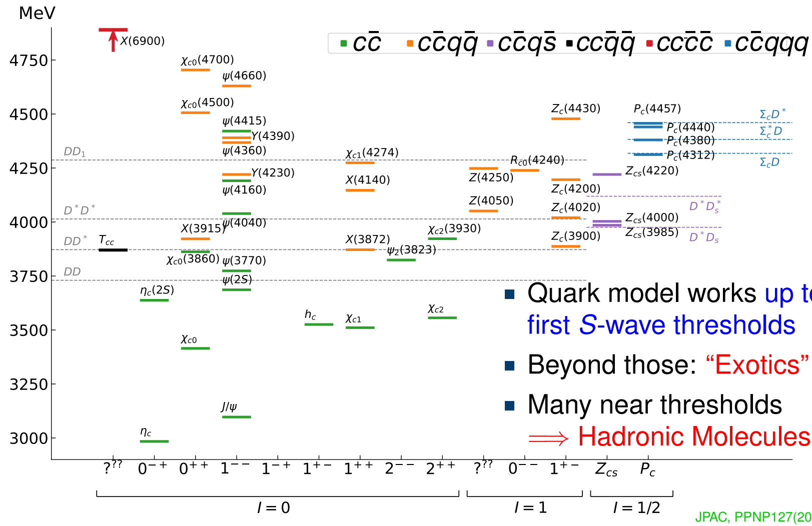
UNDERSTANDING D_0 ET AL.

by combining results from Lattice QCD, EFTs and Experiment

April 19, 2024 | Christoph Hanhart | IKP/IAS Forschungszentrum Jülich

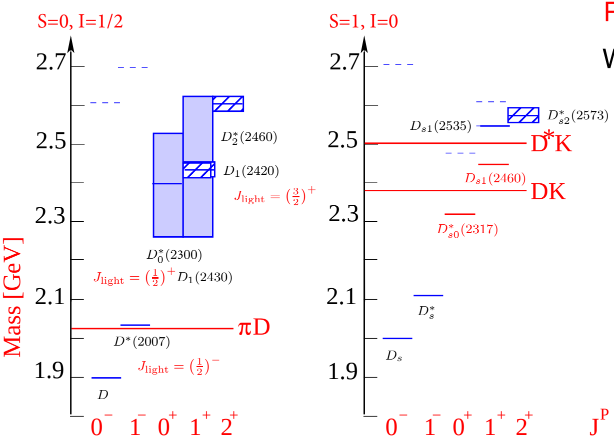
SETTING THE STAGE: XYZ ET AL.



- Quark model works up to first S-wave thresholds
- Beyond those: “Exotics”
- Many near thresholds
  $\Longrightarrow$ Hadronic Molecules?

JPAC, PPNP127(2022)103981

SETTING THE STAGE II: *D*-MESONS



Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004

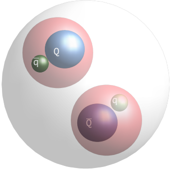
Puzzles:

Why are/is

1. $M(D_{s1})\&M(D_{s0}^*)$ so light?
2. $M(D_{s1})-M(D_{s0}^*)$ $\simeq M(D^*)-M(D)$?
3. $M(D_0^*) \simeq M(D_{s0}^*)$? $M(D_1) \simeq M(D_{s1})$?

The solution provides crucial information about the nature of these states

HADRONIC MOLECULES



- are few-hadron states, bound by the strong force
- do exist: light nuclei.
  e.g. deuteron as *pn* & hypertriton as Λd bound state
- are located typically close to relevant continuum threshold;
  e.g., for $E_B = m_1 + m_2 - M$ and $\gamma = \sqrt{2\mu E_B}$
  - $E_B^{\text{deuteron}} = 2.22$ MeV ($\gamma = 45$ MeV)
  - $E_B^{\text{hypertriton}} = (0.13 \pm 0.05)$ MeV (to Λd) ($\gamma = 13$ MeV)
- can be identified in observables (Weinberg compositeness):

$$\frac{g_{\text{eff}}^2}{4\pi} = \frac{4M^2\gamma}{\mu}(1-\lambda^2) \;\to\; a = -2\left(\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{1}{\gamma}\,; \quad r = -\left(\frac{\lambda^2}{1-\lambda^2}\right)\frac{1}{\gamma}$$

where $(1-\lambda^2)$=probability to find molecular component in bound state wave function

→ $r \gtrsim 0$ for molecule; $r < 0$ & $|r| \gg$ *range of forces* for compact state

Are there mesonic molecules?

DISCLAIMERS AND OUTLINE

The method presented is 'diagnostic' — especially,

- it does not allow for conclusions on the binding force;
- it allows one only to study individual states;
- quantitative interpretation gets lost when states get bound too deeply ('uncertainty' $\sim\ R\gamma$)

To go beyond tailor made effective field theories needed

In this talk I present how a unitarized chiral theory (UChPT) can be applied to Goldstoneboson D meson scattering and allows for a simultaneous study of experimental and lattice data to reveal the nature of $D_{s0}^*(2317)$ & $D_0^*(2300)$ and quantify the implications for other observables

CHIRAL LAGRANGIAN (1)

- The leading order Lagrangian (no free parameters)

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with $P = (D^0, D^+, D_s^+)$ for the D mesons, and the covariant derivative

$$\begin{aligned} D_\mu P &= \partial_\mu P + P\Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu) P^\dagger, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right), \end{aligned}$$

where $u_\mu = i\left[u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i l_\mu)u^\dagger\right], \quad u = e^{i\lambda_a \phi_a/(2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the Weinberg–Tomozawa term for $P\phi$ scattering:

$$\propto E_\phi + \mathcal{O}(1/M_D) \quad (S - \text{wave})$$

Interaction of kaons significantly stronger than that of pions

CHIRAL LAGRANGIAN (2)

- At the next-to-leading order p^2 (6 free parameters)

F-K Guo, CH, S. Krewald, U.-G. Meißner, PLB**666**(2008)251

$$\mathcal{L}_{\phi P}^{(2)} = P\left[-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right]P^\dagger$$
$$+ D_\mu P\left[h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right]D_\nu P^\dagger,$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \quad \chi = 2B_0\,\mathrm{diag}(m_u, m_d, m_s)$$

- Low-energy constants:

$h_1 = 0.42$: from $M_{D_s} - M_D$

Same effective operator leads to strong isospin violation

$m_{D^+} - m_{D^0} = \Delta m^{\mathrm{strong}} + \Delta m^{\mathrm{e.m.}} = ((2.5 \pm 0.2) + (2.3 \pm 0.6))$ MeV

h_0: from quark mass dependence of charmed meson masses (lattice)

$h_{2,3,4,5}$: fixed from lattice results on scattering lengths

calls for unitarisation $\Longrightarrow$ UChPT

UNITARISATION

Truong, Dorado, Pelaez, Kaiser, Weise, Oller, Oset, Lutz, Kolomeitsev, Guo, Meißner, C.H., ...

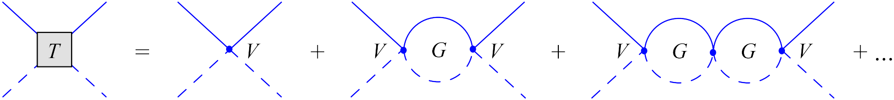
ChPT is only perturbatively consistent with unitarity.

Observe $\mathrm{Im}(t(s)) = \sigma(s)\,|t(s)|^2$ implies $\mathrm{Im}\left(t(s)^{-1}\right) = -\sigma(s)$

$\Longrightarrow$ write subtracted dispersion integral for $t(s)^{-1}$

$\Longrightarrow$ fix $\mathrm{Re}(t(s)^{-1})$ by matching to ChPT

Effectively this gives



with ChPT expression for V ... and additional parameter $a(\mu)$ (from the loop)

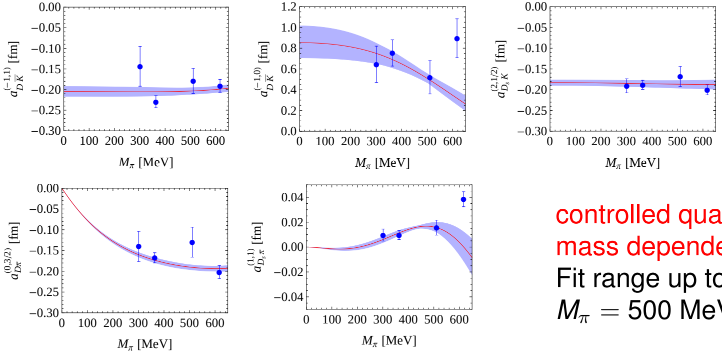
Dependence on unitarization method needs to be clarified!

FIT TO LATTICE DATA

fit LECs to lattice data for $a^{(S,I)}_{D_x\phi}$ in selected channels

Liu et al. PRD87(2013)014508

$\Longrightarrow$ 5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$



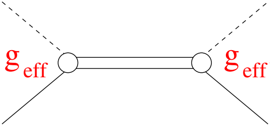
controlled quark mass dependence

Fit range up to $M_\pi = 500$ MeV

- $\pi/K/\eta$–$D^{(*)}/D_s^{(*)}$ scattering fixed
- $D^*_{s0}(2317)$ emerges as a pole with $M_{D^*_{s0}} = 2315^{+18}_{-28}$ MeV ($E_b = 47^{+28}_{-18}$);

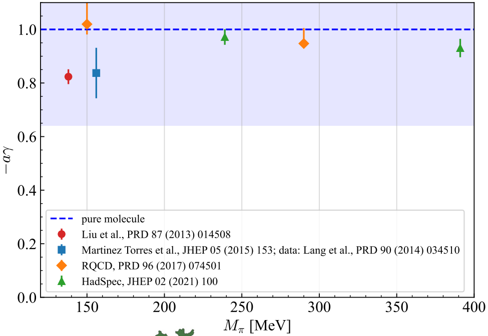
since $E_b(D_{s0}) = E_b(D^*_{s1}) + O(1/M_D)$ $\Longrightarrow$ puzzel 2 solved

INTERPRETATION A LA WEINBERG



$D^*_{s0}(2317)$: $a = g_{\rm eff}$ — $g_{\rm eff} + \mathcal{O}(1/\beta) \simeq -\left(\frac{2(1-\lambda^2)}{2-\lambda^2}\right)\frac{1}{\gamma}$

$\Longrightarrow a = -(1.05\pm0.36)\ \mathrm{fm}$ for molecule $(\lambda^2{=}0)$; smaller otherwise



Various lattice studies show under binding

study $a\gamma$ (removes E_b dep.)

All analyses consistent with purely molecular $D^*_{s0}(2317)$ (analogous for $D_{s1}(2460)$)

$\Longrightarrow$ puzzel 1 solved

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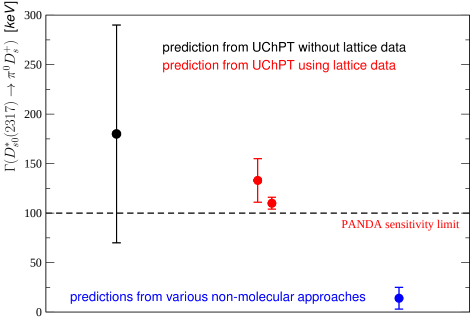
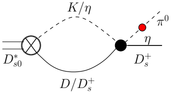
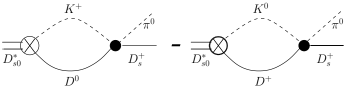
EXP. TEST: HADRONIC WIDTH



Genuine contribution:



Specific for molecules:



F.K. Guo et al., PLB666(2008)251; L. Liu et al. PRD87(2013)014508; X.Y. Guo et al., PRD98(2018)014510
and, e.g., P. Colangelo and F. De Fazio, PLB570(2003)180

Experiment needs very high resolution → PANDA

Predict $M_{B_{s0}^*} = 5722 \pm 14$ MeV and various decays

Fu et al., EPJA58(2022)70

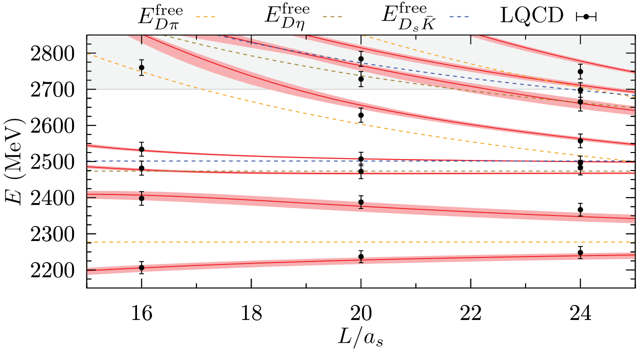
Most recent lattice result: $M_{B_{s0}^*} = 5699 \pm 14$ MeV

Hudspith & Mohler, [arXiv:2303.17295 [hep-lat]].

Next: Study multiplet structure from GB-*D*-meson scattering

THE S $=$ 0 SECTOR

Keeping parameters fixed one gets:



Albaladejo et al., PLB767(2017)465; Lattice: Moir et al. [Had.Spec.Coll.] JHEP10(2016)011

Fits directly to these data: Z. H. Guo et al., EPJC 79(2019)13; M. F. M. Lutz et al., PRD106(2022)114038

Poles for

- $M_\pi \simeq$391 MeV: (2264, 0) MeV [000] & (2468, 113) MeV [110]
- $M_\pi =$139 MeV: (2105, 102) MeV [100] & (2451, 134) MeV [110]

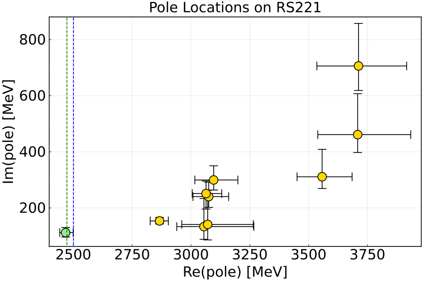
Questions $c\bar{q}$ nature of lowest lying 0^+ D state, $D_0^*(2300)$

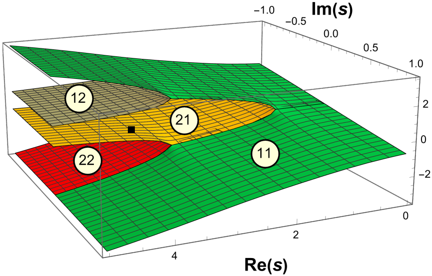
POLE STRUCTURE FROM LATTICE STUDY

Lattice study reported only bound state pole

Moir et al. [Had.Spec.Coll.] JHEP10(2016)011

Second pole was present, but location depends on amplitude model





- Poles located on hidden on sheet

A. Asokan et al., EPJC83(2023)850

- Pole locations correlated; in line with pole from UChPT
- Distance to threshold balanced by size of residue

V. Baru et al.,EPJA23(2005)523

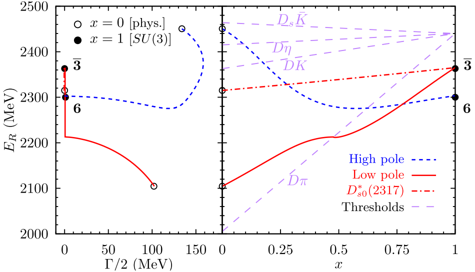
Explains correlation between Re(pole) and Im(pole)

SU(3) STRUCTURE FROM UCHPT

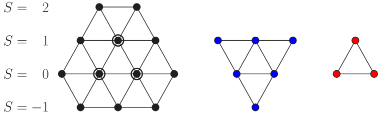
Albaladejo et al., PLB767(2017)465

$$m(x) = m^{\text{phy}} + x(m - m^{\text{phy}})$$

$m_\phi = 0.49$ GeV; $M_D = 1.95$ GeV



Multiplets: $[\bar{3}]\otimes[8]=[\overline{15}]\oplus[6]\oplus[\bar{3}]$

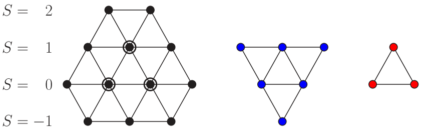


with $[\overline{15}]$ repulsive,
$[6]$ attractive,
$[\bar{3}]$ most attractive

- 3 poles give observable effect with SU(3)-breaking on
- At $SU(3)$ symmetric point $m_\phi \simeq 490$ MeV: 3 bound and 6 virtual states
- The light $D\pi$ state is the multiplet member of $D^*_{s0}(2317)$

$$\Longrightarrow \boxed{M_{D^*_{s0}(2317)} - M_{D^*_0(2100)} = 217\text{ MeV}}$$

SU(3) STRUCTURE

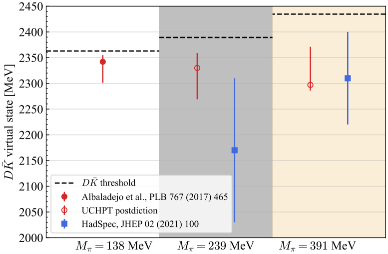


Albaladejo et al., PLB767(2017)465

- Lattice shows repulsion in $[\overline{15}]$ as predicted in UChPT
- States in [6] found in UChPT and lattice:

Hofmann and Lutz, NPA733(2004)142

  - $S = -1$



  - $S = 0$: Lattice finds virtual pole in [6] @$M_\pi \approx 600$ MeV in line with UChPT prediction

Gregory et al., [arXiv:2106.15391 [hep-ph]]+Lüscher analysis.

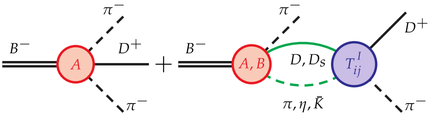
Confirmed by J.D.E. Yeo, C.E. Thomas and D.J. Wilson, [arXiv:2403.10498 [hep-lat]].

- Quark Model: $[\bar{3}] \otimes [1] = [\bar{3}]$ — the [6] is absent

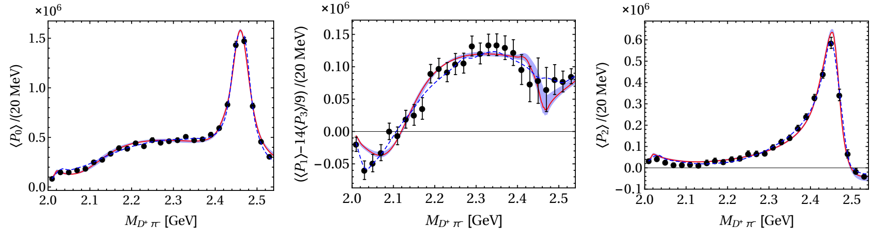
OBSERVABLE: $B^- \to D^+\pi^-\pi^-$

With ϕD amplitude fixed we can calculate production reactions:

Du et al., PRD98(2018)094018; for more results see Du et al., PRD99(2019)114002



for the *S*-wave (two free para.);
other partial waves from BW-fit

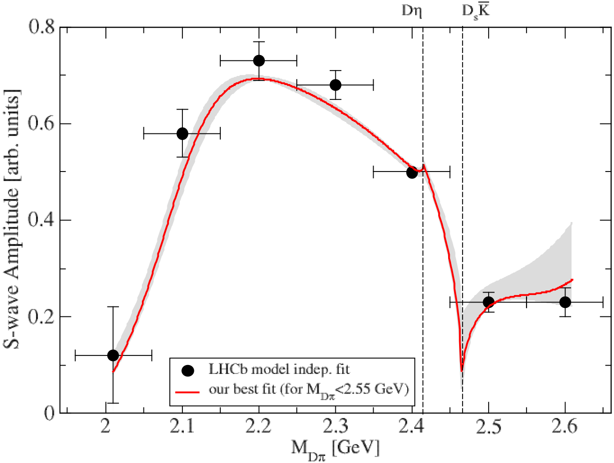


LHCb, PRD94(2016)072001

$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2\,, \quad \langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_2 - \delta_0)$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_1 - \delta_0)$$

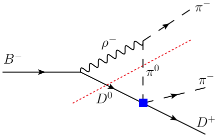
$D\pi$ S-WAVE FROM $B^- \to D^+\pi^-\pi^-$

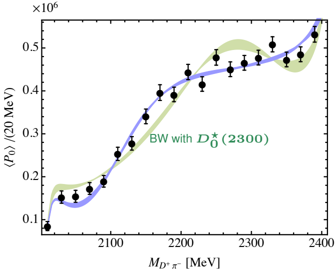


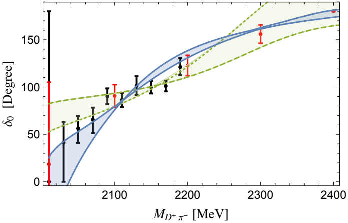
Effect of thresholds enhanced, by pole at $\sqrt{s_p} \sim (2451 - i134)$ MeV

on nearby unphysical sheet

LIGHTEST CHARMED SCALAR





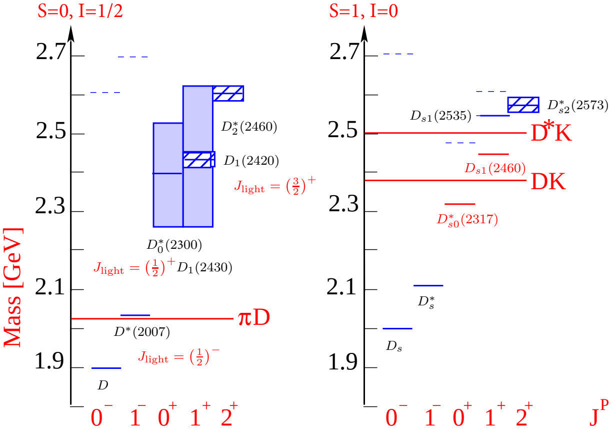


Mass of lightest charmed $J^P = 0^+$ state:

- BW with $m = 2300$ MeV incompatible with data
- UChPT with $(2105 \pm 8 - i(102 \pm 11))$ MeV is compatible Du et al., PRL126(2021)192001
- Low mass confirmed by Lattice QCD $(2196 \pm 64 - i(210 \pm 110))$ MeV at $M_\pi = 239$ MeV HadSpec, JHEP07(2021)123

Analogous picture for $J^P = 1^+$

CHARMED STATES



Quark Modell: M. Di Pierro and E. Eichten, PRD 64 (2001) 114004

Puzzles solved:

1. $M(D_{s1})\&M(D_{s0}^*)$ are DK and D^*K bound states
2. $M(D_{s1})-M(D_{s0}^*) \simeq M(D^*)-M(D)$, since spin symmetry gives equal binding
3. States with strangeness heavier
   $M(D_0^*) = 2100$ MeV
   $M(D_{s0}^*) = 2317$ MeV

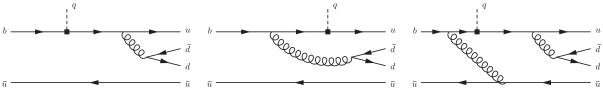
   $M(D_1) = 2247$ MeV
   $M(D_{s1}) = 2460$ MeV

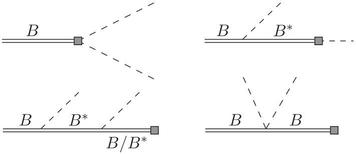
... role of left-hand cuts needs to be clarified

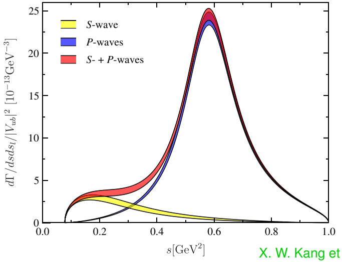
Lutz et al., PRD106(2022)114038; Korpa et al., PRD107(2023)L031505

FROM $B \to \pi\pi l\nu$ TO $B \to \bar{D}\pi l^+ l^-$

Provide input to CKM matrix elements:







X. W. Kang et al. PRD89(2014)053015

Remarks about $B \to \bar{D}\pi l^+ l^-$:

- Good control of πD system

E. J. Gustafson et al. [arXiv:2311.00864 [hep-ph]].

- Access to πD scattering from $B \to D\pi l\nu$ (see $\pi\pi$ from $K \to \pi\pi e\nu$)

J. R. Batley *et al.* [NA48/2], EPJC70(2010)635

SUMMARY AND CONCLUSION

- For near threshold states Weinberg criterion provides proper diagnostics
- View extended by studying the SU(3)$_f$ multiplet structure
  - what kinds of multiplets are there?
  - pattern of spin and flavor symmetry breaking important
- Interplay of different poles leads to
  - non-trivial line shapes
  - non-trivial phase motions

We are on a good path to identify the hadronic molecules in the spectrum

... and to exploit their imprint on various observables

Thanks a lot for your attention