Study on Proton Radius based on Lepton Pair Production in p-p Collision and Electromagnetic Form Factors

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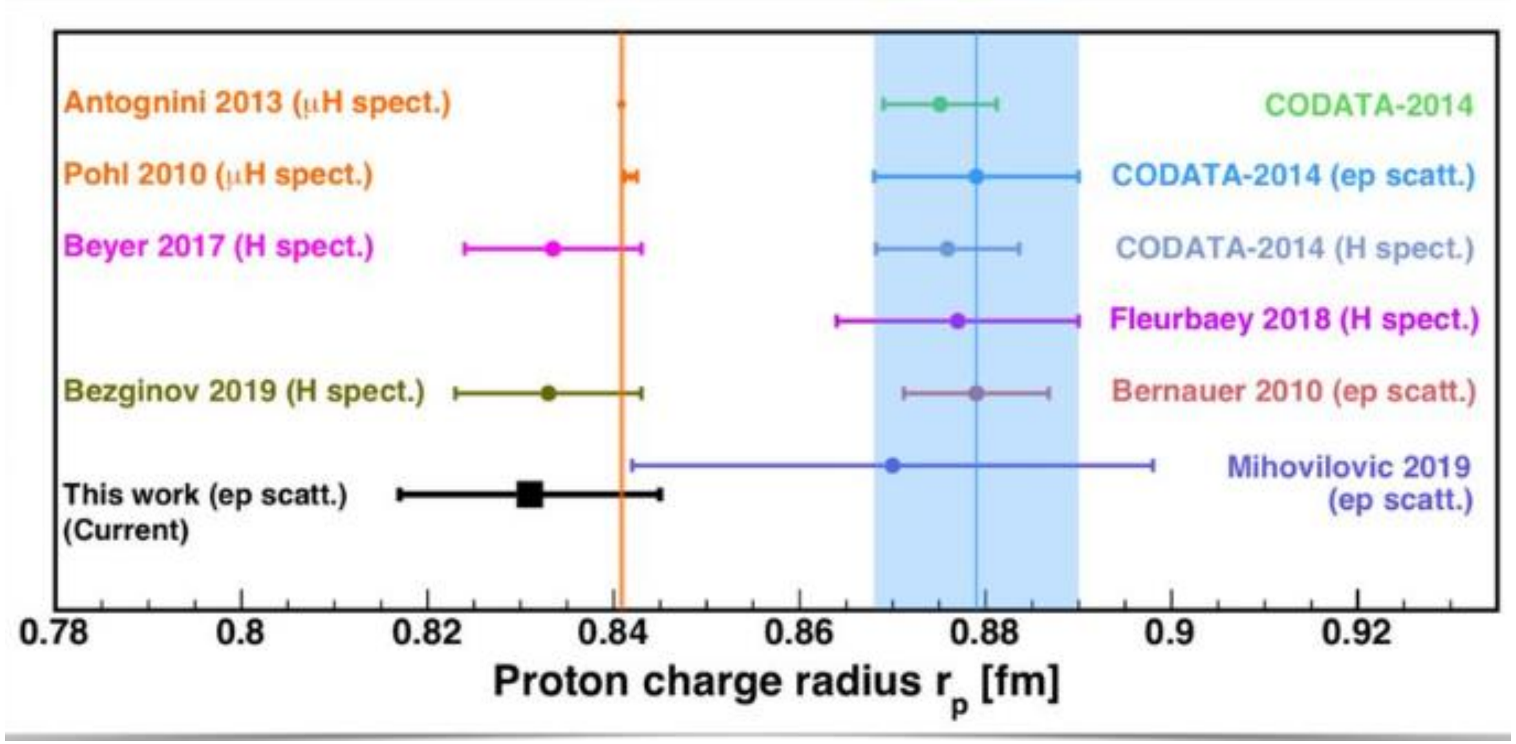
Proton Radius Measurements Over the Decades

- Since Nobel laureate Robert Hofstadter's work in the 1950s, elastic e-p scattering has been used to determine the proton radius and later atomic spectroscopy.



Electromagnetic Form Factor

- Describes the spatial distribution of electric charge and current inside a nucleon
- Most basic observable and used to study the internal structure of the nucleon
- Fourier transform of charge density

$$F(q^2) = \int \rho(\mathrm{r})\, e^{-q^2 r^2}\, d^3 r$$

- For exponential charge density- Dirac dipole form factor

$$F_D(Q^2) = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$$

where $Q^2 = -q^2$ with $\Lambda^2 = 0.71\ GeV^2$

- Similarly for Gaussian charge distribution, corresponding form factor is:

$$F_G(Q^2) = e^{-2Q^2/\Lambda^2}$$

Radius of Proton from Form Factor

Charge Distribution $(\rho(r))$	Form Factor $(F(Q^2))$	Electromagnetic Radius of Proton (R_p)
Point-like $\frac{\delta}{4\pi}$	constant 1	$\rightarrow 0$
exponential $\frac{a^3}{8\pi} e^{-ar}$	Dipole $\frac{1}{\left(1+\frac{Q^2}{a^2\hbar^2}\right)^2}$	$\frac{12}{\Lambda^2}$ (0.811 fm)
gaussian $\left(\frac{a^2}{2\pi}\right)^2 e^{-\frac{a^2r^2}{2}}$	Gaussian $e^{-\frac{Q^2}{2a^2\hbar^2}}$	$\frac{12}{\Lambda^2}$ (0.811 fm)

After expanding form factor, taking first derivative and limiting $Q^2 \rightarrow 0$

$$R_p = \frac{12}{\Lambda^2}$$

$\Lambda^2 = 4a^2\hbar^2$
(Scaling parameter)

Equivalent Photon Approximation(EPA)

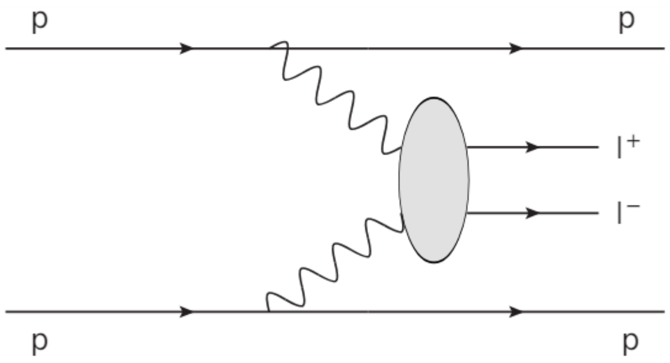
- The amplitude for lepton production depends on the interaction between photons exchanged by protons. This interaction is influenced by the form factor.
- In the EPA, flux of photons are emitted by high energy particle due to it's electromagnetic field.
- Cros-section using EPA:

$$\sigma_{(pp\to ppl^+l^-)} = \int\int f(x_1)f(x_2)\,\sigma_{\gamma\gamma\to l^+l^-}(m^2_{l^+l^-})dx_1dx_2$$

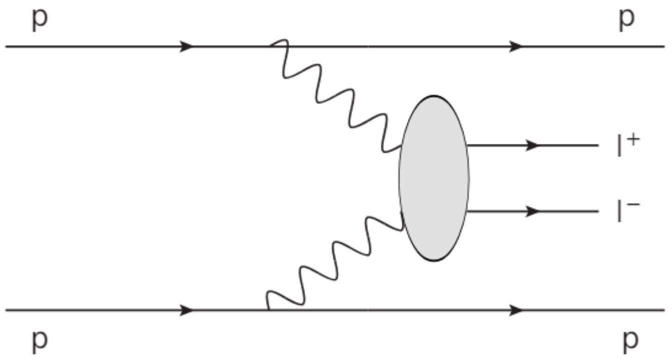
$\mathbf{f(x)} \to$ **photon density**

$x_1, x_2 \to$ **longitudinal momentum fraction**

$$\frac{m^2_{l^+l^-}}{s} = x_1x_2$$



Theoretical Approach



$$\sigma_{(pp\rightarrow ppl^{+}l^{-})} = \int |M|^2 d\xi_1 d\xi_2 d\phi_1 d\phi_2 dy_3 dy_4 dp_{mt} d\phi_{pmt}$$

$\xi = -logX$

$X = \frac{m_T}{\sqrt{s}}(e^{y_1} + e^{-y_2})$

Longitudinal momentum fraction of outgoing leptons

m_T → Transverse Mass

$p_{mt} = p_{1t} - p_{2t}$

p_{1t} and p_{2t} are transverse momentum of photons emitted from first and second proton which further interacts to create final states

$y,\ \phi,\ \phi_{pmt}$

Rapidity of outgoing lepton

Azimuthal angle

Azimuthal angle corresponding to p_{mt}

$$|M|_{\lambda_3,\lambda_4} = e^2(F(q^2))^2(p_a + p_1)^{\alpha} \frac{-ig_{\alpha\mu}}{q_1^2+i\epsilon}\ \bar{u}(p_3,\lambda_3) i\,\gamma^{\mu} \frac{i(p_3-q_1)+m_{\mu}}{(q_1-p_3)^2-m_{\mu}^2}$$

$$i\gamma^{\nu} V(p_4,\lambda_4) \frac{-ig_{\nu\beta}}{q_2^2+i\epsilon} (p_b + p_2)^{\beta}$$

DIPOLE FORM FACTOR

$\lambda_3, \lambda_4 \rightarrow$ *Helicity of outgoing leptons*

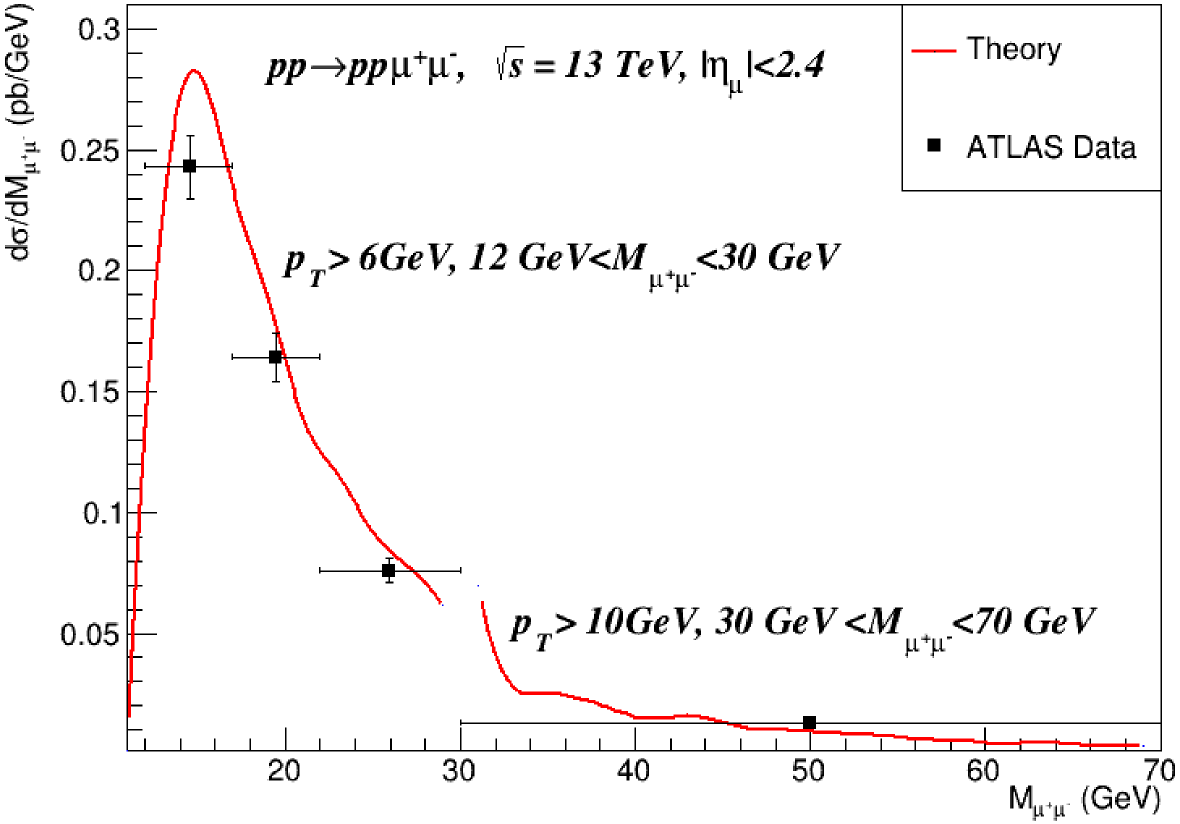
Comparison with Experimental Results

Process and kinematic limitation	σ (theory) pb	σ (experiment) pb
• $pp \to pp\mu^+\mu^-$(ATLAS) $m_{\mu^+\mu^-}$>20 GeV, p_T>10 GeV, $\|\eta\|$<2.4	0.708	0.628$\pm$0.032(stat)$\pm$0.032 (syst)
• $pp \to ppe^+e^-$(ATLAS) $m_{e^+e^-}$>24 GeV, p_T>12 GeV, $\|\eta\|$<2.4	0.435	0.428$\pm$0.035(stat)$\pm$0.018 (syst)
• $pp \to pp\mu^+\mu^-(CMS)$ $m_{\mu^+\mu^-}$>11.5 GeV, p_T>4 GeV, $\|\eta\|$<2.1	4.00	3.38$\pm$0.16(stat)$\pm$0.14(syst)
• $pp \to pp\mu^+\mu^-$(ATLAS) 12 GeV <$m_{\mu^+\mu^-} <$ 30 GeV, $p_T>$ 6 GeV, $\|\eta\|$<2.4	3.55	2.64
• $pp \to pp\mu^+\mu^-(ATLAS)$ 30 GeV <$m_{\mu^+\mu^-} <$ 70 GeV, $p_T>$ 10 GeV, $\|\eta\|$<2.4	0.49	0.52
• $pp \to pp\mu^+\mu^-$ (ATLAS) 12 GeV <$m_{\mu^+\mu^-} <$ 70 GeV, $p_T>$ 6 GeV, $\|\eta\|$<2.4	4.04	3.12$\pm$0.07(stat)$\pm$0.014(syst)

$\sqrt{s} = 7$ TeV

$\sqrt{s} = 13$ TeV

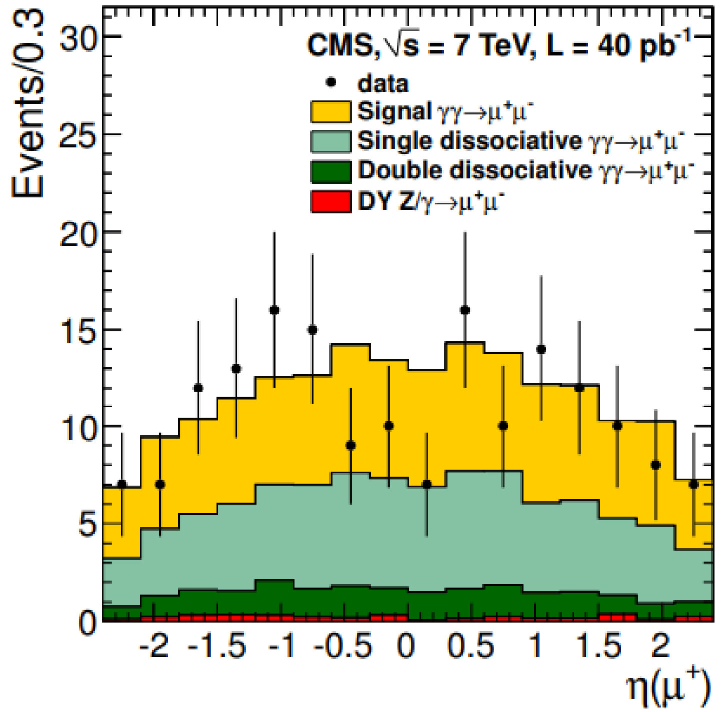
Comparison with Experimental Results

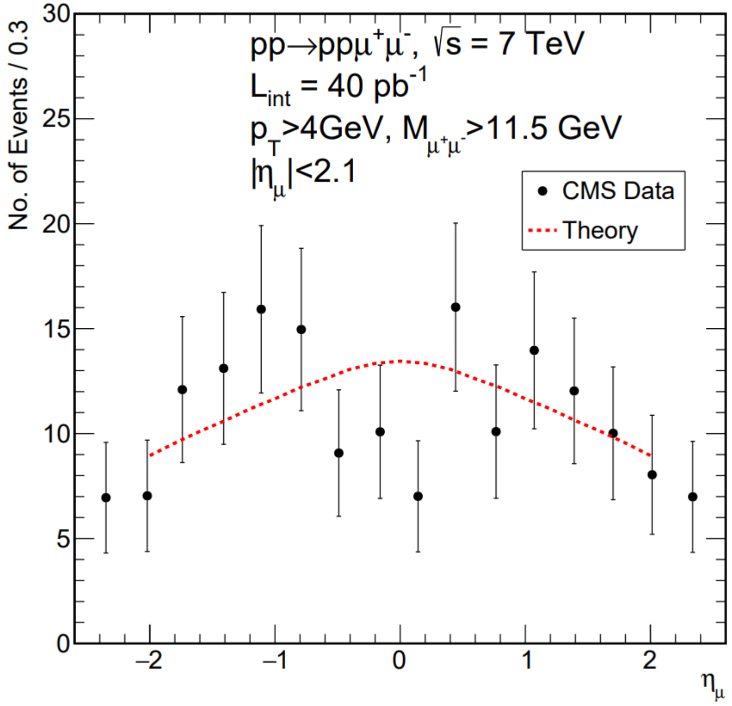


Differential cross-section as a function of invariant mass of two muons.

Experimental data points are copied from ATLAS collaboration paper: https://doi.org/10.1016/j.physletb.2017.12.043, https://arxiv.org/abs/1708.04053

Comparison with Experimental Results



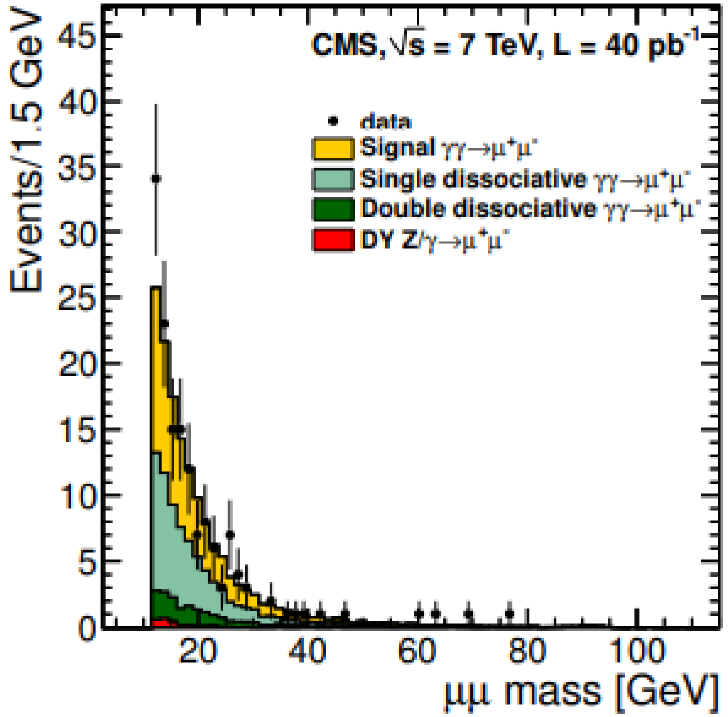


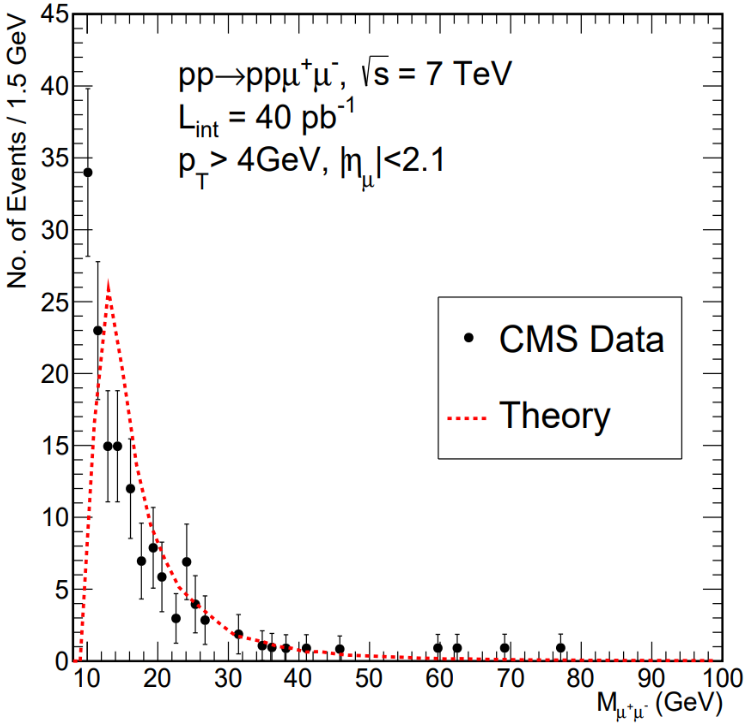
No.of events as a function of rapidity of muon

Figure: From the paper "Exclusive $\gamma\gamma \rightarrow \mu^+\mu^-$ production in proton-proton collision at $\sqrt{s} = 7\ TeV$", CMS collaboration
http://dx.doi.org/10.1007/JHEP01(2012)052

Comparison of CMS Data with theory at $\sqrt{s} = 7\ TeV$

Comparison with Experimental Results





No.of events as a function of invariant mass of muon pair

Figure: From the paper "Exclusive $\gamma\gamma \to \mu^+\mu^-$ production in proton-proton collision at $\sqrt{s} = 7\ TeV$", CMS collaboration http://dx.doi.org/10.1007/JHEP01(2012)052

Comparison of CMS Data with theory at $\sqrt{s} = 7\ TeV$, luminosity $40\ pb^{-1}$, $p_T > 4GeV$

Action Plan Based on our Findings

- Based on the strong agreement between our theory and the ATLAS and CMS experimental data, we are currently focused on calculating the proton radius to achieve more accurate results.

- We are also preparing a research article titled "***Determination of Proton Size Based on Electromagnetic Production of Lepton Pairs in Proton-Proton Collisions***," which we plan to submit to the journal *XYZ* for publication.

Summary

- Electromagnetic form factors play an important role in understanding the internal structure of nucleons.
- Calculated radius of proton theoretically from the form factors.
- We used 8 kinematic variables to find the theoretical cross-section which can give more accurate results using Dipole form factor.
- We compared the differential cross-section for invariant mass (ATLAS) and the number of events with a specified luminosity across multiple variables (ATLAS, CMS).
- The experimental results from both experiments align well with our theoretical predictions, and we look forward to further progressing these findings.

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THANK YOU FOR YOUR ATTENTION