JET TAGGING WITH LORENTZ-EQUIVARIANT GEOMETRIC ALGEBRA TRANSFORMERS

Víctor Bresó Pla

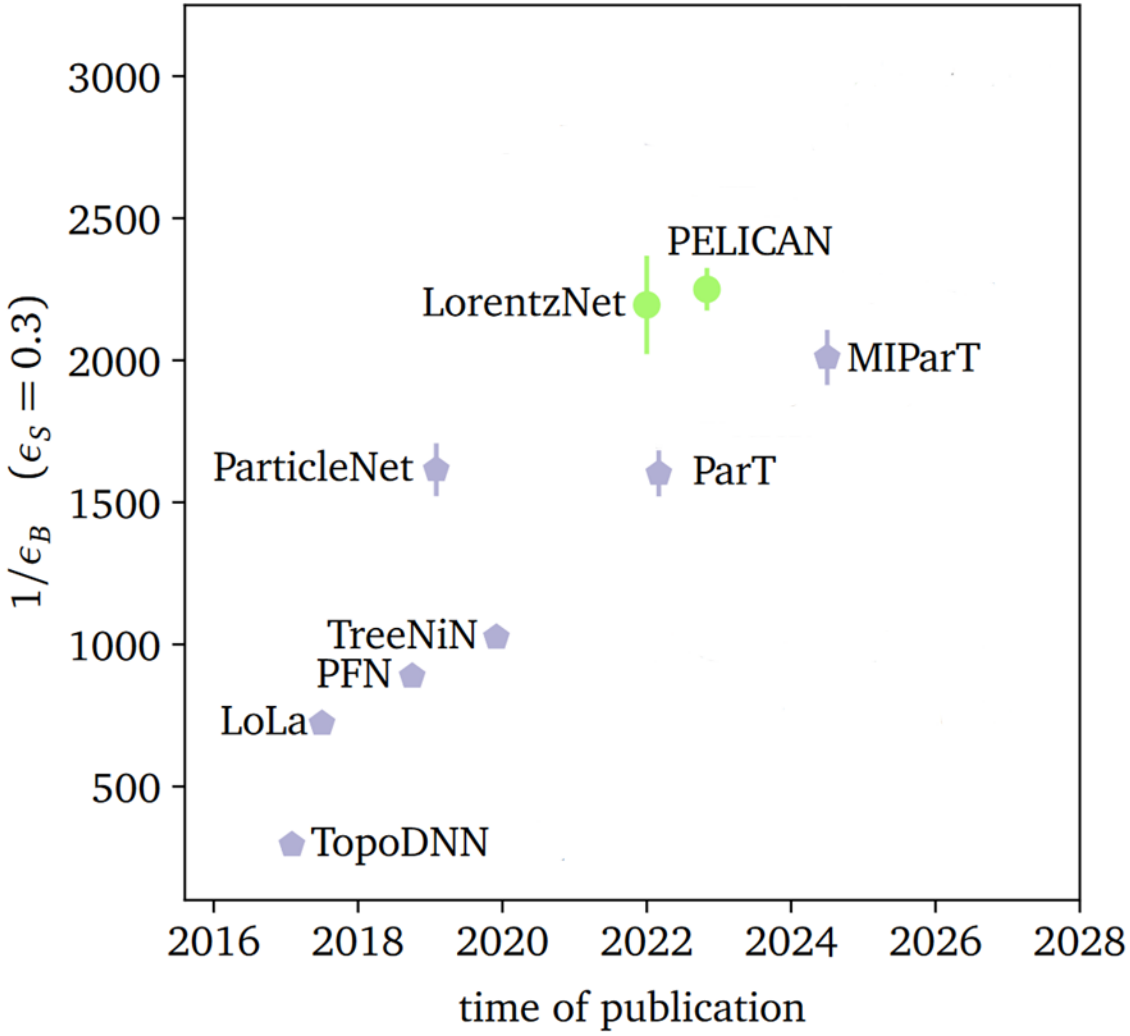
In collaboration with Jonas Spinner, Johann Brehmer, Pim de Haan, Tilman Plehn, Huilin Qu & Jesse Thaler

arXiv:2405.14806 [physics.data-an]

arXiv:2411.00104 [hep-ph, hep-ex]

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

HISTORY OF TOP TAGGING



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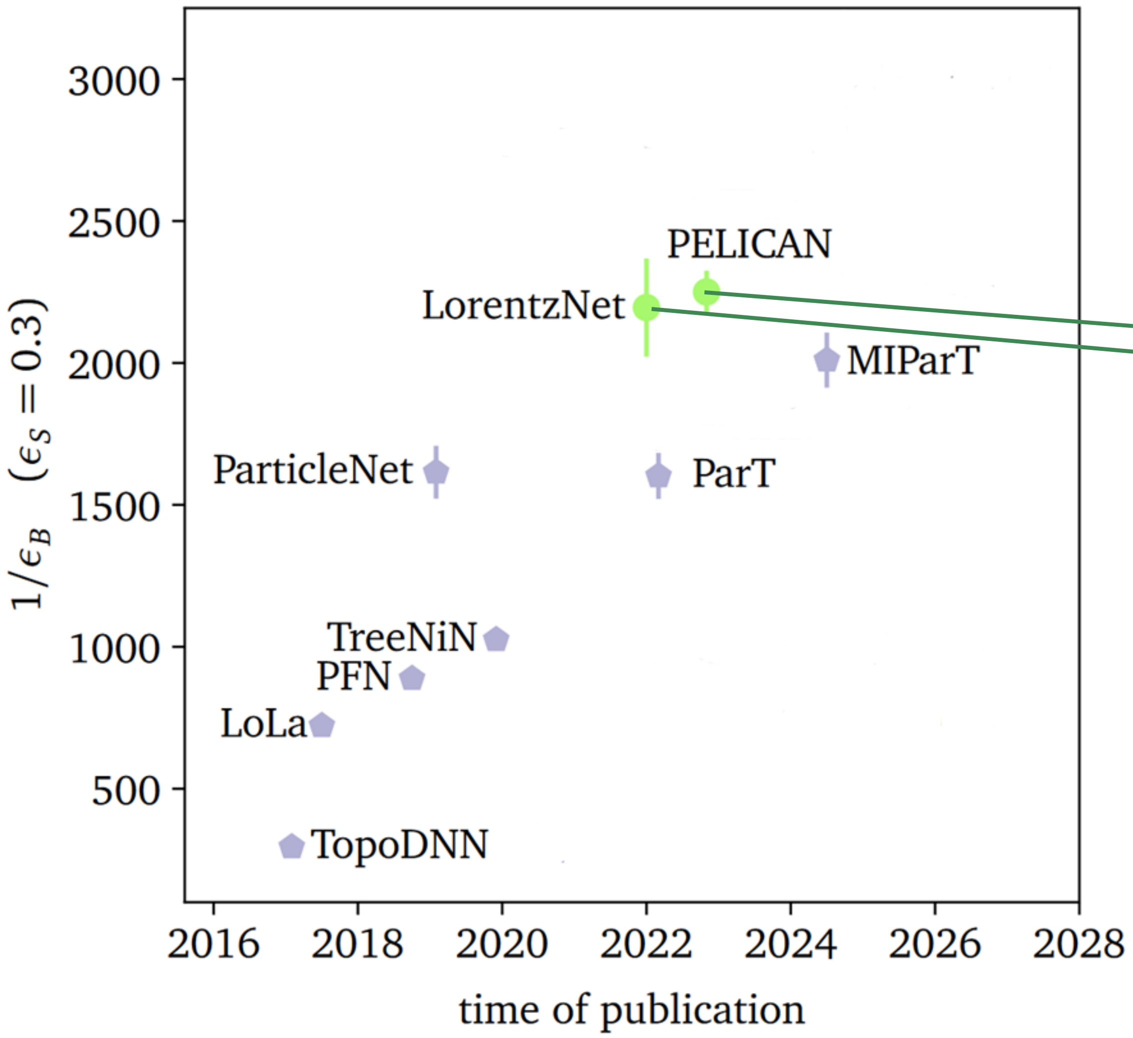

Equivariant neural networks!!

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EQUIVARIANT NEURAL NETWORKS

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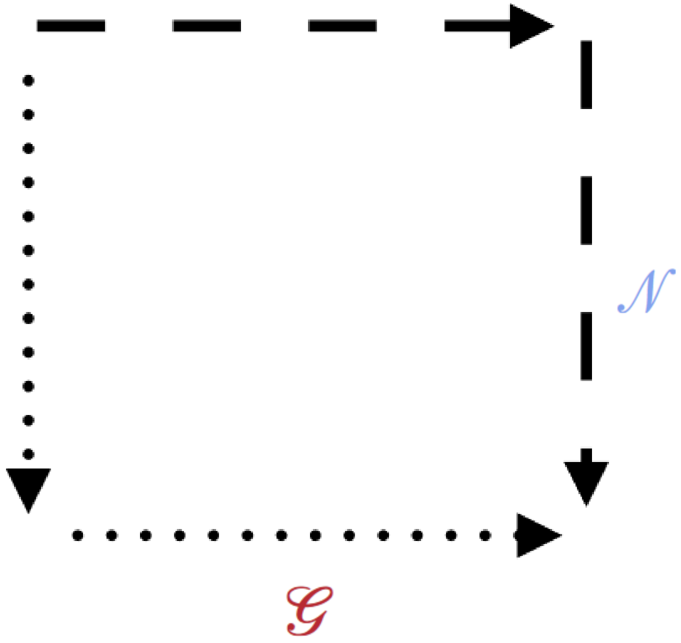
$$\mathscr{G}(\mathscr{N}(x)) = \mathscr{N}(\mathscr{G}(x))$$



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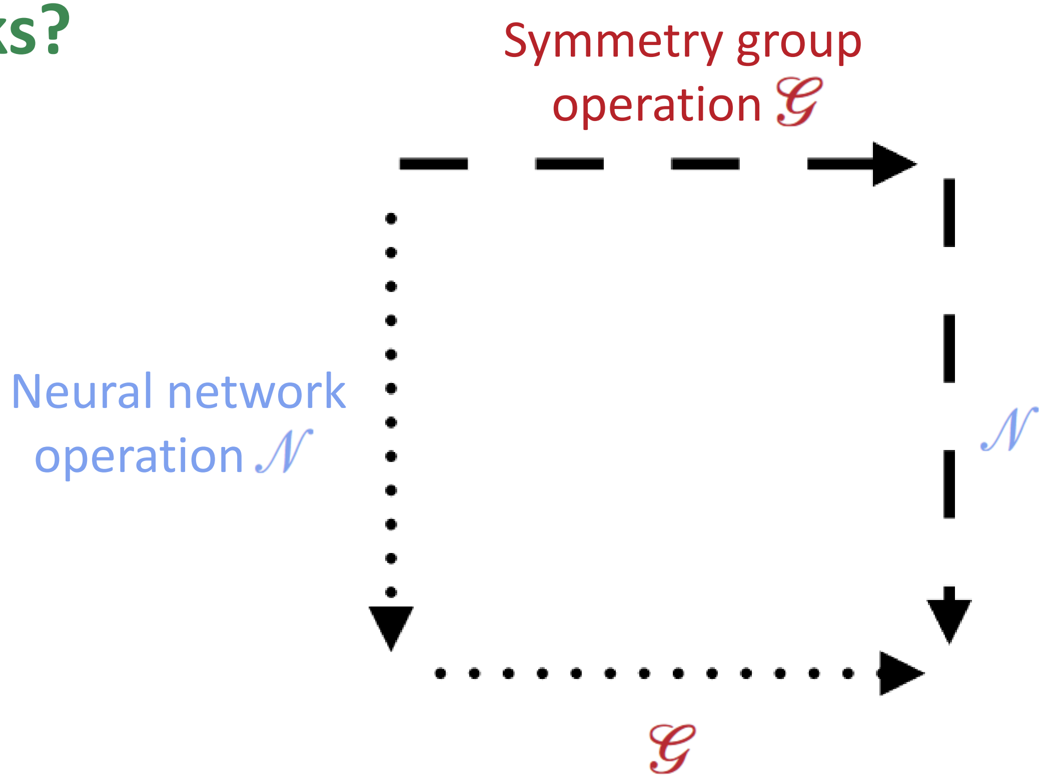

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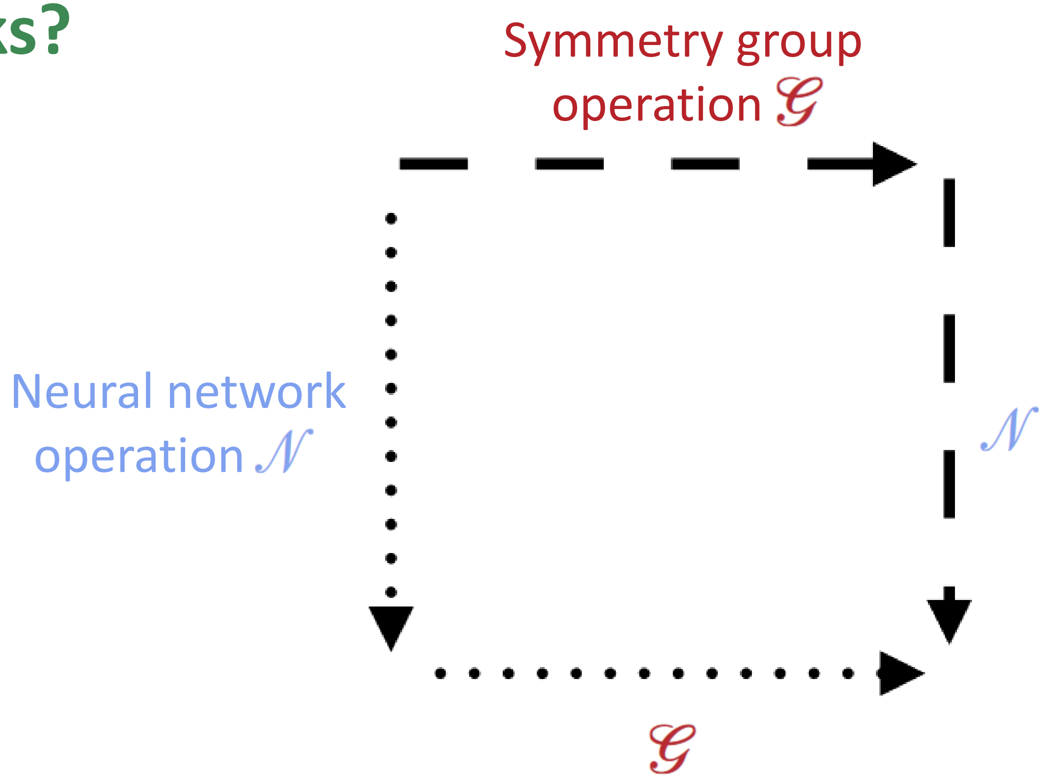

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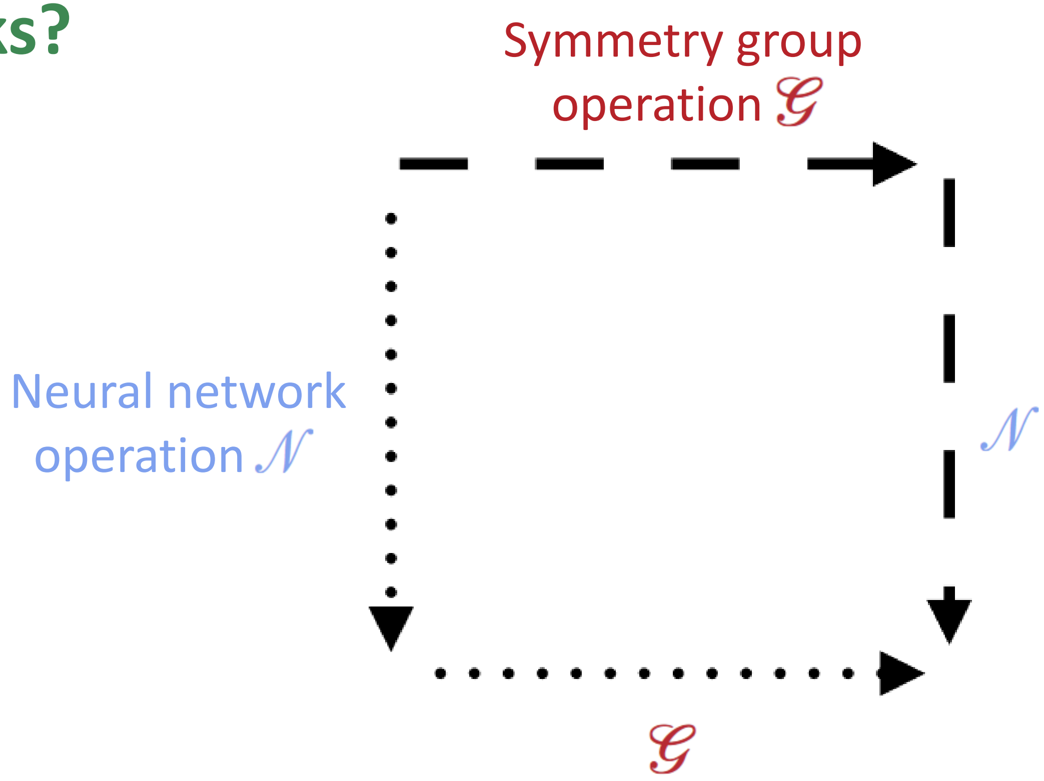

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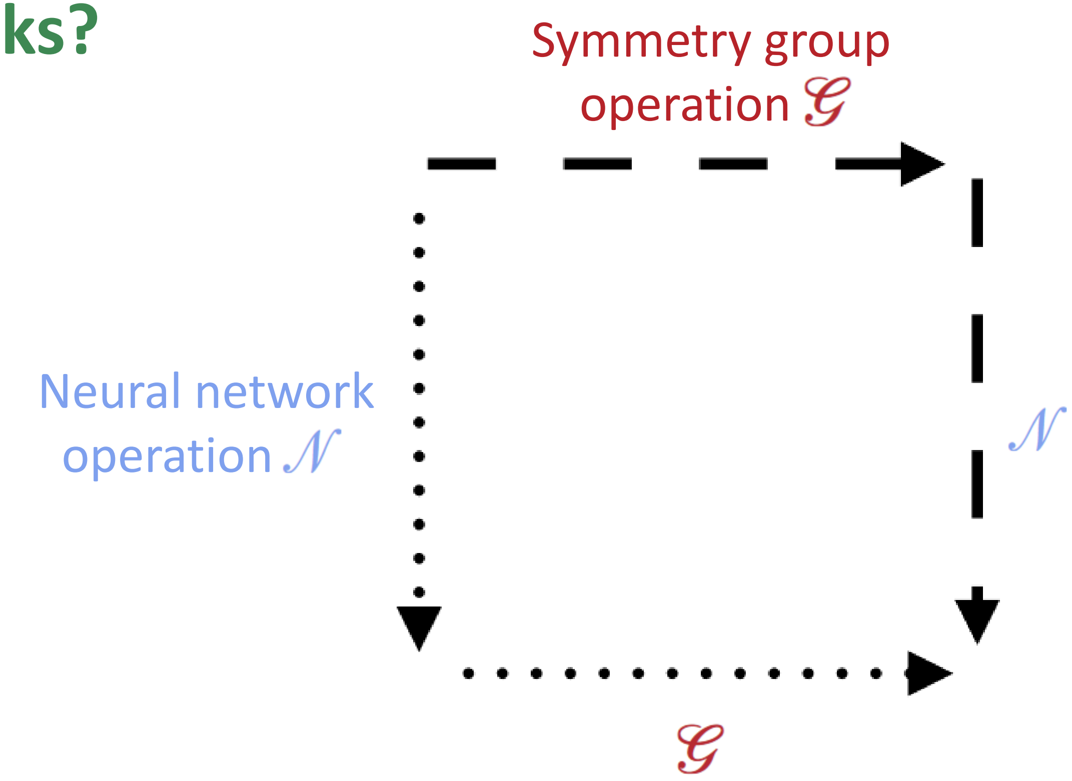

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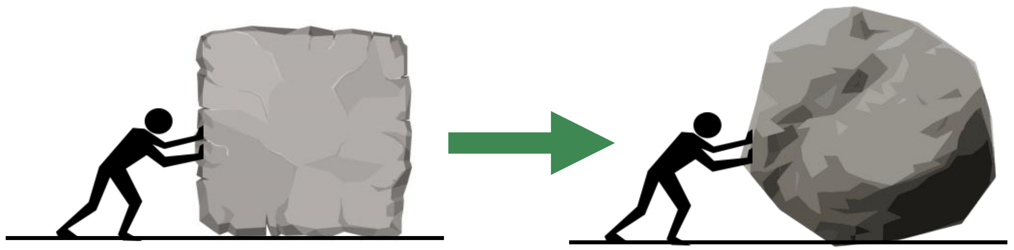
- More efficient networks



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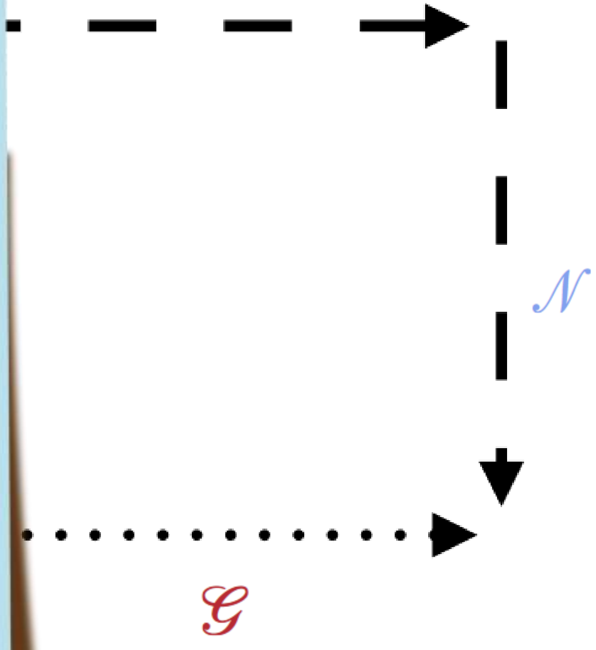
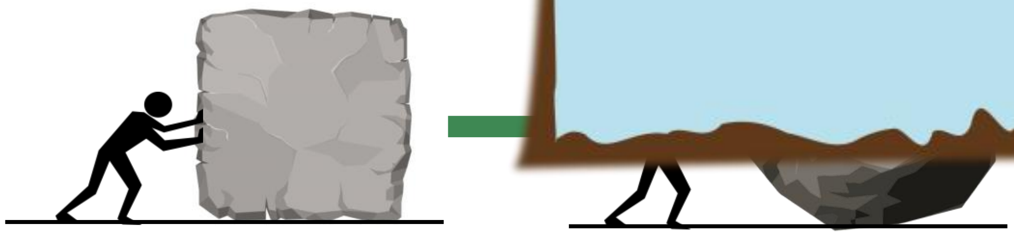
What is our recipe?

- Geometric Algebra
- Transformer

Symmetry group operation $\mathcal{G}$





SPACETIME ALGEBRA

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Vector space ✚ Geometric product

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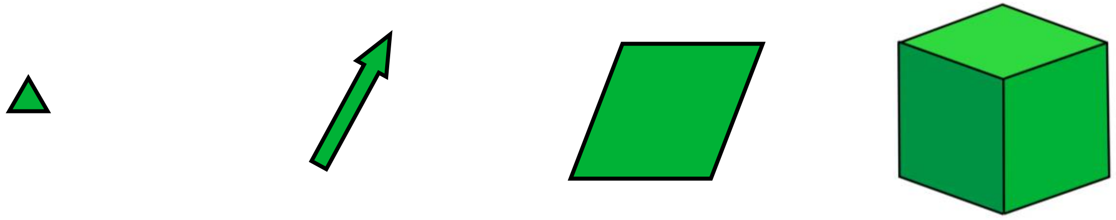

1	γ^μ	$\sigma^{\mu\nu}$	$\gamma^\mu\gamma^5$	γ^5

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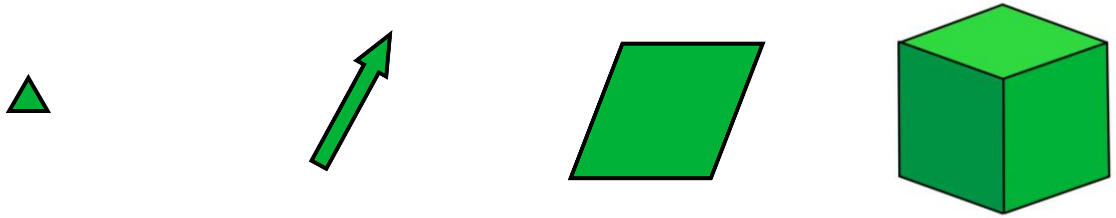
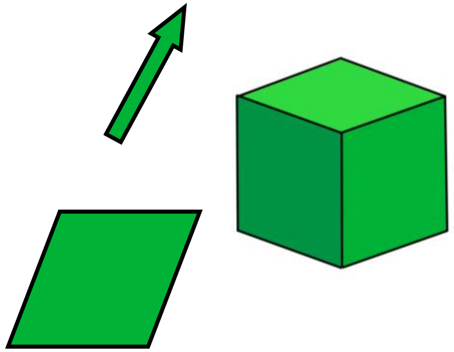


$$\underbrace{1 + \gamma^\mu + \sigma^{\mu\nu} + \gamma^\mu\gamma^5 + \gamma^5}_{\textbf{Multivector}}$$

SPACETIME ALGEBRA

- Lorentz transformations



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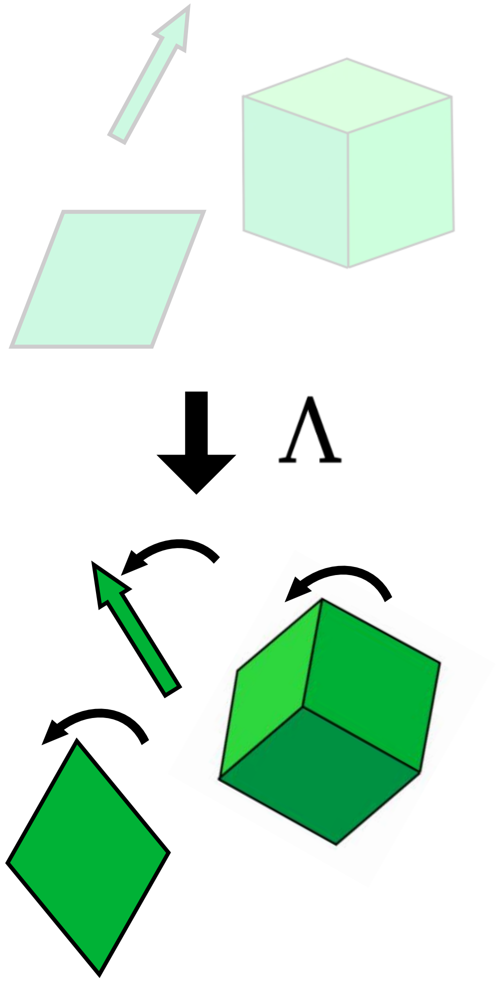

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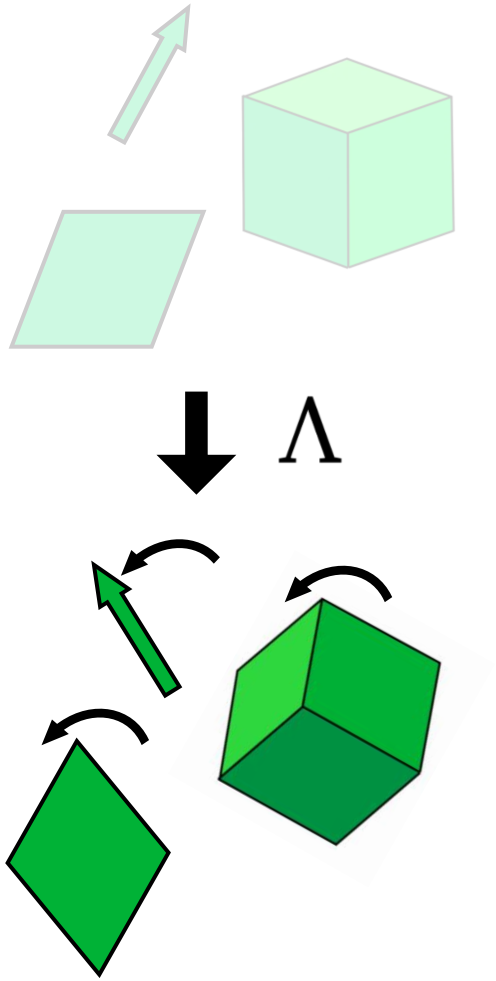

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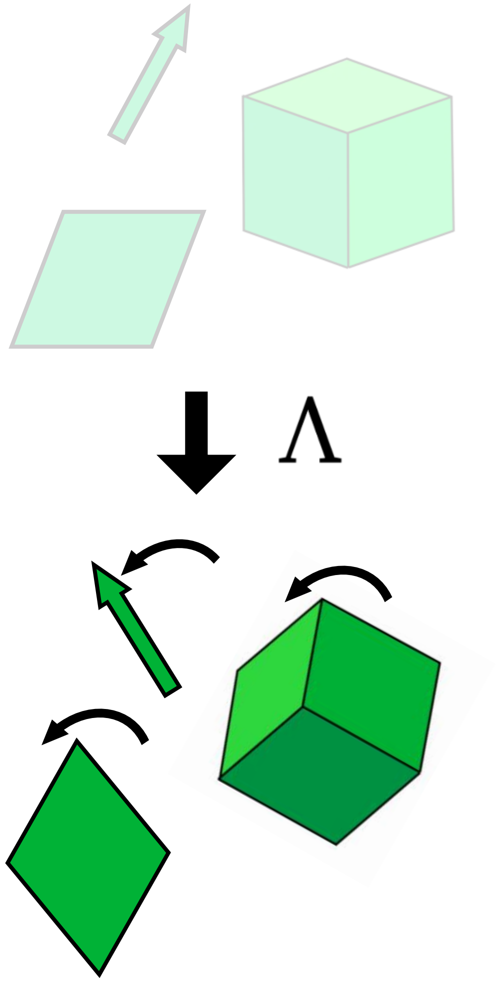

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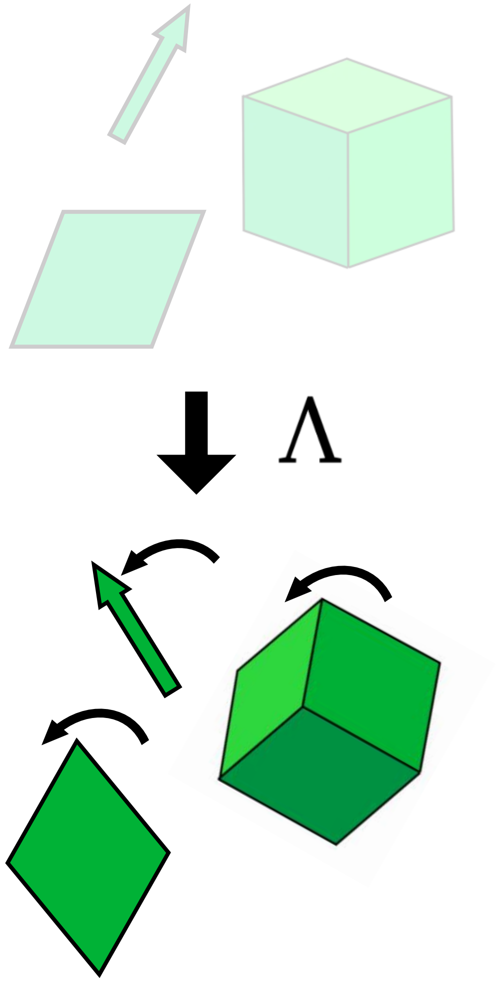

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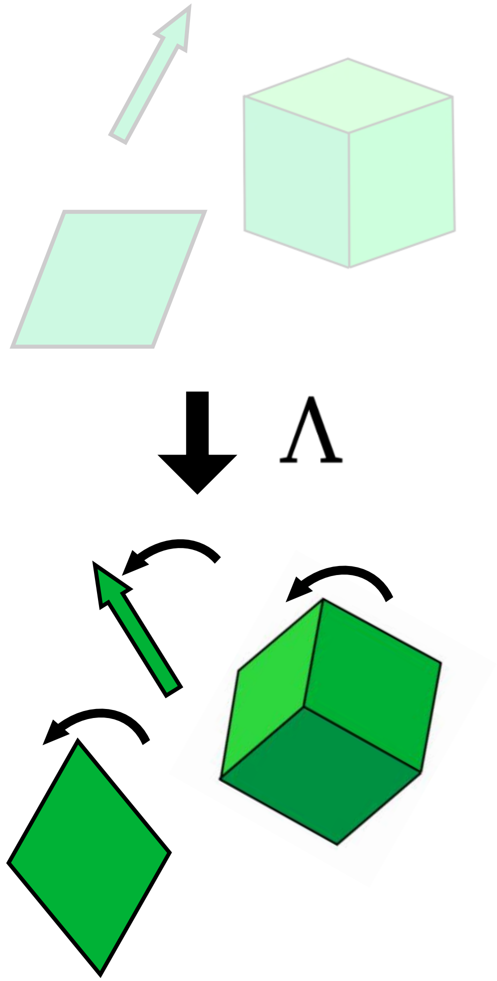

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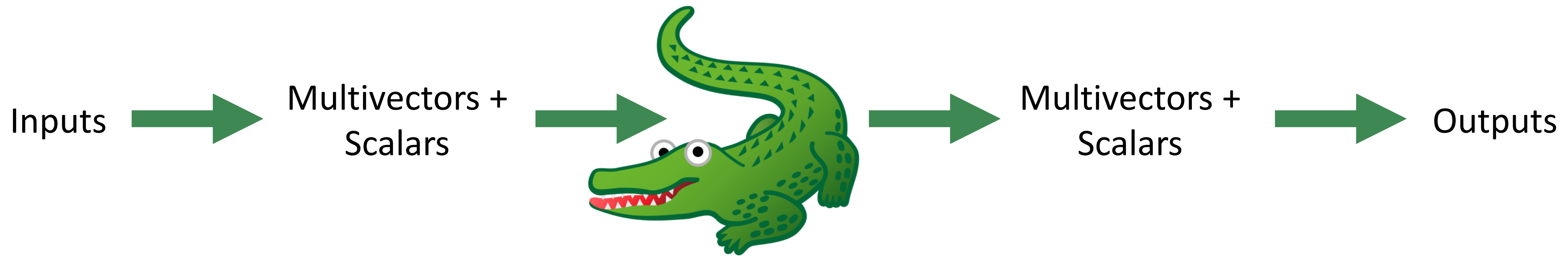
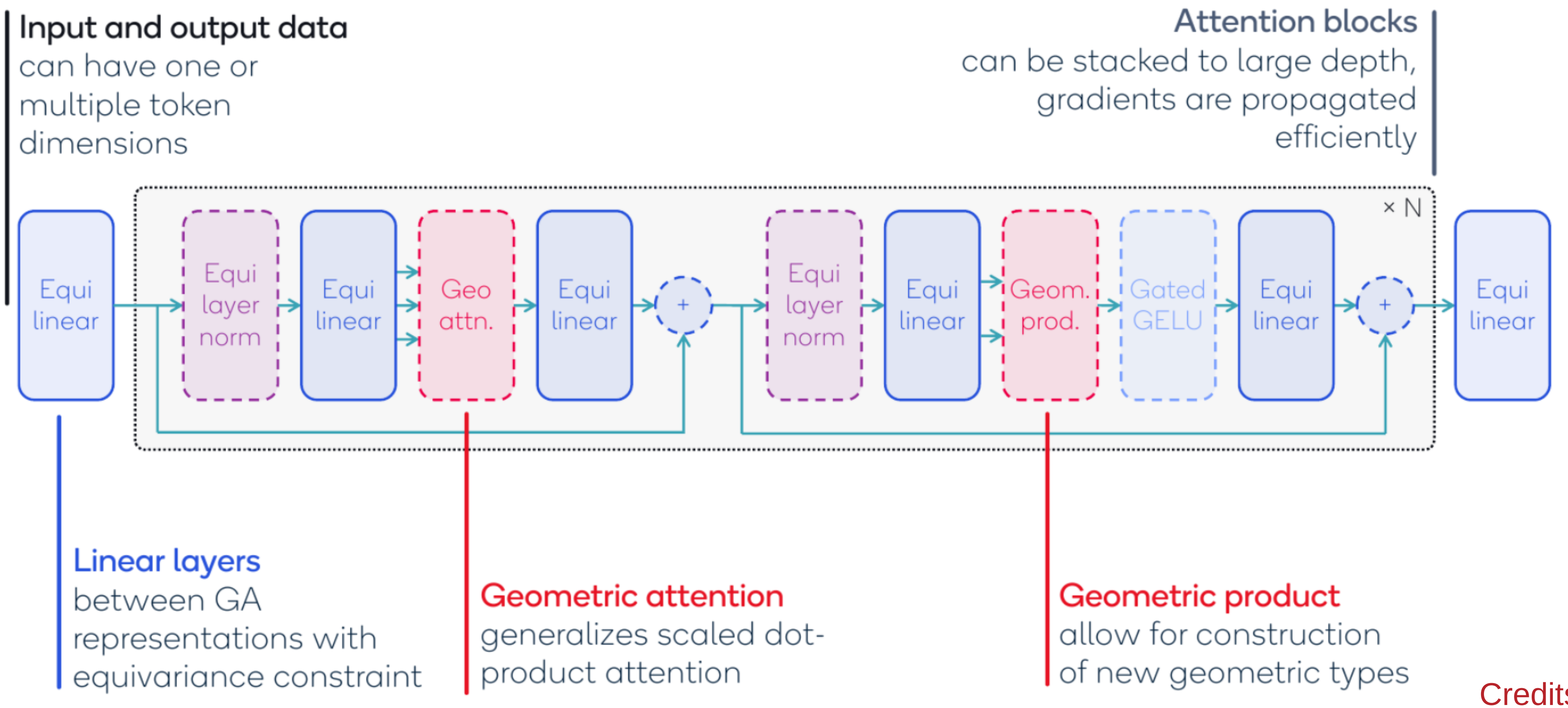
- Equivariance + Transformer = **L-GATr**

L-GATr

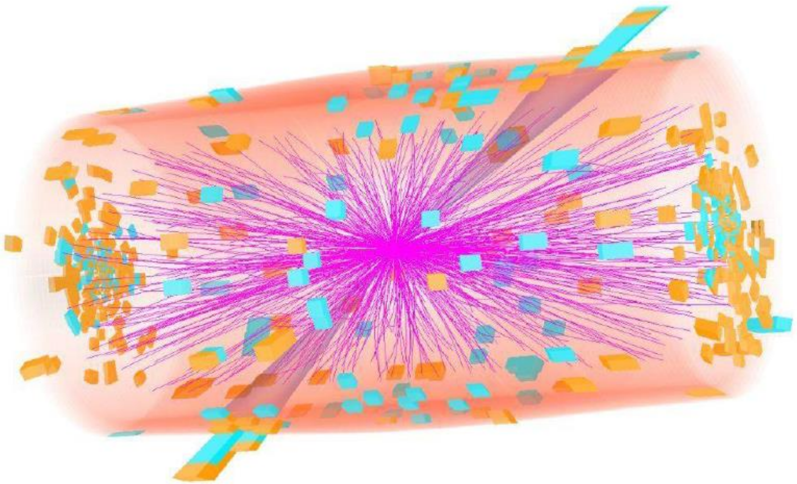




Credits to Johann Brehmer

L-GATr

- Key feature: **Symmetry breaking**



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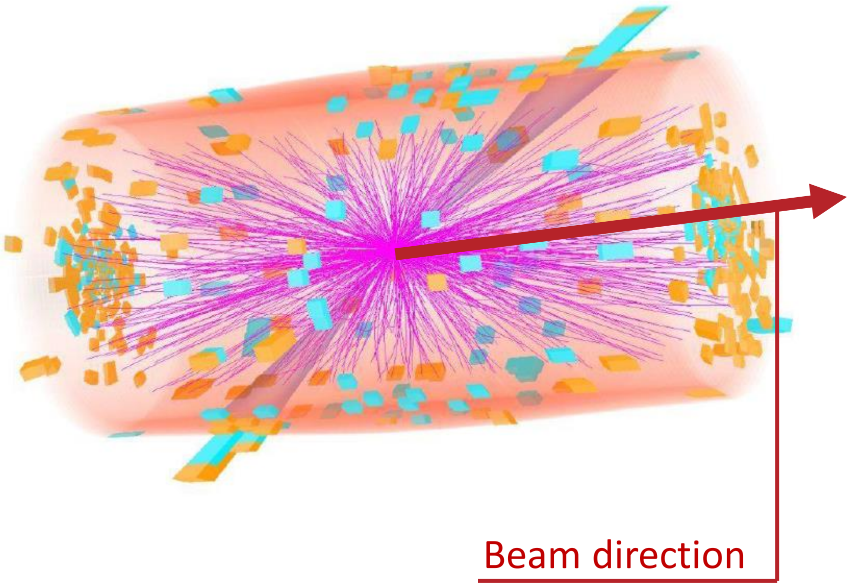

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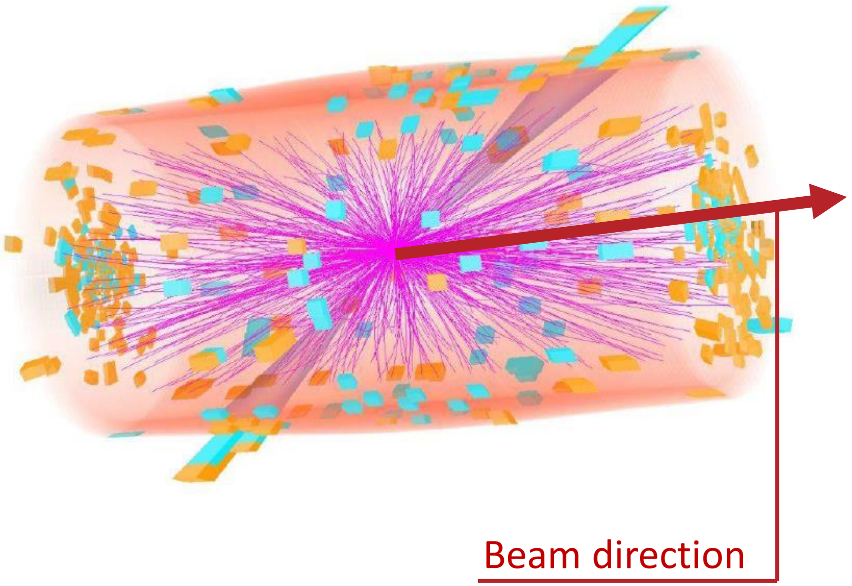

-Beam reference: $(1, 0, 0, \pm 1)$

$SO(1,3) \rightarrow$ boosts + rotations around the beam

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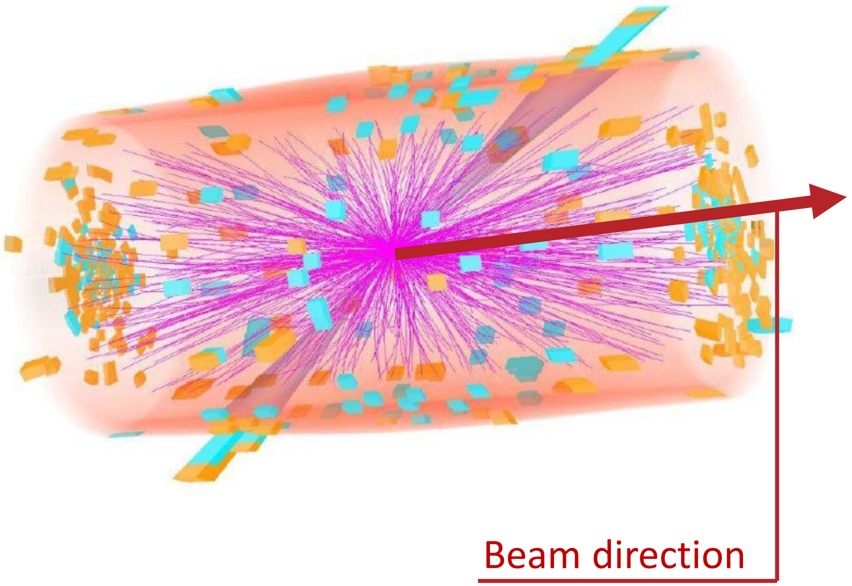

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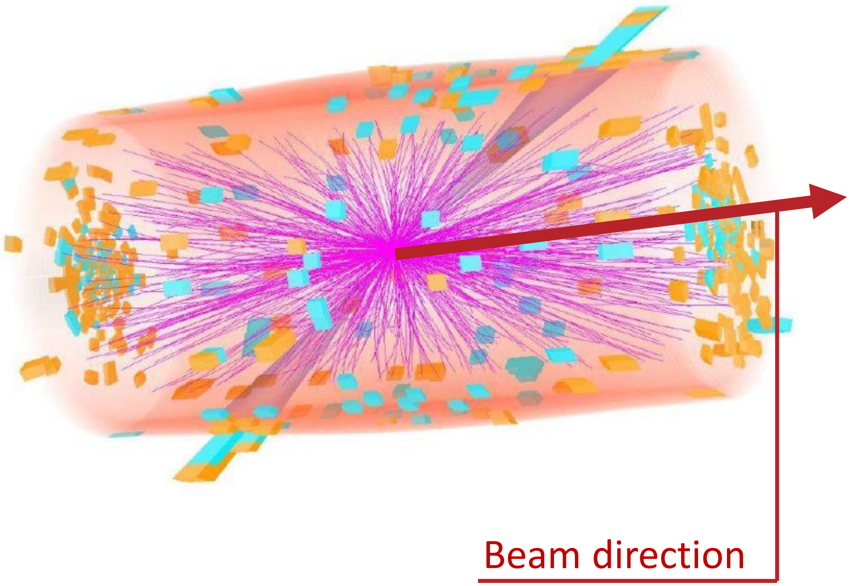

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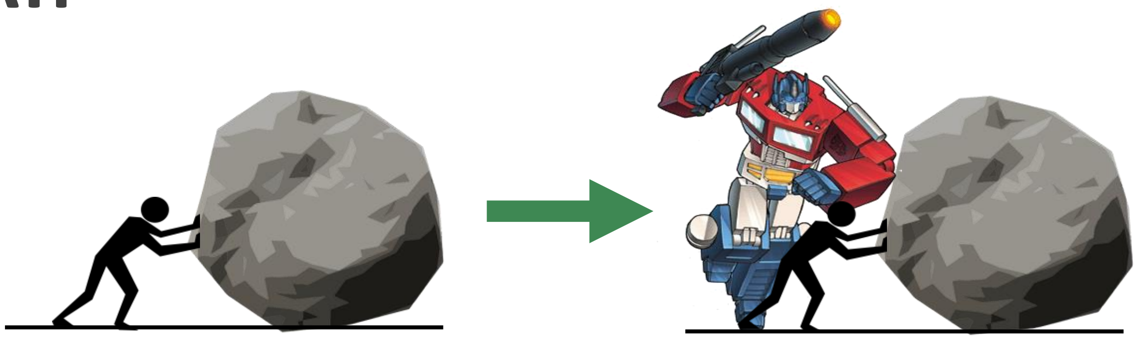
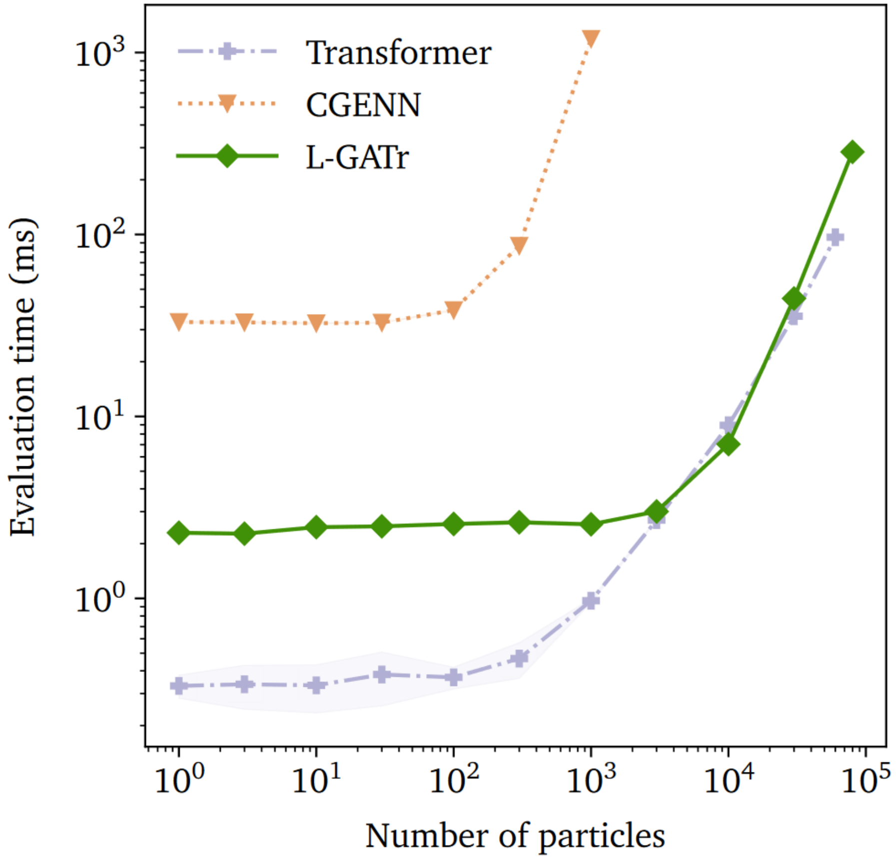
$SO(1,3) \rightarrow SO(3)$

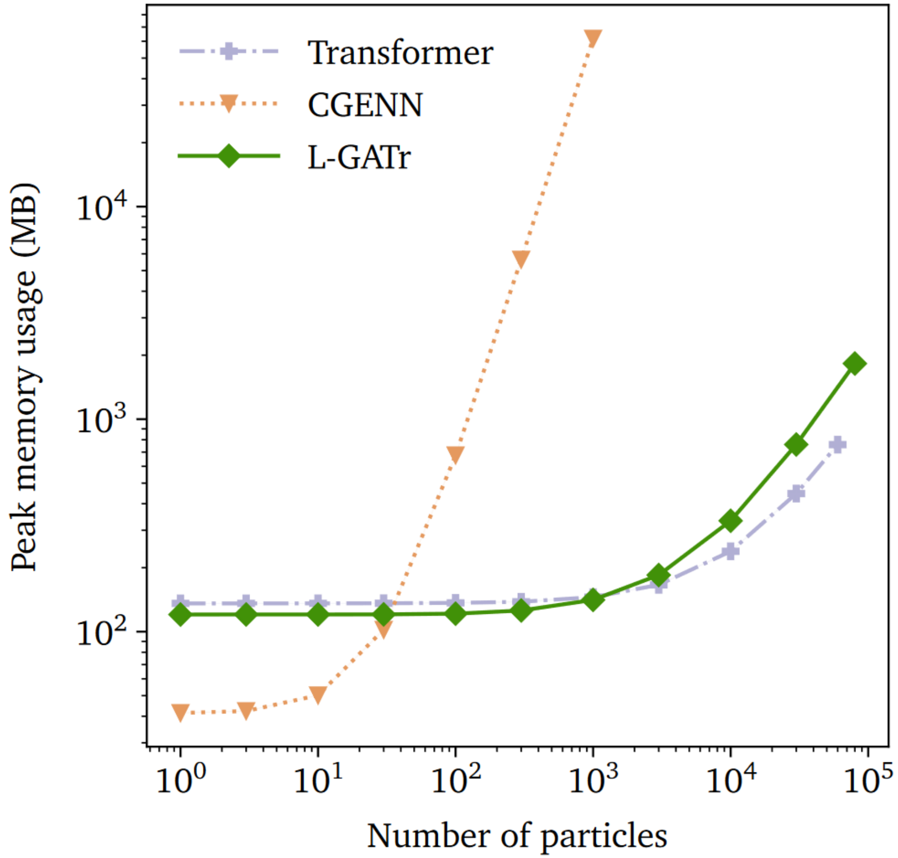
Beam	Time	Embedding	$1/\epsilon_B$ ($\epsilon_S = 0.3$)
–	✗	Particle	1422
Spacelike	✗	Particle	1905
All planes	✓	Particle	2009
–	✓	Token	1923
xy plane	✓	Channel	2060
Spacelike	✓	Particle	2152
Lightlike	✓	Particle	2114
xy plane	✓	Particle	2240

L-GATr

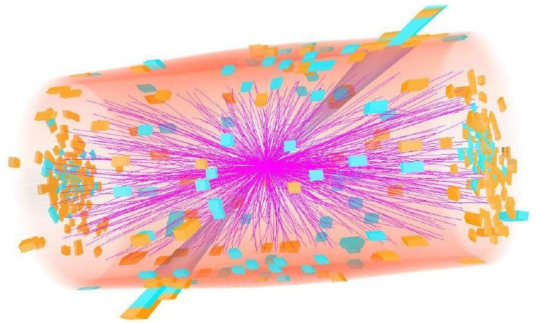
- Key feature: Transformer scaling







L-GATr TAGGING



- Experiment: **Top tagging**

Network	Accuracy	AUC	$1/\epsilon_B$ ($\epsilon_S = 0.5$)	$1/\epsilon_B$ ($\epsilon_S = 0.3$)
TopoDNN [52]	0.916	0.972	–	295 ± 5
LoLa [9]	0.929	0.980	–	722 ± 17
N-subjettiness [53]	0.929	0.981	–	867 ± 15
PFN [54]	0.932	0.9819	247 ± 3	888 ± 17
TreeNiN [55]	0.933	0.982	–	1025 ± 11
ParticleNet [56]	0.940	0.9858	397 ± 7	1615 ± 93
ParT [57]	0.940	0.9858	413 ± 16	1602 ± 81
MIParT [58]	0.942	0.9868	505 ± 8	2010 ± 97
LorentzNet* [10]	0.942	0.9868	498 ± 18	2195 ± 173
CGENN* [12]	0.942	0.9869	500	2172
PELICAN* [40]	0.9426 ± 0.0002	0.9870 ± 0.0001	–	2250 ± 75
L-GATr* [33]	0.9423 ± 0.0002	0.9870 ± 0.0001	540 ± 20	2240 ± 70

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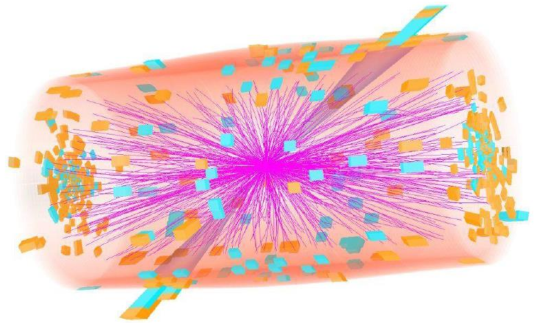

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  - Large and comprehensive jet dataset

  - 100M events

  - 10 classes

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	All classes Accuracy	AUC	$H \to b\bar{b}$ Rej$_{50\%}$	$H \to c\bar{c}$ Rej$_{50\%}$	$H \to gg$ Rej$_{50\%}$	$H \to 4q$ Rej$_{50\%}$	$H \to l\nu q\bar{q}'$ Rej$_{99\%}$	$t \to bq\bar{q}'$ Rej$_{50\%}$	$t \to bl\nu$ Rej$_{99.5\%}$	$W \to q\bar{q}'$ Rej$_{50\%}$	$Z \to q\bar{q}$ Rej$_{50\%}$
ParticleNet [56]	0.844	0.9849	7634	2475	104	954	3339	10526	11173	347	283
ParT [57]	0.861	0.9877	10638	4149	123	1864	5479	32787	15873	543	402
MIParT [58]	0.861	0.9878	10753	4202	123	1927	5450	31250	16807	542	402
L-GATr	0.865	0.9884	12195	4819	128	2304	5764	37736	19231	580	427

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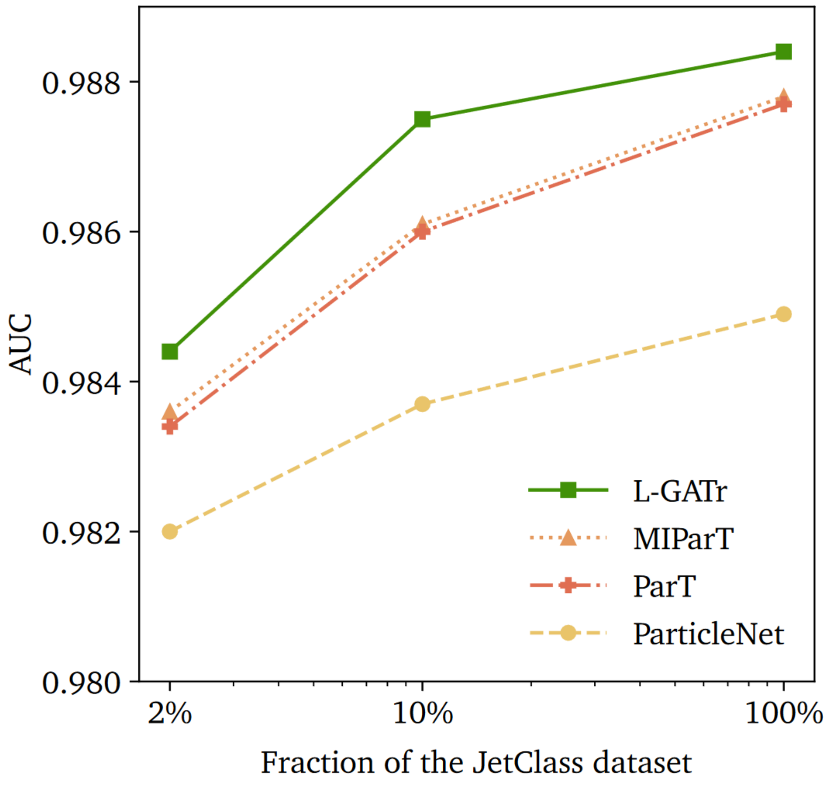

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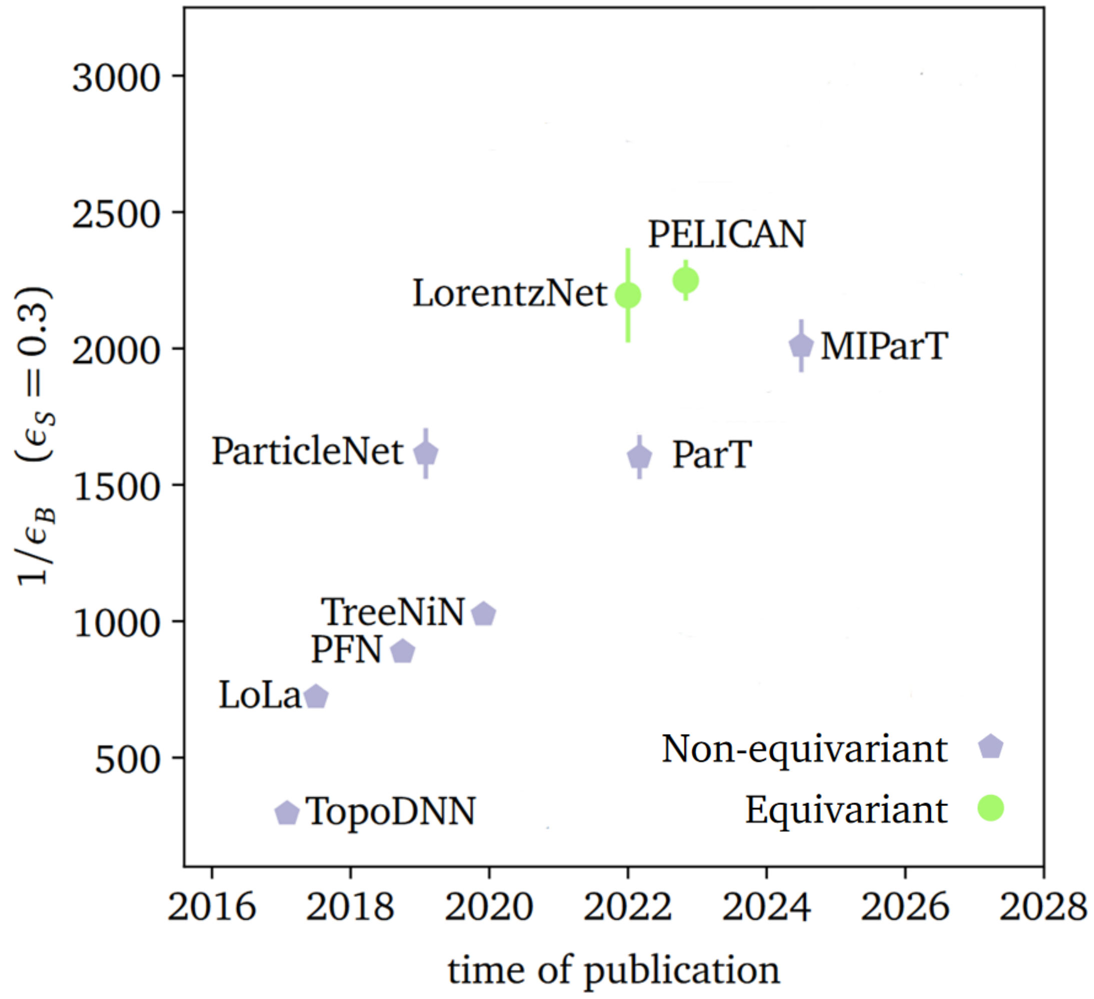

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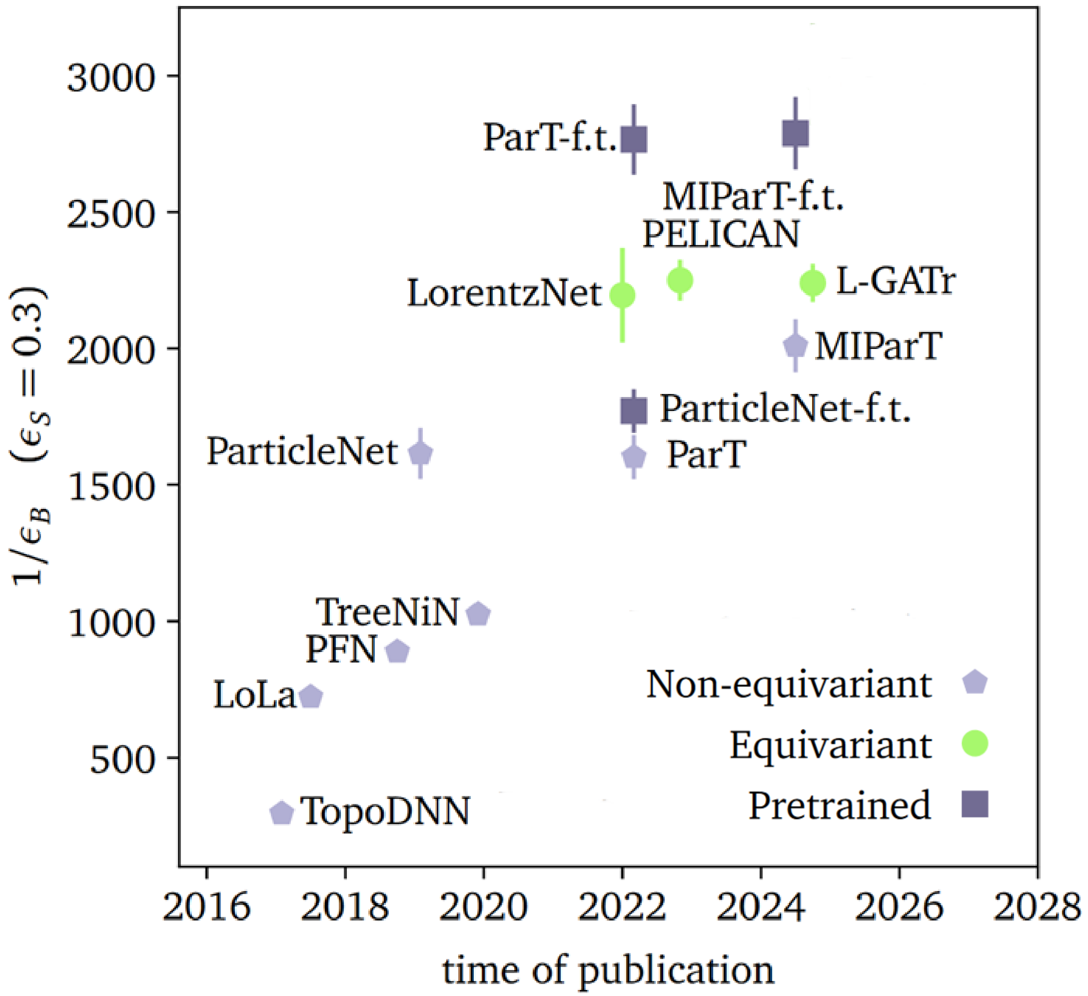

H. Qu et al., 2202,03772

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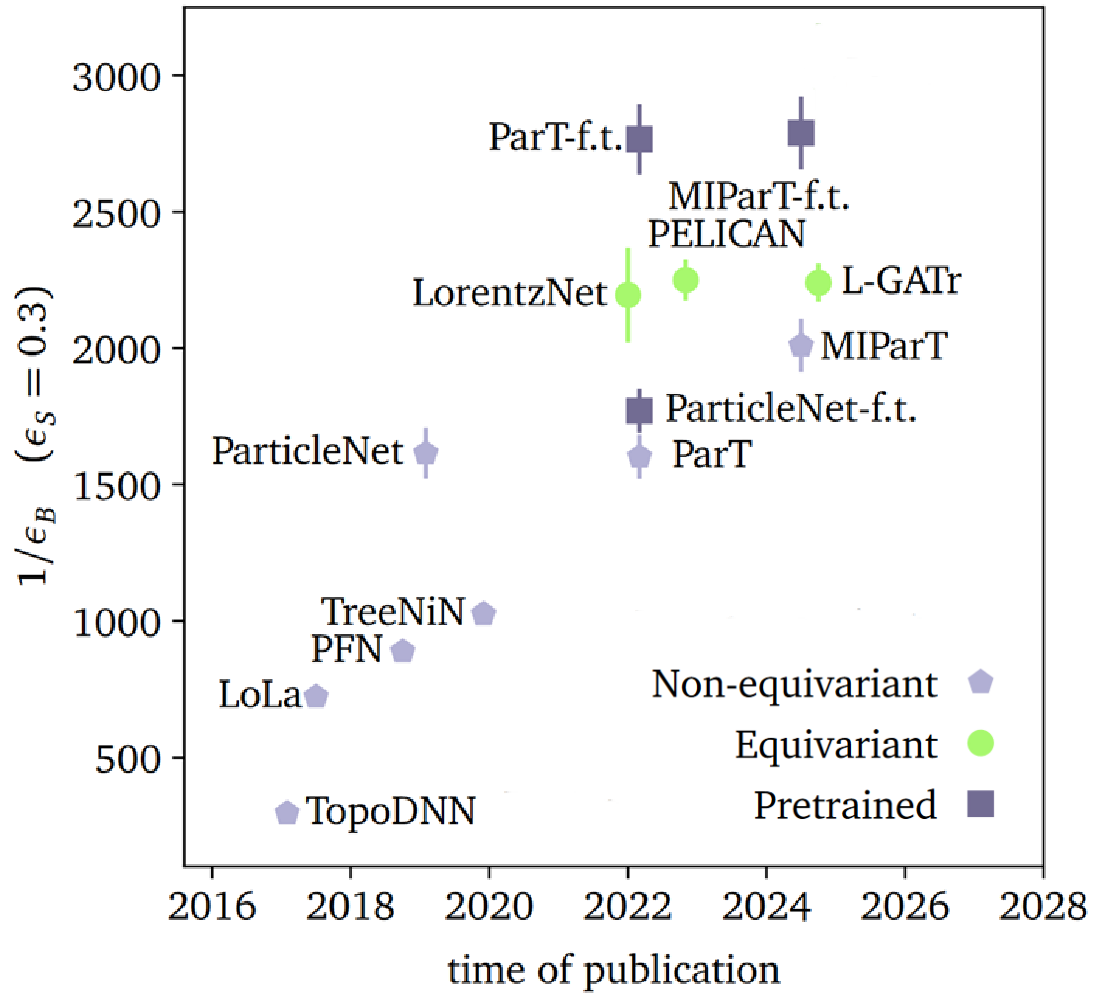

Strategy

1. Pre-train on JetClass
2. Restart the output layer
3. Fine-tune on top tagging

H. Qu et al., 2202,03772

L-GATr TAGGING

- What if we combine **pre-training** and **equivariance**?

Network	Accuracy	AUC	$1/\epsilon_B$ ($\epsilon_S = 0.5$)	$1/\epsilon_B$ ($\epsilon_S = 0.3$)
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L-GATr* [35]	0.9423 ± 0.0002	0.9870 ± 0.0001	540 ± 20	2240 ± 70
ParticleNet-f.t. [60]	0.942	0.9866	487 ± 9	1771 ± 80
ParT-f.t. [60]	0.944	0.9877	691 ± 15	2766 ± 130
MIParT-f.t. [60]	0.944	0.9878	640 ± 10	2789 ± 133
L-GATr-f.t.* (new)	0.9442 ± 0.0002	0.98792 ± 0.00004	661 ± 24	3005 ± 186

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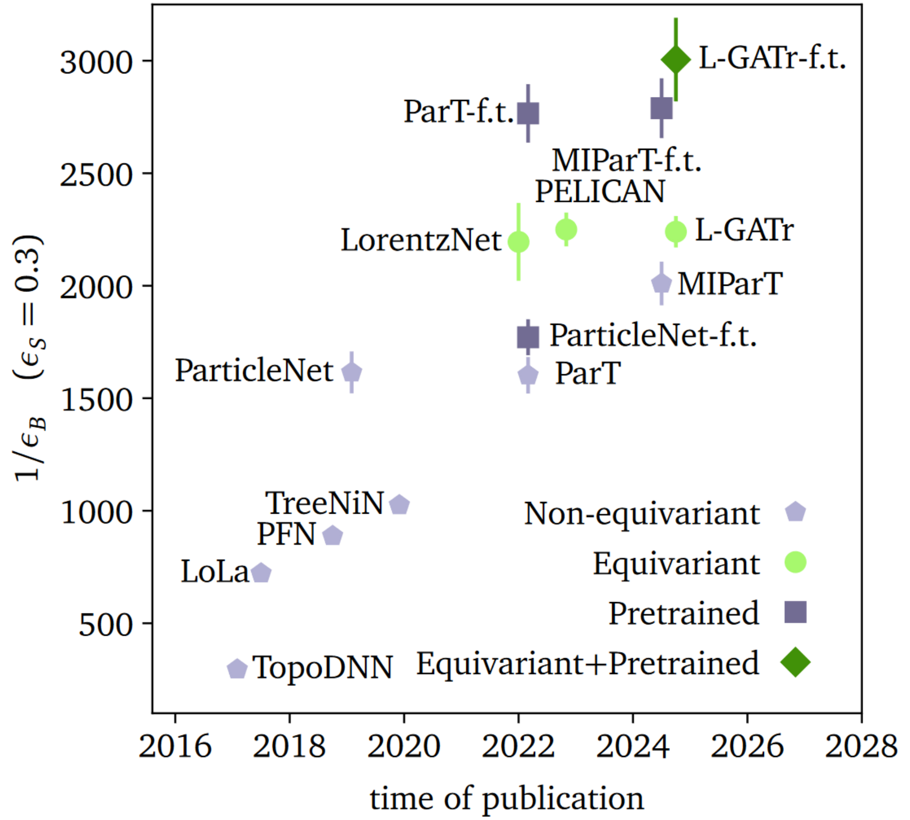
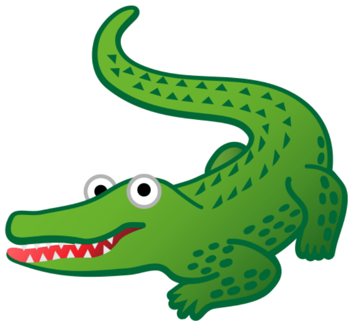


CONCLUSIONS

- L-GATr sets a new benchmark on multiple tagging tasks
- L-GATr has a better scaling behavior than competing baselines
- L-GATr has a strong performance on multiple problems, namely regression and generation

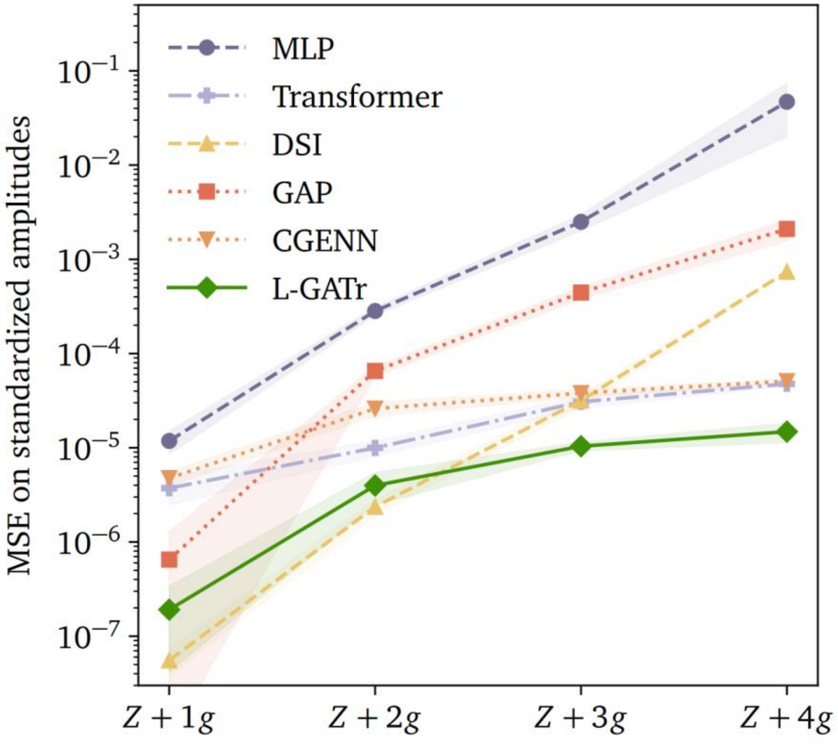


CONCLUSIONS

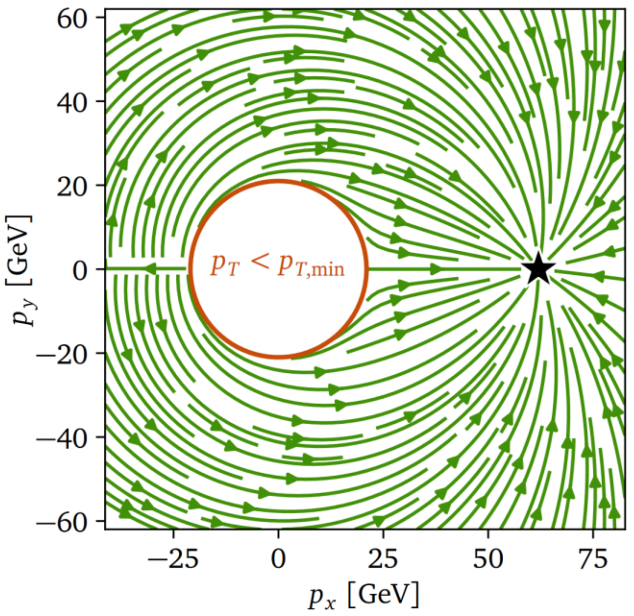
See talks by Jonas Spinner and Giovanni De Crescenzo!

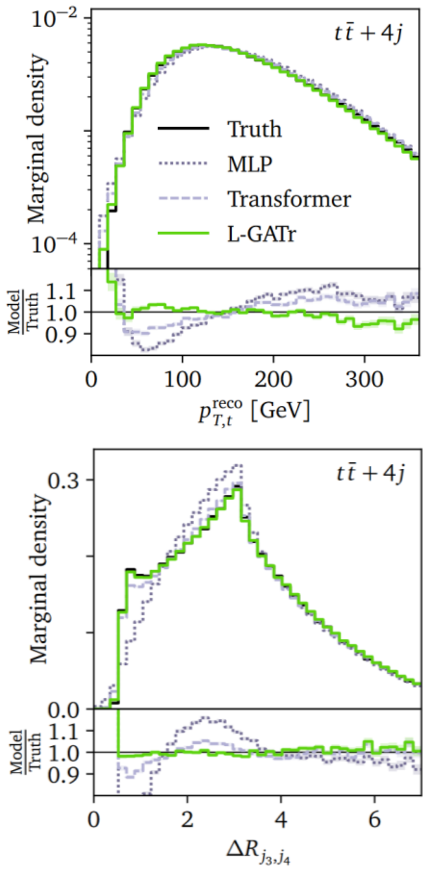
- Sneak peek at our other applications:

Amplitude regression



Event generation

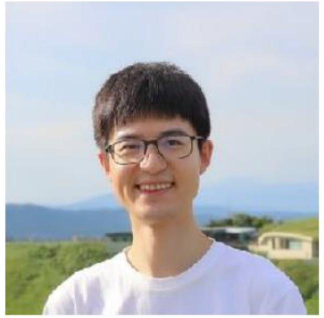





Jonas Spinner


Pim de Haan


Tilman Plehn


Huilin Qu


Jesse Thaler


Johann Brehmer

Lorentz-Equivariant Geometric Algebra Transformer for High-Energy Physics

Jonas Spinner*, Victor Breso*, Pim de Haan, Tilman Plehn, Jesse Thaler, Johann Brehmer, NeurIPS 2024, arXiv:2405.14806

CS paper

A Lorentz-Equivariant Transformer for all of the LHC

Johann Brehmer, Víctor Bresó, Pim de Haan, Tilman Plehn, Huilin Qu, Jonas Spinner, Jesse Thaler, arXiv:2411.00446

HEP paper

What will **you** use L-GATr for?

L-GATr code