Introduction to the Standard Model – part I

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Outline of the lecture

- Introduction
  1. particles of the Standard Model
  2. building hadrons from quarks and gluons
- Some basic input from Quantum Field Theory
  1. Wave equation for spinless particles
  2. Spin-1/2 particles, handedness, the Dirac equation
  3. Lagrangians in particle physics
- The gauge principle
- The Lagrangians of QED and QCD
- Towards Electroweak unification

Particle content of the Standard Model

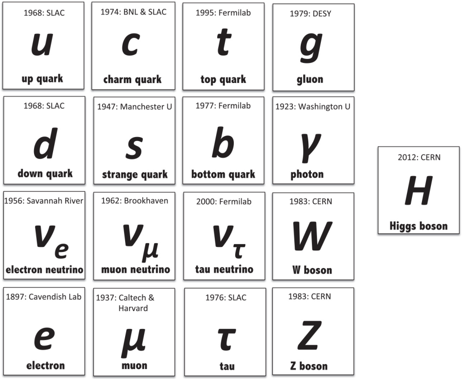


Figure: Fundamental particles of the SM

- Spin $\frac{1}{2}$ fermions: quarks and leptons, Spin 1: gauge bosons, Spin 0: Higgs boson.
- The Standard Model is based on a large body of empirical information. It is a relativistic Quantum Field Theory of the **electromagnetic, weak and strong interactions**.

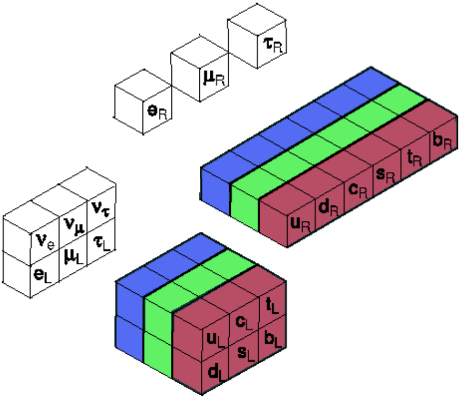


Figure: The Standard Model Fermions

- Fermions come in families with universal gauge interactions.
- Fermions in different families differ by their flavor quantum numbers and masses.

Fermions in the Standard Model

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

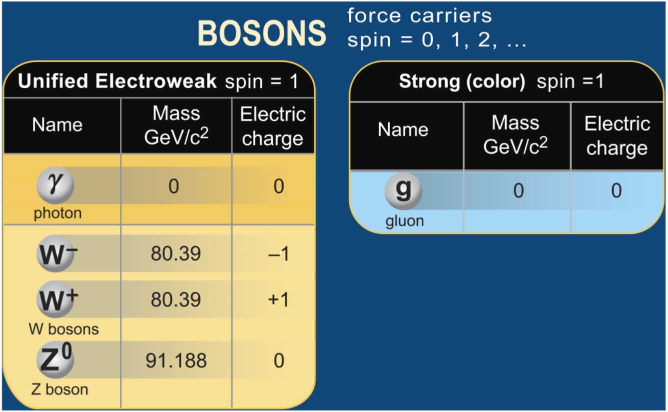
Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

Figure: The Standard Model Fermions

Gauge bosons in the Standard Model



BOSONS force carriers spin = 0, 1, 2, …

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.39	–1
W$^+$ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Figure: The Standard Model gauge bosons

Building hadrons from quarks

Mesons (spin $= 0, 1, 2, 3, \ldots$):

$$\begin{array}{llll} \pi^+ = u\bar{d}, & K^+ = u\bar{s}, & K^0 = d\bar{s}, & \pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2} \ \ldots \\ D^+ = c\bar{d}, & D^0 = c\bar{u}, & D_s^+ = c\bar{s} & \ldots \\ B^+ = u\bar{b}, & B^0 = d\bar{b}, & B_s^0 = s\bar{b}, & B_c^+ = c\bar{b} \ \ldots \end{array}$$

Baryons (spin $= 1/2, 3/2, 5/2, \ldots$):

$$\begin{array}{llll} p = uud, & n = udd, & \Sigma^+ = uus, & \Sigma^0 = uds, \ \ldots \\ \Sigma_c^+ = udc, & \Sigma_c^{++} = uuc, & \Xi_c^+ = usc, & \Xi_c^0 = dsc \ \ldots \\ \Xi_{cc}^+ = dcc, & \Xi_{cc}^{++} = ucc, & \Omega_{cc}^+ = scc & \ldots \end{array}$$

Figure: Some Mesons and Baryons with their quark content

- Quarks and gluons do not exist as free particles. They are confined in **hadrons** (strongly interacting particles).
- We know about their "existence" as degrees of freedom from studying hadrons at short distances. E.g. in high-momentum transfer scattering of electrons or the annihilation of e^+e^- at high energy.

Mesons

- Quantum numbers: Isospin: I, G-parity: G Spin: J, Parity: P Charge Conjugation:C (for neutral mesons) $m(\pi^{\pm}) \sim 140$ MeV. Large mass gap between pion & other mesons.
- Most mesons have strong decays. (The ones with masses given in brackets). They generally appear as short-lived resonances in hadronic reactions, although there are exceptions (heavy quarkonia).
- The mesons stable wrt. strong interactions decay via weak or electromagnetic interactions. There are no stable mesons.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE ($S = \pm 1$, $C = B = 0$)

	$I(J^P)$
• $K^{\pm}$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm 1$)

• $D^{\pm}$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^{\pm}$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^{\pm}$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^{\pm}$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^{\pm}$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^{\pm}$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm 1$)

	$I(J^P)$
• $D_s^{\pm}$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^{\pm}$	$0(0^+)$
• $D_{s1}(2460)^{\pm}$	$0(1^+)$
• $D_{s1}(2536)^{\pm}$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^{\pm}$	$0(1^-)$
$D_{s1}^*(2860)^{\pm}$	$0(1^-)$
$D_{s3}^*(2860)^{\pm}$	$0(3^-)$
$D_{sJ}(3040)^{\pm}$	$0(?^?)$

BOTTOM ($B = \pm 1$)

• $B^{\pm}$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^{\pm}/B^0$ ADMIXTURE	
• $B^{\pm}/B^0/B^0_s/b$ baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm 1$, $S = \mp 1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^{\pm}$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm 1$)

• B_c^+	$0(0^-)$
$B_c(2S)^{\pm}$	$0(0^-)$

$c\bar{c}$ (+ possibly non-$q\bar{q}$ states)

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$

$c\bar{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^{\pm}$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^{\pm}$	$1^-(?^{?+})$
$X(4055)^{\pm}$	$1^+(?^{?-})$
$X(4100)^{\pm}$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^{\pm}$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\bar{b}$ (+ possibly non-$q\bar{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

Baryons

- Proton is the only stable hadron.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	• $a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(0^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	• $\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$a_0(1950)$	$1^-(0^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $a_4(1970)$	$1^-(4^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\pi_2(2005)$	$1^-(2^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $h_1(1415)$	$0^-(1^{+-})$	$f_0(2020)$	$0^+(0^{++})$
$a_1(1420)$	$1^-(1^{++})$	• $f_4(2050)$	$0^+(4^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2100)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_2(2150)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	• $\phi(2170)$	$0^-(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_0(2200)$	$0^+(0^{++})$
• $f_0(1500)$	$0^+(0^{++})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$f_1(1510)$	$0^+(1^{++})$		
• $f_2'(1525)$	$0^+(2^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$\rho(1570)$	$1^+(1^{--})$	• $f_2(2300)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_4(2300)$	$0^+(4^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_0(2330)$	$0^+(0^{++})$
• $a_1(1640)$	$1^-(1^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1640)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
• $\eta_2(1645)$	$0^+(2^{-+})$	$f_6(2510)$	$0^+(6^{++})$
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
$D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$ baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\bar{c}$ (+ possibly non-$q\bar{q}$ states)

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$

$c\bar{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\bar{b}$ (+ possibly non-$q\bar{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

Ongoing research

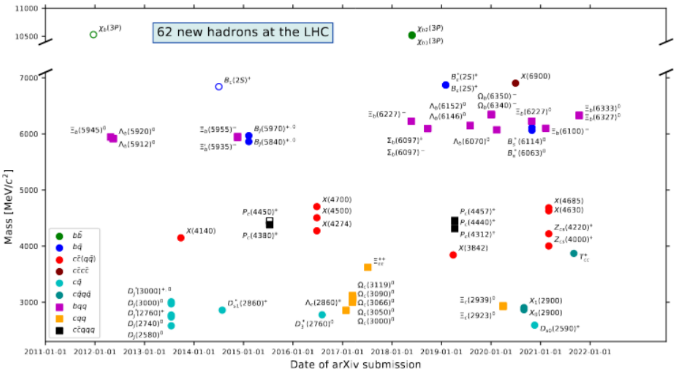


Figure 1: Hadrons discovered at the LHC, plotted as mass versus preprint submission date [1]. Only states observed with significance exceeding 5σ are included. Hollow markers indicate superseded states.

Length & energy scales, natural units

Quantity	[kg, m, s]	[$\hbar$, c, GeV]	$\hbar = c = 1$
Energy	$\mathrm{kg\,m^2\,s^{-2}}$	GeV	GeV
Momentum	$\mathrm{kg\,m\,s^{-1}}$	GeV/c	GeV
Mass	kg	GeV/c^2	GeV
Time	s	$(\mathrm{GeV}/\hbar)^{-1}$	GeV^{-1}
Length	m	$(\mathrm{GeV}/\hbar c)^{-1}$	GeV^{-1}
Area	$\mathrm{m^2}$	$(\mathrm{GeV}/\hbar c)^{-2}$	GeV^{-2}

Figure: SI vs. natural units

- In particle physics SI units will quickly yield unwieldy numbers. We want to go from SI units to natural units:

$$[kg, m, s] \to [GeV, \hbar, c].$$

$$1\,GeV = 1.602 \cdot 10^{-10}\,J, \quad 1\,J = 1\,\mathrm{Joule} = 1\,kg\,m^2\,s^{-2},$$
$$\hbar = 1.055 \cdot 10^{-34}\,J\,s, \quad c = 2.998 \cdot 10^8\,m\,s^{-1}.$$

- **Natural units**: $\hbar = c = 1$, $\hbar c = 0.1973\,GeV\,fm$, $1\,fm = 10^{-15}\,m$.

- The unit of cross section is the barn, $1\,barn = 100\,fm^2$. Proton radius is about $\sim 1\,fm$.
- $0.389\,mb \sim 1\,GeV^{-2}$.

Figure: one barn

- In quantum mechanics observables such as energy and momentum correspond to operators.

$$E \Rightarrow \hat{H} = i\hbar \frac{\partial}{\partial t} \qquad \vec{p} \Rightarrow \vec{p}_{\rm op} = -i\hbar \vec{\nabla}$$

- Free nonrelativistic particle of mass m

$$E = \frac{\vec{p}^{\,2}}{2m}$$

free Schrödinger equation

$$\hat{H}\psi(t,\vec{x}) = E\psi(t,\vec{x}) \Leftrightarrow i\hbar \frac{\partial}{\partial t}\psi(t,\vec{x}) = -\frac{\hbar^2}{2m}\vec{\nabla}^{\,2}\psi(t,\vec{x})$$

- The Schrödinger eqn. is linear in time- and quadratic in spacial derivatives. It does not lend itself for the description of relativistic phenomena.

Current and probability density

- We are used to the interpretation of $\psi^*\psi = \rho$ as a probability density. It is hidden in the equation itself.

$$
\begin{aligned}
i\frac{\partial}{\partial t}\psi(t,\vec{x}) + \frac{\vec{\nabla}^2}{2m}\psi(t,\vec{x}) &= 0 \quad \Big| \quad i\psi^* \times \text{from left} \\
(-i)\frac{\partial}{\partial t}\psi^*(t,\vec{x}) + \frac{\vec{\nabla}^2}{2m}\psi^*(t,\vec{x}) &= 0 \quad | \quad (-i)\psi \times \text{from left}
\end{aligned}
$$

- adding these two equations gives:

$$
\frac{\partial}{\partial t}\psi^*(t,\vec{x})\psi(t,\vec{x}) - \frac{i}{2m}\vec{\nabla}\cdot\Big(\psi^*(t,\vec{x})\vec{\nabla}\psi(t,\vec{x}) - \psi(t,\vec{x})\vec{\nabla}\psi^*(t,\vec{x})\Big) = 0.
$$

Continuity equation

$$
\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0
$$

$$
\rho = \psi^*\psi, \quad \vec{j} = -\frac{i}{2m}\Big(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\Big) \equiv -\frac{i}{2m}\,\psi^*\overleftrightarrow{\nabla}\psi\,.
$$

$$
\int d^3\vec{x}\,\psi^*\psi \Rightarrow \text{probability}.
$$

- Let's take the plane-wave solution

$$\psi(t, \vec{x}) = C \exp(-iEt) \exp(i\vec{p} \cdot \vec{x}), \quad E = \frac{\vec{p}^{\,2}}{2m}.$$

Then

$$\rho = |C|^2 = \text{const.}, \quad \vec{j} = \rho \, \frac{\vec{p}}{m}.$$

- We can normalize our wavefunction to describe one particle (per Volume). The Schrödinger equation is a one-body equation.
- It does not lend itself to the description of relativistic phenomena. Recall, that in relativistic kinematics, the relation between energy and momentum is

$$E = \sqrt{\vec{p}^{\,2} + m^2} \approx m + \frac{\vec{p}^{\,2}}{2m} + \dots$$

- The Schrödinger equation has its place also in the Standard model. For example heavy quarks in their bound states (the so-called Quarkonia) are governed by nonrelativistic motion. It is also applicable to much of nuclear and atomic physics, but it cannot be a starting point for the SM.
- We still require the general principles of quantum mechanics to hold! The joint description of quantum and relativistic physics is achieved by **Quantum Field Theory**.

- The fundamental laws of particle physics have the same form in all reference frames which have uniform relative velocity. The velocity of light is the same in all Lorentz frames. Coordinates in different Lorentz frames are related by Lorentz-transformations, which leave invariant $c^2 t^2 - \vec{x}^2$.
- example: The new frame moves with velocity v along the z-axis of the original frame. The transformation is called Lorentz–boost, and with $\tanh\eta = v/c$, reads

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\eta & 0 & 0 & -\sinh\eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh\eta & 0 & 0 & \cosh\eta \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} .$$

- We collect the coordinates $x^\mu = (ct, x, y, z,) = (ct, \vec{x}) \equiv (x^0, x^1, x^2, x^3)$. Any object that transforms like x^μ is called a four vector.
- Lorentz transformations leave invariant the inner product

$$g_{\mu\nu} a^\mu b^\nu = g^{\mu\nu} a_\mu b_\nu \equiv a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b}, \quad g_{00} = +1, g_{11} = g_{22} = g_{33} = -1.$$

- An example for a four vector is the four–momentum:

$$p^\mu = \Big(\frac{E}{c}, \vec{p}\Big), \quad p^\mu p_\mu = m^2 c^2 = \frac{E^2}{c^2} - \vec{p}^{\,2} .$$

Relativistic spinless particle

- For a relativistic covariant formulation, we define

$$p^\mu = i\partial^\mu = i\frac{\partial}{\partial x_\mu}, \quad x^\mu = (t, x, y, z), \quad \partial^\mu = (\frac{\partial}{\partial t}, -\vec{\nabla}), \quad \mu = 0, 1, 2, 3$$

- for the relativistic free particle energy and momentum are related by:

$$E^2 = \vec{p}^{\,2} + m^2 \Leftrightarrow p^\mu p_\mu = m^2$$

- again substitute energy and momentum (four–momentum) by operators.

Klein–Gordon equation

$$\begin{aligned} \Big(\Box + m^2\Big)\phi(x) &= 0, \quad \Box = \partial^\mu \partial_\mu \\ \Leftrightarrow \Big(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^{\,2} + m^2\Big)\phi(x) &= 0 \end{aligned}$$

This equation is insufficient for the description of particles with spin. In fact the only spinless fundamental particle in the Standard Model is the Higgs boson. There are a number of useful concepts that we can discuss with the KG equation, so we stay with it for a while.

Current for the KG eqn.

- We can obtain the current for the Klein-Gordon equation in covariant form:

$$j^\mu(x) = (\rho(x), \vec{j}(x)), \quad x = (t, \vec{x})$$

It will be a very useful quantity for the formulation of interactions.

Current

$$\begin{aligned} \rho(x) &= i\Big[\phi^*(x)\frac{\partial\phi(x)}{\partial t} - \frac{\partial\phi^*(x)}{\partial t}\phi(x)\Big] \\ \vec{j}(x) &= -i\Big[\phi^*(x)\vec{\nabla}\phi(x) - \phi(x)\vec{\nabla}\phi^*(x)\Big] \\ j^\mu(x) &= i\phi^*(x)\overleftrightarrow{\partial}^\mu\phi(x), \quad \partial_\mu j^\mu(x) = 0\,. \end{aligned}$$

- also the continuity equation can be formulated in a covariant form.
- Let us evaluate the current of the KG equation for plane wave solutions. These have the form

$$\phi(x) = N\exp(-iEt)\exp(i\vec{p}\cdot\vec{x}) = N\exp(-ip\cdot x) \Rightarrow j^\mu(x) = 2p^\mu\,|N|^2\,.$$

- This gives $\rho(x) = 2E\,|N|^2$, $\vec{j}(x) = 2\vec{p}\,|N|^2$.
- The solutions of the KG equation satisfy $E^2 = \vec{p}^{\,2} + m^2$, but to have a complete set of solutions, we must allow **both** $E = \pm\sqrt{\vec{p}^{\,2} + m^2}$. $\Rightarrow$ **negative probabilities ?**

- The "probability density" being proportional to energy gives the **correct relativistic normalization**! For example let us boost from the rest frame to the frame in which our particle has energy E. The γ-factor is $\gamma = E/m$. The volume element will contract $d^3\vec{x} \to d^3\vec{x}/\gamma$, so that

$$\rho(x) d^3\vec{x} = \text{invariant!}$$

- It appears that negative energy solutions are not amissible. Firstly we cannot allow for arbitrarily negative energies, our system needs to have a ground state. Secondly we face **negative** probability.
- Let's assume our particle carries a charge e and interpret $ej^\mu(x)$ as the corresponding current, so that $ej^0 = e\rho$ is a charge density. Then

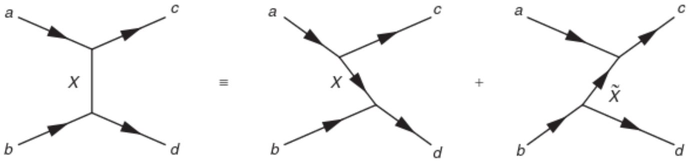
$$j^\mu(\phi^+) = 2e\,(E, \vec{p})|N|^2$$

decribes the incoming particle of charge e, but

$$\tilde{j}^\mu(\phi^-) = -2e\,(E, \vec{p})\,|N|^2 = +2e(-E, -\vec{p})\,|N|^2,$$

so that emission of ϕ^- looks like absorption of ϕ^+ with $(-E, \vec{-}p)$ propagating "backwards in time".

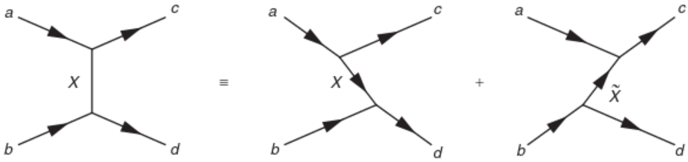
Feynman diagram



- The Feynman diagram is obtained after **adding all time orderings** of perturbation theory diagrams. Only this sum gives a relativistically invariant result. Feynman diagrams themselves have no simple space-time interpretation, they are simply a way to organize the calculation in a manifestly relativistically invariant way.
- We refer the exchanged particle in the Feynman diagram as a **virtual particle**.

$$\mathcal{M}_{fi} = \frac{g^2}{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2 - m_X^2} = \frac{g^2}{q^2 - m_X^2}$$

  Its four-momentum is $q = p_X = p_a - p_c$. The virtual particle is **off the mass shell**, $q^2 \neq m_X^2$!
- our simple example corresponds to an exchange of a spinless particle. More complicated behaviour of amplitudes will be obtained from momentum/spin dependence of **vertices** V_{ijX} and from the polarization states (or **propagators**) of the exchanged particle.



- Taking a nonrelativistic limit, it is possible to **derive** quantum mechanical potentials from Feynman diagrams for particle exchange.
- For example, let us assume, a "static" situation, in which the energy transfer can be neglected. Then our diagram gives a result

$$\mathcal{M} \propto \frac{g^2}{\vec{q}^2 + m_X^2} .$$

- after Fourier transform to space/time, this corresponds to an interaction local in time. **Yukawa Potential**:

$$V(r) = g^2 \frac{e^{-m_X r}}{r}$$

- the **mass of the exchange particle** determines the **range** of the potential. Massless particle exchange induces long-range forces. "Heavy" particles give rise to local, contact interactions.

The massless Dirac equation

- The Klein–Gordon equation does not describe **spin**. The "matter content" of the SM are leptons and quarks, Fermions of spin-1/2. Which wave equation describes these particles?
- It turns out that a simpler relativistic equation **linear** in derivatives can in fact be formulated. Let us start from the **massless** KG equation.

$$\partial^\mu \partial_\mu \psi(x) = \Big(\partial_t^2 - \vec{\nabla}^{\,2}\Big)\psi(x) = 0$$

- introduce the **Pauli-matrices**

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  Then we can write the KG equation as a product of two factors

$$\Big(\partial_t - \vec{\sigma}\cdot\vec{\nabla}\Big)\Big(\partial_t + \vec{\sigma}\cdot\vec{\nabla}\Big)\psi(x) = 0 \quad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}.$$

- We have to double the number of degrees of freedom, but have a achieved an equation **linear** in time & space derivatives.
- We can take any of the two factors as our new equation. Note that all components of the field $\psi(x)$ fulfill the ordinary (massless) KG equation.

Massless Dirac equation

$$\Big(\partial_t + \vec{\sigma}\cdot\vec{\nabla}\Big)\psi(x) = 0$$

Solutions to the massless Dirac equation

- plane wave ansatz

$$\psi(x) = \exp(-ip \cdot x)\chi, \quad p^2 = 0 \Rightarrow E^2 = \vec{p}^{\,2}.$$

- Introduce the **spin operator** $\vec{S} = \frac{1}{2}\vec{\sigma}$, and the projection of spin on the direction of momentum, **helicity**

$$\hat{h} = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}.$$

- Dirac equation(s):

$$\Big(E - \vec{\sigma} \cdot \vec{p}\Big)\chi = 0 \Leftrightarrow \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}\chi = \chi \Leftrightarrow \hat{h}\,\chi = +\tfrac{1}{2}\,\chi$$

$$\Big(E + \vec{\sigma} \cdot \vec{p}\Big)\chi = 0 \Leftrightarrow \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}\chi = -\chi \Leftrightarrow \hat{h}\,\chi = -\tfrac{1}{2}\,\chi$$

- the two versions of the Dirac equation describe particles of positive (**right**-**handed**) and negative (**left handed**) helicities, respectively.
- Let $\vec{p} = |\vec{p}|(\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\theta)$ and show, that (notice the "half angles")

$$\chi_R = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \quad \chi_L = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2}\,, \end{pmatrix}$$

Antiparticle spinors

- in the spinless case we described antiparticles by the complex conjugate solution $e^{-ip\cdot x} \to e^{ip\cdot x}$. The complex conjugate Dirac equation is

$$\left(\partial_t + \vec{\sigma}^* \cdot \vec{\nabla}\right)\psi^*(x) = 0$$

- Use the property of Pauli matrices $\sigma_y^2 = 1$, and $\sigma_y \vec{\sigma}^* \sigma_y = -\vec{\sigma}$.

$$\sigma_y\left(\partial_t + \vec{\sigma}^* \cdot \vec{\nabla}\right)\sigma_y\sigma_y\psi^*(x) = 0 \Rightarrow \left(\partial_t - \vec{\sigma} \cdot \vec{\nabla}\right)\sigma_y\psi^*(x) = 0$$

- The antiparticle is described by the spinor $\sigma_y\psi^*(x)$ and has the **opposite** helicity/handedness of the particle.
- We encountered one of the important discrete symmetry transformations. The transformation that takes a particle to its particle is called **charge conjugation**.
- Another discrete symmetry, **parity** inverts the spacial coordinates $\vec{x} = -\vec{x}$. Also momentum $\vec{p} = -i\vec{\nabla}$ is inverted: $P : \vec{p} \to -\vec{p}$.
- **Parity** takes left–handed fermions into right–handed fermions! A theory with parity invariance must have the same amount of left- and right-handed fermions.
- Fermions and anti-fermions have opposite parity!

Massive Fermions

- Helicity is a Lorentz–invariant only for massless particles. We therefore can expect that a massive fermion must be a mixture of the two helicities.

$$i\Big(\partial_t + \vec{\sigma}\cdot\vec{\nabla}\Big)\psi_\uparrow(x) = m\,\psi_\downarrow(x)$$
$$i\Big(\partial_t - \vec{\sigma}\cdot\vec{\nabla}\Big)\psi_\downarrow(x) = m\,\psi_\uparrow(x)$$

- We now come to a four-component equation

$$i\Bigg\{\begin{pmatrix}\mathbb{I} & 0\\ 0 & \mathbb{I}\end{pmatrix}\partial_t + \underbrace{\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix}}_{\vec{\alpha}}\cdot\vec{\nabla}\Bigg\}\begin{pmatrix}\psi_\uparrow(x)\\ \psi_\downarrow(x)\end{pmatrix} = m\,\underbrace{\begin{pmatrix}0 & \mathbb{I}\\ \mathbb{I} & 0\end{pmatrix}}_{\beta}\underbrace{\begin{pmatrix}\psi_\uparrow(x)\\ \psi_\downarrow(x)\end{pmatrix}}_{\Psi(x)}$$

$$\Leftrightarrow i(\partial_t + \vec{\alpha}\cdot\vec{\nabla})\,\Psi(x) = m\beta\,\Psi(x)$$

- each component fulfills the Klein-Gordon equation.

Covariant form of the Dirac equation

$$\Big(i\gamma^\mu\partial_\mu - m\Big)\Psi(x) = 0, \quad \gamma^0 = \beta, \quad \vec{\gamma} = \beta\vec{\alpha}$$

- **Dirac matrices** $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, $(\gamma^0)^2 = 1$, $(\gamma^i)^2 = -1$.

More on the Dirac equation

- The Dirac γ^μ-matrices ($\mu = 0, 1, 2, 3$) are 4×4 matrices defined by their algebra
$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$
- several realizations of the Dirac-algebra are possible, and they are all equivalent. Our choice is the so-called Weyl representation
$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$
bf need to change LH $\leftrightarrow$ RH, then sign of γ^i changes
- its is useful to introduce also the matrix $\gamma_5 = \gamma^5$, which anticommutes with all Dirac matrices.
$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}, \quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5, \quad \gamma_5^2 = \mathbb{I}\,.$$
- The projection operators
$P_L = (\mathbb{I} - \gamma_5)/2, \quad P_R = (\mathbb{I} + \gamma_5)/2, \quad P_R + P_L = \mathbb{I}, \quad P_R^2 = P_L^2 = \mathbb{I}, \quad P_R P_L = P_L P_R = 0$
serve to decompose the Dirac field into left- and right-handed components.

Left and right-handed Dirac fields

$$\psi(x) = \psi_L(x) + \psi_R(x) = \frac{\mathbb{I} - \gamma_5}{2}\psi(x) + \frac{\mathbb{I} + \gamma_5}{2}\psi(x)$$

More on Dirac fields

- We still need some kind of hermitian conjugate. It turns out the the correct conjugate to ψ is $\bar{\psi} = \psi^\dagger \gamma^0$, and $\bar{\psi}_L = \bar{\psi} P_R$, $\bar{\psi}_R = \bar{\psi} P_L$.
- The following bilinear have good transformation properties under Lorentz transformations $x^\mu \to x^{'\mu} = a^\mu_{\ \nu} x^\nu$.

Representations	Γ	Lorentz transformations
Scalar	1	$\bar{\psi}'(x')\psi'(x') = \bar{\psi}(x)\psi(x)$
Pseudoscalar	$\mathrm{i}\gamma_5$	$\bar{\psi}'(x')\mathrm{i}\gamma_5\psi'(x') = \det[a]\,\bar{\psi}(x)\mathrm{i}\gamma_5\psi(x)$
Vector	γ^μ	$\bar{\psi}'(x')\gamma^\mu\psi'(x') = a^\mu_{\ \nu}\,\bar{\psi}(x)\gamma^\nu\psi(x)$
Axial vector	$\gamma_5\gamma^\mu$	$\bar{\psi}'(x')\gamma_5\gamma^\mu\psi'(x') = \det[a]\,a^\mu_{\ \nu}\,\bar{\psi}(x)\gamma_5\gamma^\nu\psi(x)$
Tensor	$\sigma^{\mu\nu}$	$\bar{\psi}'(x')\sigma^{\mu\nu}\psi'(x') = a^\mu_{\ \alpha}\,a^\nu_{\ \beta}\,\bar{\psi}(x)\sigma^{\alpha\beta}\psi(x)$

Figure: Dirac spinor bilinears

- The conserved current for the Dirac equation is $j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x) = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L$.