PART III

The Standard Model in the fermion sector

CKM Matrix and CP Violation

Flavour Physics in the *Standard Model* (SM) in the quark sector:

~ half of the *Standard Model*

10 free parameters

6 quarks masses

4 CKM parameters

In the Standard Model, charged weak interactions among quarks are codified in a 3 X 3 unitarity matrix : the **CKM Matrix**.

The existence of this matrix conveys the fact that the quarks which participate to weak processes are a linear combination of mass eigenstates

The fermion sector is poorly constrained by SM + Higgs Mechanism

mass hierarchy and CKM parameters

The Standard Model is based on the following gauge symmetry

$$SU(2)_L \times U(1)_Y$$

Weak Isospin (symbol L because only the LEFT states are involved)

Weak Hypercharge :

(LEFT and RIGHT states)

			I	I_3	Q	Y
Leptons	doublet L	ν_e	½	½	0	-1
		e_L^-	½	-½	-1	-1
	singlet R	e_R^-	0	0	-1	-2
quarks	doublet L	u_L	½	½	2/3	1/3
		d_L	½	-½	-1/3	1/3
	singlet R	u_R	0	0	2/3	4/3
	singlet R	d_R	0	0	-1/3	-2/3

Idem for the other families

Short digression on the mass

$$E^2 = \vec{p}^{\,2} + m^2 \;\rightarrow\; \partial^\mu \partial_\mu + m^2\phi = 0 \;\leftrightarrow\; L = \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 = 0$$

$$(i\gamma^\mu \partial_\mu - m) = 0 \;\leftrightarrow\; L = i\overline{\psi}\gamma_\mu \partial^\mu \psi - m\overline{\psi}\psi$$

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}(P_L P_L + P_R P_R)\psi =$$

$$= m[(\overline{\psi}P_L)(P_L\psi) + (\overline{\psi}P_R)(P_R\psi)]\psi \qquad = m\left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

The mass should appear in a LEFT-RIGHT coupling

Wolfgang Lectures

ψ_R : SU(2) singlet

ψ_L : SU(2) doublet

The mass terms are not gauge invariant under

$SU(2)_L \times U(1)_Y$

ψ_R (I=0,Y=-2) leptoni_R

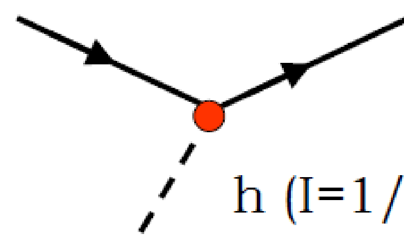
(I=0,Y=-2/3) quark d_R

(I=0,Y=4/3) quark u_R

ψ_L (I=1,Y=-1) leptoni_L

(I=1,Y=1/3) quark d_L

(I=1,Y=1/3) quark u_L



h (I=1/2,Y=1)

Adding a doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I=\frac{1}{2} \quad Y=1$$

Yukawa interaction : $\overline{\psi}_L \phi \psi_R$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$g_e(\overline{\psi}_L \phi \psi_R + \phi^+ \overline{\psi}_R \psi_L)$$

(le deuxieme terme est l'hermitien conjuge du premier)

After SSB

$$\frac{g_e v}{\sqrt{2}}(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) + \frac{g_e}{\sqrt{2}}(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) H$$

$$m_e = \frac{g_e v}{\sqrt{2}}$$

v/sqrt(2) ~natural mass (g~1)

$$g_e = \frac{\sqrt{2} m_e}{v}$$

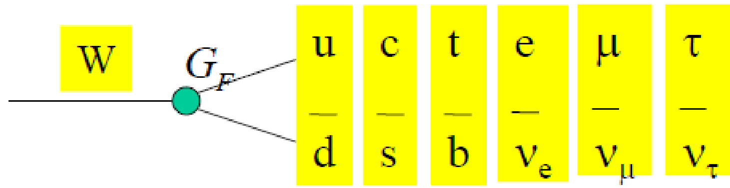
$$m_e \overline{e} e + \frac{m_e}{v} \overline{e} e H$$

$$\frac{g_e}{\sqrt{2}} = \frac{m_e}{v} \quad \text{couplage Hee}$$

Wolfgang Lectures

$$L_W = \frac{g}{2}\overline{Q}^{Int.}_{L_i}\gamma^{\mu}\sigma^{a}Q^{Int.}_{L_i}W^{a}_{\mu} \qquad a = 1,2,3 \qquad Q^{Int.}_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix} \quad L^{Int.}_{L_i} = \begin{pmatrix} \nu_{L_i} \\ l_{L_i} \end{pmatrix}$$

$$\overline{Q}^{Int.}_{L_i} Q^{Int.}_{L_i} = \overline{Q}^{Int.}_{L_i} 1_{ij} Q^{Int.}_{L_j} \quad \text{universality of gauge interactions}$$



The SM quantum numbers are I_3 and Y
→ The gauge interactions are

Flavour blind

In this basis the Yukawa interactions has the following form :

$$L_Y = Y^d_{ij}\overline{Q}^{Int.}_{L_i}\phi d^{Int.}_{R_j} + Y^u_{ij}\overline{Q}^{Int.}_{L_i}\phi u^{Int.}_{R_j} + Y^l_{ij}\overline{L}^{Int.}_{L_i}\phi l^{Int.}_{R_j}$$

$$SSB^* \rightarrow <\phi^0> = v/\sqrt{2}; \mathrm{Re}(\phi^0) \rightarrow (v + H^0)/\sqrt{2}$$

With: $\tilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^*$

To be manifestly invariant under SU(2)

Y_{ij} complex

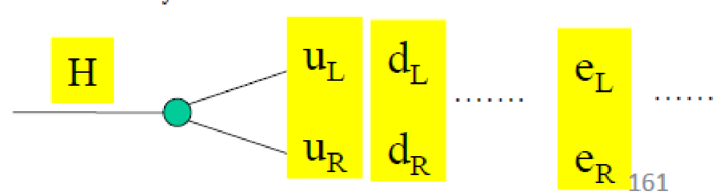
Two matrices are needed to give a mass term to the **u-type** and **d-type quarks**

....

$$L_M = M^d_{ij}\overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij}\overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij}\overline{l}^{Int.}_{L_j} l^{Int.}_{R_j}$$

We made the choice of having the Mass Interaction diagonal

where $M^f = (v/\sqrt{2})Y^f$



* SSB=Spontaneous Symmetry Breaking

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To have mass matrices diagonal and real, we have defined: $M^f_{diag} = V^f_L M^f V^{f\dagger}_R$

The mass eigenstates are:

$$d_{L_i} = (V_L^d)_{ij}\, d_{L_j}^{Int.} \quad ; \quad d_{R_i} = (V_R^d)_{ij}\, d_{R_j}^{Int.}$$

$$u_{L_i} = (V_L^u)_{ij}\, u_{L_j}^{Int.} \quad ; \quad u_{R_i} = (V_R^u)_{ij}\, u_{R_j}^{Int.}$$

$$l_{L_i} = (V_L^d)_{ij}\, l_{L_j}^{Int.} \quad ; \quad l_{R_i} = (V_R^d)_{ij}\, l_{R_j}^{Int.}$$

$$\nu_{L_i} = (V_L^l)_{ij}\, \nu_{L_j}^{Int.}$$

ν_{L_i} arbitrary (assuming ν massless)

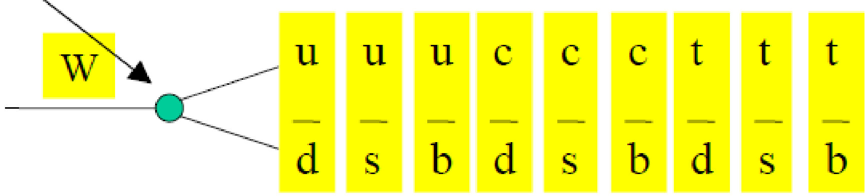
In this basis the Lagrangian for the gauge interaction is:

$$L_W = \frac{g}{2} \bar{u}_{L_i} \gamma^\mu (V_L^u V_L^{d\dagger}) d_{L_j} W_\mu^a + h.c.$$

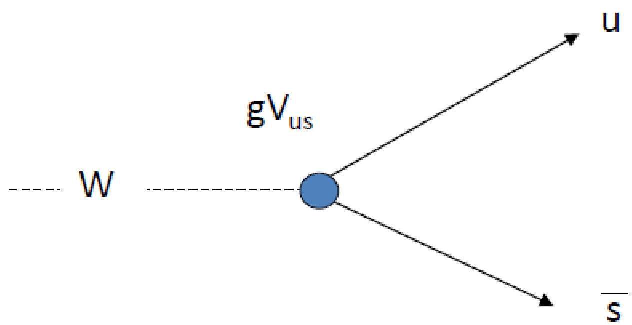
The coupling is not anymore universal

$V(CKM) = (V_L^u V_L^{d\dagger})$

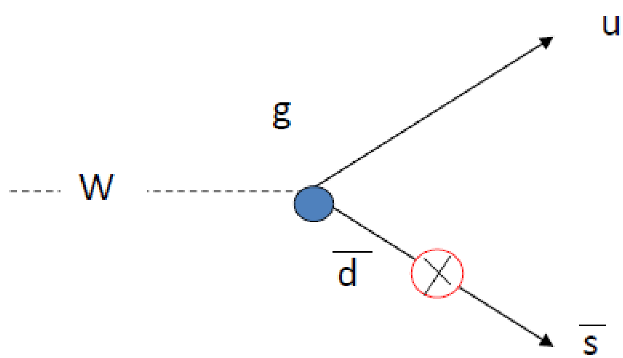
Unitary matrix



Two different way of seeing the charged interactions among quarks



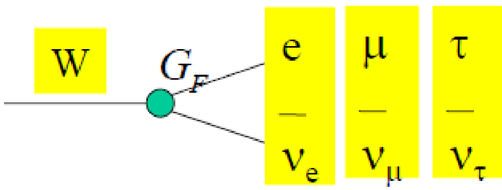
In the basis where :
the masses are real
and diagonal



In the basis where :
charged interactions are just
between members of the same family
and CKM is diagonal

If a similar procedure is applied to the lepton sector

[illegible]



Since the neutrino are (were) massless the matrix which change the basis from int-> mass is in principle arbitary
We can always choose $V_L^\nu = V_L^l$

Now the neutrino have a mass, it exists a similar matrix in the lepton sector with mixing a CP violation

For the Z^0

Facultatif

$$L_W = \frac{g}{2}\bar{Q}^{Int.}_{L_i}\gamma^\mu\sigma^a Q^{Int.}_{L_i} W^a_\mu \qquad a = 1,2,3$$

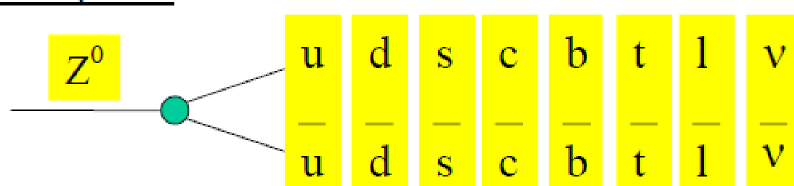
$$-L_B = g'[\frac{1}{6}\bar{Q}^{Int.}_{L_i}\gamma^\mu 1_{ij} Q^{Int.}_{L_j} + \frac{2}{3}\bar{u}^{Int.}_{R_i}\gamma^\mu 1_{ij} u^{Int.}_{R_j} - \frac{1}{3}\bar{d}^{Int.}_{R_i}\gamma^\mu 1_{ij} d^{Int.}_{R_j}]B_\mu$$

for the Z^0 $\quad Z^\mu = \cos\vartheta_W W_3^\mu - \sin\vartheta_W B^\mu \;\; ; \;\; \tan\vartheta_W = g'/g$

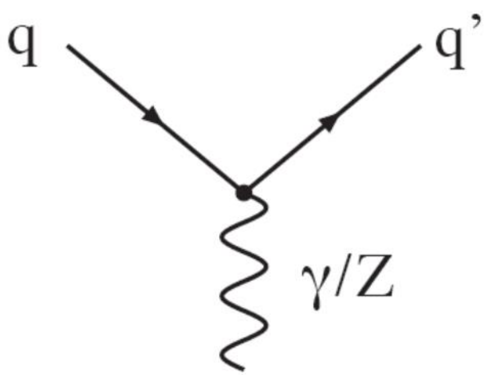
in the mass basis (example for d_L)

$$-L_Z = \frac{g}{\cos\vartheta_W}(-\frac{1}{2}+\frac{1}{3}\sin^2\vartheta_W)\bar{d}_{L_i}\gamma^\mu (V^\dagger_{dL} V_{dL}) d_{L_i} Z_\mu = \frac{g}{\cos\vartheta_W}(-\frac{1}{2}+\frac{1}{3}\sin^2\vartheta_W)\bar{d}_{L_i}\gamma^\mu d_{L_i} Z_\mu$$

The neutral currents stay universal, in the mass basis :
we do not need extra parameters for their complete description

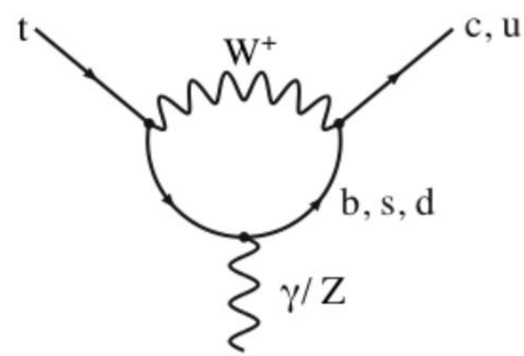


$u\bar{u}$ $d\bar{d}$ $s\bar{s}$ $c\bar{c}$ $b\bar{b}$ $t\bar{t}$



<u>NEUTRAL CURRENTS with Z0.</u>
DO NOT CHANGE THE FLAVOUR

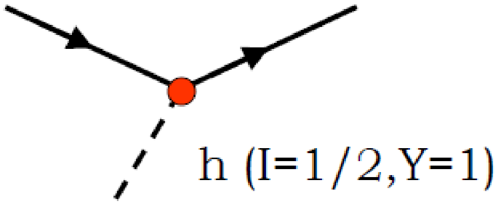
$c\bar{u}$ $s\bar{d}$ $b\bar{s}$ $t\bar{c}$ $b\bar{b}$ $b\bar{d}$



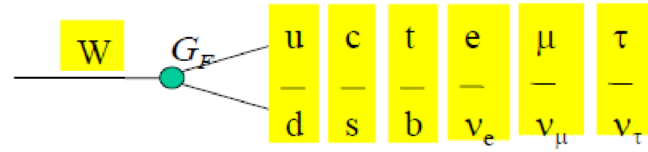
<u>Flavour Changing Neutral Current (FCNC) occurs with W exchange.</u>
THEY ARE SUPPRESSED IN THE SM SINCE OCCURS AT SECOND ORDER.

SUMMARY

The mass is a LEFT-RIGHT coupling and has to respect the gauge invariance $SU(2)_L \times U(1)_Y$



$$\overline{\psi}_L \phi \psi_R \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I = \frac{1}{2} \quad Y = 1$$



$$M^D = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \qquad M^U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}$$

9+9 Complex parameters

$$M^{D,U}_{DIAG} = V^{D,U}_L M^{D,U} (V^{D,U}_R)^+$$

$$M^D_{DIAG} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \qquad M^U_{DIAG} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$V(CKM) = V^U_L (V^D_L)^+ = \begin{pmatrix} \text{4 parameters} \\ \lambda, A, \rho, \eta \end{pmatrix}$$

$$L_M = M^d_{ij} \overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij} \overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij} \overline{l}^{Int.}_{L_j} l^{Int.}_{R_j}$$

To have mass matrices diagonal and real, we have defined: $M^f(diag) = V^f_L M^f V^f_R$

The mass eigenstates are:

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The Lagrangian for the gauge interaction is:

$$L_W = \frac{g}{2} \overline{u}_{L_i} \gamma^\mu (V^u_L V^{d\dagger}_L) d_{L_j} W^a_\mu + h.c.$$

Pattern	U	D	$\lvert V_{us}\rvert$ ($Exp.$ 0.22)	$\lvert V_{ub}\rvert$ ($Exp.$ 0.0036)	$\lvert V_{cb}\rvert$ ($Exp.$ 0.040)	
1 M_7, M_3	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17, 0.28)	$\sqrt{\frac{m_d m_u}{m_b m_c}}$ 0.0023	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
2 M_8, M_3	$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u}{m_c}}\left[\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}\right]$ (0.0011, 0.0058)	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022, 0.10)	No (V_{ub}, V_{cb})
3 M_6, M_3	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}}$ 0.040	OK
4 M_3, M_7	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u^2}{m_c m_t}}$ 0.00021	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{ub}, V_{cb})
5 M_2, M_7	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \frac{m_u}{m_c}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})

Pattern	U	D	$\lvert V_{us}\rvert$ ($Exp.$ 0.22)	$\lvert V_{ub}\rvert$ ($Exp.$ 0.0036)	$\lvert V_{cb}\rvert$ ($Exp.$ 0.040)	
1 M_1, M_7	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})
2 M_2, M_3	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.036,0.043)	OK
3 M_2, M_4	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_d m_s}{2m_b^2}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.0013,0.0085)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})
4 M_3, M_4	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_d m_s}{2m_b^2}} \pm \sqrt{\frac{m_u^2}{m_c m_t}}$ (0.0047, 0.0051)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{ub}, V_{cb})
5 M_4, M_3	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}}$ 0.040	OK
6 M_5, M_3	$\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{2m_u}{m_t}}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022,0.10)	? (V_{cb})
7 M_6, M_1	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}} \pm 2\sqrt{\frac{m_d{}^2}{m_s m_b}}$ (0.014,0.021)	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
8 M_7, M_1	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$2\sqrt{\frac{m_d{}^2}{m_s m_b}} \pm \sqrt{\frac{m_d m_u}{m_b m_c}}$ (0.015,0.020)	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
9 M_8, M_1	$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$2\sqrt{\frac{m_d{}^2}{m_s m_b}} \pm \sqrt{\frac{m_d m_u}{m_b m_c}}$ (0.015,0.020)	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022,0.10)	No (V_{ub}, V_{cb})

The matrix $(V_{uL}V_{dL}^\dagger)$ is the mixing matrix for 2 quark generations. It is a 2×2 unitary matrix. As such, it generally contains 4 parameters, of which one can be chosen as a real angle, θ_C, and 3 are phases:

$$(V_{uL}V_{dL}^\dagger) = \begin{pmatrix} \cos\theta_C \; e^{i\alpha} & \sin\theta_C \; e^{i\beta} \\ -\sin\theta_C \; e^{i\gamma} & \cos\theta_C \; e^{i(-\alpha+\beta+\gamma)} \end{pmatrix}. \tag{4.11}$$

By the transformation

$$(V_{uL}V_{dL}^\dagger) \to V = P_u(V_{uL}V_{dL}^\dagger)P_d^*, \tag{4.12}$$

with

$$P_u = \begin{pmatrix} e^{-i\alpha} & \\ & e^{-i\gamma} \end{pmatrix}, \quad P_d = \begin{pmatrix} 1 & \\ & e^{i(-\alpha+\beta)} \end{pmatrix}, \tag{4.13}$$

we eliminate the three phases from the mixing matrix. (We redefine the mass eigenstates $u_{L,R} \to P_u u_{L,R}$ and $d_{L,R} \to P_d d_{L,R}$, so that the mass matrices remain unchanged. In particular, they remain real.) Notice that there are three independent phase differences between the elements of P_u and those of P_d, and three phases in $(V_{uL}V_{dL}^\dagger)$. Consequently, there are no physically meaningful phases in V, and hence no CP violation:*

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}. \tag{4.14}$$

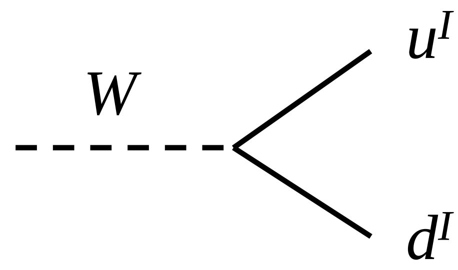
For two generations, V is called the Cabibbo matrix [1]. If $\sin\theta_C$ of (4.14) is different from zero, then the $W^\pm$ interactions mediate generation-changing currents.

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

Recap

$$-L_{Yuk} = Y^{d}_{ij}\,(\overline{u^{I}_{L}}, \overline{d^{I}_{L}})_i \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix} d^{I}_{Rj} + \ldots$$

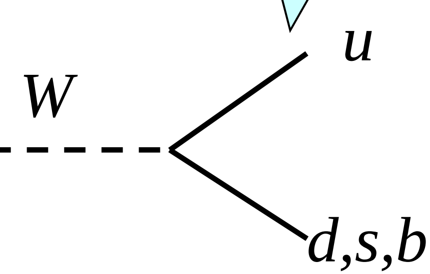
$$L_{Kinetic} = \frac{g}{\sqrt{2}}\overline{u^{I}_{Li}}\gamma^{\mu}W^{-}_{\mu}d^{I}_{Li} + \frac{g}{\sqrt{2}}\overline{d^{I}_{Li}}\gamma^{\mu}W^{+}_{\mu}u^{I}_{Li} + \ldots$$



Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^{I} \\ s^{I} \\ b^{I} \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-L_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}}\overline{u_i}\gamma^{\mu}W^{-}_{\mu}V_{ij}\left(1-\gamma^{5}\right)d_j + \frac{g}{\sqrt{2}}\overline{d_j}\gamma^{\mu}W^{+}_{\mu}V^{*}_{ij}\left(1-\gamma^{5}\right)u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

M(diag) is unchanged if $V_L^{'f} = P^f V_L^f$; $V_R^{'f} = P^f V_R^f$ $V(CKM) = P^u V(CKM') P^{*d}$

P^f = phase matrix

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} e^{-i\varphi_1} & 0 \\ 0 & e^{-i\varphi_2} \end{pmatrix} \begin{pmatrix} V'_{11} & V'_{12} \\ V'_{21} & V'_{22} \end{pmatrix} \begin{pmatrix} e^{-i\chi_1} & 0 \\ 0 & e^{-i\chi_2} \end{pmatrix} = \begin{pmatrix} V'_{11} e^{-i(\varphi_1-\chi_1)} & V'_{12} e^{-i(\varphi_1-\chi_2)} \\ V'_{21} e^{-i(\varphi_2-\chi_1)} & V'_{22} e^{-i(\varphi_2-\chi_2)} \end{pmatrix}$$

$u \to u e^{i\phi_{11}}$

Redifine the quark field

$V_{11} e^{i\phi_{11}} e^{-i(\varphi_1-\chi_1)}$

I choose $\varphi_1 - \chi_1$ such than V_{11} real

I choose $\varphi_1 - \chi_2$ such than V_{12} real

I choose $\varphi_2 - \chi_1$ such than V_{21} real

BUT: $(\varphi_2 - \chi_2) = (\varphi_2 - \chi_1) + (\varphi_1 - \chi_2) - (\varphi_1 - \chi_1)$

I cannot play the same game with all four fields
but only with 3 over 4

(2n-1) irreducible phases

APPENDIX

JARSLOG DISCRIMINANT

UT area and condition for CP violation (formal)

The standard representation of the CKM matrix is:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad \begin{aligned} c_{ij} &\equiv \cos\theta_{ij} \\ s_{ij} &\equiv \sin\theta_{ij} \end{aligned}$$

However, many representations are possible. What are the invariants under re-phasing?

- Simplest: $U_{\alpha i} = |V_{\alpha i}|^2$ is independent of quark re-phasing
- Next simplest: Quartets: $Q_{\alpha i\beta j} = V_{\alpha i}\, V_{\beta j}\, V_{\alpha j}^*\, V_{\beta i}^*$ with $\alpha \neq \beta$ and $i \neq j$
  - "Each quark phase appears with and without *"
- $V^{\dagger}V=1$: Unitarity triangle: $V_{ud}\, V_{cd}^* + V_{us}\, V_{cs}^* + V_{ub}\, V_{cb}^* = 0$
  - Multiply the equation by $V_{us}^*\, V_{cs}$ and take the imaginary part:
  - $Im\,(V_{us}^*\, V_{cs}\, V_{ud}\, V_{cd}^*) = -\,Im\,(V_{us}^*\, V_{cs}\, V_{ub}\, V_{cb}^*)$
  - $J = Im\, Q_{udcs} = -\,Im\, Q_{ubcs}$
  - The imaginary part of each Quartet combination is the same (up to a sign)
  - In fact it is equal to 2x the surface of the unitarity triangle

    Area = ½ $|V_{cd}|\,|V_{cb}|$ h ; $h=|V_{ud}|\,|V_{ub}| \sin \arg(-V_{ud}V_{cb}V_{ub}^*V_{cb}^*)|$

    $=1/2\ |Im(V_{ud}V_{cb}V_{ub}^*V_{cb}^*)|)|$
- $Im[V_{\alpha i}\, V_{\beta j}\, V_{\alpha j}^*\, V_{\beta i}^*] = J \sum \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ijk}$ where J is the universal Jarlskog invariant
- Amount of CP Violation is proportional to J

The Amount of CP Violation

Using Standard Parametrization of CKM:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

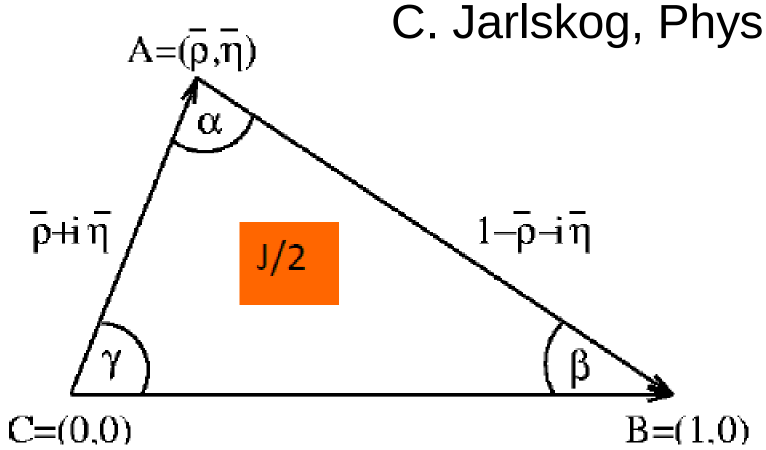
$$c_{ij} \equiv \cos\theta_{ij} \qquad s_{ij} \equiv \sin\theta_{ij}$$

$$J \equiv c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta = (3.0 \pm 0.3)\times 10^{-5} = \lambda^6 A^2 \eta$$

(eg.: $J=Im(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$)

(The maximal value J might have = $1/(6\sqrt{3}) \sim 0.1$)

C. Jarlskog, Phys. Rev. Lett.55, 1039 (1985)



More details

CP Violation at the Lagrangian level

$$L_W = \frac{g}{2}\overline{Q}^{Int.}_{L_i}\gamma^\mu\sigma^a Q^{Int.}_{L_i} W^a_\mu \qquad a=1,2,3 \qquad Q^{Int.}_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i}\end{pmatrix} \quad L^{Int.}_{L_i} = \begin{pmatrix} \nu_{L_i} \\ l_{L_i}\end{pmatrix}$$

$$L_M = M^d_{ij}\overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij}\overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij}\overline{l}^{Int.}_{L_j} l^{Int.}_{R_j} \qquad \text{where} \quad M^f = (v/\sqrt{2})Y^f$$

Accept that (or verify) the most general CP transformation which leave the lagrangian invariant is

$$d^{Int.}_L -> W_L C d^{Int.*}_L \quad ; \quad d^{Int.}_R -> W^d_R C d^{Int.*}_R$$

$$u^{Int.}_L -> W_L C u^{Int.*}_L \quad ; \quad u^{Int.}_R -> W^u_R C u^{Int.*}_R$$

$$(C = i\gamma^2\gamma^0 \qquad W_L, W^u_R, W^d_R \quad \text{unitarity matrices})$$

In order to have L_M to be invariant under CP, the M matrices should satisfy the following relations :

$$W_L^\dagger M_u W^u_R = M^*_u \qquad W_L^\dagger H_u W_L = H^*_u \qquad \text{where} \quad H_u = M_u M_u^\dagger \text{ and } \quad W^u_R = M_u^\dagger W_L$$

$$W_L^\dagger M_d W^d_R = M^*_d \qquad W_L^\dagger H_d W_L = H^*_d \qquad \text{where} \quad H_d = M_d M_d^\dagger \text{ and } \quad W^d_R = M_d^\dagger W_L$$

in this form, these conditions are of little use. A way of doing is :

$$W_L^\dagger H_u H_d W_L = H^T_u H^T_d$$

$$W_L^\dagger H_d H_u W_L = H^T_d H^T_u$$

- The existence of charged current contrains u_L,d_L to trasform in the same way under CP while the absence of right charged current allow u_R,d_R to tranform differentely under CP

More details

Substracting these two equations

$$W_L^\dagger [H_u\, H_d] W_L \quad = -[H_u H_d]^T$$

If one evaluates the traces of both sides, they vanish identically and no constraints is obtained. In order to obtain no trivial contrain, we have to multiply the previous equation a odd number of times :

$$W_L^\dagger [H_u\, H_d]^r W_L \quad = -\{[H_u H_d]^r\}^T \qquad (r \quad \text{odd})$$

Taking the traces one obtain :

$$Tr[H_u\, H_d]^r \quad = 0$$

For n=1, and n=2 the previous equations are automatically satified for harbitrary hermitian H matrices (it is the same as the counting of the physical phase of the CKM matrix). For n=3 or larger the previous eq. provides non trivial contraints on the H matrix. It can be shown that for n=3 it implies

$$Tr[H_u\, H_d]^3 \quad = 6\Delta_{21}\Delta_{31}\Delta_{32}\,\mathrm{Im}\,Q$$
$$\Delta_{21} = (m_s^2 - m_d^2)\times(m_c^2 - m_u^2)$$
$$\Delta_{31} = (m_b^2 - m_d^2)\times(m_t^2 - m_u^2)$$
$$\Delta_{32} = (m_b^2 - m_s^2)\times(m_t^2 - m_c^2)$$

CP violation vanish in the limit where any two quarks of the same charge become degenerate. But it does not necessarily vanish in the limit where one quark is massless (m_u=0) or even in the chiral limit (m_u=m_d=0)

CP violation vanish if the triangle has area equal to 0

CP Violation in the Standard Model

Requirements for CP violation

$$\left(m_t^2 - m_c^2\right)\left(m_t^2 - m_u^2\right)\left(m_c^2 - m_u^2\right)$$
$$\times \left(m_b^2 - m_s^2\right)\left(m_b^2 - m_d^2\right)\left(m_s^2 - m_d^2\right) \times J_{CP} \neq 0$$

where

$$J_{CP} = \left|\mathrm{Im}\left\{V_{i\alpha} V_{j\beta} V_{i\beta}^{*} V_{j\alpha}^{*}\right\}\right| \left(i \neq j, \alpha \neq \beta\right)$$

Jarlskog determinant

Using above parameterizations

$$J_{CP} = s_{12} s_{13} s_{23} c_{12} c_{23} c_{13} \sin\delta = \lambda^6 A^2 \eta = O\left(10^{-5}\right)$$

➡ CP violation is small in the Standard Model