

# Triple Higgs production at hadron colliders

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A. Papaefstathiou, M. Zaro, GTX: *Eur.Phys.J.C* 79 (2019) 11, 947 (1909.09166)

A. Papaefstathiou, T. Robens, GTX: *JHEP* 05 (2021) 193 (2101.00037)

A. Papaefstathiou, GTX: *JHEP* 06 (2024) 124 (2312.13562)

O. Karkout, A. Papaefstathiou, M. Postma, GTX, J. van de Vis, T du Pree: *JHEP* 11 (2024) 077 (2404.12425)

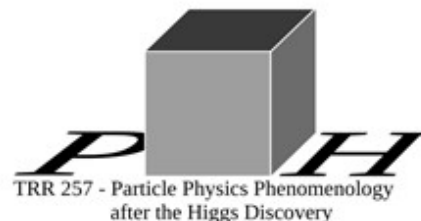
A. Papaefstathiou, GTX: (2501.14866)

B. Fuks, A. Papaefstathiou, GTX: (2509.16364)

**CPPS, Theoretische Physik 1,  
Universität Siegen**

## HHH Workshop

Dubrovnik/Croatia 29<sup>th</sup> September-1<sup>st</sup> October 2025



# Higgs Self-Interactions in the SM

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda_{SM} (\Phi^\dagger \Phi)^2$$

$$\Phi = (0, v_0 + h)^T / \sqrt{2}$$

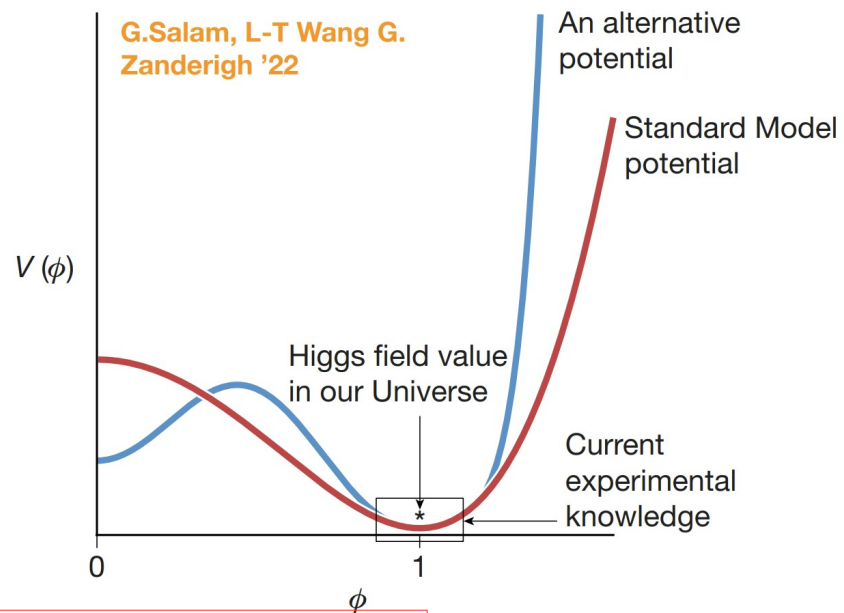
$$V(\Phi^\dagger \Phi) \supset \frac{1}{2} m_h^2 h^2 + \lambda_{SM} v_0 h^3 + \frac{\lambda_{SM}}{4} h^4$$

In the SM  $m_h^2 = \lambda_{SM} v_0^2 / 2$        $v_0^2 = -\mu^2 / \lambda_{SM}$

# Why study triple Higgs production?

- The **triple Higgs self coupling** is sensitive to **New Particles**.
- It also gives the opportunity to **test the Higgs quartic self couplings**.

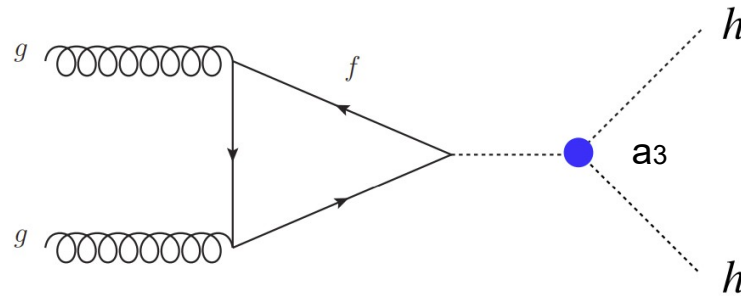
We do not know much about the shape of the Higgs potential.



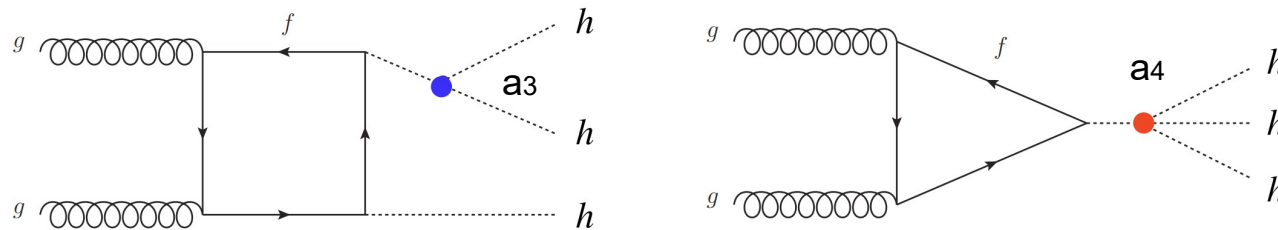
$$V(\Phi^\dagger \Phi) \supset \frac{1}{2} m_h^2 h^2 + a_3 h^3 + a_4 h^4$$

# Why study triple Higgs production?

**Double Higgs** production is the lowest multiplicity to probe for  $a_3$ .



**Triple Higgs** production is the lowest multiplicity to probe for  $a_4$ .



# Possible Final states

$$h h h \longrightarrow X$$

$X$ (Final State)	Br(%)
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21
$(b\bar{b})(b\bar{b})(W W_{1l})$	7.20
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31
$(b\bar{b})(\tau\bar{\tau})(W W_{1l})$	1.58
$(b\bar{b})(b\bar{b})(W W_{2l})$	0.98
$(b\bar{b})(W W_{1l})(W W_{1l})$	0.90
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23

Papaefstathiou, GTX, Zaro: 1909.09166  
 A. Papaefstathiou, GTX: (2501.14866)  
 Fuks, Papaefstathiou, GTX: 2509.16364

Fuks, Kim, Lee: 1510.07697  
 1704.04298

Killian et al.: 1702.03554

Papaefstathiou, Sakurai.: 1508.06524  
 Chen et al.:1510.04013  
 Fuks, Kim, Lee: 1510.07697

*6-b final state has the largest Branching Fraction*

*This is the channel we are focusing on in this talk*

# Backgrounds vs Signal

In the FCC (pp @ 100 TeV)

## Signal and Irreducible backgrounds

## Reducible backgrounds

Process	$\sigma_{\text{GEN}} \times k_{\text{fac}} \times \text{BR} \times \mathcal{P}(6b)$ [fb]
$hhh$ (SM)	$4.29 \times 10^{-1}$
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	$1.07 \times 10^4$
$q\bar{q} \rightarrow Z(b\bar{b})(b\bar{b})$	$3.61 \times 10^2$
$q\bar{q} \rightarrow ZZb\bar{b}$	$1.14 \times 10^1$
$q\bar{q} \rightarrow ZZZ$	$1.80 \times 10^{-1}$
$q\bar{q} \rightarrow hZb\bar{b}$	2.04
$q\bar{q} \rightarrow hZZ$	$1.48 \times 10^{-1}$
$q\bar{q} \rightarrow hhZ$	$8.11 \times 10^{-2}$
$q\bar{q} \rightarrow hh(b\bar{b})$	$1.80 \times 10^{-2}$
$q\bar{q} \rightarrow h(b\bar{b})(b\bar{b})$	$7.25 \times 10^{-1}$
$gg \rightarrow ZZZ$	$5.18 \times 10^{-3}$
$gg \rightarrow hZZ$	$3.59 \times 10^{-2}$
$gg \rightarrow hhZ$	$6.41 \times 10^{-2}$

Process	$\sigma_{\text{GEN}}$ [fb]	$\sigma_{\text{GEN}} \times k_{\text{fac}} \times \mathcal{P}(6b)$ [fb]
$(b\bar{b})(b\bar{b})(c\bar{c})$	$73.0 \times 10^3$	762
$(b\bar{b})(c\bar{c})(c\bar{c})$	$72.0 \times 10^3$	10.4
$(c\bar{c})(c\bar{c})(c\bar{c})$	$21.8 \times 10^3$	$4.36 \times 10^{-2}$
$(b\bar{b})(b\bar{b})(jj)^*$	$3.09 \times 10^6$	161
$(b\bar{b})(jj)(jj)^*$	$1.07 \times 10^8$	1.54
$(c\bar{c})(c\bar{c})(jj)^*$	$1.73 \times 10^5$	$3.46 \times 10^{-3}$
$(c\bar{c})(jj)(jj)^*$	$6.67 \times 10^7$	$6.67 \times 10^{-3}$
$(jj)(jj)(jj)^*$	$3.04 \times 10^9$	$6.08 \times 10^{-3}$

Assuming a K-factor of 2

Maltoni, Vryonidou, Zaro: 1408.6542

# Details on the study of the 6b final state

- Parton level events (signal/background) generated with [MadGraph5\\_aMC@NLO](#).
- The **source of background with the highest XS is QCD-6b-Jets**.
- The **production of the 6b-final state is challenging**, it was generated in the [Siegen computer cluster](#) using the gridpack option available in [MadGraph5\\_aMC@NLO](#).
- Parton shower and non-perturbative effects included with [Herwig 7](#).
- The [analysis was performed using HwSim](#). [*Papaefsathiou*, <https://bitbucket.org/andreasp/hwsim>].
- Two selection analysis approaches are followed: **traditional cuts selection and gradient boosting using XGBoost**.

# Smearing effects

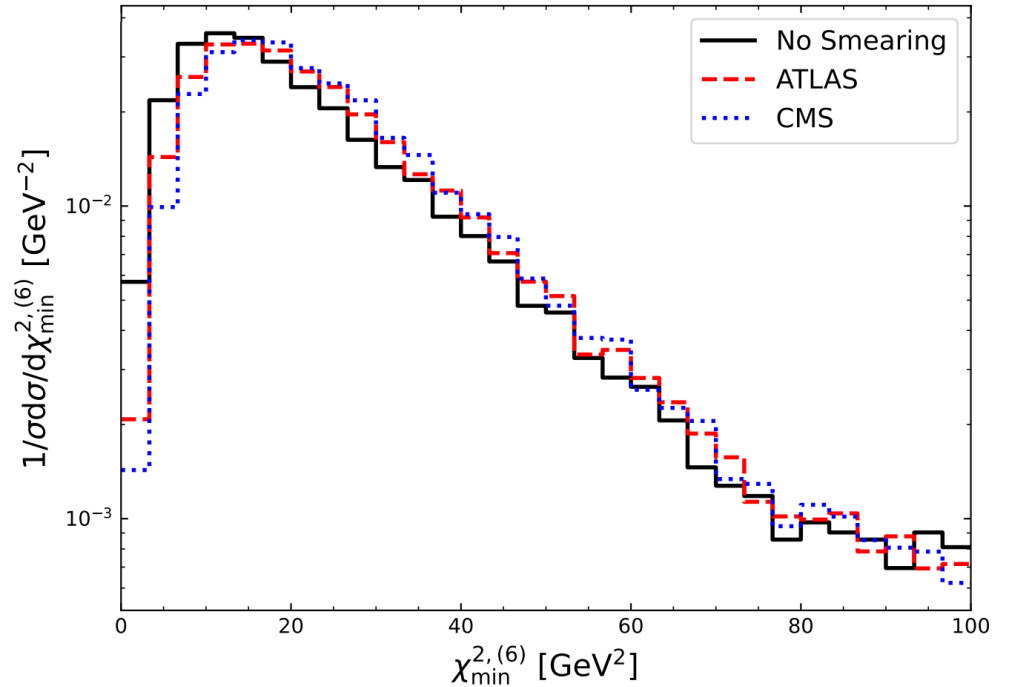
We introduce smearing effects based on *ATLAS and CMS like scenarios*

$$E_{\text{smearred}} = E + \Delta E, \quad \Delta E \sim \mathcal{N}(0, \sigma_E),$$

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$$E_{\text{smearred}} = E + \Delta E \quad p_{T,\text{smearred}} = \frac{E + \Delta E}{\cosh \eta}$$

$$\sigma_E^{\text{ATLAS}} = \begin{cases} \sqrt{(0.0302E)^2 + 0.5205^2 E + 1.59^2} & |\eta| \leq 1.7 \\ \sqrt{(0.05E)^2 + 0.706^2 E} & |\eta| \in [1.7, 3.2] \\ \sqrt{(0.0942E)^2 + E} & |\eta| \in [3.2, 4.9] \end{cases}, \quad \sigma_E^{\text{CMS}} = \begin{cases} \sqrt{(0.05E)^2 + 1.5^2 E} & |\eta| \leq 3.0 \\ \sqrt{(0.130E)^2 + 2.7^2 E} & 3.0 < |\eta| \leq 5.0 \end{cases},$$



# Selection Analysis

- *Require 6 b-tagged jets*
- *Construct all the possible combinations of 3-pairs of b-jets:  $I$ .*
- *For each combination  $I$  calculate the observable*

$$\chi^{2,(6)} = \sum_{qr \in I} (M_{qr} - m_h)^2$$

- *Select the event based on the value of the combination which minimizes  $\chi^{2,(6)}$*
- *The combination determining  $\chi_{min}^{2,(6)}$  defines the best candidates for the set of 3-Higgs bosons in the event.*

# Selection Analysis

Set of **observables and optimized cuts** applied during the *traditional* selection analysis (FCC)

Observable	Threshold
$p_{T,b} >$	35.0 GeV
$ \eta_b  <$	3.0
$\Delta R_{bb} >$	0.3
<hr style="border-top: 1px dashed black;"/>	
$p_{T,b_i} >$	[170.0, 135.0, 35.0] GeV
$\chi^{2,(6)} <$	26.0 GeV
$\Delta m_i <$	[8, 8, 8] GeV
$\Delta R_{bb}(h^i) <$	[3.5, 3.5, 3.5]
$\Delta R(h^i, h^j) <$	[3.5, 3.5, 3.5]
$p_T(h^i) >$	[200.0, 190.0, 20.0] GeV

$h^i$  : Higgs boson candidate

$i=1,2,3$

# Sensitivity to self couplings

We consider a generalized version of the SM scalar potential

$$\mathcal{L} \supset -\frac{m_h^2}{2v} (1 + c_3) h^3 - \frac{m_h^2}{8v^2} (1 + d_4) h^4$$

To assess the sensitivity towards the anomalous couplings we construct the following test statistic

$$\chi_{\text{tot}}^2(c_3, d_4) = \chi^2(c_3, d_4) + \left( \frac{c_3}{\delta c_3} \right)^2 \quad \chi^2(c_3, d_4) = \left[ \frac{S_{\text{SM}} - S(c_3, d_4)}{\delta_{\text{SM+B}}} \right]^2$$

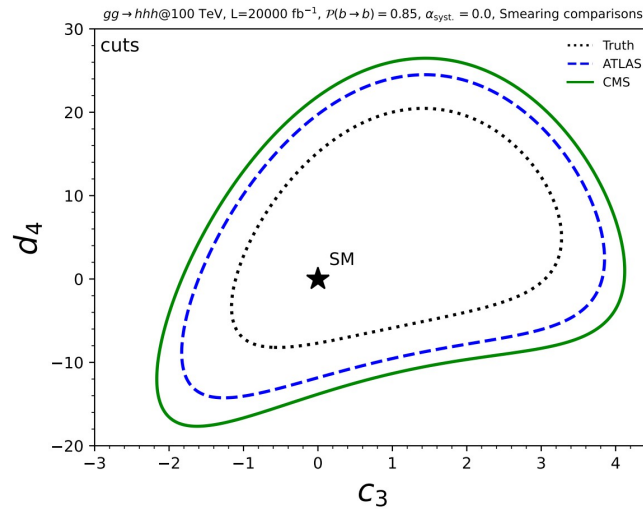
Gaussian prior to introduce information on the cubic coupling

Uncertainty in the number of expected SM events

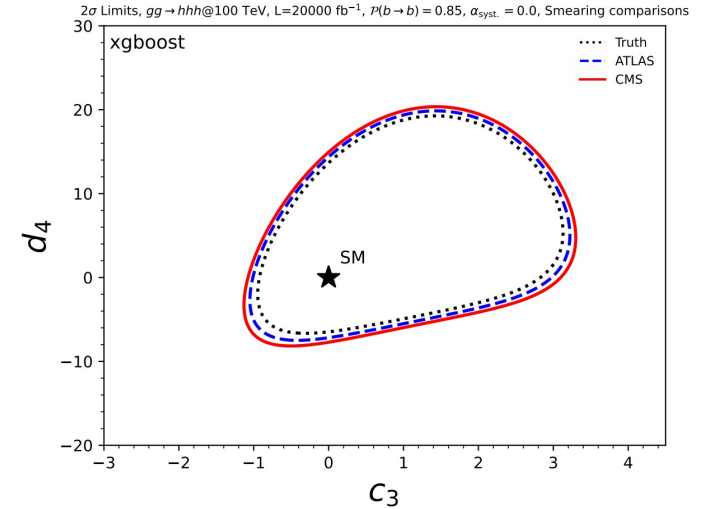
$$\delta_{\text{SM+B}} = \sqrt{S_{\text{SM}} + B + (\alpha B)^2}$$

Systematic uncertainties

# Sensitivity to self couplings



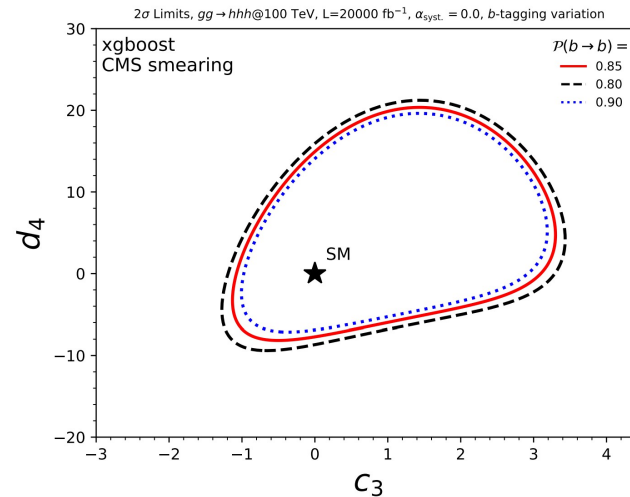
Traditional Cuts



XGBoost

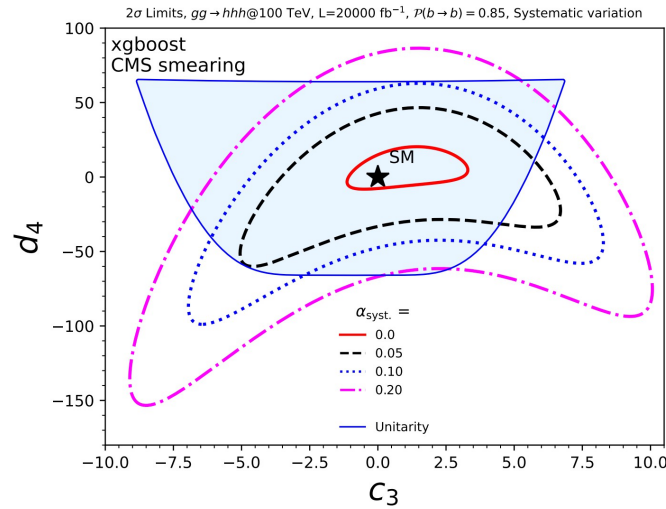
95% C. L.  
Regions

B. Fuks, A. Papaefstathiou, GTX:  
2509.16364

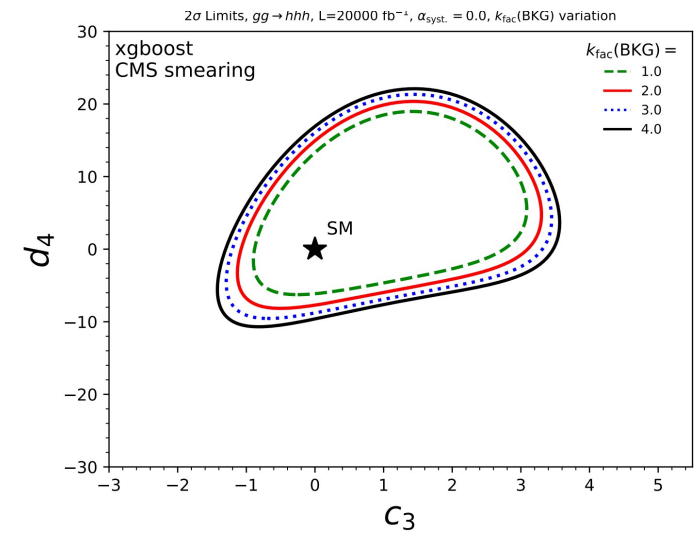


*b*-Tagging impact

# Sensitivity to self couplings



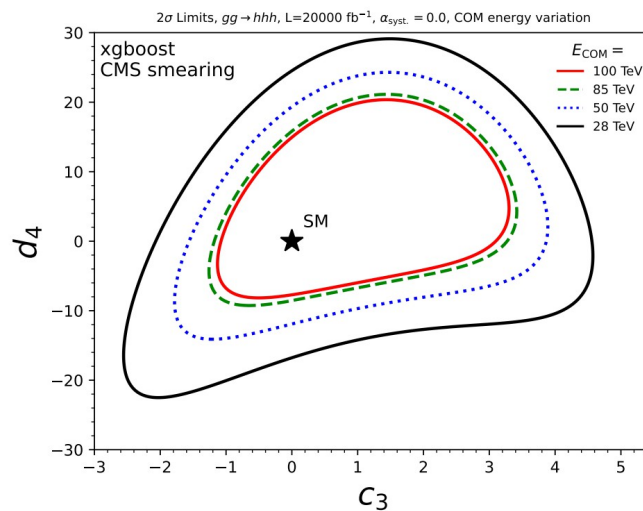
Systematic-error  
impact



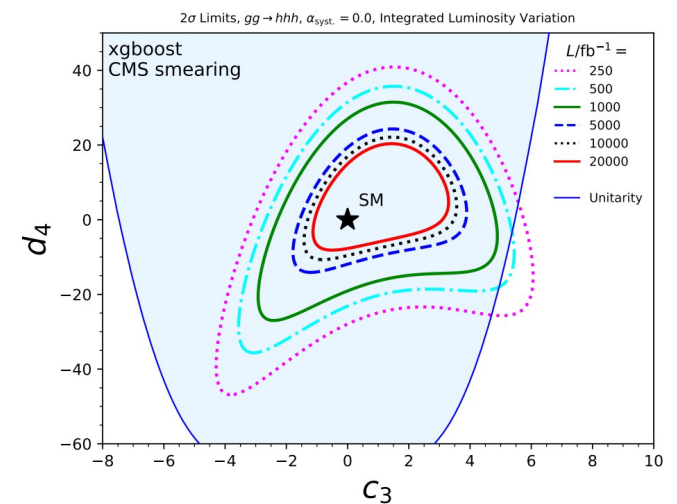
K-factor impact

95% C. L.  
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2509.16364

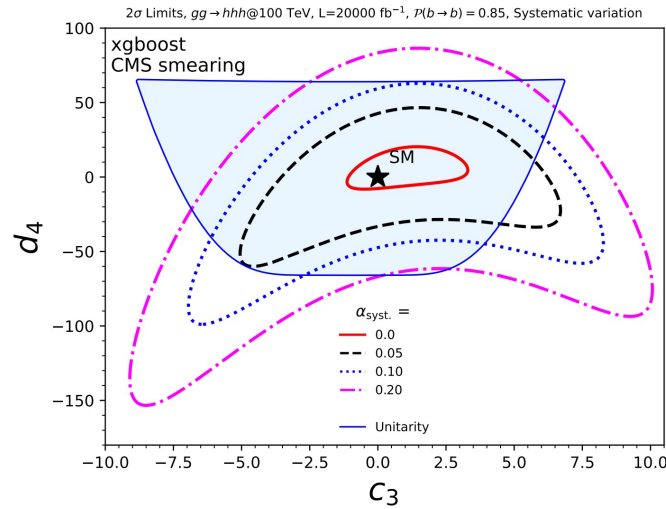


CM-impact

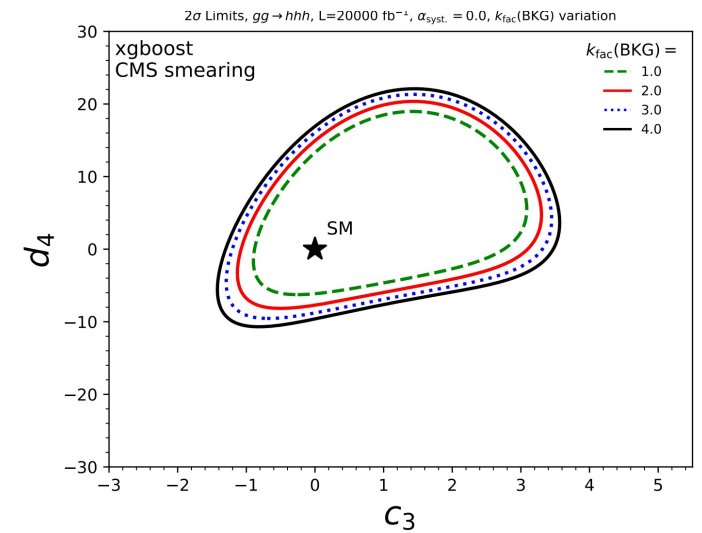


Luminosity impact

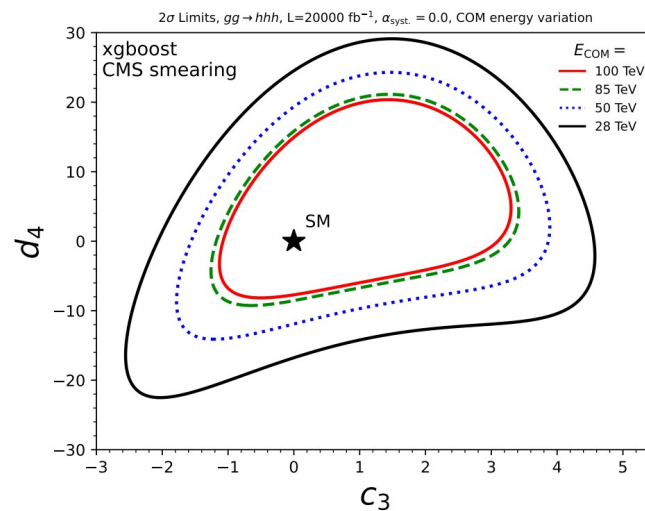
# Sensitivity to self couplings



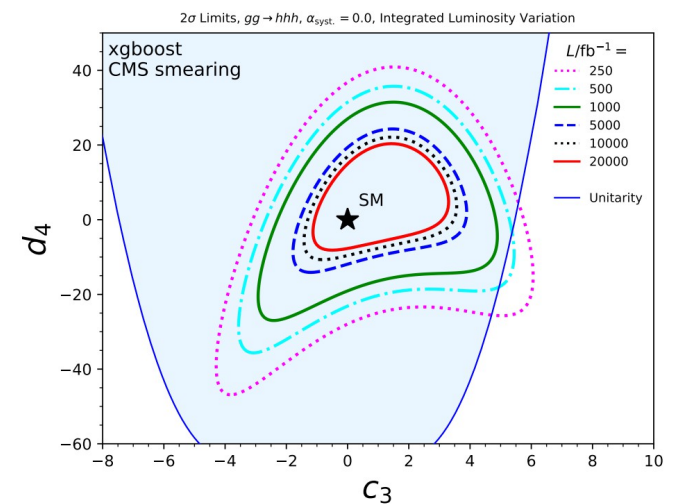
Systematic-error  
impact



K-factor impact



CM-impact



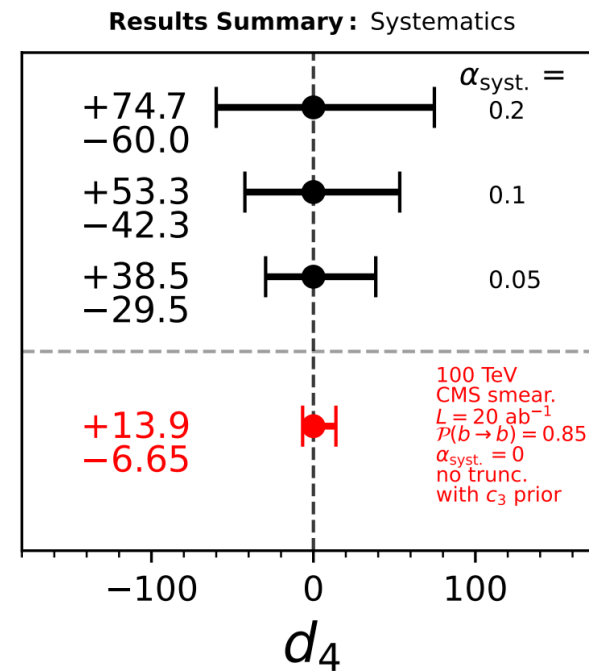
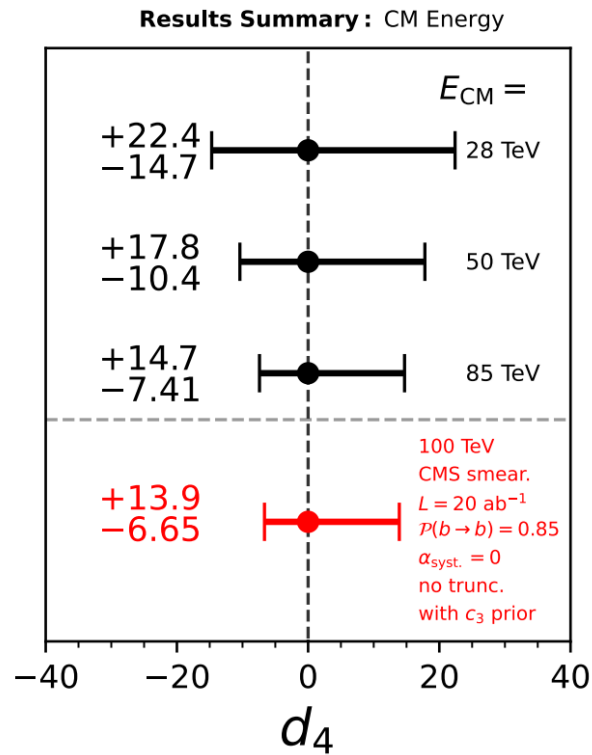
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# Sensitivity to self couplings

## Summaries



$$\delta c_3 = 0.05$$

# Anomalous couplings

Relevant phenomenological Lagrangian  
to test anomalous couplings

$$\begin{aligned}\mathcal{L}_{\text{PhenoExp}} = & -\lambda_{\text{SM}}v(1+d_3)h^3 - \frac{\lambda_{\text{SM}}}{4}(1+d_4)h^4 \\ & + \frac{\alpha_s}{12\pi} \left( c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[ \frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[ \frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[ \frac{m_t}{v^3} \left( \frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left( \frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right],\end{aligned}$$

Obtained by considering D=6 EFT operators (SILH, 0703164) and breaking correlations (ATLAS and CMS)

Can also be obtained from the Electroweak chiral  
Lagrangian

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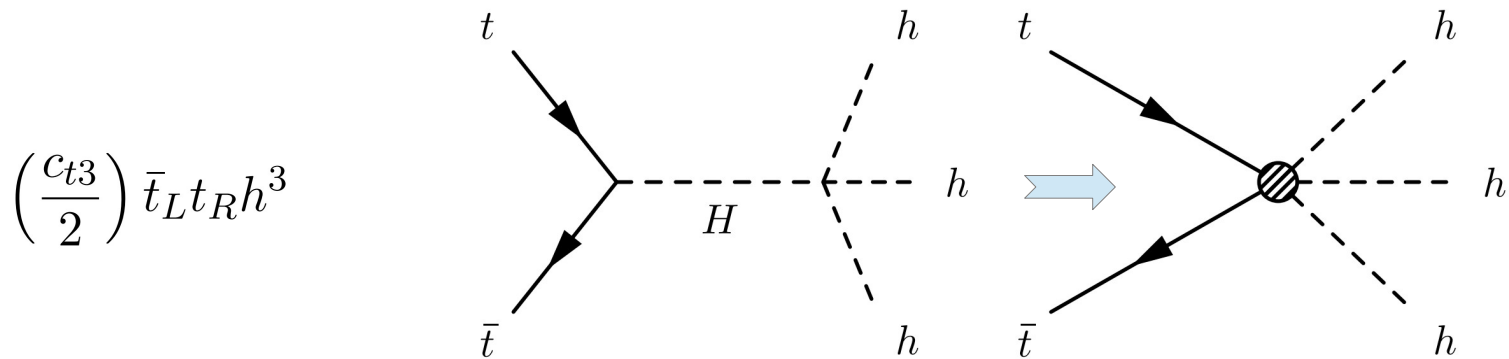
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# Anomalous couplings

Current and expected bounds on the anomalous couplings

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[1905.03764]
$\delta(c_{g1})$	2.3	0.49	[1905.03764]
$\delta(c_{g2})$	5	1	[1502.00539]
$\delta(c_{t1})$	3.3	1.0	[1905.03764]
$\delta(c_{t2})$	30	10	[1502.00539]
$\delta(c_{b1})$	3.6	0.43	[1905.03764]
$\delta(c_{b2})$	30	10	[1502.00539]

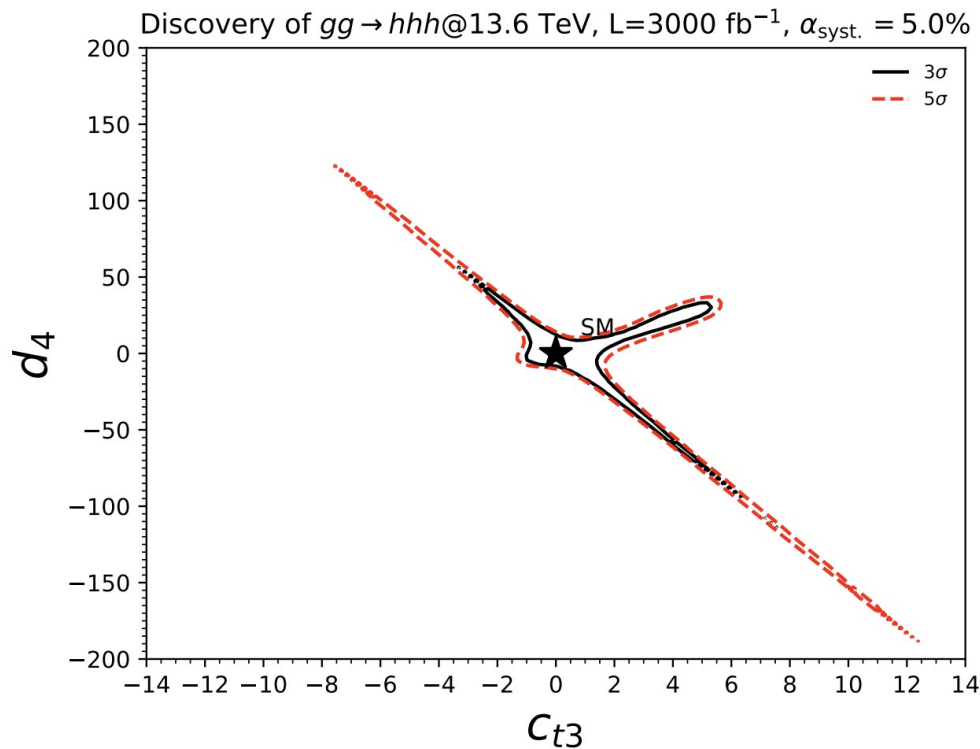


The couplings  $d_4$  and  $c_{t3}$  can be bounded by triple Higgs production

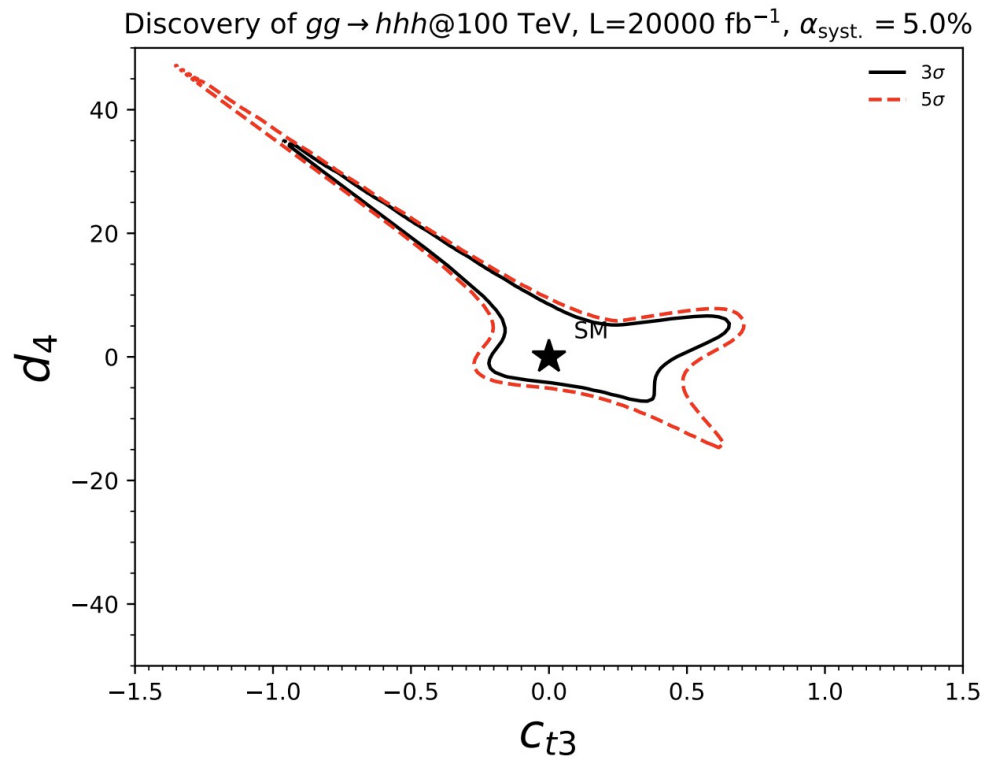
# Anomalous couplings

Evidence and discovery regions for triple Higgs production at proton-proton colliders

HL-LHC



FCC



*Evidence and discovery contours at proton colliders*

# Two Real Singlet Extension of the SM TRSM

$$V(\Phi, \phi_i) = V_{SM}(\Phi) + V(\Phi, S, X)$$

Reduce the number  
of parameters by  
imposing

$$\mathbb{Z}_2^S: S \rightarrow -S, X \rightarrow X$$

$$\mathbb{Z}_2^X: S \rightarrow S, X \rightarrow -X$$

$$V(\Phi, X, S) = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 \\ + \mu_X^2 X^2 + \lambda_X X^4 + \lambda_{\Phi S} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2$$

$$S = (\phi_S + v_S) / \sqrt{2}$$

$$X = (\phi_X + v_X) / \sqrt{2}$$

Change to  
the physical  
basis

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R(\theta_X, \theta_S) \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

$h_1 = h$  is the SM Higgs boson

$$M_1 = 125 \text{ GeV}$$

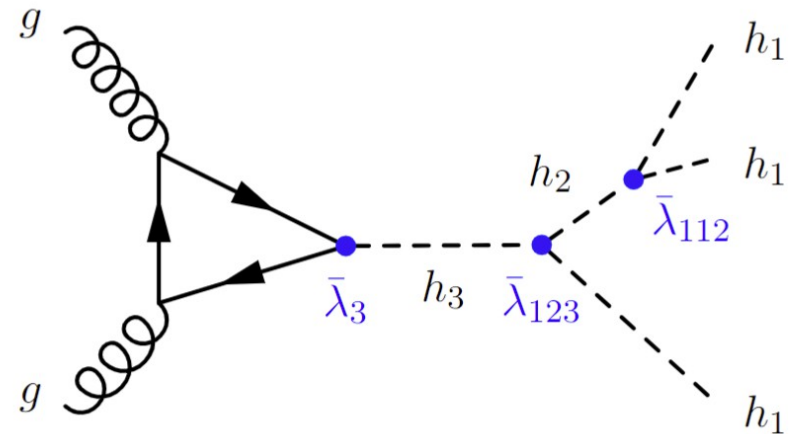
Free independent parameters

$$M_2, M_3, \theta_{hS}, \theta_{hX}, \theta_{SX}, v_S, v_X$$

# Old Benchmark Scenario of Study BP3

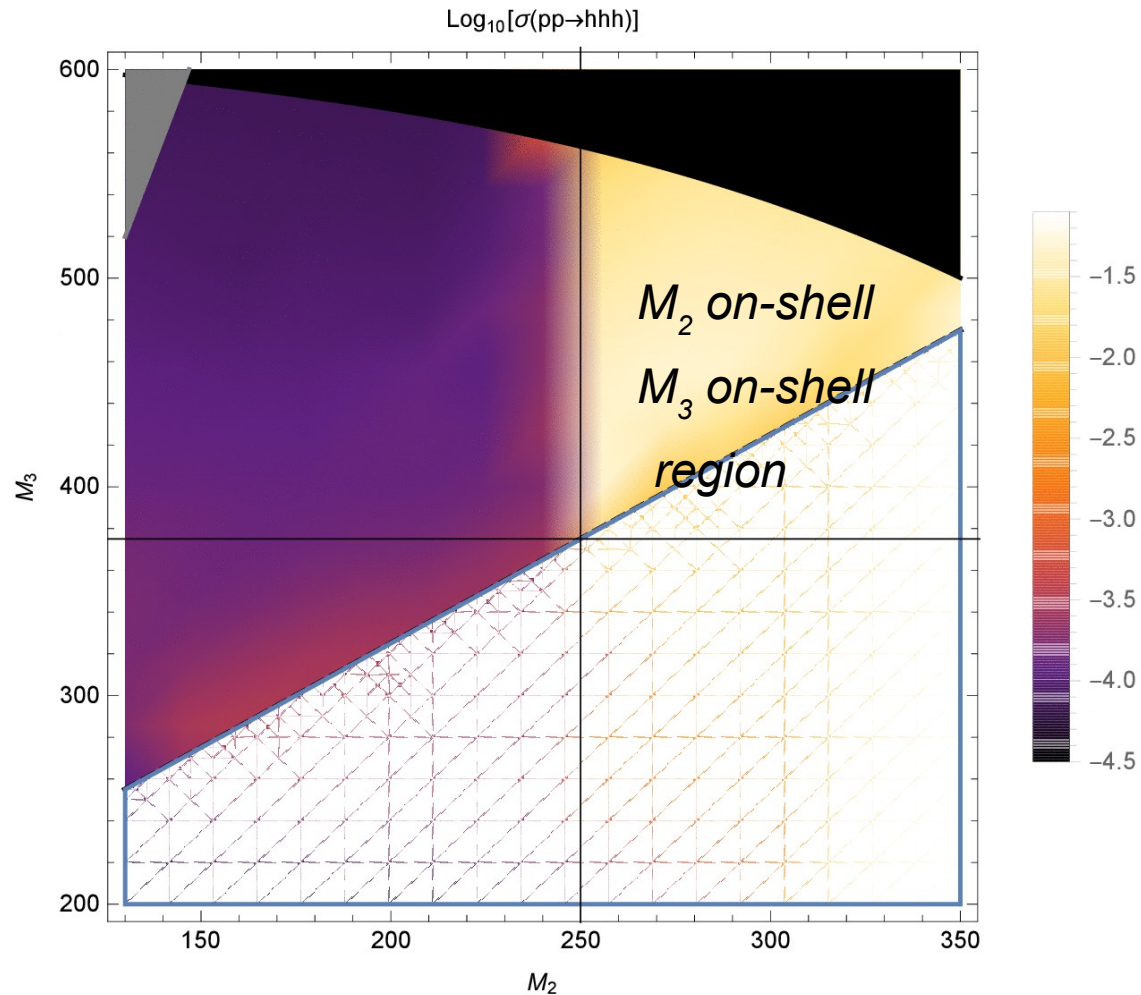
*The BP3 Scenario introduced in 1908.08554 which allows for a large  $h_1 h_1 h_1$  production while obeying current theoretical and experimental constraints.*

Parameter	Value
$M_1$	125.09 GeV
$M_2$	[125, 500] GeV
$M_3$	[255, 650] GeV
$\theta_{hS}$	-0.129
$\theta_{hX}$	0.226
$\theta_{SX}$	-0.899
$v_S$	140 GeV
$v_X$	100 GeV



We consider the mass hierarchy  $M_1 < M_2 < M_3$

# Production cross section



*The X-Section can reach up to 50 fb for  $M_2 \sim (263, 280)$  GeV and  $M_3 \sim 450$  GeV*

# Old benchmark points

Label	$(M_2, M_3)$ [GeV]	$\epsilon_{\text{Sig.}}$	$S _{300\text{fb}^{-1}}$	$\epsilon_{\text{Bkg.}}$	$B _{300\text{fb}^{-1}}$	$\text{sig} _{300\text{fb}^{-1}}$	$\text{sig} _{3000\text{fb}^{-1}}$
<b>A</b>	(255, 504)	0.025	14.12	$8.50 \times 10^{-4}$	19.16	2.92	9.23
<b>B</b>	(263, 455)	0.019	17.03	$3.60 \times 10^{-5}$	8.11	4.78	15.11
<b>C</b>	(287, 502)	0.030	20.71	$9.13 \times 10^{-5}$	20.60	4.01	12.68
<b>D</b>	(290, 454)	0.044	37.32	$1.96 \times 10^{-4}$	44.19	5.02	15.86
<b>E</b>	(320, 503)	0.051	32.54	$2.73 \times 10^{-4}$	61.55	3.76	11.88
<b>F</b>	(264, 504)	0.028	18.18	$9.13 \times 10^{-5}$	20.60	3.56	11.27
<b>G</b>	(280, 455)	0.044	38.70	$1.96 \times 10^{-4}$	44.19	5.18	16.39
<b>H</b>	(300, 475)	0.054	41.27	$2.95 \times 10^{-4}$	66.46	4.64	14.68
<b>I</b>	(310, 500)	0.063	41.42	$3.97 \times 10^{-4}$	89.59	4.09	12.94
<b>J</b>	(280, 500)	0.029	20.67	$9.14 \times 10^{-5}$	20.60	4.00	12.65

*These points are associated with large couplings which can break perturbativity at the energy scale  $M_Z$*

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*Determine phase space that enhances triple Higgs production in the TRSM  
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$$\lambda_{11} < \frac{\pi^2}{3} \approx 3.3, \quad \lambda_{22}, \lambda_{33} < \frac{4\pi^2}{9} \approx 4.4, \quad \lambda_{12}, \lambda_{13}, \lambda_{23} < 2\pi^2 \approx 20$$

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Relevant HiggsBounds Experimental Analyses			
Processes	Experiment	Int. Luminosity	arXiv ref.
$gg \rightarrow S \rightarrow W^+W^-, ZZ$	ATLAS	139 fb <sup>-1</sup>	2004.14636 [57]
$gg \rightarrow S \rightarrow ZZ$	ATLAS	139 fb <sup>-1</sup>	2009.14791 [58]
$gg \rightarrow S \rightarrow h_1 h_1 \rightarrow (b\bar{b})(\tau^+\tau^-)$	CMS	137 fb <sup>-1</sup>	2106.10361 [59]
$(b\bar{b}, \tau^+\tau^-, W^+W^-, ZZ, \gamma\gamma)(b\bar{b})$		35.9 fb <sup>-1</sup>	1811.09689 [60]
$gg \rightarrow S \rightarrow h_1 h_1 \rightarrow (b\bar{b}, \tau^+\tau^-, W^+W^-, \gamma\gamma)^2$	ATLAS	36.1 fb <sup>-1</sup>	1906.02025 [61]
$gg \rightarrow S \rightarrow h_1 h_1 \rightarrow (b\bar{b})(\gamma\gamma)$	ATLAS	36.1 fb <sup>-1</sup>	1807.04873 [62]
$gg \rightarrow S \rightarrow W^+W^-, ZZ$	ATLAS	36.1 fb <sup>-1</sup>	1808.02380 [63]
$pp \rightarrow S \rightarrow ZZ$ (incl. VBF)	CMS	35.9 fb <sup>-1</sup>	1804.01939 [64]
$gg \rightarrow S \rightarrow h_1 h_1 \rightarrow (b\bar{b})(b\bar{b})$	CMS	35.9 fb <sup>-1</sup>	1806.03548 [65]
$gg \rightarrow S \rightarrow h_1 h_1 \rightarrow (b\bar{b})(b\bar{b})$	ATLAS	36.1 fb <sup>-1</sup>	1806.03548 [65]

# New Benchmark points (LHC)

*Determine phase space that enhances triple Higgs production in the TRSM based on*

*Perturbative conditions*

$$\lambda_{11} < \frac{\pi^2}{3} \approx 3.3, \quad \lambda_{22}, \lambda_{33} < \frac{4\pi^2}{9} \approx 4.4, \quad \lambda_{12}, \lambda_{13}, \lambda_{23} < 2\pi^2 \approx 20$$

*Boundedness from below*

*Experimental constraints from HiggsTools (HiggsSignals and HiggsBounds)*

*We consider the threshold*

$$\sigma_{3h_1} > 100 \sigma_{3h_1}^{\text{SM}},$$

*Our analysis entailed 530,000 phase space points*

*Only 130 points fulfilled all the conditions*

*See Osama Karkout talk*

# New Benchmark points (LHC)

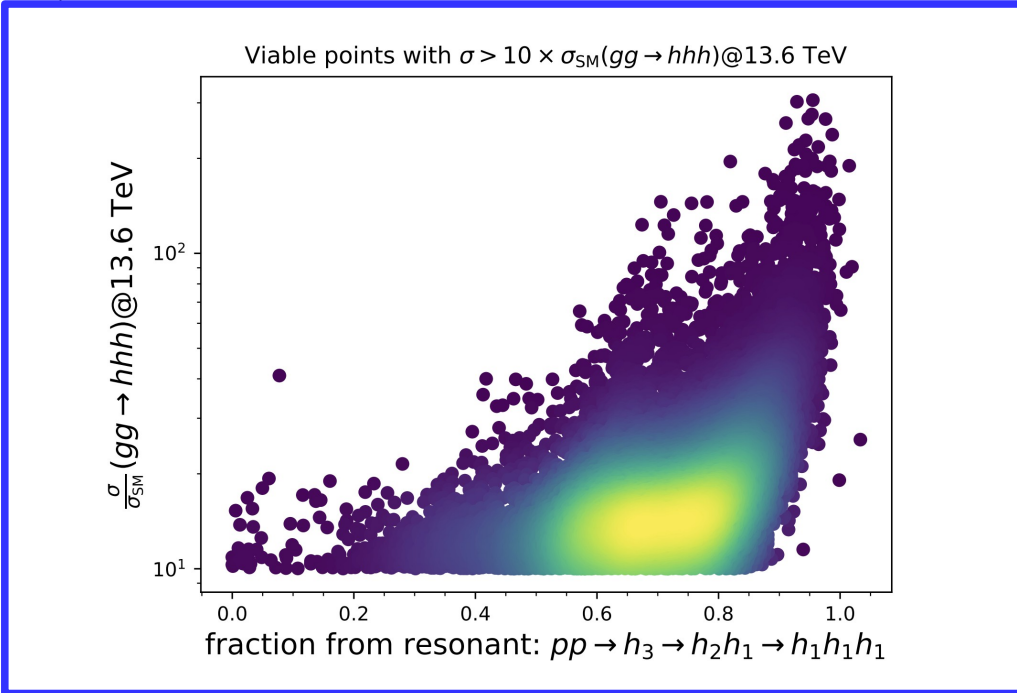
Benchmark points for enhanced triple Higgs production										
$M_2$	$M_3$	$v_2$	$v_3$	$\theta_{12}$	$\theta_{13}$	$\theta_{23}$	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\text{pert}}$ GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	$2.7 \times 10^2$	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	$1.8 \times 10^2$	7.4
268.6	452.7	137.8	784.8	0.263	0.023	0.645	275.616	0.954	$2.4 \times 10^2$	7.3
272.6	480.7	928.3	143.7	3.098	2.9	2.375	267.245	0.948	$1.4 \times 10^2$	7.3
269.0	409.8	138.0	599.4	0.244	0.004	0.773	266.439	0.976	$2.4 \times 10^2$	7.4
269.1	486.9	227.5	307.9	0.074	6.149	2.631	157.583	0.956	$4.3 \times 10^2$	7.3
259.2	577.0	289.0	275.6	0.137	6.148	2.324	145.470	0.781	$1.2 \times 10^4$	7.3
283.7	575.0	259.4	330.4	0.137	6.152	2.299	122.546	0.779	$3.0 \times 10^3$	7.3
264.3	469.3	207.3	359.5	0.285	6.277	0.692	119.121	0.999	$5.4 \times 10^3$	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	$5.3 \times 10^4$	7.4
259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	$1.2 \times 10^5$	7.4

Update of  
A. Papaefstathiou, T. Robens, GTX: 2101.00037/ JHEP 05 (2021), 193

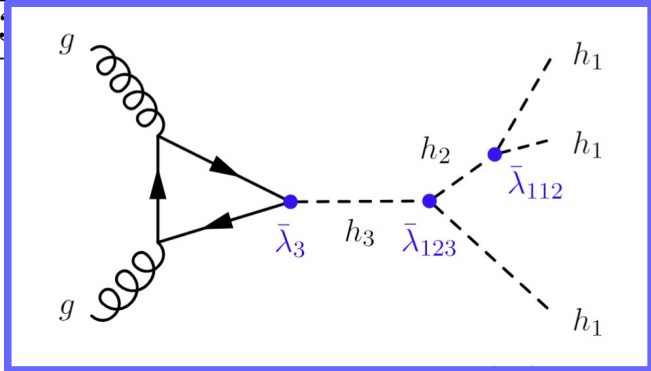
O. Karkout, A. Papaefstathiou, M. Postma, GTX, J. van de Vis, T du Pree:  
2404.12425

# New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production



	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\text{pert}}$ GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$						
23										
691	306.025	0.955	$2.7 \times 10^2$	7.3						
522	302.361	0.929	$1.8 \times 10^2$	7.4						
645	275.616	0.954	$2.4 \times 10^2$	7.3						
375	267.245	0.948	$1.4 \times 10^2$	7.3						
773	266.439	0.976	$2.4 \times 10^2$	7.4						
631	157.583	0.956	$4.3 \times 10^2$	7.3						
324	145.470	0.781	$1.2 \times 10^4$	7.3						
299	122.546	0.779	$3.0 \times 10^3$	7.3						
692	119.121	0.999	$5.4 \times 10^3$	7.3						
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	$5.3 \times 10^4$	7.4
259.2					46	1.047	103.717	0.973	$1.2 \times 10^5$	7.4



# New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production

$M_2$	$M_3$	$v_2$	$v_3$	$\theta_{12}$	$\theta_{13}$	$\theta_{23}$	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\text{pert}}$ GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
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# New Benchmark points (LHC)

**Benchmark points for enhanced triple Higgs production**

$M_2$	$M_3$	$v_2$	$v_3$	$\theta_{12}$	$\theta_{13}$	$\theta_{23}$	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\text{pert}}$ GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
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*In practice our points fulfil the following theoretical relationship*

$$\ln(\mu_{\text{pole}}/\mu_{\text{pert}}) = 2$$

$$\mu_{\text{pole}} \approx 7.4\mu_{\text{pert}}$$

# A simplified approach to double resonant triple Higgs production (LHC)

*When the double resonant channel is dominant*

$$\sigma(m_2, m_3, \Gamma_2, \Gamma_3, \kappa_3, \lambda_{123}, \lambda_{112}) = \hat{\sigma}_u(m_2, m_3) \times \frac{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}{\Gamma_2 \Gamma_3} = \hat{\sigma}_u(m_2, m_3) \times \rho^2$$

*“Unity” cross section*

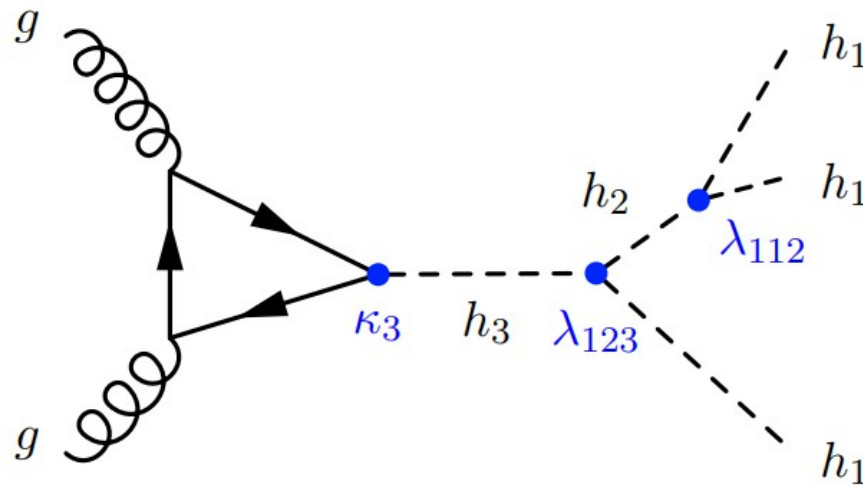
$$\kappa_3 = 1, \lambda_{123} = \lambda_{112} = 1 \text{ GeV}$$

$$\Gamma_2 = \Gamma_3 = 1 \text{ GeV}$$

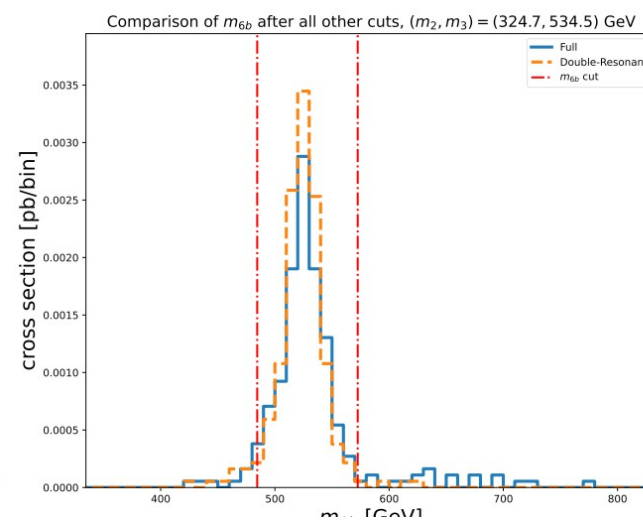
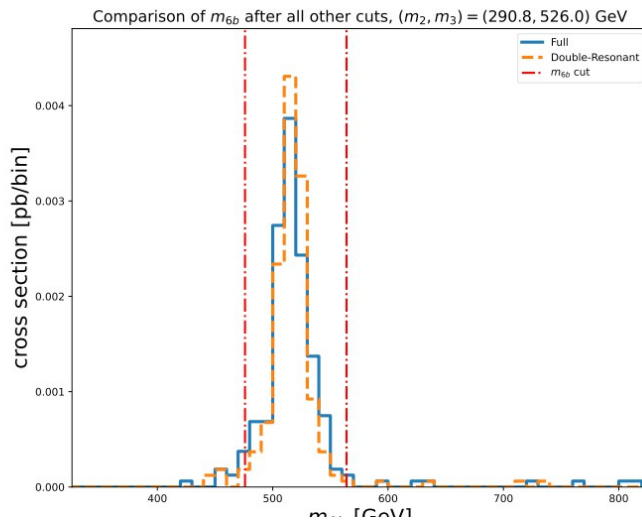
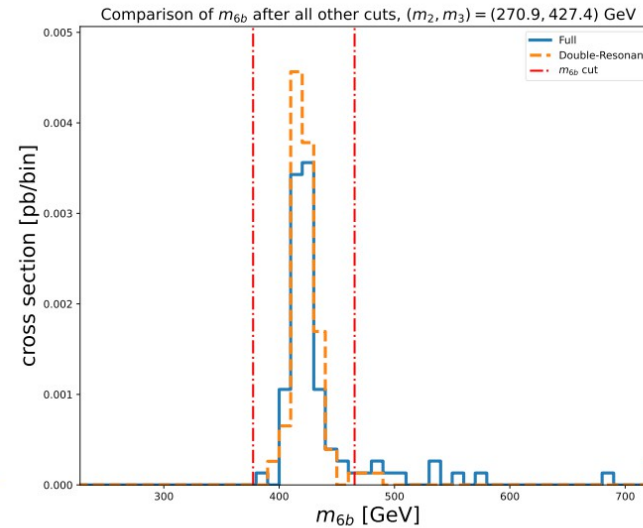
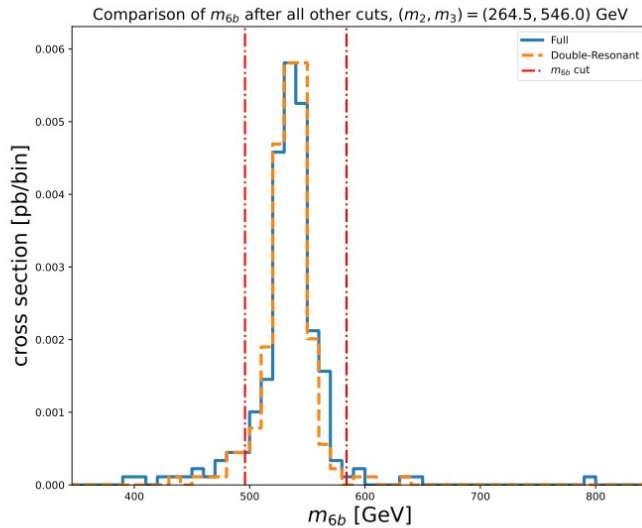
$$\Gamma_i \ll M_i$$

$$\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3) .$$

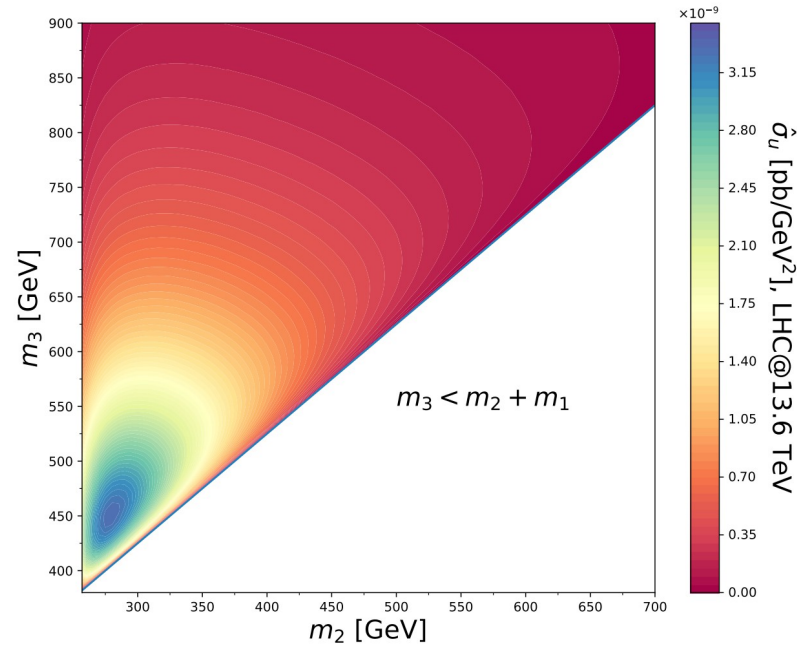
*Model dependent factor*



# A simplified approach to double resonant triple Higgs production (LHC)



# A simplified approach to double resonant triple Higgs production (LHC)



Define a “universal” set of selection cuts by optimizing

$$\Sigma_{\Pi} \equiv \prod_{i=1}^N \Sigma_i = \prod_{i=1}^N \frac{S_i}{\sqrt{B}} \quad \Rightarrow \quad \Sigma_{\Pi} = \left( \prod_{j=1}^N \frac{\sqrt{\mathcal{L}} \sigma_j}{\sqrt{\sigma_B}} \right) \left( \prod_{i=1}^N \frac{\varepsilon_i}{\sqrt{\varepsilon_{B,i}}} \right)$$

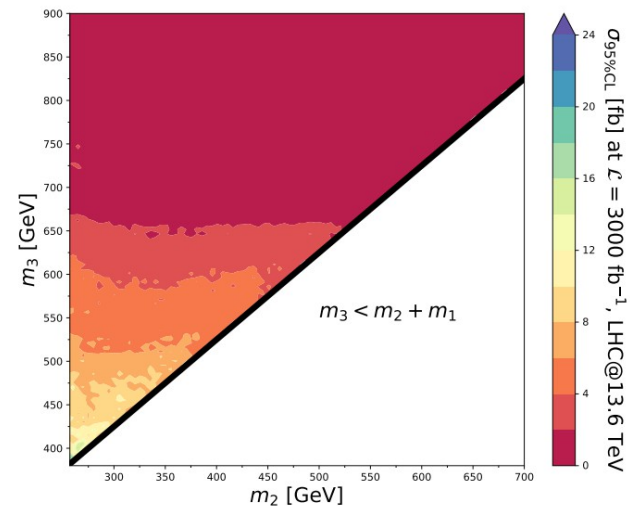
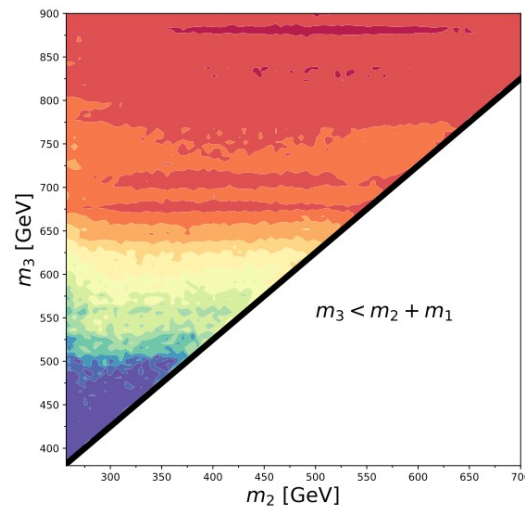
$$\Sigma = \frac{\varepsilon_S \sigma_S \mathcal{L}}{\sqrt{\varepsilon_B \sigma_B \mathcal{L}}} \quad \Sigma = 2 \quad \Rightarrow \quad \sigma_{95\%CL} = 2 \frac{\sqrt{\varepsilon_B \sigma_B}}{\varepsilon_S \sqrt{\mathcal{L}}}$$

# A simplified approach to double resonant triple Higgs production (LHC)

“Universal” set  
of selection cuts

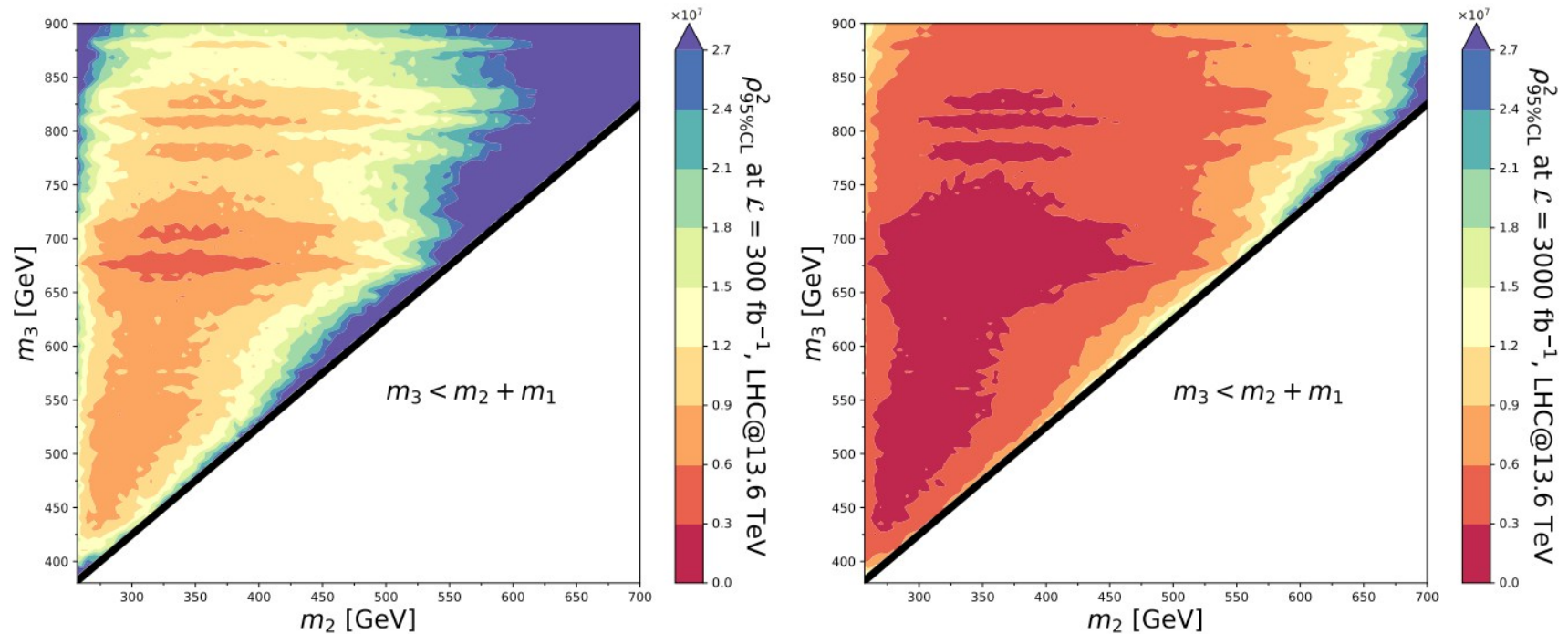


Observable	Constraint
$p_{Tmin,b}$	37.0
$ \eta_{b,max} $	2.95
$\chi^{2,(6)}$	12.0
$\chi^{2,(4)}$	34.0
$\Delta m_{6b}^{inv}$	+38.0 -50.0
$m_{4b}^{inv}$	$\leq m_3$
$p_T(h_1^i)$	$\geq [50, 50, 0]$
$\Delta m_{min,med,max}$	$\geq [15, 14, 20]$
$\Delta R(h_1^i, h_1^j)$	$\leq 3.5$
$\Delta R_{bb}(h_1)$	$\leq 3.5$



# A simplified approach to double resonant triple Higgs production (LHC)

For the TRSM



A. Papaefstathiou, GTX:  
2501.14866

# Closing Remarks

- The *6-b jets final state* is a good candidate to *search for  $h_1 h_1 h_1$  within and beyond the SM*
- *Extended scalar sectors can be probed through  $h_1 h_1 h_1$  even in the HL-LHC (consider for instance the TRSM).*
- *We have presented projections on the potential values that future hadron colliders can explore on the cubic and quartic self couplings.*
- *The double resonance dominance allows to find a **universal simplified approach** which allows to get bounds on potential optimal selection points in a model independent way.*

# Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 945422



Backup

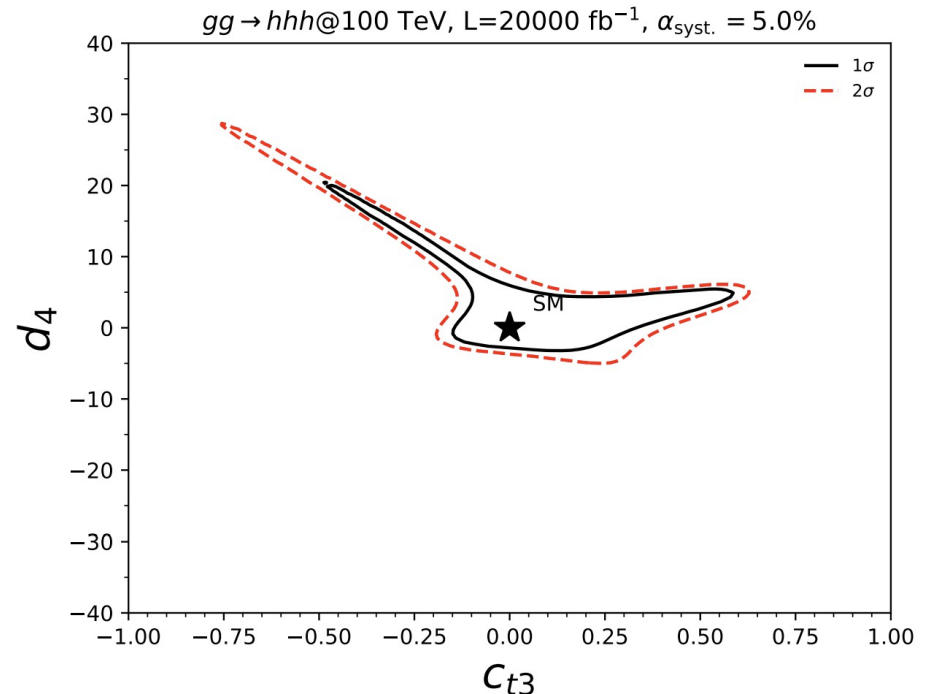
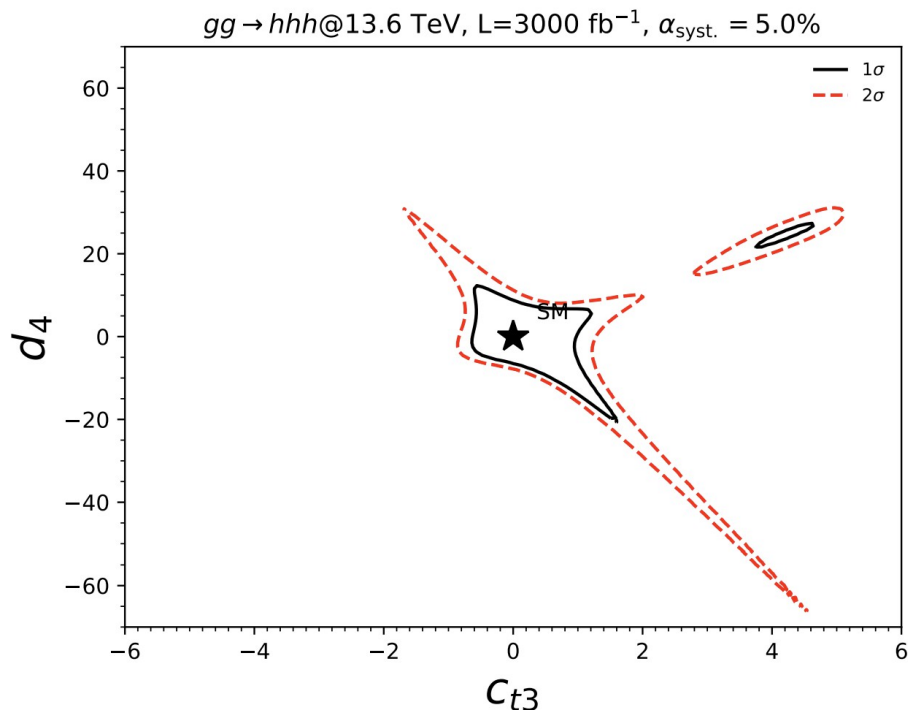
$$\begin{aligned}
\mathcal{L}_{h^n} = & -\mu^2 |H|^2 - \lambda |H|^4 - (y_t \bar{Q}_L H^c t_R + y_b \bar{Q}_L H b_R + \text{h.c.}) \\
& + \frac{c_H}{2\Lambda^2} (\partial^\mu |H|^2)^2 - \frac{c_6}{\Lambda^2} \lambda_{\text{SM}} |H|^6 + \frac{\alpha_s c_g}{4\pi \Lambda^2} |H|^2 G_{\mu\nu}^a G_a^{\mu\nu} \\
& - \left( \frac{c_t}{\Lambda^2} y_t |H|^2 \bar{Q}_L H^c t_R + \frac{c_b}{\Lambda^2} y_b |H|^2 \bar{Q}_L H b_R + \text{h.c.} \right),
\end{aligned}$$

# Anomalous couplings

Confidence regions on the anomalous couplings at proton-proton colliders

HL-LHC

FCC



*In this plot it is assumed that the SM is the underlying theory*

# Adding an Extra-Scalar Singlet

## The x-SM potential

$$V(\Phi, S) = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \left(\frac{a_1}{2}\right) (\Phi^\dagger \Phi) S$$

*Kotwal et al. 1605.06123*

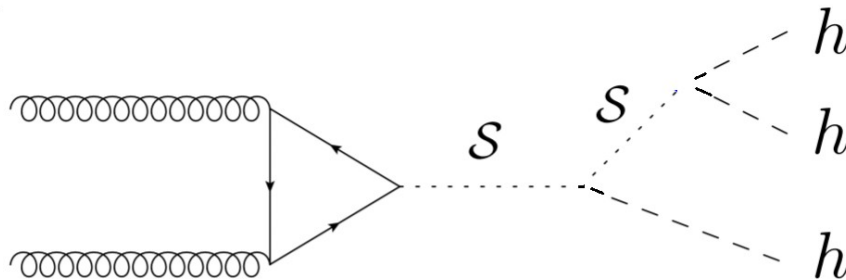
$$+ \left(\frac{a_2}{2}\right) (\Phi^\dagger \Phi) S^2 + \left(\frac{b_2}{2}\right) S^2 + \left(\frac{b_3}{3}\right) S^3 + \left(\frac{b_4}{4}\right) S^4$$

Mass  
Eigenstates

$$h_1 = h \cos \theta + \phi_s \sin \theta$$

$$h_2 = -h \sin \theta + \phi_s \cos \theta$$

$$S = (\phi_s + v_s) / \sqrt{2}$$



Triple Higgs production in  
the presence of an  
extra-scalar

# Analysis results

Benchmark points which lead to a Strong-First Order EW Phase Transition

Benchmark	$\cos \theta$	$\sin \theta$	$m_2$ (GeV)	$\Gamma_{h_2}$ (GeV)	$x_0$ (GeV)	$\lambda$	$a_1$ (GeV)	$a_2$	$b_3$ (GeV)	$b_4$	$\frac{\sigma(h_1 h_1)}{\sigma(hh)_{SM}}$	$\frac{\sigma(h_1 h_1 h_1)}{\sigma(hhh)_{SM}}$
B1max	0.976	0.220	341	2.42	257	0.92	-377	0.392	-403	0.77	22.44	60.55
B2max	0.982	0.188	353	2.17	265	0.99	-400	0.446	-378	0.69	22.43	56.69
B3max	0.983	0.181	415	1.59	54.6	0.17	-642	3.80	-214	0.16	6.43	3.01
B4max	0.984	0.176	455	2.08	47.4	0.18	-707	4.63	-607	0.85	5.19	3.37
B5max	0.986	0.164	511	2.44	40.7	0.18	-744	5.17	-618	0.82	3.49	2.94
B6max	0.988	0.153	563	2.92	40.5	0.19	-844	5.85	-151	0.083	2.79	3.60
B7max	0.992	0.129	604	2.82	36.4	0.18	-898	7.36	-424	0.28	2.51	4.70
B8max	0.994	0.113	662	2.97	32.9	0.17	-976	8.98	-542	0.53	2.28	4.91
B9max	0.993	0.115	714	3.27	29.2	0.18	-941	8.28	497	0.38	1.98	2.68
B10max	0.996	0.094	767	2.83	24.5	0.17	-920	9.87	575	0.41	1.95	2.35
B11max	0.994	0.105	840	4.03	21.7	0.19	-988	9.22	356	0.83	1.76	1.03

Identification of the  
Extra-scalar at 100 TeV

Benchmark	Significance
B1max	46.6
B2max	42.9
B3max	2.9
B4max	3.7
B5max	3.0
B6max	3.8
B7max	5.3
B8max	7.8
B9max	5.9
B10max	4.9
B11max	2.3