_j} + n\frac{\partial n\langle v_i v_j\rangle}{\partial x_i} = 0$$

Can we recover the full 6D information somehow?

Radon Transformation

- Can we recover the full 6D information somehow?
  → Yes, if we have a 3D projected snapshot of the dSph from all the direction





- This tomographic reconstruction is possible (e.g. **MRI imaging**), but we only have a snapshot from only one direction…
  → Classic solution: assume **spherical symmetry**.

Toward free-form Spherical Jeans analaysis...

Conventional methods assumes

- **Symmetry assumptions (Spherical symmetry)**

$$\frac{\partial n\sigma_{v_r}^2}{\partial r} + \frac{2\beta_{\rm ani}}{r} n\sigma_{v_r}^2 = -n\frac{\partial \Phi}{\partial r}, \qquad \frac{\partial \Phi(r)}{\partial r} = \frac{GM_<(r)}{r^2}$$

- **Assumed families of stellar and DM halo profiles**

$$n(r) = n_0 \left(\frac{r}{r_*}\right)^{-\gamma_*} \left[1 + \left(\frac{r}{r_*}\right)^{\alpha_*}\right]^{(\gamma_* - \beta_*)/\alpha_*}$$

- **Velocity anisotropy profile (Mass-Anisotropy degeneracy)**

$$\beta_{\rm ani} = 1 - \frac{\sigma_{v_t}^2}{\sigma_{v_r}^2}$$

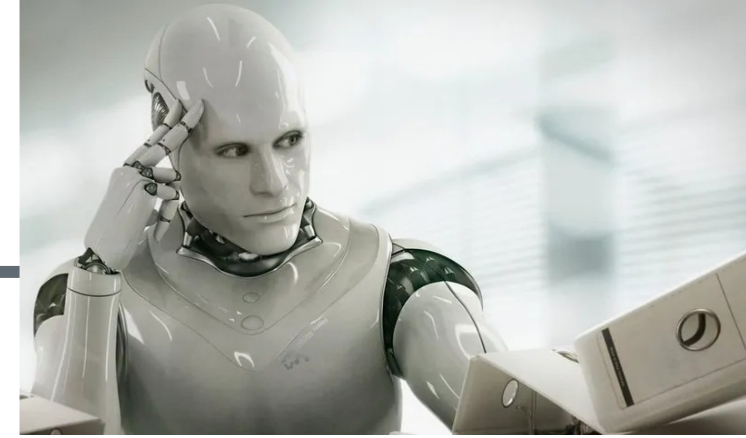
- **Data binning**

Constraining the analysis setup gives more confident results, but if the assumption is differ from the true, result can be less accurate. (Bias-Variance tradeoff)

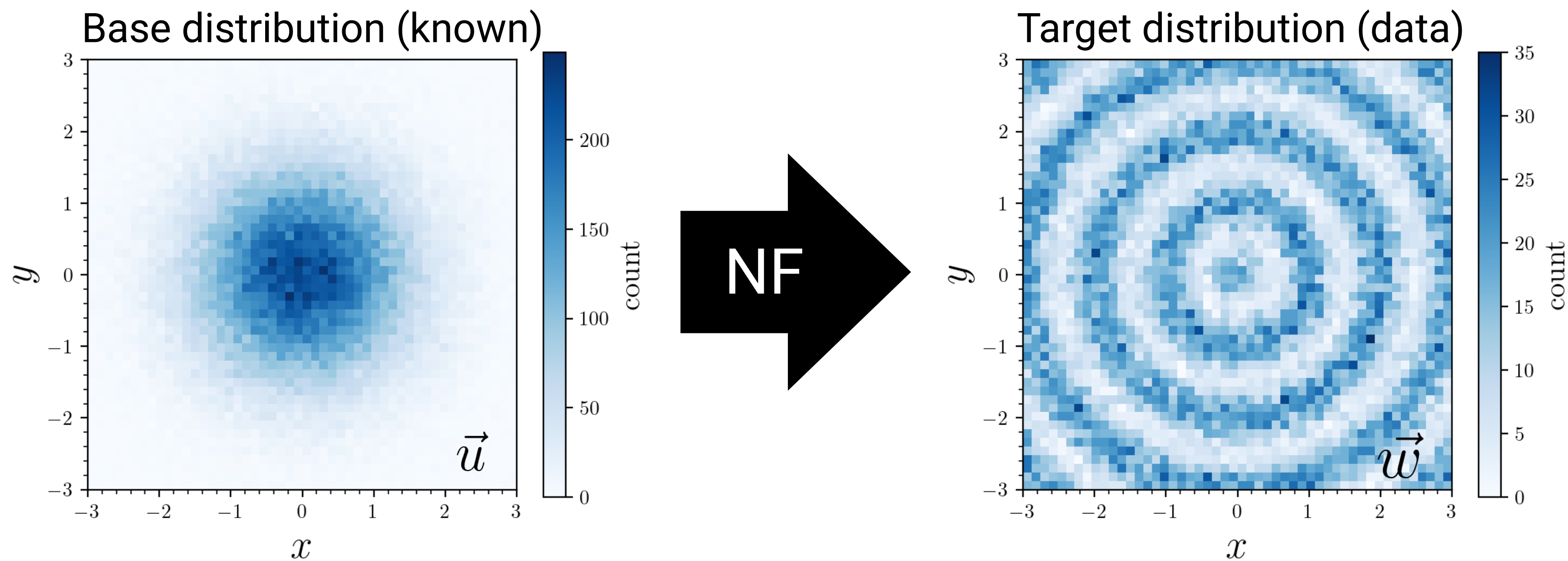
We introduce model-independent, unbinned Jeans analysis using neural density estimator:

→ Normalizing Flows $n(r)$ $\sigma_{v_r}^2(r)$

Normalizing Flows: Neural Density Estimator



<u>Normalizing Flows</u> (NFs) is an artificial neural network that learns a transformation of random variables.



Main idea: if we could find out such transformation, we can use the transformation formula for the density estimation:

$$p_W(\vec{w}) = p_U(\vec{u}) \cdot \left| \frac{d\vec{u}}{d\vec{w}} \right|$$

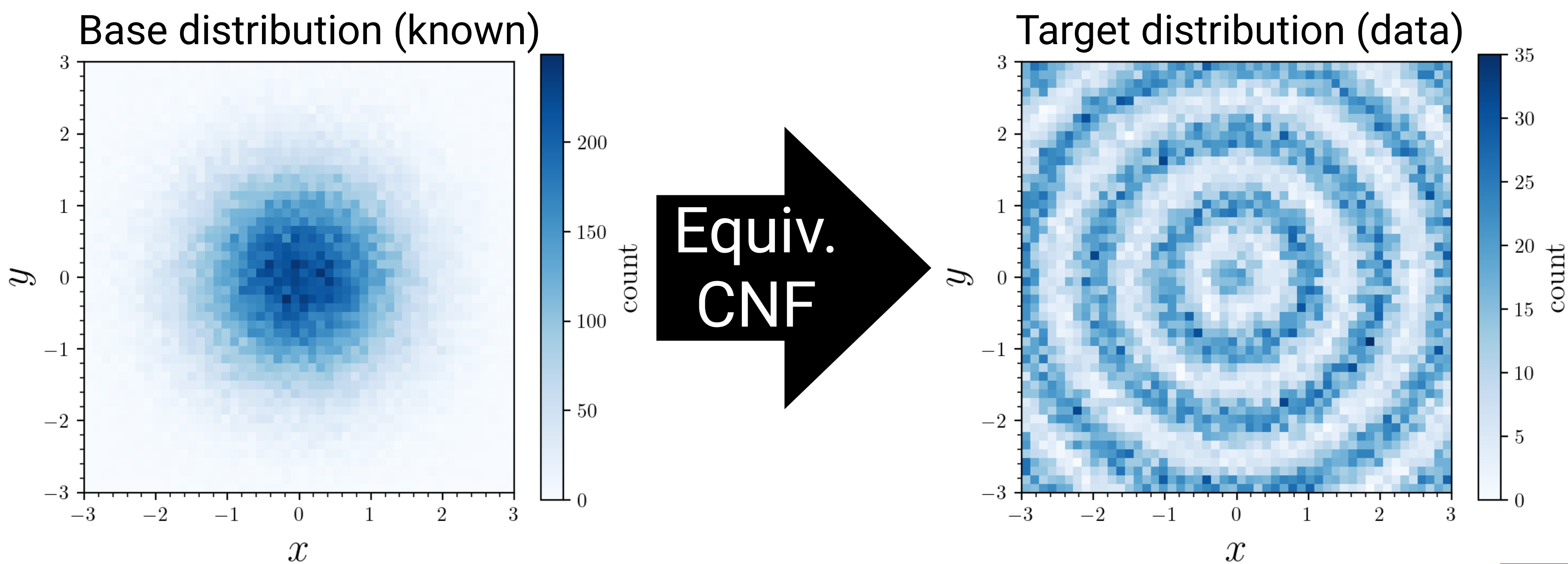
We will use this model for estimating the phase space density f(x,v) from the data.

Equivariant Continous Normalizing Flows

How to model spherically symmetric density using normalizing flows?
→ Use Equivariant Continuous Normalizing Flows!

$$\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t)$$

- Invariant (Gaussian) base distribution
- Equivariant vector field



Cored Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at r << 0)
- **Cuspy** density

ex) plummer sphere:

$$p(r) = \left(1 + \frac{r^2}{r_0^2}\right)^{-5/2}$$

cored density profile

Equivariant CNF for modeling **cored** density profile

$$\frac{d\vec{x}}{dt} = \hat{r} f(\vec{x}, t) \longrightarrow \frac{d\vec{x}}{dt} = \hat{r} \tanh\left(\frac{|\vec{x}|}{r_0}\right) f(\vec{x}, t)$$

Transformation at the origin is suppressed, remaining as Gaussian-shape. → cored density

Cuspy Spherical Density Model

In dSph analysis, we may further constrain the density model as conventional analysis often only consider the following type of densities.

- **Cored** density (constant density at r << 1)
- **Cuspy** density

Equivariant CNF for modeling **cuspy** density profile

ex) NFW profile:

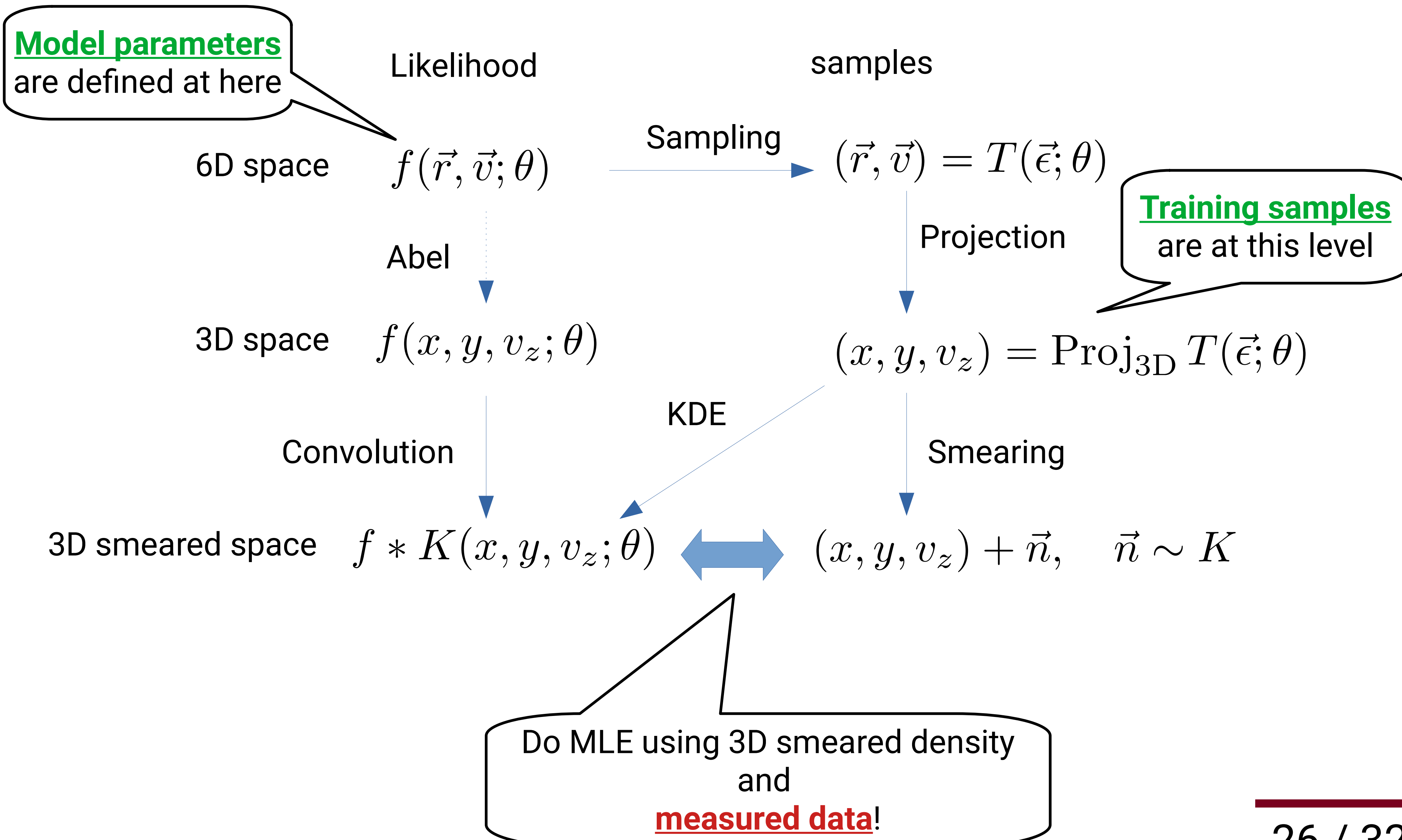
$$p(r) = \left(\frac{r}{r_0}\right)^{-1}\left(1 + \frac{r}{r_0}\right)^{-2} \to \frac{1}{r}$$

Apply power-law transform to radial component

$$|r| \to |r|^{c+1} \qquad \text{Jacobian} \propto r^{-\frac{3c}{1+c}}$$

to **cored** spherical symmetric density model

How to train this model?



Loss Function for Modeling Dwarf Spheroidal Galaxy

- In order to train the normalizing flow with spherical symmetry using limited kinematic information, we minimize the following entropy:

$$\mathcal{L}(\theta) = \int d\vec{w}_\perp \; p * K_h(\vec{w}_\perp) \; \log \hat{p} * K_h(\vec{w}_\perp; \theta)$$

- Importance sampling: N_T training sample (stars) ~ p, N_K noise samples ~ K_h

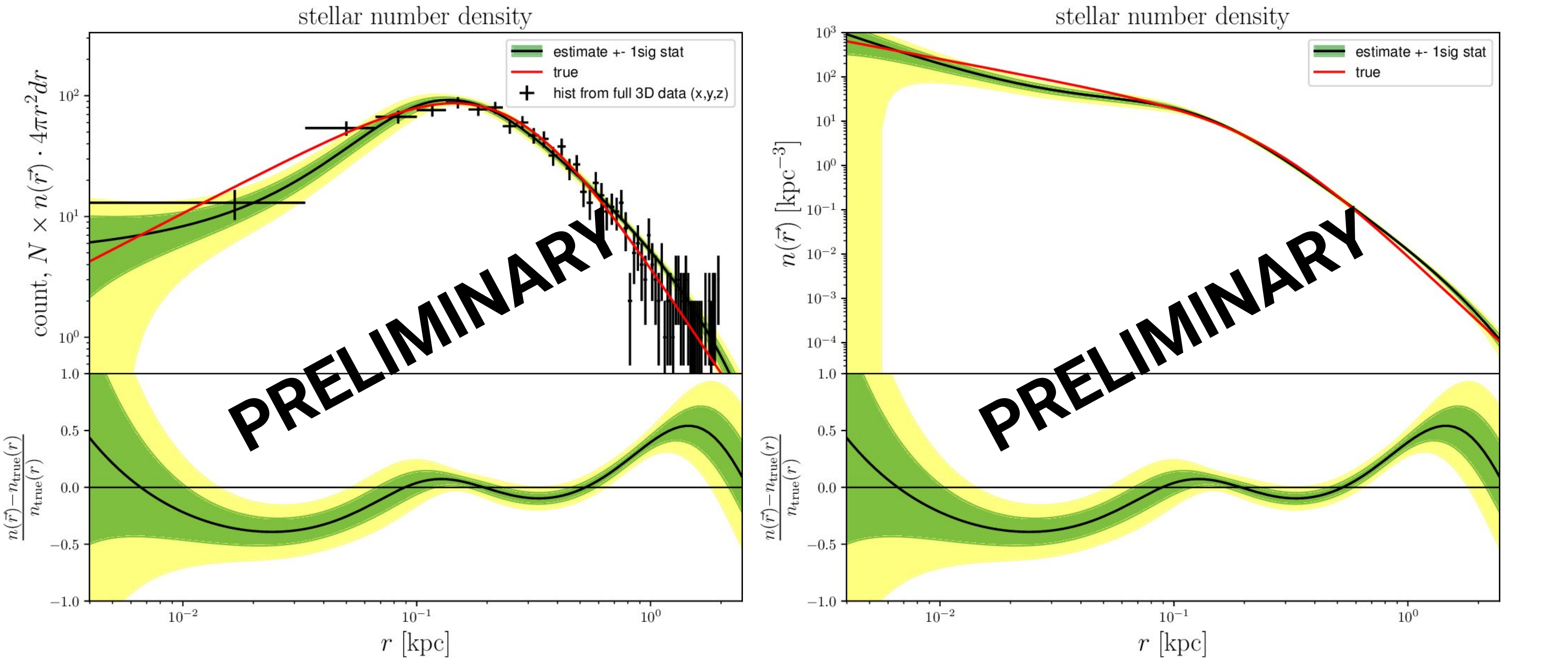
$$\mathcal{L}(\theta) = \frac{1}{N N_K} \sum_{a=1}^{N} \sum_{b=1}^{N_K} \log \hat{p} * K_h(\vec{w}_\perp^{(a)} + \vec{\epsilon}^{(b)}; \theta)$$

- KDE for the smeared likelihood model:
  N_G generated stars from the normalizing flows~ \hat{p}

$$\mathcal{L}(\theta) = \frac{1}{N N_K} \sum_{a=1}^{N} \sum_{b=1}^{N_K} \log \frac{1}{N_G} \sum_{c=1}^{N_G} K_h \left[\vec{w}_\perp^{(a)} + \vec{\epsilon}^{(b)} - \vec{T}(\vec{z}^{(c)}; \theta) \right]$$

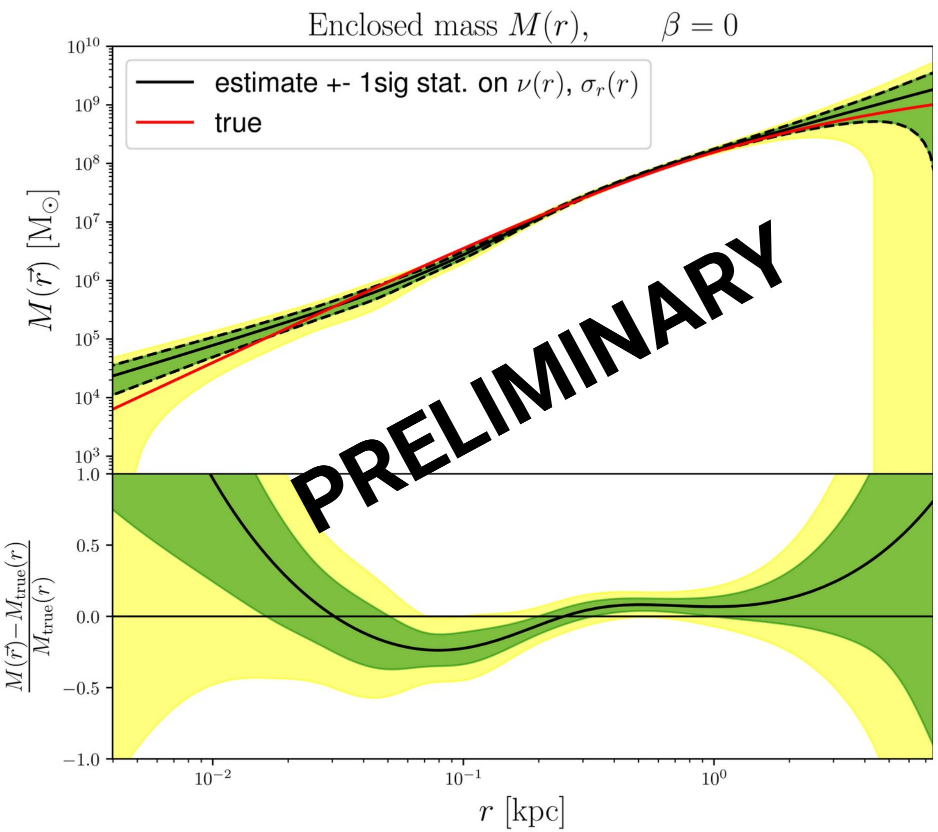
Results: stellar number density

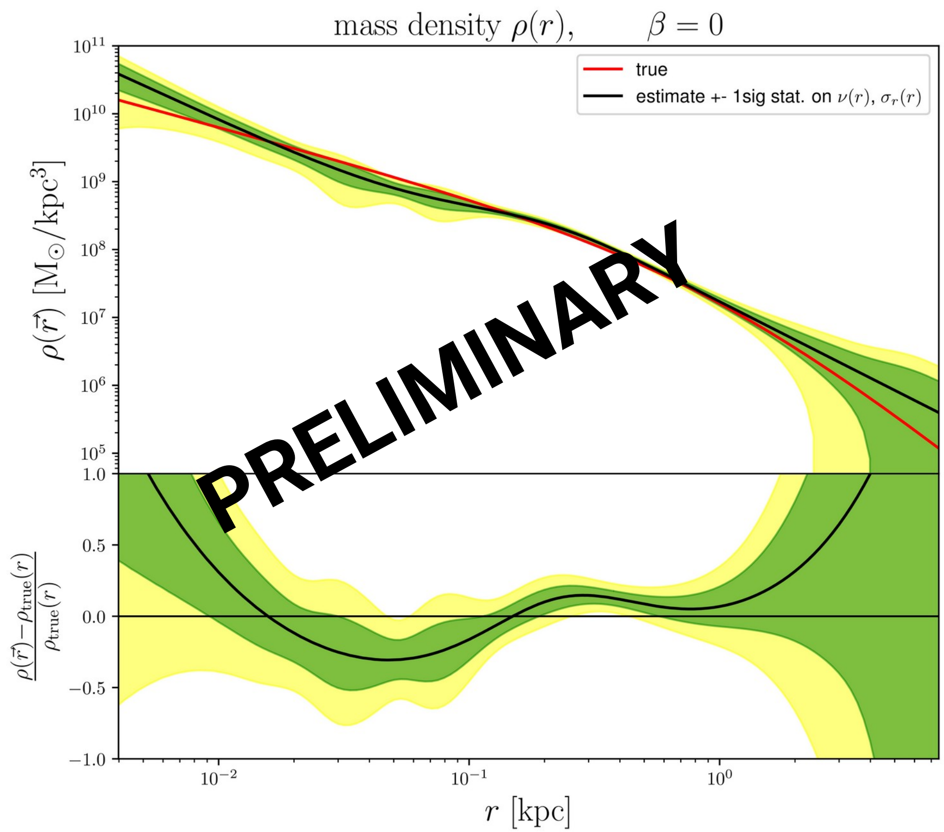
Here we present inferred stellar number density trained on 2D position information (x, y).



Results: dark matter mass density

Here we present inferred mass density calculated from stellar density and velocity dispersion trained on 3D information (x, y, vz).



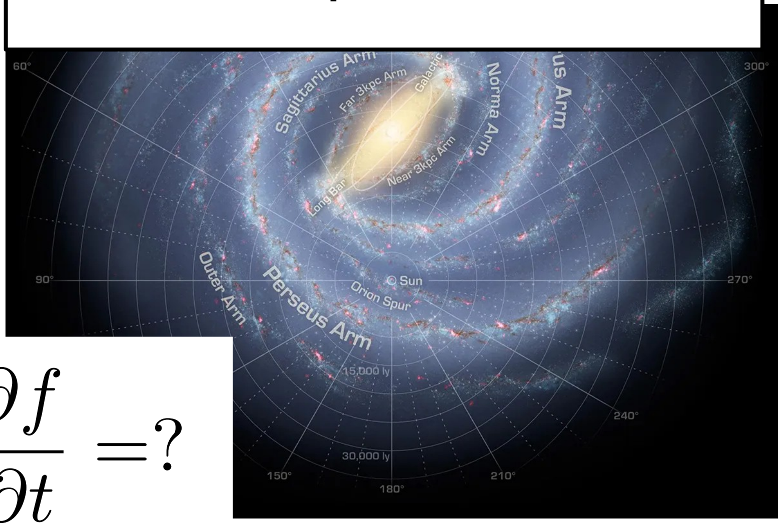


Conclusions

- We introduce a model-independent and unbinned spherical Jeans analysis using **normalizing flows**, a neural density estimator utilizing transformation of random variables.
- We **invented a loss function** for training normalizing flows modeling dSphs only using projected information, without performing Abel transformation.
- Using a mock spherical galaxy from Gaia Challenge dataset, we demonstrated that normalizing flows are capable of estimating **phase-space density** information for required solving Jeans equation.
- To do?:
  - Generalizing the framework to axisymmetric system.
  - Applying our analysis to real dwarf spheroidal galaxies, and estimate the effect to J-factors when the assumptions are relaxed.

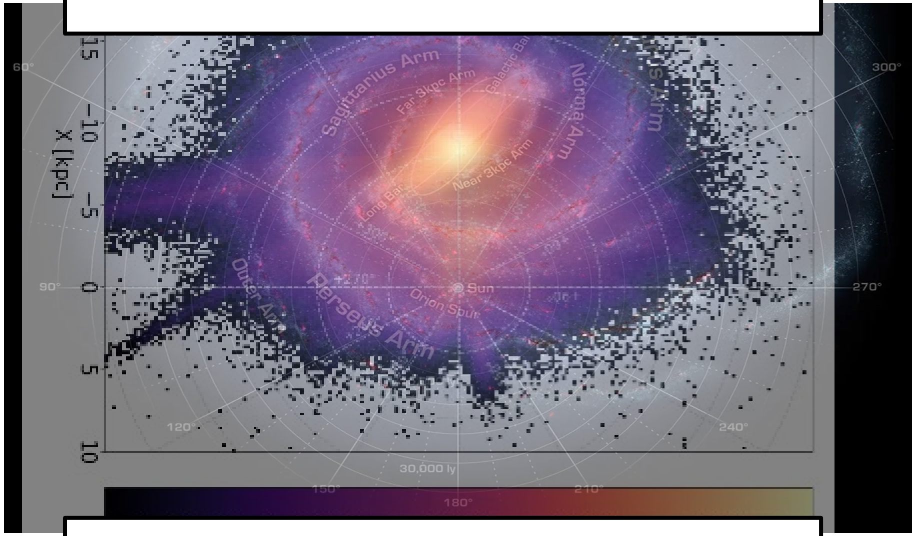
Various challenging incompleteness!

Disequilibrium

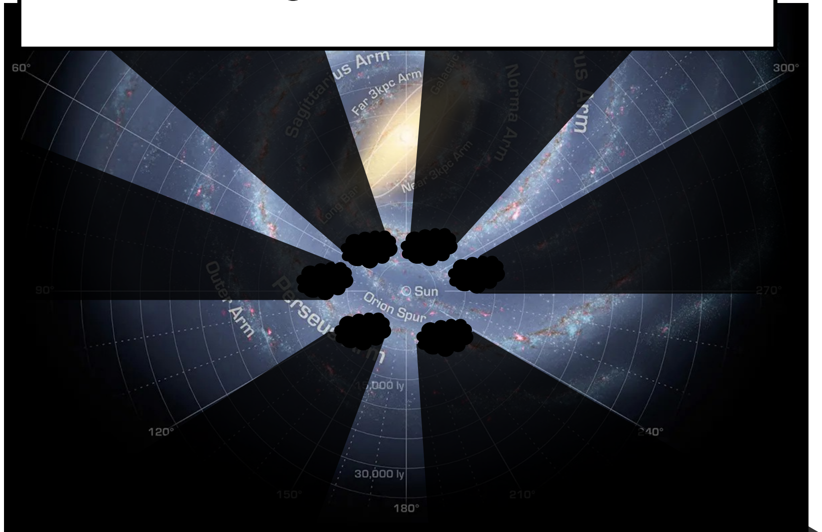


$$\frac{\partial f}{\partial t} = ?$$

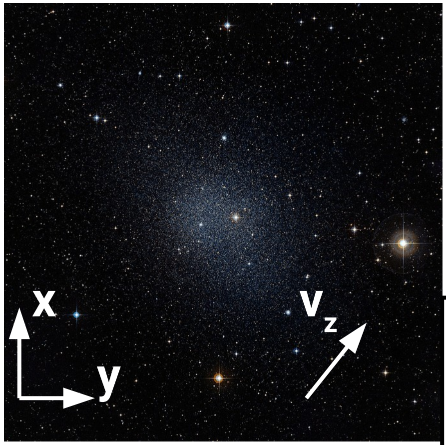
Spatial Incompleteness



Intergalactic Dust



Lack of information



Only 3D info. available, not the full 6D PS info.

More challenges are waiting!

Thank you for listening!

Shameless advertisements

Tianji and I are planning to organize a small (regional) ML4HEP workshop in Korea next Feb.

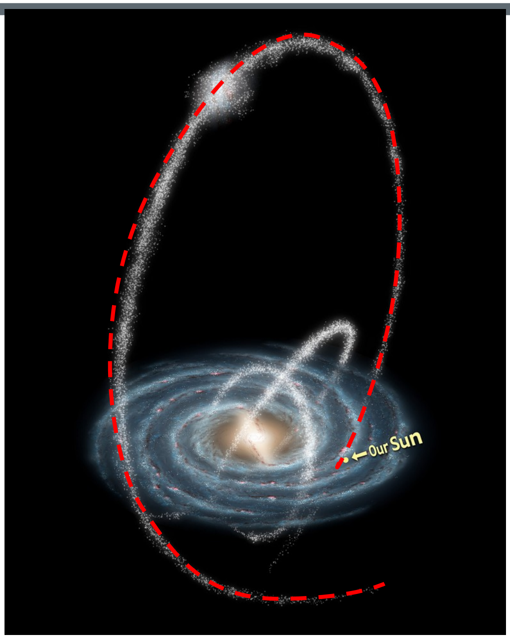
Please contact us at: tianji@slac.stanford.edu, sunghak.lim@rutgers.edu.

IBS has a few postdoc positions opening this year!

Please contact at sunghak.lim@rutgers.edu

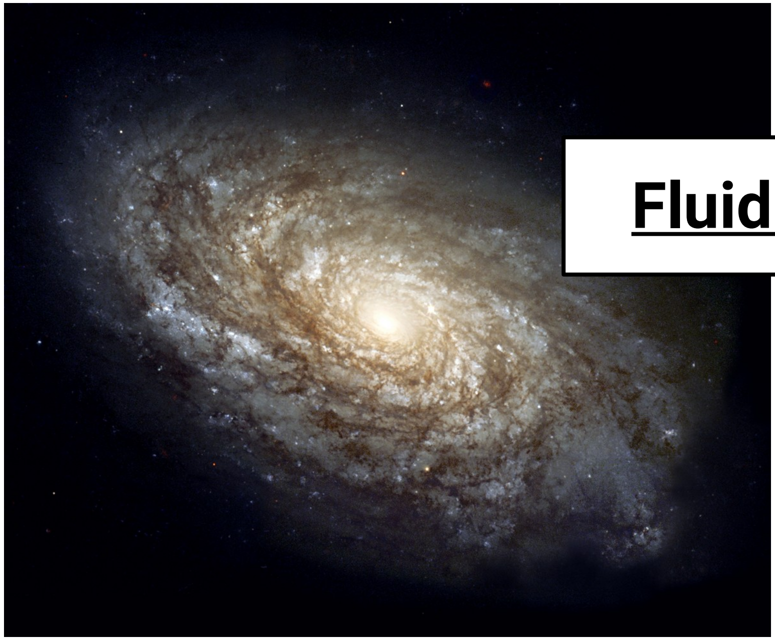
Backups

How we infer mass density? → Gravity!



Orbital mechanics

Gravitational Acceleration and Mass Density



Fluid mechanics

And so on!

Q. Why not just do density estimation on the radial component?

- A: spherical coordinate is useful, but there is a coordinate singularity at r=0. This will introduce numerical instability near the origin.

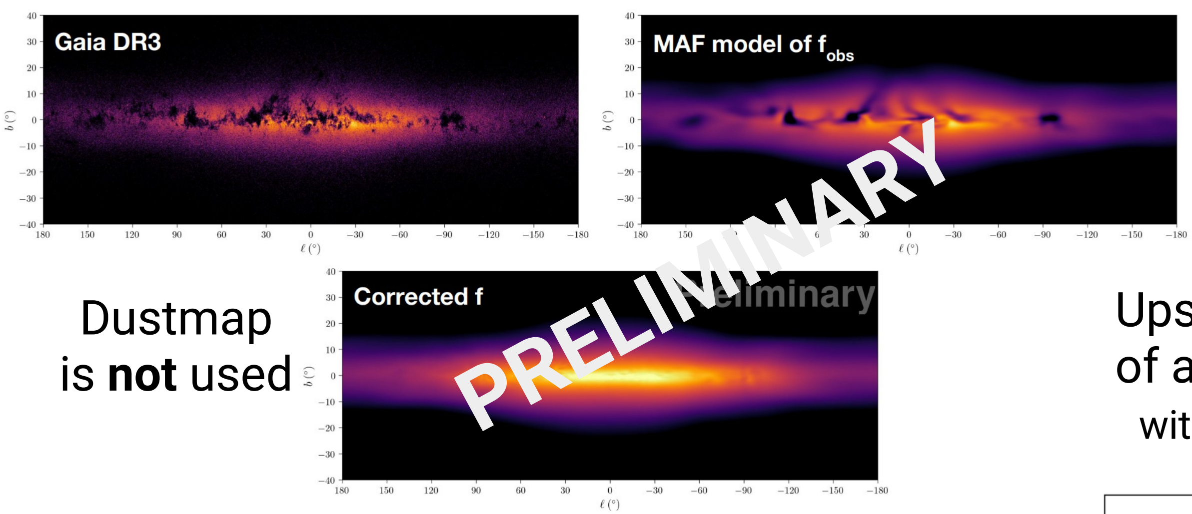
  In dSph analysis, inner DM density profile is important since it tells the characteristic of DM interaction (cored-cuspy halo problem).

  We want to maintain precision at the origin,
  so we stick the Cartesian coordinate, which does not have a singularity at the origin.

Other ongoing projects!

Sweeping the Dust Away -
Removing extinction bias from the Milky Way's potential using Gaia DR3
and unsupervised machine learning
with E. Putney, M. Buckley, D. Shih (Rutgers)

Unbiased Phase Space Density

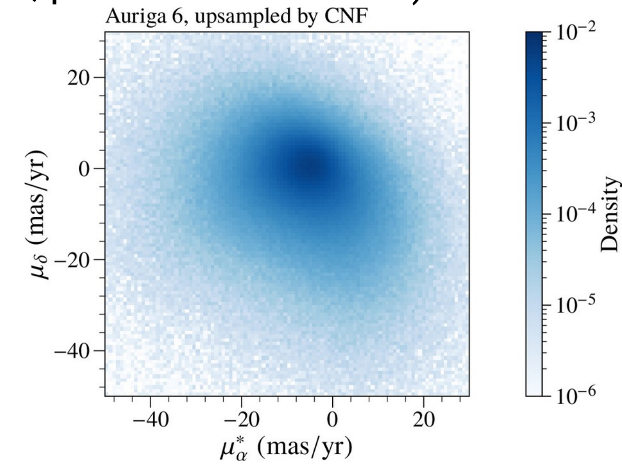


Dustmap is **not** used

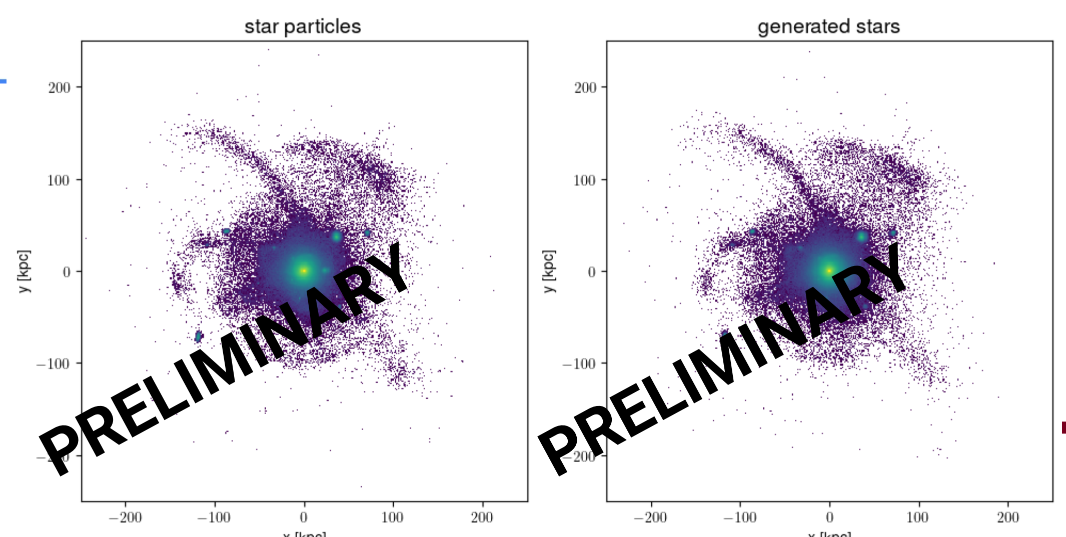
Figure by **Eric Putney**

And so on! :)

Upsampling hydrodynamic simulation of a galaxy in a neighborhood of Solar system
with K. Raman , M. Buckley, D. Shih (Rutgers)
(arXiv: 2211.11765, published in MNRAS)



Upsampling hydrodynamic simulation of a full galaxy
with A. Brooks, M. Buckley, C. Riggs, D. Shih (Rutgers)

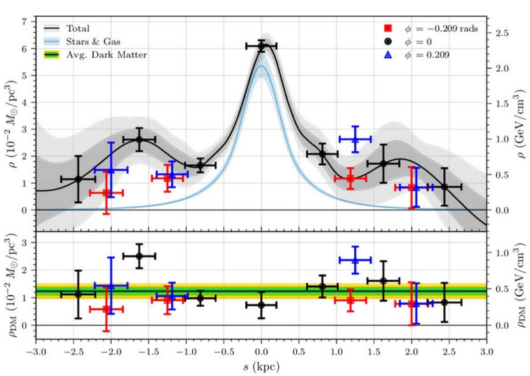


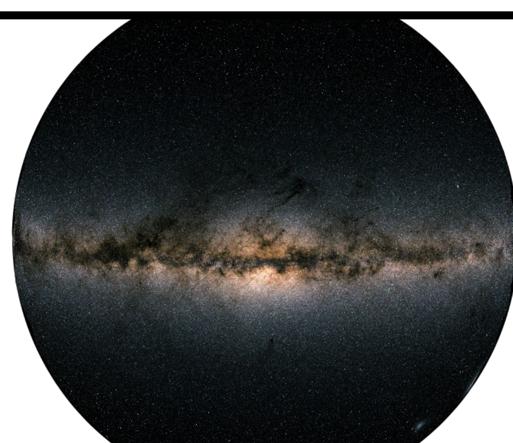
Future Timeline

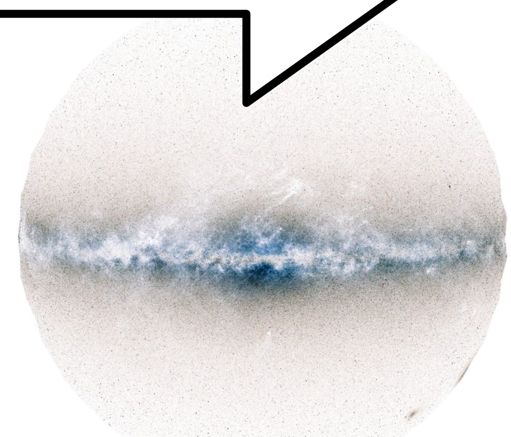
2023	2024	2025	2026

Gaia DR3

Gaia DR4, SuperPFS, and so on!







Understanding DM in dust-free region (halo, solar neighborhood)

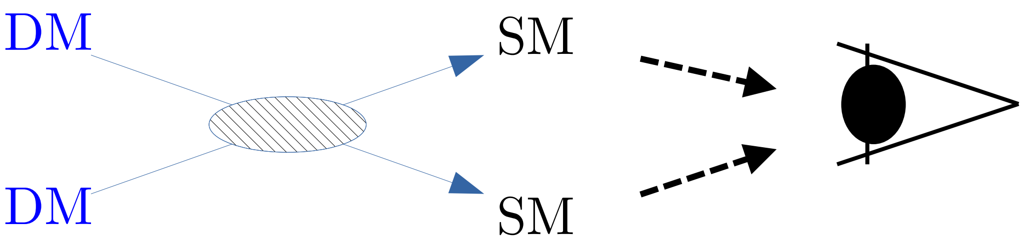
Understanding DM in dusty region (disk)

Publishing Full DM map of the Milky Way

More precision and new opportunities!

Why Dwarf Spheroidal Galaxy (dSph) is an Interesting Object for DM study?

- No star formation activity, consisting of old stars.
  → kinetic equilibrium dynamics based approach is viable approach for describing the system and inferring DM density.
- Low stellar density, mass density of stars are not sufficient to hold them together.
  → DM dominated system. Stars are tracing gravitational potential of DM halo. Potential of stars are negligible.
- No baryonic activity, great object for DM direct detection



Good system for understanding DM interaction!

Benchmark on Gaia Challenge Dataset

Stellar number density profile: variant of Plummer profile

$$n(r) = n_0 \left(\frac{r}{r_*}\right)^{-\gamma_*} \left[1 + \left(\frac{r}{r_*}\right)^{\alpha_*}\right]^{(\gamma_* - \beta_*)/\alpha_*}$$

$$(\alpha_*, \beta_*, \gamma_*) = (2, 5, 1)$$

$$r_* = 0.25 \text{ kpc}$$

Dark matter halo mass density: cuspy profile

$$\rho_{\text{DM}}(r) = \rho_0 \left(\frac{r}{r_{\text{DM}}}\right)^{-\gamma_{\text{DM}}} \left[1 + \left(\frac{r}{r_{\text{DM}}}\right)^{\alpha_{\text{DM}}}\right]^{(\gamma_{\text{DM}} - \beta_{\text{DM}})/\alpha_{\text{DM}}}$$

$$(\alpha_{\text{DM}}, \beta_{\text{DM}}, \gamma_{\text{DM}}) = (1, 3, 1)$$

$$r_{\text{DM}} = 1.0 \text{ kpc} \qquad \rho_0 = 6.4 \times 10^7 \ M_\odot/\text{kpc}^3$$

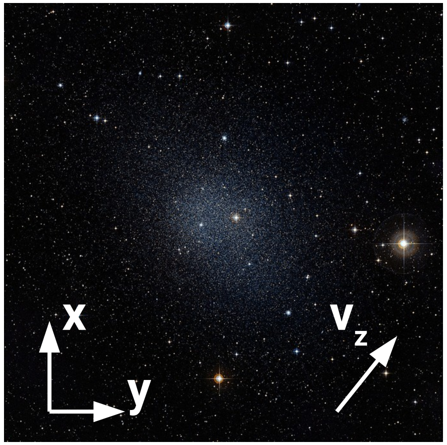
Velocity anisotropy: Isotropic, we provided the correct anisotropy profile when solving the Jeans equation.

$$\beta_{\text{ani}} = 0$$

Dataset statistics: 1,000 stars

Systematic Uncertainties on DM distribution measurement of dSph..

But because stars in dSph are distant objects, available information is often limited to:
- celestial coordinate, and radial velocity
- distance and proper motion is not available



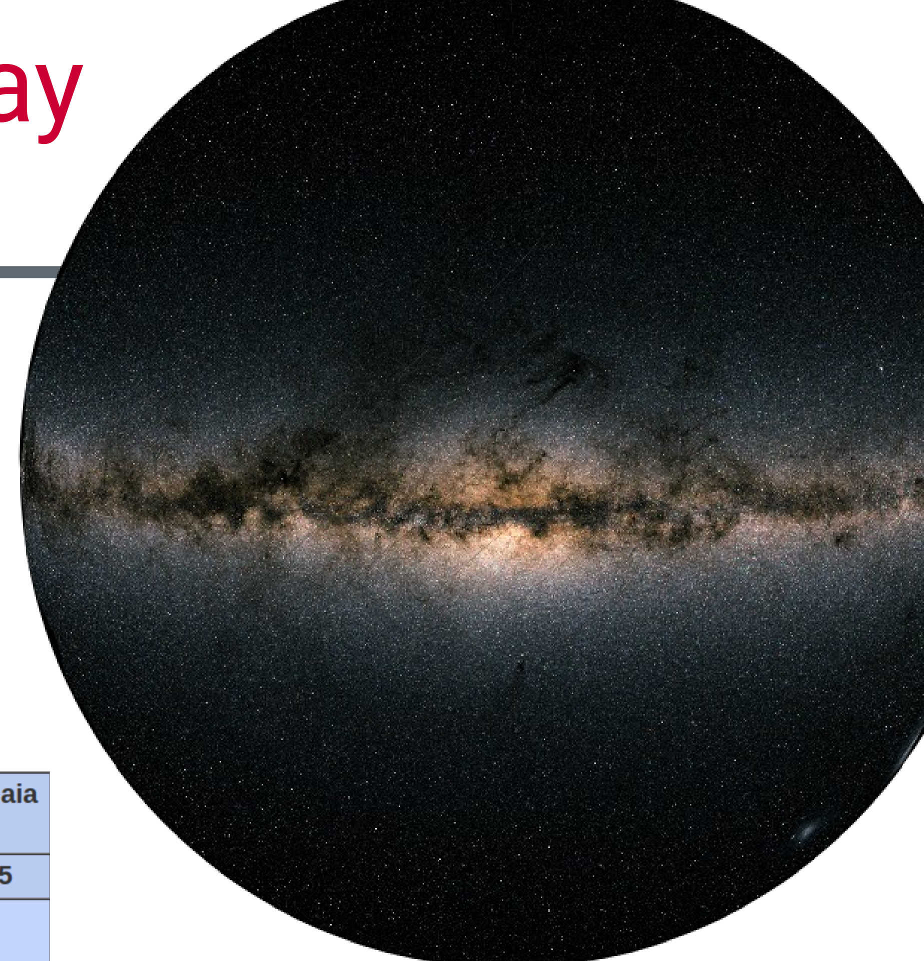
$$(x, y, z, v_x, v_y, v_z) \rightarrow (x, y, v_z)$$

We need full 6D information in order to solve the equation of motion (Jeans equation)

$$\frac{\partial n \langle v_j \rangle}{\partial t} + n \frac{\partial \Phi}{\partial x_j} + n \frac{\partial n \langle v_i v_j \rangle}{\partial x_i} = 0$$

Conventional methods introduce in order to simplify the problem and make it solvable only with limited information
- **Symmetry assumptions (Spherical symmetry)**
- Assumed families of stellar and DM halo profiles
- Velocity anisotropy profile (Mass-Anisotropy degeneracy)
- Data binning
- ...

A Snapshot of Milky Way from Gaia



Recently, Gaia mission from European Space Agency (ESA) released a new catalog containing very detailed measurement of stars in the Milky Way that can be used for various physics analysis.

	# sources in Gaia DR3	# sources in Gaia DR2
Total number of sources	**1,811,709,771**	**1,692,919,135**
	Gaia Early Data Release 3	
Number of sources with full astrometry	1,467,744,818	1,331,909,727
Number of 5-parameter sources	585,416,709	
Number of 6-parameter sources	882,328,109	
Number of 2-parameter sources	343,964,953	361,009,408
Gaia-CRF sources	1,614,173	556,869
Sources with mean G magnitude	1,806,254,432	1,692,919,135
Sources with mean G_{BP}-band photometry	1,542,033,472	1,381,964,755
Sour	1,554,997,939	1,383,551,713
	New in Gaia Data Release 3	Gaia DR2
Sources with radial velocities	33,812,183	7,224,631
Sources with mean G_{RVS}-band magnitudes	32,232,187	-
Sources with rotational velocities	3,524,677	-

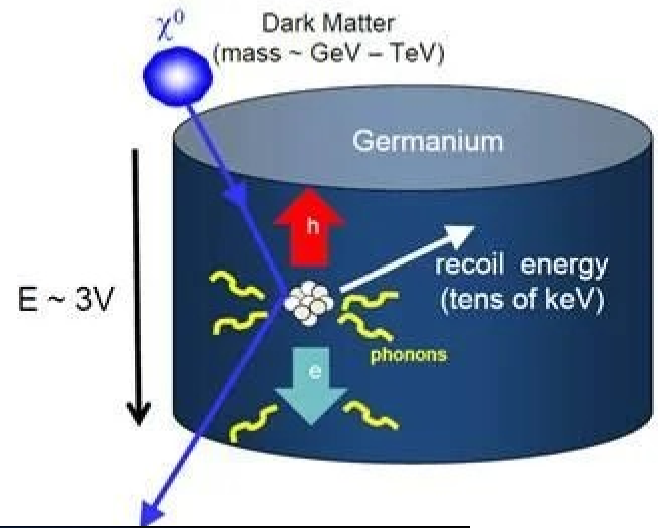
# of stars with full kinematic information

We could use this dataset to understand structure of the **Milky Way** very precisely.

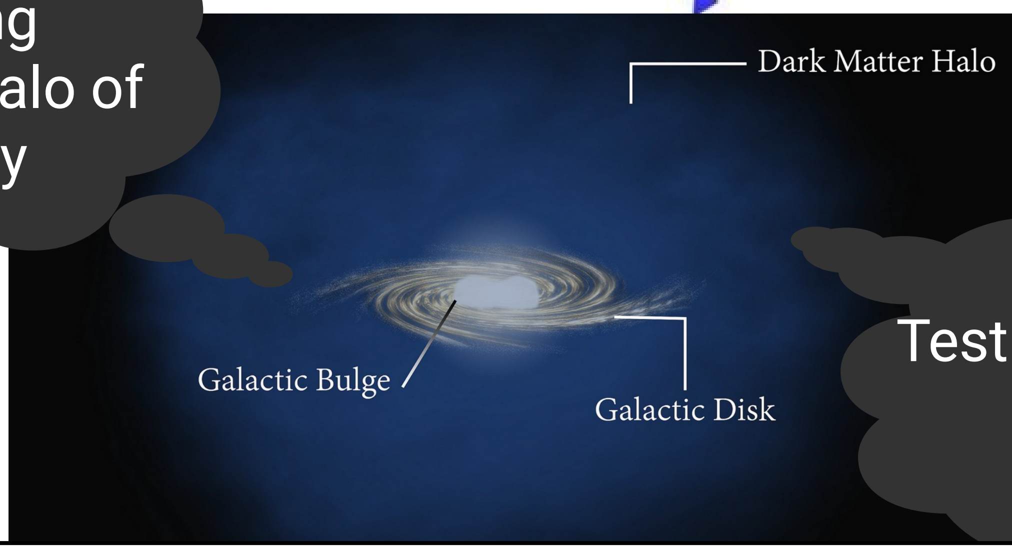
Image from https://gea.esac.esa.int/archive/visualization/

Why understanding galactic dark matter is important?

Inputs to Direct Detection experiments



Understanding the dark matter halo of the Milky Way

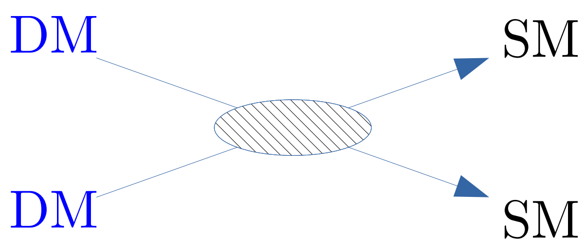


Testing modified gravity solutions?

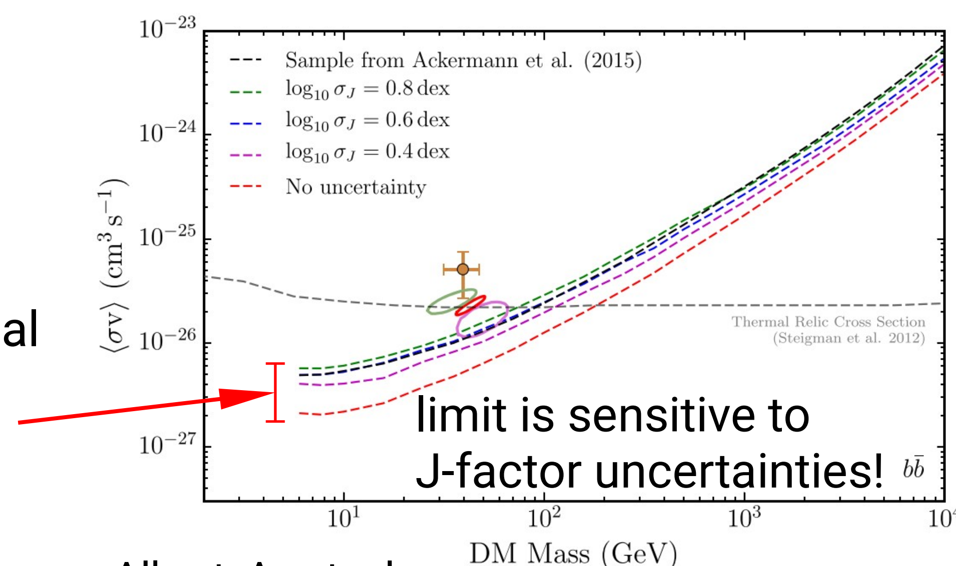
Of course, there have been huge efforts on measuring DM density using various techniques...

Dwarf Spheroidal Galaxy and Dark matter indirect detection

- Dwarf Spehroidal Galaxy (dSph):
  a small, faint satellite galaxy with little dust, and old stellar population.
- Great object for studying annihilating dark matter

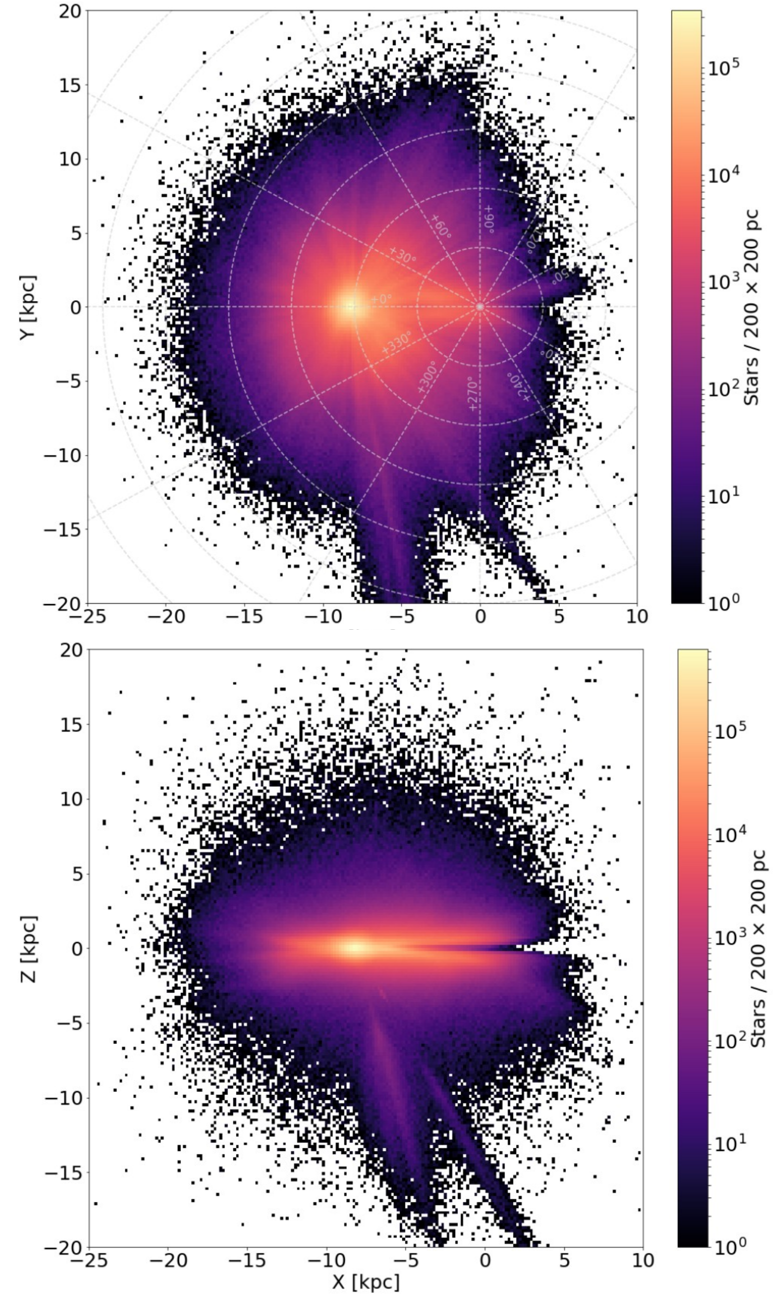


  - old stellar population in equilibrium
    → Gravity field can be obtained by solving equations of motion (Boltzmann/Jeans)
  - less stars and gas, DM dominated system:
    → Mass density can be directly interpreted as DM density.
- Photon flux from DM anhilation is proportional to DM density squared (J-factor), so understanding the DM distribution and its uncertainties are important for the measurment.



Albert, A., et. al., Astrophys. J. 834, 110 (2017).

Milky Way vs. Distant Galaxy



(+) Stars are closer, we can observe full kinematics in high precision.
(-) Stars with full kinematics info. are limited to nearby stars.

Milky Way vs. Distant Galaxy

(+) Stars are closer, we can observe full kinematics in high precision.
(-) Stars with full kinematics info. are limited to nearby stars.

(+) Whole galaxy is visible
(-) Only limited kinematic information is available:
- position on the sky
- radial velocity

Results using wrong anisotropy

Here we present inferred mass density calculated from
stellar density and velocity dispersion trained on 3D information (x, y, vz).