Introduction to the Standard Model – part IV

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Hidden gauge symmetry in a superconductor

- Ginzburg–Landau phenomenological description of superconductivity (microscopic: Bardeen–Cooper–Schrieffer (BCS) theory):
  1. there exists an order parameter, the macroscopic "wave function" ψ. It describes a condensate of correlated electron pairs (the Cooper pairs). We assing it a charge $-2e$ and a mass $2m_e$.
  2. Below the critical temperature, the order parameter, $|\psi|^2$ is nonzero, and equal to (half) the density of superconducting electrons n_{sc}.
  3. In general $\psi = |\psi| \exp(i\phi(\vec{x}))$, with almost constant and nonzero $|\psi|$ in the superconducting phase.

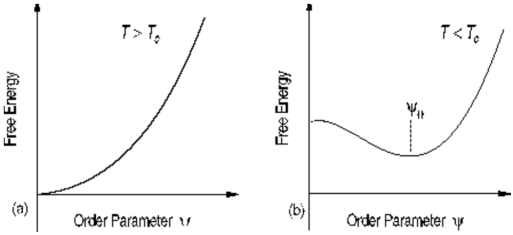

Figure: Free energy as a function of order parameter below and above critical temperature

Photon acquires mass inside superconductor

- Consider the quantum-mechanical e.m. current associated with ψ in the presence of a static magnetic field described by a vector potential $\vec{A}$:

$$\vec{j}_{\rm em} = \frac{-2e}{4m_e i}\Big(\psi^*(\vec{\nabla} + 2ie\vec{A})\psi - \psi((\vec{\nabla} + 2ie\vec{A})\psi)^*\Big)\,, \quad \psi = |\psi| \exp(i\phi(\vec{x}))\,.$$

- Only the phase ϕ of the condensate WF has a variation with $\vec{x}$. We then obtain:

$$\vec{j}_{\rm em} = -\frac{2e^2|\psi|^2}{m_e}\Big(\vec{A} + \vec{\nabla}\phi\Big), \quad \Rightarrow \quad \vec{\nabla} \times \vec{j}_{\rm em} = -\frac{2e^2|\psi|^2}{m_e}\,\vec{B}.$$

- Using the static Maxwell's equation, we obtain the massive Klein–Gordon equation for the magnetic field.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \vec{j}_{\rm em} \quad \Rightarrow \quad \vec{\nabla}^2 \vec{B} = \underbrace{\frac{2e^2|\psi|^2}{m_e}}_{m^2_{\rm eff}}\,\vec{B}\,.$$

- within the superconducting material the magnetic field decays $B \propto \exp(-m_{\rm eff}|\vec{x}|)$. The magnetic field is expelled from the superconductor (Meissner effect).
- The photon has **effectivly acquired a mass** in the presence of the superconducting condensate.

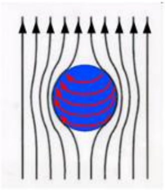

Figure: Meissner effect

- Superconductivity gives a hint how a hidden symmetry phase can lead to a massive gauge boson.
- We need to generate masses for $W^{\pm}, Z$ the photon needs to stay massless.
- Introduce a new scalar field, the Higgs field.

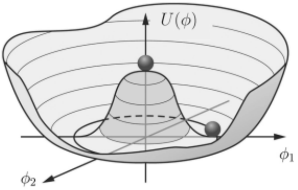

Figure: Mexican hat potential

- The Higgs field is a complex $SU(2)_L$ doublet and has $U(1)_Y$ hypercharge $Y = 1/2$:

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \begin{pmatrix} \Re e\, \phi^+(x) + i\Im m\, \phi^+(x) \\ \Re e\, \phi^0(x) + i\Im m\, \phi^0(x) \end{pmatrix}.$$

The components of the Higgs field have charges and weak isospin T_3

$$Q(\phi^+) = +1, \quad T_3(\phi^+) = +\tfrac{1}{2} \quad Q(\phi^0) = 0, \quad T_3(\phi^0) = -\tfrac{1}{2}.$$

Higgs mechanism

- Now, we want to let the Higgs field participate in the electroweak interactions, in a gauge-invariant way. We simply take the Lagrangian for the complex scalar field and replace derivatives ∂_μ by **covariant derivatives** D_μ:

$$\mathcal{L}_{\rm Higgs} = (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \,, \quad \mu^2 < 0, \lambda > 0.$$

$$D_\mu \phi = (\partial_\mu + ig \frac{\sigma^i}{2} W^i_\mu + ig' y_\phi B_\mu) \phi.$$

- SSB for $\mu^2 < 0$: infinite set of degenerate states minimize the potential. The Higgs field has a vev:

Higgs vev

$$|\langle 0 | \phi^0 | 0 \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}.$$

- Only the neutral part of the scalar field, ϕ^0 can develop a vev, as charge is conserved and the vacuum is neutral.

- choosing a particular ground state, we break the $SU(2)_L \times U(1)_Y$ symmetry. There remains however a $U(1)_{\rm em}$ symmetry intact. This is the subgroup generated by $Q = T_3 + Y$. We write

$$SU(2)_L \otimes U(1)_Y \to U(1)_{\rm em}.$$

- The Goldstone theorem now tells us (?) that because the symmetries generated by three out of the four generators are broken, we should have three massless Goldstone bosons. There are no such particles in Nature, so these degrees of freedom **should be unphysical**.
- Let us parametrize the Higgs field as:

$$\phi(x) = \exp\left(i\frac{\sigma^i}{2}\theta^i(x)\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + H(x)\end{pmatrix}.$$

- Now from our previous discussion, we realize that the three real fields θ^i are the would-be Goldstone bosons. But we have a local $SU(2)_L$ invariance! Hence we can "rotate" the phase away, or in other words choose the gauge $\theta^i(x) \equiv 0, i = 1, 2, 3.$
- The gauge $\theta^i = 0$ is called "unitary gauge". In unitary gauge the Higgs field reads:

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + H(x)\end{pmatrix}.$$

- From the "kinetic piece" of the scalar Lagrangian we obtain in unitary gauge:

$$(D_\mu \phi)^\dagger D^\mu \phi \to \tfrac{1}{2}\partial_\mu H \partial^\mu H + (v+H)^2 \Big(\frac{g^2}{4} W^\dagger_\mu W_\mu + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu \Big) \, .$$

- The vev v of the scalar field generates a quadratic term for $W^\pm$ and Z. **They acquire the masses** (proportional to the vev):

$$M_W = \tfrac{1}{2} v g \, , \quad M_Z = \frac{M_W}{\cos\theta_W} \, .$$

- In unitary gauge we "lost" the three degrees of freedom θ^i. But we gained three longitudinal polarization states of the massive $W^\pm, Z$. "The Goldstone bosons get eaten by the gauge bosons."
- There are also couplings between Higgs boson and gauge bosons:

$$\mathcal{L}_{HWW,HZZ} = \Big(\frac{1}{v} H + \frac{1}{2v^2} H^2 \Big) \Big(2 M_W^2 W^\dagger_\mu W^\mu + M_Z^2 Z^\mu Z_\mu \Big) \, .$$

- The Higgs itself is also massive. Its mass is also $\propto v$, but depends on the parameter λ of the potential. In the SM $m_H^2 = 2\lambda v^2$. There are also triple and quartic Higgs couplings in the SM. But remember that **the exact form of the potential is not important for the SSB mechanism alone**.

- Because $M_W = M_Z \cos\theta_W$ the Standard Model predicts, that $M_Z > M_W$, in agreement with experiment:

$$M_Z = 91.1876(21)\,\mathrm{GeV}, \quad M_W = 80.377(12)\,\mathrm{GeV}\,.$$

From these values, one would obtain:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223\,.$$

- The energy release in the decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is much smaller than M_W:

$$q^2 = (p_\mu - p_{\nu_\mu})^2 = (p_e + p_{\nu_e})^2 < M_\mu^2 \ll M_W^2$$

- We can therefore neglect q^2 in the W–propagator. In fact it "shrinks to a point" and we deal with a pointlike four–fermion interaction with the **dimensionful Fermi coupling** G_F.

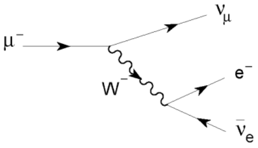

Figure: Weak decay of a muon

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} = 4\sqrt{2} G_F \,.$$

- From the muon lifetime

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} f(m_e^2/m_\mu^2)(1+\delta_{\rm RC}) \Rightarrow \tau_\mu = 2.197019(21) \cdot 10^{-6} s$$

we get the Fermi coupling $G_F = 1.1663 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$, and therefore the vev

$$v = (\sqrt{2} G_F)^{1/2} = 246\,\mathrm{GeV}\,.$$

This is the **electroweak energy scale**.

Decay modes for W and Z

- From tree–level diagrams we get the partial decay widths (neglecting all fermion masses):

$$\Gamma(W^- \to \ell^- \bar{\nu}_\ell) = \frac{G_F M_W^3}{6\pi\sqrt{2}}, \quad \Gamma(W^- \to \bar{u}_i d_j) = N_c \, |V_{ij}| \frac{G_F M_W^3}{6\pi\sqrt{2}}$$

$$\Gamma(Z \to f\bar{f}) = \frac{G_F M_W^3}{6\pi\sqrt{2}} \left(|v_f|^2 + |a_f|^2 \right) \times N_f$$

Here $N_f = 1$ for leptons and $N_f = N_c = 3$ for quarks.

- For the W all leptons have the same partial width, quarks involve the mixing factor V_{ij} (CKM matrix).
- For the Z boson each decay mode has a different partial width, but couplings do not distinguish families.
- The total widths obtained from summing over all modes are:

$$\Gamma_W = 2.09\,\mathrm{GeV}, \qquad \Gamma_Z = 2.49\,\mathrm{GeV}\,.$$

in good agreement with experiment

$$\Gamma_W = 2.085(42)\,\mathrm{GeV}, \quad \Gamma_Z = 2.4952(23)\,\mathrm{GeV}$$

Notice also, that $\Gamma_W \ll M_W, \Gamma_Z \ll M_Z$, they are very narrow resonances.

Figure: Tree level diagrams for the decays

- The universality of couplings gives equal branching fractions into all lepton channels for the W:

$$\mathrm{BR}(W^- \to \ell^- \bar{\nu}_\ell) = \frac{1}{3 + 2 \times N_c} = 11.1\%$$

but for the Z boson $\Gamma(Z \to \ell\bar{\ell}) = 84.85\,\mathrm{MeV} \ll \Gamma_Z$.

- we obtain a very good picture of gauge boson properties predicted by EW theory already at the tree level:

lepton	e	μ	τ	average
$\mathrm{BR}(W^- \to \ell\bar{\nu}_\ell)$	10.71(16)	10.63(15)	11.38(21)	10.86(9)
$\Gamma(Z \to \ell^+\ell^-)$ [MeV]	83.92(12)	83.99(18)	84.08(22)	83.984(86)

Fermion mass terms

- Previously we could not construct a mass term as it couples left and right handed fermions. Now we have the isodoublet Higgs field, and $\bar{L}\phi$ is an $SU(2)_L$ singlet that can be coupled to a right–handed fermion field.
- Let us concentrate on one family only. The gauge invariant Lagrangian reads:

$$\mathcal{L}_{Yuk} = -c_1 \bar{Q}_L \phi d_R - c_2 \bar{Q}_L i\sigma_2 \phi^* u_R - c_3 \bar{\ell}_L \phi e_R - c_4 \bar{\ell}_L i\sigma_2 \phi^* \nu_R \,.$$

  Here the doublet fields are

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \ell_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad i\sigma_2 \phi^* = \phi^c = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}.$$

- after symmetry breaking we obtain the very simple form:

$$\mathcal{L}_{Yuk} = -\frac{1}{\sqrt{2}}(v + H)(c_1 \bar{d}d + c_2 \bar{u}u + c_3 \bar{e}e + c_4 \bar{\nu}\nu)\,.$$

- The SSB also can generate fermion masses, again proportional to the vev.

$$m_d = c_1 \frac{v}{\sqrt{2}} \quad m_u = c_2 \frac{v}{\sqrt{2}} \quad m_e = c_3 \frac{v}{\sqrt{2}} \quad m_\nu = c_4 \frac{v}{\sqrt{2}}\,.$$

- The scale of masses is set by the vev $v/\sqrt{2} \approx 174\,\mathrm{GeV}$. The parameters c_i are completely undetermined by theory.
- The Yukawa couplings of fermions to the Higgs are also proportional to their masses $H/v \cdot (m_d \bar{d}d + \dots)$

Masses of SM particles

- Masses vary over 5 orders of magnitude (14 orders is neutrinos are included).

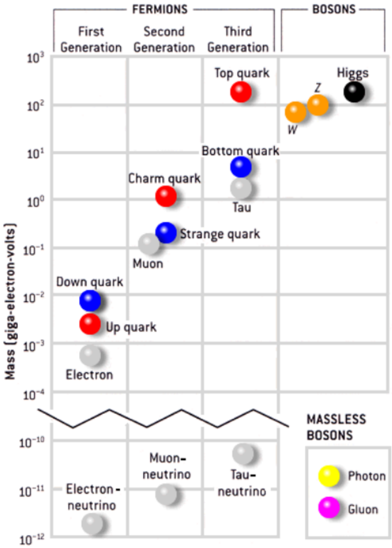

Figure: Fermion masses are a mystery

Fermion masses – Yukawa couplings

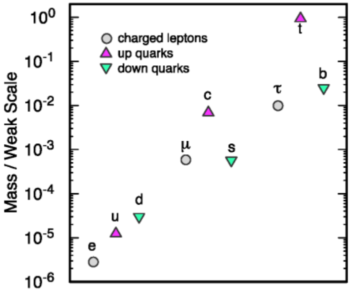

Figure: Yukawa couplings

- The Yukawa couplings $c_i = \sqrt{2}m_i/v$ range from ~ 1 for the top quark to $\sim 3 \cdot 10^{-6}$ for the electron.
- Two different mass generation mechanisms in the SM:
  1. Masses of gauge bosons are directly related to SSB, they are on the order of the EW scale.
  2. Fermion masses depend on Yukawa couplings which are not predicted by the SM. Their large variation suggests that the fermion masses involve physics beyond the standard model.
- While EW gauge boson couplings are universal over families, the Higgs field and Higgs boson coupling clearly distinguish among families.

Incompleteness of the Standard Models

- (too) many free parameters.
- does not predict masses of quarks and leptons and mixing parameters
- does not explain origin of CP violation.
- amount of CP violation is not sufficient to describe matter–antimatter asymmetry in the universe
- "hierachy problem": radiative corrections to the Higgs mass grow quadratically with the cutoff $\delta m_H^2 \propto \Lambda^2$, so that very large cancellations between "bare mass" and radiative corrections are required ("fine tuning").
- dark matter? ...
- conceptual problems:
  1. Higgs potential gives rise to a vacuum energy density $\rho_H = \frac{M_H^2 v^2}{8} \sim 10^8\,\mathrm{GeV}$. About 54 orders of magnitude bigger than the lower bound derived by cosmologists.
  2. no gravity

References

- Some open access sources which have been used in the preparation of these lectures are:

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