Quantum control and enhancement of superconducting pairing in one-dimensional Fermi-Hubbard chains

Oleksandr Povitchan

last-year bachelor student

V. N. Karazin Kharkiv National University

Kharkiv, Ukraine

Fermi – Hubbard model

- In 1963, it was introduced by Martin Gutzwiller, John Hubbard and Junjiro Kanamori as a **simple model** capable of describing **electron correlations** in transition metals

tunnelling from j to i tunnelling from i to j

$$\hat{\mathcal{H}} = -t_h \sum_{\sigma} \sum_{\langle i,j \rangle} (\hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + h.c) + U \sum_{\sigma} \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

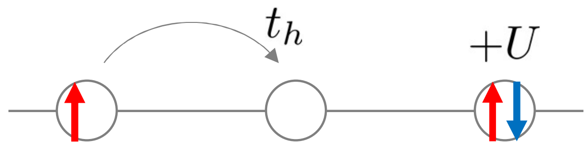
$t_h \to 1$

on-site pair density

fermionic creation and annihilation operators

- periodic quantum spin lattice
- single localised orbital level at each site
- on-site coulomb repulsion (pairs increase energy)
- tunnelling between the nearest neighbours

1-D chain



- Electron correlations lead to insulating, magnetic, and **superconducting** effects

Superconducting correlations in a discrete system

REVIEWS OF MODERN PHYSICS VOLUME 34 NUMBER 4 OCTOBER 1962

Concept of Off-Diagonal Long-Range Order and the Quantum Phases of Liquid He and of Superconductors

W

We def

VOLUME 63, NUMBER 19 PHYSICAL REVIEW LETTERS 6 NOVEMBER 1989

η Pairing and Off-Diagonal Long-Range Order in a Hubbard Model

Chen Ning Yang
State University of New York, Stony Brook, New York 11794-3840
and Chinese University of Hong Kong, Hong Kong
(Received 22 August 1989)

PHYSICAL REVIEW LETTERS **122,** 077002 (2019)

Photoinduced η Pairing in the Hubbard Model

Tatsuya Kaneko,[1] Tomonori Shirakawa,[2,1,3,4] Sandro Sorella,[2,5,3] and Seiji Yunoki[1,3,4]
[1]*Computational Condensed Matter Physics Laboratory, RIKEN Cluster for Pioneering Research (CPR), Wako, Saitama 351-0198, Japan*
[2]*SISSA–International School for Advanced Studies, Via Bonomea 265, 34136 Trieste, Italy*

- Mechanism of superconducting correlations: **η-pairing**
  - We can introduce the following **order parameter** to track the level of correlations:

$$\langle \hat{\eta}^2 \rangle / L$$

$$\hat{\eta}^2 = \frac{1}{2}(\hat{\eta}^+\hat{\eta}^- + \hat{\eta}^-\hat{\eta}^+) + \hat{\eta}_z^2$$

$$\left.\frac{\langle \hat{\eta}^2 \rangle}{L}\right|_{\max} = \frac{1}{2}\left(\frac{L}{2} + 1\right)$$

System subjected to external electromagnetic field

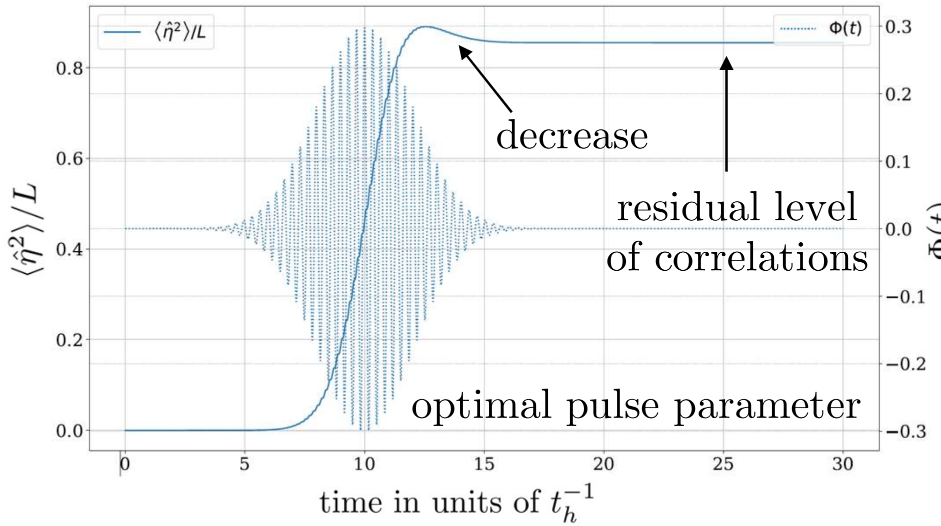
- Time-dependent Hamiltonian:

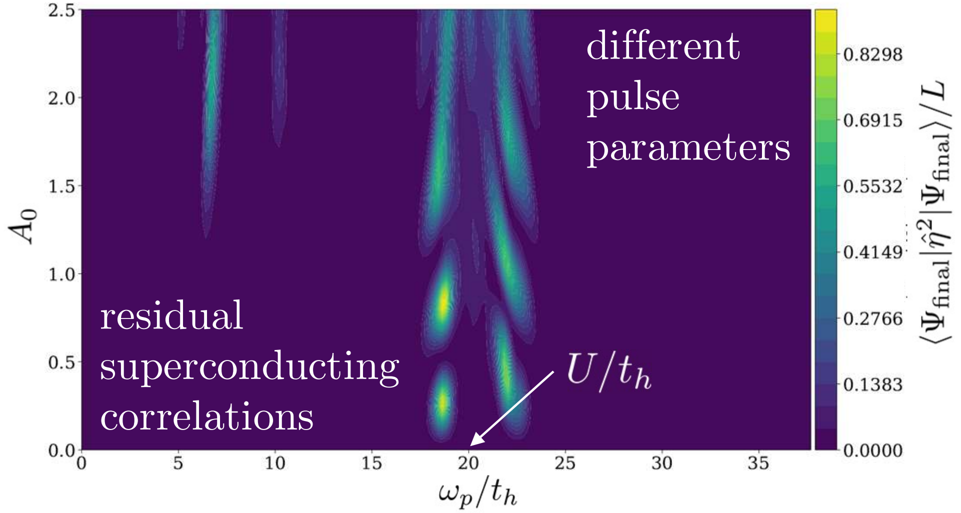
$$\hat{\mathcal{H}} = -t_h \sum_{\sigma,\langle i,j\rangle} (e^{i\Phi(t)} \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{j,\sigma} + h.c) + U \sum_{\sigma,i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

- Driving field:

$$\Phi(t) = aA_0 \cos[\omega_p(t - t_p)] \exp\left[-\frac{(t-t_p)^2}{2\sigma_p^2}\right]$$

- Problems:
  - The majority of pulses do not lead to excitation of the nontrivial superconducting properties
  - Residual correlations are weaker than the strongest achievable.
  - The strongest possible correlations are not reached





Improving evolution with Lyapunov control

- The idea: $\frac{\partial}{\partial t}\langle \hat{\eta}^2 \rangle \geq 0$

- Ehrenfest theorem:

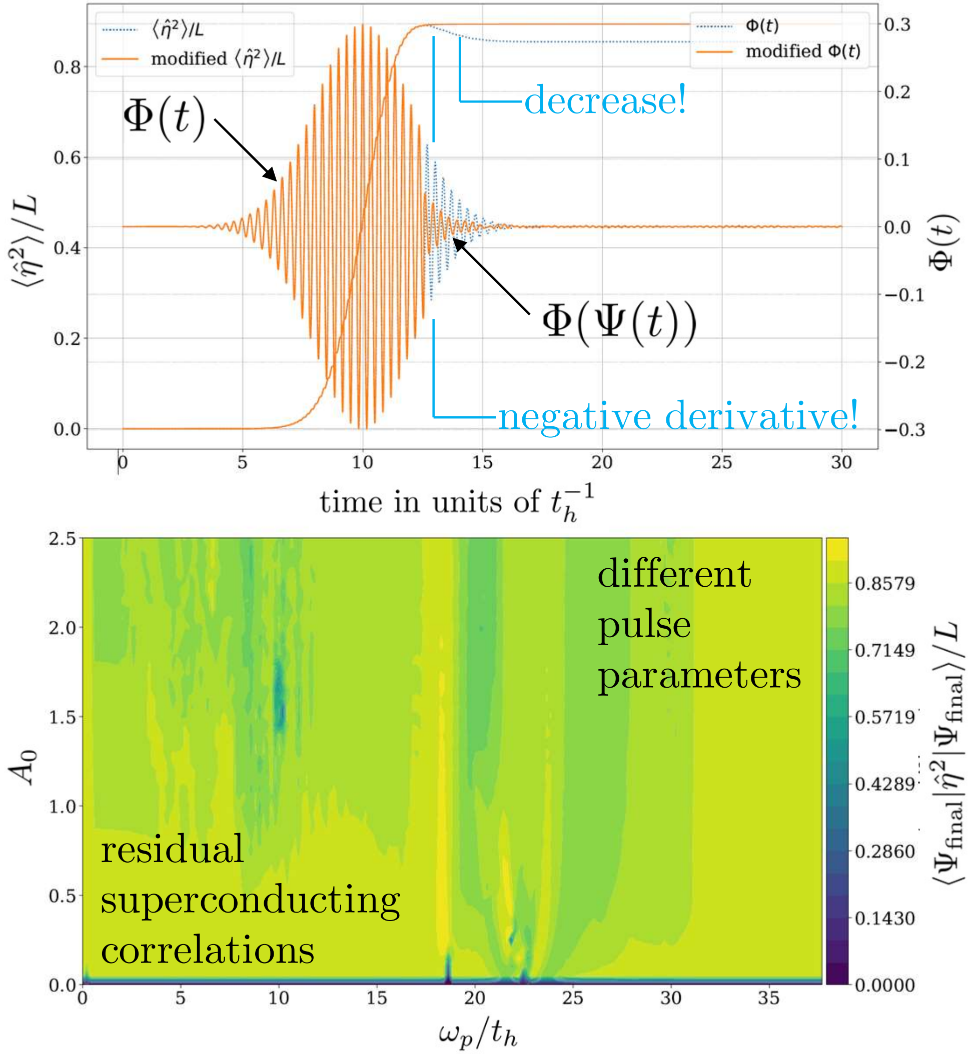
$$\frac{\partial}{\partial t}\left\langle \hat{\eta}^2 \right\rangle = \underbrace{-i\left\langle \left[\hat{\eta}^2, \hat{\mathcal{H}}\right] \right\rangle}_{t_h \langle \hat{\mathcal{Q}} \rangle \sin[\Phi(t)]} + \left\langle \frac{\partial \hat{\eta}^2}{\partial t} \right\rangle$$

- Control field:

$$\Phi(t) \to \arcsin \frac{\langle \Psi(t)|\hat{\mathcal{Q}}|\Psi(t)\rangle}{\max(|\{\mathcal{Q}_i\}_{\text{eigen}}|)}$$

$$\frac{\partial}{\partial t}|\Psi(t)\rangle = -i\hat{\mathcal{H}}(\Psi(t))|\Psi(t)\rangle$$

- Unsolved problem:
  - The strongest possible correlations are not reached



Capabilities of tracking control

- The idea: $\langle \hat{J} \rangle(t) = J_T(t)$
- Control field:

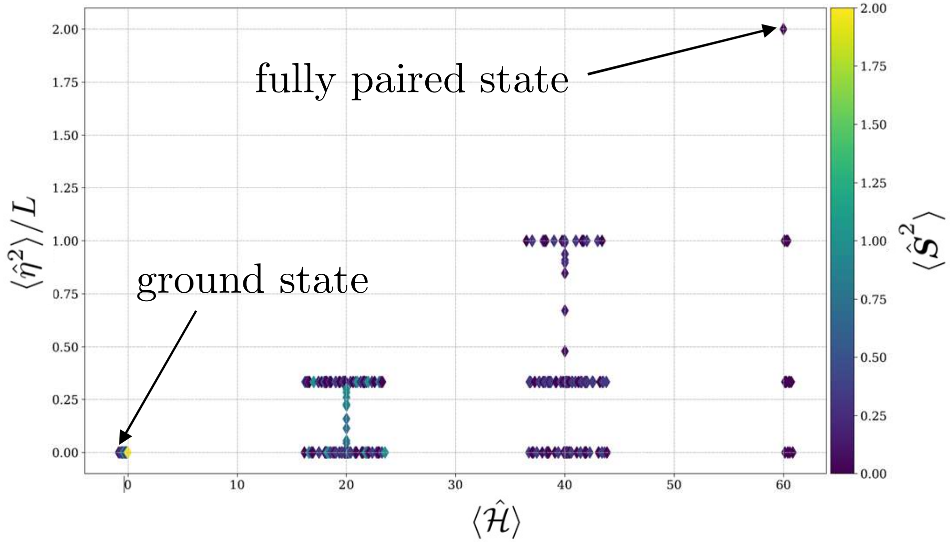
$$\Phi(t) \to \arcsin\left(\frac{J_T(t)}{2at_h R(\Psi(t))}\right) - \theta(\Psi(t))$$

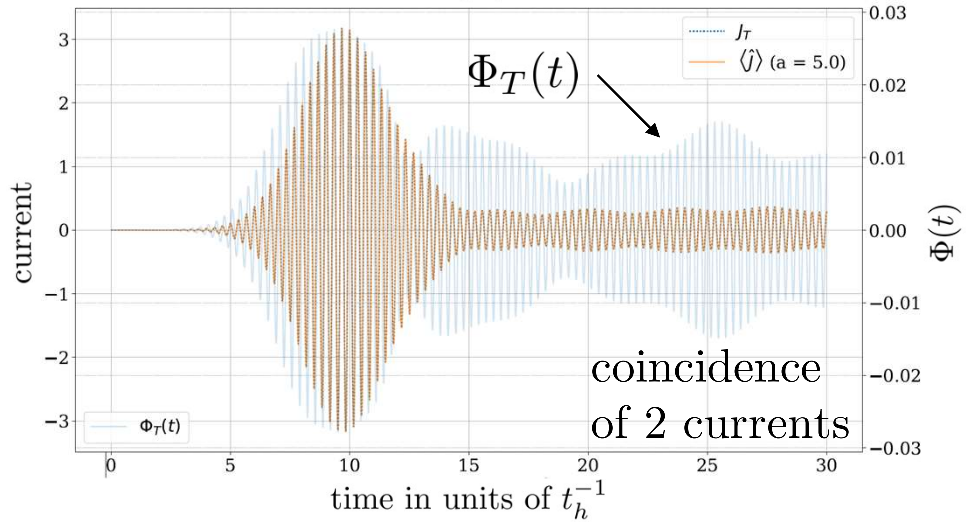
$$\frac{\partial}{\partial t}|\Psi(t)\rangle = -i\hat{\mathcal{H}}(t, \Psi(t))|\Psi(t)\rangle$$

- Constraints:

$$\left|\frac{J_T(t)}{2at_h R(\Psi(t))}\right| < 1 - \epsilon_1$$

$$|R(\Psi(t))| > 0 + \epsilon_2$$





Conclusions

- A brief look at the **Fermi – Hubbard model** and **Superconducting correlations**

- System under **external** driving **field**

- Lyapunov quantum control to **improve excitation** of superconducting correlations

- Tracking control to **mimic unreachable** system behaviour