



Particle Accelerator Physics

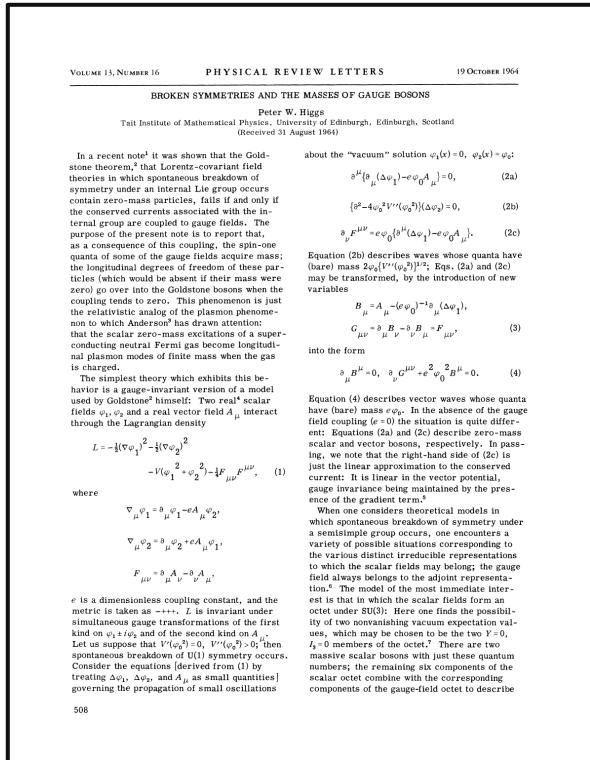
How to run a Supercollider
Part I

Pascal Hermes
CERN

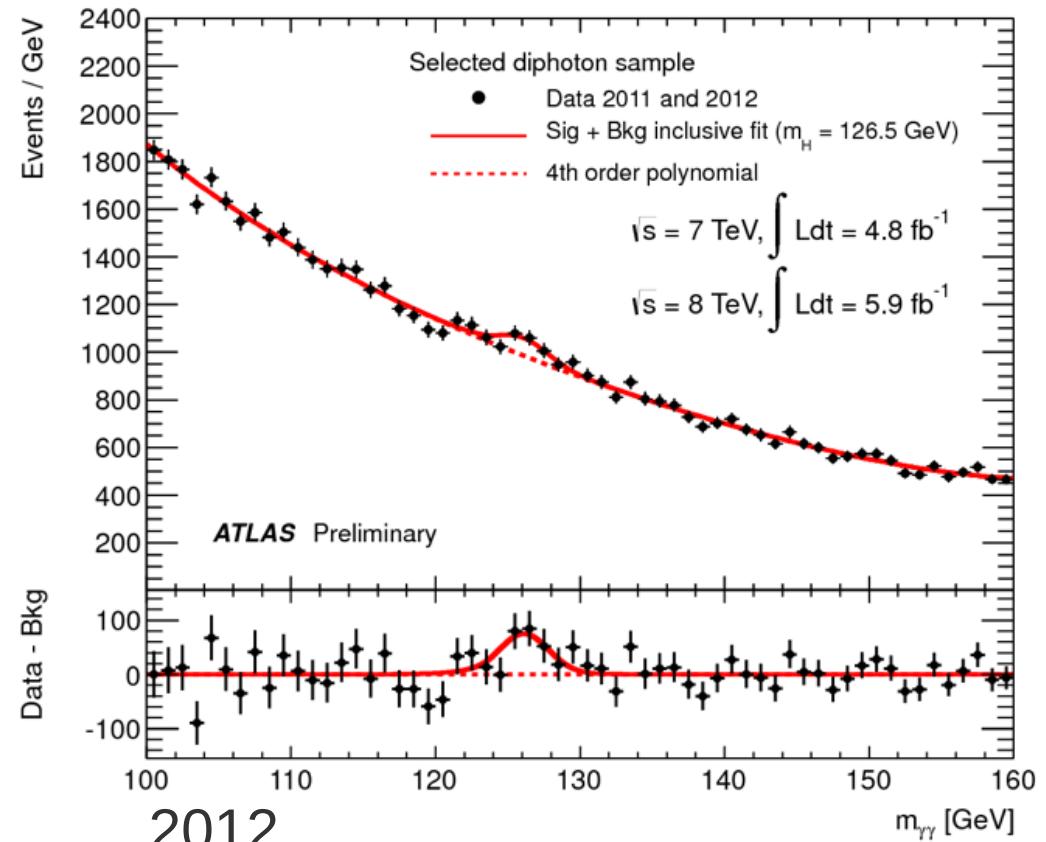
Trans-European School of High Energy Physics

15.07.2024

1964 – 2012: The Higgs Boson Adventure



1964

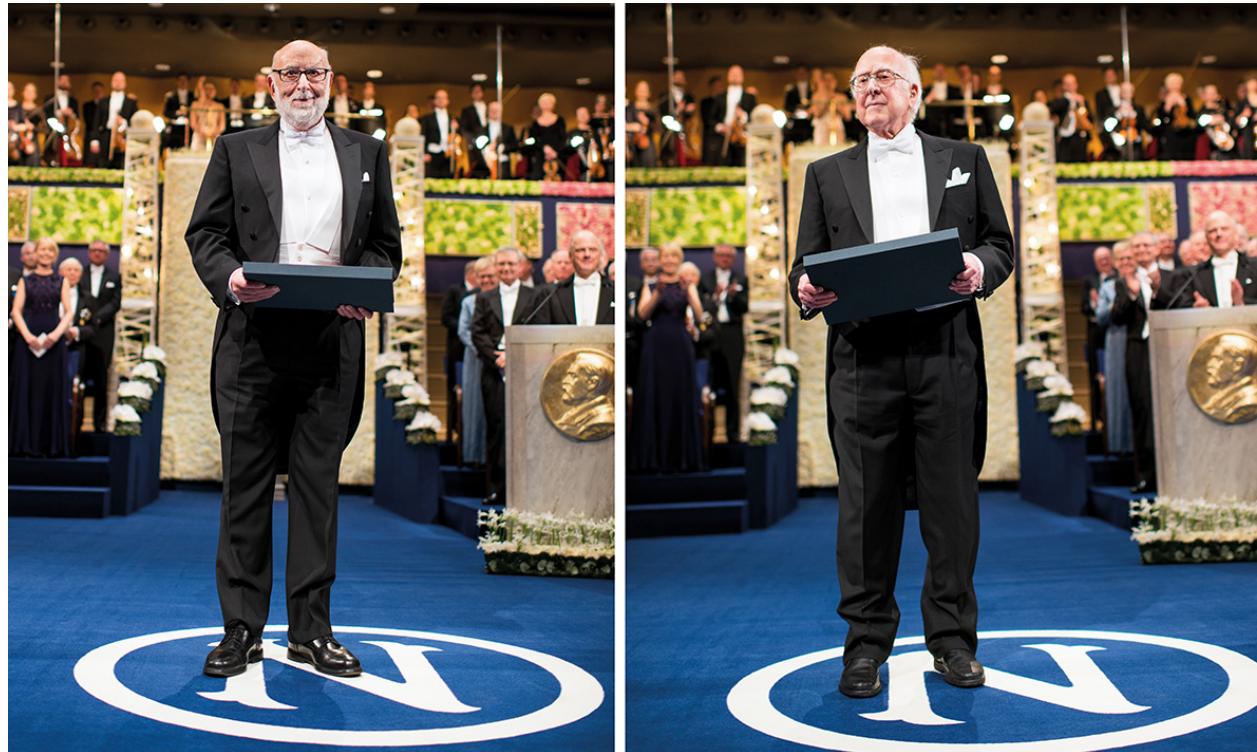


2012

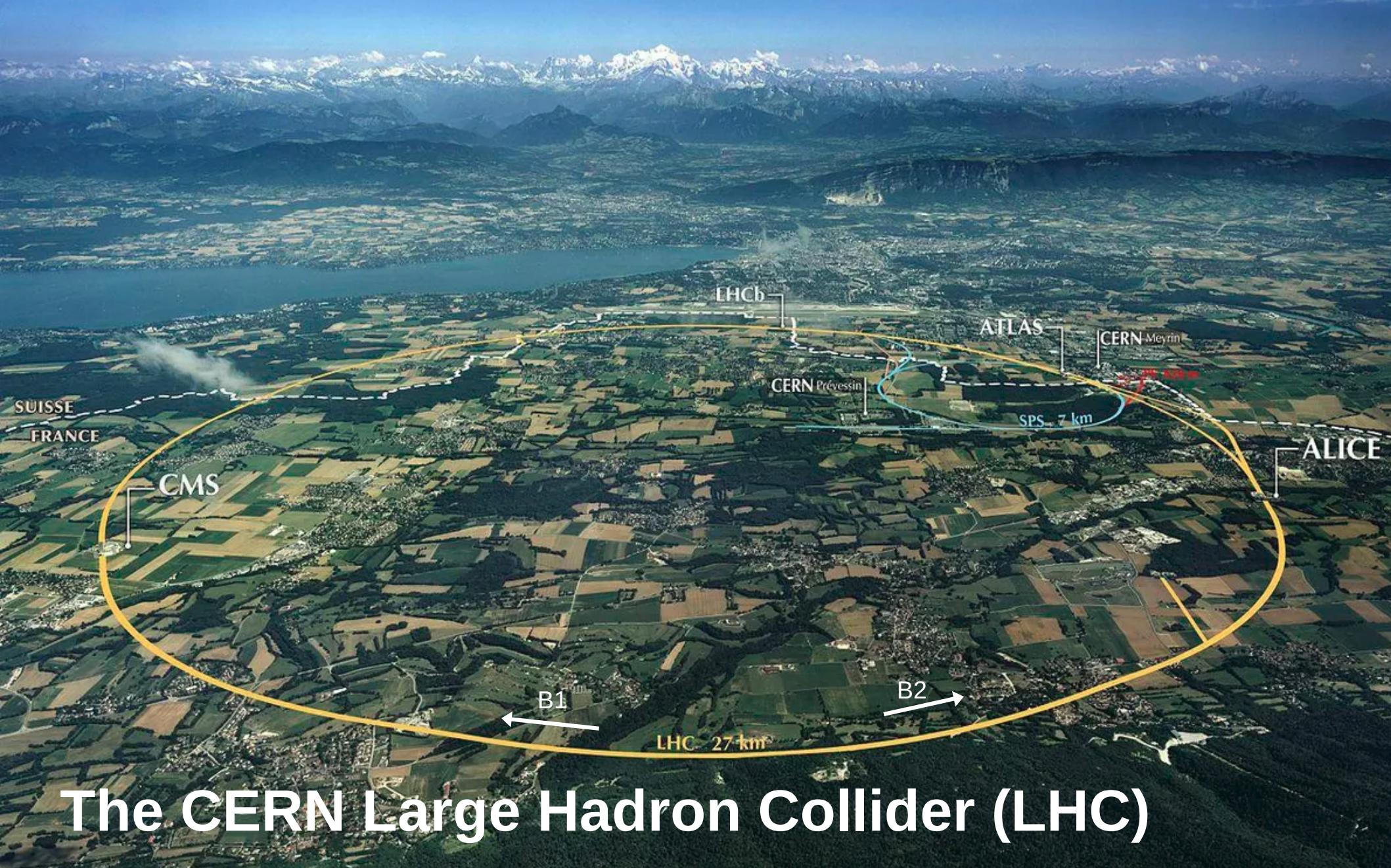
Introduction

48 years of search between prediction and discovery of the Higgs Boson

Why was it so challenging?

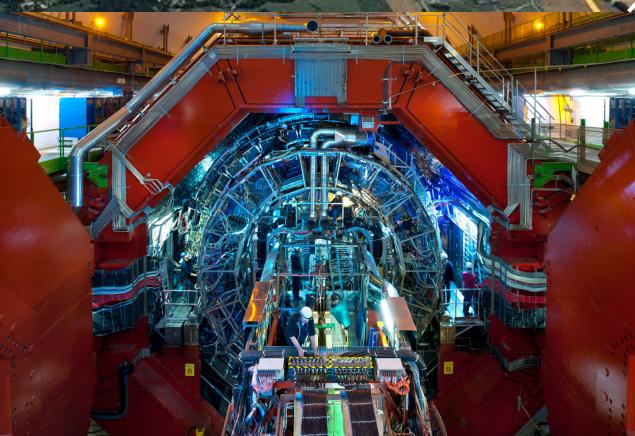
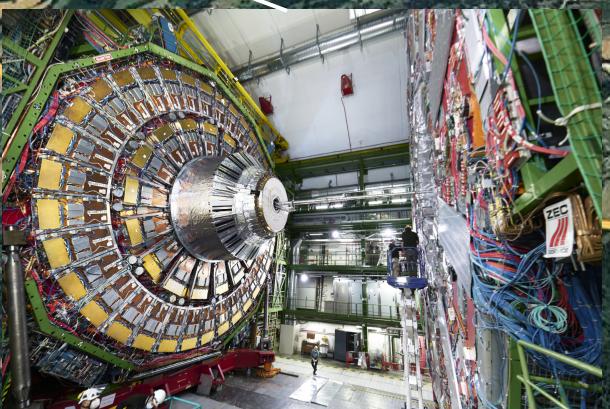
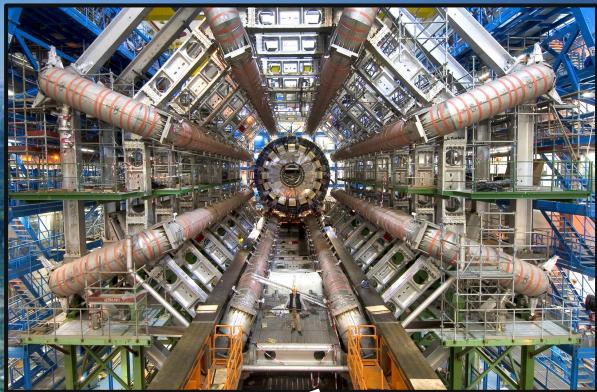
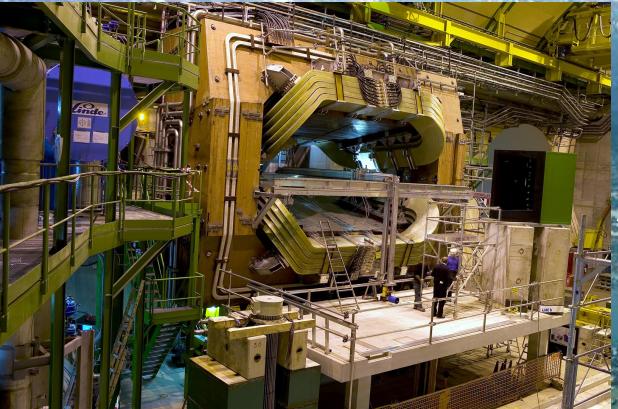


Credit: A. Mahmout / Nobel Media



LHC Physics Goals

- What is the origin of mass? (Higgs Boson) – **ATLAS and CMS**
- Will we discover evidence for supersymmetry? - **ATLAS and CMS**
- What are dark matter and dark energy? – **ATLAS, CMS**
- Why is there far more matter than antimatter in the universe? - **LHCb**
- How does the quark-gluon plasma give rise to the particles that constitute the matter of our Universe? - **ALICE**
- Smaller experiments FASER, ATLAS-ALFA, TOTEM, LHCf...



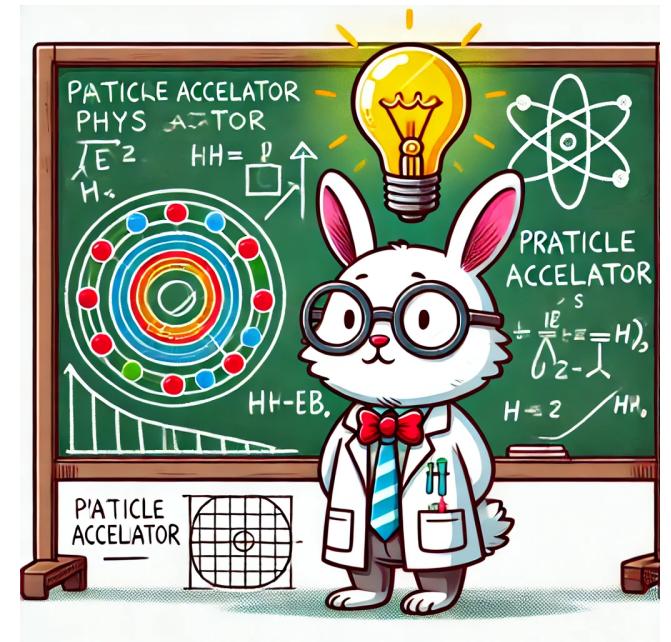
LHC Design Parameters	Unit	Protons		Lead Ions	
		Injection	Collision	Injection	Collision
Energy	[GeV]	450	7000	36900	574000
Relativistic γ		479.6	7461	190.5	2963.5
Max. Luminosity ^a	[cm ⁻² s ⁻¹]		$1.0 \cdot 10^{34}$		$1.0 \cdot 10^{27}$
Num. of bunches			2808		592
Bunch spacing	[ns]		24.95		118.58
Part. per bunch			$1.15 \cdot 10^{11}$		$6.7 \cdot 10^7$
Beam current	[A]		0.582		0.00612
Norm. emittance	[$\mu\text{m rad}$]	3.50	3.75	1.40	1.50
Bunch length σ_l	[cm]	11.24	7.55	9.97	7.94
Momentum spread	[10^{-3}]	1.90	0.45	0.39	0.11
β^* at IP2	[m]	10	10	10	0.55

This Lecture Series: Accelerator Physics

- **What do we need to generate LHC beams needed for Higgs search et al.?**
 - Magnets for focusing and steering
 - Particle beam acceleration
 - Beam Instrumentation
 - Pre-accelerators (“Injectors”)
- **What are the biggest challenges in operation of the LHC?**
 - Performance reach
 - Machine safety and operational efficiency

Main takeaways

- Concept of beam steering, focusing and acceleration
- Basics of linear accelerators and synchrotrons (like the LHC)
- Betatron motion and phase advance
- Concept of emittance
- Concept of β^* , crossing angle and luminosity
- Most important beam instrumentation devices
- We will illustrate all with LHC examples!



Magnets: Dipoles

Dipole Magnets

Moving coordinate system !

Spacially homogeneous magnetic field exploiting Lorentz

$$F = q E + q(v \times B)$$

Must be identical to centrifugal force!

$$\downarrow = \frac{\gamma mv^2}{\rho}$$

$$\frac{1}{\rho} [\text{m}^{-1}] = 0.2995 \frac{B_{\perp} [\text{T}]}{cp [\text{GV}]}$$

Bending radius ρ for perpendicular B-field

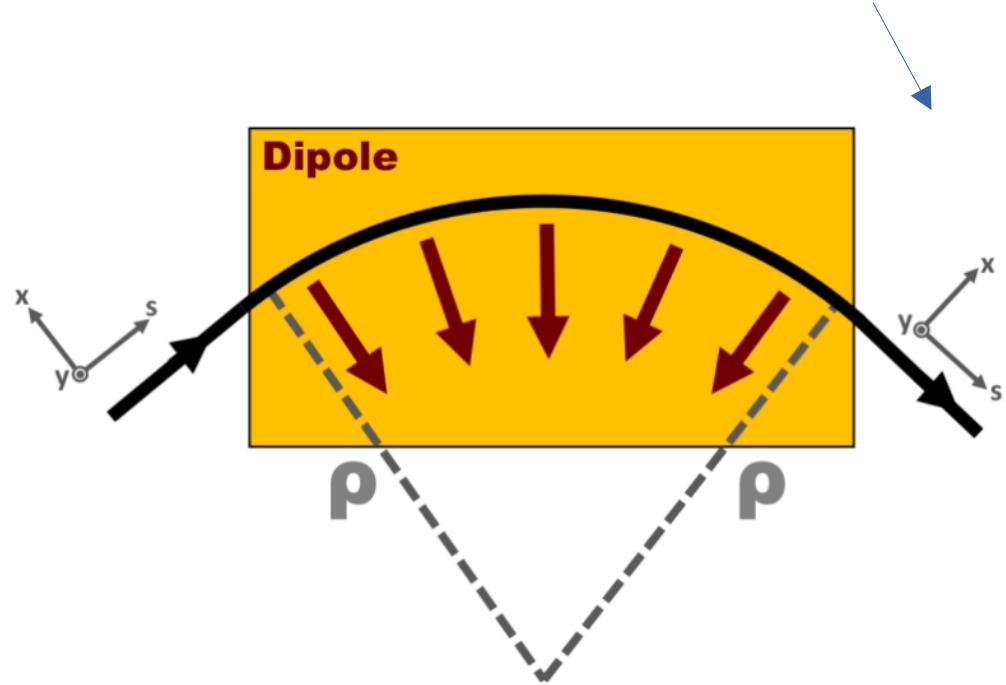
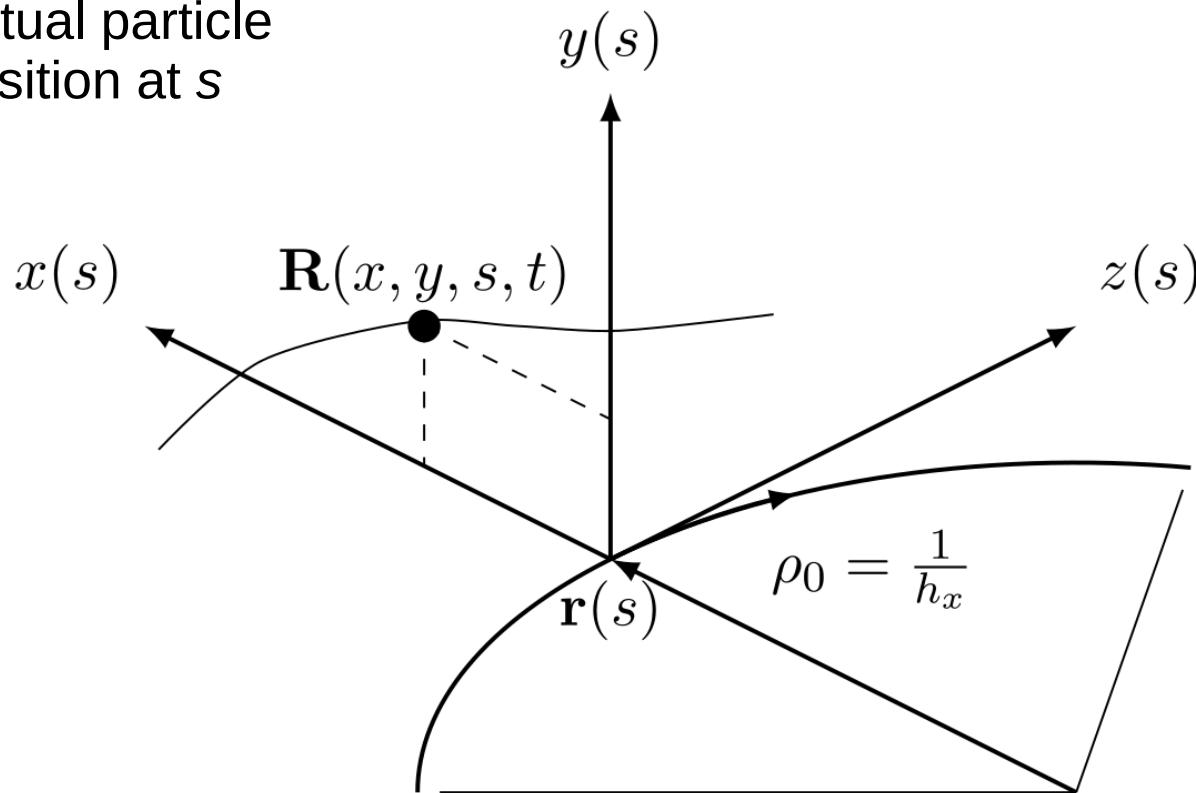


Figure courtesy of J. Dilly

Coordinate system

Actual particle
position at s



Reference
particle trajectory
→ function of s

Dipole Magnets

Can we steer the beam
also with an E-field?

$$F = q E + q (\boxed{v} \times B)$$

At the LHC:

$$v \approx c = 2.998 \times 10^8 \text{ m/s}$$

Equivalent force from
electric and magnetic field



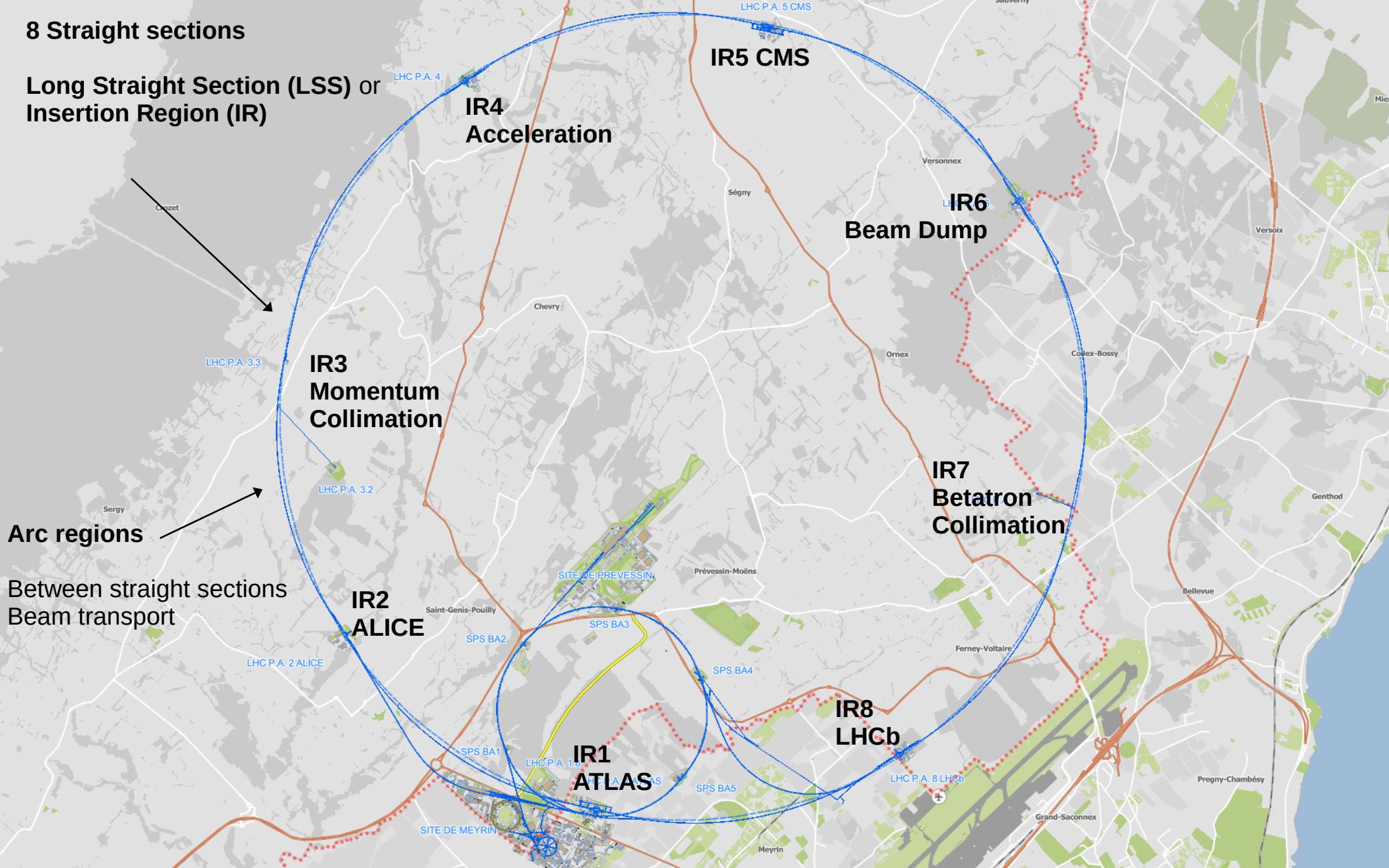
$$1 \text{ T} \cong 3 \times 10^8 \text{ V/m}$$



Only magnets are used for
beam steering!

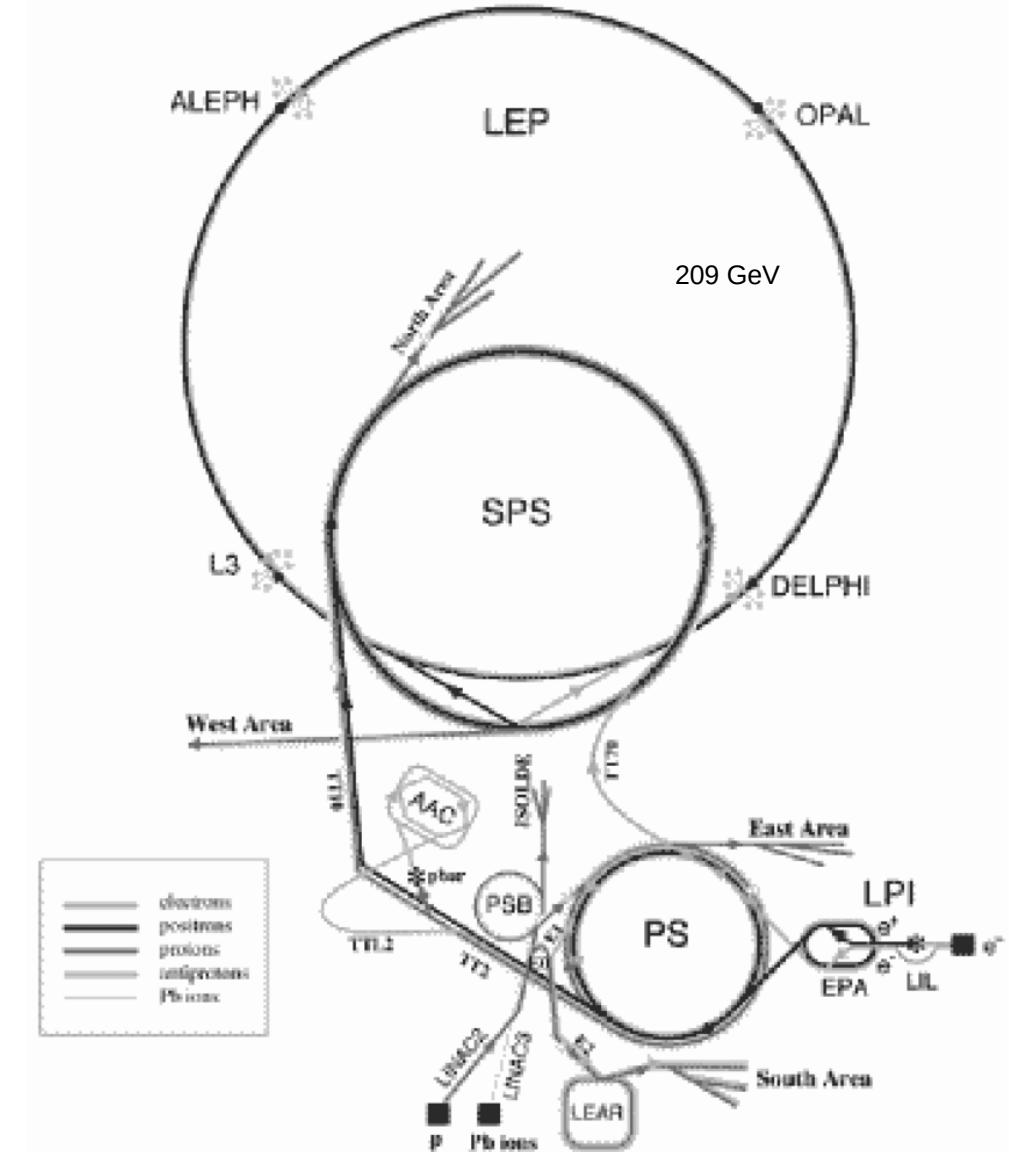
8 Straight sections

Long Straight Section (LSS) or
Insertion Region (IR)



The LHC Magnets

- Design of LHC was constraint by dimensions of existing LEP tunnel



LHC Dipoles

Bending of beam trajectory in arcs

Target Energy: 7 TeV

$\rho = 2800 \text{ m}$
(Existing Tunnel Geometry)

LHC Dipole Magnets

$$\frac{1}{\rho} = 0.2995 \frac{B[T]}{cp[GeV]}$$

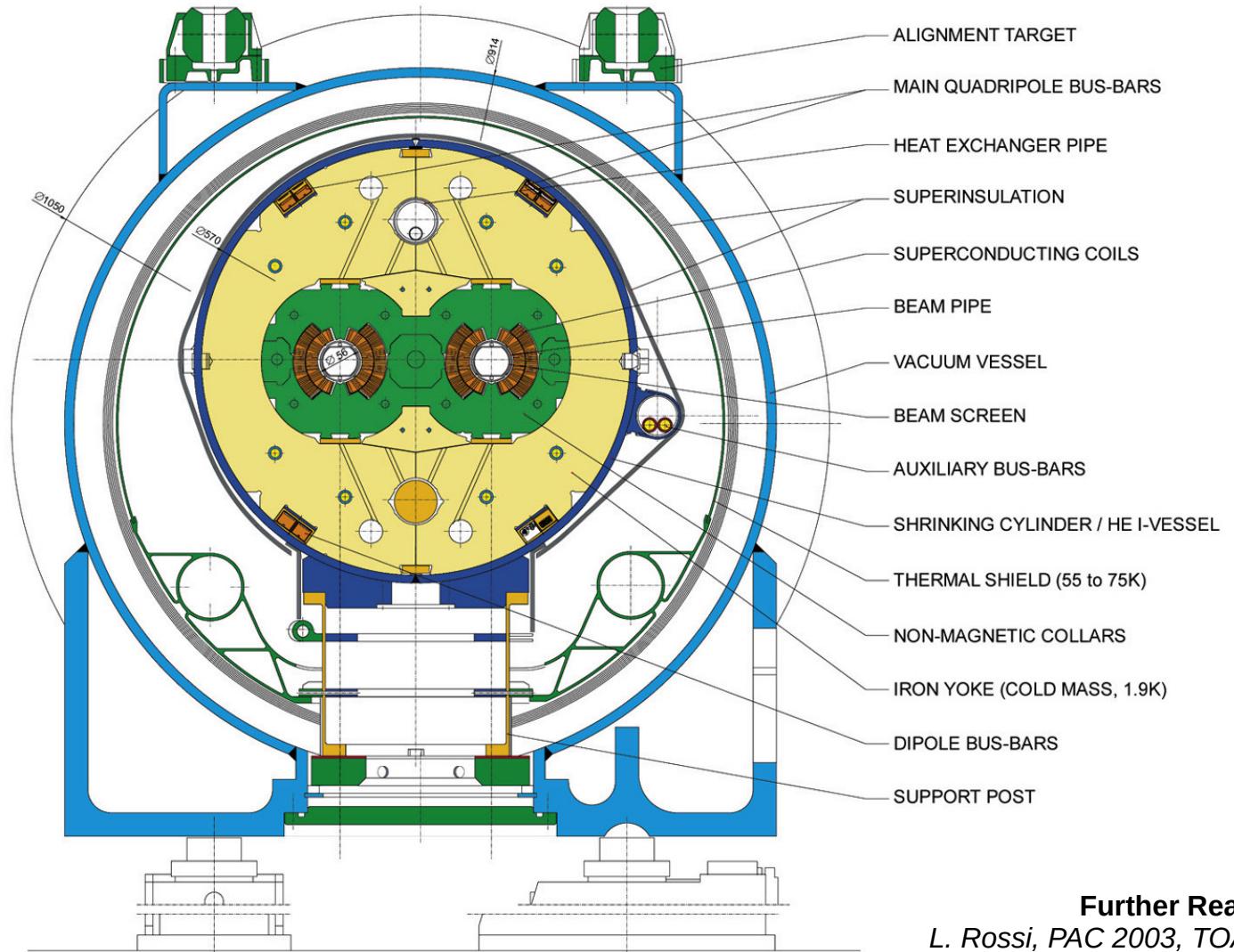
Target Proton Energy = 7 TeV
 $\rho = 2800$ m
(Existing Tunnel!)

→ Required Magnetic Field of 8.3T

LHC DIPOLE : STANDARD CROSS-SECTION

CERN AC/DI/MM - HE107 - 30 04 1999

- Double bore magnet
- Nominal B -field of 8.3T
- Nominal operating $T = 1.9\text{K}$
- $I = 11850 \text{ A}$
- 1232 x in the LHC
- $L = 15\text{m}$



Further Reading:
L. Rossi, PAC 2003, TOAB001

LHC Dipoles: Thermal expansion

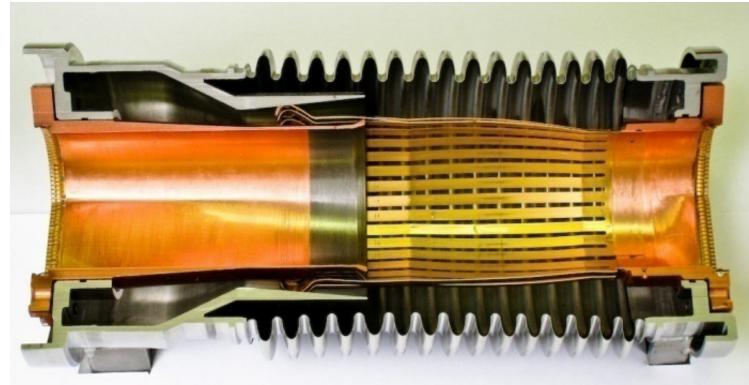
Thermal Contraction of an LHC dipole?

15m long

Thermal expansion coefficient (steel) $11 \times 10^{-6} \frac{\text{m}}{\text{m K}}$

$$\Delta T \approx 291 \text{ K}$$

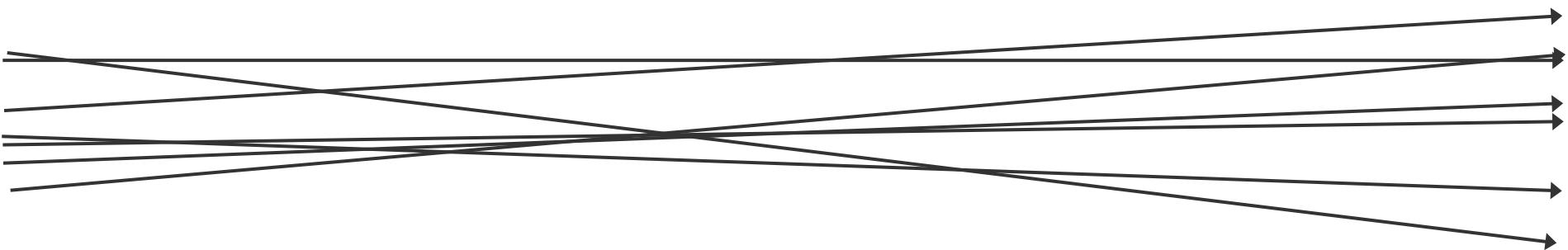
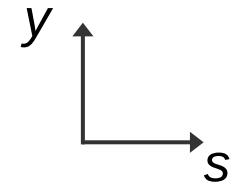
$$\rightarrow \Delta L \approx 5 \text{ cm}$$



D. Ramos, Proceedings of EPAC08, Genoa, TUPD035

Magnets: Quadrupoles

The need for beam focusing



Beam particles will not move straight

We need focusing!

Quadrupoles

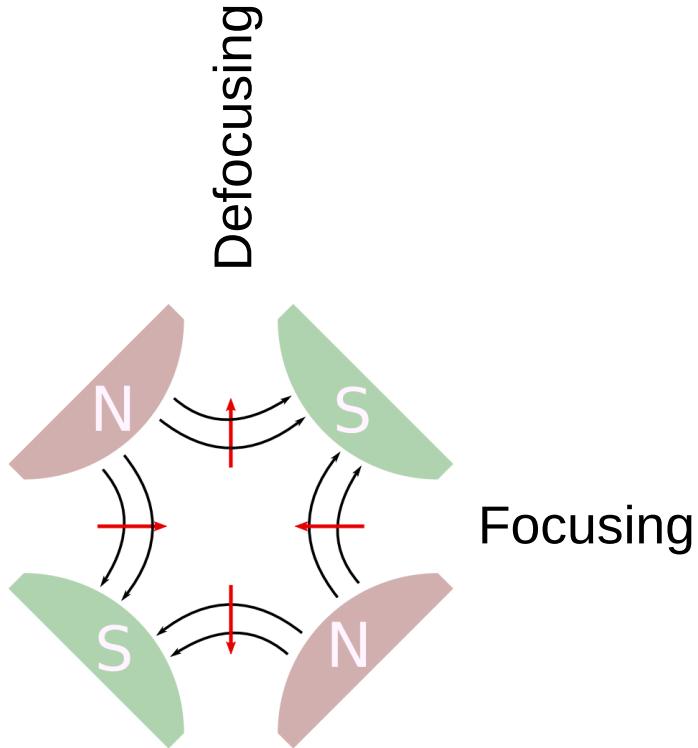
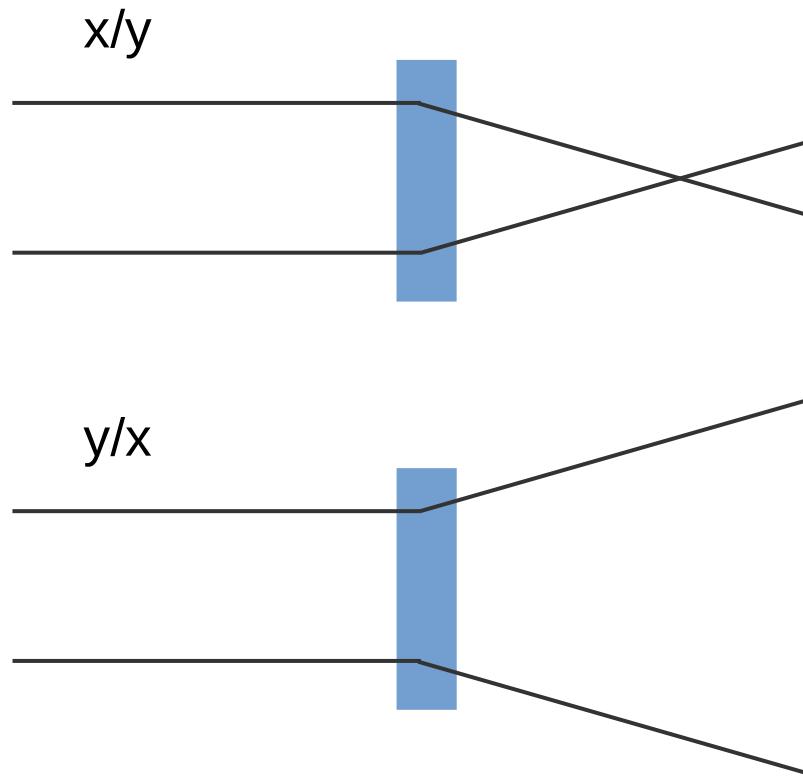
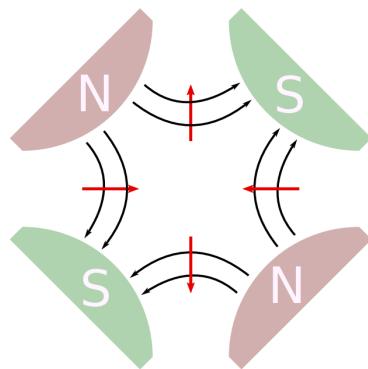


Figure: Courtesy of J. Dilly





Gradient

$$B_x = -g y$$

$$B_y = -g x$$

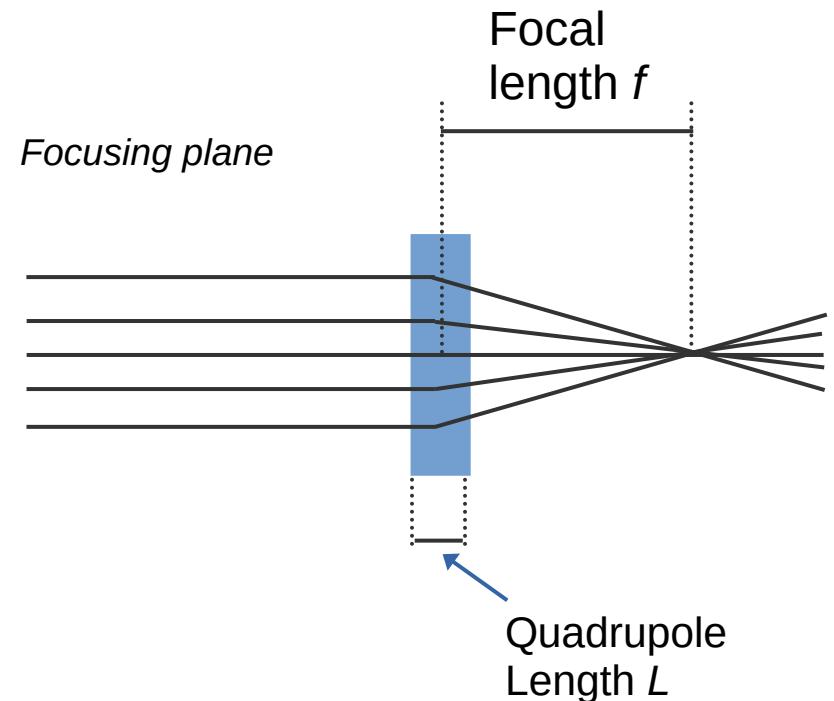
After some derivation:

$$f_x = -\frac{p}{q g L} = -\frac{1}{k L}$$

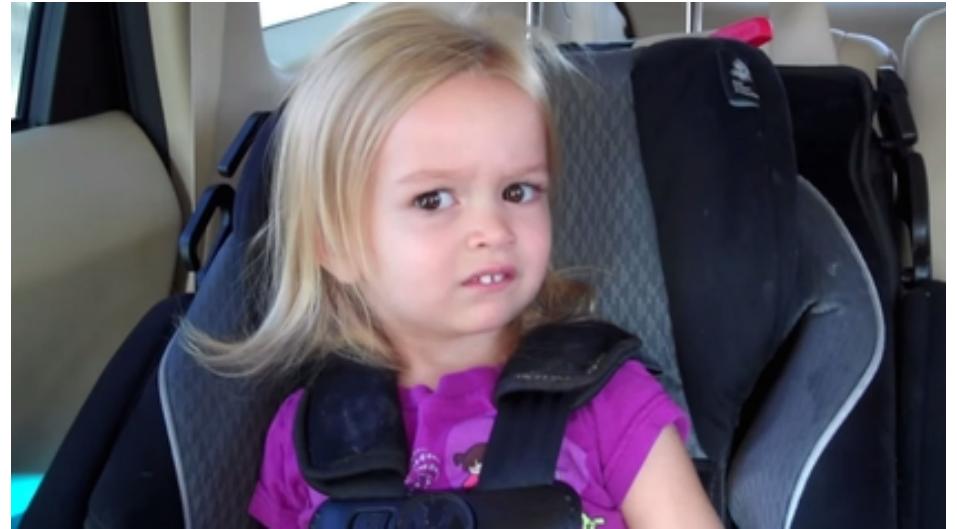
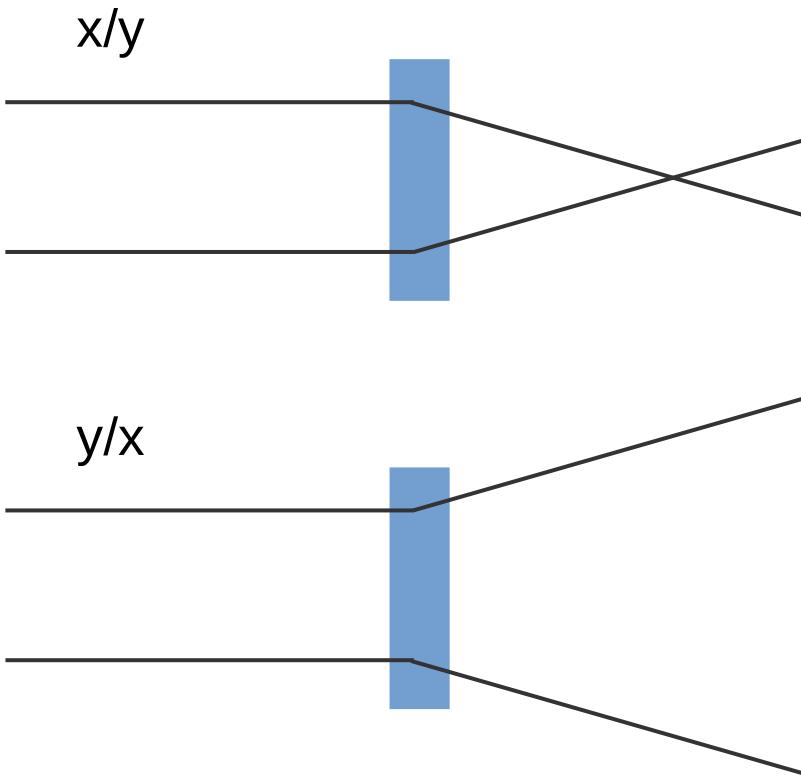
We define k as the **normalized quadrupole gradient**

- Energy independent
- Charge independent

Quadrupoles



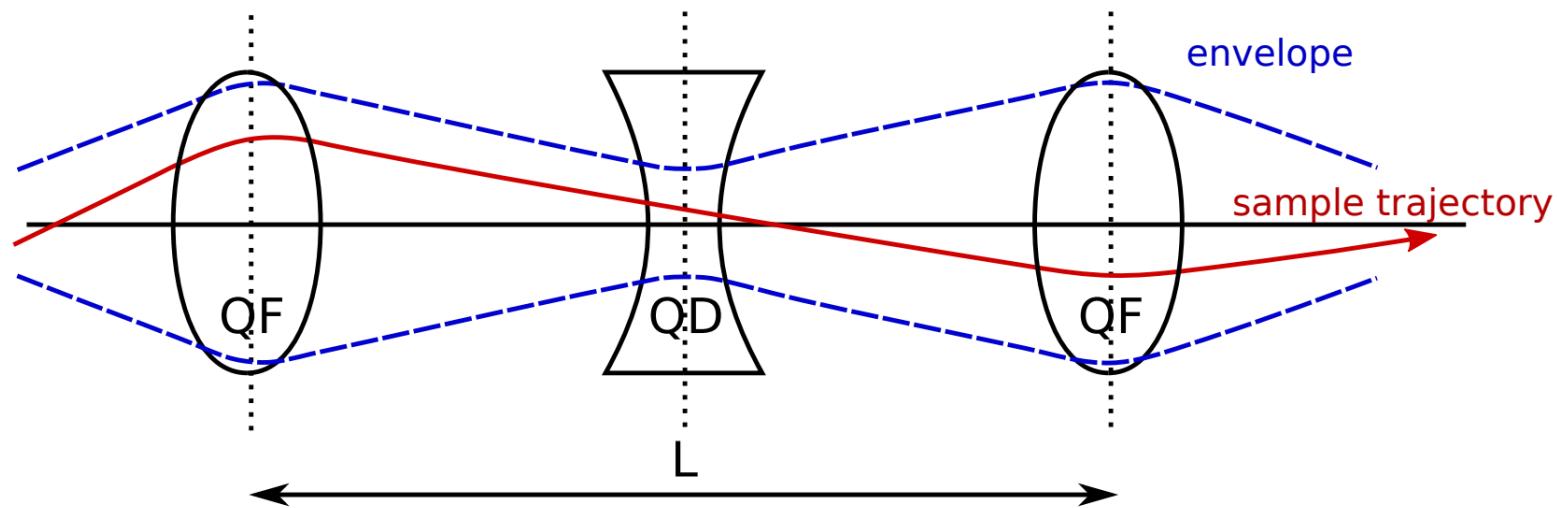
Quadrupoles



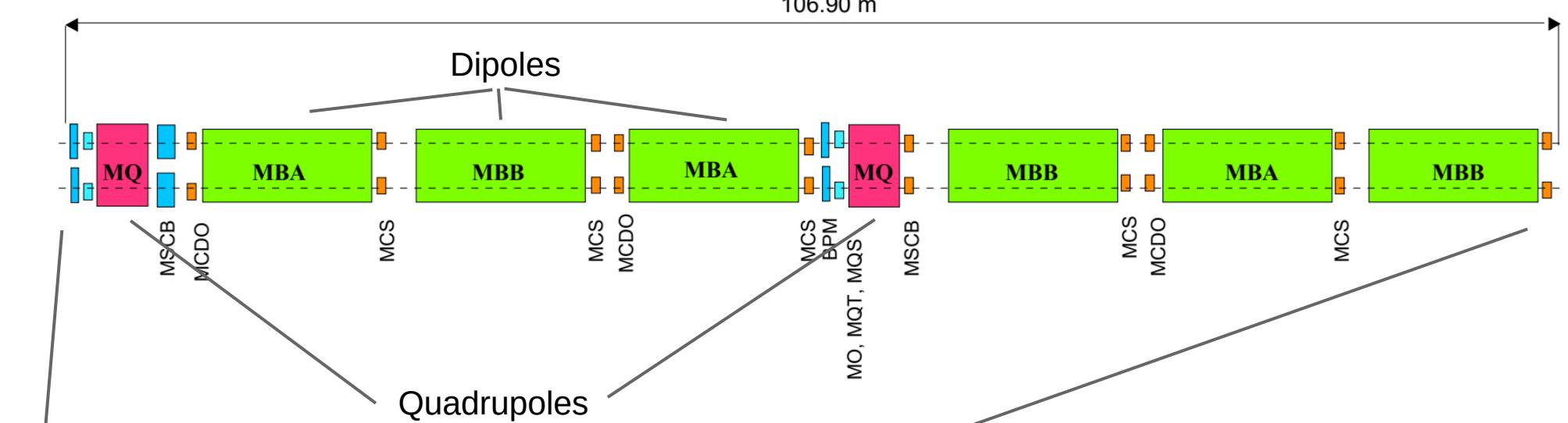
"How can we use quadrupoles for focusing if they are also defocusing in the orthogonal plane?"

Optical quadrupole lattice

Figure: Courtesy of J. Dilly



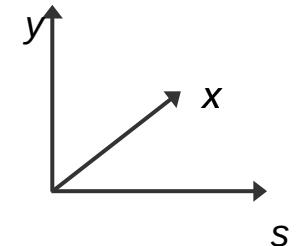
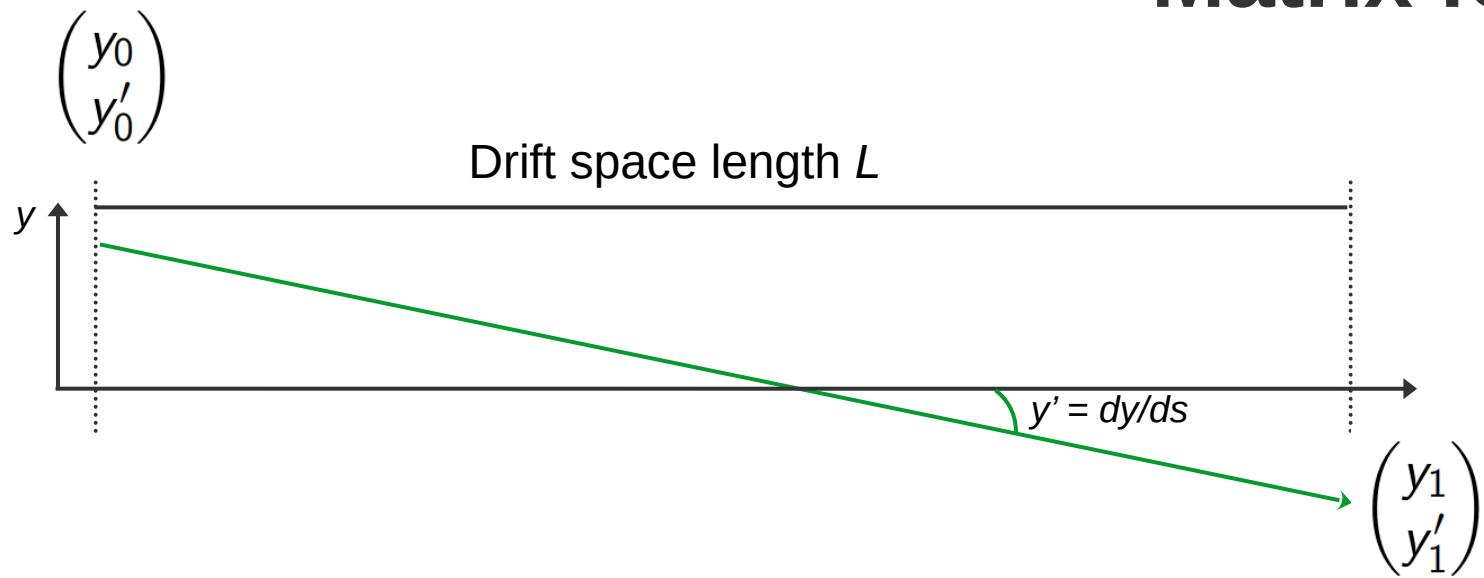
- Quadrupoles must be used in combination
- Fully analogous to an optical system for photons
- **Beam optics**



- LHC arc regions with **FODO lattice**:
Alternating lattice of focusing + defocusing Quads
- Used for “beam transport” between straight IRs

Matrix Formalism

Matrix formalism



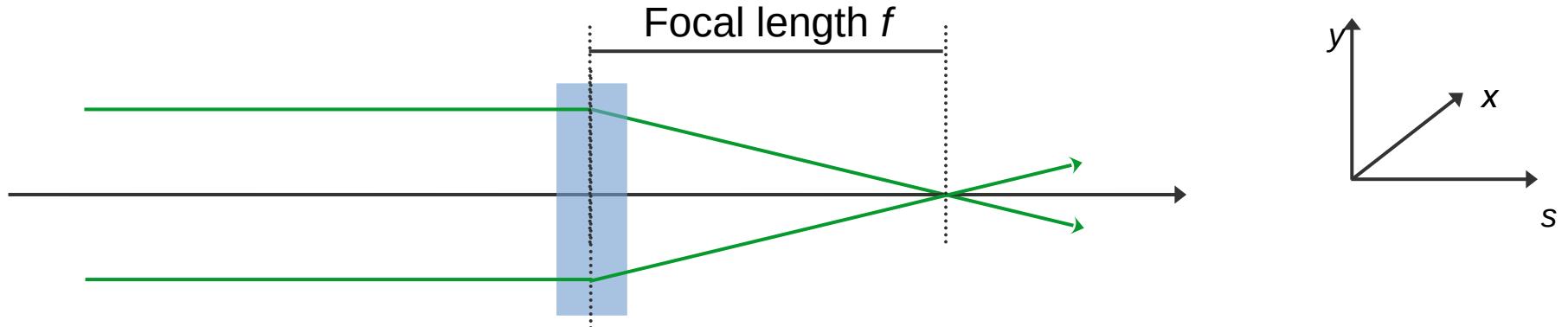
$$\begin{aligned}y_1 &= y_0 + y'_0 \cdot L \\y'_1 &= y'_0\end{aligned}$$



$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Transfer Matrix M_D

Matrix Formalism - Quadrupole



Thin-lens
approximation

$$f = -\frac{1}{k L}$$

$$y_1 = y_0$$

$$y'_1 = y'_0 \pm \frac{1}{f} y_0$$

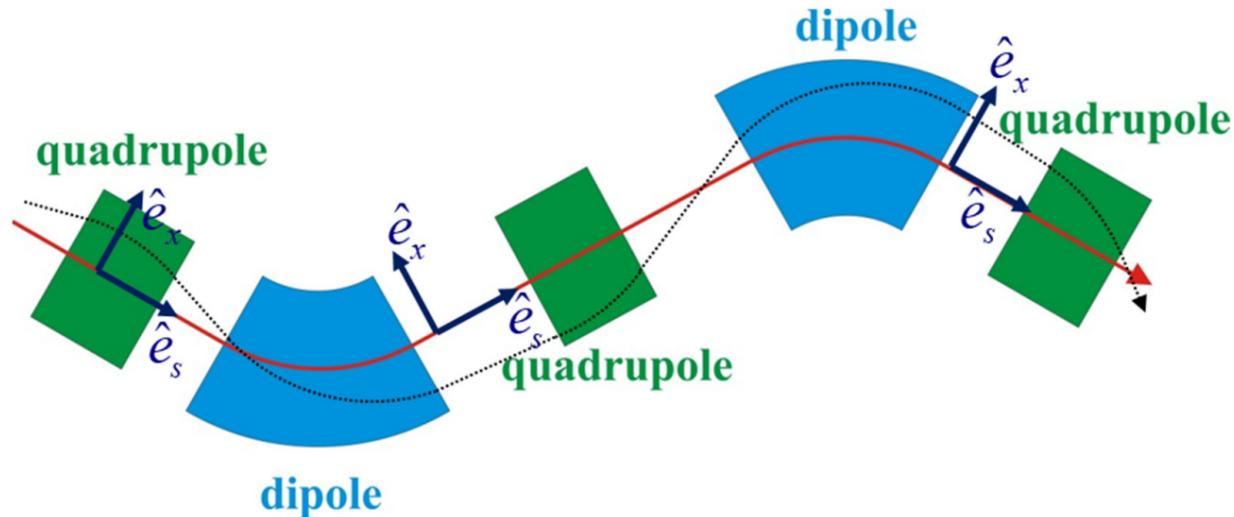


$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \pm \frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Transfer Matrix M_Q

Matrix Formalism – Transfer Matrix

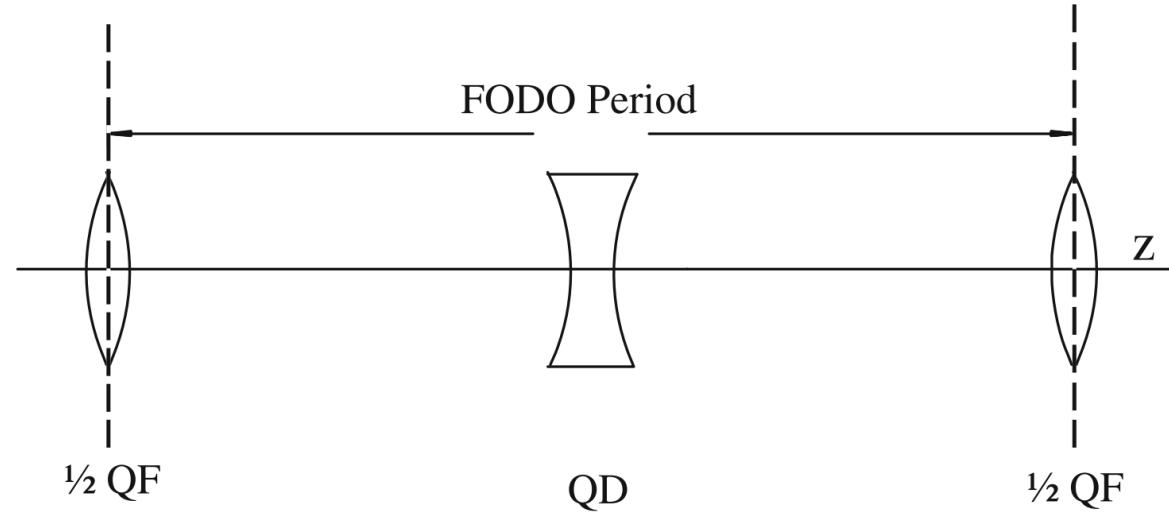
Can combine matrices → calculate
 \mathbf{M} for combination of elements



$$\vec{x} = \underbrace{\mathbf{M}_d \cdot \mathbf{M}_Q \cdot \mathbf{M}_d \cdot \mathbf{M}_D \cdot \mathbf{M}_d \cdot \mathbf{M}_Q \cdot \mathbf{M}_d \cdot \mathbf{M}_D \cdot \mathbf{M}_d \cdot \mathbf{M}_Q \cdot \mathbf{M}_d \cdot \vec{x}_0}_{= \text{Transfer Matrix } \mathbf{M}}$$

Figure: Hillert, CAS

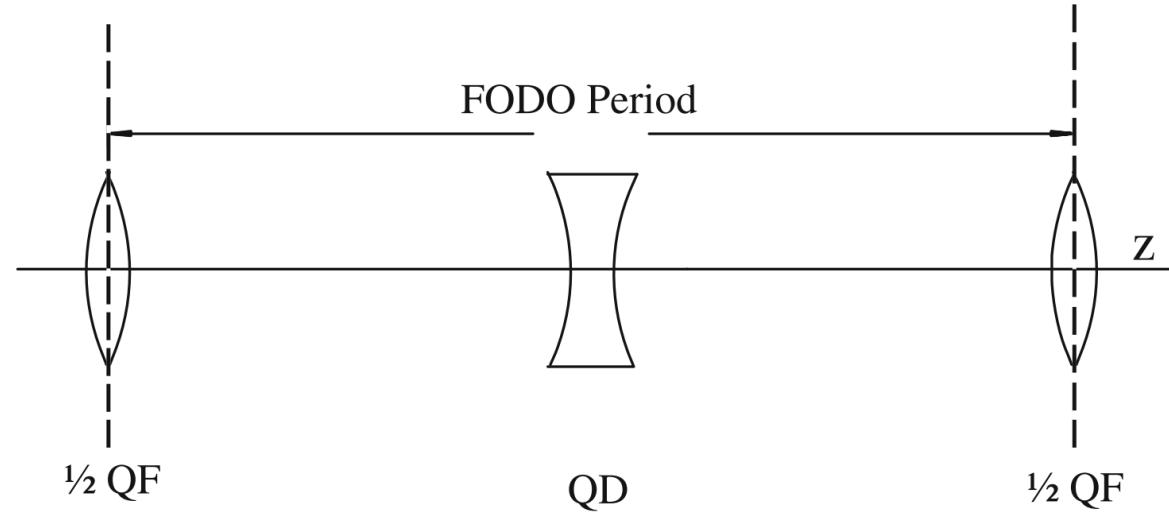
Matrix Formalism



$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \pm\frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Quad}$$

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Drift}$$

Matrix Formalism

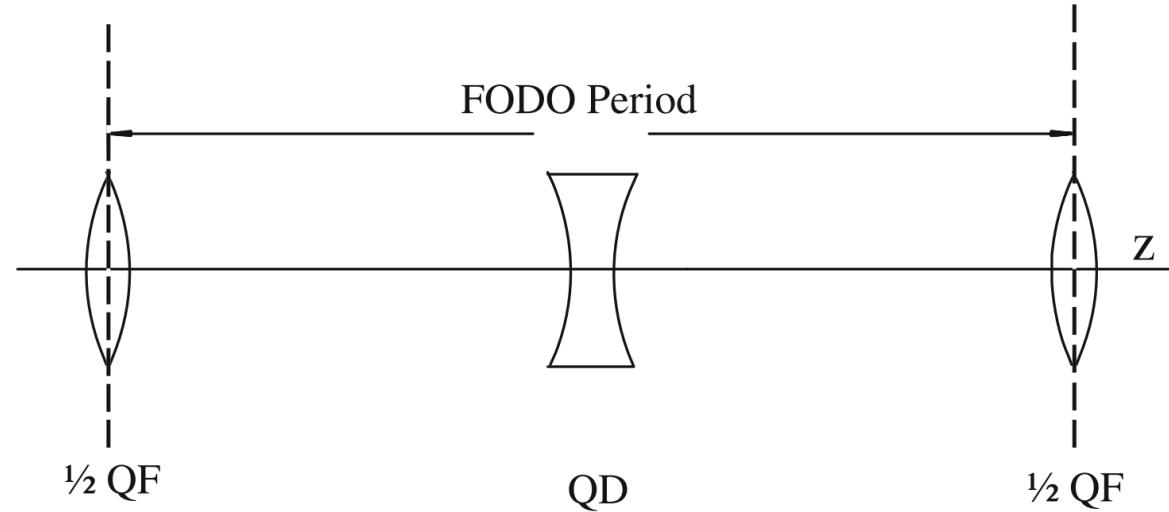


$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \pm \frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Quad}$$

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Drift}$$

Exercise: Calculate transfer matrix for FODO cell

Matrix Formalism



$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \pm\frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Quad}$$

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} \quad \text{Drift}$$

$$\mathcal{M}_{\text{FODO}} = \begin{pmatrix} 1 - 2\frac{L^2}{f^2} & 2L\left(1 + \frac{L}{f}\right) \\ -\frac{1}{f^*} & 1 - 2\frac{L^2}{f^2} \end{pmatrix}$$

$$1/f^* = 2(1 - L/f)L/f^2$$

Equation of Motion

Equation of motion

$$\begin{aligned}x'' - k x &= 0 \\y'' + k y &= 0\end{aligned}$$

Hill's Equation

Simplest linear equation of motion for particles in magnetic lattice

Simplifications:

- “Weak focusing” from dipoles ignored
- Particle momentum offsets ignored (see tomorrow)

$$f = -\frac{1}{k L}$$

Equation of motion: Quadrupole

Solution for
quadrupole

$$\begin{aligned}x'' - k x &= 0 \\y'' + k y &= 0\end{aligned}$$



$$\begin{aligned}\mathcal{M}_{Q,f} &= \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}L) \\ -\sqrt{K} \sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}, \\ \mathcal{M}_{Q,d} &= \begin{pmatrix} \cosh(\sqrt{|K|}L) & \frac{1}{\sqrt{|K|}} \sinh(\sqrt{|K|}L) \\ \sqrt{|K|} \sinh(\sqrt{|K|}L) & \cosh(\sqrt{|K|}L) \end{pmatrix}.\end{aligned}$$



$$L \rightarrow 0 \quad \text{with} \quad K L = \text{const.}$$

$$\begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \pm \frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Thin lens approximation

$$k = k(z) = k(z + L_P)$$

Magnetic fields periodic
with revolution length



Equation of motion: Periodic Lattice

Here: we use z instead of s

$$k = k(z) = k(z + L_p)$$

Periodicity Length L_p

$$\begin{aligned}x'' - k x &= 0 \\y'' + k y &= 0\end{aligned}$$

Generalized u
 $u = x/y$

$$\begin{aligned}u_1(z) &= w(z) e^{i \mu z / L_p}, \\u_2(z) &= w^*(z) e^{-i \mu z / L_p}\end{aligned}$$

Two independent solutions

Select real solutions
 $w^*(z) = w(z)$

Periodic in L_p

$$w(z + L_p) = w(z)$$

Equation of motion: Periodic Lattice

$$\begin{aligned}x'' - k x &= 0 \\y'' + k y &= 0\end{aligned}$$

$$w(z + L_p) = w(z)$$

Two independent solutions

$$\begin{aligned}u_1(z) &= w(z) e^{i \mu z / L_p}, \\u_2(z) &= w^*(z) e^{-i \mu z / L_p}\end{aligned}$$

Transformation over one period:

$$u(z + L_p) = u(z) e^{\pm i \mu} = u(z) (\cos \mu \pm i \sin \mu)$$

Equation of motion: Periodic Lattice

$$\begin{aligned}x'' - k x &= 0 \\y'' + k y &= 0\end{aligned}$$

Solution for one period

$$u(z + L_p) = u(z) e^{\pm i \mu} = u(z) (\cos \mu \pm i \sin \mu)$$



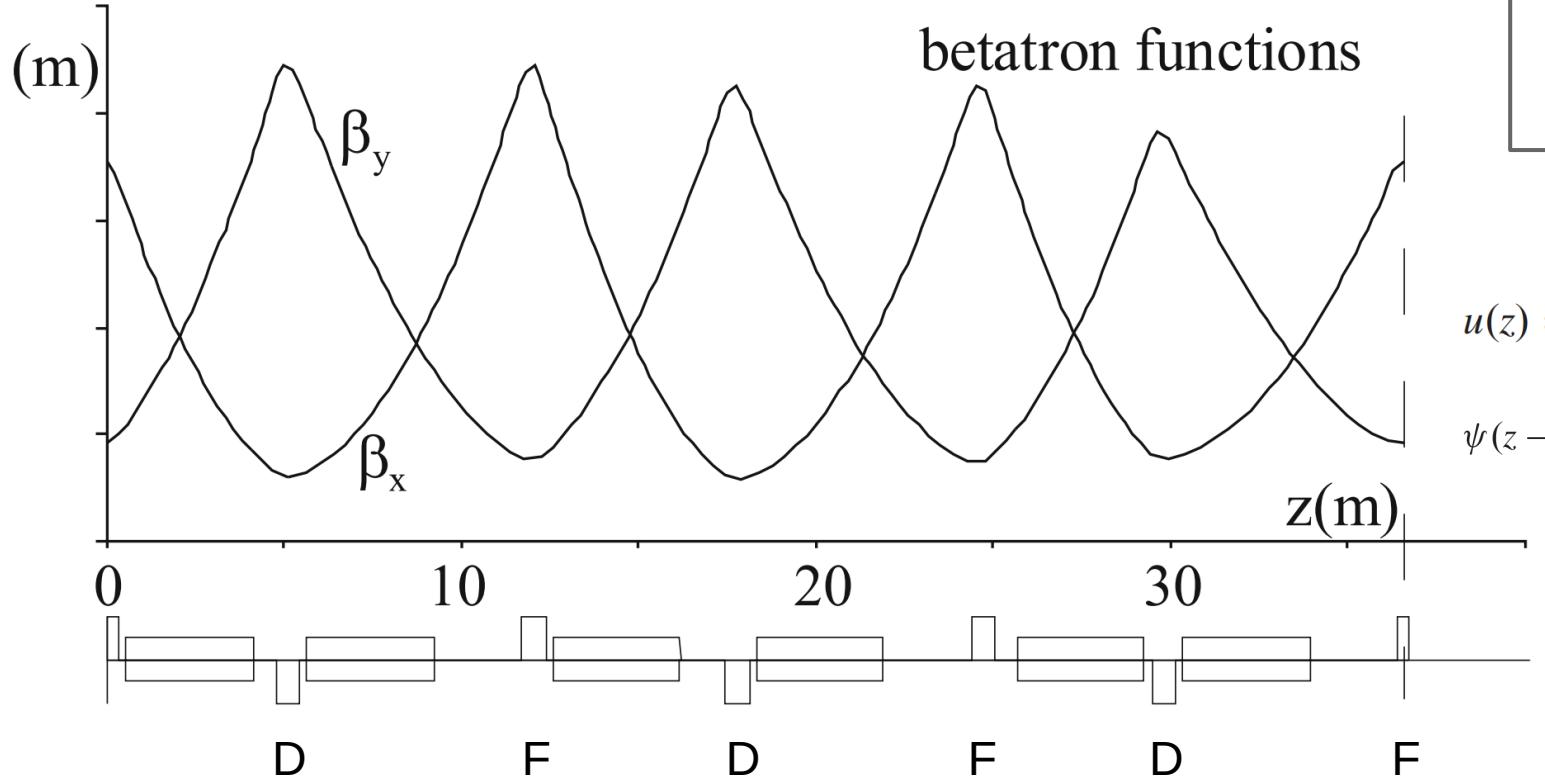
Some derivation

General Solution
incl. stability criterion

$$u(z) = a \sqrt{\beta(z)} e^{\pm i \psi}$$

$$\psi(z - z_0) = \int_{z_0}^z \frac{d\xi}{\beta(\xi)}$$

Betatron function (FODO)



$$x'' - k x = 0$$
$$y'' + k y = 0$$

$$u(z) = a \sqrt{\beta(z)} e^{\pm i \psi}$$

$$\psi(z - z_0) = \int_{z_0}^z \frac{d\xi}{\beta(\xi)}$$

Betatron function

$$u(z) = a \sqrt{\beta(z)} e^{\pm i \psi}$$

Maximum amplitude at z

$$u_{\max}(z) = a \sqrt{\beta(z)}$$

Property of the
particle considered

Property of the machine
lattice: “optics”

Betatron phase

$$\psi(z - z_0) = \int_{z_0}^z \frac{d\xi}{\beta(\xi)}$$

Defines the number of betatron oscillations
across a given length

Betatron function

$$\mathcal{M}(z + L_p \mid z) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$$

Individual Particles
Transfer matrix for u, u'



Beam lattice

Transfer matrix for *betatron parameters* α, β, γ

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + C'S - SS' \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} = \mathcal{M}_\beta \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

Twiss Parameters

$$\beta(z)$$
$$\alpha(z) = -\frac{1}{2} \frac{d\beta(z)}{dz}$$

$$\gamma(z) = \frac{1 + \alpha^2(z)}{\beta(z)}$$

Courant-Snyder Invariant

$$u(z) = \sqrt{2 J \beta(z)} \cos(\psi(z) + \psi_0)$$

$$u'(z) = \sqrt{\frac{2 J}{\beta(z)}} [\sin(\psi(z) + \psi_0) + \alpha(z) \cos(\psi(z) + \psi_0)]$$

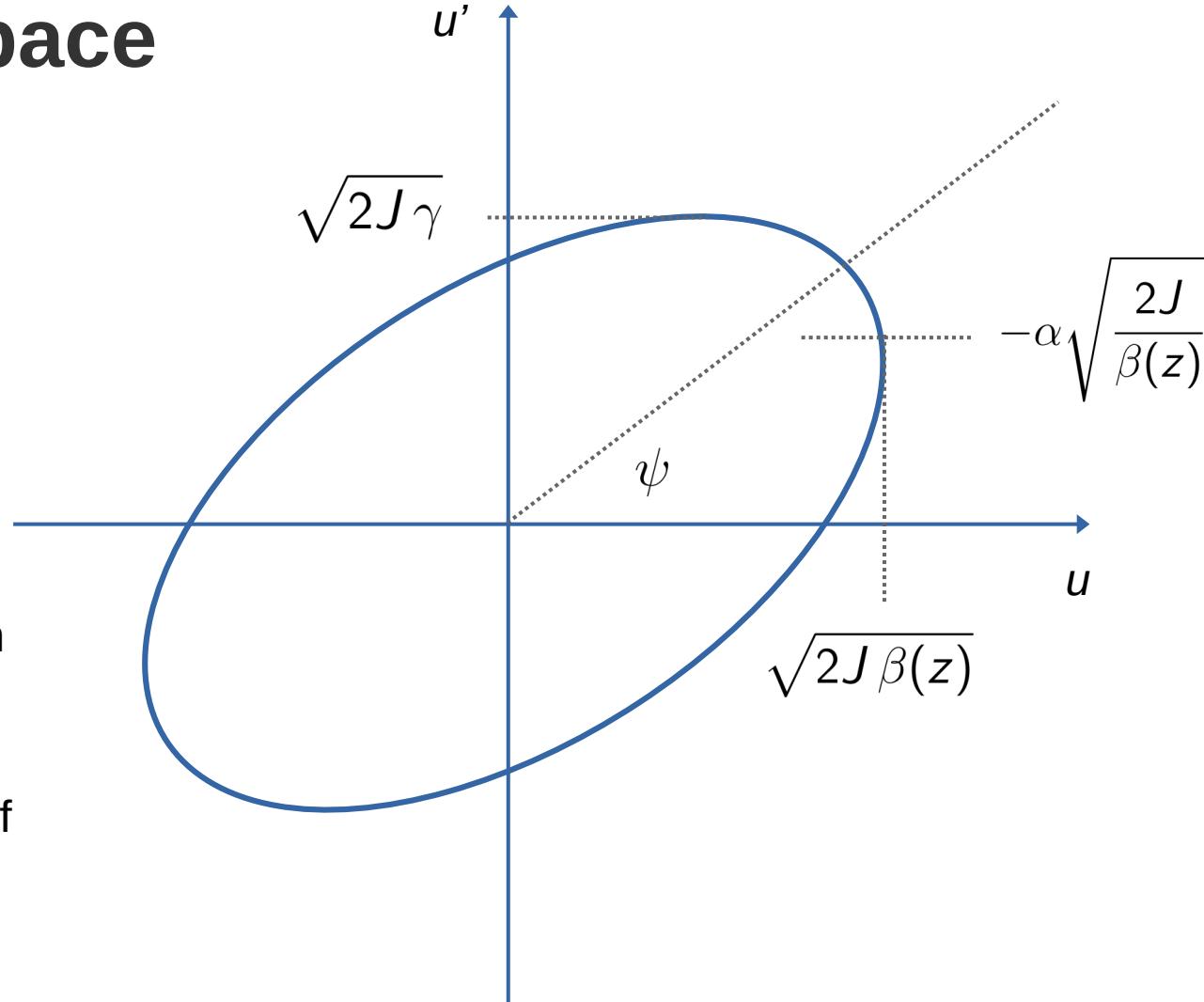


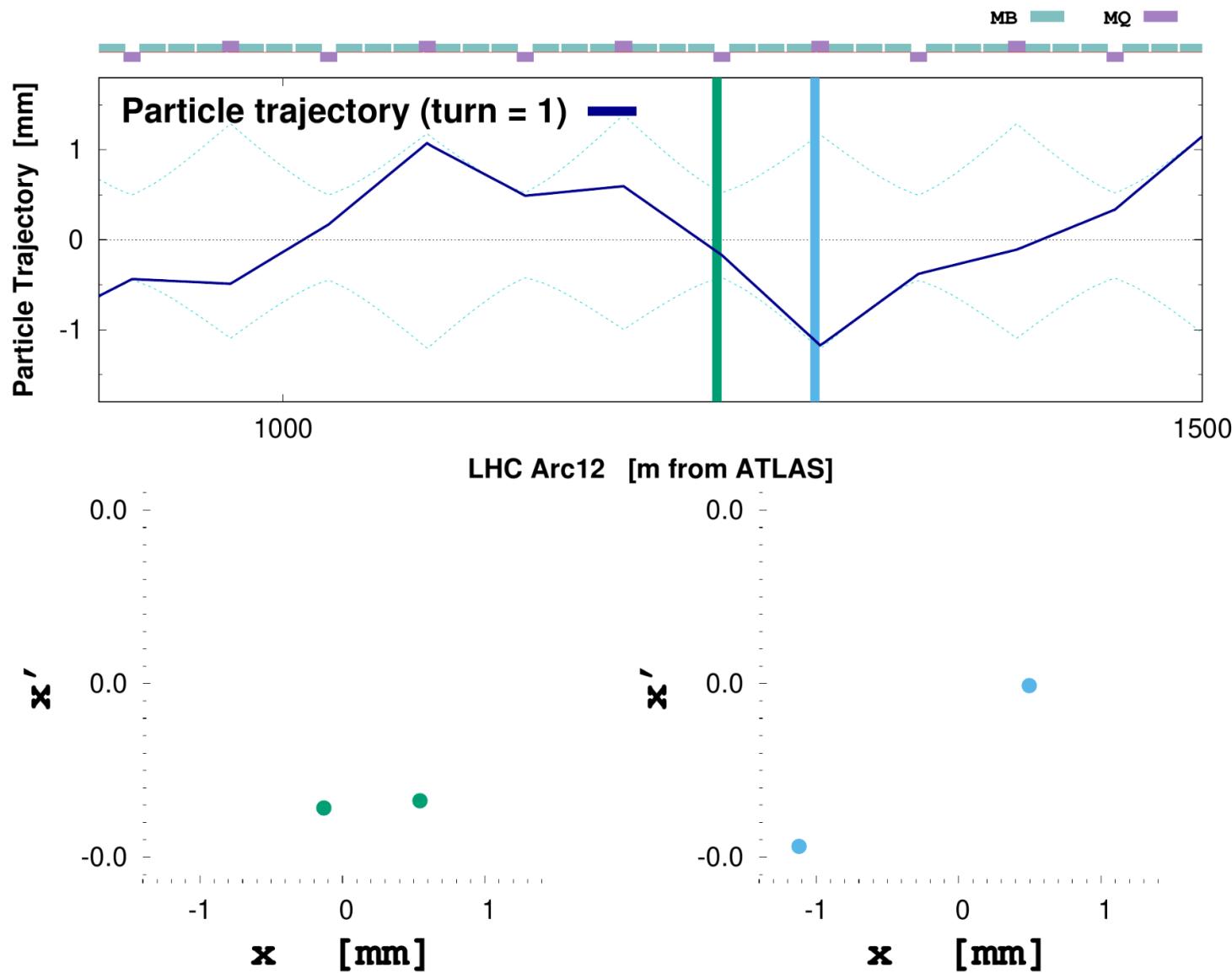
$$\gamma u^2 + 2\alpha uu' + \beta u' = 2 J$$

Courant-Snyder Invariant

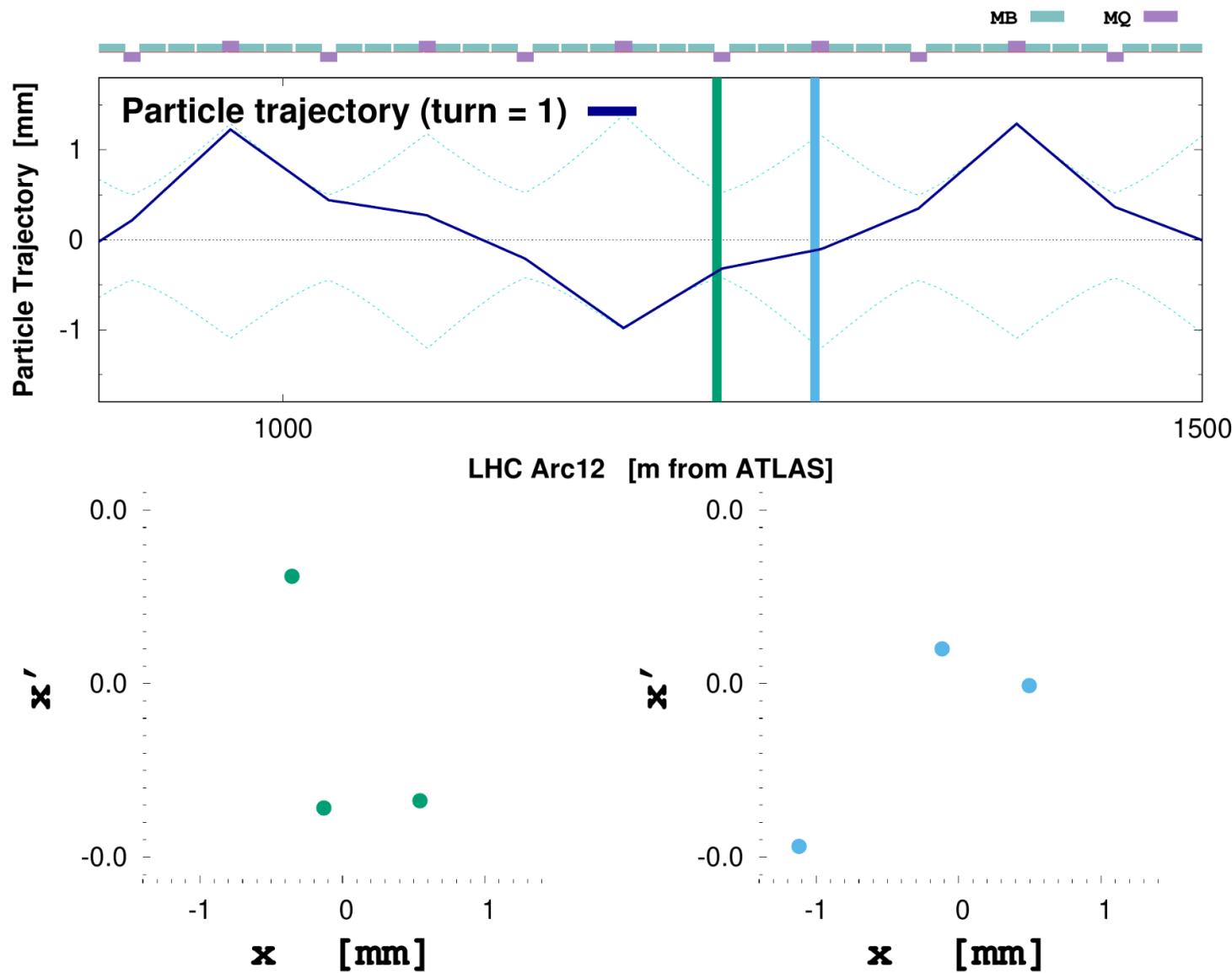
- J is called the **particle action**
- Linear magnetic fields: constant for each particle!
(Liouville theorem)

Phase space ellipse

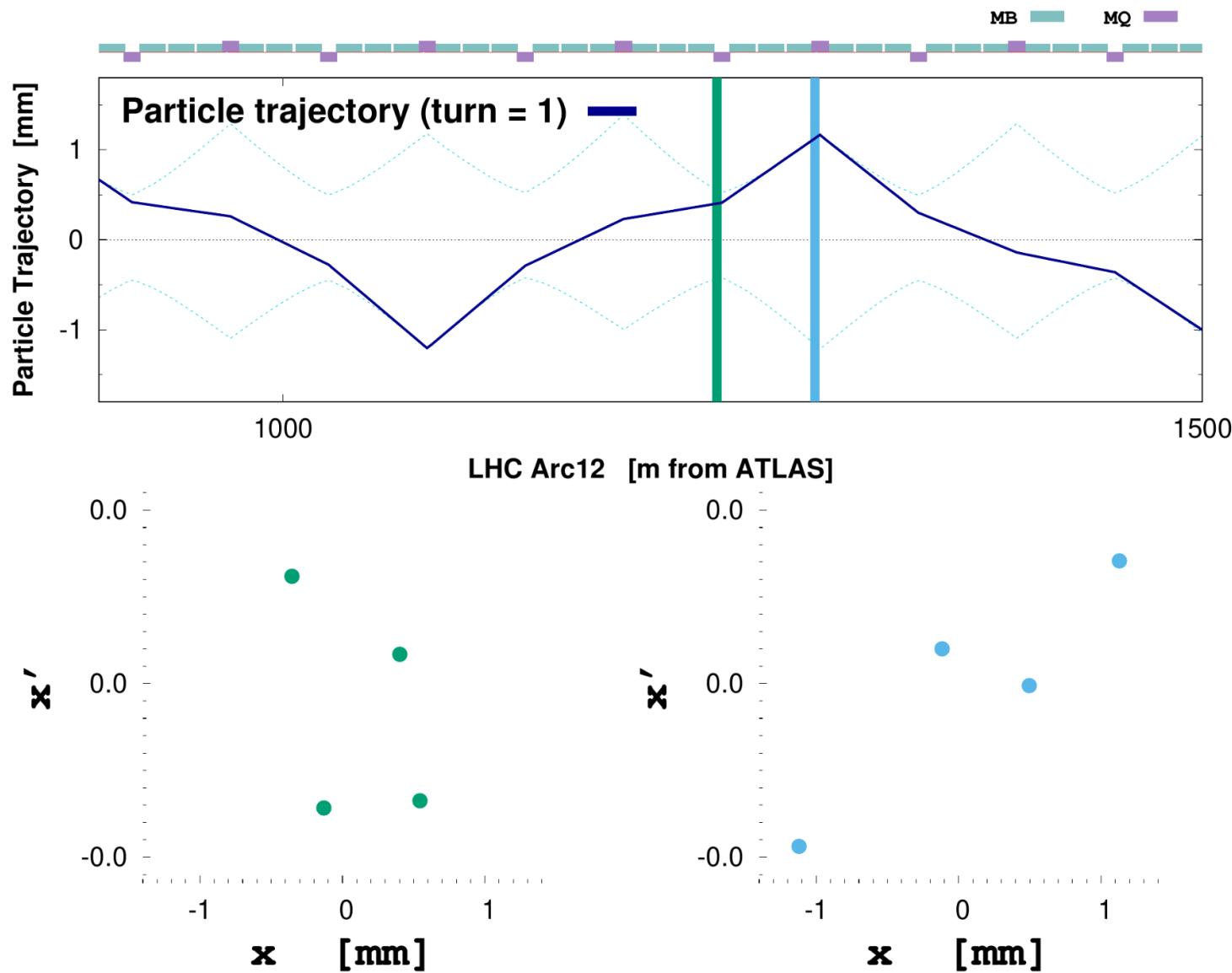


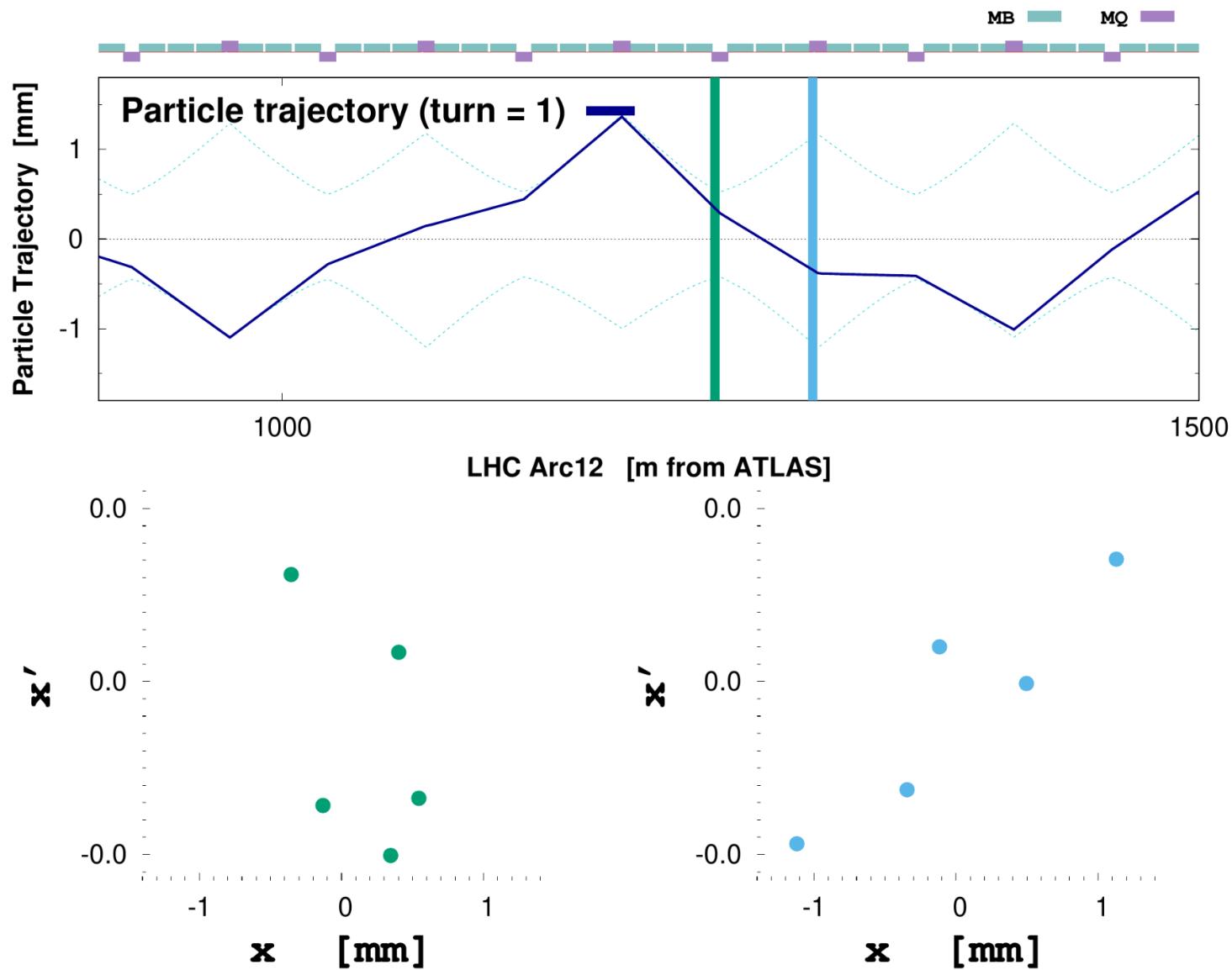


Courtesy of J. Dilly

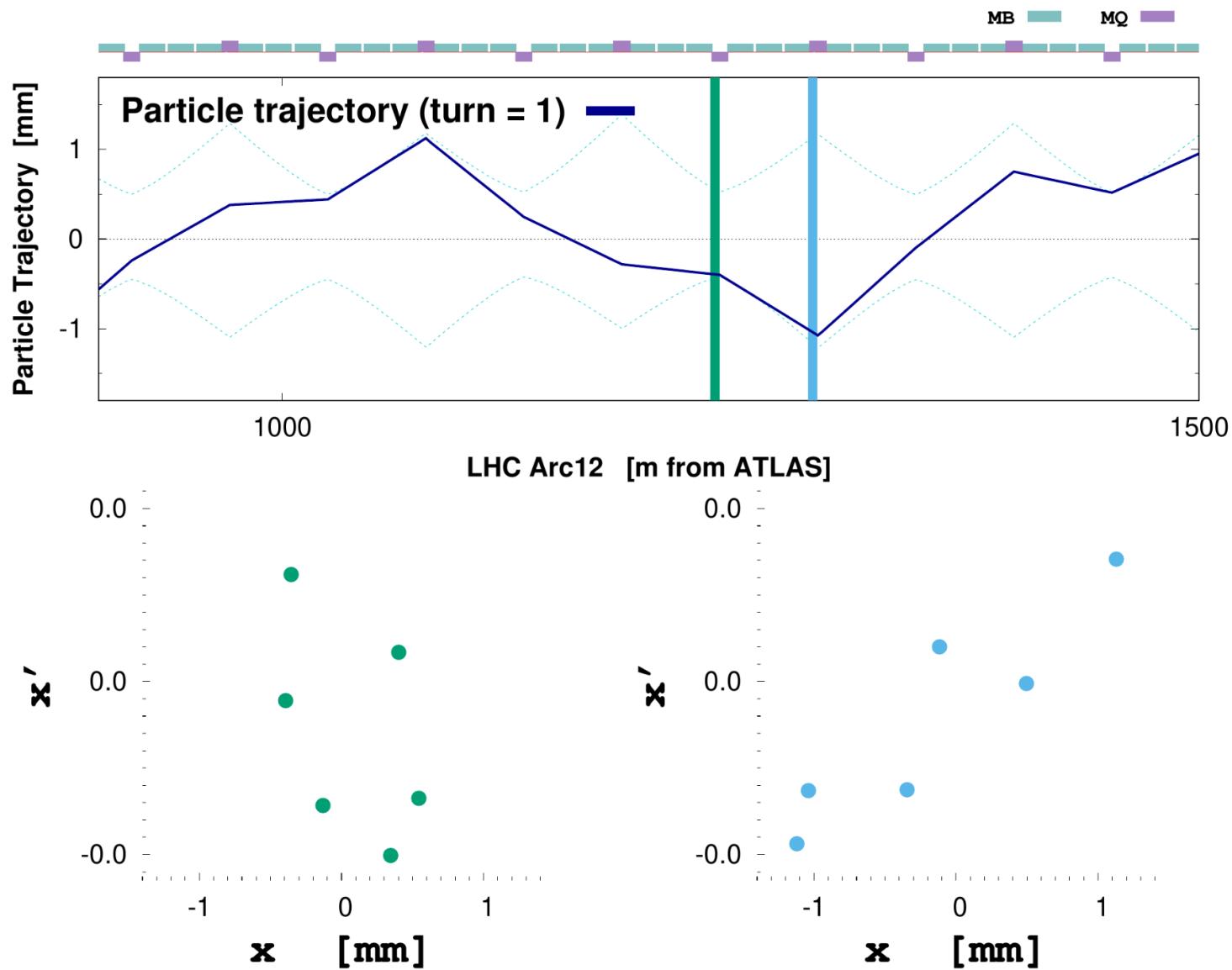


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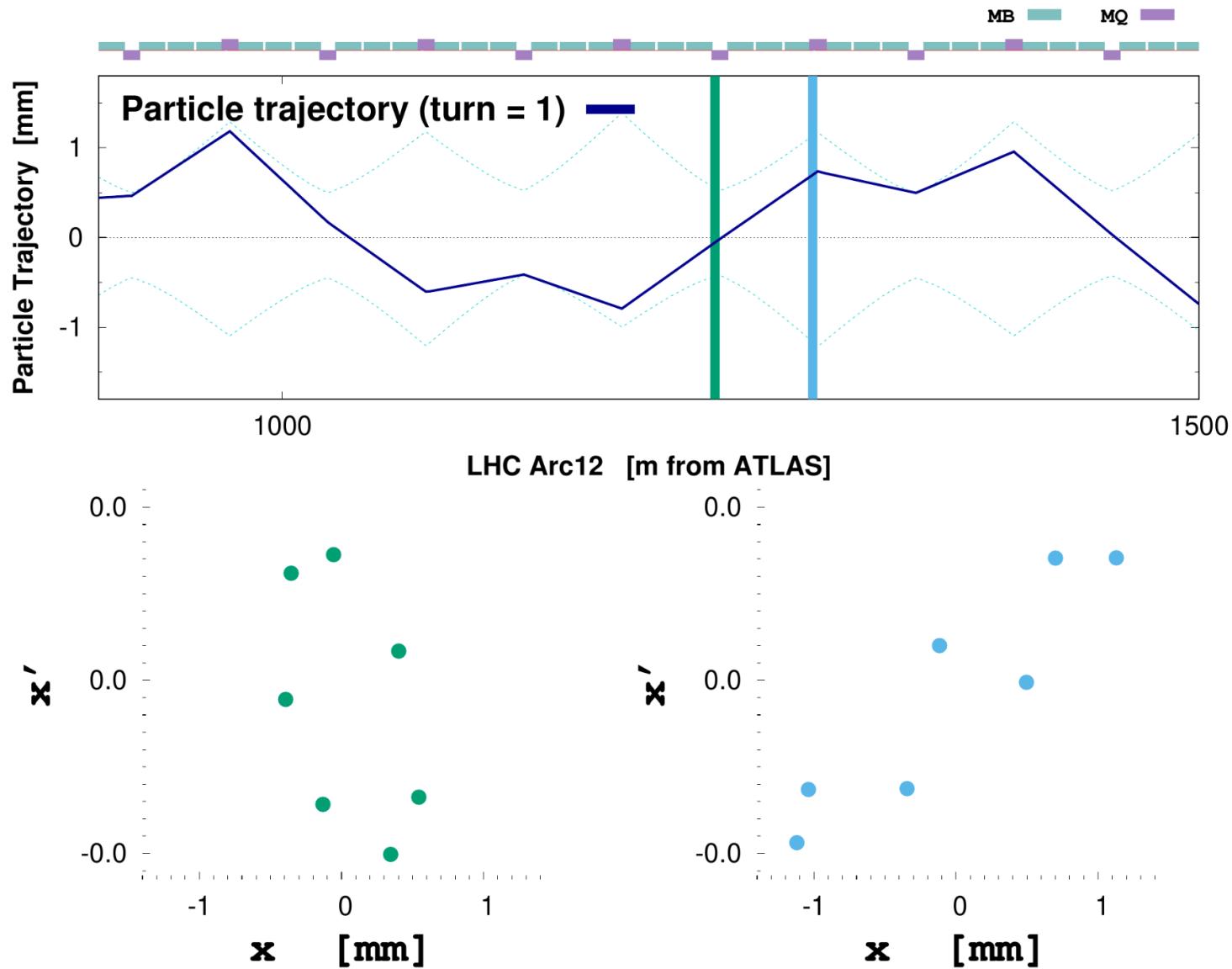




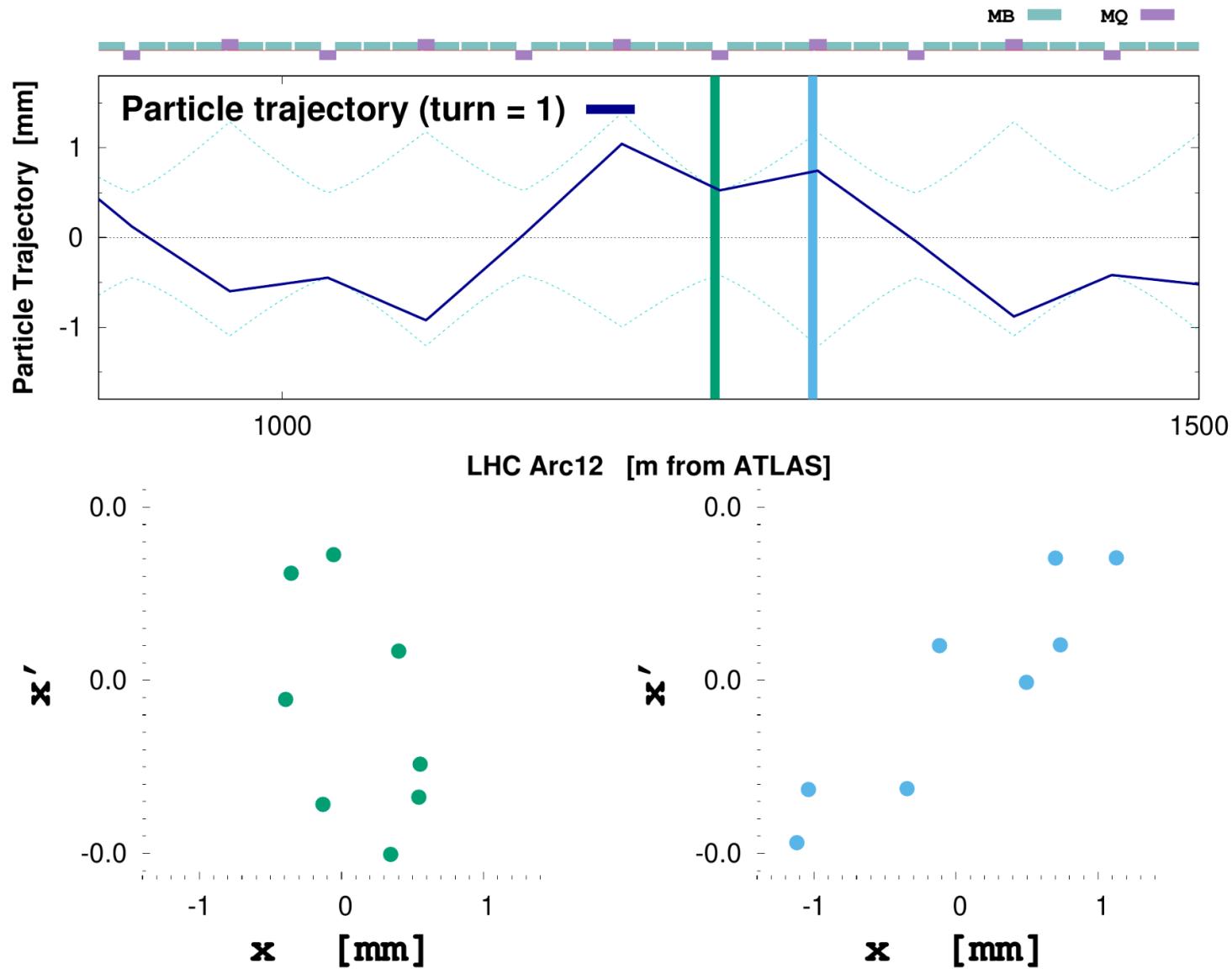
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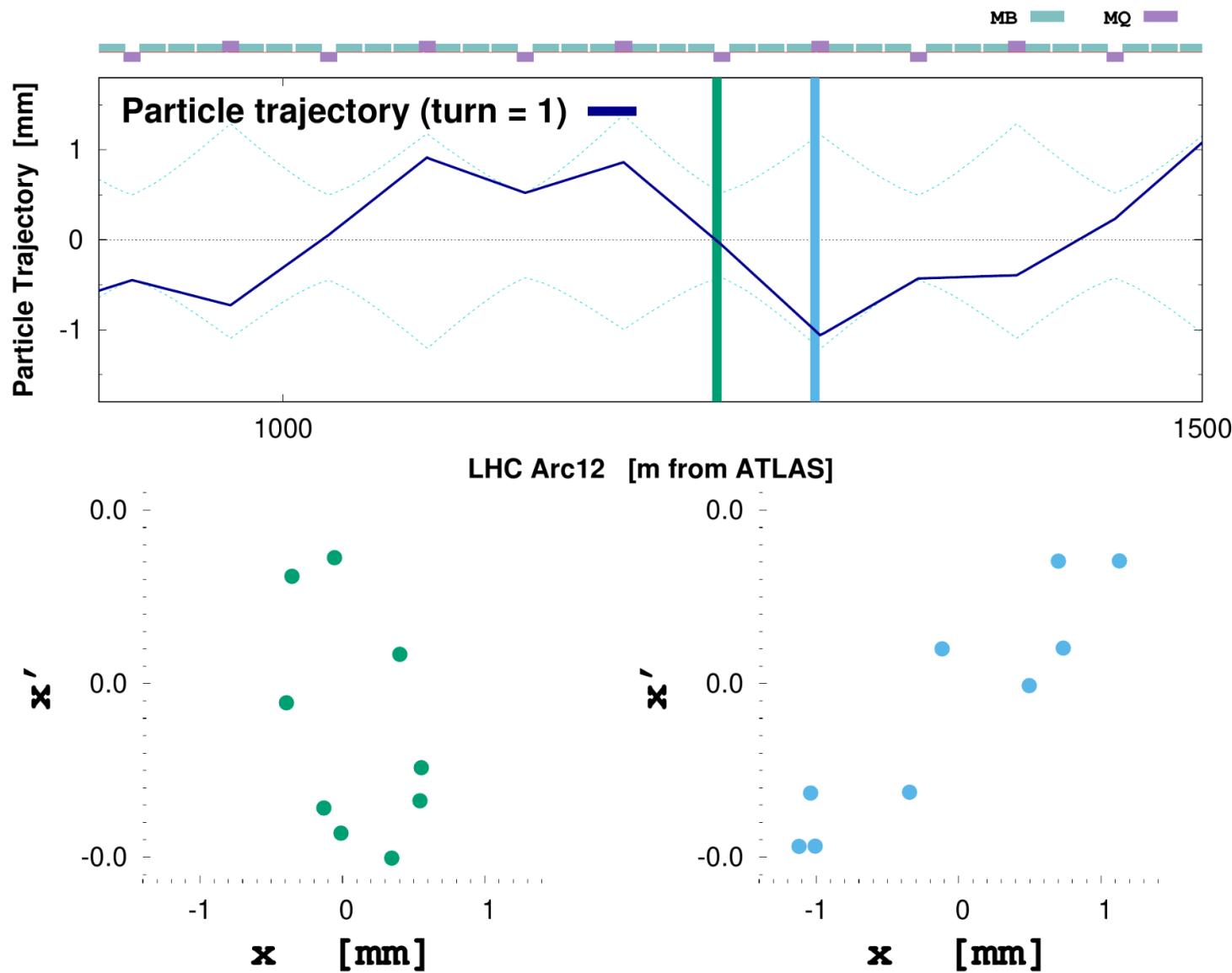
Courtesy of J. Dilly



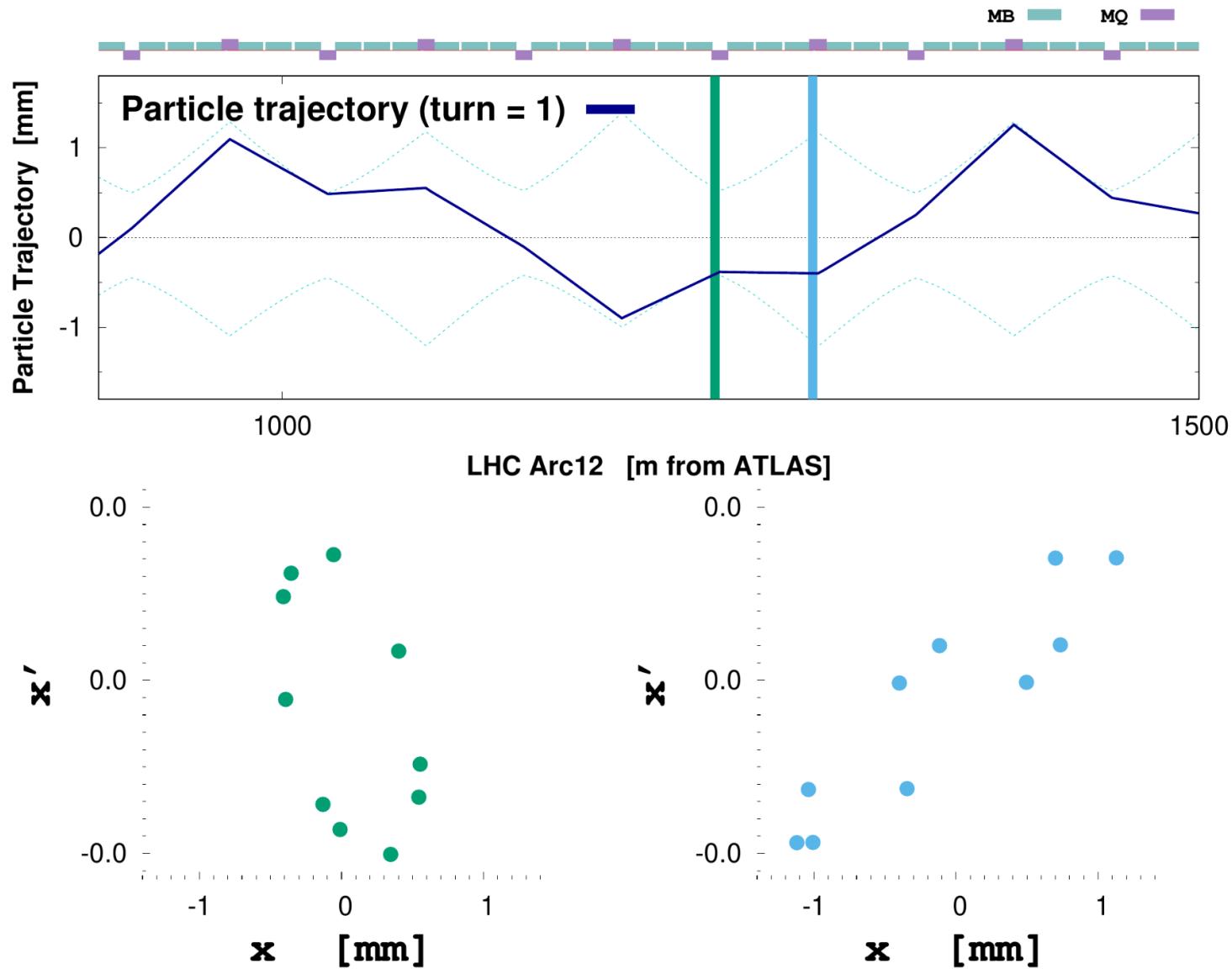
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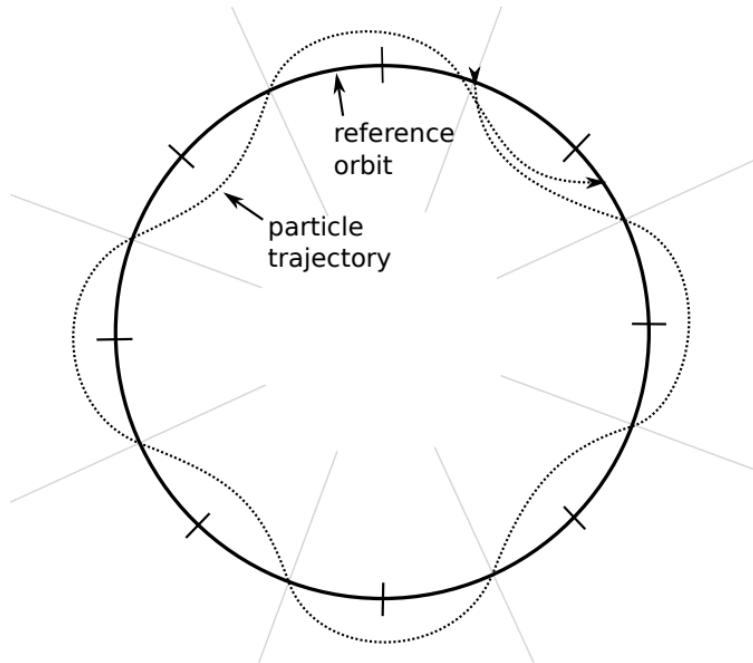
Courtesy of J. Dilly



$$\sqrt{2J\beta(z)}$$

Courtesy of J. Dilly

Betatron function



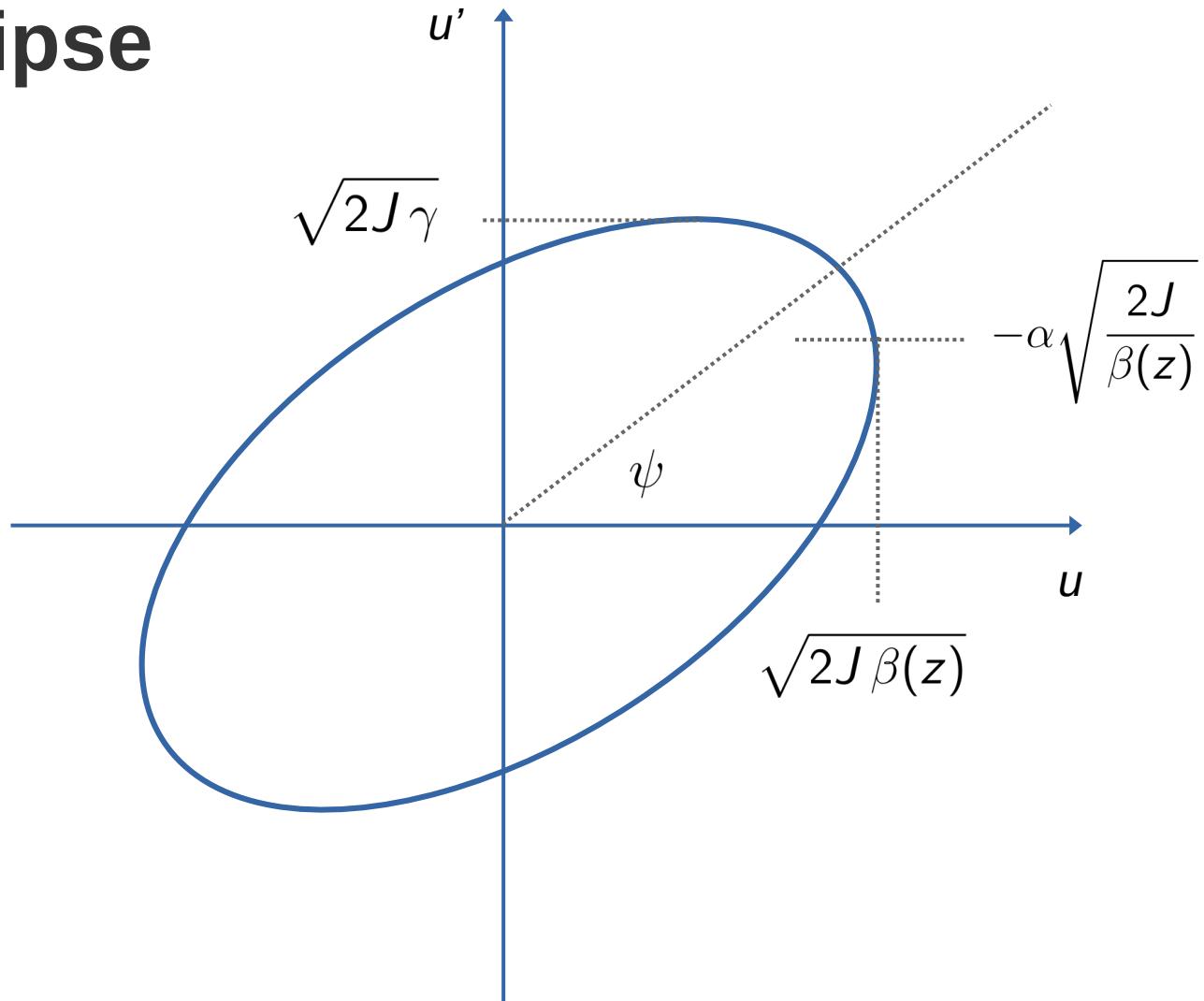
- “**Tune**” (ν): Number of betatron oscillations for one turn

$$\psi(z - z_0) = \int_{z_0}^z \frac{d\xi}{\beta(\xi)} \quad \rightarrow \quad \nu_{x,y} = \frac{1}{2\pi} \oint \frac{d\xi}{\beta_{x,y}(\xi)}.$$

- Fractional part of tune important to avoid resonances (can you imagine why?)
- LHC: $\nu_x = 62.31$ and $\nu_y = 60.32$

Phase space ellipse

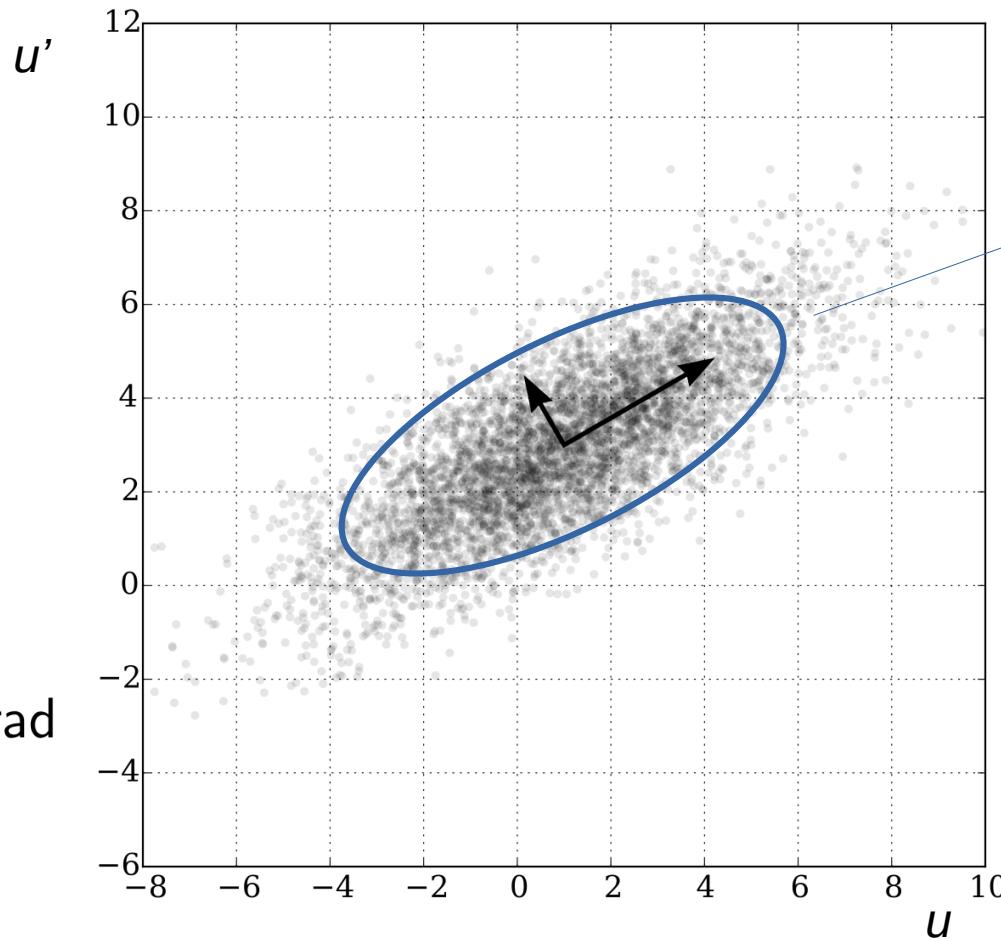
- Apparently \mathbf{J} determines the size of the ellipse **for one particle**
- Doesn't it make sense to look at the **\mathbf{J} of the entire beam?**



Emittance

LHC top energy
(6.8 TeV):

$$\epsilon = 5 \times 10^{-10} \text{ m rad}$$



**Beam
emittance**

$$\epsilon = \langle J \rangle$$

**Definition:
Beam size σ**

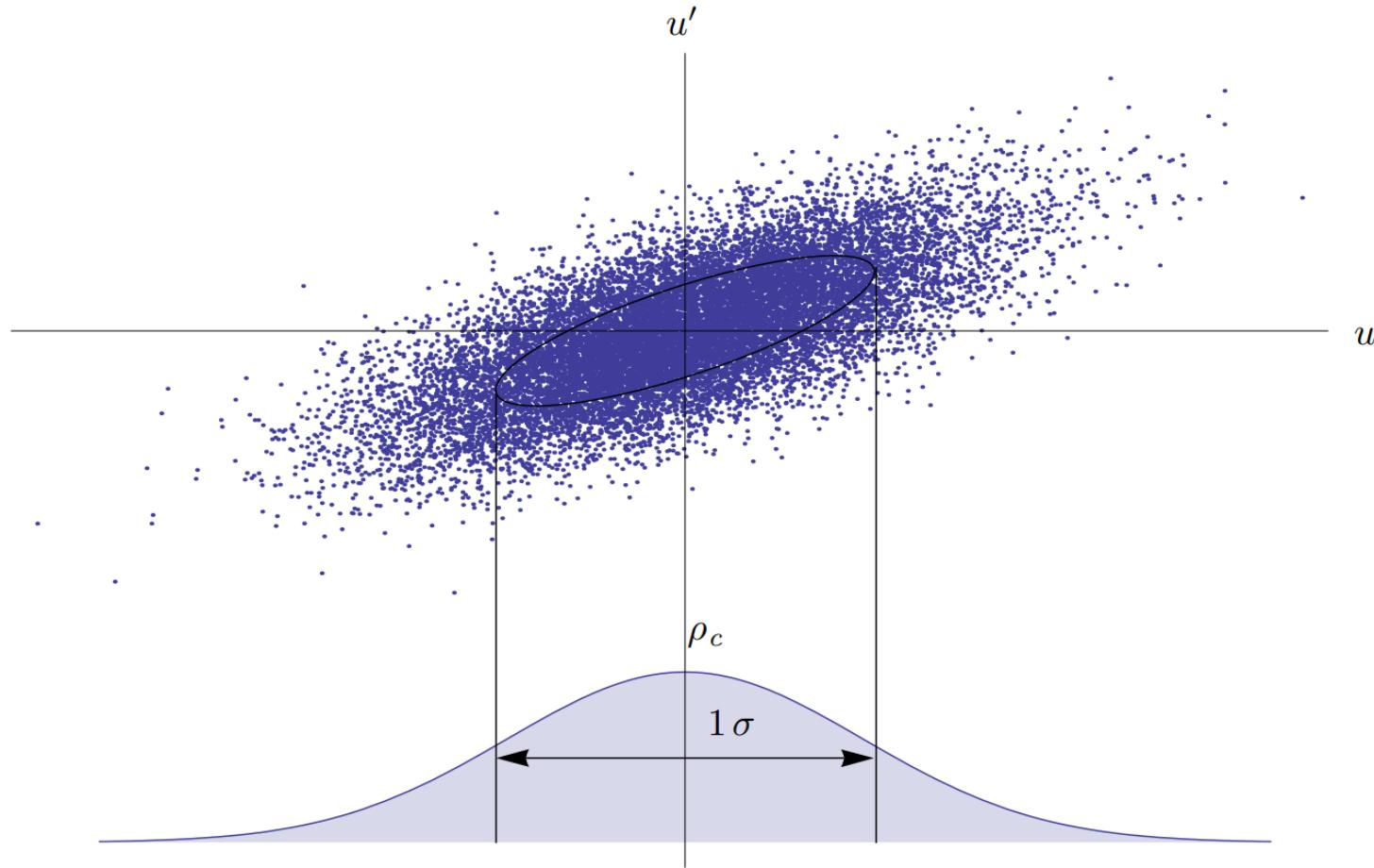
$$\sigma = \sqrt{\epsilon \beta(z)}$$

Crucial
indicator for
beam quality!

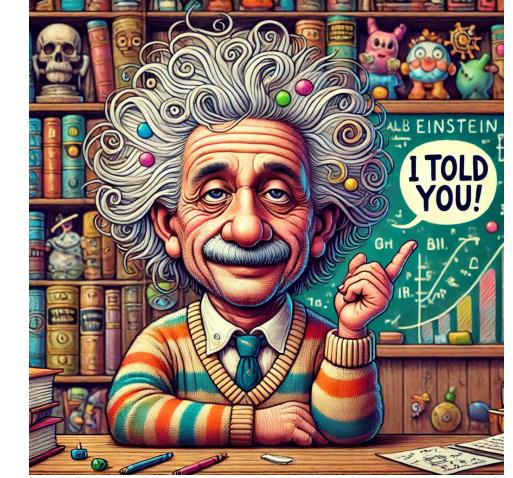
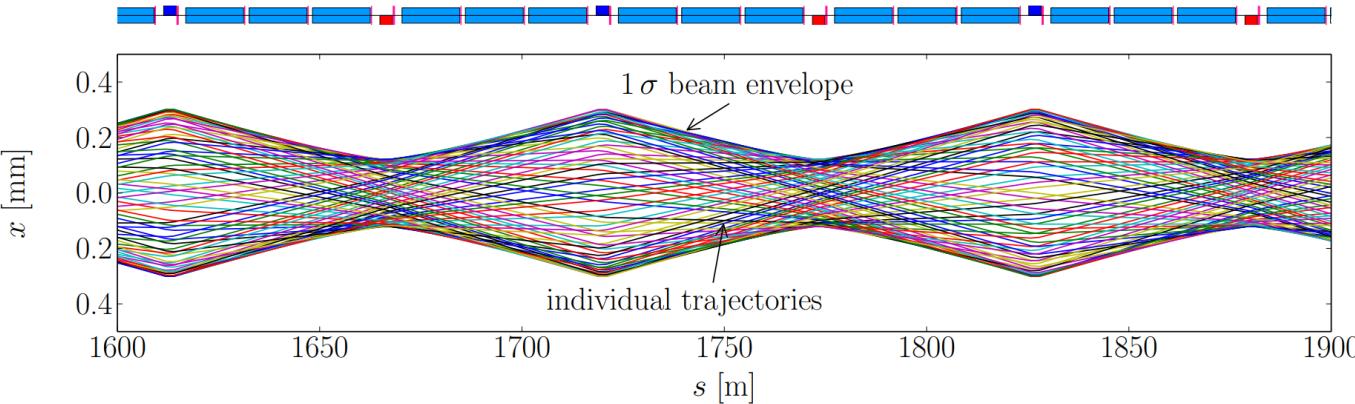
By Nicoguaro - Own work, CC BY 4.0, <https://commons.wikimedia.org/w/index.php?curid=46871195>



Emittance



Emittance



Acceleration:

- Increase of relativistic γ leads to beam shrinking!
- Observed u' in lab frame decreases
- Emittance decreases with **relativistic $\beta\gamma$**

Definition

Normalized emittance

$$\epsilon_N = \beta \gamma \epsilon$$

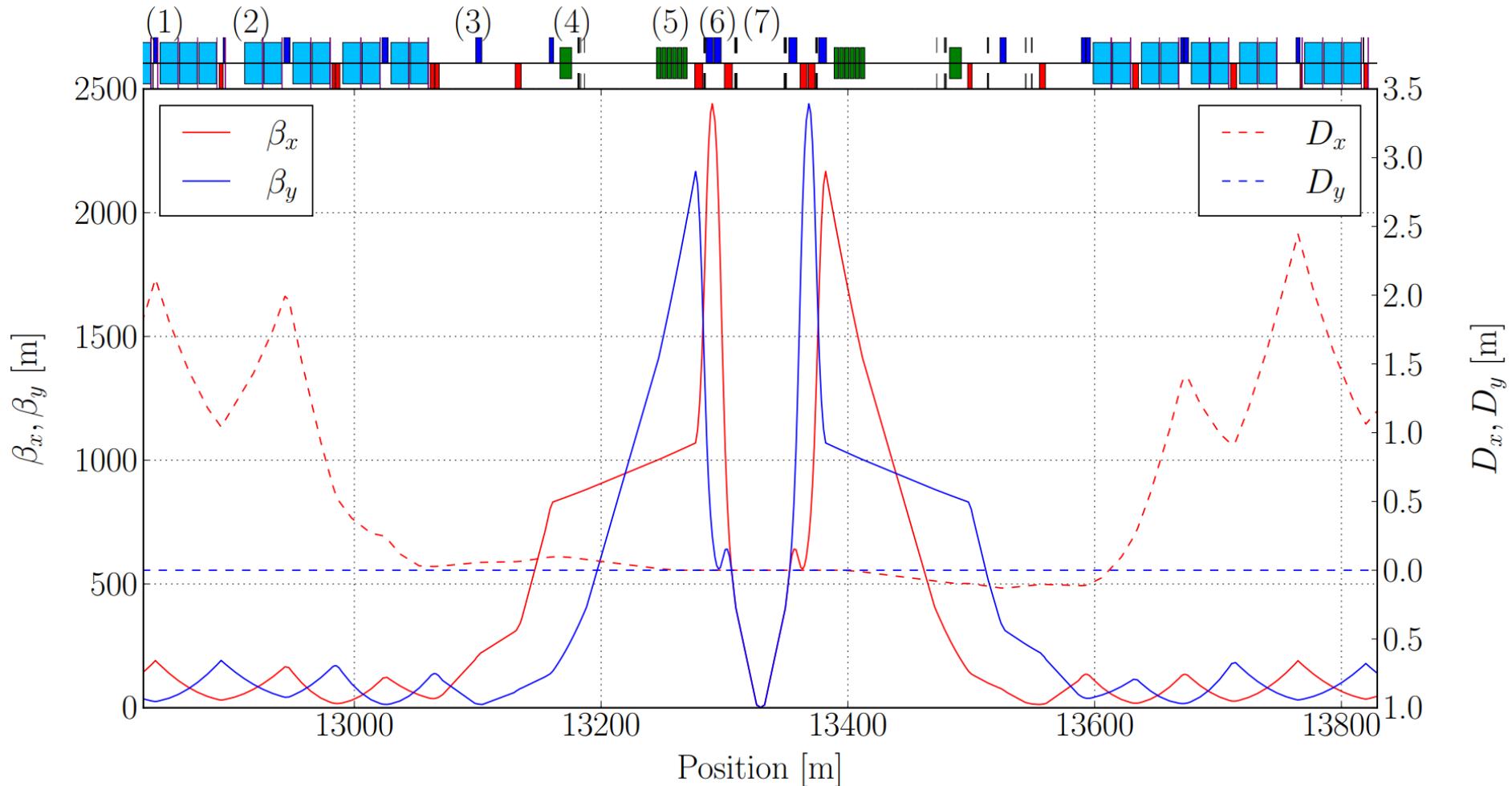
Constant!

LHC: $\epsilon_N = 3.5 \mu\text{mrad}$

Bringing the beams into collision

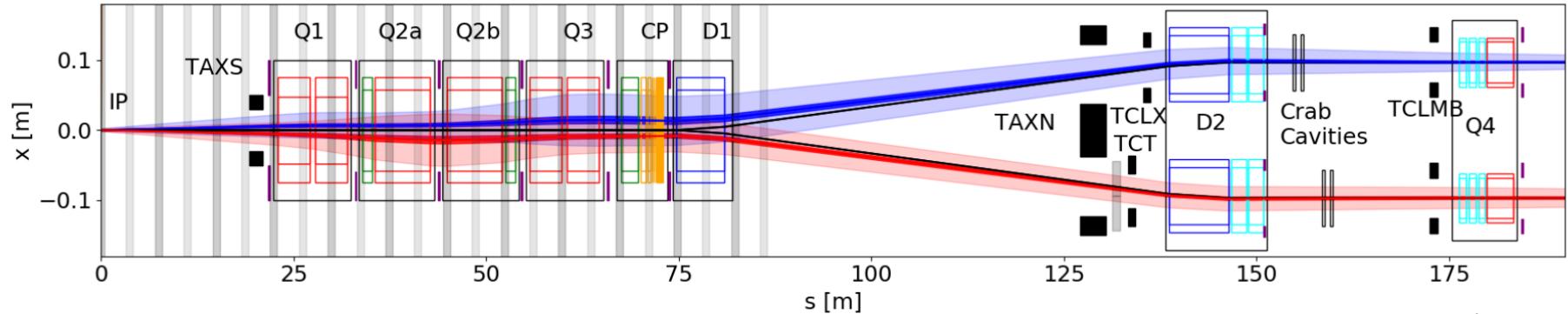
Collision Point Optics

$$\sigma = \sqrt{\epsilon \beta(z)}$$



Collision Point Optics

$$\sigma = \sqrt{\epsilon \beta(z)}$$



Collision Points:

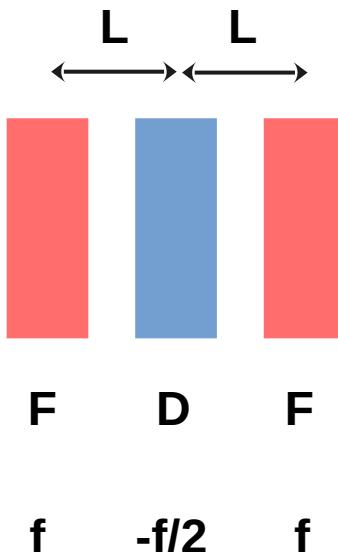
- Need synchronous focusing in both planes!
- How can we achieve this?

Figure taken from HL-LHC Technical DR



Collision Point Optics: Triplet

Quadrupole triplet
for 2D focusing



$$\mathcal{M}_{\text{tr}} = \mathcal{M}_r \mathcal{M} = \begin{pmatrix} 1 - 2L^2/f^2 & 2L(1 + L/f) \\ -1/f^* & 1 - 2L^2/f^2 \end{pmatrix}$$

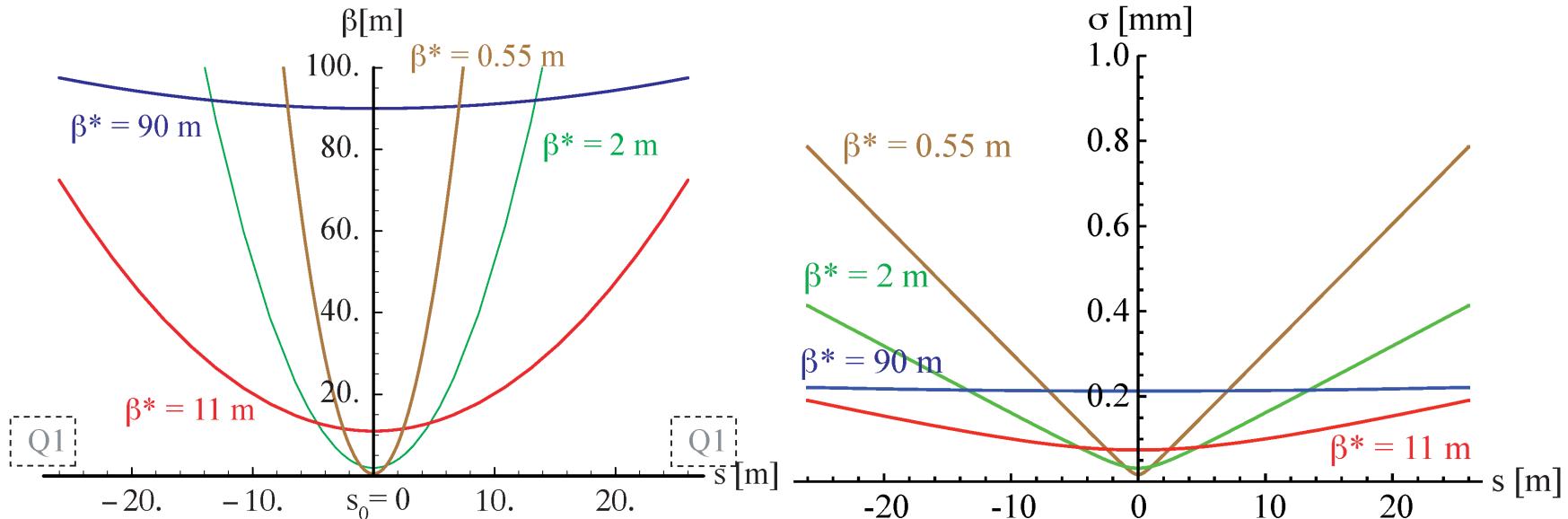
$f \gg L$

↓

Lens with focal
length in x/y

$$\frac{1}{f^*} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

Collision Point Optics



High-Beta Optics and Running Prospects by Helmut Burkhardt, Instruments 2019, 3(1), 22

Collision Point Optics

$$\sigma = \sqrt{\epsilon \beta(z)}$$

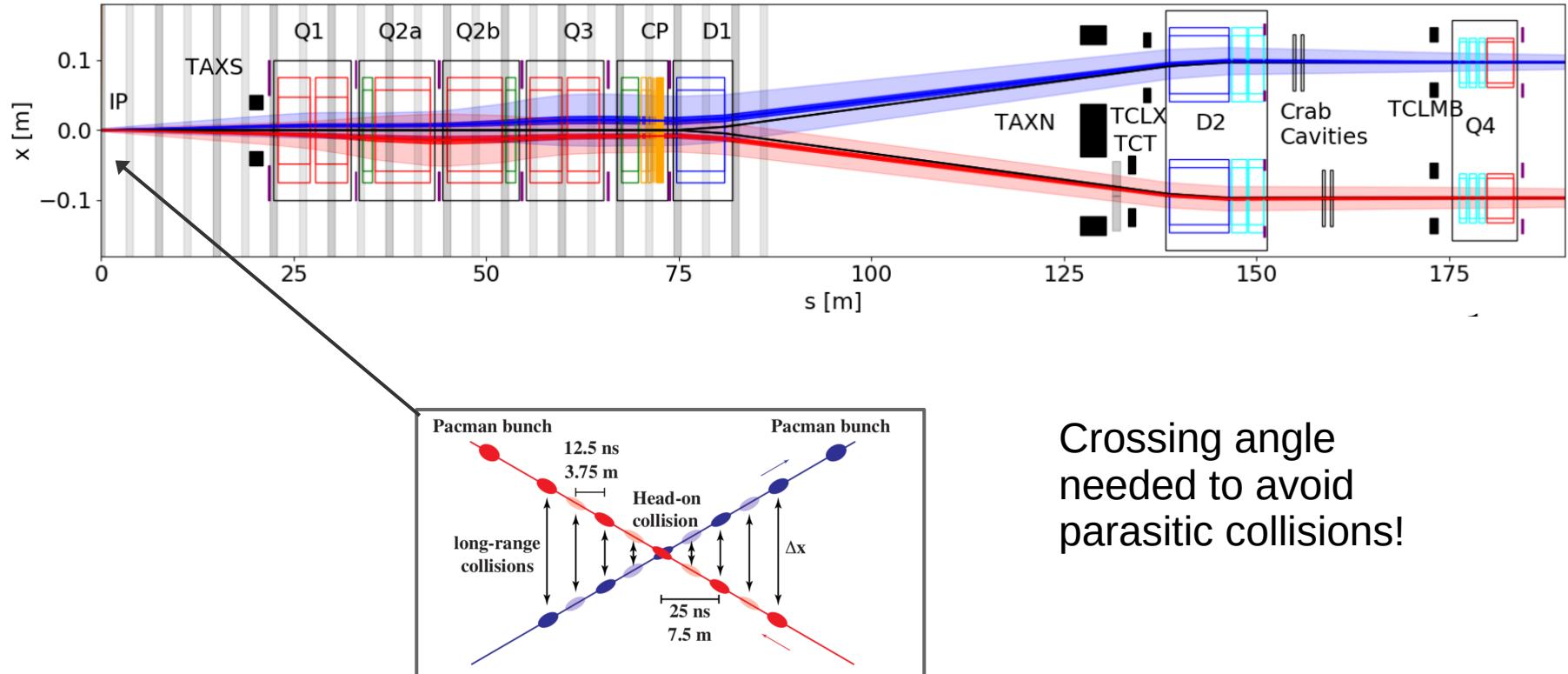


Figure taken from HL-LHC Technical DR



Collision Point Optics

$$\sigma = \sqrt{\epsilon \beta(z)}$$

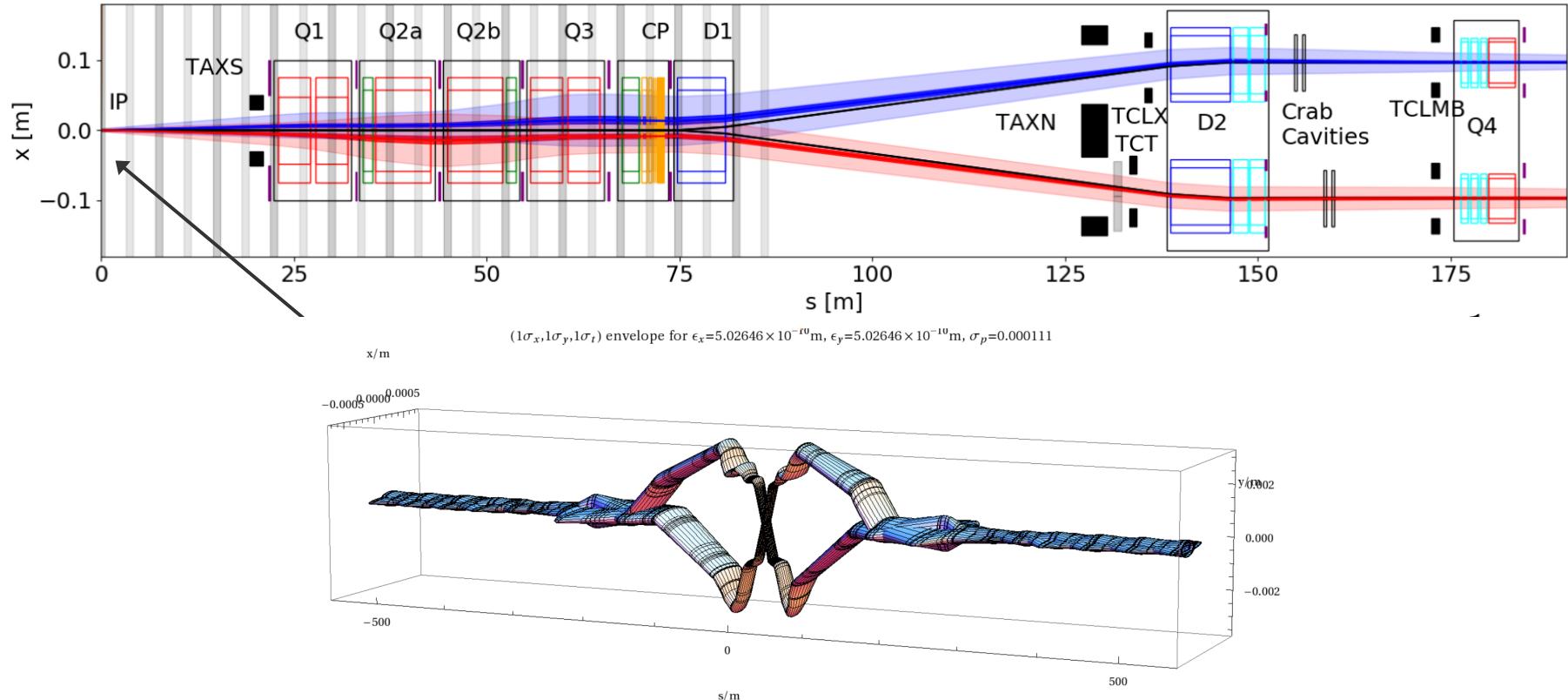


Figure taken from HL-LHC Technical DR



Luminosity

$$\frac{dN_i}{dt} = \mathcal{L} \sigma$$

Interaction rate for physics process with cross-section σ

Particles per bunch
 1.6×10^{11}

Revolution Frequency
11245 Hz

Bunch Number
 ~ 2800

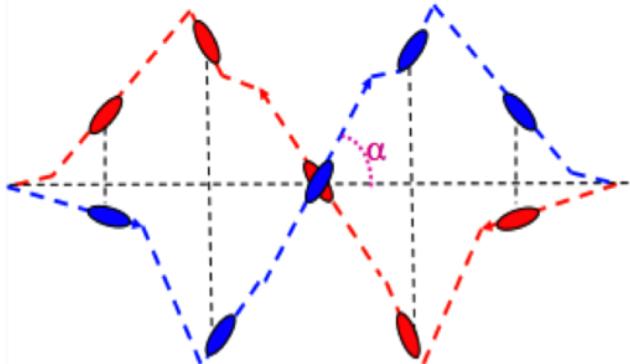
$$\mathcal{L}_0 = f k_b \frac{N_1 N_2}{4 \pi \epsilon \beta^*}$$

$\epsilon_N = 3.5 \mu\text{mrad}$

0.3m to 10m

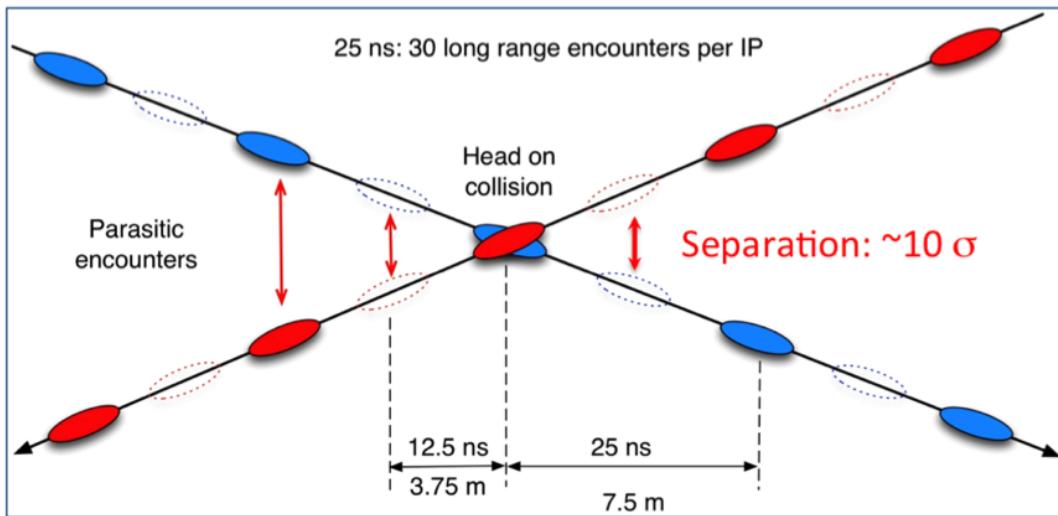
Quiz: What is the luminosity with these parameters? What is the unit?

Luminosity



$$\mathcal{L} = \mathcal{L}_0 F_C$$

$$F_C \approx \frac{1}{1 + \left(\frac{\sigma_l}{\sigma^*} \frac{\theta_C}{2} \right)^2}$$



Luminosity is reduced to crossing angle!

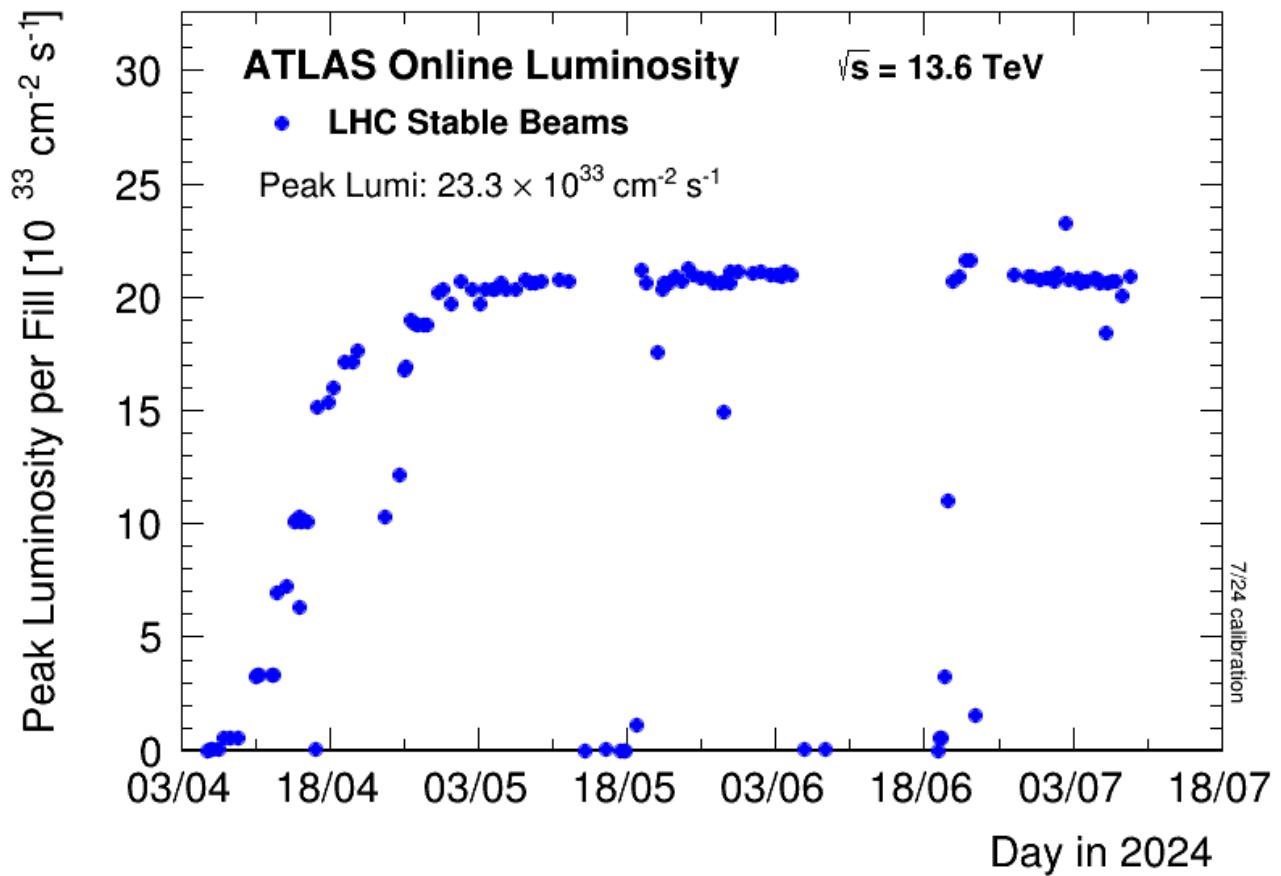
Figure taken from <https://cds.cern.ch/journal/CERNBulletin/2016/38/News%20Articles/2216373>

The LHC Parameters

Parameter		LHC (nominal)	HL-LHC (standard)
Beam energy in collision	[TeV]	7	7
Particles per bunch	[10^{11}]	1.15	2.2
Bunches per beam		2808	2760
Collisions in IP1 and IP5		2808	2748
Half-crossing angle in IP1 and IP5	[μrad]	142.5	250
Minimum β^*	[m]	0.55	0.15
Normalized emittance ϵ_n	[μm]	3.75	2.5
Peak luminosity w/o crab cavities	[$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	1.00	8.11
Peak luminosity w/ crab cavities	[$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	–	17.0
Events / Crossing		27	131

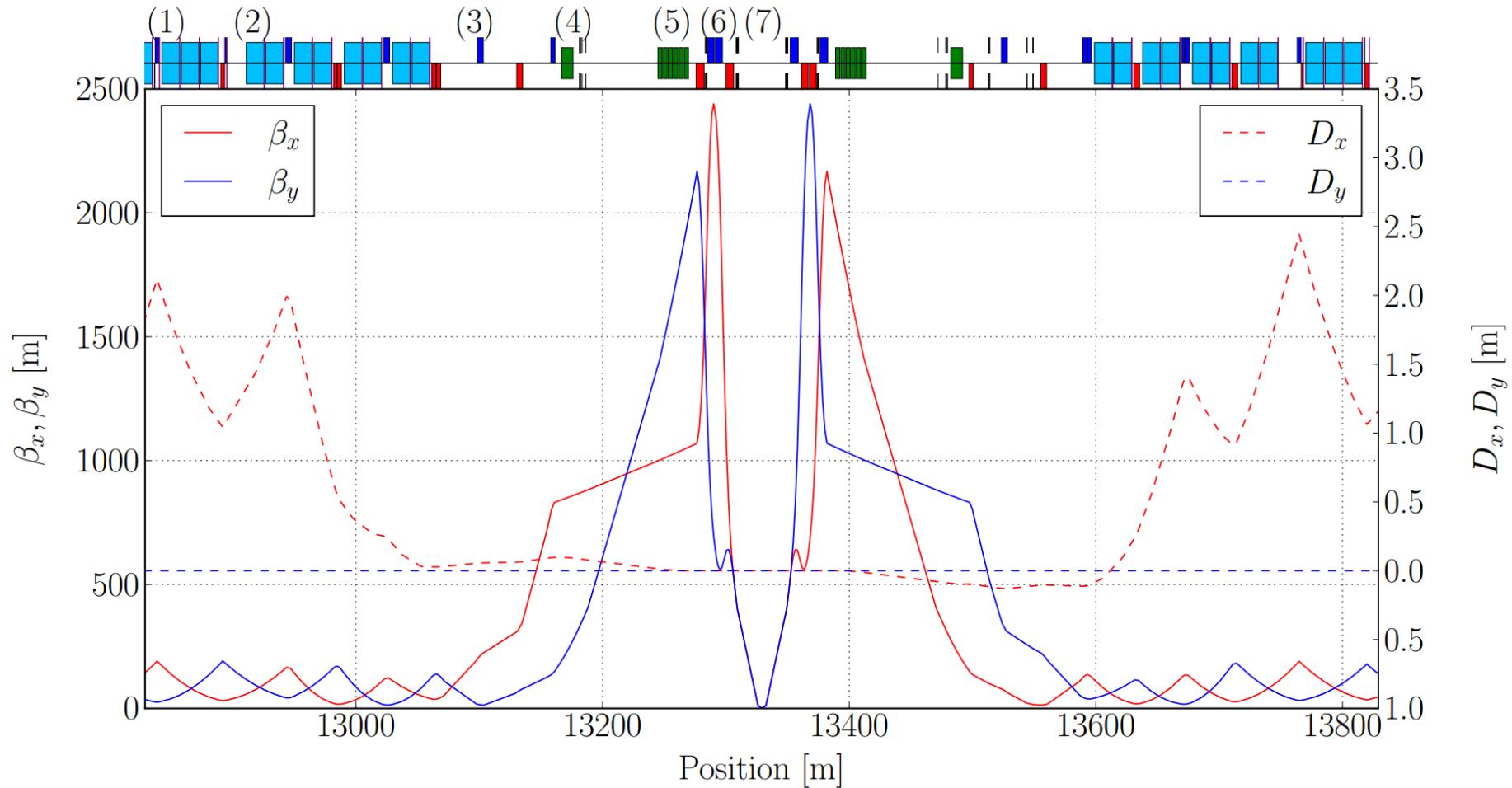
Courtesy of J. Dilly

Luminosity

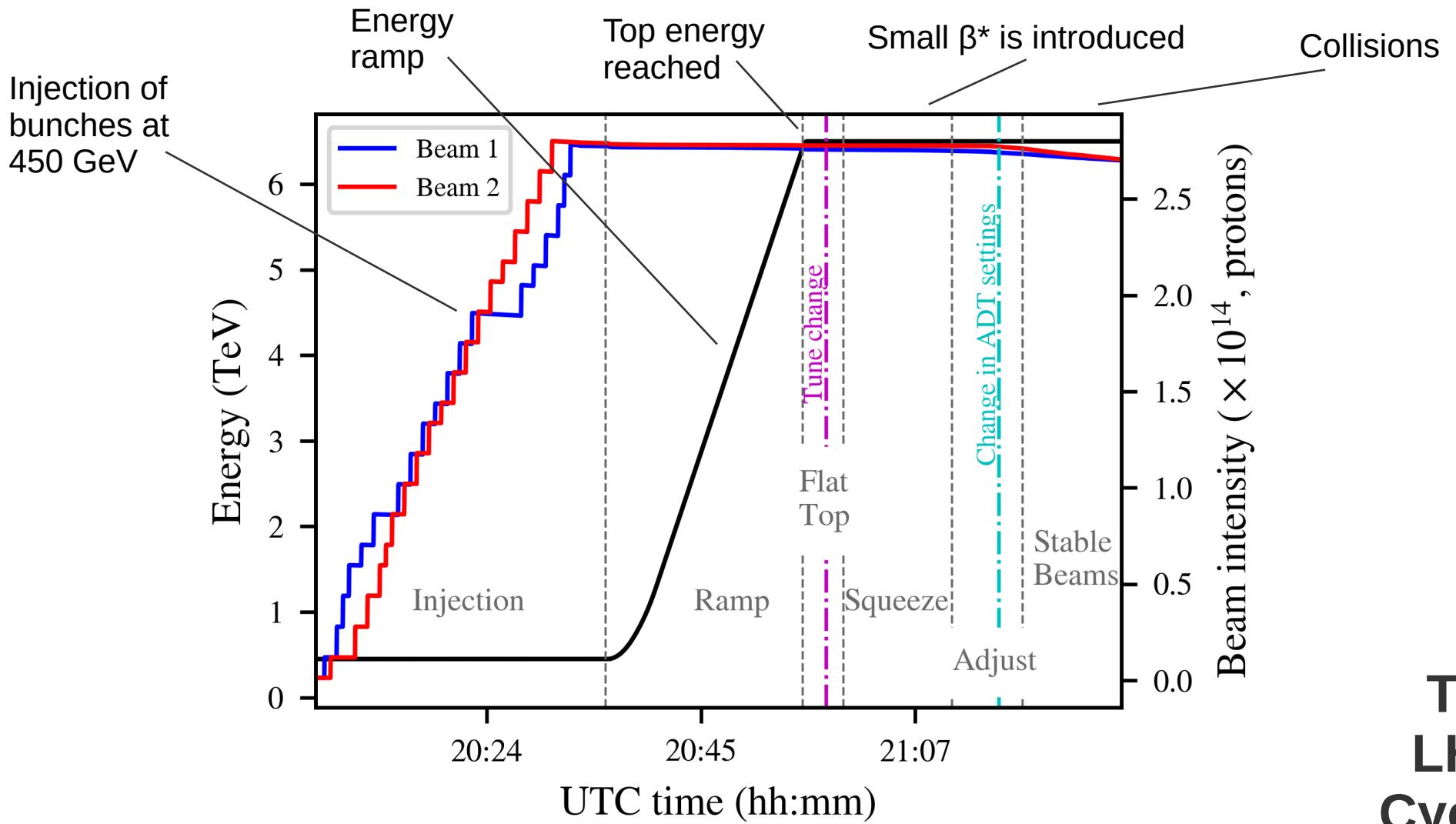


Collision Point Optics

Quiz: Can we apply the “squeezed” optics from the beginning?



The LHC Cycle

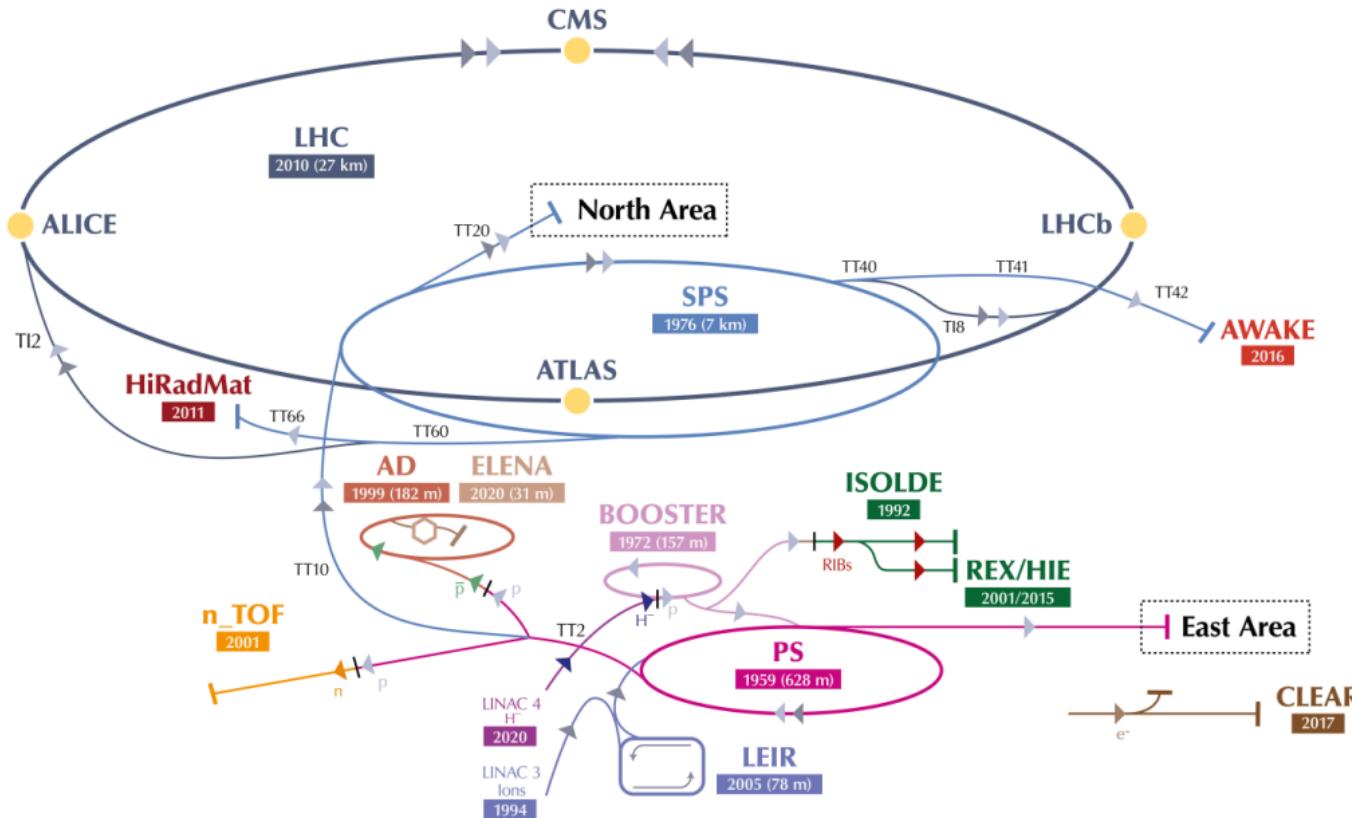


S. Kostoglou et al., Origin of the 50 Hz harmonics in the transverse beam spectrum of the Large Hadron Collider



Outlook for tomorrow

CERN Accelerator Complex



Beam energy along the
LHC injector chain

Linac 4	160 MeV
PSB	2 GeV
PS	25 GeV
SPS	450 GeV
LHC	6.5-7 TeV