Introduction to the Standard Model – part III

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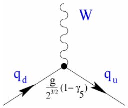


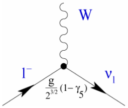


Figure: The charged current interaction of fermions

- In the matrix of gauge bosons

$$\hat{W}_\mu = \frac{1}{2}\frac{\sigma^i}{2} W^i_\mu = \frac{1}{2}\begin{pmatrix} W^3_\mu & \sqrt{2}W^\dagger_\mu \\ \sqrt{2}W_\mu & -W^3_\mu \end{pmatrix}, \quad W_\mu \equiv W^1_\mu + iW^2_\mu, \quad W^\dagger_\mu = W^1 - iW^2_\mu \, ,$$

  the **off-diagonal** fields $W_\mu, W^\dagger_\mu$ connect doublet members of charge differing by one unit, they therefore correspond to the charged bosons $W^\pm$.

- For **a single family of quarks and leptons**,

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\Big(W^\dagger_\mu[\bar{u}\gamma^\mu(1-\gamma_5)d + \bar{\nu}_e\gamma^\mu(1-\gamma_5)e] + W_\mu[\bar{d}\gamma^\mu(1-\gamma_5)u + \bar{e}\gamma^\mu(1-\gamma_5)\nu_e]\Big).$$

- **Universality of quark and lepton interactions.**

Structure of the neutral current, the Weinberg rotation

- There remain the fields W^3_μ and B_μ, which both do not carry charge.

$$\mathcal{L}_{\rm NC} = -g W^3_\mu \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g' B_\mu \sum_{j=1}^{3} y_i \bar{\psi}_i \gamma^\mu \psi_i \, .$$

- Can B_μ be the photon, and W^3_μ the Z-boson? **No, they cannot!**. We know the photon has the same coupling to both chiralities, hence we would have to require

$$y_1 = y_2 = y_3 = y, \quad \text{and} \quad g' y = e Q_u = e Q_d \, ,$$

  which cannot be true, as $Q_u = +2/3$ and $Q_d = -1/3$.

- But we can try with a linear combination, introducing the weak mixing angle (**Weinberg angle**) θ_W:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

- Now, in terms of the fields Z_μ and A_μ, the neutral–current Lagrangian can be rewritten as

$$\mathcal{L}_{\rm NC} = -\sum_{j=1}^{3} \bar{\psi}_j \gamma^\mu C_\mu \psi_j$$

with $(T_3 \equiv \sigma_3/2,\ Y\psi_j \equiv y_j \psi_j)$:

$$C_\mu = A_\mu (g T_3 \sin\theta_W + g' Y \cos\theta_W) + Z_\mu (g T_3 \cos\theta_W - g' Y \sin\theta_W)$$

- To obtain QED, we impose the conditions

$$g \sin\theta_W = g' \cos\theta_W = e, \quad \text{and} \quad Y = Q - T_3,$$

where Q is the em charge operator for quarks and leptons:

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix}, \quad Q_2 = Q_u, \quad Q_3 = Q_d \quad \text{quarks}$$

$$Q_1 = \begin{pmatrix} Q_{\nu_e} & 0 \\ 0 & Q_e \end{pmatrix}, \quad Q_2 = Q_{\nu_e}, \quad Q_3 = Q_e \quad \text{leptons}.$$

Here $Q_u = 2/3$, $Q_d = -1/3$, $Q_{\nu_e} = 0$, $Q_e = -1$.

Neutral Current interaction

- Now, the hypercharges are fixed by the electric charges and weak isospin T_3 components, $Y = Q - T_3$. The right handed fields have $T_3 = 0$.
- This means a hypothetical **right handed neutrino** has $Y = Q = 0$, so that it does not couple neither to photon nor Z-boson. It also does not couple to $W^{\pm}$ (only left handed fields do!). Such a particle which has no SM interactions is called a **sterile neutrino**.
- We summarize the NC Lagrangian: $\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}^{Z}_{\rm NC}$:

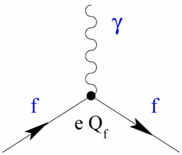
$$\mathcal{L}_{\rm QED} = -eA_\mu J^\mu_{\rm em} = -eA_\mu \Big(\frac{2}{3} \bar{u}\gamma^\mu u - \frac{1}{3} \bar{d}\gamma^\mu d - \bar{e}\gamma^\mu e \Big)$$

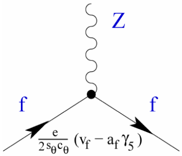
$$\mathcal{L}^{Z}_{\rm NC} = -\frac{e}{2 \sin\theta_W \cos\theta_W} J^\mu_Z Z_\mu \quad J^\mu_Z = J^\mu_3 - 2\sin^2\theta_W J^\mu_{\rm em} \, .$$

Explicitly, in terms of the fermion fields, $\mathcal{L}^{Z}_{\rm NC}$ has the form:

$$\mathcal{L}^{Z}_{\rm NC} = -\frac{e}{2 \sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu (v_f - a_f \gamma_5) f$$

where $v_f = T^f_3 - 2Q_f \sin^2\theta_W$ and $a_f = T^f_3$.





	u, c, t	d, s, b	ν_e, ν_m, ν_τ	e, μ, τ
$2v_f$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$
$2a_f$	1	-1	1	-1

Table: vector and axial vector NC couplings

$$\mathcal{L}^Z_{\rm NC} = -\frac{e}{2\sin\theta_W\cos\theta_W} Z_\mu \sum_f \bar{f}\gamma^\mu(v_f - a_f\gamma_5)f$$

Outline of the lecture

- Spontaneous Symmetry Breaking (SSB)
- Goldstone theorem
- SSB for the complex scalar field
- Hidden gauge symmetry in a superconductor. The Meissner effect and an effective photon mass.
- The Higgs mechanism in the Standard Model, masses of gauge bosons
- Some basic phenomenology
- Generation of Fermion mass terms, Yukawa couplings.

EW gauge theory of $SU(2)_L \otimes U(1)_Y$

- Up to now we have
  1. derived the charged- ($W^\pm$) and neutral (γ, Z) current interactions that allow us to describe weak decays.
  2. we have incorporated electromagnetic interactions (unified weak and electromagnetic interactions), they emerge from a common gauge group.
- We have obtained additional interactions between $W^\pm$ and Z bosons and of $W^\pm$ and γ.
- Problem: the gauge bosons are still massless:

$$M_{W^\pm} = 0, \quad M_Z = 0, \quad m_\gamma = 0\,.$$

- This is fine for the photon, but not for $W^\pm, Z$: weak interactions are **short-range**, they should not be mediated by massless exchanges.
- Gauge theories with massless gauge bosons have the very attractive property of being renormalizable. (All the gauge coupling constants are dimensionless).
- Adding mass terms "by hand" breaks the gauge symmetry. Somehow we need to "break" the gauge symmetry by still **maintaining a symmetric (gauge invariant) Lagrangian**.

- **Landau-Ginzburg theory** of continuous ("2$^{\mathrm{nd}}$ order") phase transitions. Near T_C magnetization $\vec{M}$ is small, and we can expand the free energy in low powers of $\vec{M}$.

$$u(\vec{M}) = (\nabla_i \vec{M})^2 + V(\vec{M})$$
$$V(\vec{M}) = \alpha_1(T)\vec{M}\cdot\vec{M} + \alpha_2(\vec{M}\cdot\vec{M})^2$$

- $\alpha_2 > 0$, and $\alpha_1(T) = \alpha \cdot (T - T_C)\,,\ \alpha > 0$.

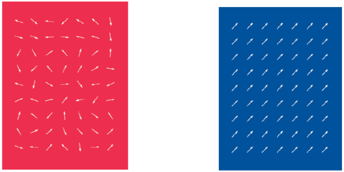


Figure: Magnet above and below T_C

- The minimum of the free energy (ground state) is realized at

$$M_i\Big(\alpha_1 + 2\alpha_2 \vec{M}\cdot\vec{M}\Big) = 0$$

- For $T > T_C$, the bracket never vanishes, and we must have $\vec{M} = 0$. However for $T < T_c$ coefficient $\alpha_1 < 0$, and the mimimum of the free energy is at a **finite magnetization**:

$$|\vec{M}| = \sqrt{\frac{\alpha\cdot(T_C - T)}{\alpha_2}}$$

- The direction of the magnetization is unspecified by the theory. The ground state with $\vec{M}$ in a particular direction is one of an infinitely degenerate set. Its direction is fixed by external conditions/boundary condistions and "breaks the rotational symmetry".

This donkey from a review by A. Pich illustrates everything about spontaneous symmetry breaking (SSB):

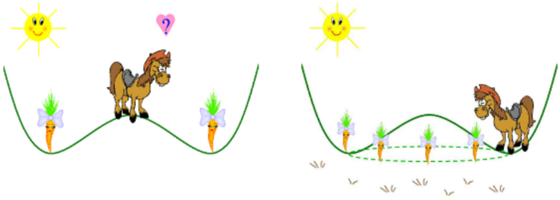
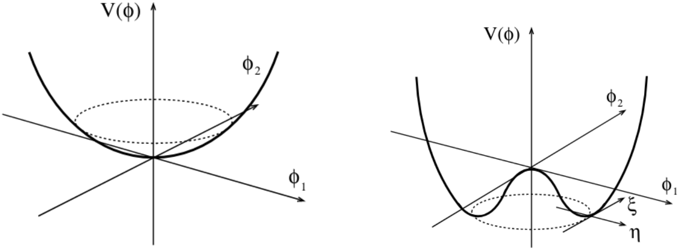


Figure: Spontaneous symmetry breaking and "Goldstone mode"

- The existence of flat directions connecting the degenerate states of minimal energy (free energy etc...) is a general property of SSB.
- In many-body theory/relativistic QFT it implies the existence of gapless/massless degrees of freedom, the Goldstone bosons.
- The fact that in a relativistic QFT for every broken direction there is a massless Goldstone boson is called the Goldstone theorem.

Spontaneous symmetry breaking for a complex scalar field



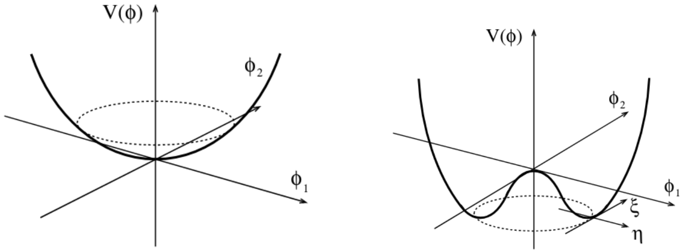
- complex scalar field with Lagrangian:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi)\,,\; V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \;= \lambda \Big(\phi^* \phi + \frac{\mu^2}{2\lambda} \Big)^2 + \mathrm{const}\,.$$

- For $\mu^2 > 0$ we have the potential on the LHS. μ is the mass of the particle, and in the ground state we have $\phi = 0$, or $\langle 0|\phi|0\rangle = 0$.
- The situation completely changes, if we allow $\mu^2 < 0$. Then we **cannot interpret the quadratic term as a mass term**. Minima of potential:

$$|\phi|^2_{\mathrm{min}} = -\frac{\mu^2}{2\lambda}\,.$$

- Now we have **infinitely many** degenerate ground states with $\phi_{\mathrm{min}} = |\phi|_{\mathrm{min}} e^{i\alpha}$.



- We can parametrize $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$. Let's choose for the ground state the mimimum for $\alpha = 0$ (ϕ_1-direction). This "breaks" the global $U(1)$ symmetry.
- Now the minimum is at $\phi_1 = \sqrt{-\mu^2/\lambda} = v$. Then we shift the fields

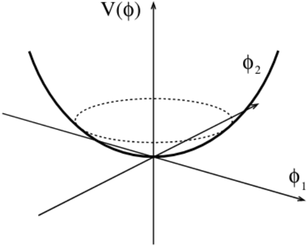
$$\phi_1 = v + \eta(x), \quad \phi_2 = \xi(x)$$

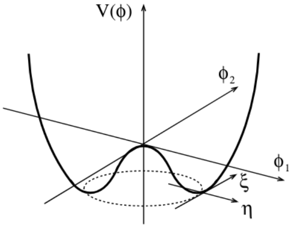
- The Lagrangian becomes:

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta + \tfrac{1}{2}\partial_\mu\xi\partial^\mu\xi - \tfrac{1}{2}(-2\mu^2)\eta^2 - \frac{\lambda}{4}(\eta^2+\xi^2)^2 - \lambda v(\eta^2+\xi^2)\eta$$

- η is a massive particle, $m_\eta = -2\mu^2$.
- We have a **massless mode**. ξ is a Nambu-Goldstone boson.

Spontaneous symmetry breaking for a complex scalar field





- Despite the spontaneous breaking of the symmetry, the Noether current is still conserved.

$$j_\mu = -i\Big(\phi^* \partial_\mu \phi - \partial_\mu \phi^* \phi\Big) = v\partial_\mu \xi + \eta \partial_\mu \xi - \xi \partial_\mu \eta$$

- The Noether current has a nonzero matrix element between a 1-Goldstone boson state and the vacuum

$$\langle \xi(p) | j_\mu | 0 \rangle = v\, p_\mu \, \exp(-ip \cdot x)$$

- It means that the Noether charge does not annihilate the vacuum. It creates a zero-momentum Goldstone boson.

$$\hat{Q}|0\rangle \propto \delta^{(3)}(\vec{p})|\xi(p)\rangle$$

Exponential parametrization

- a subtlety of field theory is that we have a freedom to redefine fields, while maintaining all observables (on-shell amplitudes). It is a certain freedom in the choice of "coordinates" in the field space.
- For the case at hand, there is a **more convenient parametrization** of $\phi(x)$ which highlights another property of Goldstone boson physics.

$$\phi = \frac{1}{\sqrt{2}}(v+\eta)\,\exp\left[i\frac{\theta}{v}\right]$$

- Now, the field θ will be the Goldstone degree of freedom, while η still is a massive mode.

$$\mathcal{L} = \tfrac{1}{2}\left(1+\frac{\eta}{v}\right)^2\partial_\mu\theta\partial^\mu\theta + \tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta - \tfrac{1}{2}(-2\mu^2)\eta^2 + a\eta^3 + b\eta^4$$

- **The Goldstone particle does not appear in the potential terms**. It only has derivative interactions with the η-field. $\propto \eta\partial_\mu\theta\partial^\mu\theta$ and $\eta^2\partial_\mu\theta\partial^\mu\theta$.
- **Goldstone bosons are weakly coupled at low momenta**. A new possibility for a perturbative expansion emerges.