Particle Accelerator Physics

How to run a Supercollider
Part II

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CERN

Trans-European School of High Energy Physics

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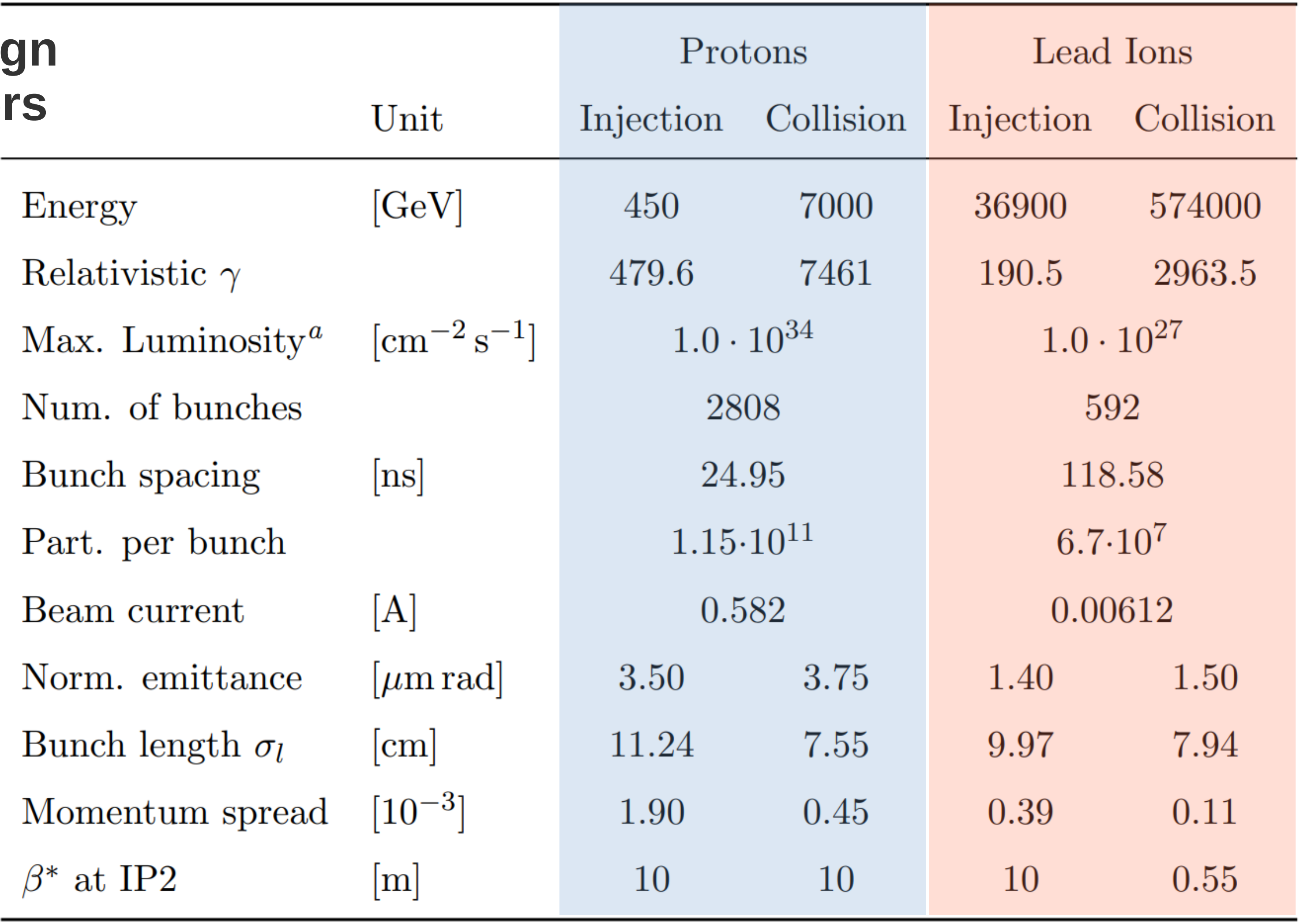
Recap Part I

LHC Design Parameters

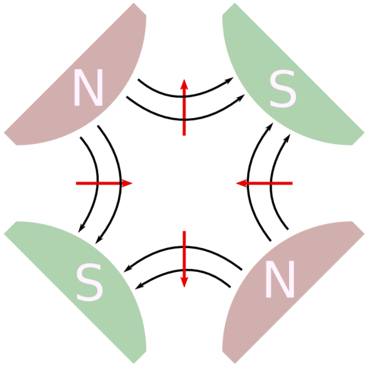


$^{208}\mathrm{Pb}^{82+}$

		Protons		Lead Ions	
	Unit	Injection	Collision	Injection	Collision
Energy	[GeV]	450	7000	36900	574000
Relativistic γ		479.6	7461	190.5	2963.5
Max. Luminosity[a]	[$\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$]	$1.0 \cdot 10^{34}$		$1.0 \cdot 10^{27}$	
Num. of bunches		2808		592	
Bunch spacing	[ns]	24.95		118.58	
Part. per bunch		$1.15 \cdot 10^{11}$		$6.7 \cdot 10^{7}$	
Beam current	[A]	0.582		0.00612	
Norm. emittance	[μm rad]	3.50	3.75	1.40	1.50
Bunch length σ_l	[cm]	11.24	7.55	9.97	7.94
Momentum spread	[10^{-3}]	1.90	0.45	0.39	0.11
β^* at IP2	[m]	10	10	10	0.55


LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SUISSE
FRANCE
SPS 7 km
ALICE
CMS
LHC 27 km

Quadrupoles



Gradient

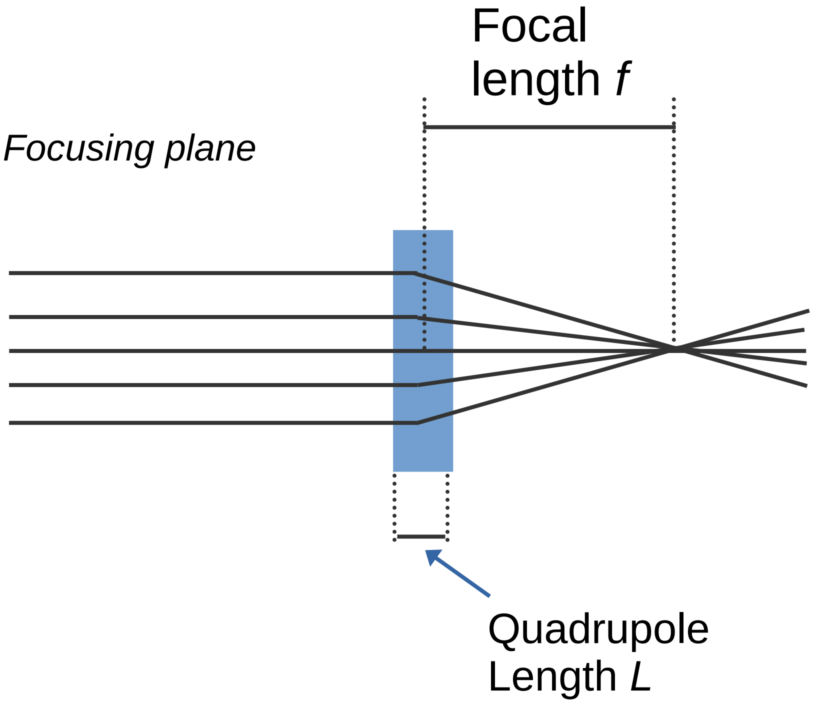
$$B_x = -g\,y$$
$$B_y = -g\,x$$

After some derivation:

$$f_x = -\frac{p}{q\,g\,L} = -\frac{1}{k\,L}$$

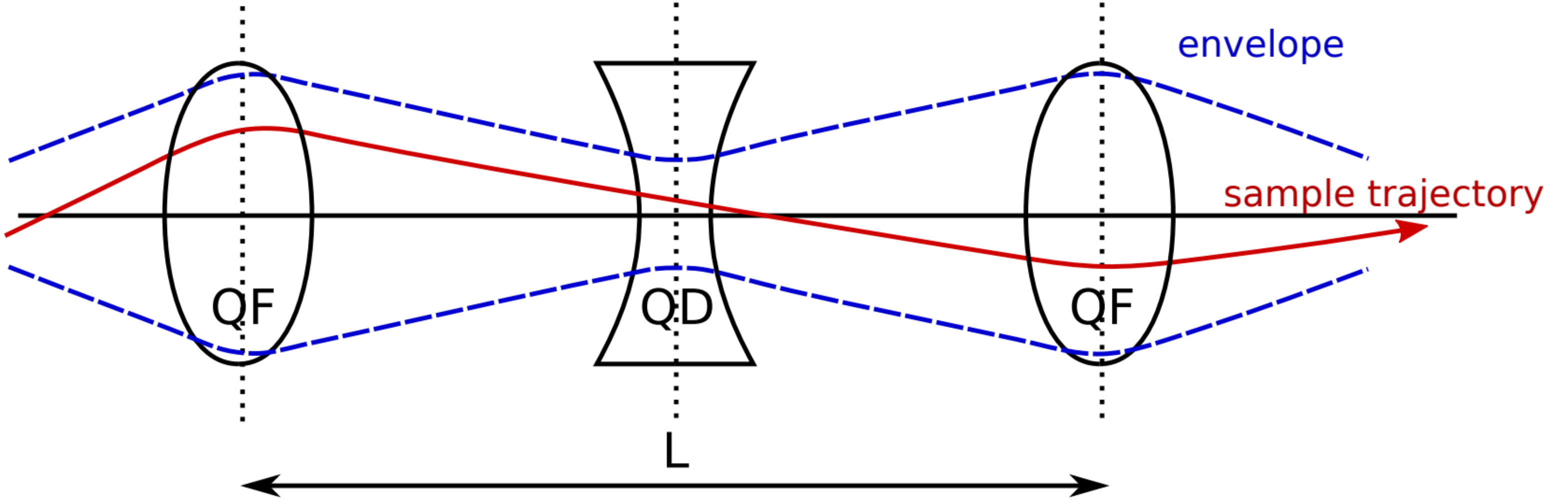
We define *k* as the **normalized quadrupole gradient**

- Energy independent
- Charge independent



Optical quadrupole lattice

Figure: Courtesy of J. Dilly



- Quadrupoles must be used in combination
- Fully analogous to an optical system for photons
- ***<u>Beam optics</u>***

Betatron function

$$\mathcal{M}(z + L_p \,|\, z) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$$

Individual Particles
Transfer matrix for *u, u'*

Beam lattice
Transfer matrix for *betatron parameters α, β, γ*

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + C'S & -SS' \\ C'^{\,2} & -2C'S' & S'^{\,2} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} = \mathcal{M}_\beta \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

Twiss Parameters

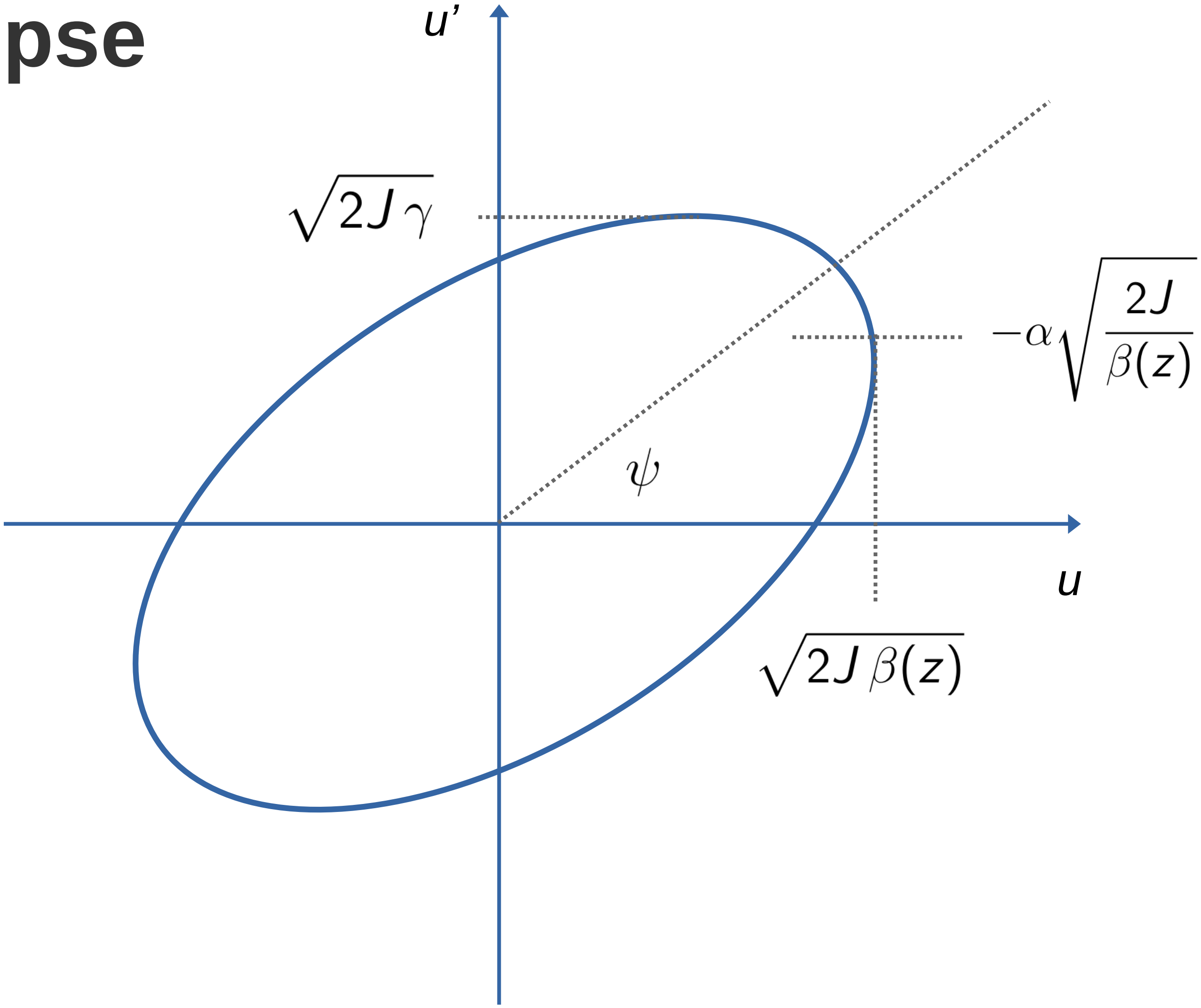
$$\beta(z)$$

$$\alpha(z) = -\frac{1}{2}\frac{d\beta(z)}{dz}$$

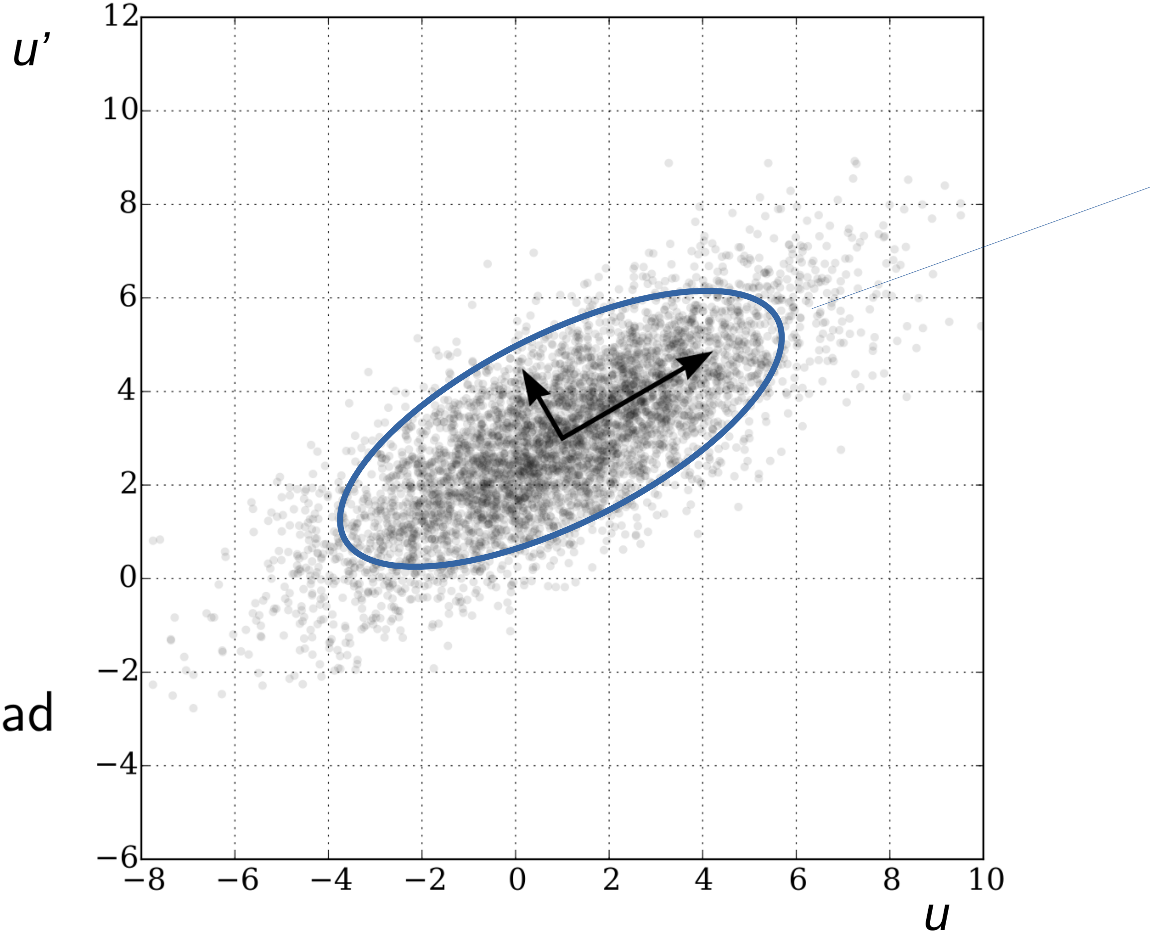
$$\gamma(z) = \frac{1 + \alpha^2(z)}{\beta(z)}$$

Phase space ellipse

- Apparently ***J*** determines the size of the ellipse **for one particle**
- Doesn't it make sense to look at the ***J* of the entire beam**?



Emittance



Beam emittance

$$\boxed{\epsilon = \langle J \rangle}$$

Definition:
Beam size σ

$$\boxed{\sigma = \sqrt{\epsilon\,\beta(z)}}$$

Crucial indicator for beam quality!

LHC top energy (6.8 TeV):

$$\epsilon = 5 \times 10^{-10}\ \mathrm{m\ rad}$$

By Nicoguaro - Own work, CC BY 4.0, https://commons.wikimedia.org/w/index.php?curid=46871195

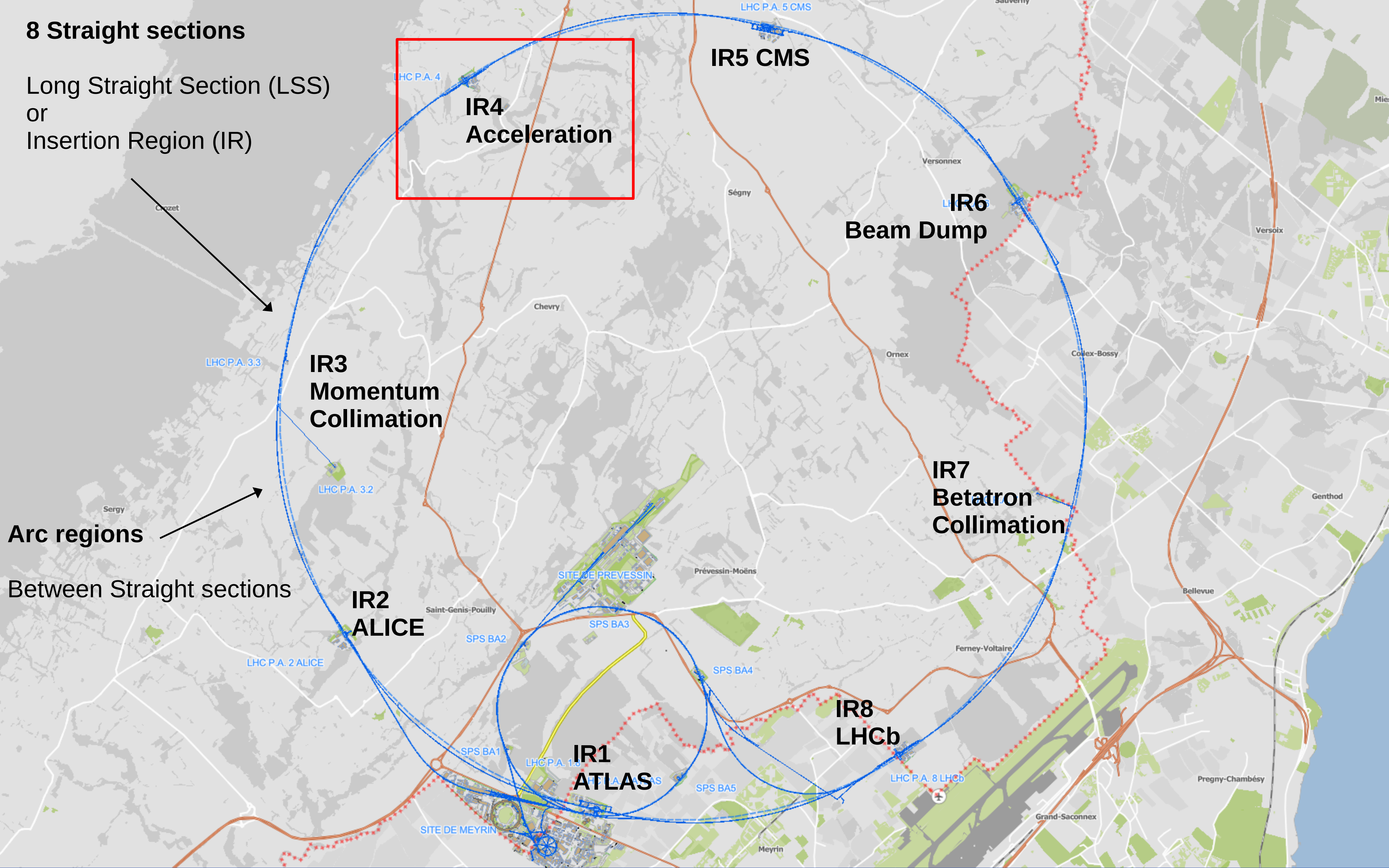
Today’s Outlook

- So far: transverse beam dynamics

Today:

- Longitudinal dynamics
- LHC pre-accelerators (injectors)
- Linear accelerators
- Transfer between accelerators

Longitudinal Dynamics



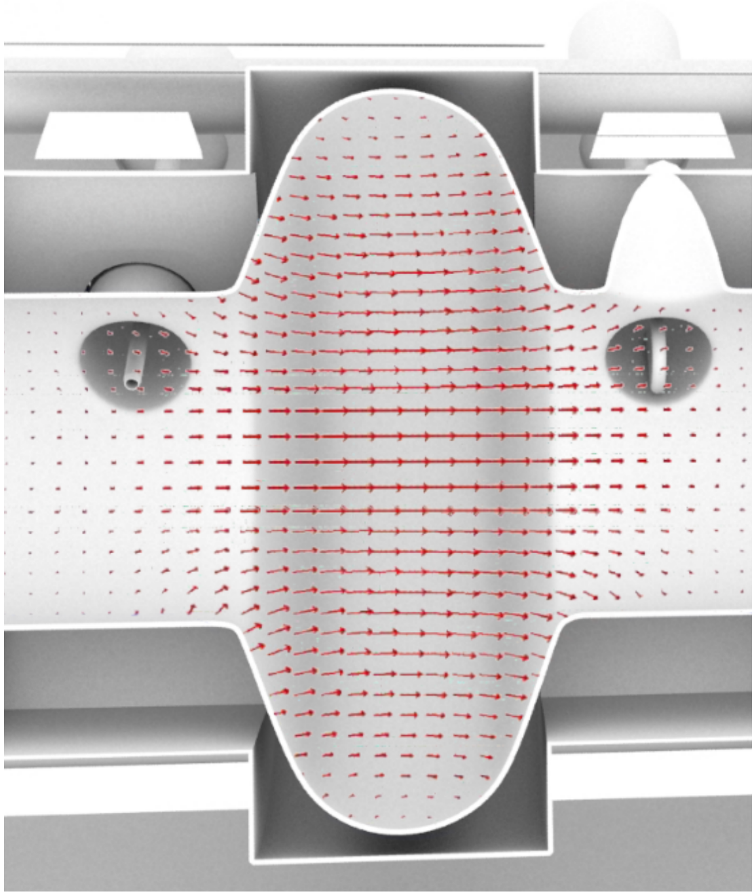
8 Straight sections

Long Straight Section (LSS)
or
Insertion Region (IR)

Arc regions

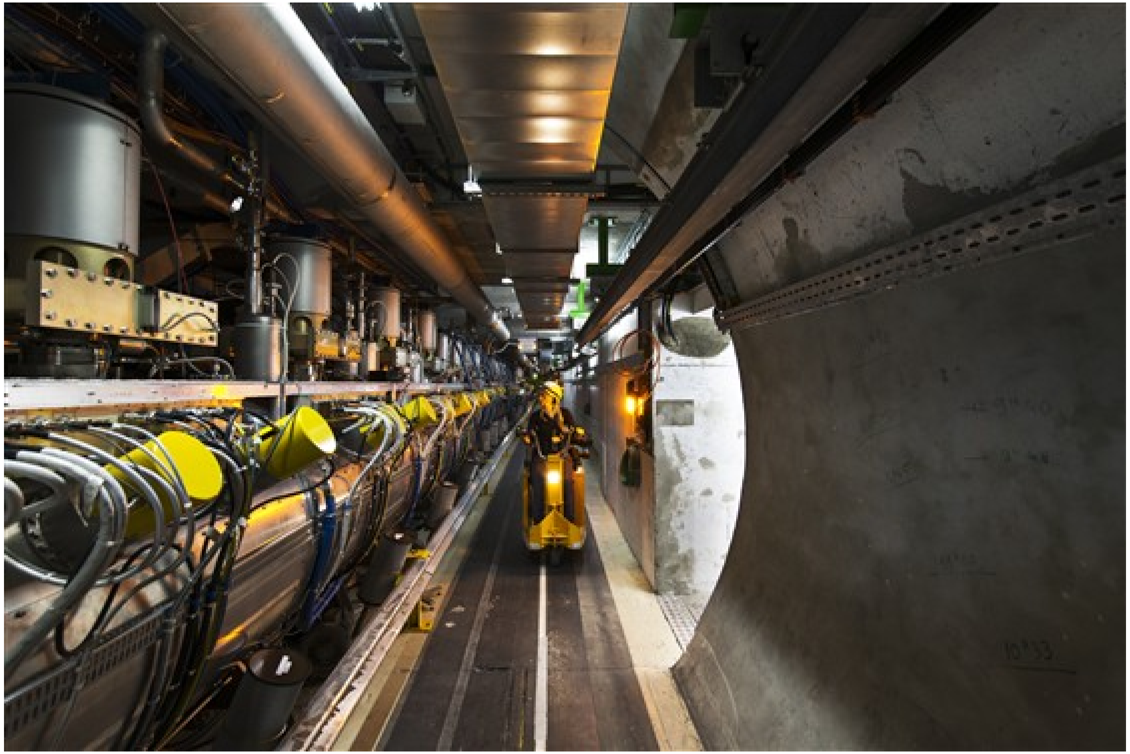
Between Straight sections

RF cavities



http://anim3d.web.cern.ch/content/equipement

Image: Maximilien Brice/CERN



- **8 RF cavities per beam max. 2MV each**
- **E-field for longitudinal acceleration**

Longitudinal Dynamics

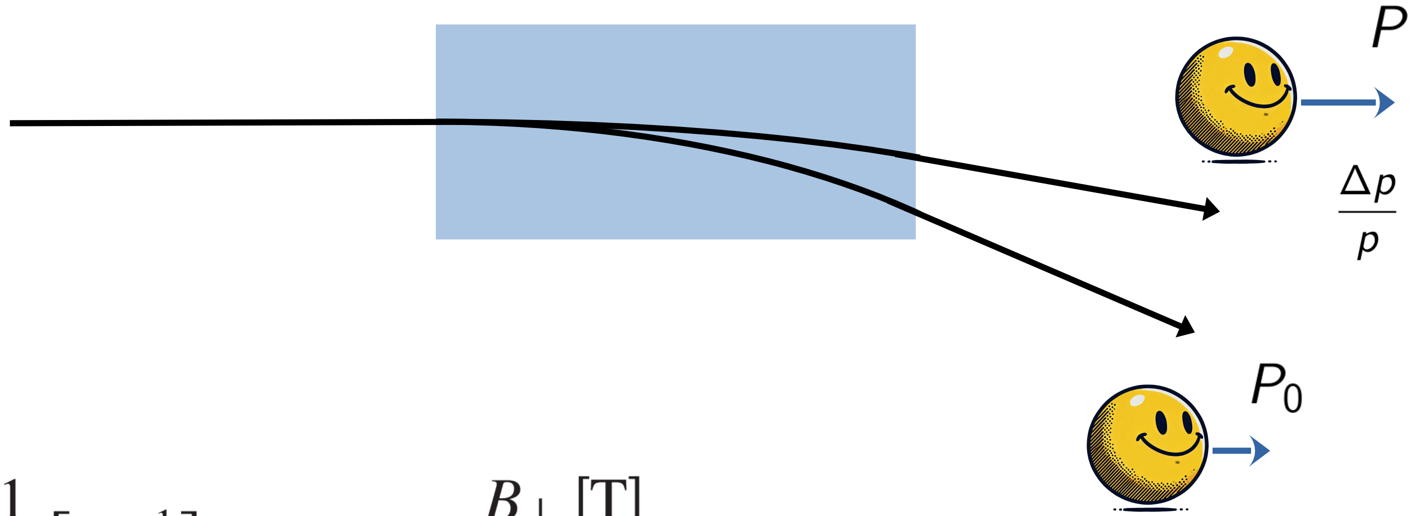
Momentum offset

$$\frac{\Delta p}{p} = \frac{P - P_0}{P_0}$$

Question:
Is there any impact on transverse motion? Yes/No?

Dispersion

Magnets act as spectrometer!



$$\frac{\Delta p}{p} = \frac{P - P_0}{P_0} > 0$$

$$\frac{\Delta p}{p} = \frac{P - P_0}{P_0} = 0$$

$$\frac{1}{\rho}\left[\mathrm{m}^{-1}\right] = 0.2995\frac{B_\perp\,[\mathrm{T}]}{cp\,[\mathrm{GV}]}$$

Dispersion

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} + \left(\frac{1}{\rho^2} - k\right) x = \frac{1}{\rho}\frac{\Delta p}{p_0}$$

Generalized equation of motion

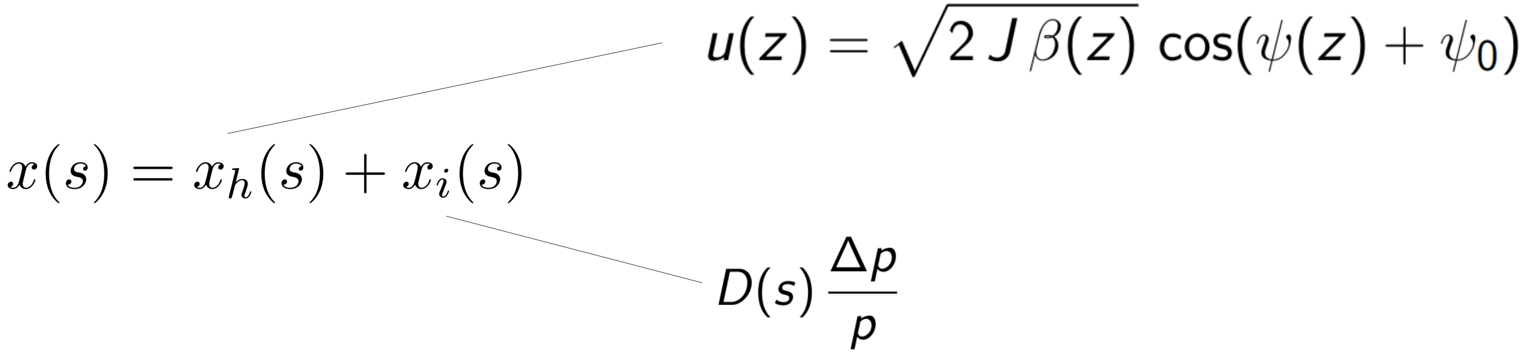
$$u(z) = \sqrt{2\,J\,\beta(z)}\,\cos(\psi(z) + \psi_0)$$

Solution $$x(s) = x_h(s) + x_i(s)$$

$$D(s)\,\frac{\Delta p}{p}$$

Dispersion function *D*: off-momentum offset in x/y

Dispersion



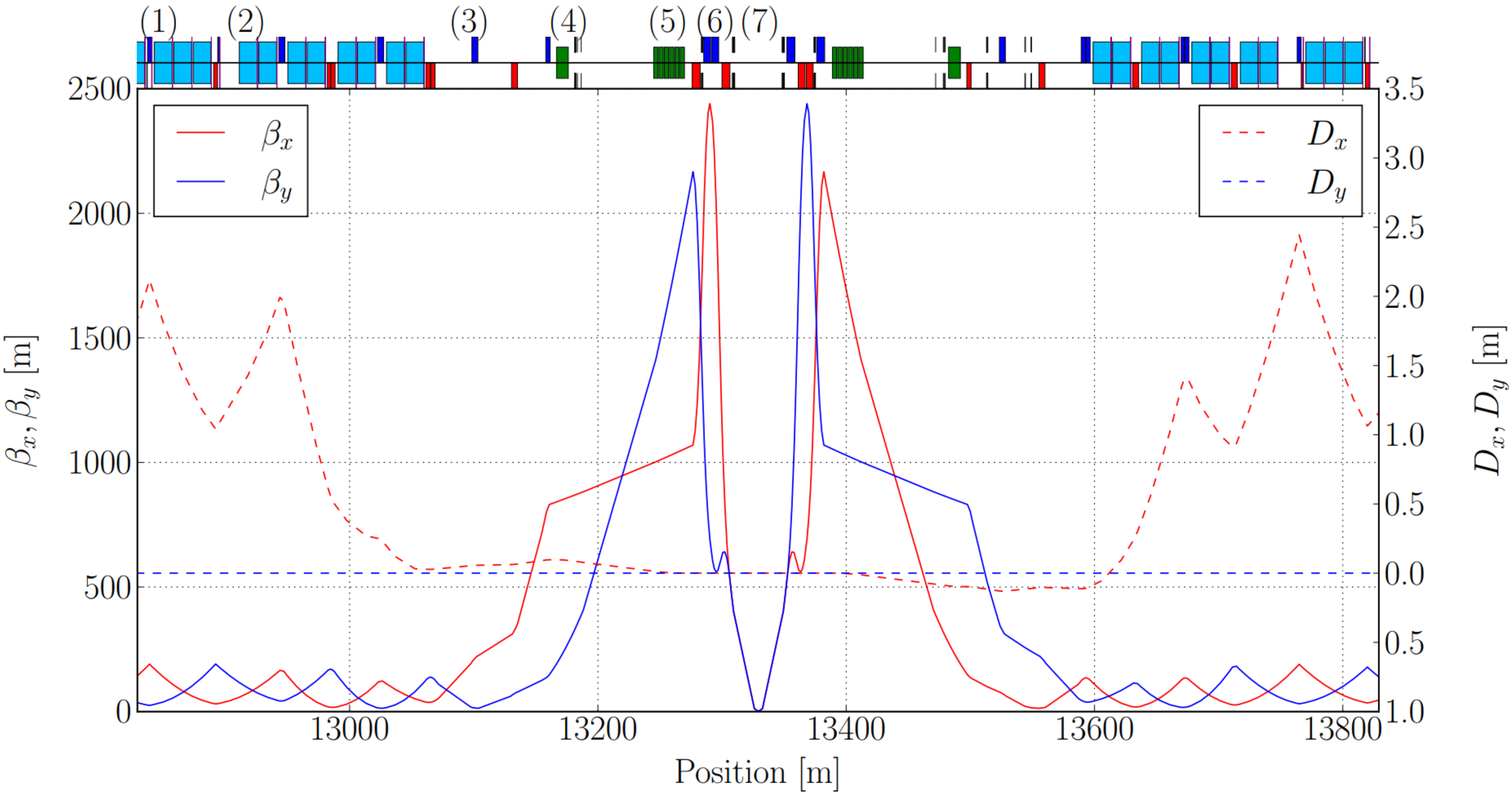
$$u(z) = \sqrt{2\,J\,\beta(z)}\,\cos(\psi(z) + \psi_0)$$

$$x(s) = x_h(s) + x_i(s)$$

$$D(s)\,\frac{\Delta p}{p}$$

- Transverse motion & orbit of off-momentum particles is different
- Periodic solution from equation of motion
- Property of beam optics → can & must be optimized in lattice design
- Momentum offset must be kept under control

Dispersion



(1)
(2)
(3)
(4)
(5)
(6)
(7)
β_x
β_y
D_x
D_y
β_x, β_y [m]
D_x, D_y [m]
Position [m]
0
500
1000
1500
2000
2500
13000
13200
13400
13600
13800
1.0
0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Synchrotron

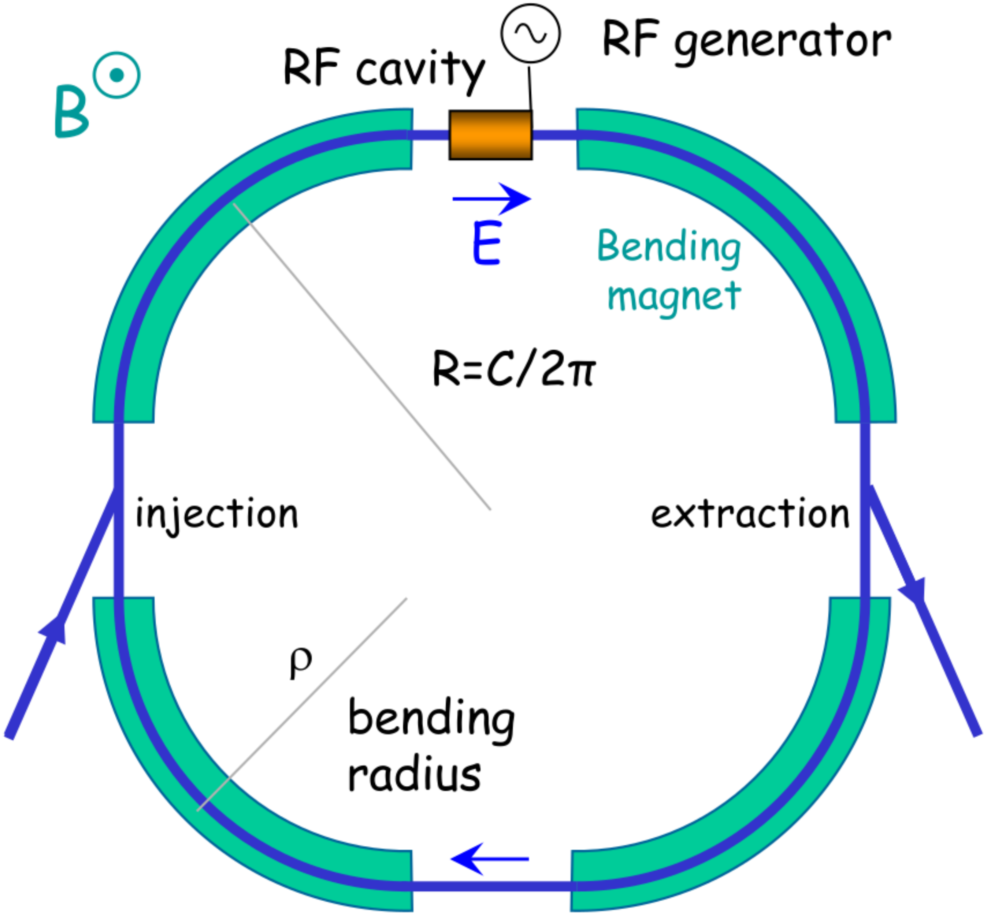
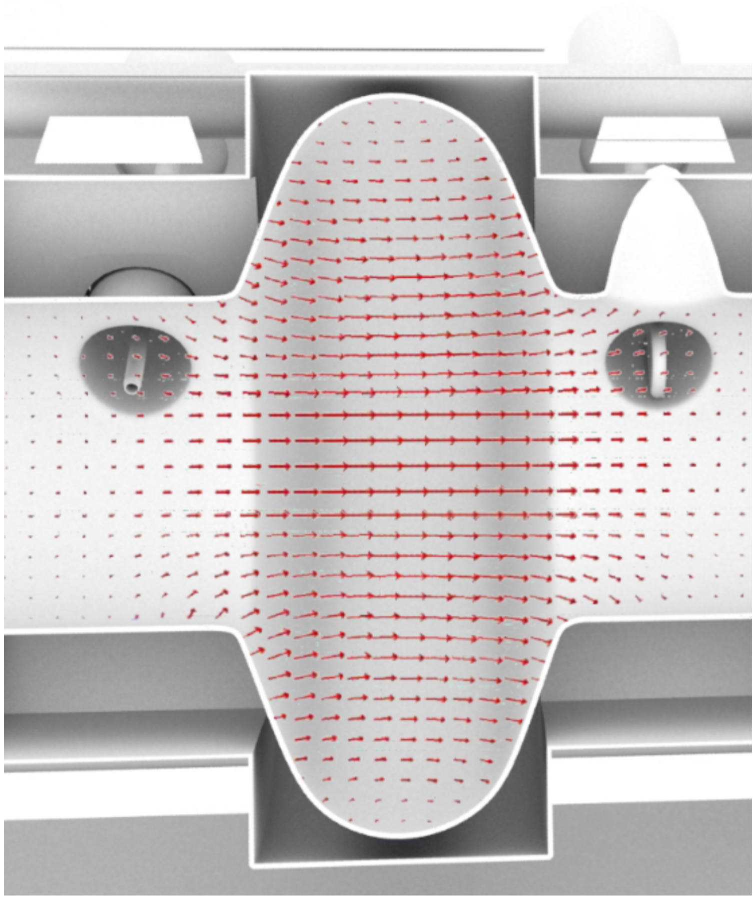


Figure from F. Tecker, CERN Accelerator School

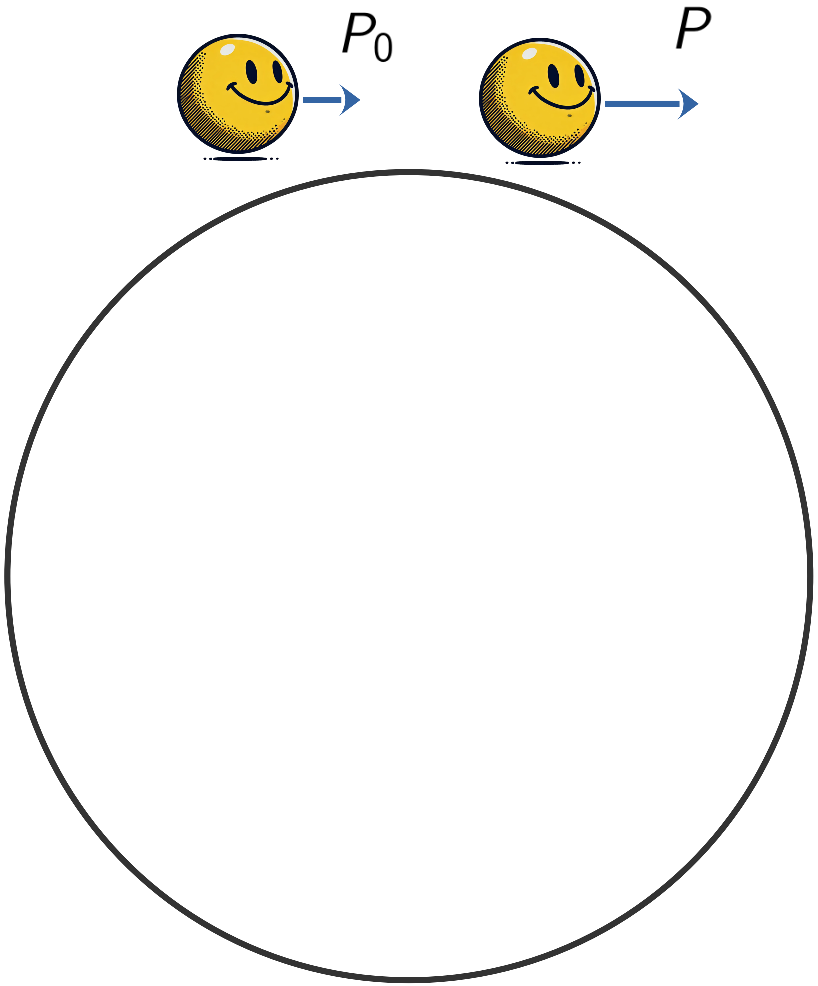
LHC RF system



http://anim3d.web.cern.ch/content/equipement

- Frequency 400 MHz
- Voltage 2MV per cavity, 8 cavities
- For acceleration and longitudinal stability of ultrarelativistic particles
- Reminder: at injection into the LHC relativistic β~0.999998
- RF system works differently for relativistic and non-relativistic particles: why?

Non-relativistic system



Let's go to phantasy world

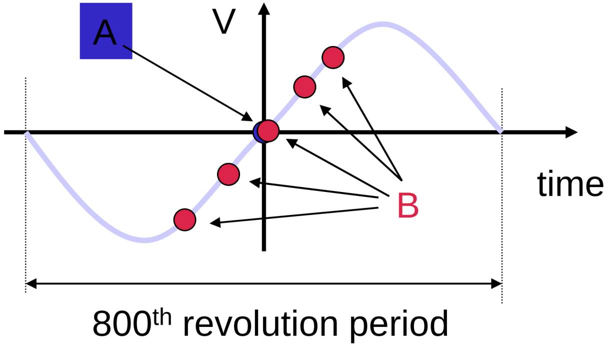
No dispersion D = 0

How will the revolution frequency change with dp/p?

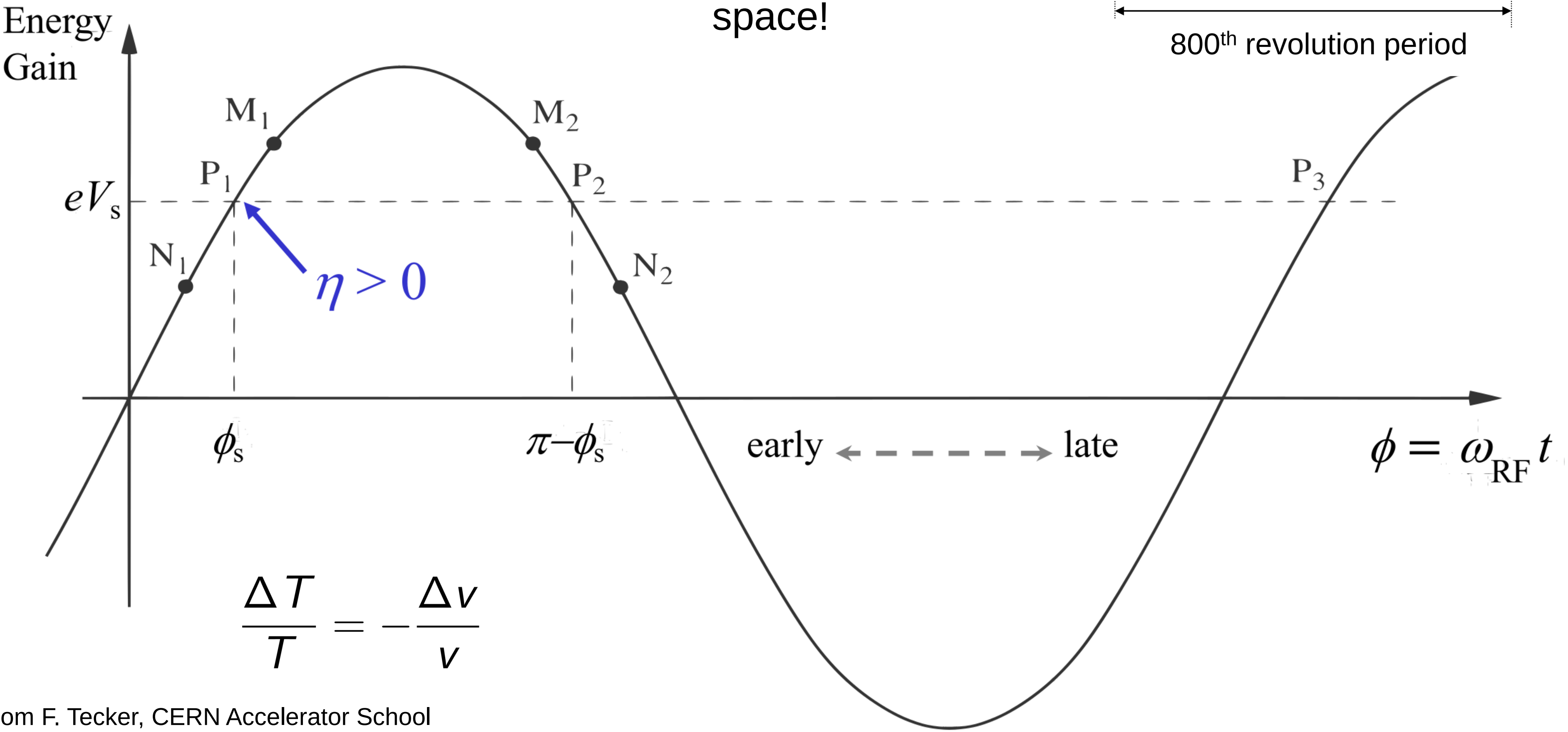
$$T = \frac{L}{v} = \frac{2\pi R}{v}$$

$$f_r = \frac{1}{T} = \frac{v}{2\pi R}$$

Non-relativistic particles



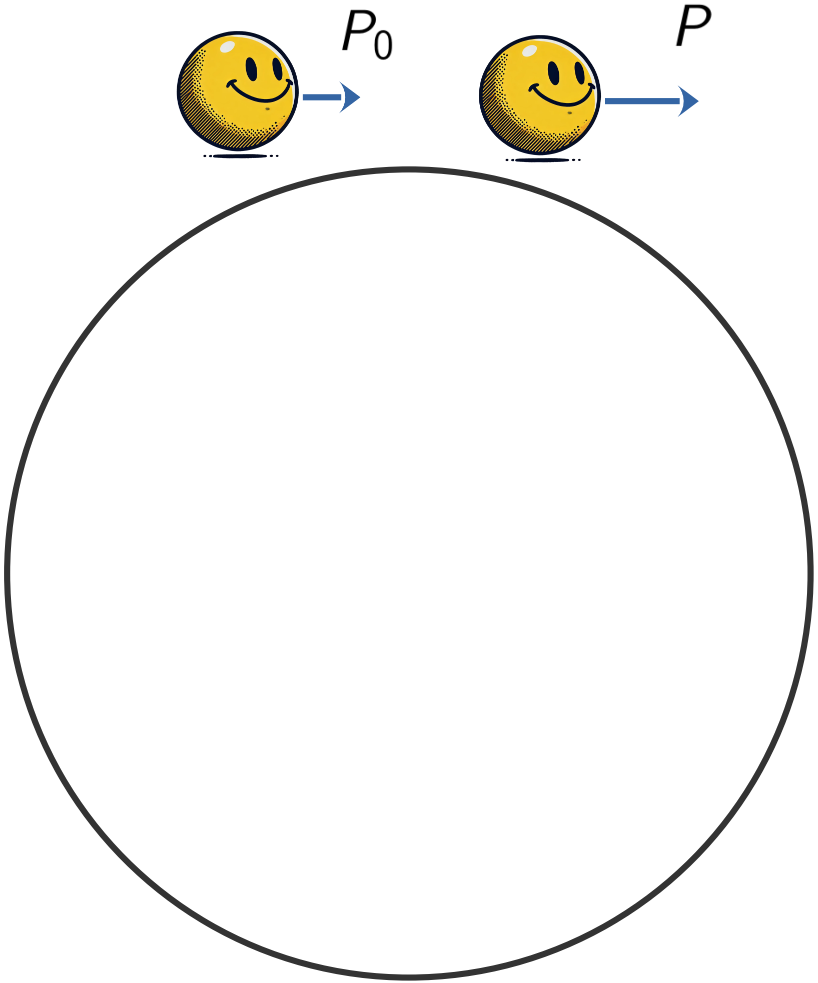
Stable oscillation in longitudinal phase space!



$$\frac{\Delta T}{T} = -\frac{\Delta v}{v}$$

Figure from F. Tecker, CERN Accelerator School

Longitudinal motion



No more phantasy world

Including dispersion → *R* no longer constant

$$T = \frac{L}{v} = \frac{2\pi R}{v}$$

$$\Downarrow$$

$$\frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta v}{v}$$

Longitudinal motion

$$\frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta v}{v}$$

Dominates for relativistic beams

Dominates for non-relativistic beams

$$\alpha_c = \frac{dL/L}{dp/p}$$

$$\alpha_c = \frac{p}{L}\frac{dL}{dp}$$

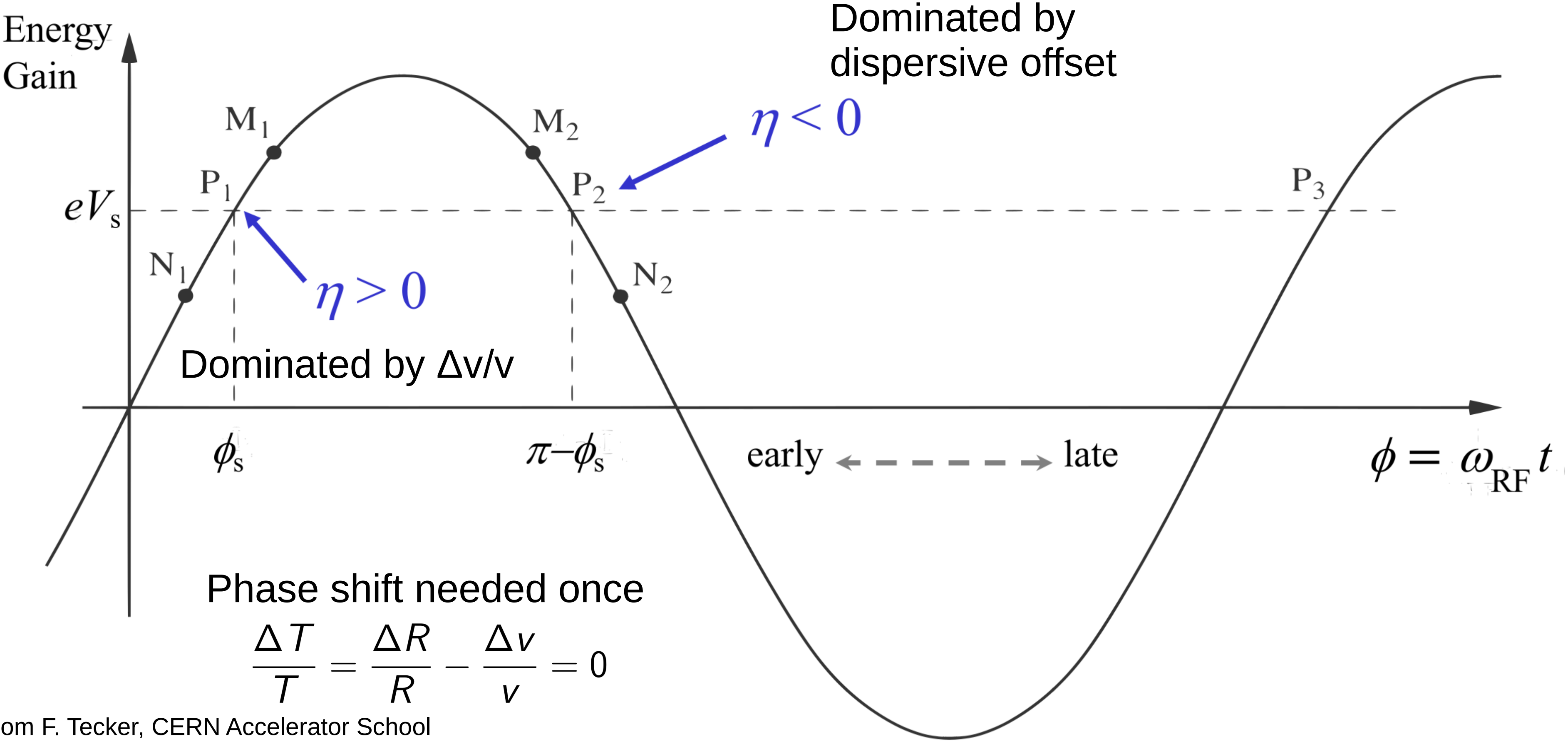
Momentum compaction factor

$$\alpha_c = \frac{\langle D_x \rangle_m}{R}$$

→ depends on transverse optics!

$$L = 2\pi R$$

For both types of beams



Dominated by dispersive offset

Dominated by Δv/v

Phase shift needed once

$$\frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta v}{v} = 0$$

Figure from F. Tecker, CERN Accelerator School

Transition energy

Phase shift needed once

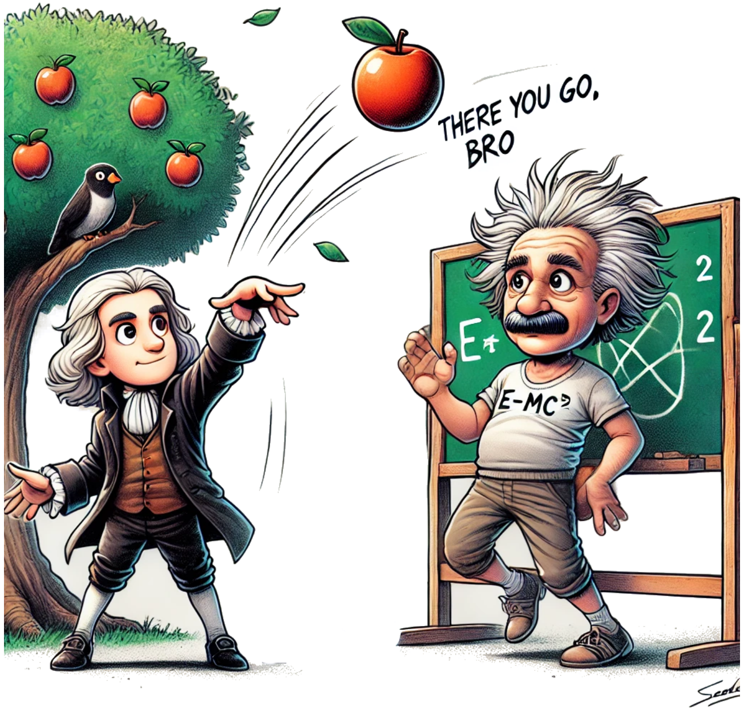
$$\frac{\Delta T}{T} = \frac{\Delta R}{R} - \frac{\Delta v}{v} = 0$$

Can be calculated as $$\frac{\mathrm{d}T}{T} = \left(\alpha_c - \frac{1}{\gamma^2}\right) \frac{\mathrm{d}p}{p}$$

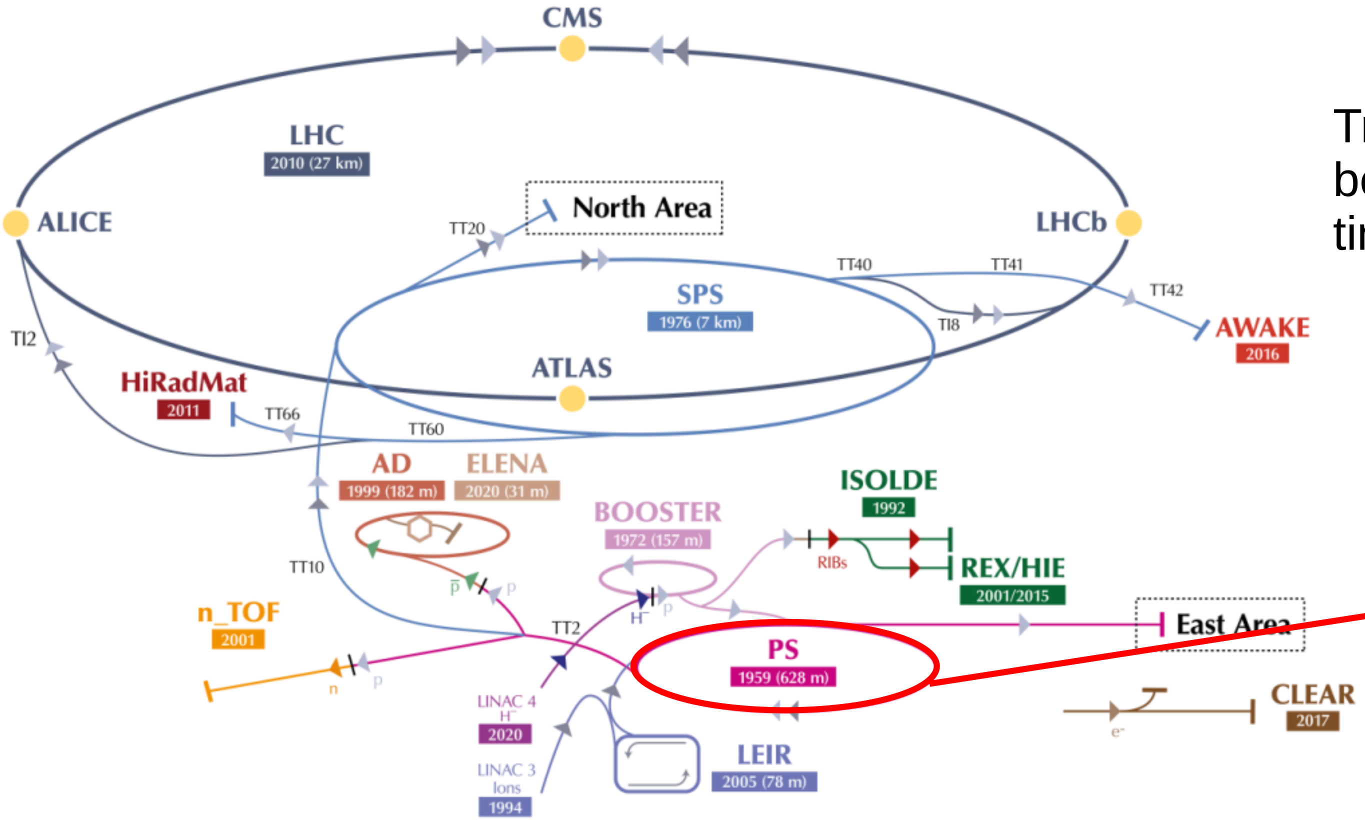
Depends on machine optics and relativistic ɣ

→ **Transition energy**

It is the energy at which relativity takes over!



Transition energy



Transition crossing between ~2 to 25 times proton mass

	Final Energy
Linac 4	160 MeV
PSB	2 GeV
PS	25 GeV
SPS	450 GeV
LHC	6.5-7 TeV

Synchrotron oscillations: No Acceleration

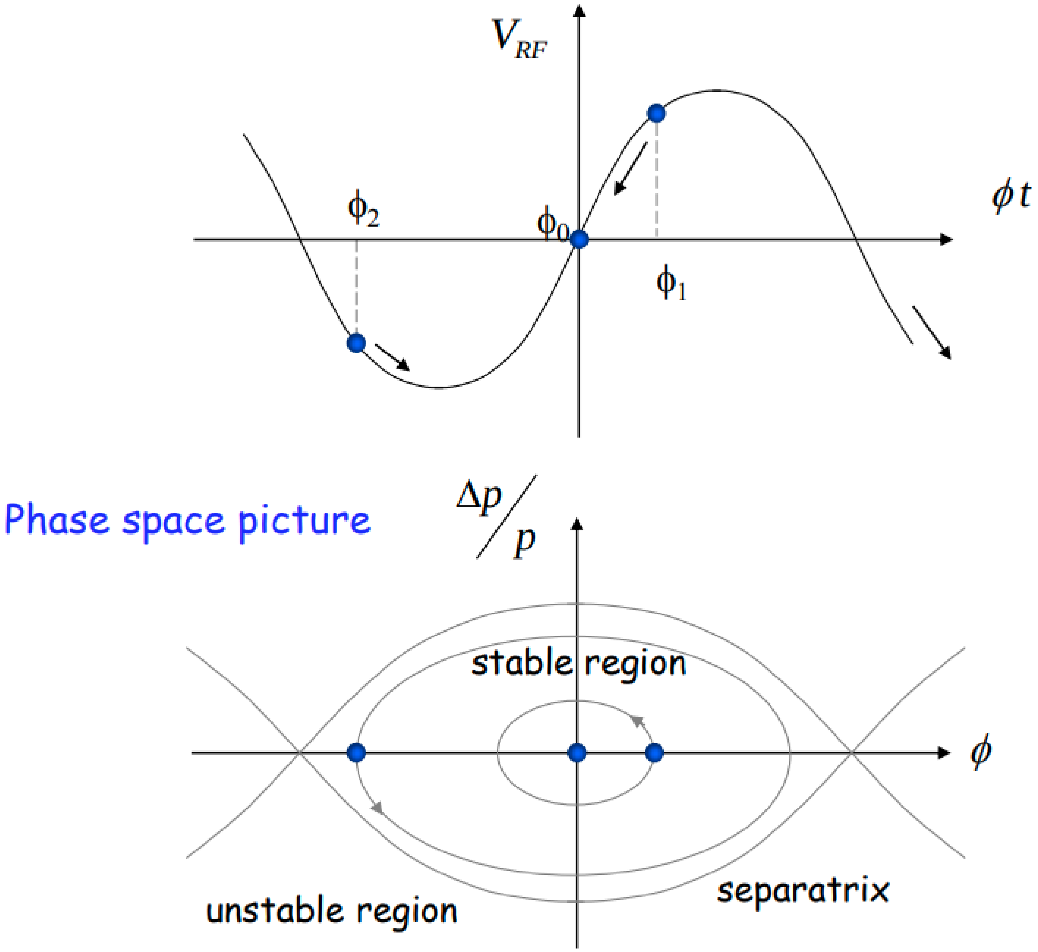
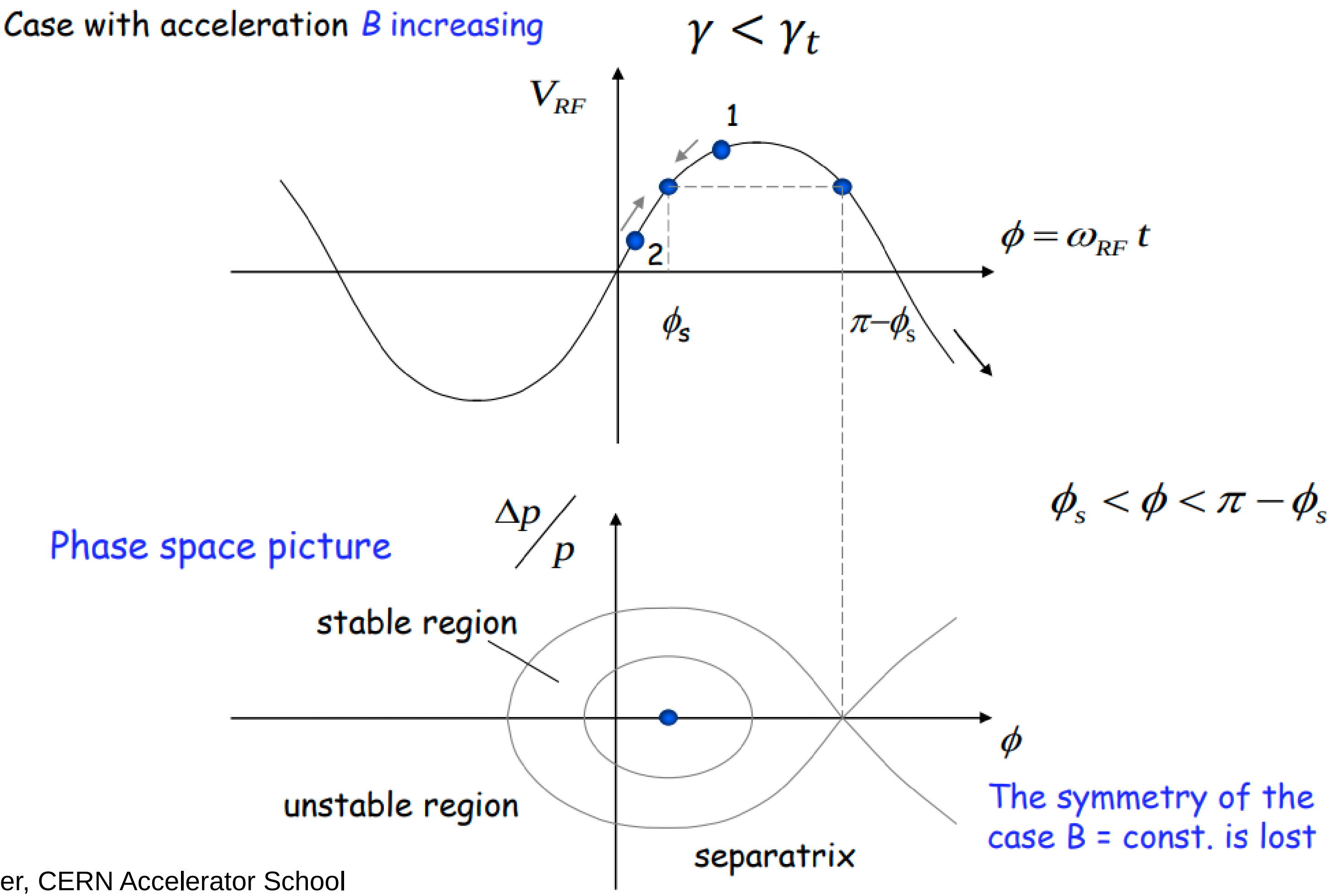


Figure from F. Tecker, CERN Accelerator School

Synchrotron oscillations: Acceleration



Case with acceleration *B* increasing

$\gamma < \gamma_t$

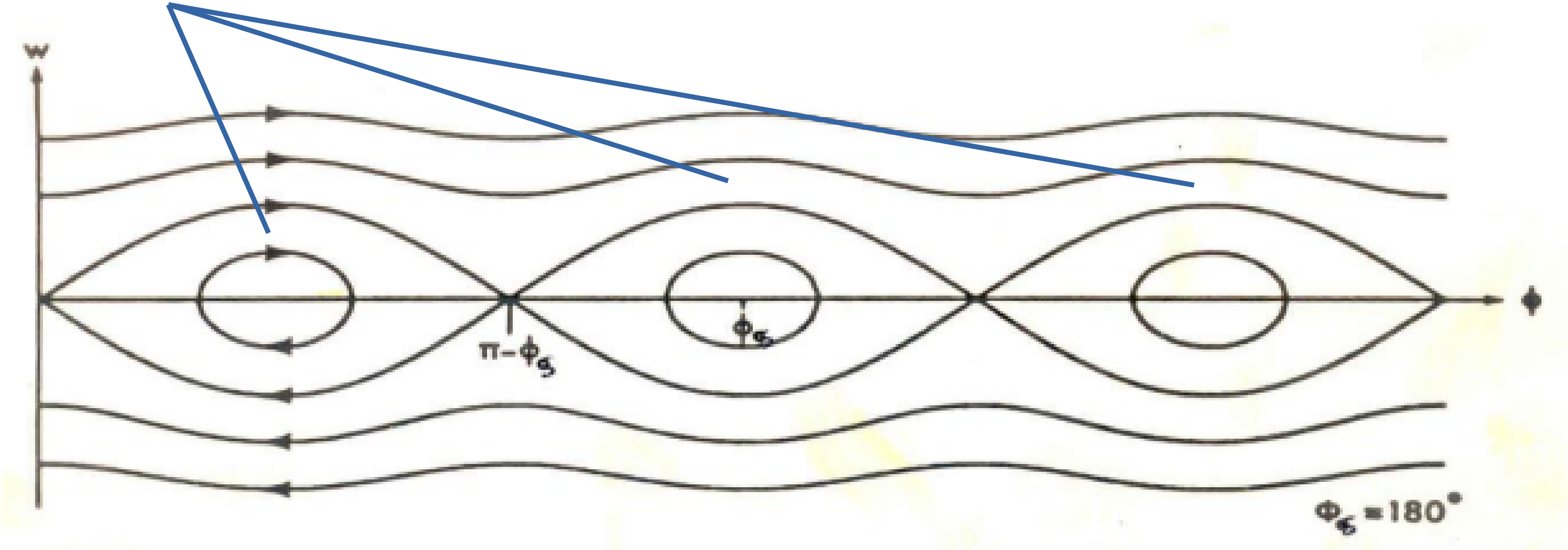
$\phi_s < \phi < \pi - \phi_s$

Phase space picture

The symmetry of the case B = const. is lost

Figure from F. Tecker, CERN Accelerator School

Buckets

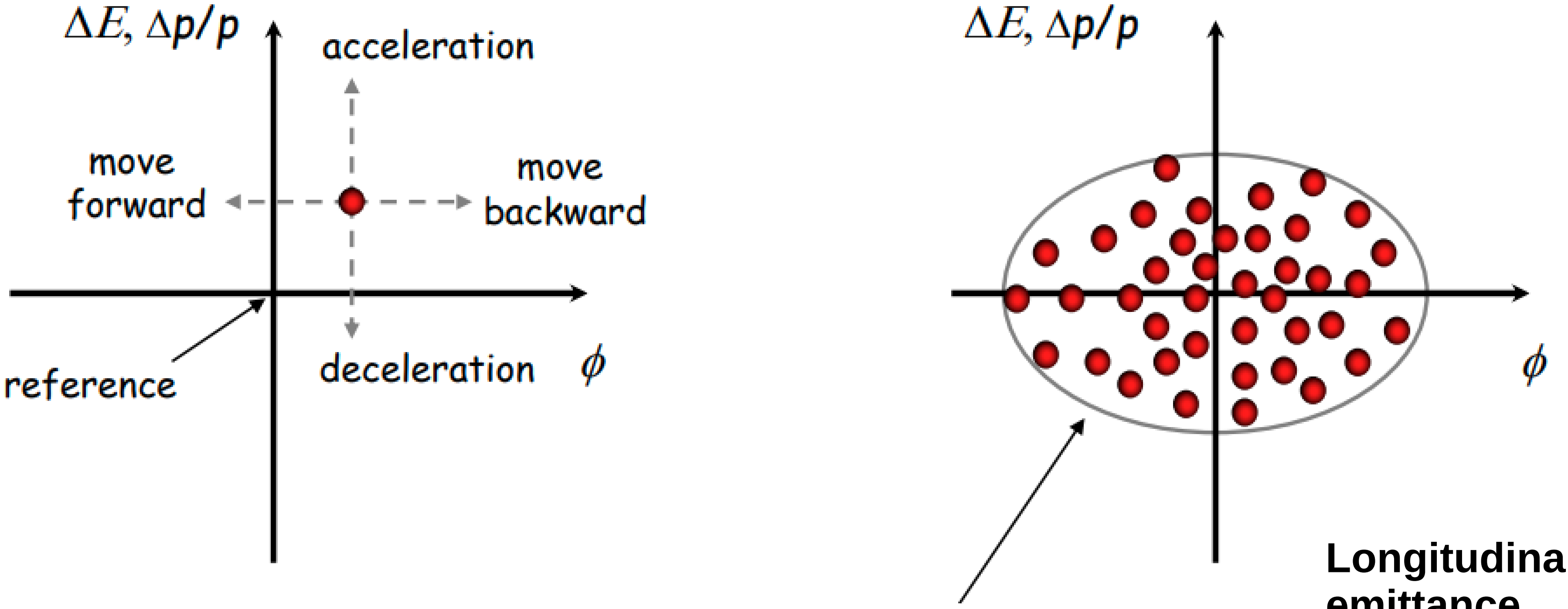


That's the reason why beams are bunched and not continuous!

LHC: 2808 bunches (25ns spacing), but ten times more buckets

Figure from F. Tecker, CERN Accelerator School

Longitudinal phase space

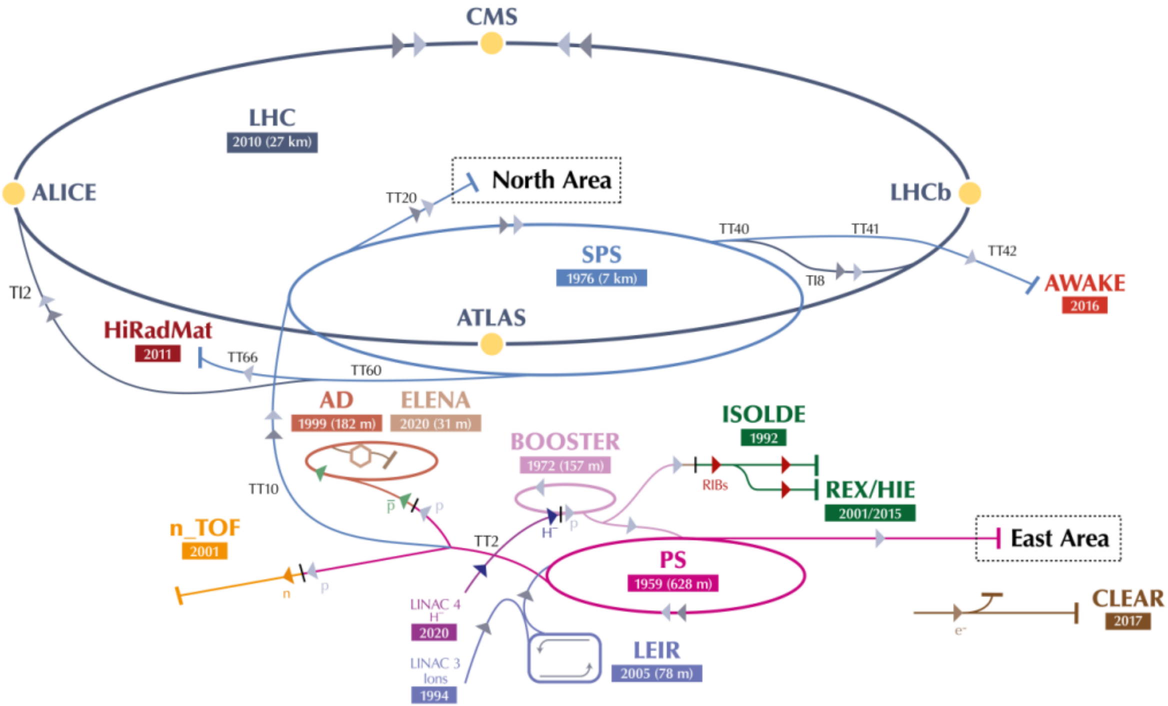


Longitudinal emittance

LHC: 2.5eV

Figure from F. Tecker, CERN Accelerator School

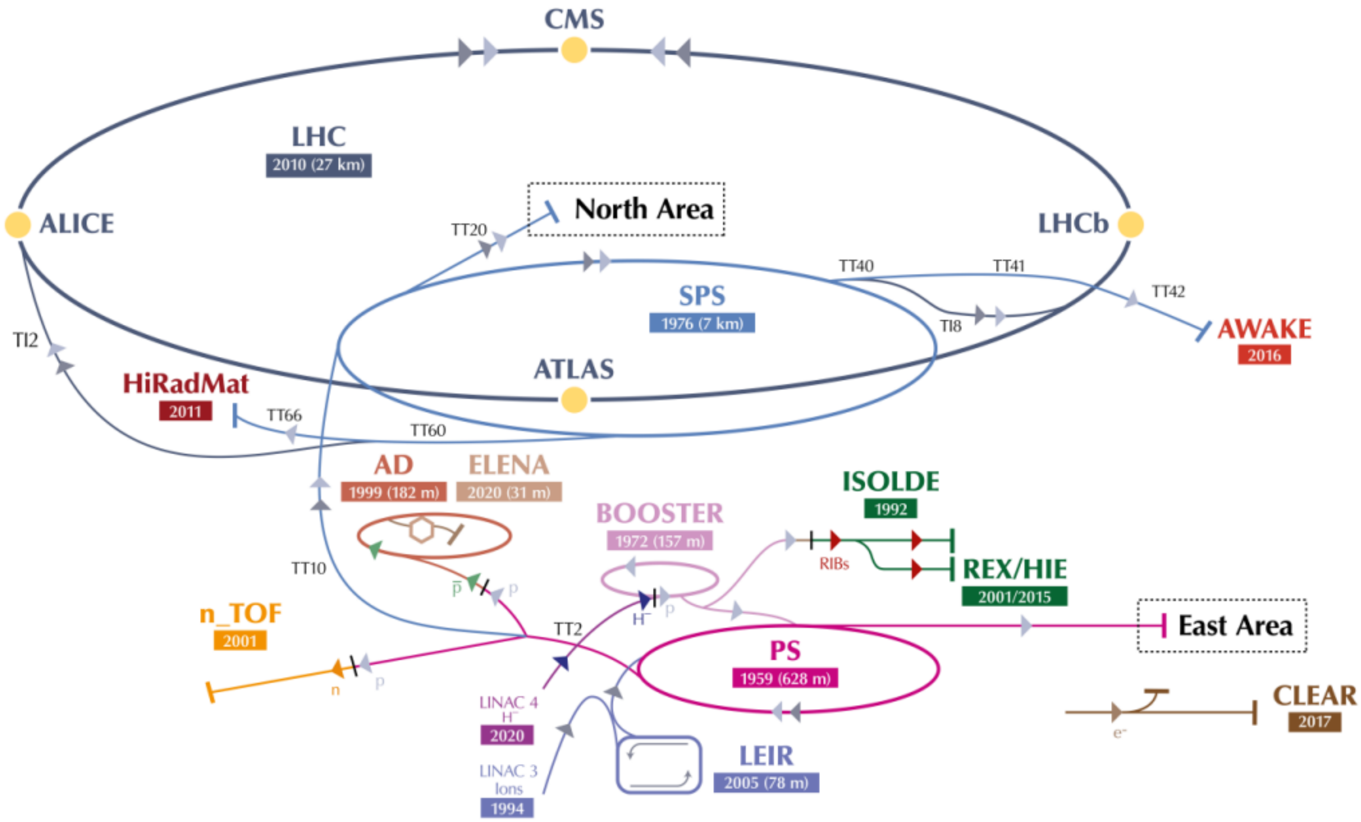
Along the injector chain



	Energy *(relativistic β)*
LINAC4	**0 – 160 MeV** ***(0 – 0.52)***
PSB	**0.16 – 2.0 GeV** ***(0.52 – 0.948)***
PS	**2.0 – 25.4 GeV** ***(0.948 – 0.9994)***
SPS	**25.4 – 450 GeV** ***(0.948 – 0.9994)***
LHC	**0.45 – 7 TeV** ***(0.9994 – 0.999999991)***

Linear Accelerators

LINACs

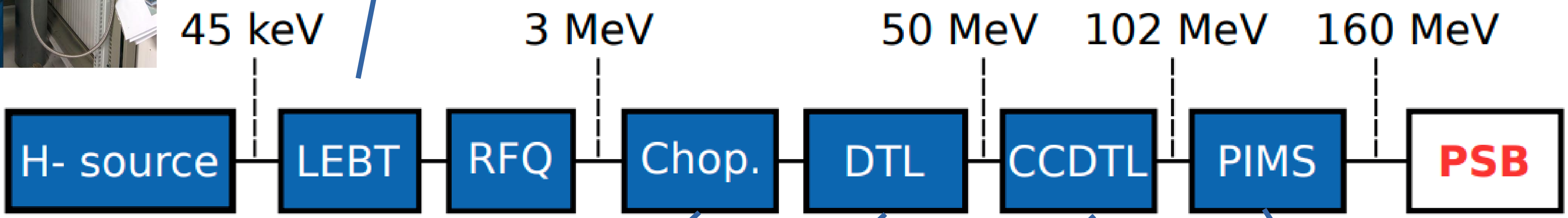


The journey of LHC protons starts at a Linear Accelerator (LINAC)

Why?

Final Energy	
Linac 4	160 MeV
PSB	2 GeV
PS	25 GeV
SPS	450 GeV
LHC	6.5-7 TeV

CERN LINAC4



Low Energy Beam Transport

Radio-Frequency Quadrupole

β=0

Chopper and buncher

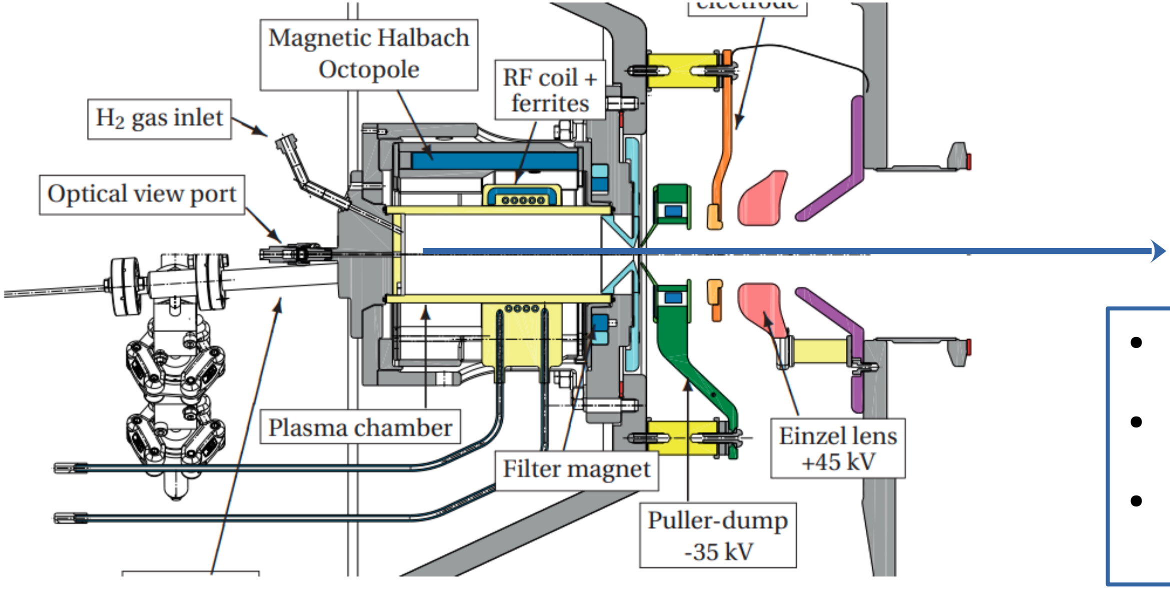
Drift tube linac

Cell Coupled Drift Tube Linac

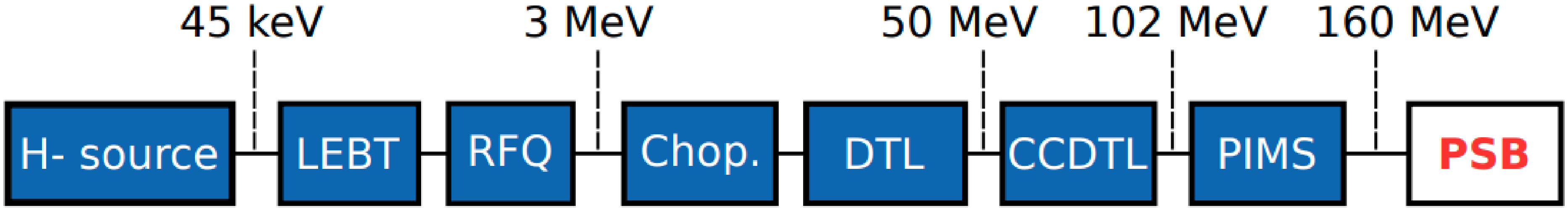
Pi-mode structure

Linear Accelerators, CAS 2021, David Alesini

CERN LINAC4



- 50 mA of H− beam
- Energy of 45 keV
- RMS emittance of 0.25 π mm mrad.



45 keV
3 MeV
50 MeV
102 MeV
160 MeV

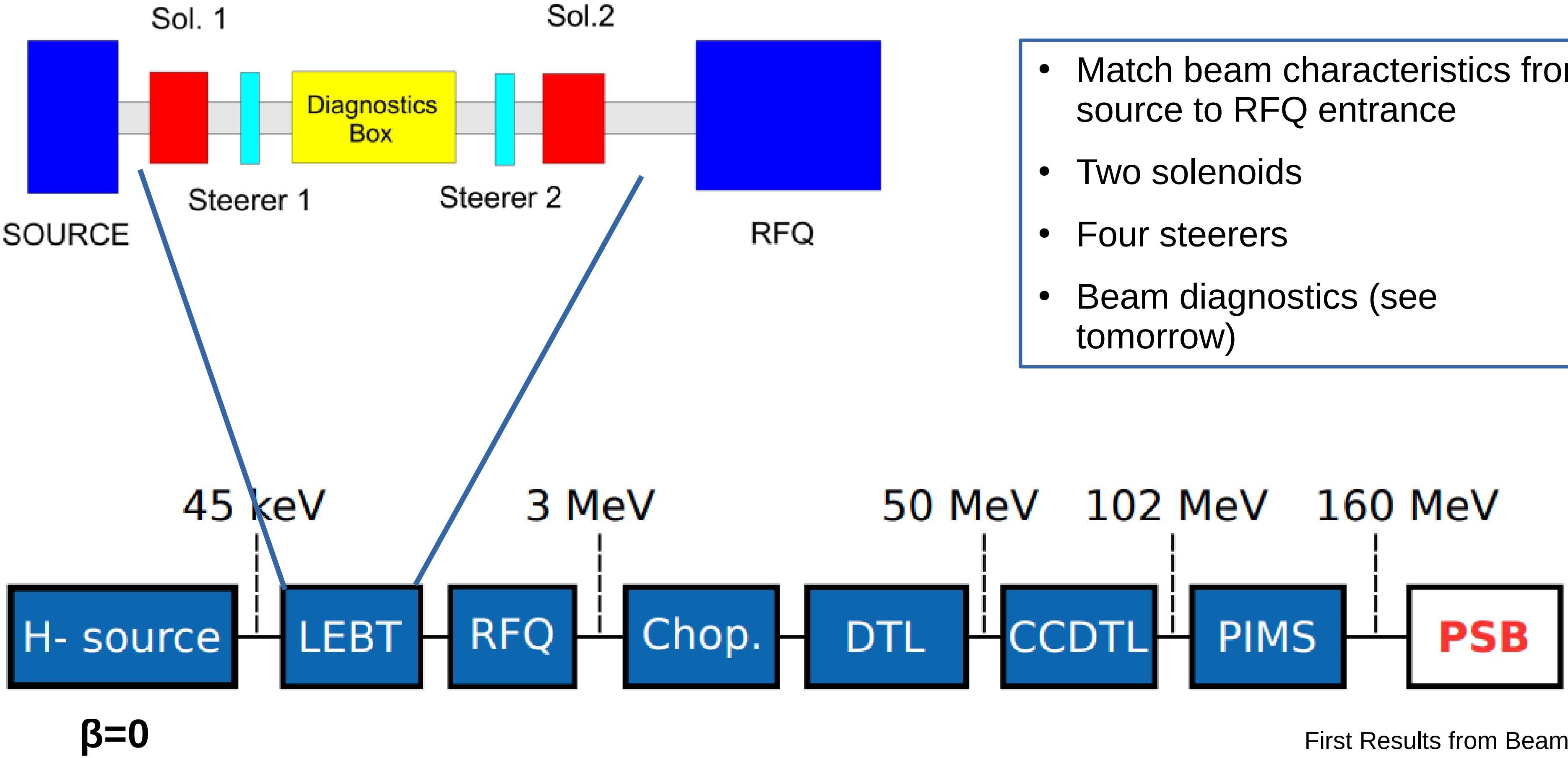
H- source
LEBT
RFQ
Chop.
DTL
CCDTL
PIMS
PSB

β=0

Linear Accelerators, CAS 2021, David Alesini

CERN LINAC4

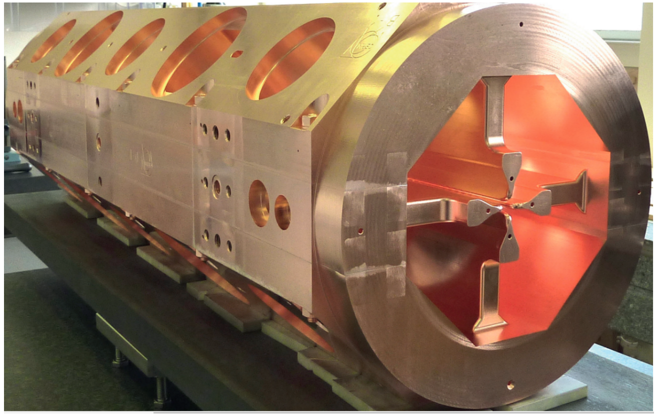
- Match beam characteristics from source to RFQ entrance
- Two solenoids
- Four steerers
- Beam diagnostics (see tomorrow)

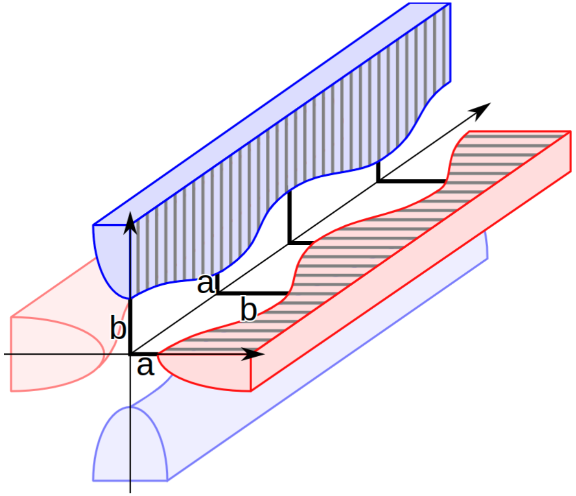


Linear Accelerators, CAS 2021, David Alesini

First Results from Beam Measurements at the 3 MeV Test Stand for CERN LINAC4

CERN LINAC4

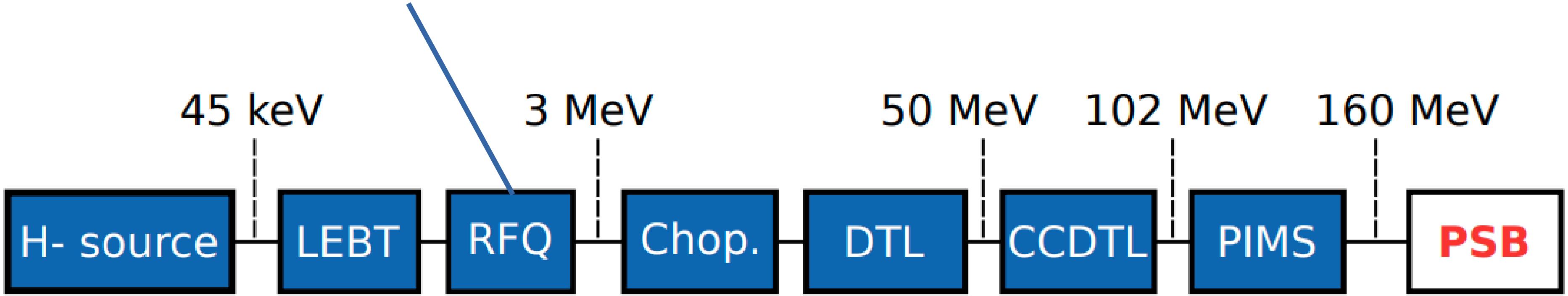




Combined-function component

Accelerates and focuses the beam

Geometry and electrodes: force resonance in specific TE mode



45 keV | 3 MeV | 50 MeV | 102 MeV | 160 MeV

H- source | LEBT | RFQ | Chop. | DTL | CCDTL | PIMS | PSB

β=0

Linear Accelerators, CAS 2021, David Alesini

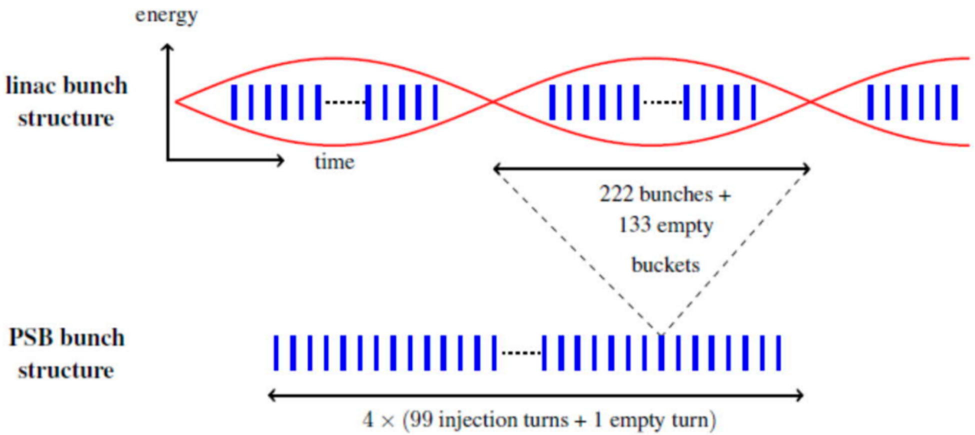
First Results from Beam Measurements at the 3 MeV Test Stand for CERN LINAC4

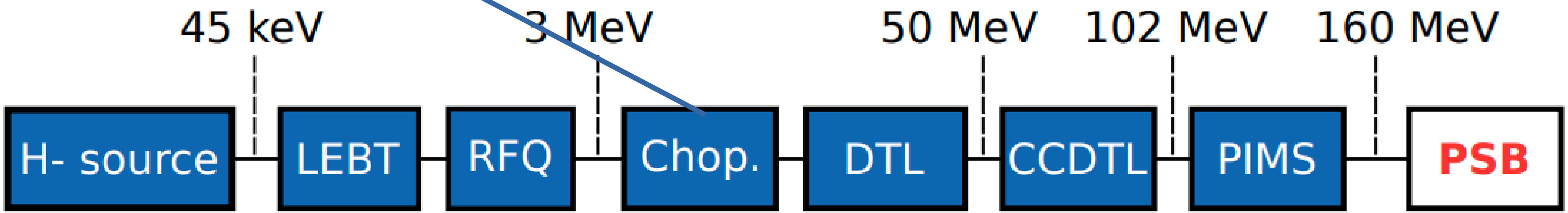
CERN LINAC4

Select only bunches accepted by the PSB

Fast-switching electrostatic plates

Powered to deflect the unwanted bunches onto a beam dump





β=0

LINAC4 DR, CERN-2020-006

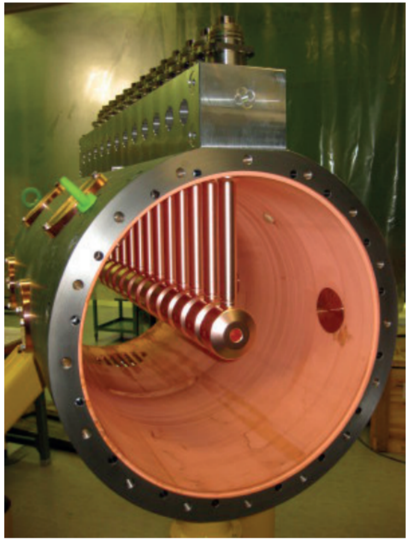
Linear Accelerators, CAS 2021, David Alesini

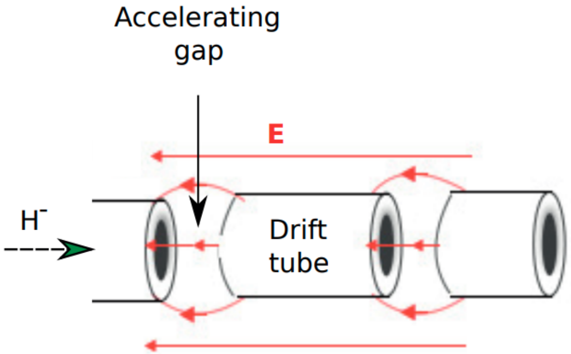
CERN LINAC4

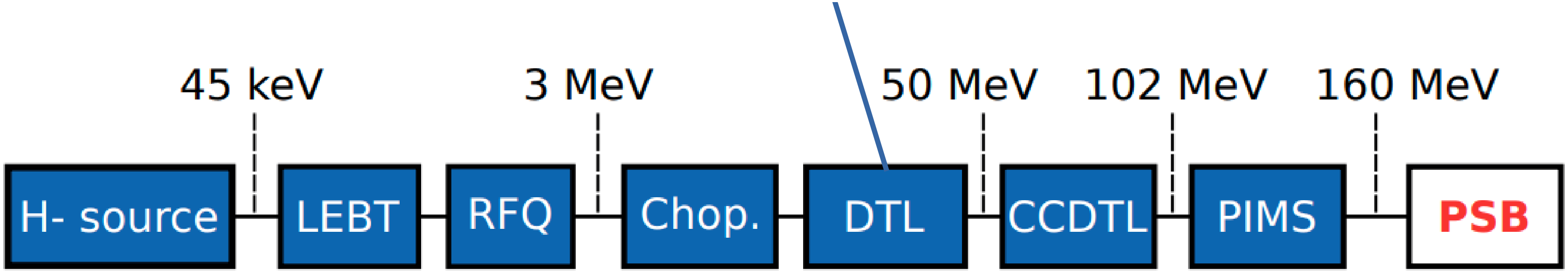
Drift Tube LINAC

Acceleration through E-field in gaps between annular Cu rings

Length of 108 drift tubes increases with particle *v*





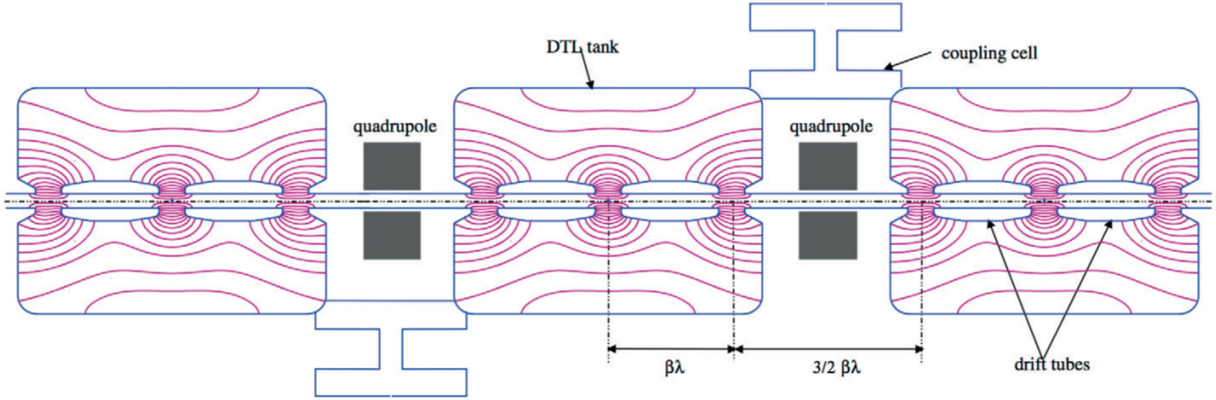


β=0

LINAC4 DR, CERN-2020-006

Linear Accelerators, CAS 2021, David Alesini

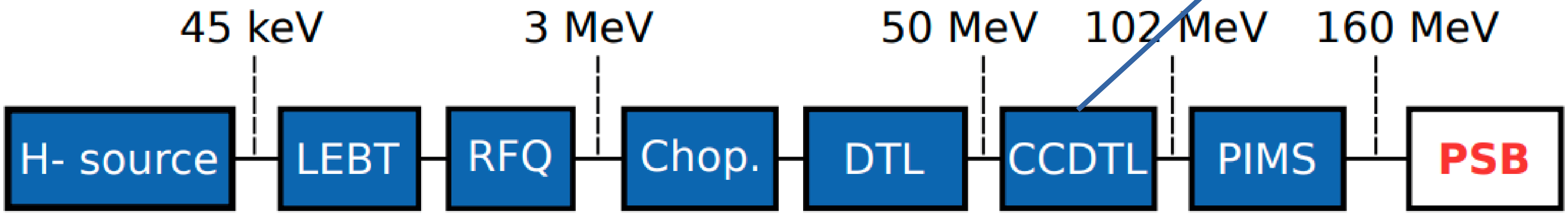
CERN LINAC4



Cell Couple Drift Tube LINAC

Less limitation from space charge at higher energies

Quadrupoles between tanks : more cost effective acceleration



45 keV | 3 MeV | 50 MeV | 102 MeV | 160 MeV

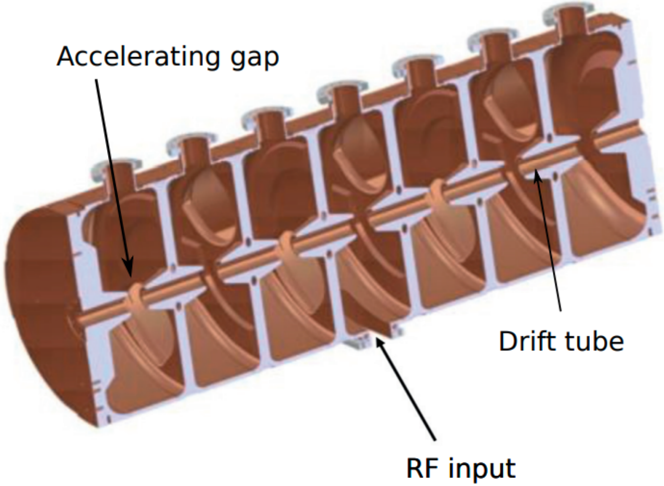
H- source → LEBT → RFQ → Chop. → DTL → CCDTL → PIMS → **PSB**

β=0

LINAC4 DR, CERN-2020-006

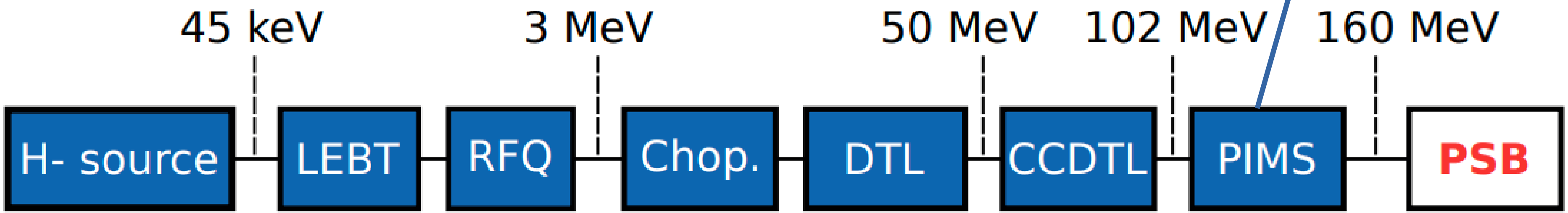
Linear Accelerators, CAS 2021, David Alesini

CERN LINAC4



Pi-mode structures

Simpler design and more economic construction compared to other methods


45 keV
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H- source
LEBT
RFQ
Chop.
DTL
CCDTL
PIMS
PSB

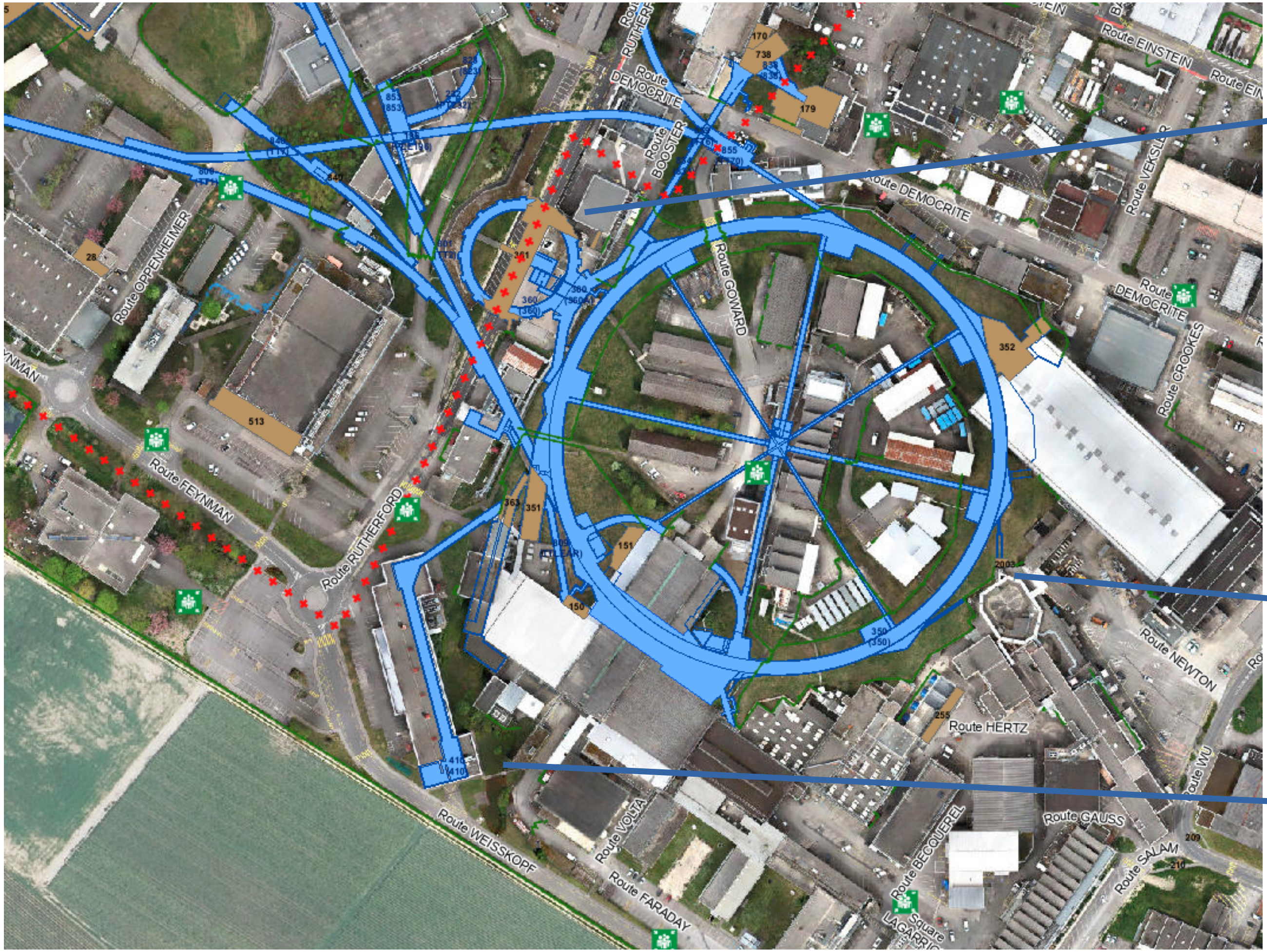
β=0

LINAC4 DR, CERN-2020-006

Linear Accelerators, CAS 2021, David Alesini

Transfer lines

Transfer lines

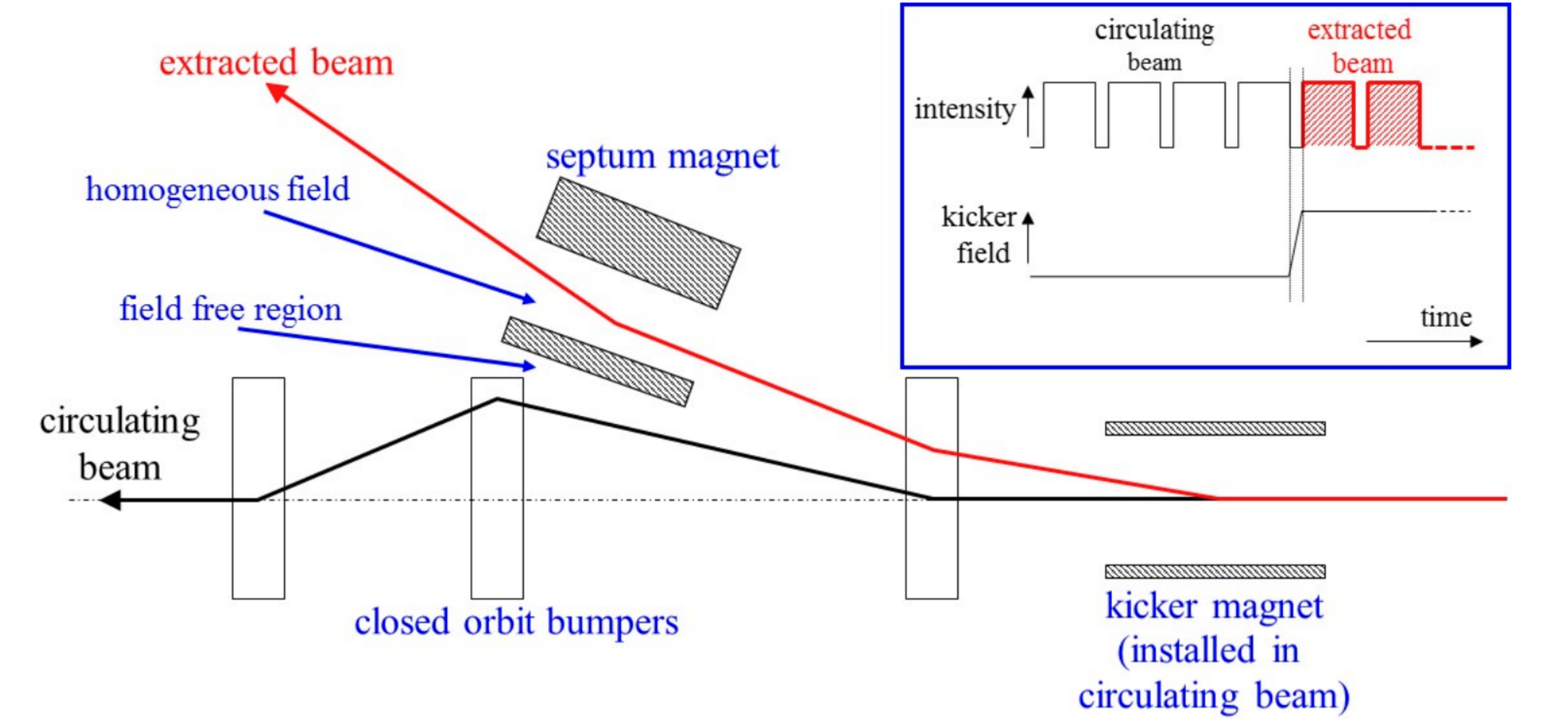


Proton Synchrotron Booster (PSB)

Proton Synchrotron (PS)

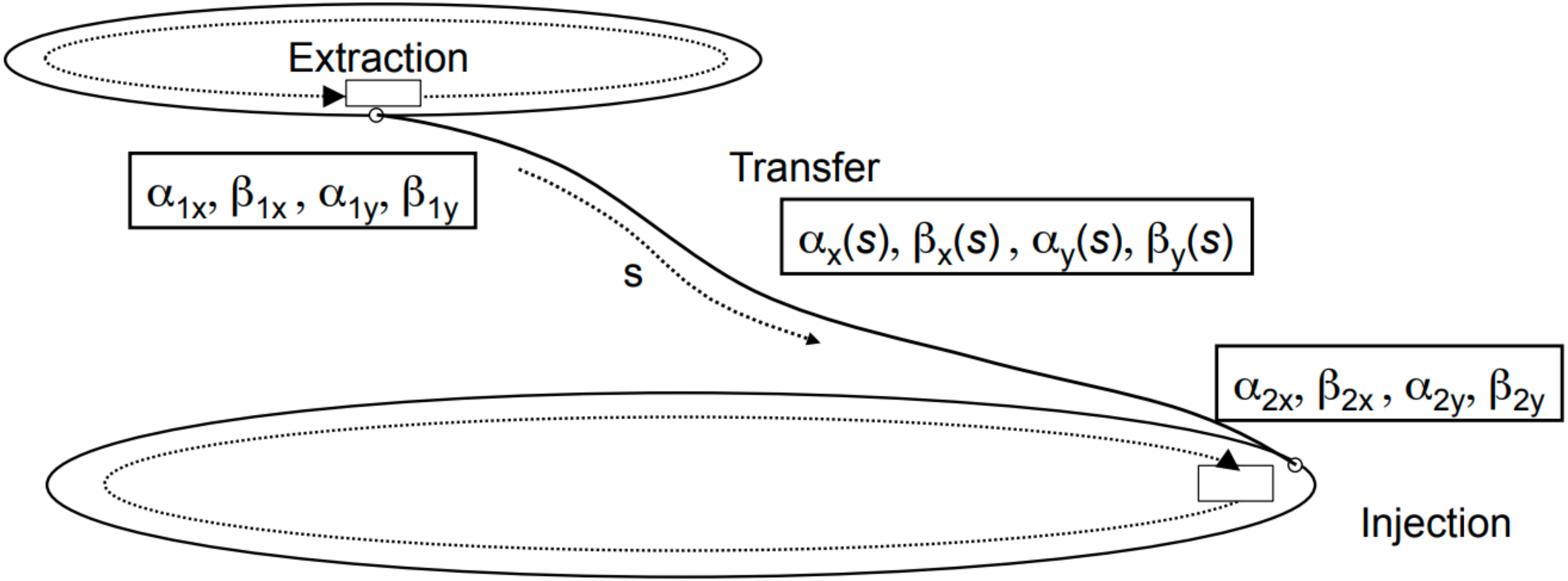
LINAC4

Kicker Magnets



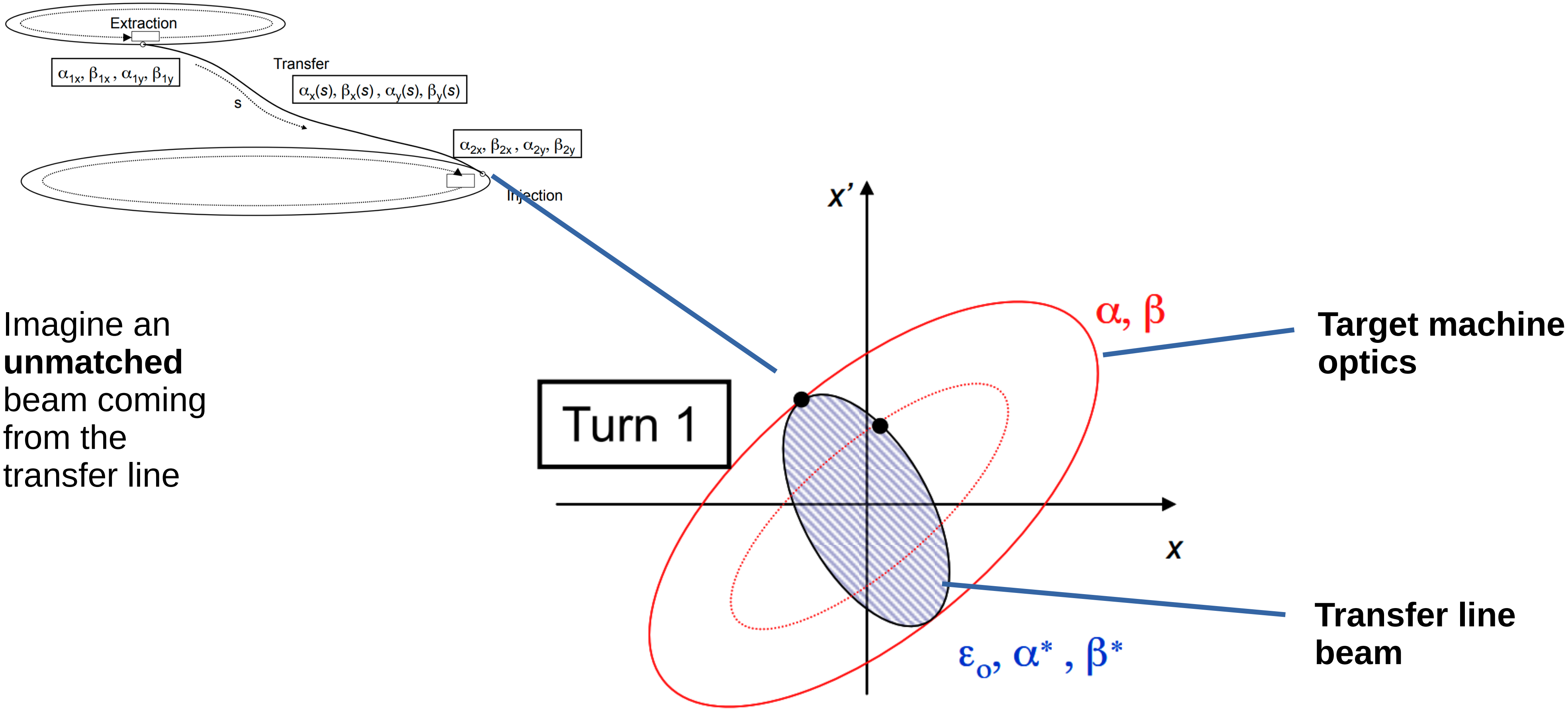
M. Barnes, Kicker Systems, CAS 2017

Beam optics matching



Vera Kain, CAS 2014, Transfer Lines

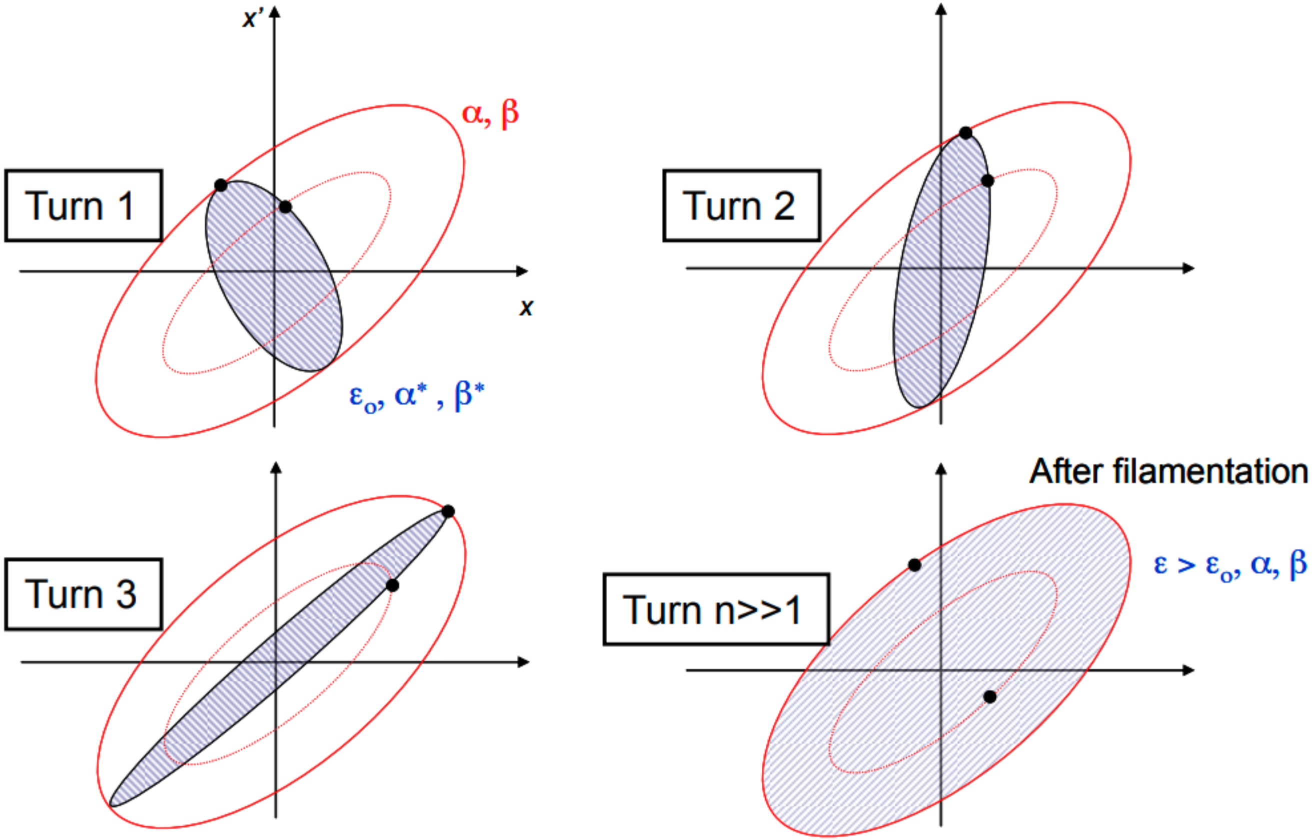
Beam optics matching



Imagine an **unmatched** beam coming from the transfer line

Verena Kain, CAS 2014, Transfer Lines

Beam optics matching



Emittance has increased!

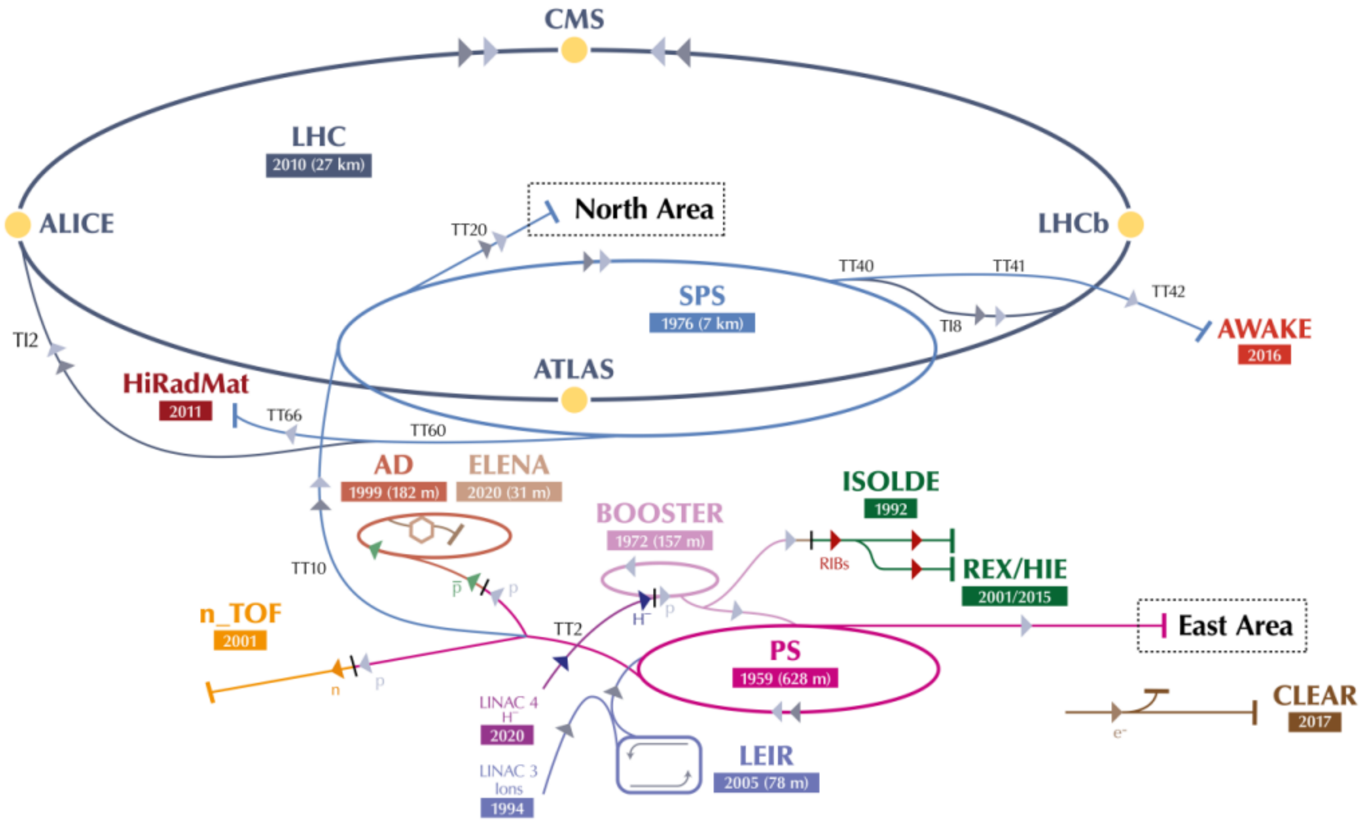
Detrimental to beam quality!

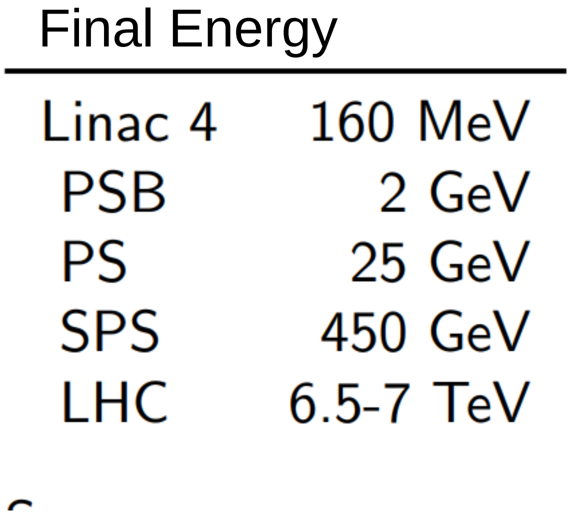
Phase space matching at beam transfer crucial!

Verena Kain, CAS 2014, Transfer Lines

Recap and Outlook

The CERN Accelerator Complex





Final Energy

Linac 4	160 MeV
PSB	2 GeV
PS	25 GeV
SPS	450 GeV
LHC	6.5-7 TeV

Next session's program