Precision Physics and BSM: high-low energy connections

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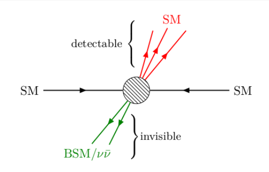
18 July 2024, Bezmiechowa Górna, Poland

'Precise measurements of known particles and interactions are just as important as finding new particles'
– Fabiola Gianotti



Supported by NATIONAL SCIENCE CENTRE POLAND

First, addendum to the yesterday's discussion

WHAT'S THE USE OF BASIC SCIENCE?

Christopher Llewellyn Smith,
Director-General of CERN from 1994-1998

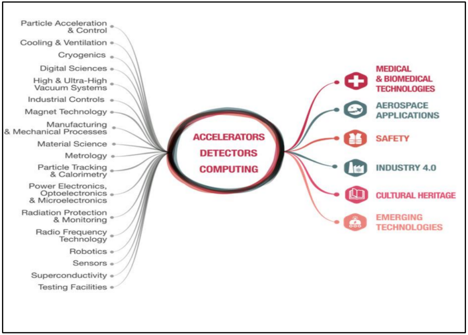
by C.H. Llewellyn Smith,
former Director-General of CERN
Original: The use of basic science

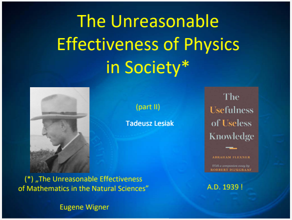
Content:

Video: https://cds.cern.ch/record/388110?ln=en

https://www-zeuthen.desy.de/ jknapp/JK/Reading_files/basic_science.html

DG Fabiola Gianotti, CERN vision and goals until next strategy update, → pdf





Tadeusz Lesiak Polish Physical Society (pdf): Nieracjonalna użyteczność fizyki dla społeczeństwa"

German Rodrigo, MTTD2021, The future of particle physics

"Forecasting the Socio-Economic Impact of the Large Hadron Collider: a Cost-Benefit Analysis to 2025 and Beyond"

https://inspirehep.net/literature/1425942,

"The socio–economic impact of a breakthrough in the particle accelerators' technology: a research agenda"

https://arxiv.org/pdf/1802.00352

BSM - terra incognita: Energy frontiers

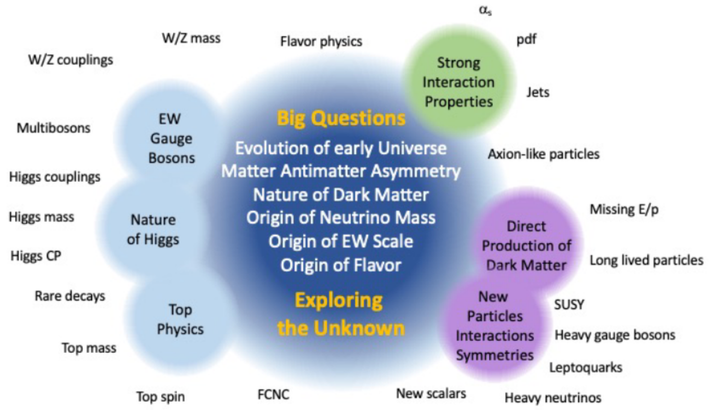
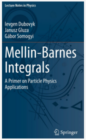


Figure 6-1. *Six categories of Probes and a multiple of Signatures accessible at energy-frontier collid address the Big Questions that are at the center of the EF pursuit.*

SM, BSM, CPV, LNV, LFV, intensity frontiers [disclaimer]

	11/7	12/7	13/7	14/7	15/7	16/7	17/7	18/7	19/7
Breakfast									
9:00-10:00		Introduction	SM 2	Accelerators 1	Accelerators 2	Accelerators 3	Topical : flavours & CPV	BSM	Future physics challenges
10:00-11:00		SM 1	SM 3	SM 4	Intro to QCD	Intro to QCD	Topical : flavours & CPV	BSM	Future physics challenges
Break									
11:30-12:30		Detectors 1	Detectors 2	Detectors 3	Detectors 4	Topical : charm 50 years	Topical : flavours & CPV	Topical : Cosmic rays and CREDO	Future physics challenges
12:30-13:30		Seminar 1 (Tadek)	Statistics	Statistics	Detectors 5	Topical : charm 50 years	Topical : flavours & CPV	Topical : Cosmic rays and CREDO	Seminar 4 (Mariola)
Lunch									
15:00-16:00		Hands-on Team 2 / Work with Profs Team 1	Seminar 2 (Marta)	Hands-on Team 1 / Work with Profs Team 2	Excursion	Topical : charm 50 years	Seminar 3 (Iwona)	Topical : Cosmic rays and CREDO	Students conference
16:00-17:00		Hands-on Team 2 / Work with Profs Team 1	Hot topic discussions (Achille)	Hands-on Team 1 / Work with Profs Team 2	Excursion	Topical : charm 50 years	Hot topic discussions (Achille)	Topical : Cosmic rays and CREDO	Students conference
Break									
17:30-18:30	Conference Charpak (17h-19h)	Hands-on Team 1 / Work with Profs Team 2	Work with Profs	Hands-on Team 2 / Work with Profs Team 1	Excursion	Students-Profs work	« Football »	Students-Profs work	Students conference
18:30-19:30	Conference Charpak (17h-19h)	Hands-on Team 1 / Work with Profs Team 2	Work with Profs	Hands-on Team 2 / Work with Profs Team 1	Excursion	Students-Profs work	« Football »	Students-Profs work	Summary
Dinner					**School dinner**				
Evening session			Statistics games	Statistics games		Intro to CERN			

particles.us.edu.pl



Janusz Gluza Homepage

SOLIDARNI Z UKRAINĄ

Particle Group in Katowice

Main Page
Selected Talks
Research Activity
Publications
Teaching (in Polish)

Research

Present:

- Future Circular Collider FCC
- AMBRE: construction of Mellin-Barnes representations for Feynman integrals (with I. Dubovyk)
- Polish grant NCN Opus (2021-2025): "Non-standard neutrinos and CP-violating effects in the leptonic sector", Popular description

BSM

- Polish grant NCN Maestro (2024-2029): "Precision Studies for particle-collider physics", Popular description

SM

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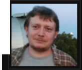

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Alain Blondel, Epiphany 2021 https://indico.cern.ch/event/934666

FCC

The Physics Landscape

We are in a fascinating situation: where to look and what will we find?

For the first time since Fermi theory, WE HAVE NO SCALE

The next facility must be versatile with as broad and powerful reach as possible, as there is no precise target

→ more Sensitivity, more Precision, more Energy

FCC , thanks to synergies and complementarities, offers the most versatile and adapted response to today's physics landscape,

Zobaczmy co takie niezerowe minimum daje w przypadku transformacji cechowania U(1)

$$\mathcal{L} = D_\mu^\dagger \phi^* D^\mu \phi - \mu^2 \phi^* \phi - \lambda(\phi^*\phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Niezmienniczy gdy:

$$\phi \rightarrow e^{i\Theta(x)}\phi$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \Theta(x)$$

Wokół minimum

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x))$$

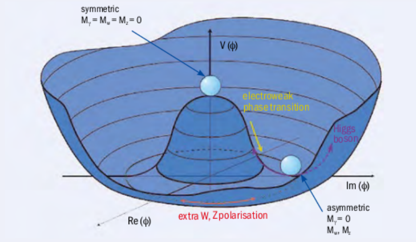
$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \zeta)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu + \ldots$$

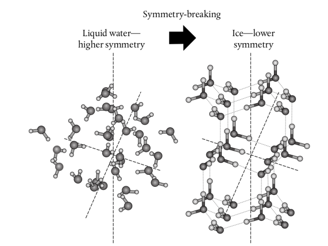
$$m_\zeta = 0, \quad m_\eta = \sqrt{2\lambda v^2}, \quad m_A = ev$$

Bezmasowa i masywna cząstka skalarna

Masywny bozon

Spontanous symmetry breaking (SSB) and scalar potential shape

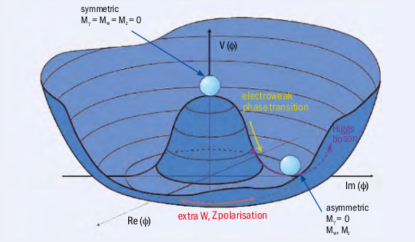


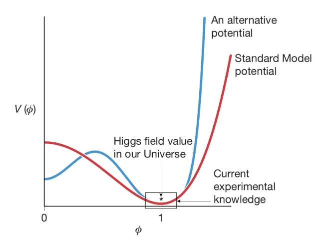


Jim Baggott "Mass", Oxford U. Press, 2017,

"Hand on heart, we never really understood it. Now we discover that it may not actually exist"

Spontanous symmetry breaking (SSB) and potential shape





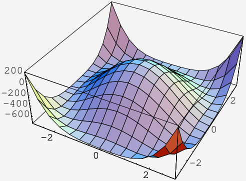
$$\Phi \equiv \Phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

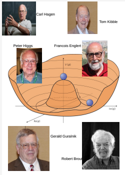
$$V = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \Leftrightarrow y = ax + bx^2, y \equiv V, x \equiv \Phi^\dagger \Phi$$

$$V_{min} = v/\sqrt{2}, v = \sqrt{\mu^2/\lambda} \simeq 250 \text{ GeV}$$

250 GeV: This is our study lab. scale! (So do heavy SM particle masses).

Can the "landscape" of the potential be so simple? Mayon mountain ("perfect cone")





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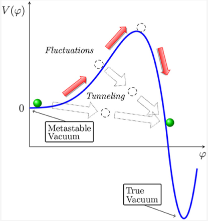

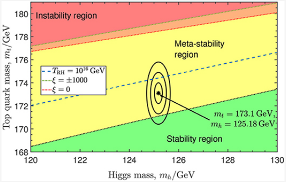

Precision directs towards discoveries! Higgs & Z bosons, top-quark, neutrinos

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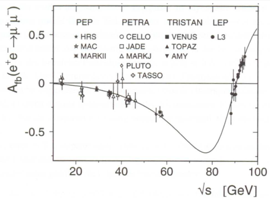
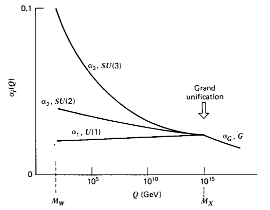
Electro-magnetism $\Longrightarrow$ electro-weak unification $\Longrightarrow$?

Weinberg angle

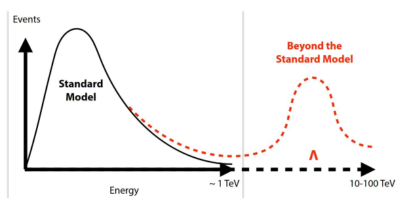


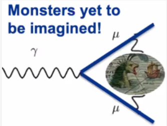


XX century's success!

Not possible without colliders.

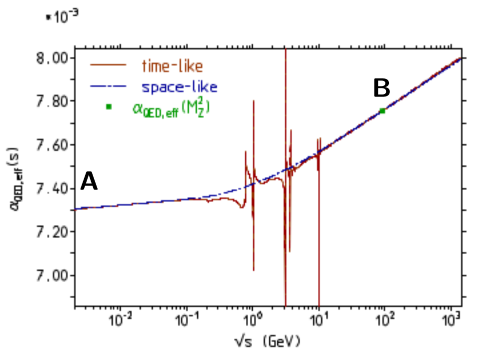
The general strategy for finding traces from "there":

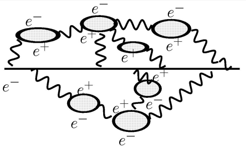




New excited states

$\alpha_{QED}(s)$, vacuum polarisation





F. Jegerlehner, http://dx.doi.org/10.23731/CYRM-2020-003.9

$$\mathbf{A} : \alpha_{\mathbf{QED}}(\mathbf{0}) \simeq \mathbf{1/137},\ \mathbf{B} :\ \alpha_{\mathbf{QED}}(\mathbf{M_Z^2}) \simeq \mathbf{1/128}.$$

Two ways for discoveries (in both cases precision is crucial):

1. within the known theory (anomalies[1])
2. new processes and (rare) phenomena;

[1]'I have always suspected that, one day, (...) they [JG: experimentalists] would like to see what would happen, just for the fun of it, if they falsely report that there exists a certain bump, or an oscillation in a certain curve, and see how the theorists predict it. I know these men so well that the moment I thought of that possibility I have honestly always been concerned that some day they will do just that. Then you can imagine how absurd the theoretical physicists would sound, making all these complicated calculations to demonstrate the existence of such a bump, while these fellows are laughing up their sleeves.' – R.P. Feynman)

In quest of new elusive particles and interactions (I)

https://physicsworld.com/a/and-so-to-bed-for-the-750-gev-bump/

http://resonaances.blogspot.com/2016/06/game-of-thrones-750-gev-edition.html

Résonaances

Particle Physics Blog

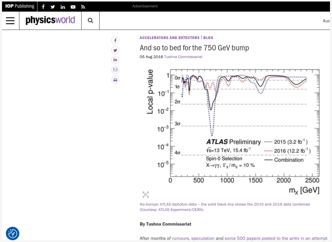
Saturday 18 June 2016

Game of Thrones: 750 GeV edition

The 750 GeV diphoton resonance has made a big impact on theoretical particle physics. The number of papers on the topic is already legendary, and they keep coming at the rate of order 10 per week. Given that the Backović model is falsified, there's no longer a theoretical upper limit. Does this mean we are not dealing with the classical ambulance chasing scenario? The answer may be known in the next days.

So who's leading this race? What kind of question is that, you may shout, of course it's Strumia! And you would be wrong, independently of the metric. For this contest, I will consider two different metrics: the *King Beyond the Wall* that counts the number of papers on the topic, and the *Iron Throne* that counts how many times these papers have been cited.

In the first category, the contest is much more fierce than one might expect: it takes 8 papers to be the leader, and 7 papers may not be enough to even get on the podium! Among the 3 authors with 7 papers the final classification is decided by ~~trial by combat~~ the citation count. The result is (*drums*):



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ACCELERATORS AND DETECTORS | BLOG

And so to bed for the 750 GeV bump

05 Aug 2016 Tushna Commissariat

No bumps: ATLAS diphoton data – the solid black line shows the 2015 and 2016 data combined. (Courtesy: ATLAS Experiment/CERN)

By Tushna Commissariat

After months of rumours, speculation and some 500 papers posted to the *arXiv* in an attempt

In quest of new elusive particles and interactions (II)

Simpson's 17-keV neutrino, $\frac{\Delta K}{K} \sim \sqrt{1 - \frac{M^2}{(Q-E)^2}}$

A. Franklin, The appearance and disappearance of the 17-keV neutrino

https://journals.aps.org/rmp/pdf/10.1103/RevModPhys.67.457

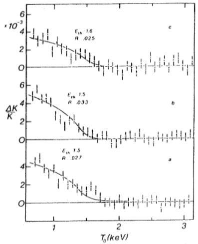


FIG. 1. Data of three runs presented as $\Delta K/K$ (the fractional change in the Kurie plot) as a function of the kinetic energy of the β particles. $E_{\rm th}$ is the threshold energy, the difference between the end-point energy and the mass of the heavy neutrino. A kink is clearly seen at $E_{\rm th}=1.5$ keV, or at a mass of 17.1 keV. Run (a) included active pileup rejection, whereas runs (b) and (c) did not. (c) was the same as (b) except that the detector was housed in a soundproof box. No difference is apparent. From Simpson (1985).

[5]In a normal beta-decay spectrum the quantity $K = \{N(E)/[f(Z,E)(E^2-1)^{1/2}E]\}^{1/2}$ is a linear function of E, the energy of the electron. A plot of that quantity as a function of E, the energy of the decay electron, is called a Kurie plot.

TABLE II. Experimental evidence for a 17-keV neutrino (Simpson, 1992).

Iostope	ν Mass (keV)	Mixing angle θ^a	Reference
^{3}H [Si(Li)]	17.1±0.2	0.105±0.015	Hime and Simpson (1989), Simpson (1985)
^{3}H in Ge	16.9±0.1	0.105±0.015	Hime and Simpson (1989)
^{35}S	16.9±0.4	0.082±0.008	Hime and Simpson (1989)
	16.95±0.35	0.088±0.005	Simpson and Hime (1989)
^{14}C in Ge	17.0±0.5	0.114±0.015	Sur *et al.* (1991)
^{63}Ni	16.75±0.38	0.101±0.011	Hime, Oxford report (OUNP-91-20).

Dwa teoretyczne wzory

$$m\left(\nu_e\right) = \left[\left|U_{e1}\right|^2 m_1^2 + \left|U_{e2}\right|^2 m_2^2 + \left|U_{e3}\right|^2 m_3^2\right]^{1/2} \leq m_\beta,$$

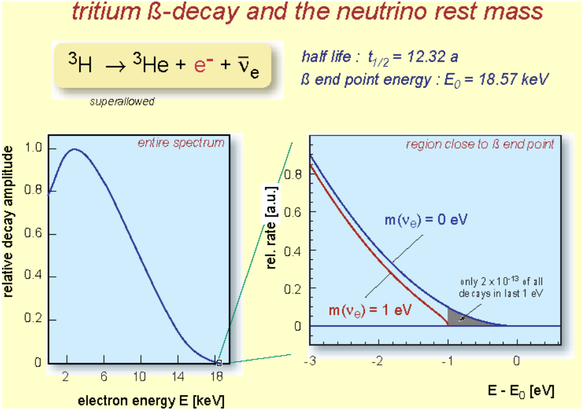
$$m_\beta = 2.7 \text{ eV } [18] \quad 3.4 \text{ eV}$$

Rozpad trytu

$$|\langle m_\nu \rangle| \equiv \left|\sum U_{ei}^2 m_i\right| < 0.2 \text{ eV}$$

Bezneutrinowy rozpad beta

Back to roots: rozpad trytu



Wykład V: Neutrina, J. Gluza

‘The reasonable man adapts himself to the world.
The unreasonable one persists in trying to adapt the world to himself.
Therefore all progress depends on the unreasonable man.’

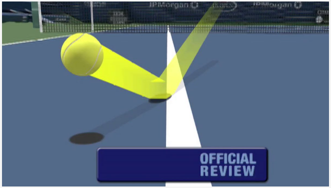
– George Bernard Shaw, Man and Superman

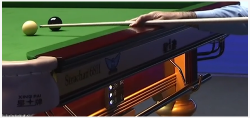
Transport Katrin

In quest of new elusive particles and interactions (II)

PRECISION

In general, and consequences





"Whoever has only a hammer will see nothing but nails."

BBC series, *Precision: The Measure of All Things*

Precision, true inception

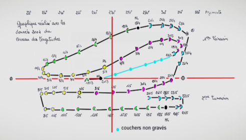
- Tycho de Brahe ($\sim$ 1601) Mars orbits, Rudolphine tables;

  https://archive.org/details/tabulrudolphinqu00kepl/page/n5/mode/2up

  https://indico.cern.ch/event/958085/contributions/4329017/attachments/2245318/3807690/Physics

  $\longrightarrow$ Johannes Kepler ($\sim$ 1609) - planets laws of motion;

  $\longrightarrow$ Isaac Newton ($\sim$ 1686) - gravitation



Abri Branchard bone ($\sim$ 30 000 years BC),

Alexander Marschack, 'Cognitive Aspects of Upper Paleolithic Engraving' Current Anthropology (1972),

Interpretation: Chantal Jegues-Wolkiewicz - probably the first Moon calendar,

https://www.dailymotion.com/video/x8044kz, Prehistoric Astronomy

Similar paintings, Lascaux caves, $\sim$ 17 000 BC.

Precision changes history: justice, law, crime, trade, economy, social, ...

To be just was precisely to use balance.

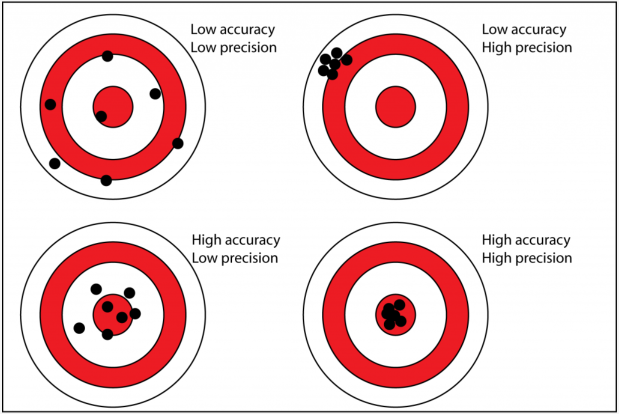
Wisdom 11:20

'By weight, measure and number, God made all things'

Code of Hammurabi 1772 BC - any taverner using false weights could be served up with the death penalty

PRECISION

Particle Physics and Standard Model



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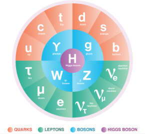
LEP ($W^{\pm}, Z$), LHC (H^0) - shaping the Standard Model

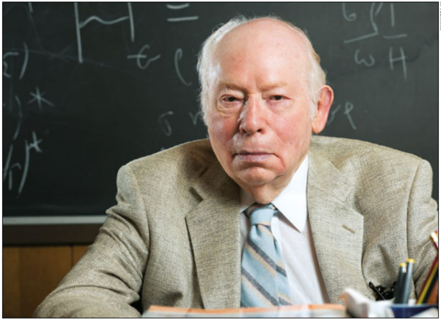


Steven Weinberg 1933–2021

A mind to rank with the greatest

Steven Weinberg, one of the greatest theoretical physicists of all time, passed away on 23 July, aged 88. He revolutionised particle physics, quantum field theory and cosmology with conceptual breakthroughs that still form the foundation of our understanding of physical reality.

Weinberg is well known for the unified theory of weak and electromagnetic forces, which earned him the Nobel Prize in Physics in 1979, jointly awarded with Sheldon Glashow and Abdus Salam, and led to the prediction of the Z and W vector bosons, later discovered at CERN in 1983. His breakthrough was the realisation that some new theoretical ideas, initially believed to play a role in the description of nuclear strong interactions, could instead explain the nature of the weak force. "Then it suddenly occurred to me that this was a perfectly good sort of theory, but I was applying it to the wrong kind of interaction. The right place to apply these ideas was not to the strong interactions, but to the weak and electromagnetic interactions," as he later recalled. With his work, Weinberg had made the next step in the unification of physical laws, after Newton understood that the motion of apples on Earth and planets in the sky are governed by the same gravitational force, and Maxwell understood that electric and magnetic phenomena are the expression of a single force.



Steven Weinberg radically changed the way we look at the universe.

In my life, I have built only *one* model

physicists, and will certainly continue to serve future generations.

Steven Weinberg is among the very few individuals who, during the course of the history

>50 years of the Z-boson theory (1967)

S. Weinberg

"A MODEL OF LEPTONS"

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \quad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \quad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \cong M_1^2/2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere[6] without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,[7] so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle\varphi\rangle = \lambda\binom{1}{0}. \quad (6)$$

The first four terms in $\mathfrak{L}$ remain intact, while the rest of the Lagrangian becomes

$$-\tfrac{1}{8}\lambda^2 g^2[(A_\mu{}^1)^2 + (A_\mu{}^2)^2] - \tfrac{1}{8}\lambda^2(gA_\mu{}^3 + g'B_\mu)^2 - \lambda G_e \bar{e}e. \quad (7)$$

We see immediately that the electron mass is λG_e. The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu{}^1 + iA_\mu{}^2) \quad (8)$$

and has mass

$$M_W = \tfrac{1}{2}\lambda g. \quad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu{}^3 + g'B_\mu), \quad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu{}^3 + gB_\mu). \quad (11)$$

Their masses are

$$M_Z = \tfrac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \quad (12)$$

$$M_A = 0, \quad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{i(g^2+g'^2)^{1/2}}{4}\left[\left(\frac{3g'^2-g^2}{g'^2+g^2}\right)\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu. \quad (14)$$

Many puzzles, e.g.

If

$$\rho_t = \frac{m_Z m_t}{m_H^2},$$

then (for ATLAS, CMS combined $m_H = 125.6 \pm 0.4 \pm 0.5$)

$$\rho_t^{(exp)} = 1.0022 \pm 0.007 \pm 0.009$$

Separately,

$$\rho_t^{(exp)} = 1.0077 \pm 0.007 \pm 0.009 \qquad (m_{h,ATLAS}),$$
$$\rho_t^{(exp)} = 0.9965 \pm 0.007 \pm 0.007 \qquad (m_{h,CMS})$$

E. Torrente-Lujan, https://inspirehep.net/literature/1184358

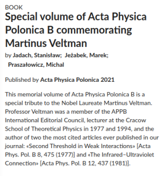
- (i) muon discovery, J/Ψ
  (ii) $(g-2)_e$, $(g-2)_\mu$
  (iii) V-A, parity;
  Note the 100^{th} Birthday Anniversary of Prof. Chen Ning Yang, link

EXPERIMENT $\rightarrow$ THEORY

- (i) $\tau^\pm$ (tau lepton);
  (ii) Tevatron - top quark discovery;
  (iii) H^0 (scalar Higgs-Englert boson)

THEORY $\rightarrow$ EXPERIMENT

SM corrections matters! LEP, SLAC, LHC
M. Veltman (1977) ρ-parametr $\sim m_t^2, \ln(m_H^2)$;
$\longrightarrow$ Acta Physica Polonica B


BOOK
Special volume of Acta Physica Polonica B commemorating Martinus Veltman
by **Jadach, Stanisław; Jeżabek, Marek; Praszałowicz, Michal**

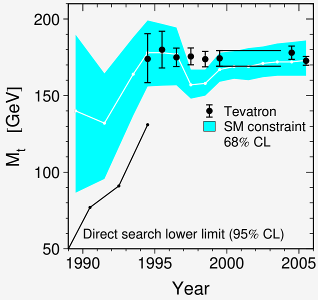
Published by **Acta Physica Polonica 2021**

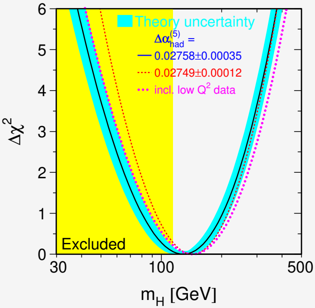
This memorial volume of Acta Physica Polonica B is a special tribute to the Nobel Laureate Martinus Veltman. Professor Veltman was a member of the APPB International Editorial Council, lecturer at the Cracow School of Theoretical Physics in 1977 and 1994, and the author of two the most cited articles ever published in our journal: «Second Threshold in Weak Interactions» [Acta Phys. Pol. B 8, 475 (1977)] and «The Infrared–Ultraviolet Connection» [Acta Phys. Pol. B 12, 437 (1981)].

- Neutrinos (masses, mixing angles, CP phase(s));
  Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, Dune, SNO+, Daya Bay, Double Chooz, RENO, ...

Future Colliders; THEORY $\leftrightarrow$ EXPERIMENT

Indirect top and Higgs precision search





Small deviations matters!

Z.Ligeti

Aside: factor-of-2 improvements can matter!

Search for $K_L \to \pi\pi$

ANNALS OF PHYSICS: 5, 156–181 (1958)

Long-lived Neutral K Mesons*

M. BARDON, K. LANDE, AND L. M. LEDERMAN

Columbia University, New York, New York, and Brookhaven National Laboratories, Upton, New York

AND

WILLIAM CHINOWSKY

Brookhaven National Laboratories, Upton, New York

set an upper limit <0.6% on the reactions

< 0.6%

$$K_2^0 \to \begin{cases} \mu^\pm + e^\mp \\ e^+ + e^- \\ \mu^+ + \mu^- \end{cases}$$

and on $K_2^0 \to \pi^+ + \pi^-$.

VOLUME 6, NUMBER 10 PHYSICAL REVIEW LETTERS MAY 15, 1961

DECAY PROPERTIES OF K_2^0 MESONS*

D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov
Joint Institute of Nuclear Research, Moscow, U.S.S.R.
(Received April 20, 1961)

Combining our data with those obtained in reference 7, we set an upper limit of 0.3% for the relative probability of the decay $K_2^0 \to \pi^- + \pi^+$. Our

< 0.3%

"At that stage the search was terminated by administration of the Lab."

[Okun, hep-ph/0112031]

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

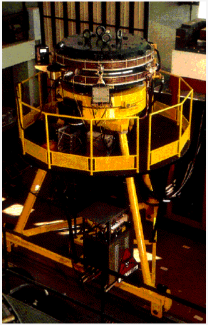
EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

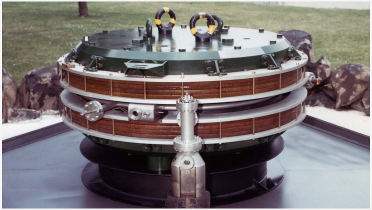
J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

= 0.2 ± 0.04 %

We would conclude therefore that K_2^0 decays to two pions with a branching ratio $R = (K_2 \to \pi^+ + \pi^-)/(K_2^0 \to \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$ where the error is the standard deviation. As empha-

The first circular e^+e^- accelarator





ADA/ADONE, 1969-1993, Frascati, $\sqrt{s} \leq 3$ GeV

To be lucky is an important life/research factor

2 PRLs in 1974 for J/Ψ discovery

SPEAR at SLAC

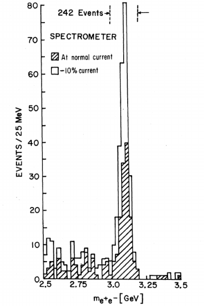


FIG. 2. Mass spectrum showing the existence of J.

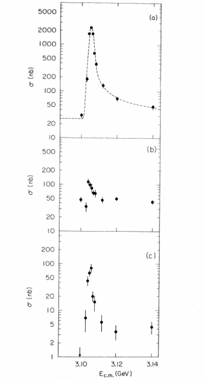


FIG. 1. Cross section versus energy for (a) multi-hadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\pi^+\pi^-$, and K^+K^- final states. The curve in (a) is the ex-

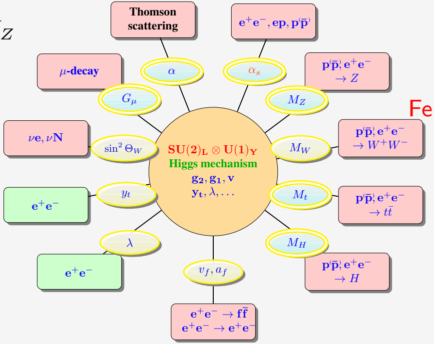
"J"

"Psi"

Input and calculated/measured parameters

Schemes: G_μ vs M_W,...

G_μ, $\sin^2\theta^\ell_{eff}$, M_Z



Fermilab!

Fig. from the FCC-ee report 'α_{QED}' by F. Jegerlehner in 1905.05078

Introduction to Precision Electroweak Analysis by J. Welss, 0512342

Input and calculated/measured parameters

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$

$$\frac{\delta \mathbf{G}_\mu}{\mathbf{G}_\mu} \sim 8.6 \times 10^{-6}$$

$$\frac{\delta \mathbf{M_Z}}{\mathbf{M_Z}} \sim 2.4 \times 10^{-5}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 0.9 \div 1.6 \times 10^{-4}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5 \times 10^{-5} \quad (\mathtt{FCC-ee/ILC\ requirement})$$

$$\longrightarrow \frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}\,, \; \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}\,, \; \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}\,,$$

Among the basic input parameters $\alpha(M_Z), G_\mu, M_Z$, $\alpha(M_Z)$ is the least precise and requires a major effort of improvement.

Electromagnetism: atoms, chemistry, biology

$$F = k\frac{qQ}{r^2} \equiv \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2} \quad \to \alpha \equiv \frac{\phi \mathbf{e}}{\mathbf{mc^2}} = \frac{\frac{\mathbf{e^2}}{4\pi\epsilon_0 \mathbf{L}}}{\mathbf{mc^2}} = \frac{\mathbf{e^2}}{4\pi\epsilon_0 \hbar \mathbf{c}},\ L = \frac{\hbar}{mc}$$

Fine structure **constant** α 1/137 [137.035999206(11)]

1/136 or 1/138 makes a difference

Percentage changes of α
$\to$ changes stars evolution (red or blue stars) ("Gravitation", Misner, Thorne, Wheeler)

$\to$ key input parameter in the Standard Model

2020's result from the Paris lab on $\alpha_{QED}(0)$

REPORT

Measurement of the fine-structure constant as a test of the Standard Model

Richard H. Parker[1,*], Chenghui Yu[1,*], Weicheng Zhong[1], Brian Estey[1], Holger Müller[1,2,†]
+ See all authors and affiliations

Science 13 Apr 2018:
Vol. 360, Issue 6385, pp. 191-195
DOI: 10.1126/science.aap7706

Article | Published: 02 December 2020

Determination of the fine-structure constant with an accuracy of 81 parts per trillion

Léo Morel, Zhibin Yao, Pierre Cladé & Saïda Guellati-Khélifa

Nature **588**, 61–65(2020) | Cite this article

$\alpha^{-1}(Cs)$= 137.035 999 046(27)

$\alpha^{-1}(Rb)$= 137.035 999 206(11)

$\alpha^{-1}(a_e)$= 137.035 999 139(31)

Remarks:
(i) new result - deviation from SM in the same direction as in $(g-2)_\mu$,
(ii) substantial disagreement with Cs ($\sim 5.4\sigma$).

Over 2 decades of improvements
https://www.nature.com/articles/s41586-020-2964-7 [02 December 2020]

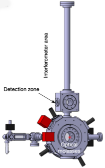
$\alpha_{QED}(0)$ and BSM

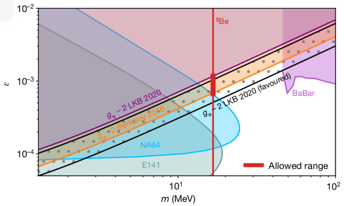
Table 1 | Error budget on α

Source	Correction ($\times 10^{-11}$)	Relative uncertainty ($\times 10^{-11}$)
Gravity gradient	−0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman light shift	2.3	2.3
Index of refraction	0	<0.1
Internal interaction	0	<0.1
Light shift (two-photon transition)	−11.0	2.3
Second-order Zeeman effect		0.1
Phase shifts in Raman phase-lock loop	−39.8	0.6
Global systematic effects	**64.2**	**6.8**
Statistical uncertainty		**2.4**
Relative mass of ^{87}Rb[a]: 86.9091805310(60)		3.5
Relative mass of the electron[b]: 5.48579909065(16) $\times 10^{-4}$		1.5
Rydberg constant[b]: 10,973,731.568160(21) m^{-1}		0.1
Total: α^{-1} = 137.035999206(11)		**8.1**

For each systematic effect, more discussion can be found in Methods.
[a]From ref. 13.
[b]From https://pml.nist.gov/cuu/Constants/.

Morel, L., Yao, Z., Cladé, P. et al. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. Nature 588, 61–65 (2020).



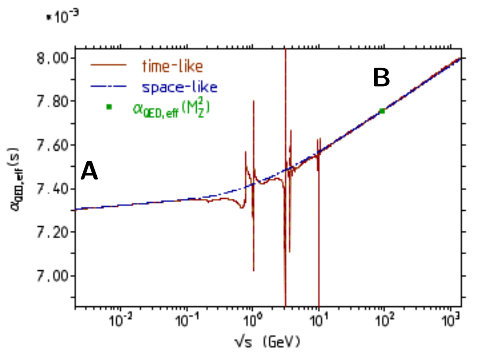


Substructure: $\alpha_{QED}(0) \longrightarrow$ modification of $\delta a_e \simeq m_e/m^*$
Excluded (light, states, weakly coupled):

$$m^* < 520 \text{ GeV}.$$

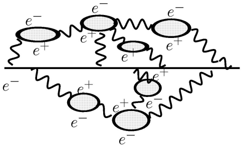
Future δa_e improvement by an order of magnitude in next years, sensitivity similar as for $(g-2)_\mu$.

$\alpha_{QED}(s)$, vacuum polarisation



A: $\alpha_{QED}(0) \simeq 1/137$

B: $\alpha_{QED}(M_Z^2) \simeq 1/128$



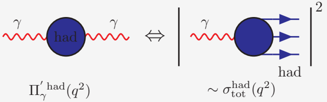
F. Jegerlehner, http://dx.doi.org/10.23731/CYRM-2020-003.9

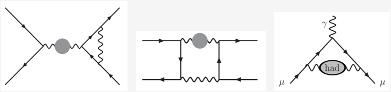
The effective $\alpha(s)$ in terms of the photon vacuum polarization (VP) self-energy correction $\Delta\alpha(s)$ by

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)} \; ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s) \, .$$

R-data evaluation of $\alpha(M_Z^2)$

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)} \; ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)\,.$$





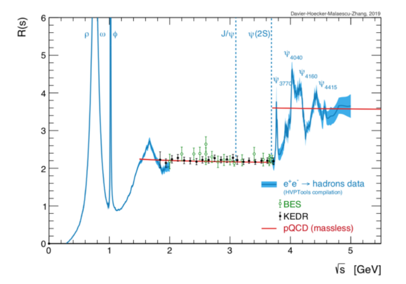
The non-perturbative hadronic piece from the five light quarks $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)^{(5)}_{\rm had}$ can be evaluated in terms of $\sigma(e^+e^- \to {\rm hadrons})$ data via the dispersion integral **(s can be any, also negative!)**

$$\Delta\alpha^{(5)}_{\rm had}(s) \quad = \quad -\frac{\alpha\, s}{3\pi}\left(\int_{m^2_{\pi^0}}^{E^2_{\rm cut}} {\rm ds}'\, \frac{{\rm R}^{\rm data}_\gamma({\rm s}')}{{\rm s}'({\rm s}'-{\rm s})} + \int_{{\rm E}^2_{\rm cut}}^{\infty} {\rm ds}'\, \frac{{\rm R}^{\rm pQCD}_\gamma({\rm s}')}{{\rm s}'({\rm s}'-{\rm s})}\right),$$

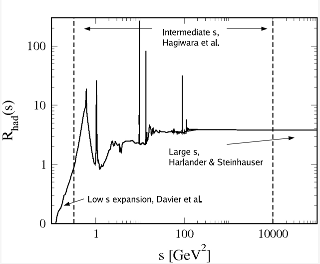
$$a_\mu^{\rm had} \quad = \quad \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{R(s)\;\hat{K}(s)}{s^2} \quad , \hat{K}(s) \in 0.63 \div 1.$$

$$R_\gamma(s) \quad \equiv \quad \sigma^{(0)}(e^+e^- \to \gamma^* \to {\rm hadrons}) / \left(\frac{4\pi\alpha^2}{3s}\right)$$

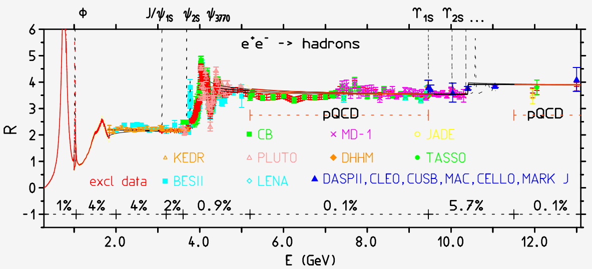
Davier et al



E.g. Parametrization used for Bhabha
Phys.Rev.D 78 (2008) 085019



The compilation of $R(s)$-data utilized by F. Jegerlehner for $\Delta\alpha_{\rm had}$.



EWPOs and

N^xLO SM CORRECTIONS

LO (tree), NLO, NNLO, NNNLO (N^3LO, ... (loops)

Collider physics ... magic of math world!

Annals of Mathematics, **141** (1995), 443-551

Pierre de Fermat

Modular elliptic curves and Fermat's Last Theorem

By Andrew John Wiles*

For Nada, Claire, Kate and Olivia

Andrew John Wiles

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatum in duos ejusdem nominis fas est dividere: cujes rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

Communications in
Number Theory and Physics
Volume 12, Number 2, 193–251, 2018

Feynman integrals and iterated integrals of modular forms

Luise Adams and Stefan Weinzierl

In this paper we show that certain Feynman integrals can be expressed as linear combinations of iterated integrals of modular forms to all orders in the dimensional regularisation parameter ε. We discuss explicitly the equal mass sunrise integral and the kite integral. For both cases we give the alphabet of letters occurring in the iterated integrals. For the sunrise integral we present a compact formula, expressing this integral to all orders in ε as iterated integrals of modular forms.

Analytic solutions for multiloop massive integrals which describes scattering processes/decays goes beyound elliptic functions - how far? 

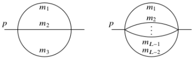


Fig. 2.4: The two-loop banana diagram (also called the sunrise or the sunset diagram, depending on the author's mood) with generic internal masses (on left) and its generalization to the $L-1$ loop integral (on right).

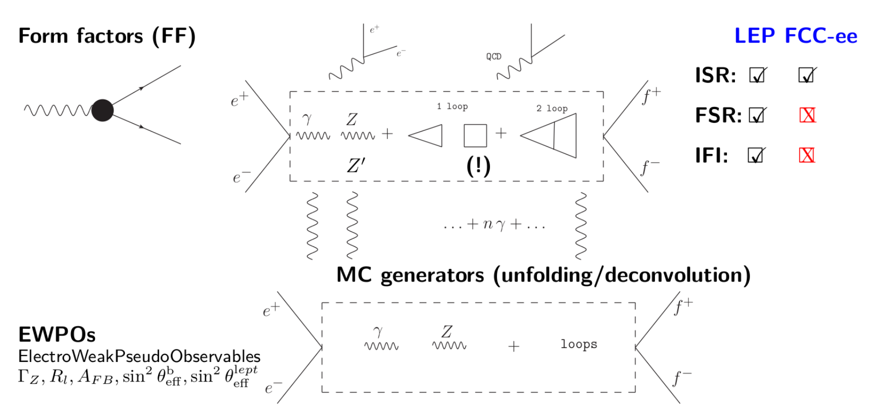
2.7 Gamma and Hypergeometric Functions, Hypergeometric Integrals

Due to their sophistication, the hypergeometric functions are more often than we think present in solutions to the exact physical problems. For instance, in classical mechanics an exact solution for the pendulum period is

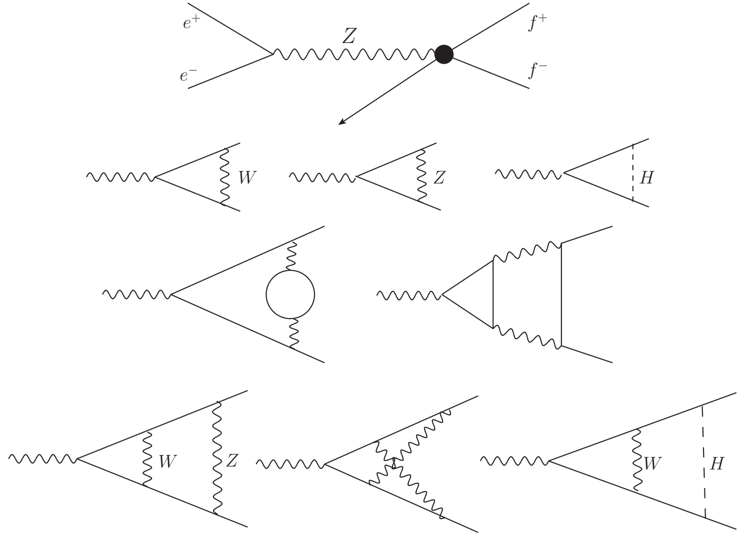
$$T = 2\pi\sqrt{\frac{l}{g}} \times {}_2F_1\left[\frac{1}{2}, \frac{1}{2}; 1; \sin^2\theta\right], \qquad (2.169)$$

and in the limit of small θ, the classical formula emerges $T = 2\pi\sqrt{\frac{l}{g}}$. For more examples, see [32].

Experimental measurements at Z-pole: after unfolding



Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections



EWPOs, Z pole

$$\sigma^0_{\rm had} = \sigma[e^+e^- \to \textsf{hadrons}]_{s=M_Z^2},$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$

$$R_\ell = \frac{\Gamma[Z \to \textsf{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau,$$

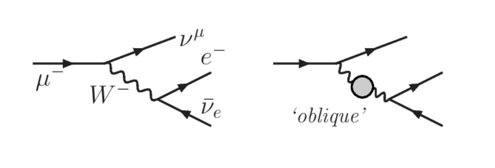
$$R_q = \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \textsf{hadrons}]}, \quad q = u, d, s, c, b.$$

The remaining EWPOs are cross section asymmetries, measured at the Z pole, e.g., forward-backward asymmetry

$$A^f_{\rm FB} = \frac{\sigma_f\left[\theta < \frac{\pi}{2}\right] - \sigma_f\left[\theta > \frac{\pi}{2}\right]}{\sigma_f\left[\theta < \frac{\pi}{2}\right] + \sigma_f\left[\theta > \frac{\pi}{2}\right]},$$

where θ is the scattering angle between the incoming e^- and the outgoing f.

Shaping SM, oblique corrections also not sufficient



$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

$$\frac{(\hat{G}_F)^{\mathrm{th}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0} = \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right].$$

Primary role of SM radiative corrections, F. Jegerlehner, in 1905.05078

$$\sin^2\Theta_i\ \cos^2\Theta_i\ = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\,\frac{1}{1-\Delta r_i}\quad \Delta r_i=\Delta r_i(\alpha,G_\mu,M_Z,m_H,m_{f\neq t},m_t)\,,$$

$$\Delta r_i=\Delta\alpha-\frac{c_W^2}{s_W^2}\,\Delta\rho+\Delta r_{i\,\mathrm{reminder}}\,,$$

$$\Delta\rho=\frac{3\,m_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}$$

$$\hat{\alpha}(m_Z)=\frac{\hat{\alpha}}{1-\Delta\alpha(m_Z)}=\frac{e^2}{4\pi}\left[1+\frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2}\right]\sim 128\ (137\ \text{at the Thomson limit})$$

Still, well visible disagreement between SM prediction and experiment for EWPOs without subleading SM corrections, and only with the leading corrections $\Delta\alpha(m_Z)$ and $\Delta\rho$.

$r_{i\,\mathrm{reminder}}$ **matters!**

How can I compute - part I ($\rightarrow$ BSM)?

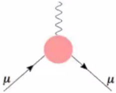
Examples
(FeynArts, FeynMast, FeynCalc, Feynrules, ...)

SM vs BSM, Compositeness - tests of hypothetical substructures

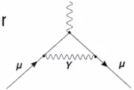
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

Dirac: $g = 2$

quantum effects



Julian Schwinger
[1948]



$$g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$

$$\alpha \approx \frac{1}{137}$$

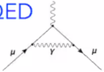
Anomalous magnetic moment:

$$a \equiv \frac{g-2}{2} = 0.00116\ldots$$

Compositeness - tests of hypothetical substructures, $(g-2)_\mu$

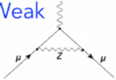
$$\boxed{a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{Weak}) + a_\mu(\mathrm{Hadronic})}$$

QED



$+\ldots$ $\quad 116\,584\,718.9\,(1) \times 10^{-11}$ $\quad$ 0.001 ppm

Weak

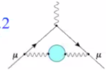


$+\ldots$ $\quad 153.6\,(1.0) \times 10^{-11}$ $\quad$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

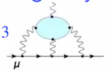
α^2



$+\ldots$ $\quad 6845\,(40) \times 10^{-11}$ [0.6%] $\quad$ 0.37 ppm

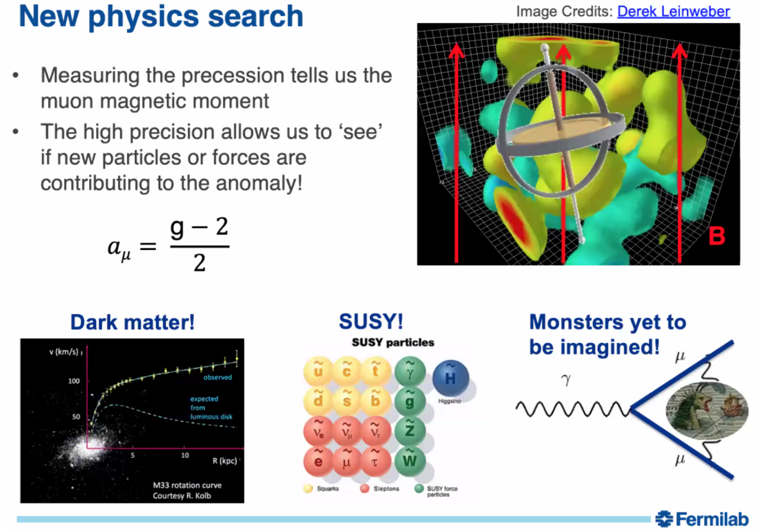
…Light-by-Light (HLbL)

α^3

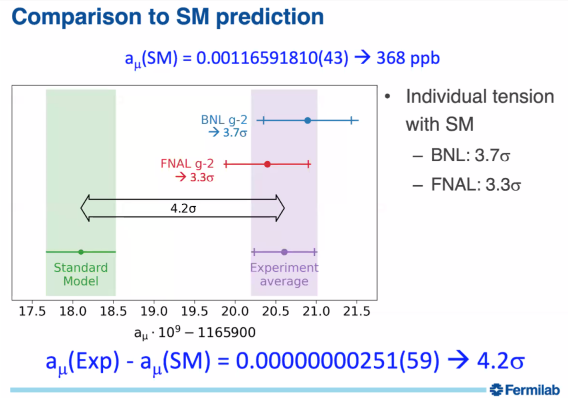


$+\ldots$ $\quad 92\,(18) \times 10^{-11}$ [20%] $\quad$ 0.15 ppm

Compositeness - vacuum structure, https://youtu.be/81PfYnpuOPA



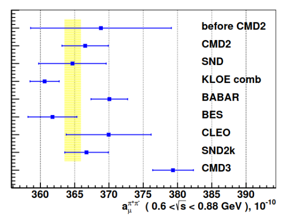
Fermilab 2021

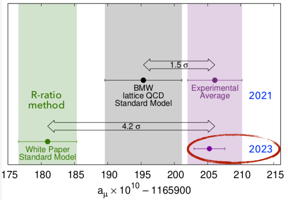


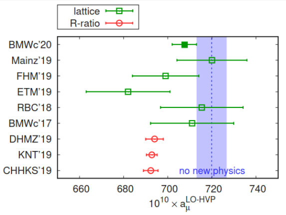
CMD3, new $\pi^+\pi^-$ results, latice QCD, smaller tensions

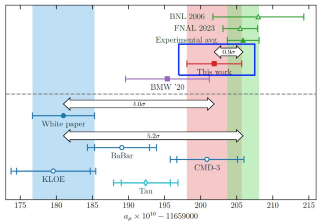
CMD3: https://arxiv.org/abs/2302.08834

"The CMD-3 result reduces the tension between the experimental value of the a_μ and its Standard Model prediction."



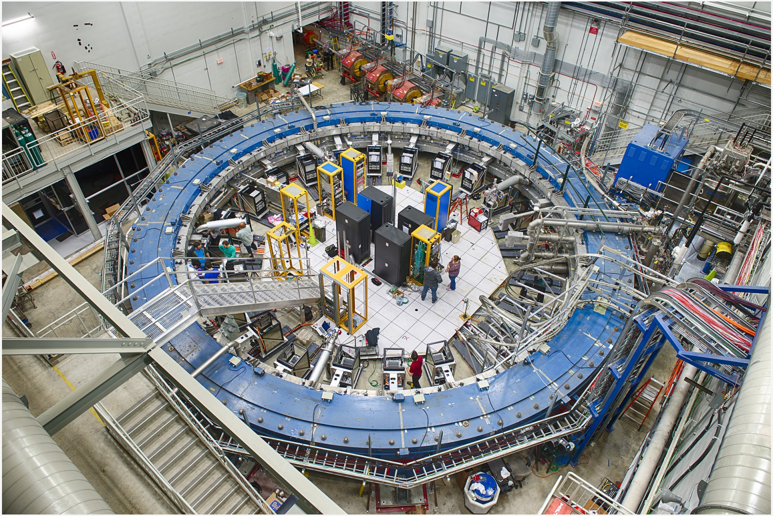






New lattice: https://arxiv.org/abs/2407.10913

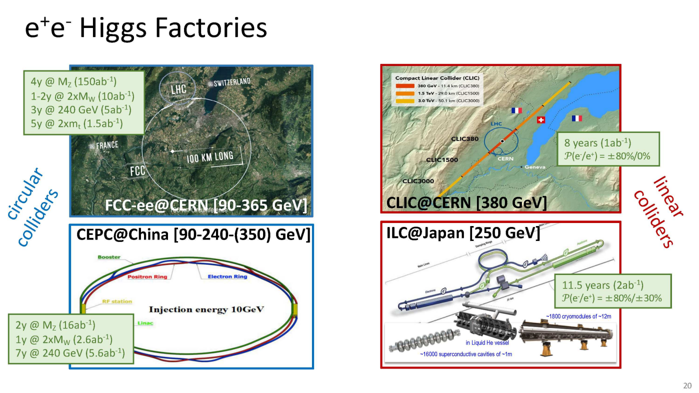
Pretty compact experiment



NEEDS FOR PRECISION: THE FUTURE

Jorgen D'Hondt, "Strategies and plans for particle physics in Europe",

Epiphany 2021, https://indico.cern.ch/event/934666



Future Circular Collider – A key to new physics

The Future Circular Collider (FCC) study is an international collaboration aimed at designing the particle accelerator that will replace the LCH once it has completed its operational lifetime. The FCC will expand the current energy and luminosity frontiers in order to help answer the most fundamental questions in science: What is dark matter? Are there extra dimensions in the universe? Are there other forces in nature?

The FCC collaboration, hosted by CERN, is open to universities, research institutes and high-tech companies. A conceptual design will be delivered before the end of 2018, in time for the next update of the European Strategy for Particle Physics.

FCC-hh – A discovery machine

The 100 TeV proton-proton collider (FHC-hh) will have an energy seven times higher than the LHC. Such a collider will give access to the smallest scales and the most energetic phenomena in nature.

New fundamental forces and particles can be discovered, extending the reach for searching dark matter particles, supersymmetric partners of quarks and gluons, and possible substructure inside quarks.

Billions of Higgs bosons and trillions of top quarks will be produced, creating new opportunities for the study of rare decays, flavor physics, and the mechanism of electroweak symmetry breaking.

The FCC-hh collider provides also the opportunity to push the exploration of the collective structure of matter at the most extreme density and temperature conditions to new frontiers through the study of heavy-ion collisions.

FCC-ee – A machine for precision

The second scenario of the FCC design study (FCC-ee) is a high-luminosity, high-precision electron-positron collider with center-of-mass collision energies between 90 and 350 GeV. Located in the same 100 km long tunnel as the FCC-hh it is considered a potential intermediate step towards the realization of the hadron facility, and complementary to it.

Clean experimental conditions give electron-positron colliders the capability to measure known particles with the highest precision.

FCC-ee would measure the properties of the Z, W, Higgs and top particles with unequalled accuracy, offering the potential for discovering dark matter or heavy neutrinos. The FCC-ee could enable profound investigations of electroweak symmetry breaking and open a broad indirect search for new physics over several orders of magnitude in energy.

FCC-he – New opportunities

With the huge energy provided by the 50 TeV proton beam and the potential availability of an electron beam with energies of the order of 60 GeV, new horizons open up for the physics of deep inelastic electron-proton scattering.

The FCC-he collider would be both a high-precision Higgs factory and a powerful microscope to discover new particles. It would be the most accurate tool for studying quark-gluon interactions, possible substructure of matter and unprecedented measurements of strong and electroweak interaction phenomena. The hadron-electron collider is a unique complement to the exploration of nature at high energies within the FCC complex.

The FCC study explores three different scenarios: a hadron-had collider (FCC-hh), an electron-positron collider (FCC-ee), an hadron-lepton (FCC-he) collider. The hadron-hadron collider def the overall infrastructure for the FCC. With a target center-of-n energy of 100 TeV, and 16-Tesla bending magnets, such a mach will have a circumference of 100 km.

Main parameters and geometrical aspects

	LHC	FCC
Circumference [km]	26.7	100
Dipole field [T]	8.33	16
Straight sections	8 × 528 m	6 ×1400 m + 2 × 4200 m
Number of IPs	2 + 2	2 + 2
Injection energy [TeV]	0.45	3.3

FCC-hh compared with LHC and High-Luminosity LH

	LHC	HL-LHC	FCC-hh baseline		FCC-hh ultim	
Energy at center of mass [TeV]	14	14	100		100	
Bunch spacing [ns]	25	25	25	5	25	
Number of bunches	2808	2808	10600	53000	10600	53
Transverse emittance [mm]	3.75	2.5	2.2	0.44	2.2	0.
Beam current [A]	0.584	1.12	0.5		0.5	
Peak luminosity [10^{34} $cm^{-2}s^{-1}$]	1.0	5.0	5.0		< 30.0	

FCC-ee compared with the Large Electron-Positron collider (LEP2)

The main center-of-mass operating points with strong physics interest for FCC-ee a 91 GeV (Z pole), 160 GeV (W pair production threshold), 240 GeV (Higgs resonanc and 350 GeV ($t\bar{t}$ threshold).

	LEP2	FCC-ee				
		Z		W	H	t
Energy at center of mass [GeV]	208	91		160	240	3
Bunch spacing [ns]	247 / 494	7.5	2.5	50	400	40
Number of bunches	4	30180	91500	5260	780	8
Emittance (horizontal) [nm]	22	0.2	0.09	0.26	0.61	1
Emittance (vertical) [pm]	250	1		1	1.2	2
Beam current [mA]	3.04	1450		152	30	6
Peak luminosity (for 2 IPs) [10^{34} $cm^{-2}s^{-1}$]	0.012	207	90	19.1	5.1	1

Contacts and further information

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EuroCirCol - Prof. Carsten P. Welsc
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http://fcc.web.cern.ch

http://www.eurocircol.eu

This project has received funding from the European Union's Horiz research and innovation programme under grant No 654305. The info herein reflects only the views of its authors and the Research Executive is not responsible for any use that may be made of the information con

To get to the experimental precision, we must improve very much!

Expected precision in 2040

J. Gluza

- **Conclusion of the 2018 Workshop**

> "We anticipate that, at the beginning of the FCC-ee campaign of precision measurements, the theory will be precise enough not to limit their physics interpretation. This statement is however conditional to sufficiently strong support by the physics community and the funding agencies, including strong training programmes".

- **Numerical evaluation with three-loops calculations:**

arXiv:1901.02648

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta \sin^{2,l}_{eff}\theta$ $[10^{-6}]$
Present EWPO theoretical uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO theoretical uncertainties when FCC-ee will start				
EXP-FCC-ee	~~0.1~~ 0.025	10	$2 \div 6$	~~6~~ 3
TH-FCC-ee	0.07	7	3	7

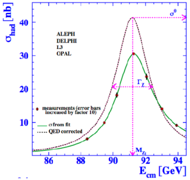
$0.5 \to 0.4$

Five years!

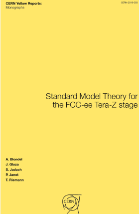
- 500 person-years needed over 20 years – Recognized as strategic priority.

Patrick Janot | FCC Week, Brussels 28 June 2019 | 23

- Cross section : Z mass and width



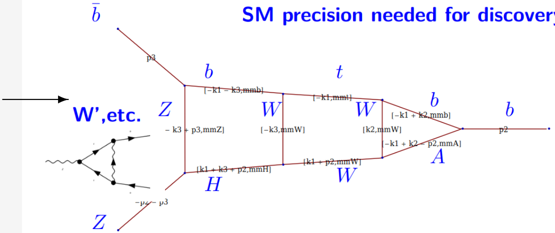
- ~30% QED corrections (ISR)



$M_Z = 91$ GeV

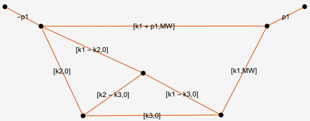
Becoming narrow resonance!

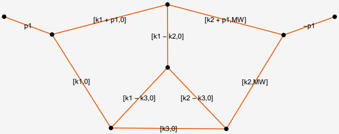
SM precision needed for discovery studies (indirect effects)



Four scales :

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{s+i\varepsilon}{M_Z^2} \right\}$$





1-dim

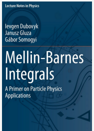
$$-18.779406962 - 6.390785027i$$

4-dim

$$-22.5213 + 4.74442i \pm (0.001 + 0.001i)$$

$$I = -\frac{1}{(-s)^{1+3\epsilon}} \int_{-i\infty}^{+i\infty} \prod_{i=1}^{4} dz_i \left(-\frac{M_W^2}{s}\right)^{z_3} \frac{\Gamma(-\epsilon - z_1)\Gamma(-z_1)\Gamma(1+2\epsilon+z_1)}{\Gamma(1-2\epsilon)\Gamma(1-3\epsilon-z_1)}$$

$$\times \frac{\Gamma(-2\epsilon - z_{12})\Gamma(1-\epsilon+z_2)\Gamma(1+z_{12})\Gamma(1+\epsilon+z_{12})\Gamma(1+3\epsilon+z_3)\Gamma(1-\epsilon-z_4)}{\Gamma(1-z_2)\Gamma(2+\epsilon+z_{12})} \ldots$$



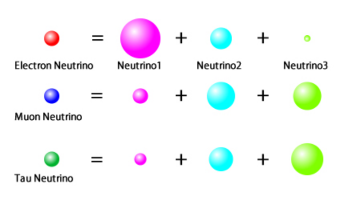
+ differential equations, numerical methods, ...

SM and BSM: NEUTRINOS*

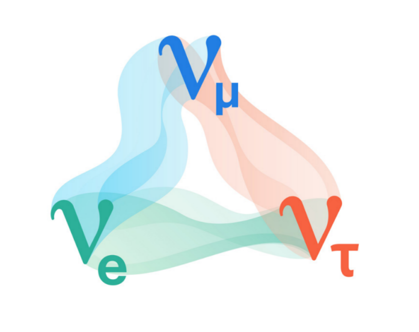
* Neutrino physics itself enters the precision era (mass ordering, C-nature, CP phases).

The Number 3 Stays with Us For Long: Neutrino Oscillations

$$\nu_\alpha^{(f)} = (U_{\texttt{PMNS}})_{\alpha i}\nu_i^{(m)}$$



Neutrino oscillations



Mixing matrix

$$U_{\texttt{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

source: http://www.hyper-k.org/en/index.html; https://neutrinos.fnal.gov

Present main Issues

• **Neutrino Mixing:** Flavor eigenstates and mass eigenstates are related, Pontecorvo-Maki-Nakagawa-Sakata parametrization

$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$.

• **Neutrino mixing parameters:** the known and unknowns

Three mixing angles: θ_{12}, θ_{23} and θ_{13}

Dirac CP-violating phase: δ_{CP}

Two mass squared differences: $\Delta m^2_{\odot} = m_2^2 - m_1^2$, $\Delta m^2_A = |m_3^2 - m_1^2|$

6 parameters involved in neutrino oscillation, still ambiguity over 3 parameters

Two Majorana phase: α_{21} and α_{31} (Not sensitive to oscillation experiments)

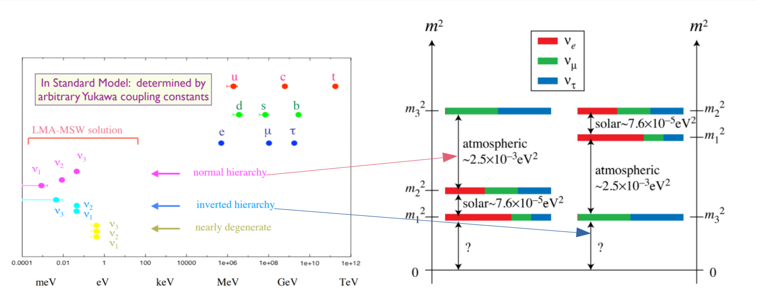
Current main questions in neutrino oscillation physics:

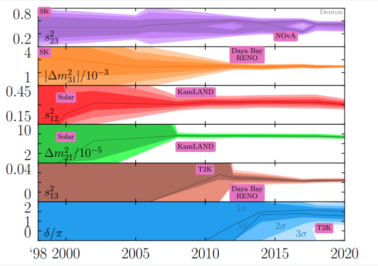
(i) what is the mass ordering of the neutrinos (i.e sign of $|\Delta m^2_{32(1)}|$

(ii) what is the octant of θ_{23}

(iii) is CP symmetry violated in the leptonic sector?

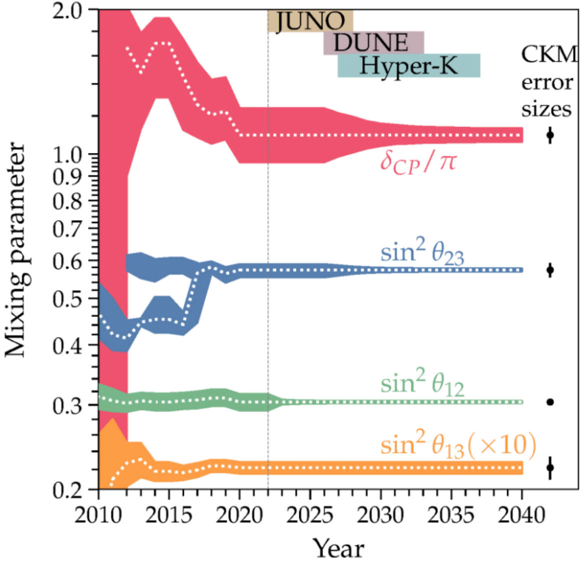
Neutrino parameters and the known unknowns





	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

Neutrino parameters and the known unknowns: 'Big' Data



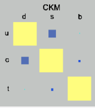
courtesy of Shirley Li

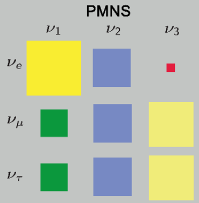
Neutrino Physics Enters Precisoin Era

Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, Dune, SNO+, Daya Bay, Double Chooz, RENO, ...

$$U_{PMNS} = \begin{pmatrix} \{0.810, 0.829\} & \{0.539, 0.562\} & \{0.147, 0.169\} \\ \{-0.485, -0.479\} & \{0.467, 0.563\} & \{0.669, 0.743\} \\ \{0.278, 0.339\} & \{-0.683, -0.626\} & \{0.647, 0.728\} \end{pmatrix}$$

$$\theta_{12} = 33.9^o \pm 1.0^o$$
$$\theta_{23} = 36^o - 54^o$$
$$\theta_{13} = 9.12^o \pm 0.63^o$$
$$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ [eV^2]$$
$$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ [eV^2]$$

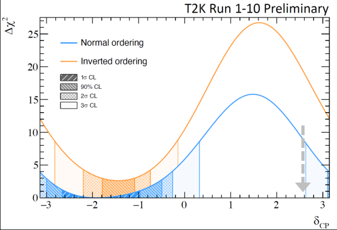


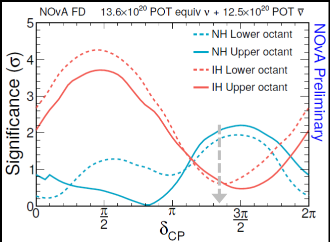


Conclusion: Neutrino Physics stepped in the precision era.

Till 2030: mass hierarchy, δ_{CP} (maybe), absolute masses, Majorana-Dirac, L. Wen, EPS2021.

The CP Phase



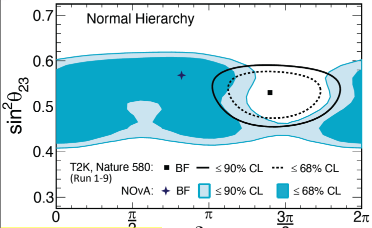


T2K

- $\delta = -\pi/2$ favored
- Large range of values of δ_{CP} around $+\pi/2$ are excluded at 99.7%

NOvA

- Best-fit $\delta = 0.82\,\pi$
- Exclude **IH** $\delta = \pi/2$ at $>3\sigma$
- Disfavor NH $\delta = 3\pi/2$ at ~2σ



Clear tension exists

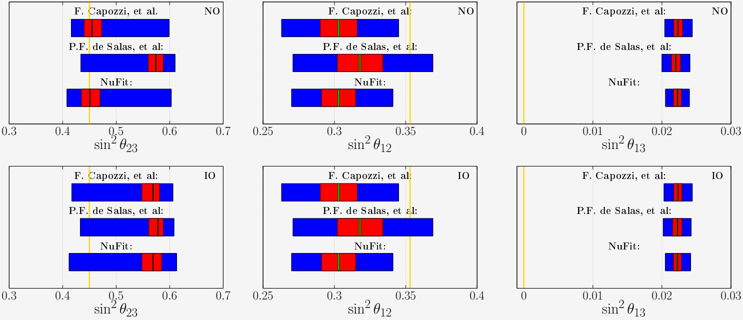
NOvA + T2K joint analysis is underway

Alex Himmel @ Neutrino 2020

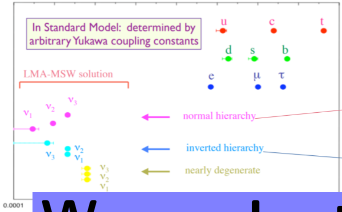
8

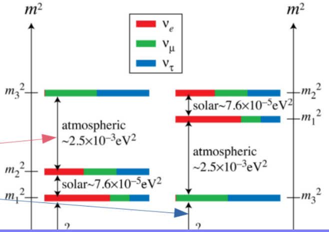
'Big' Data

Parameter	Ordering	NuFit 5.2		de Salas et al.		Capozzi et al.	
		bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range
$\sin^2\theta_{12}/10^{-1}$	NO, IO	$3.03^{+0.12}_{-0.12}$	$2.70-3.41$	$3.18^{+0.16}_{-0.16}$	$2.71-3.69$	$3.03^{+0.13}_{-0.13}$	$2.63-3.45$
$\sin^2\theta_{23}/10^{-1}$	NO	$4.51^{+0.19}_{-0.16}$	$4.08-6.03$	$5.74^{+0.14}_{-0.14}$	$4.34-6.10$	$4.55^{+0.18}_{-0.15}$	$4.16-5.99$
	IO	$5.69^{+0.16}_{-0.21}$	$4.12-6.13$	$5.78^{+0.10}_{-0.17}$	$4.33-6.08$	$5.69^{+0.12}_{-0.21}$	$4.17-6.06$
$\sin^2\theta_{13}/10^{-2}$	NO	$2.225^{+0.056}_{-0.059}$	$2.052-2.398$	$2.200^{+0.069}_{-0.062}$	$2.000-2.405$	$2.23^{+0.07}_{-0.06}$	$2.04-2.44$
	IO	$2.223^{+0.058}_{-0.058}$	$2.048-2.416$	$2.225^{+0.064}_{-0.070}$	$2.018-2.424$	$2.23^{+0.06}_{-0.06}$	$2.03-2.45$
δ/π	NO	$1.29^{+0.20}_{-0.14}$	$0.80-1.94$	$1.08^{+0.13}_{-0.12}$	$0.71-1.99$	$1.24^{+0.18}_{-0.13}$	$0.77-1.97$
	IO	$1.53^{+0.12}_{-0.16}$	$1.08-1.91$	$1.58^{+0.15}_{-0.16}$	$1.11-1.96$	$1.52^{+0.15}_{-0.11}$	$1.07-1.90$
$\Delta m^2_{21}/10^{-5}\text{eV}^2$	NO, IO	$7.41^{+0.21}_{-0.20}$	$6.82-8.03$	$7.50^{+0.22}_{-0.20}$	$6.94-8.14$	$7.36^{+0.16}_{-0.15}$	$6.93-7.93$
$\lvert\Delta m^2_{atm}\rvert/10^{-3}\text{eV}^2$	NO	$2.507^{+0.026}_{-0.027}$	$2.427-2.590$	$2.55^{+0.02}_{-0.03}$	$2.47-2.63$	$2.485^{+0.023}_{-0.031}$	$2.401-2.565$
	IO	$2.486^{+0.028}_{-0.025}$	$2.406-2.570$	$2.45^{+0.02}_{-0.03}$	$2.37-2.53$	$2.455^{+0.030}_{-0.025}$	$2.376-2.541$

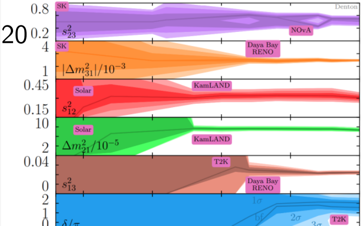


Neutrino parameters and the known unknowns



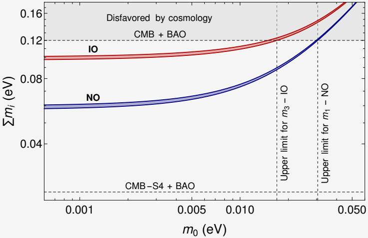


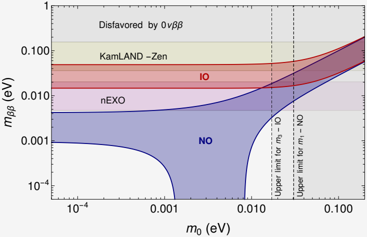
We need a theory of flavor!!



	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$

Neutrino Mass : Cosmology to $0\nu\beta\beta$





- Absolute neutrino mass : $m_\nu^2 < 0.9$ eV^2 (The KATRIN Collaboration 2022)

Determination of U_{PMNS} entries

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & \mathbf{U_{e3}} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Appearence/Disappearence, SBL/LBL experiments sensitive to different U_{PMNS} entries or their combinations.

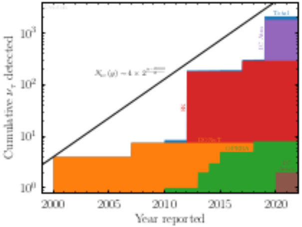
- E.g., $\mathbf{U_{e3}}$ - Daya Bay ($\bar{\nu}_e$ disappearence).
- Least knowledge about the τ entries.

Unitarity violation: tau row

Leptons: tau row is the weakest

1. Existing global analyses use OPERA and SNO
2. More data from atmospheric ν_τ appearance!

PBD 2109.14576



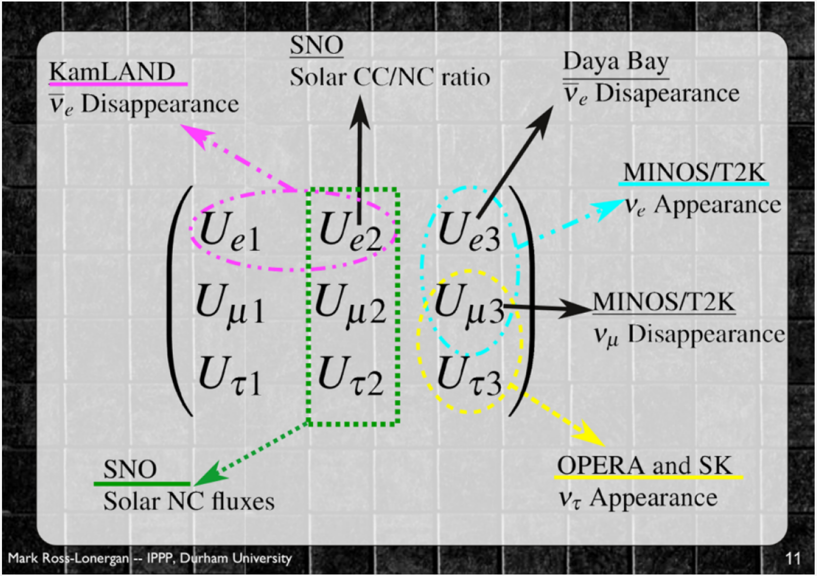
Also astrophysical ν_τ appearance; weak but distinct!

PBD, J. Gehrlein 2109.14575

Works because τ is in direct region

Tau neutrino data set doubles every two years

PBD, et al. 2203.05591 (whitepaper)


KamLAND
$\bar{\nu}_e$ Disappearance
SNO
Solar CC/NC ratio
Daya Bay
$\bar{\nu}_e$ Disapearance
MINOS/T2K
ν_e Appearance
MINOS/T2K
ν_μ Disappearance
U_{e1} U_{e2} U_{e3}
$U_{\mu 1}$ $U_{\mu 2}$ $U_{\mu 3}$
$U_{\tau 1}$ $U_{\tau 2}$ $U_{\tau 3}$
SNO
Solar NC fluxes
OPERA and SK
ν_τ Appearance
Mark Ross-Lonergan -- IPPP, Durham University
11

Setting

Experimental values of mixing parameters

$$\theta_{12} \in [31.61^\circ, 36.27^\circ], \quad \theta_{23} \in [41.1^\circ, 51.3^\circ],$$
$$\theta_{13} \in [8.22^\circ, 8.98^\circ], \quad \delta \in [144^\circ, 357^\circ]$$

Interval matrix build up from unitary matrices U_{PMNS} (3σ C.L.)

$$|U|_{int} = \begin{pmatrix} [0.797, 0.842] & [0.518, 0.585] & [0.143, 0.156] \\ [0.243, 0.490] & [0.473, 0.674] & [0.651, 0.772] \\ [0.295, 0.525] & [0.493, 0.688] & [0.618, 0.744] \end{pmatrix}$$

includes non-unitary matrices. $\delta \neq 0$: complex intervals, $\longrightarrow U_{int}$

- **Can we get from $|U|_{int}$ an additional information on existence and structure of hypothetical $N_\nu > 3$ states?**
- We explore U_{int} from matrix theory perspective.

Heavy neutrinos: see-saw type-I, type-II, type-III

Seesaw I: right handed singlets

$$
\begin{aligned}
\mathcal{L}_{\mathsf{Y}} &= -Y_{ij}\,\overline{L'_{iL}}N'_{jR}\,\tilde{\phi} + \text{H.c.} \\
\mathcal{L}_{\mathsf{M}} &= -\frac{1}{2}M_{ij}\overline{N'_{iL}}N'_{jR} + mH.c.\,, \\
\mathcal{L}_{\text{mass}} &= -\frac{1}{2}\,(\bar{\nu}'_L\ \bar{N}'_L)\begin{pmatrix} 0 & \frac{v}{\sqrt{2}}Y \\ \frac{v}{\sqrt{2}}Y^T & M \end{pmatrix}\begin{pmatrix} \nu'_R \\ N'_R \end{pmatrix} + \text{H.c.}\,.
\end{aligned}
$$

The neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R(v_R) \end{pmatrix}$$

with $M_D \ll M_R$.

$$
\begin{aligned}
m_N &\sim M_R \\
m_{\text{light}} &\sim M_D^2/M_R
\end{aligned}
$$

$M_D \sim \mathcal{O}(1)$ GeV $\rightarrow \mathrm{M_R} \sim 10^{15}$ GeV, if light neutrino masses of the order of 0.1 eV.

CP phases, complex mixing elements

$$\begin{pmatrix} \nu^{(f)}{}_\alpha \\ \widetilde{\nu}^{(f)}{}_\beta \end{pmatrix} = \begin{pmatrix} U_{\rm PMNS} & V_{lh} \\ V_{hl} & V_{hh} \end{pmatrix} \begin{pmatrix} i \\ \widetilde{\nu}_j^{(m)} \end{pmatrix} \equiv \mathcal{U} \begin{pmatrix} i \\ \widetilde{\nu}^{(m)}{}_j \end{pmatrix} .$$

The SM flavor states $\nu^{(f)}{}_\alpha$ are then given by

$$\nu^{(f)}{}_\alpha = \sum_{i=1}^{3} \underbrace{(U_{\rm PMNS})_{\alpha i}\, i}_{\rm SM\ part} + \sum_{j=1}^{n_R} \underbrace{(V_{lh})_{\alpha j} \widetilde{\nu}^{(m)}{}_j}_{\rm BSM\ part} .$$

The mixing matrix $\mathcal{U}$ in (75) diagonalizes a general neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix},$$

using a congruence transformation $\mathcal{U}^T M_\nu \mathcal{U} \simeq \mathrm{diag}(m_i, M_j)$

$$\begin{aligned} U_{\rm PMNS} &= U(\theta_{23})U(\theta_{13},\delta_{\rm CP})U(\theta_{12})U_M(\alpha_1,\alpha_2) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\times \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Non-unitary Matrices and a Notion of Contractions

$$\|A\| \leq 1$$

Operator norm (spectral norm)

$$\|A\| := \sup_{\|x\|=1} \|Ax\| = \sigma_{\max}(A)$$

E.g. A singular value of a real matrix A is the positive square root of an eigenvalue of the symmetric matrix AAT or AT A.

Contractions as submatrices of the unitary matrix

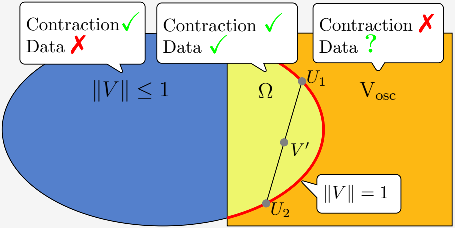
$$\textbf{If} \quad UU^\dagger = 1 \Longrightarrow \left\| \begin{pmatrix} U_{3\times 3} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} \right\| = 1 \Longrightarrow \|U_{3\times 3}\| \leq 1.$$

PRD'2018.

Contractions allow us to determine the set of physically admissible mixing matrices $\Omega \subset U_{int}$

(I) U_{int} and the Physical Region of Mixing (Convex Hull of U_{PMNS})

$$\Omega := \mathrm{conv}(U_{\mathtt{PMNS}}) = \{\sum_{i=1}^{m} \alpha_i U_i \mid U_i \in U(3), \alpha_1, ..., \alpha_m \geq 0, \sum_{i=1}^{m} \alpha_i = 1,$$
$$\theta_{12}, \theta_{13}, \theta_{23} \text{ and } \delta \text{ given by experimental values}\}$$



We proved that the Carathéodory's number is $\mathbf{m} \leq 4$, instead of 10(19) for CP ($\not{CP}$) cases, e.g., for the 3+1 scenario, two U_{PMNS} matrices are enough to span the corresponding subset of Ω region.

Fig. from PRD2018, $V_{osc} \equiv U_{int}$

(II) Physical Region Can Be Divided into Non-Overlapping Subregions !

Not all $U_{3\times 3}$ entries known well (precision) - - hard to avoid analysis based on Euler angles.
Nonetheless, we can use the knowledge of Ω differently.

Ω is divided into four disjoint subsets by *singular values* (PRD2018)

Ω_1 : 3+**1** scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \boldsymbol{\sigma_3 < 1.0}\}$,

Ω_2 : 3+**2** scenario: $\Sigma = \{\sigma_1 = 1.0, \boldsymbol{\sigma_2 < 1.0}, \boldsymbol{\sigma_3 < 1.0}\}$,

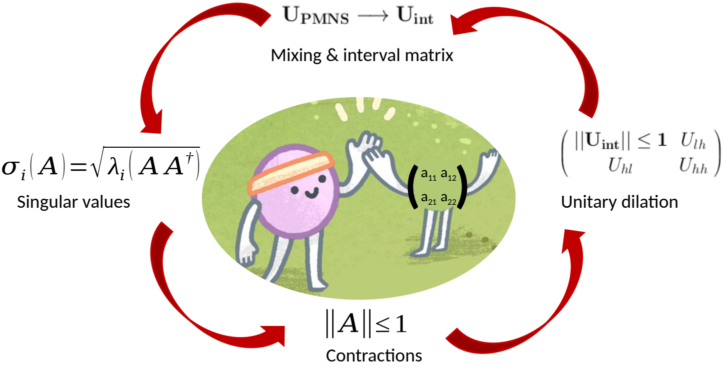
Ω_3 : 3+**3** scenario: $\Sigma = \{\boldsymbol{\sigma_1 < 1.0}, \boldsymbol{\sigma_2 < 1.0}, \boldsymbol{\sigma_3 < 1.0}\}$,

Ω_4 : PMNS scenario: $\Sigma = \{\sigma_1 = 1, \sigma_2 = 1, \sigma_3 = 1\}$.

$$\sigma_i(A) = \sqrt{\lambda_i(AA^\dagger)}$$

The connection between $\Sigma = (\sigma_1, \sigma_2, \sigma_3)$ and $3 + N$ scenarios, with N additional νs, goes by the *dilation* procedure.

Matrix Theory and Neutrinos: Summary



source (inside picture): https://www.symmetrymagazine.org

Questions, based on the knowledge of U_{int}

Q1 How much space do we have for the additional neutrinos and how quantify it within our approach?

Q2 Can we distinguish between $\Omega_1 - \Omega_3$ (3+n models) using U_{int}?

Q3 Can we estimate active-sterile mixing using singular values and U_{int}?

Q3: Can We Estimate Active-Sterile Mixing Using Singular Values and U_{int}?

Ω_1 : 3+1 scenario: $\Sigma = \{\sigma_1 = 1.0, \sigma_2 = 1.0, \sigma_3 < 1.0\}$

$$\begin{pmatrix} \not{U}_{\mathtt{PMNS}} & U_{lh} \\ U_{hl} & U_{hh} \end{pmatrix} = \begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c & -s \\ \hline 0 & 0 & s & c \end{array}\right) \begin{pmatrix} Q_1^\dagger & 0 \\ 0 & Q_2^\dagger \end{pmatrix}.$$

We are interested in the estimation of the light-heavy mixing sector which is given by

$$U_{lh} = W_1 S_{12} Q_2^\dagger,$$

where $W_1 \in \mathbb{C}^{3\times 3}$ is unitary, $S_{12} = (0, 0, -s)^T$ and $Q_2 = e^{i\theta}, \theta \in (0, 2\pi]$. Taking into account exact values of the W_1 we can estimate the light-heavy mixing by the analytical formula

$$|U_{i4}| = |w_{i3}| \cdot |\sqrt{1 - \sigma_3^2}|, \quad i = e, \mu, \tau.$$

PMNS data analysis (Nonunitarity), 3+1

New limits on neutrino non-unitary mixings based on prescribed singular values,
W. Flieger, JG, K. Porwit, JHEP 03 (2020) 169

- (I): $m > EW$.

  $$Ours: |U_{e4}| \in [0, 0.021]\,, \quad |U_{\mu 4}| \in [0.00013, 0.021]\,, \quad |U_{\tau 4}| \in [0.0115, 0.075]\,.$$
  $$Others: |U_{e4}| \leq 0.041\;, \quad |U_{\mu 4}| \leq 0.030\;, \quad |U_{\tau 4}| \leq 0.087$$ [J. de Blas, 2013]

- (II): $\Delta m^2 \gtrsim 100 eV^2$.

  $$Ours: |U_{e4}| \in [0, 0.082]\,, \quad |U_{\mu 4}| \in [0.00052, 0.099]\,, \quad |U_{\tau 4}| \in [0.0365, 0.44]\,.$$

- (III): $\Delta m^2 \sim 0.1 - 1 eV^2$.

  $$Ours: |U_{e4}| \in [0, 0.130]\,, \quad |U_{\mu 4}| \in [0.00052, 0.167]\,, \quad |U_{\tau 4}| \in [0.0365, 0.436]\,.$$
  $$Others: |U_{e4}| \in [0.114, 0.167]\;, \quad |U_{\mu 4}| \in [0.0911, 0.148]\;, \quad |U_{\tau 4}| \leq 0.361\;.$$
  [C. Giunti et al., 2017] [M. Dantler et al., 2018]

$\longrightarrow$ In some cases we improved (blue), in some not (red).

e^+e^-, example

Alain Blondel, André de Gouvêa, Boris Kayser, 2105.06576

$$B(Z \to \nu_4 \nu_{\rm light}) = 2|U_4|^2 \frac{B(Z \to \text{invisible})}{3}\left(1+\frac{m_4^2}{2M_Z^2}\right)\left(1-\frac{m_4^2}{M_Z^2}\right)^2; \quad \sum_{\alpha=e,\mu,\tau}|U_{\alpha 4}|^2 \equiv |U_4|^2,$$

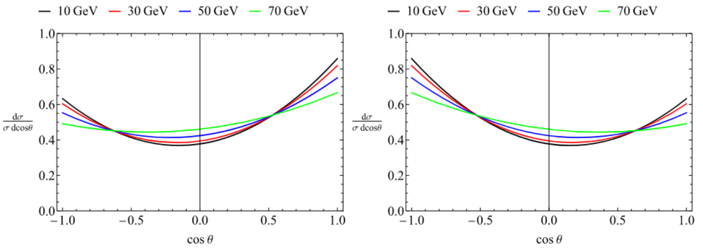


FIG. 1. Normalized differential cross-section for $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ (left) and $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ (right) as a function of the direction of the heavy (anti)neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Dirac fermions.

"We estimate semiquantitatively that around 400 events are required to establish the Majorana or Dirac nature of the heavy neutrinos using the potential forward-backward asymmetry alone"

N_{eff}: LEP and Now

ALEPH, OPAL, L3, DELPHI, MARKII (SLC): $N_\nu = 3.12 \pm 0.19$
CERN, 13.10.1989, Video (∼12,000 Z decays)
[LEP, 2006] (∼17 mln Z decays)

$$N_\nu = 2.9840 \pm 0.0082$$

Update: [P. Janot and S. Jadach, 2019](only 1σ off from N=3)

$$N_\nu = 2.9963 \pm 0.0074$$

Theorem: [C. Jarlskog, 1990]
In the Standard Model with n left-handed lepton doublets and $N - n$ right-handed neutrinos, the effective number of neutrinos, N_ν, defined by

$$\Gamma(Z \to \nu' s) \equiv N_\nu \Gamma_0,$$

where Γ_0 is the standard width for one masseless neutrino, satisfies

$$N_\nu \leq n.$$

$$\nu_\alpha^{(f)} = \underbrace{(\mathrm{V}_{\mathrm{osc}})_{\alpha i}\, \nu_i^{(m)}}_{\text{SM part}} + \underbrace{(\mathrm{V}_{lh})_{\alpha j}\, \widetilde{\nu}_j^{(m)}}_{\text{BSM part}}$$

Cosmology: $N_{eff} = 3.044$. J. Froustey, C. Pitrou, M. Volpe, JCAP 12 (2020) 015,
J. Bennett, G. Buldgen, M. Drewes, Y. Wong, JCAP 03 (2020) 003, JCAP 03 (2021) A01

How Light and Heavy Masses (Eigenvalues) Influence Active-Sterile Mixings (Eigenvectors)?

$$M_{SS} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} + \begin{pmatrix} 0 & M_D \\ M_D^T & 0 \end{pmatrix} \equiv \mathcal{M}_R + \mathcal{M}_D,$$

$\begin{pmatrix} 100 & -95 \\ -95 & 90 \end{pmatrix} \to \begin{matrix} \lambda_1 = 190.131 \\ \lambda_2 = -0.131 \end{matrix}$ When 3 light νs? SS-I, II, III, ESS, ISS, LSS

- we can rearange to the same structure, W. Flieger, JG, Chin.Phys.C 45 (2021) 2, 023106

$$|m_D| \ll \lambda(M_R),\ \lambda(M_{SS}) \simeq \lambda(\mathcal{M}_R) \pm |_D|$$

A relation between light and heavy masses and their mixings

$$\|\sin\Theta(V_{light}, V_{heavy}^{'})\| \leq \frac{1}{\delta}\|M_{SS} - \mathcal{M}_R\| = \frac{1}{\delta}\|\mathcal{M}_D\|,$$

$\delta = min(M_{N_i}) - max(m_{\nu_j})$

P. Denton et al, Bull.Am.Math.Soc. 59 (2022) 1

$$|v_{i,j}|^2 \prod_{k=1;k\neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) \ .$$

RHNs at hadron collider, 1983

VOLUME 50, NUMBER 19 PHYSICAL REVIEW LETTERS 9 MAY 1983

Majorana Neutrinos and the Production of the Right-Handed Charged Gauge Boson

Wai-Yee Keung and Goran Senjanović
Physics Department, Brookhaven National Laboratory, Upton, New York 11973

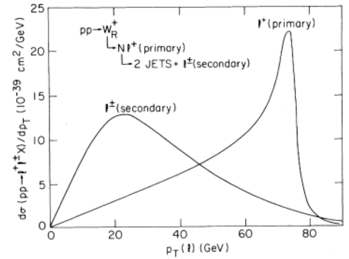
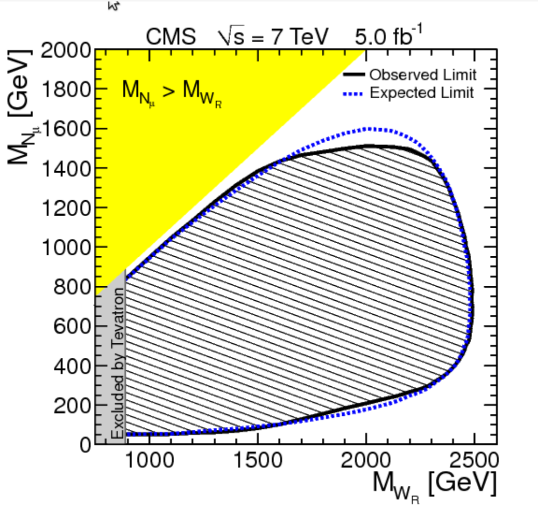


FIG. 2. Transverse momentum distributions of the primary and secondary leptons from W_R production for pp collision at $\sqrt{s} = 800$ GeV. The case $M_R = 200$ GeV, $m_N = 100$ GeV, and $\sin^2\theta_W = 0.25$ is illustrated.

$$pp \to l^{\pm} l^{\pm} jj$$

M_{W_2} and M_N



CMS, arXiv:1210.2402

PDG 2012

A REVIEW GOES HERE – Check our WWW List of Reviews

MASS LIMITS for *W′* (Heavy Charged Vector Boson Other Than *W*) in Hadron Collider Experiments

Couplings of W' to quarks and leptons are taken to be identical with those
The following limits are obtained from $p\overline{p} \to W'X$ with W' decaying to the
indicated in the comments. New decay channels (*e.g.*, $W' \to WZ$) are assun
be suppressed. The most recent preliminary results can be found in the "W'-
searches" review above.

VALUE (GeV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
>2150	95	AAD	11Q	ATLS	$W' \to e\nu,\ \mu\nu$
none 180–690	95	[1] ABAZOV	11H	D0	$W' \to WZ$
> 863	95	[2] ABAZOV	11L	D0	$W' \to tb$
>1510	95	CHATRCHYAN	11Y	CMS	$W' \to q\overline{q}$

HTTP://PDG.LBL.GOV Page 1 Created: 6/18/2012

Note, differcence with low energy

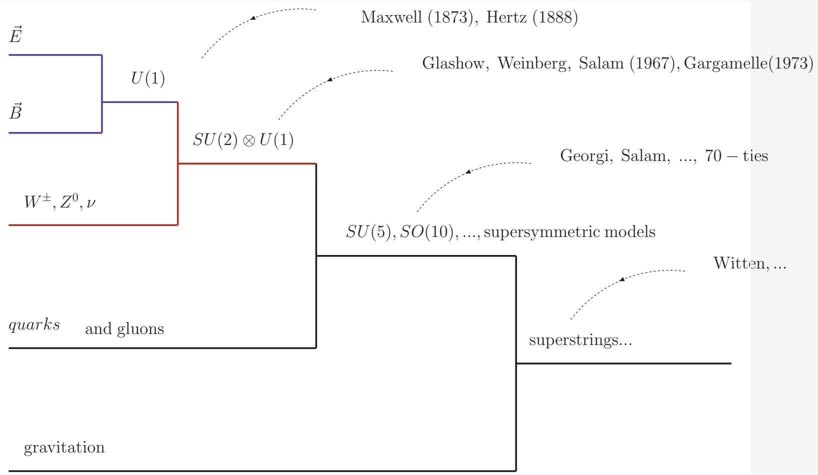
W_R (Right-Handed W Boson) MASS LIMITS

Assuming a light right-handed neutrino, except for BEALL 82, LANGACKER 8[illegible] and COLANGELO 91. $g_R = g_L$ assumed. [Limits in the section MASS LIMITS W' below are also valid for W_R if $m_{\nu_R} \ll m_{W_R}$.] Some limits assume manif left-right symmetry, *i.e.*, the equality of left- and right Cabibbo-Kobayashi-Maska matrices. For a comprehensive review, see LANGACKER 89B. Limits on the W_L-V mixing angle ζ are found in the next section. Values in brackets are from cosmologi and astrophysical considerations and assume a light right-handed neutrino.

VALUE (GeV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
> 715	90	[18] CZAKON	99	RVUE	Electroweak
• • • We do not use the following data for averages, fits, limits, etc. • • •					
> 245	90	[19] WAUTERS	10	CNTR	^{60}Co β decay
> 180	90	[20] MELCONIAN	07	CNTR	^{37}K β^+ decay
> 290.7	90	[21] SCHUMANN	07	CNTR	Polarized neutron decay
[> 3300]	95	[22] CYBURT	05	COSM	Nucleosynthesis; light ν_R
> 310	90	[23] THOMAS	01	CNTR	β^+ decay
> 137	95	[24] ACKERSTAFF	99D	OPAL	τ decay

Looking for more (gauge) symmetries

Simple picture again?



Simple picture again?

Start: 1973-1974,

Pati, Salam, Senjanovic, Mohapatra

gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

(i) restores left-right symmetry to e-w interactions

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \qquad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

(ii) hypercharge interpreted as a difference of baryon and lepton numbers

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$\begin{matrix} W_L^\pm, W_L^0 & & W_1^\pm, W_2^\pm \\ W_R^\pm, W_R^0 & \to [SSB?] & Z_1, Z_2 \\ B^0 & & \gamma \end{matrix}$$

however, when going into details...

breaking chains $G \to G^{(1)} \to G^{(2)} \ldots \to G^{(n)} \to G_{SM}$

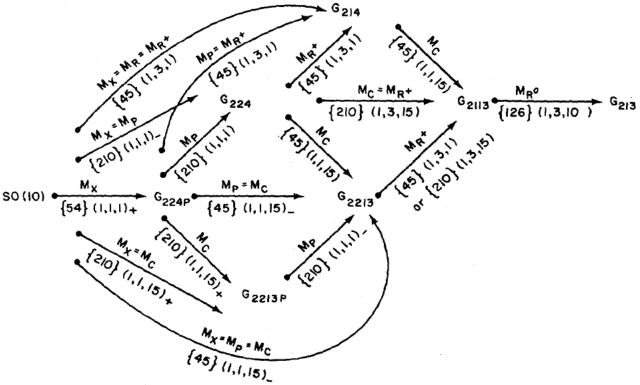
TABLE I. – *E_6 and its subgroups which contain G_{SM}. Here we use the inclusion relation* $SO(7) \supset SU(4) \supset SU(3) \times U(1)$.

E_6	F_4 $SO(10) \times U(1)$ $SU(2) \times SU(6)$ $SU(3) \times SU(3) \times SU(3)$
F_4	$SO(9)$ $SU(3) \times SU(3)$
$SO(9)$	$SU(2) \times SU(4)$
$SO(10)$	$SU(5) \times U(1)$ $SU(2) \times SU(2) \times SU(4)$ $SU(2) \times SO(7)$
$SU(6)$	$SU(5) \times U(1)$ $SU(2) \times U(1) \times SU(4)$ $SU(3) \times SU(3) \times U(1)$
$SU(5)$	$SU(3) \times SU(2) \times U(1)$

Extra gauge bosons

TABLE II. – *Group hierarchies which allow unification. Here the dots indicate that the hierarchy chains break directly into G_{SM} and $G_{\mathrm{LR}} = SU(3) \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)$ indicates the left-right–symmetric gauge group.*

E_6	G_{LR}	$\rightarrow \ldots$	$\rightarrow \ldots$	
	$SU(2) \times SU(2) \times SU(4)$	$\rightarrow \ldots$	$\rightarrow \ldots$	
		G_{LR}	$\rightarrow \ldots$	
	$SO(10) \times U(1)$	$SU(2) \times SU(2) \times SU(4) \times U(1)$	$\rightarrow \ldots$	G_{SM}
			G_{LR}	
		G_{LR}	$\rightarrow \ldots$	
	$SU(2) \times SU(6)$	$SU(2) \times SU(3) \times SU(3) \times U(1)$	G_{LR}	
		$SU(2) \times SU(2) \times SU(4) \times U(1)$	$\rightarrow \ldots$	
			G_{LR}	
		G_{LR}	$\rightarrow \ldots$	
	$SU(3) \times SU(3) \times SU(3)$	G_{LR}	$\rightarrow \ldots$	
$SO(10)$	$SU(2) \times SU(2) \times SU(4)$	$\rightarrow \ldots$	$\rightarrow \ldots$	
		G_{LR}	$\rightarrow \ldots$	
	G_{LR}	$\rightarrow \ldots$	$\rightarrow \ldots$	



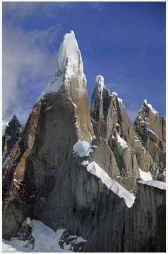
Diagrammatic sketch of 18 symmetry-breaking chains in SO(10).

Chang et al, PRD31, 1718 (1985)

Deshpande, Gunion, Kayser, Olness, 1991

$$
\begin{aligned}
&\mathcal{L}_{Higgs} = \\
&-\mu_1^2 Tr[\Phi^\dagger\Phi] - \mu_2^2(Tr[\tilde{\Phi}\Phi^\dagger] + Tr[\tilde{\Phi}^\dagger\Phi]) - \mu_3^2(Tr[\Delta_L\Delta_L^\dagger] + Tr[\Delta_R\Delta_R^\dagger]) \\
&+\lambda_1 Tr[\Phi\Phi^\dagger]^2 + \lambda_2(Tr[\tilde{\Phi}\Phi^\dagger]^2 + Tr[\tilde{\Phi}^\dagger\Phi]^2) + \lambda_3(Tr[\tilde{\Phi}\Phi^\dagger]Tr[\tilde{\Phi}^\dagger\Phi]) \\
&+\lambda_4(Tr[\Phi\Phi^\dagger](Tr[\tilde{\Phi}\Phi^\dagger] + Tr[\tilde{\Phi}^\dagger\Phi])) + \rho_1(Tr[\Delta_L\Delta_L^\dagger]^2 + Tr[\Delta_R\Delta_R^\dagger]^2) \\
&+\rho_2(Tr[\Delta_L\Delta_L]Tr[\Delta_L^\dagger\Delta_L^\dagger] + Tr[\Delta_R\Delta_R]Tr[\Delta_R^\dagger\Delta_R^\dagger]) + \rho_3(Tr[\Delta_L\Delta_L^\dagger]Tr[\Delta_R\Delta_R^\dagger]) \\
&+\rho_4(Tr[\Delta_L\Delta_L]Tr[\Delta_R^\dagger\Delta_R^\dagger] + Tr[\Delta_R\Delta_R]Tr[\Delta_L^\dagger\Delta_L^\dagger]) + \alpha_1(Tr[\Phi\Phi^\dagger](Tr[\Delta_L\Delta_L^\dagger] + Tr[\Delta_R\Delta_R^\dagger])) \\
&+\alpha_2(Tr[\Phi\tilde{\Phi}^\dagger]Tr[\Delta_R\Delta_R^\dagger] + Tr[\tilde{\Phi}\Phi^\dagger]Tr[\Delta_L\Delta_L^\dagger])) + \alpha_2^*(Tr[\Phi^\dagger\tilde{\Phi}]Tr[\Delta_R\Delta_R^\dagger] + Tr[\tilde{\Phi}^\dagger\Phi]Tr[\Delta_L\Delta_L^\dagger])) \\
&+\alpha_3(Tr[\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger] + Tr[\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger]) + \beta_1(Tr[\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger] + Tr[\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger]) \\
&+\beta_2(Tr[\tilde{\Phi}\Delta_R\Phi^\dagger\Delta_L^\dagger] + Tr[\tilde{\Phi}^\dagger\Delta_L\Phi\Delta_R^\dagger]) + \beta_3(Tr[\Phi\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger] + Tr[\Phi^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger]),
\end{aligned}
$$

invariant under the symmetry $\Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^\dagger, \quad \beta_i = 0.$



The minimal Higgs sector consists of two triplets and one bidoublet

$$\Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix},$$

$$\Phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}.$$

with vacuum expectation values allowed for the neutral particles

$$\frac{v_L}{\sqrt{2}} = \langle \delta^0_L \rangle,$$

$$\text{new HE scale}: \frac{v_R}{\sqrt{2}} = \langle \delta^0_R \rangle,$$

$$\text{SM VEV scale}: \sqrt{\kappa_1^2 + \kappa_2^2}$$

$$\frac{\kappa_1}{\sqrt{2}} = \langle \phi^0_1 \rangle,$$

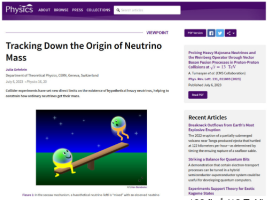
$$\frac{\kappa_2}{\sqrt{2}} = \langle \phi^0_2 \rangle.$$

- The result is 20 real scalar fields, of which 14 are physical (the rest are Goldstone bosons):
  - 4 neutral scalars: $H_0^0, H_1^0, H_2^0, H_4^0$, (the first can be considered to be the light Higgs of the SM at tree level),
  - 2 neutral pseudo-scalars: A_1^0, A_2^0,
  - 2 charged scalars: $H_1^{\pm}, H_2^{\pm}$,
  - 2 doubly-charged scalars: $H_1^{\pm\pm}, H_2^{\pm\pm}$.
- see-saw mechanism for the generation of light neutrino masses, with specific SB sectors. The neutrino mass matrix

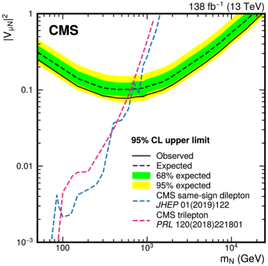
$$M_\nu = \begin{pmatrix} M_L(v_L) & M_D(\kappa_{1,2}) \\ M_D^T & M_R(v_R) \end{pmatrix}$$

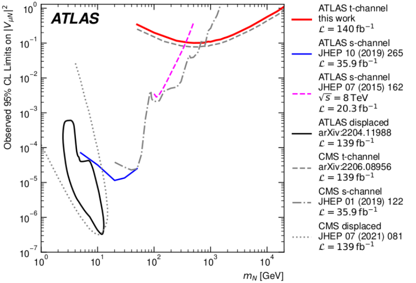
with $M_L \ll M_D \ll M_R$.

RHNs and LHC, present day



Search for $W^{\pm}W^{\pm} \to \ell^{\pm}\ell'^{\pm}$ quickly adopted by LHC groups!





ATLAS ('23) [2305.14931]

← CMS ('22) [2206.08956]

Talk by R. Ruiz, MTTD 2023, link

Origin of neutrino mass? Dirac or Majorana Particle?

A theorist's commentary

Why are neutrino masses still Beyond the Standard Model Physics?

We do not know how to write neutrino masses:

- Are ν data described by left-handed Majorana masses?

$$\Delta\mathcal{L} = \frac{1}{2} m_L \overline{\nu_L^c} \nu_L \quad \text{(maybe!)}$$

- Are ν data described by right-handed Majorana masses?

$$\Delta\mathcal{L} = \frac{1}{2} m_R \overline{\nu_R^c} \nu_R \quad \text{(not by itself!)}$$

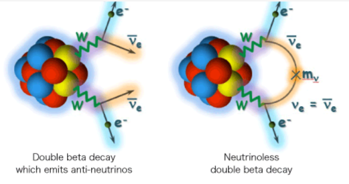
- Are ν data described by Dirac masses?

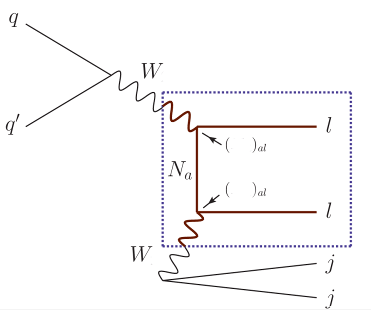
$$\Delta\mathcal{L} = m_D \overline{\nu_L} \nu_R + \text{H.c.} \quad \text{(maybe, but I hope not!)}$$

Experimentally establishing 1/2 is probably worth a prize... 

$e^-e^- \to W^-W^-$, $W^-W^- \to e^-e^-$, $pp \to lljj$, $(\beta\beta)_{0\nu}$

Lepton number violation and 'Diracness' of massive neutrinos composed of Majorana states, PRD'2016 1604.01388





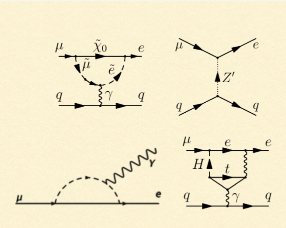
e.g. $e^-e^- \to W^-W^-$, PLB'1995, hep-ph/9507269

$$\sigma(m_N(a) \gg \sqrt{s} \gg M_W) = \frac{G_F^2 s^2}{4\pi} \Big| \sum_{\nu(a)} (V_{ae})^2 \frac{m_a}{s} + \sum_{N(a)} (V_{ae})^2 \frac{1}{m_a} \Big|^2$$

$$\Psi_{Dirac} = e^{\pm i\alpha} \frac{1}{\sqrt{2}}(N_1 \pm iN_2) \longrightarrow \sigma(e^-e^- \to W^-W^-) = 0.$$

LNV, Majorana neutrinos, further constraints

Process	Present limits	Future	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	5×10^{-14}	MEG II
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	10^{-16}	Mu3e
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}$	$< 6.1 \times 10^{-13}$	10^{-17}	Mu2e, COMET
$\mu^-\mathrm{Si/C} \to e^-\mathrm{Si/C}$	–	5×10^{-14}	DeeMe
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	5×10^{-9}	Belle II, FC
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	10^{-9}	Belle II , FC
$\tau \to eee$	$< 2.7 \times 10^{-8}$	5×10^{-10}	Belle I I, FC
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	5×10^{-10}	Belle II, FC
$\tau \to e$ had	$< 1.8 \times 10^{-8}$	3×10^{-10}	Belle II, FC



Sensitivity up to 10 000 TeV!

What about CP effects in the heavy neutrino sector?

- Effects crucial for $(\beta\beta)_{0\nu}$ and colliders studies
- Needed for leptogenesis (standard way)
- Elegant theory for that.

Heavy neutrinos, CP-parity, neutrino mixings

- The nonzero eigenvalues of a real symmetric matrix can be either positive or negative.

$$m'_k = \rho_k m_k$$

where $m_k = |m'_k|$ and $\rho_k = \pm 1$

- Using the identity $\rho_k = e^{i(\pi/2)(\rho_k - 1)}$, we find

$$M = (U^\dagger)^T m U^\dagger, \;\; U_{\ell k} = O_{\ell k} e^{i(\pi/4)(\rho_k - 1)}$$

- With $\chi_{kL} = \sum_{e,\mu,\tau\ldots} = U^*_{\ell K} \nu_{\ell K}, \;\; U^*_{\ell K} = U_{\ell K} \rho_k$, the CP parity of the Majorana fields can be written as

$$\eta_{CP}(\chi_k) = i\rho_k$$

- Thus, the CP parity of the field of a Majorana neutrino with mass m_k is determined by the sign of the corresponding eigenvalue of the neutrino mass matrix and CP parities are reflected in $U_{\ell k}$.

E.g., Bilenky, Petcov, Rev. Mod. Phys. 1989

Constraints and the space of allowed light-heavy mixings

$$(i) \qquad \sum_{N(heavy)} \mid V_{Ne} \mid^2 \leq \kappa^2, \quad [0.0030]$$

$$(ii) \qquad \mid \sum_{\nu(light)} V_{\nu e}^2 m_\nu \mid < \kappa_{\mathbf{light}}^{\mathbf{2}}, \quad [0.68 \text{ eV}]$$

$$(iii) \qquad \mid \sum_{N(heavy)} V_{Ne}^2 \frac{1}{m_N} \mid < \omega^{\mathbf{2}}, \quad [5 \times 10^{-5} \text{ TeV}^{-1}]$$

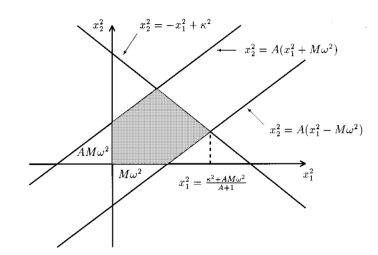
$$(iv) \qquad \sum_{\nu(light)} \mid V_{\nu e} \mid^2 + \sum_{N(heavy)} \mid V_{Ne} \mid^2 = 1.$$

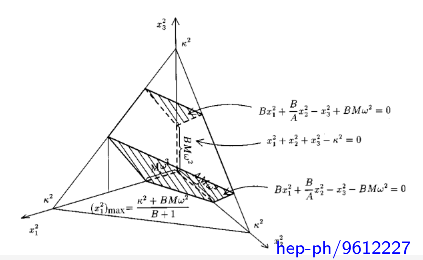
$$(v) \qquad \sum_a V_{ae}^2 m_a = (M_L)_{\nu_e \nu_e} = 0 \Longrightarrow \sum_{\nu(\mathbf{light})} \mathbf{V}_{\nu \mathbf{e}}^{\mathbf{2}} \mathbf{m}_\nu = - \sum_{\mathbf{N(heavy)}} \mathbf{V}_{\mathbf{Ne}}^{\mathbf{2}} \mathbf{m}_{\mathbf{N}}$$

$$\begin{pmatrix} U_{\text{PMNS}} & V_{lh} \\ V_{hl} & V_{hh} \end{pmatrix}, \begin{pmatrix} M_L = 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

For CP-conserving cases, the theory constraints diminish the maxima of the LH mixings, e.g. for

$$M_{N_1} = M,\ M_{N_2} = AM,\ M_{N_3} = BM,$$
$$\eta_{CP}(N_1) = \eta_{CP}(N_2) = -\eta_{CP}(N_3) = +i,$$
$$V_{eN_1} \equiv x_1,\ V_{eN_2} \equiv x_2,\ V_{eN_3} \equiv ix_3,$$



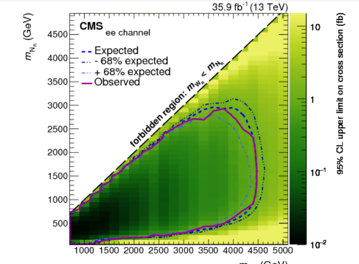


$$| V_{Ne} |^2_{max} \rightarrow \frac{\kappa^2 + M[TeV]\omega^2}{2} \xrightarrow{M \leq 1\ TeV} \frac{\kappa^2}{2}$$

Largest mixing for almost degenerate heavy neutrinos with not the same CP-parities (to avoid $\beta\beta_{0\nu}$ Majorana constraint), $A \rightarrow 1$ for n=2, $A \gg B$, $B \rightarrow 1$ for n=3.

LHC analysis = the same CPs of RHNs (real mixings)

$$| \sum_{\nu(a)} (V_{ae})^2 \frac{m_a}{s} + \sum_{N(a)} (V_{ae})^2 \frac{1}{m_a} |^2$$



GLUZA, JELIŃSKI, and SZAFRON

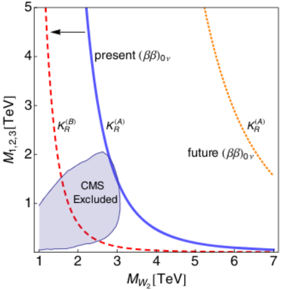
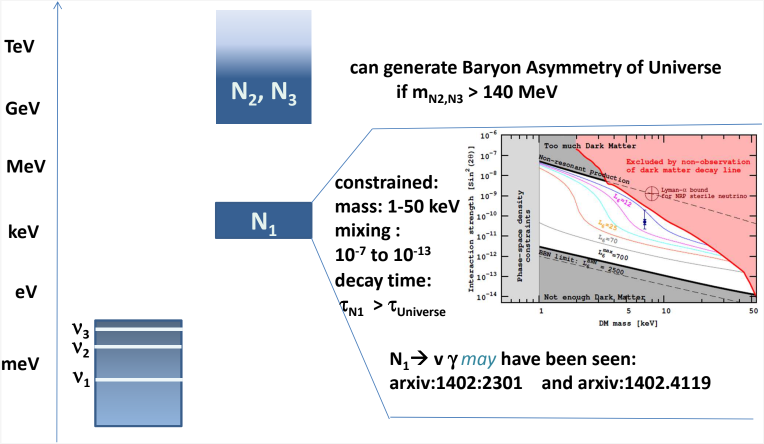


FIG. 3. The CMS vs m^N dominant $(\beta\beta)_{0\nu}$ exclusion limits on the masses of W_2 and N_a in the case when $(K_R)_{aj} = \delta_{aj}$ as in the (A) scenario. The shaded region is excluded by the CMS data related to $pp \to eejj$ at the LHC Run 1 [47]. Present $(\beta\beta)_{0\nu}$ experiments exclude the region under the blue solid curve. The dotted orange curve corresponds to a future bound on $T^{0\nu}_{1/2}$ [25]. For comparison, when the mixing matrix K_R is of the form (5) with $\theta_{13} = 0.9 \times \pi/4$ and $\phi_3 = \pi/2$, only the region under the dashed red curve is excluded. There are no available LHC data exclusion analyses for such "almost" Dirac neutrinos.

RHNs in Cosmology

A. Blondel et al. 1411.5230



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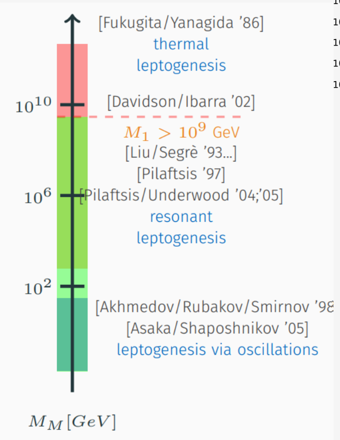
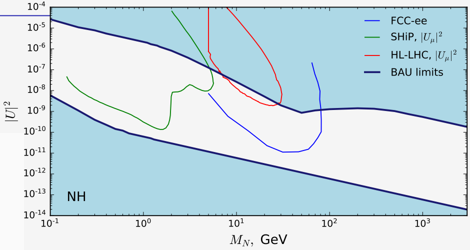
RHN: Leptogenesis



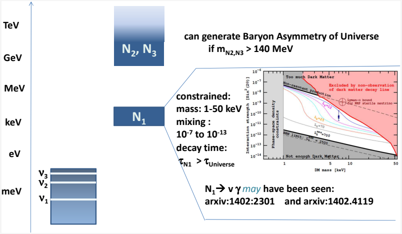


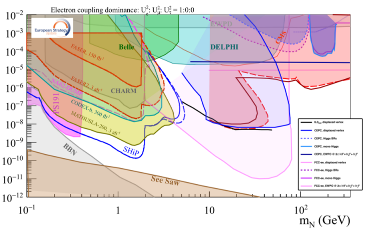
Juraj Klarić et. al., 2008.13771

BSM and RHNs, FCC-ee CDR vol.1

LFV Z-decays: $(10^{-6} \div 10^{-5})$. FCC-ee $\longrightarrow \sim 10^{-9}$ branching fractions.

A. Blondel et al. 1411.5230 ESPPU Briefieng Book 1910.11775



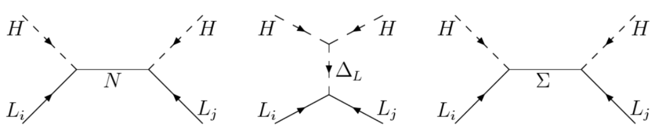


Low-scale leptogenesis with flavour and CP symmetries, M. Drewes et al, 2203.08538

Discrete Flavor Symmetries and Lepton Masses and Mixings, G. Chauhan, et al, 2203.08538 (Snowmass contribution)

Resonant Leptogenesis, Collider Signals and Neutrinoless Double Beta Decay from Flavor and CP Symmetries, G. Chauhan, B. Dev, 2203.08538

Low scale CP and leptogenesis from RHNs sector

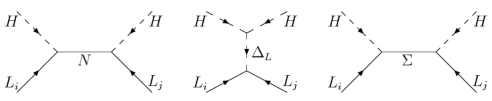


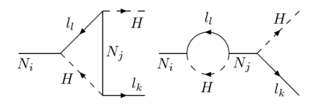
	Type of seesaw model		
	Type-I	Type-II	Type-III
Seesaw states	N	$\Delta_L = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$	$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$
Kin. term	$\mathrm{i}\overline{N}\not\partial N$	$\mathrm{Tr}[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)]$	$\mathrm{Tr}[\overline{\Sigma} i \not{D} \Sigma]$
Mass term	$-\frac{1}{2}\mathrm{Tr}[\overline{N} m_N N^c + \overline{N^c} m_N^* N]$	$-m_\Delta^2 \mathrm{Tr}[\Delta_L^\dagger \Delta_L]$	$-\frac{1}{2}\mathrm{Tr}[\overline{\Sigma} m_\Sigma \Sigma^c + \overline{\Sigma^c} m_\Sigma^* \Sigma]$
Interactions	$-\tilde{\phi}^\dagger \overline{N} Y_N L - \overline{L} Y_N{}^\dagger N \tilde{\phi}$	$-L^{\mathrm{T}} Y_\Delta C \mathrm{i}\tau_2 \Delta_L L + \mu \tilde{H}^{\mathrm{T}} \mathrm{i}\tau_2 \Delta_L \tilde{H}$	$-\tilde{\phi}^\dagger \overline{\Sigma} \sqrt{2} Y_\Sigma L - \overline{L} \sqrt{2} Y_\Sigma{}^\dagger \Sigma \tilde{\phi}$
ν masses	$\mathcal{M}_\nu^N = -\frac{v^2}{2} Y_N^{\mathrm{T}} \frac{1}{m_N} Y_N$	$\mathcal{M}_\nu^\Delta = 2 Y_\Delta v_{\Delta_L} = Y_\Delta \mu^* \frac{v^2}{m_\Delta^2}$	$\mathcal{M}_\nu^\Sigma = -\frac{v^2}{2} Y_\Sigma^{\mathrm{T}} \frac{1}{m_\Sigma} Y_\Sigma$
CP asym.	$\varepsilon_N \equiv \frac{\Gamma(N\to LH) - \Gamma(N\to \bar{L}\bar{H})}{\Gamma(N\to LH) + \Gamma(N\to \bar{L}\bar{H})}$	$\varepsilon_\Delta \equiv 2\,\frac{\Gamma(\bar{\Delta}_L \to LL) - \Gamma(\Delta_L \to \bar{L}\bar{L})}{\Gamma_\Delta + \Gamma_{\bar{\Delta}}}$	$\varepsilon_\Sigma \equiv \frac{\Gamma(\Sigma\to LH) - \Gamma(\bar{\Sigma}\to \bar{L}\bar{H})}{\Gamma(\Sigma\to LH) + \Gamma(\bar{\Sigma}\to \bar{L}\bar{H})}$

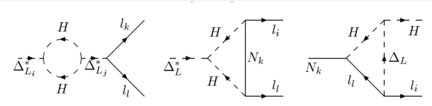
Leptogenesis: beyond the minimal type I seesaw scenario, Thomas Hambye, 1212.2888

Low scale CP and leptogenesis from RHNs sector

Minimal setup: > 1 RHNs needed (or Δ_L), complex couplings.







$$\varepsilon_N = -\frac{3}{32\pi^2}\frac{m_N^3}{\Gamma_N v^4} Im[(\mathcal{M}_\nu^N)_{\beta\alpha}(\mathcal{M}_\nu^H)^\dagger_{\alpha\beta}] \longrightarrow \text{induced by decaying N and "H" in loop}$$

$$\varepsilon_\Delta = -\frac{1}{16\pi^2}\frac{m_\Delta^3}{\Gamma_\Delta v^4} Im[(\mathcal{M}_\nu^\Delta)_{\beta\alpha}(\mathcal{M}_\nu^H)^\dagger_{\alpha\beta}] \longrightarrow \text{by decaying } \Delta_L \text{and heavy "H" in loop}$$

$$\varepsilon_\Sigma = -\frac{1}{32\pi^2}\frac{M_\Sigma^3}{\Gamma_\Sigma v^4} Im[(\mathcal{M}_\nu^\Sigma)_{\beta\alpha}(\mathcal{M}_\nu^H)^\dagger_{\alpha\beta}] \longrightarrow \text{by decaying } \Sigma \text{ and heavy "H" in loop}$$

Not discussed: Footprints of CP from the heavy sector at low scale $\longrightarrow$ our PPNP review arXiv:2310.20681

Take away, RHNs

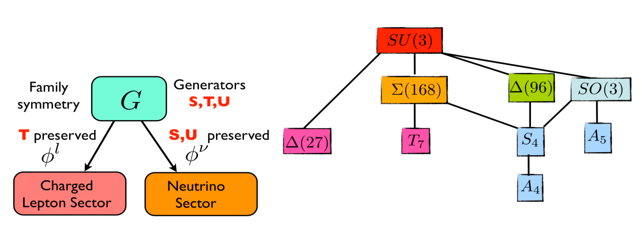
- RHNs are promising candidates for BSM signals discovery at lepton and hadron colliders.
- Light-heavy mixings are sensitive to (heavy) neutrino CP-parities.

In this context:

- It is worth studying further seesaw and non-decoupling mixing models with $Z \to l_i l_j$ (LFV and LFC decays) and $Z \to \nu N_i$, NLO effects, Dirac/Majorana cases, consistency with low energy LFV/LFC/LNV effects, leptogenesis, ...

What is flavor symmetry?

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.



S. F. King 1301.1340

$G_f \to G_e, G_\nu$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

$\Downarrow$

(Prior to 2012)

$s_{23} = 1/\sqrt{2}$ ($\theta_{23} = 45^\circ$) and $\theta_{13} = 0$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Bimaximal Mixing	Tribimaximal Mixing	Golden Ratio Mixing	Hexagonal Mixing
$\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$	$\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$	$\theta_{12} = 31.7^\circ$	$\theta_{12} = 30^\circ (s_{12} = 1/2)$

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1+\sqrt{5})/2$)

Simple example: $\mu - \tau$ permutation symmetry and TBM

$$m_\nu = U_0^\star \mathrm{diag}(m_1, m_2, m_3) U_0^\dagger,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}\ i.e., \theta_{12} = 35.26^\circ$

- Observed mixing matrix :

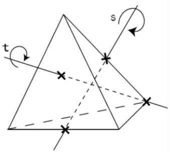
$$U_{\rm PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

Example of Flavor Symmetry: A_4

A_4 **is the minimal group which contains 3 dim. representation**

- **A_4** → Even Permutation of 4 objects
  Group of order 12
  invariant group of a tetrahedron
- **minimal group** → It has $1, 1', 1'', 3$
- **3 dim. representation** → Can accommodate three flavors of leptons $\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} \sim 3$; $e_R \sim 1, \mu_R \sim 1'', \tau_R \sim 1'$

- Product rule: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$
- $1 \otimes 1 = 1,\ 1' \otimes 1' = 1'', 1' \otimes 1'' = 1$
  $1'' \otimes 1'' = 1'$ etc



- SM scalars (flavons) introduced: $\langle \phi_S \rangle = v_S\ (1,1,1)^T\ \langle \xi \rangle = v_\xi,\ \langle \phi_T \rangle = v_T\ (1,0,0)^T$
- Neutrino mass follows from:

$$\frac{\ell_i H \ell_j H}{\Lambda} \left(\frac{\phi_S}{\Lambda} + \frac{\xi}{\Lambda} \right)$$

- Light neutrino mass matrix

$$(m_\nu)_0 = \begin{pmatrix} a - 2b/3 & b/3 & b/3 \\ b/3 & -2b/3 & a + b/3 \\ b/3 & a + b/3 & -2b/3 \end{pmatrix}, \quad \begin{matrix} a = y_1 (v^2/\Lambda)\epsilon \\ b = y_2 (v^2/\Lambda)\epsilon \end{matrix} \quad , \epsilon = v_\xi/\Lambda = v_S/\Lambda$$

yet another example:

- Let us consider $G_f = S_4$ as a guiding symmetry.
- Geometrically, it's a symmetry group of a rigid cube (group of permutation 4 objects).
- the order of the group is $4! = 24$ and the elements can be conveniently generated by the generators S, T and U satisfying the relation

$$S^2 = T^3 = U^2 = 1 \text{ and } ST^3 = (SU)^2 = (TU)^2 = 1.$$

- irreducible triplet representations:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}; T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \text{ and } U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T^\dagger M_\ell^\dagger M_\ell T = M_\ell^\dagger M_\ell, \; S^T M_\nu S = M_\nu \text{ and } U^T M_\nu U = M_\nu$$

$$[T, M_\ell^\dagger M_\ell] = [S, M_\nu] = [U, M_\nu] = 0$$

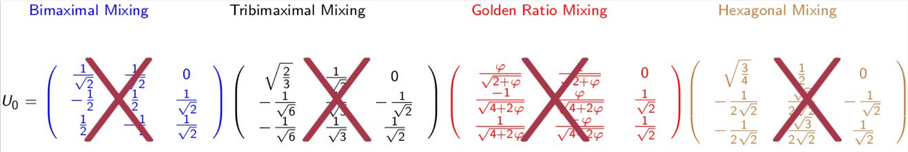
- The non-diagonal matrices S, U can be diagonalized by

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

Tribimaximal Mixing: A_4- Ma, Rajasekaran 0106291; Altarelli, Feruglio 0504165; $\Delta(27)$-Varzielas, King, Ross- 0607045; Bimaximal Mixing: Frampton, Petcov, Rodejohann 0401206; Golden Ratio Mixing: Feruglio, Paris 1101.0393; Hexagonal Mixing: Albright, Dueck, Rodejohann-1004.2798.

Non-zero θ_{13}

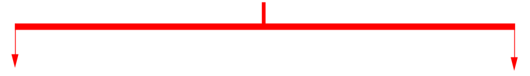
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$



Decendents of fixed pattern mixing schemes

Non-zero θ_{13}: Decendents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{\rm PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$



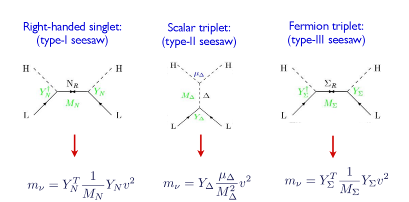
$$|U_{\rm TM_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\rm TM_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

- If S_4 is considered to be broken spontaneously into $Z_3 = \{1, T, T^2\}$ (for the charged lepton sector) $Z_2 = \{1, SU\}$ (for the neutrino sector) such that it satisfies : $[T, M_\ell^\dagger M_\ell] = [SU, M_\nu] = 0$

$$U_{\rm TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}, U_{\rm TM_2} = \begin{pmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix},$$

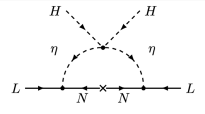
Neutrino Mass Generation

Seesaw frameworks



• **Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, etc.:** Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi; Schechter, Valle 81; Schechter, Valle 80; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Mohapatra Valle 86; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass



• **Radiative models, started in 80s:** Zee 80, Cheng, Li 80; Zee 86; Babu 88; Babu, Ma, Valle, 02; Ma 06;

• **For a review of radiative models:** Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

Hybrid Scenarios??

- **Review article: Progress of Particle and Nuclear Physics 138 (2024) 104126**

In this review, we present a detailed discussion on the viability of flavor symmetric models in the context of current neutrino oscillation data and provide a wide range of phenomenological implications related to energy, intensity, and cosmic frontiers.

ELSEVIER

Progress in Particle and Nuclear Physics

Volume 138, June 2024, 104126

Review

Phenomenology of lepton masses and mixing with discrete flavor symmetries

Garv Chauhan [a], P.S. Bhupal Dev [b], Ievgen Dubovyk [c], Bartosz Dziewit [c], Wojciech Flieger [d], Krzysztof Grzanka [c], Janusz Gluza [c], Biswajit Karmakar [c], Szymon Zięba [c]

Show more

Flavor Symmetry and Hybrid Mass Mechanisms: Why?

- Ratio of solar to atmospheric mass difference :

$$r = \frac{\Delta m^2_{\rm SOL}}{\Delta m^2_{\rm ATM}} \simeq \frac{7.41 \times 10^{-5}\ {\rm eV}^2}{2.51 \times 10^{-3}\ {\rm eV}^2} \simeq 3 \times 10^{-2}$$

- Two different mass scales that might originate from **entirely separate mechanisms !!**

- Minimal **Scoto** **Seesaw** scenario: Rojas, Srivastava, Valle 1807.11447

Greek word **'skótos'** $\longrightarrow$ **'darkness'**

$$\mathcal{L} = -Y^k_N \bar{L}^k i\sigma_2 H^* N_R + \frac{1}{2} M_R \bar{N}^c_R N_R + Y^k_f \bar{L}^k i\sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f$$

The number of right-handed neutrinos added to the SM is not fixed as they do not carry any anomaly
Schechter, Valle 1980

- The total neutrino mass reads:

$$M^{ij}_\nu = -\frac{v^2}{M_N} Y^i_N Y^j_N + \mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f\ Y^i_f Y^j_f .$$

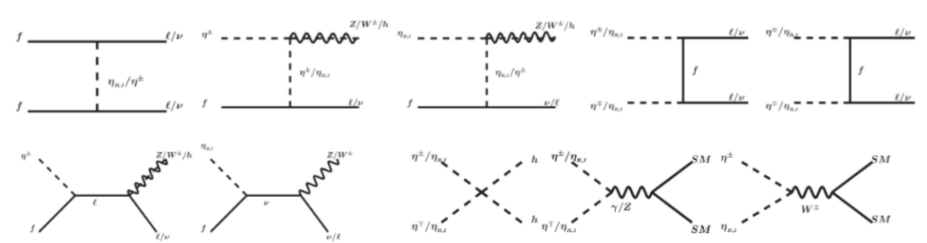
where

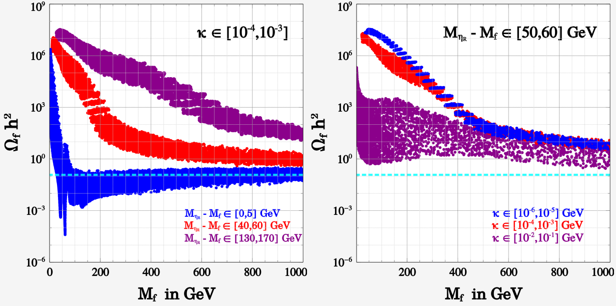
$$\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) = \frac{1}{32\pi^2}\Big[\frac{M^2_{\eta_R}\log\left(M^2_f/M^2_{\eta_R}\right)}{M^2_f - M^2_{\eta_R}} - \frac{M^2_{\eta_I}\log\left(M^2_f/M^2_{\eta_I}\right)}{M^2_f - M^2_{\eta_I}}\Big],$$

where M_{η_R} and M_{η_I} are the masses of the neutral component of η.

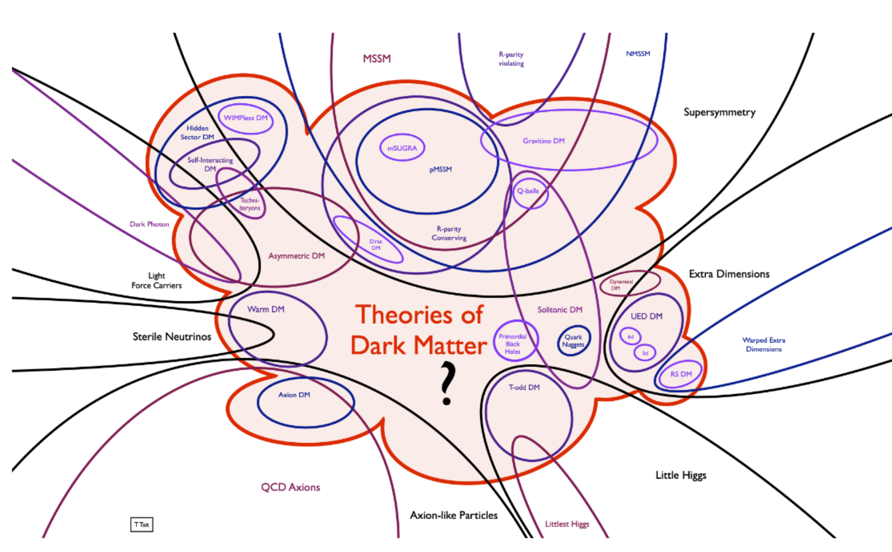
FSS_1 (scoto-seesaw) phenomenology: dark matter

- 2 viable DM candidates ⇒ the **lightest neutral scalar** (Mandal, Srivastava, Valle, 2104.13401)
  ⇒ the **singlet fermion** (Ganguly, Gluza, Karmakar, Mahapatra 2311.15997).



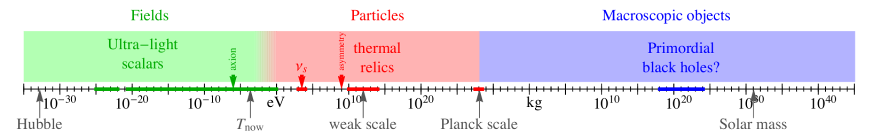


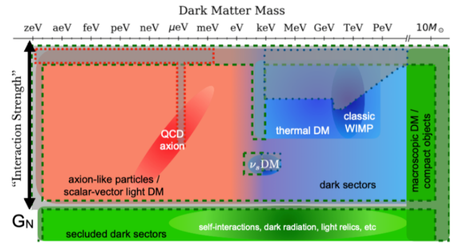
BSM - terra incognita: DM & RHN, ...



BSM - terra incognita: DM & RHN, ...

- 10^{-22} eV$\lesssim m_\chi \lesssim 1$ eV: "Ultralight" Dark Matter
- 1 eV$\lesssim m_\chi \lesssim 1$ GeV: "Light" particle Dark Matter
- 1 GeV$\lesssim m_\chi \lesssim 100$ TeV: "Heavy" particle Dark Matter
- $m_\chi \gtrsim 100$ TeV: "Ultra-Heavy" Dark Matter (UHDM)





BSM - terra incognita: DM & CMS

J. Luo, Investigating the Physics of the Dark Sector with CMS

https://indico.cern.ch/event/1403078/attachments/2892939/5071574/DarkSector_CMS_v3.pdf

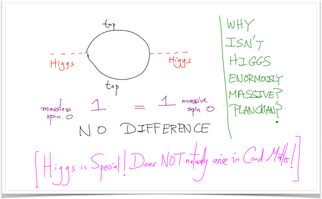
The Higgs boson — a new territory

❖**The Higgs boson itself is in fact "new" physics**

- The first (possibly) elementary scalar we have ever discovered in nature

> "*There is today a wide spread view that … scalar field theories with ^{4} interactions, are not mathematically consistent.*"
> — **Steven Weinberg**, *The Quantum Theory of Fields, vol. 2*

- The Hierarchy Problem — m_H v.s. Planck
- No Higgs boson in condensed matter systems



(Nima Arkani-Hamed, Higgs turns 10 celebration@CERN)

Exhaustively examining the Higgs boson is extremely important

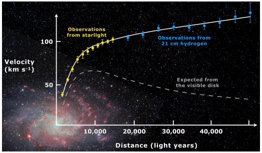
A unique window into new "dark" sectors. [Brain Patt & Frank Wilczek, 2006]

BSM - terra incognita: DM & CMS

Astrophysical and cosmological evidences of dark matter

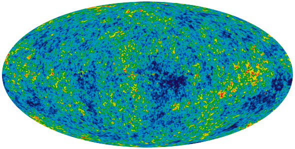
❖**Evidences of dark matter are overwhelming**

Galaxy rotation curves



Galaxy Messier 33 — 21cm line

Cosmic microwave background (CMB)



Planck space observatory

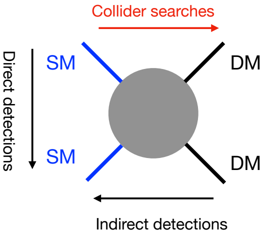
Bullet cluster

Chandra/Magellan/Hubble telescopes

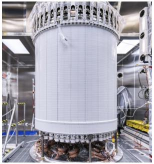
- Dark matter is 5x more abundant than ordinary matter (according to e.g. CMB fit);
- Standard model doesn't provide dark matter candidates;
- The particle nature of the dark matter remains a mystery.

Searches for dark matter/sectors

❖**Extensive searches have been performed with different experimental techniques**

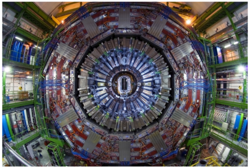


EFT



LZ experiment

AMS experiment



CMS experiment

Direct detections

- *DM-nucleon scatterings;*
- *Axion conversions.*

Indirect detections

- *DM annihilations*

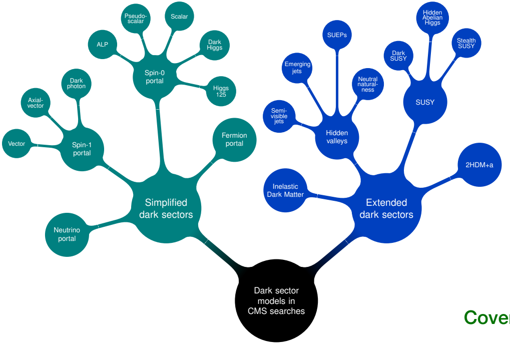
Collider Searches

- *Invisible/visible final states;*
- *Unconventional signatures.*

BSM - terra incognita: DM & CMS

Dark matter/sector searches at CMS

❖**Map of CMS searches for dark sectors**



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2024-106
2024/05/24

CMS-EXO-23-005

Dark sector searches with the CMS experiment

The CMS Collaboration*

arXiv: 2405.13778, submitted to Physics Reports

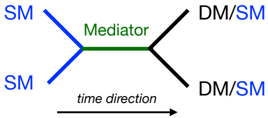
- **Huge community efforts within CMS**
  - ~**20** editors
  - ~**40** analyses
  - ~**500** authors
  - **145** pages
  - **10** updated summary plots
  - **27** new reinterpretations

Covered mass range from ~GeV to multiple TeV

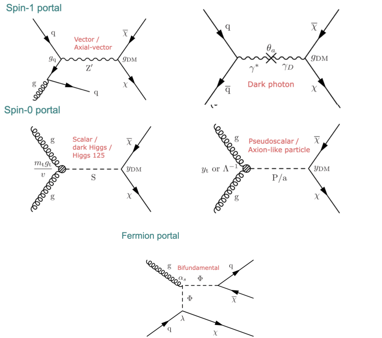
Dark sector benchmarks

❖Simplified dark sectors

- One DM candidate + one mediator (portal)
- Additional states in the dark sector are assumed to be decoupled

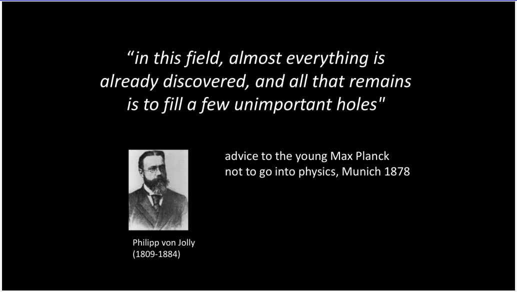


Categorized according to the mediator/portal



- **Typical signatures:**
  - **Invisible final states:** DM productions
  - **Fully visible final states:** mediator resonances

Summary: Where we are in basic research: half-empty or half-full glass of water?



Albert Michelson (1894):

`"It seems probable that most of the grand underlying principles have been firmly established (...) the future truths of physical science are to be looked for in the sixth place of decimals"`

Q: Dear Albert: What about special and general relativity, and quantum mechanics, particle physics, ...?