N-Jettiness and LHC Jet Masses at NNLL

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MIT & Harvard

Boost workshop, Princeton
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N-Jettiness Event Shape

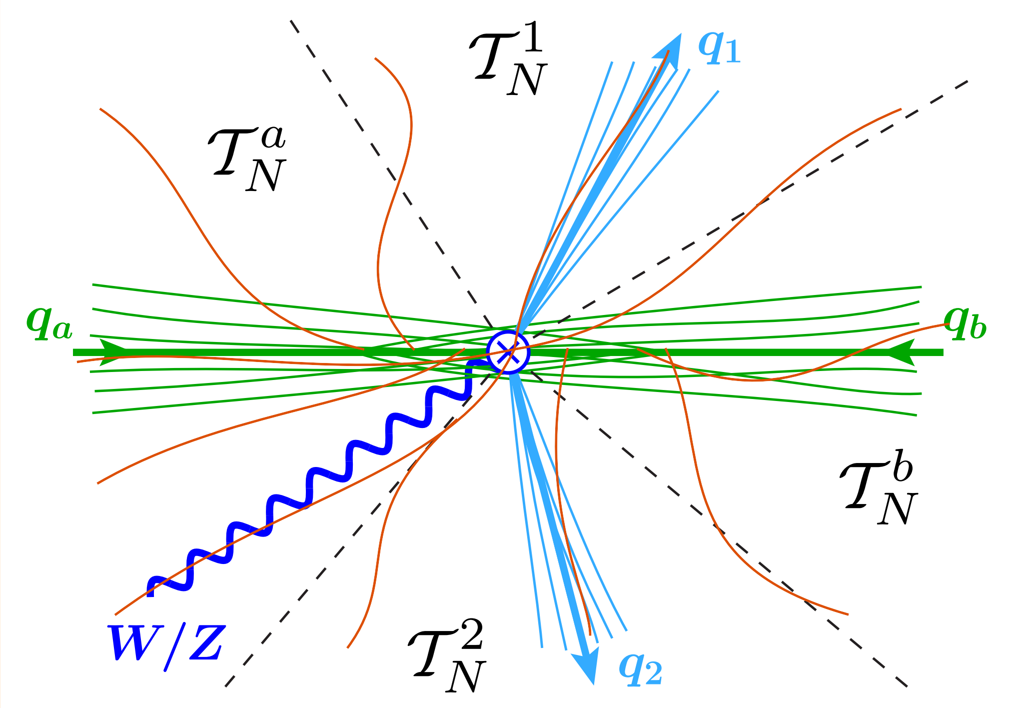
IS, Tackmann, Waalewijn
arXiv: 1004.2489

$$\mathcal{T}_N = \mathcal{T}_N(q_a, q_b, q_1, \ldots, q_N)$$

$\mathcal{T}_N \to 0$ for N-jets

Factorization Friendly

$$\mathcal{T}_N = \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \ldots + \mathcal{T}_N^N$$



Want to calculate N-jet exclusive cross-sections.

eg. differential jet masses

$$\frac{d\sigma}{d\mathcal{T}_N^a \cdots d\mathcal{T}_N^N}$$

Jouttenus, IS, Tackmann, Waalewijn
arXiv: 1102.4344

Why?

- sum logs beyond the parton shower (up to NNLL)
- realistic estimates for theory errors
- test and tune Monte Carlo
- reweight Monte Carlo (eg. Higgs Search)

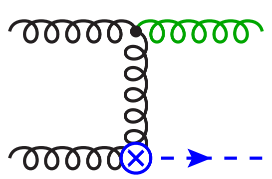
Exclusive Jet Measurements

- signal may prefer N-jets (eg. top is 2, 4, or 6)
- backgrounds vary with # of jets

$\Rightarrow$ Be exclusive in the number of jets

- $pp \to H(\to WW^*) + 0, 1, 2$ jets
- Also relevant for $H \to \gamma\gamma$

- study jet substructure, study exclusive sub-jets

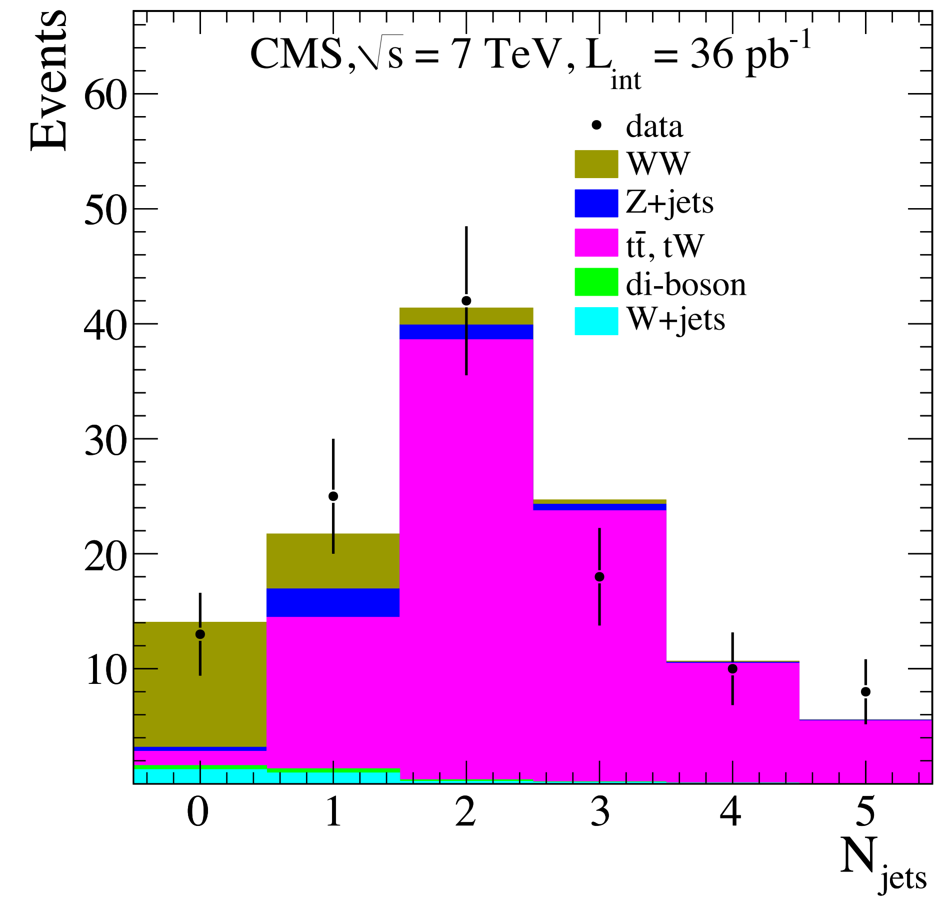


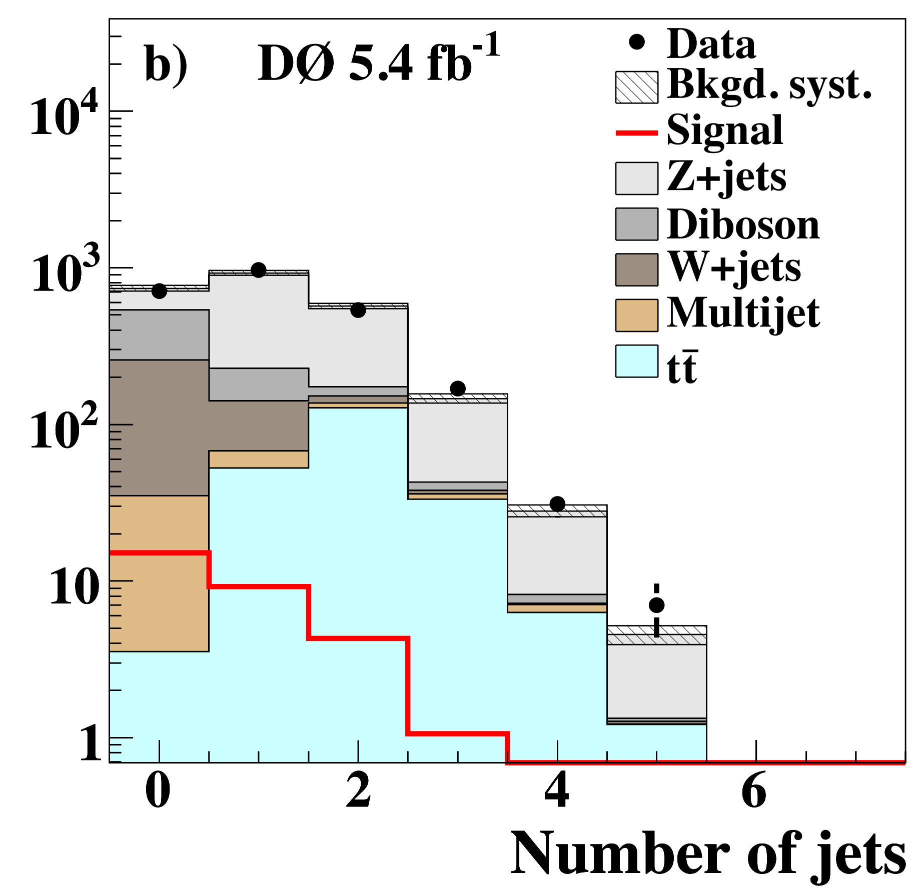
$\Rightarrow$ t-channel singularities

produce Sudakov double logarithms

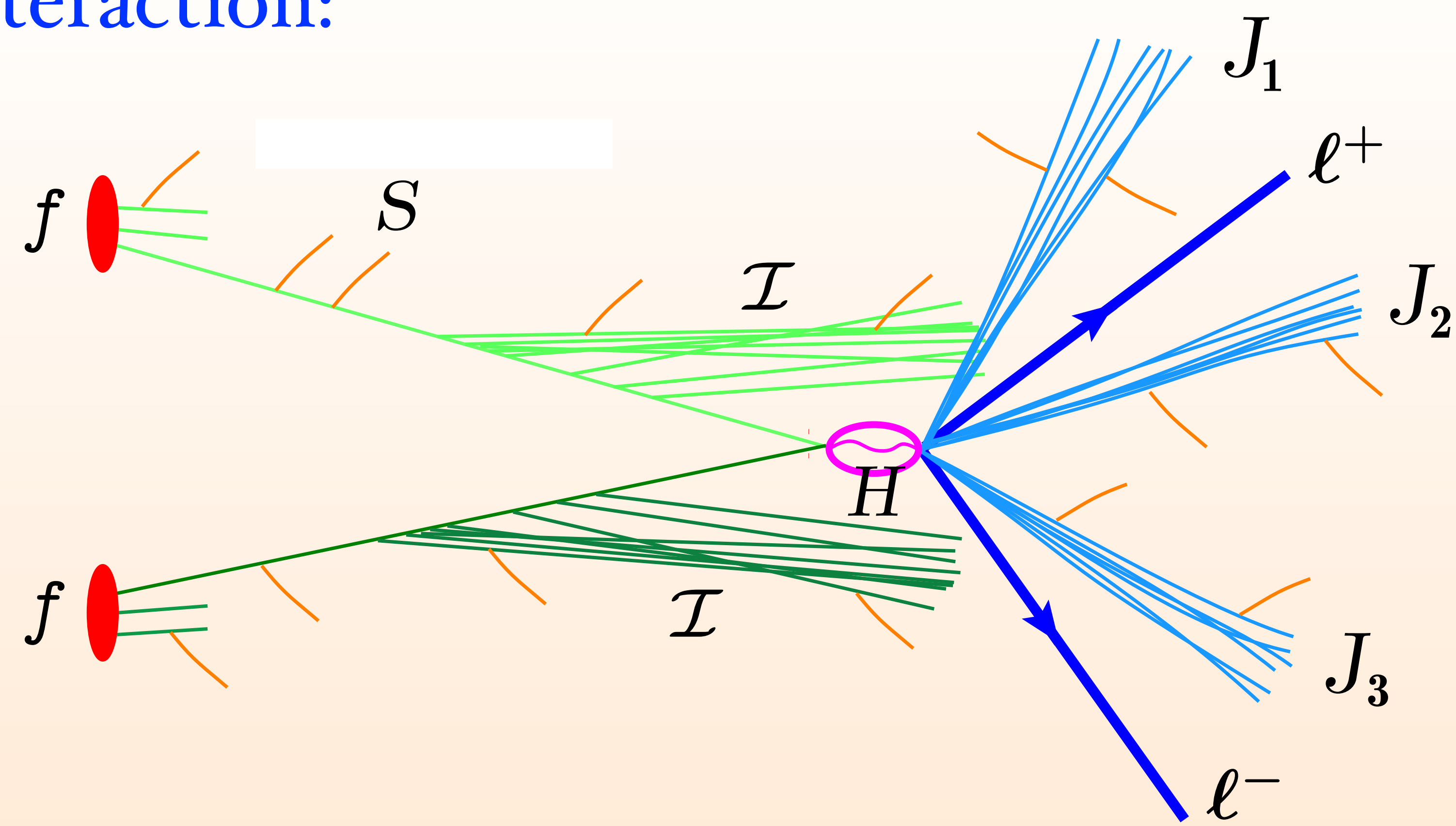
$$\sigma(p_T^{\rm cut}) \propto 1 - \frac{3\alpha_s}{\pi} 2 \ln^2 \frac{p_T^{\rm cut}}{m_H} + \cdots$$

$$\sigma(\mathcal{T}^{\rm cut}) \propto 1 - \frac{3\alpha_s}{\pi} \ln^2 \frac{\mathcal{T}^{\rm cut}}{m_H} + \cdots$$





Typical Event with Hard Interaction:



Factorization:

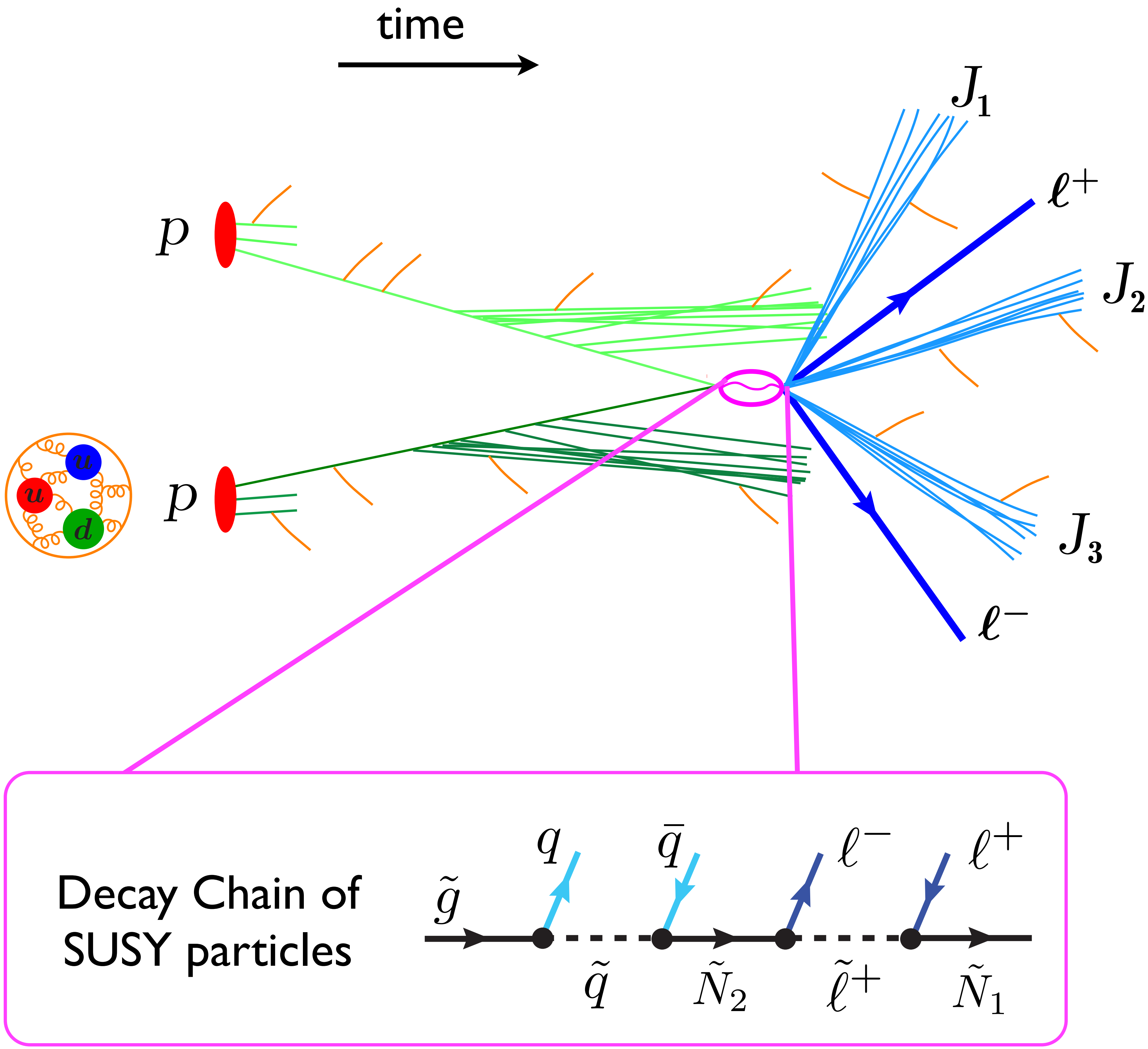
"cross section can be computed as product of independent pieces"

Shower MC programs assume factorization:

$d\sigma =$ [initial state parton shower] $\otimes$ [hard scattering fixed order perturbative computation] $\otimes$ [final state parton showers] $\otimes$ [hadronization model, underlying event, ...]

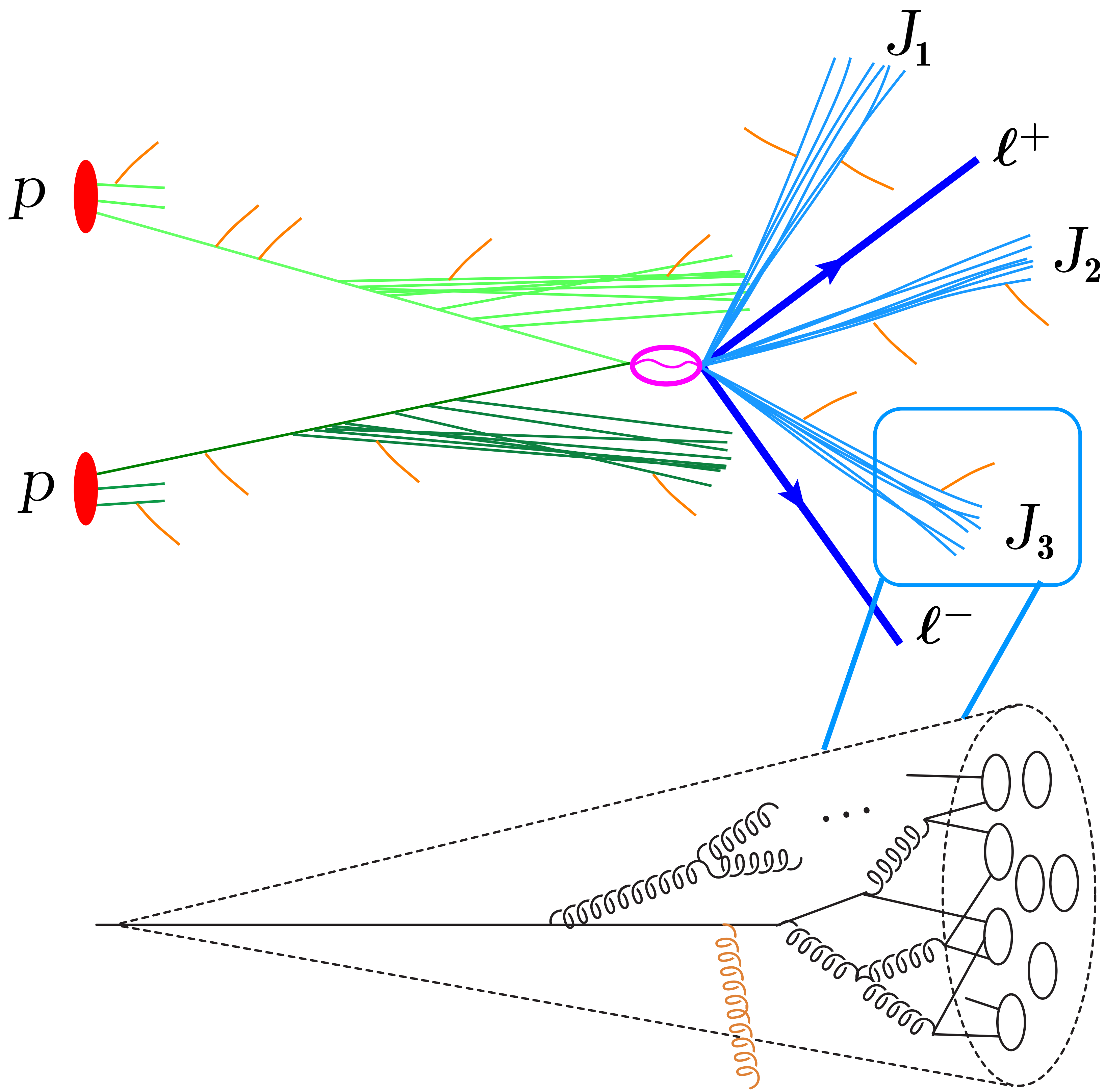
(with parton distributions)

Events with a Hard Interaction:



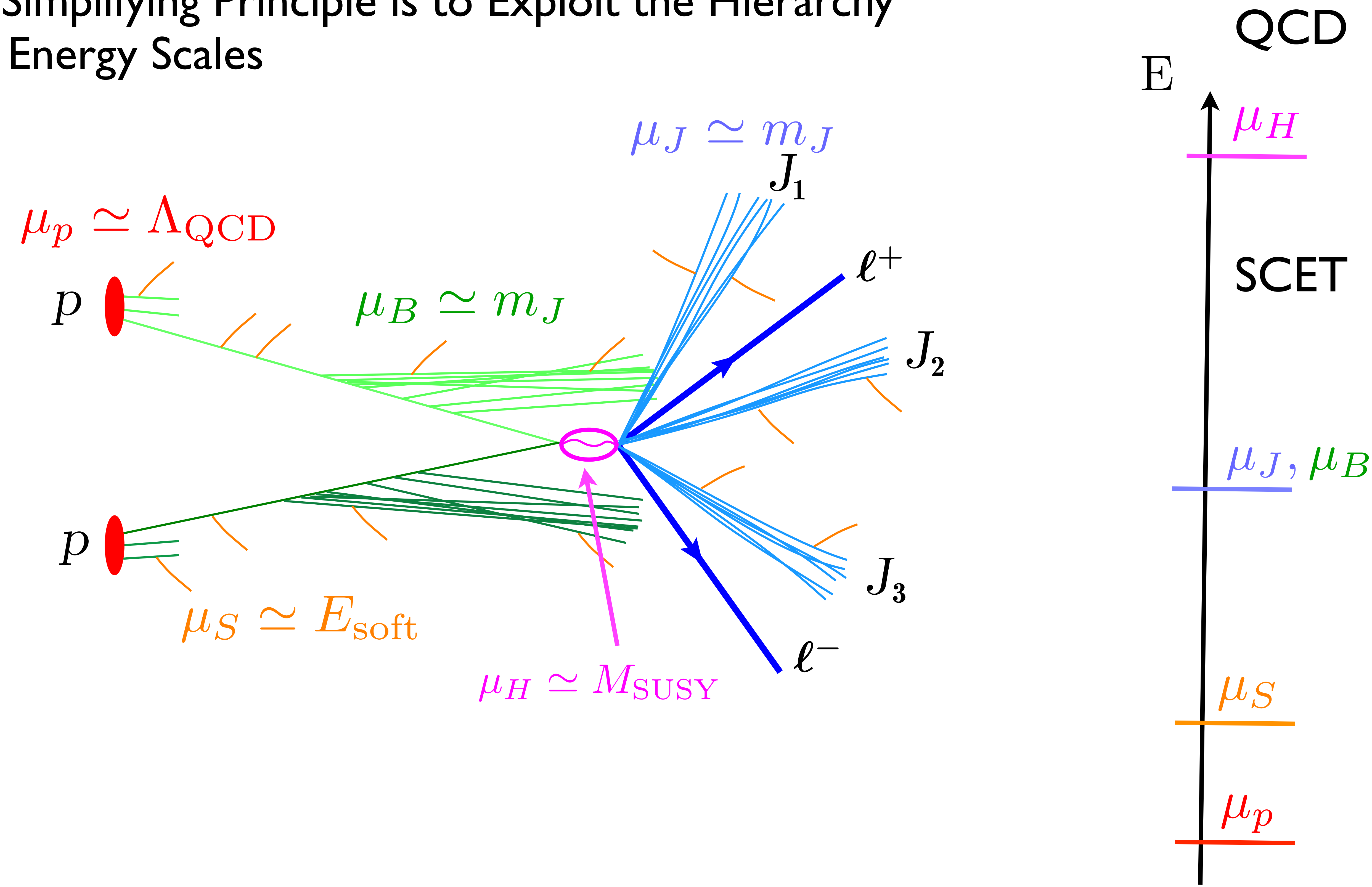
Search for New
Heavy Particles
at short distances

Events with a Hard Interaction:



Quarks and Gluons
Form Jets

Key Simplifying Principle is to Exploit the Hierarchy of Energy Scales



SCET = Soft-Collinear Effective Theory

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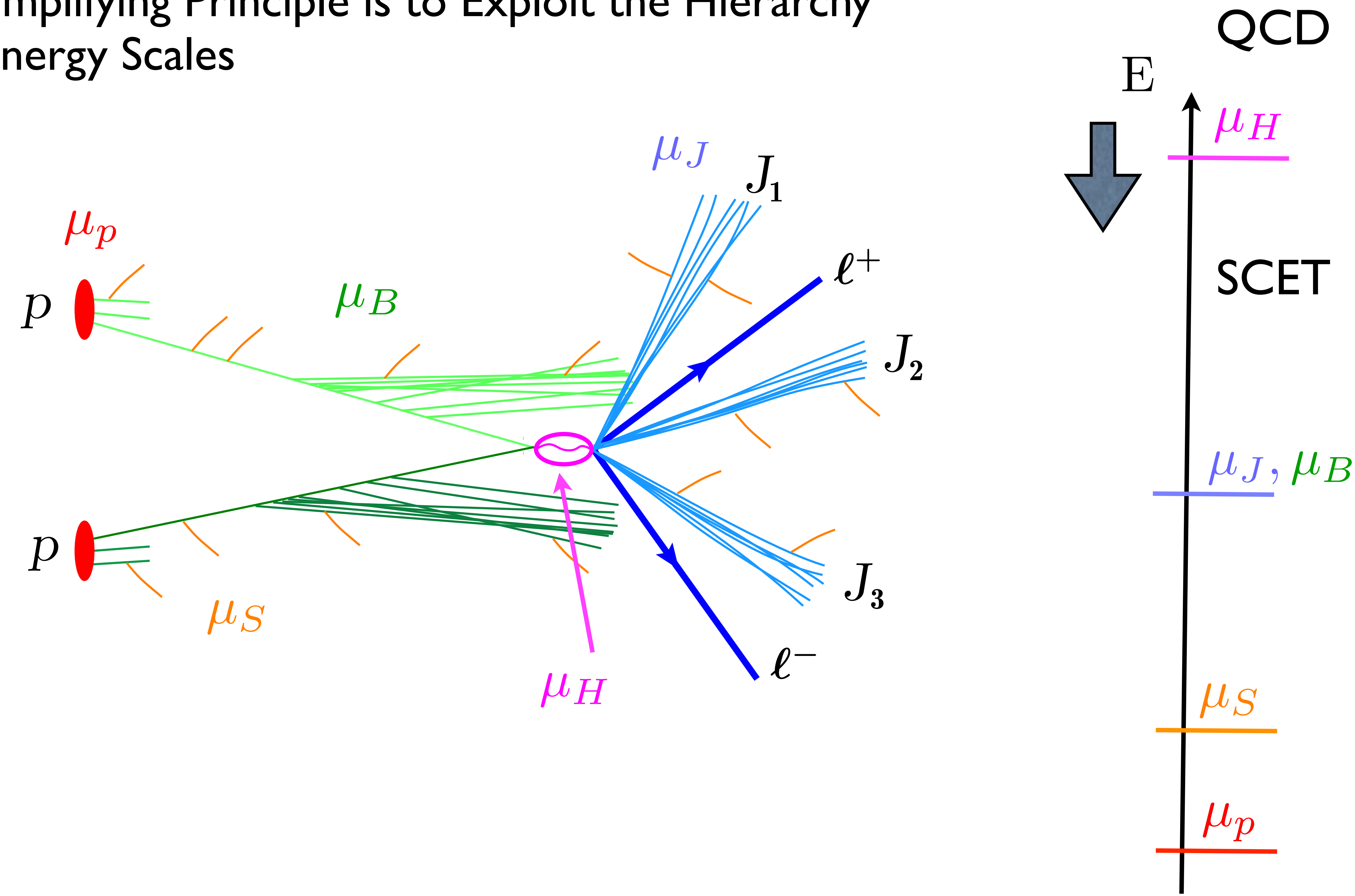

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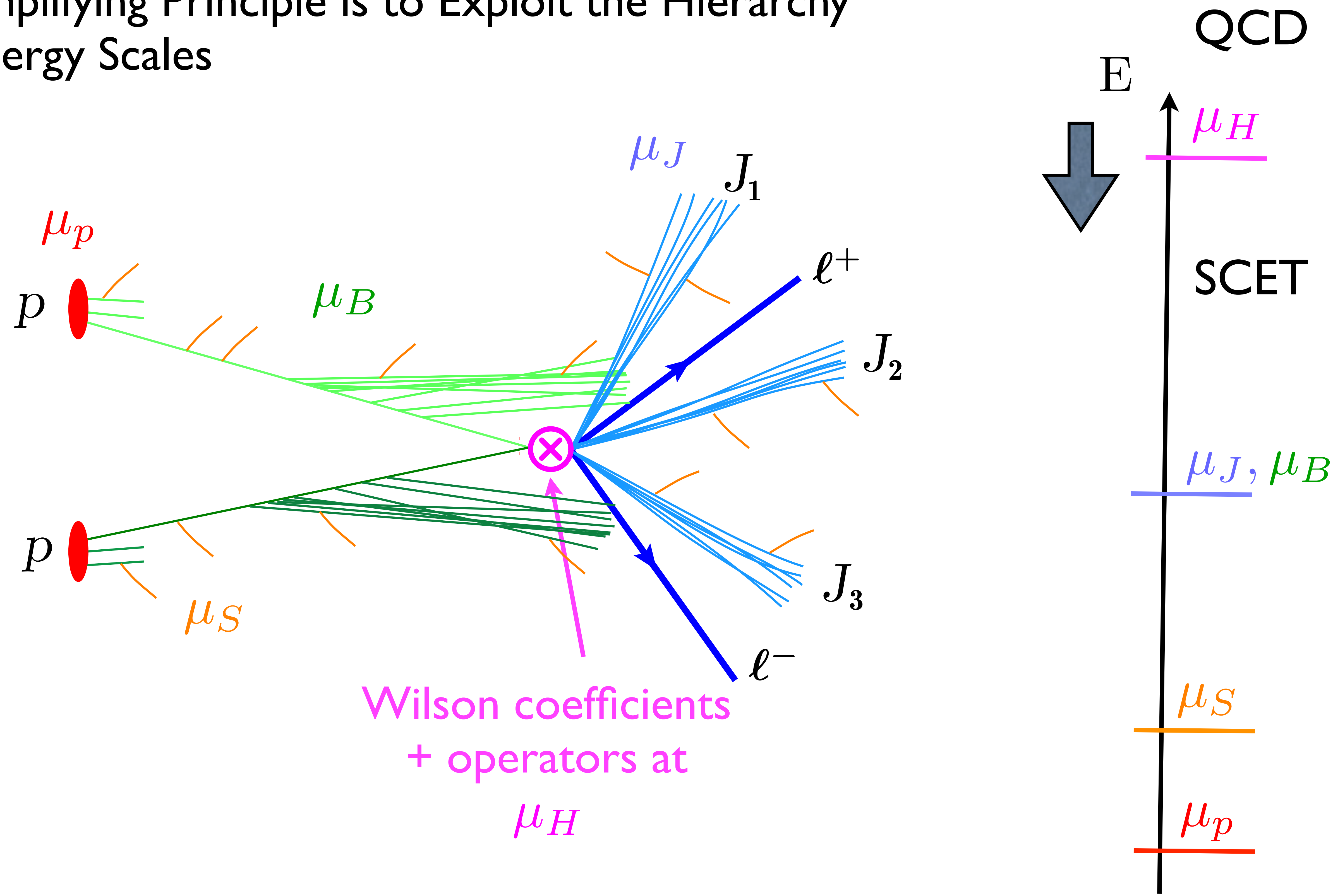

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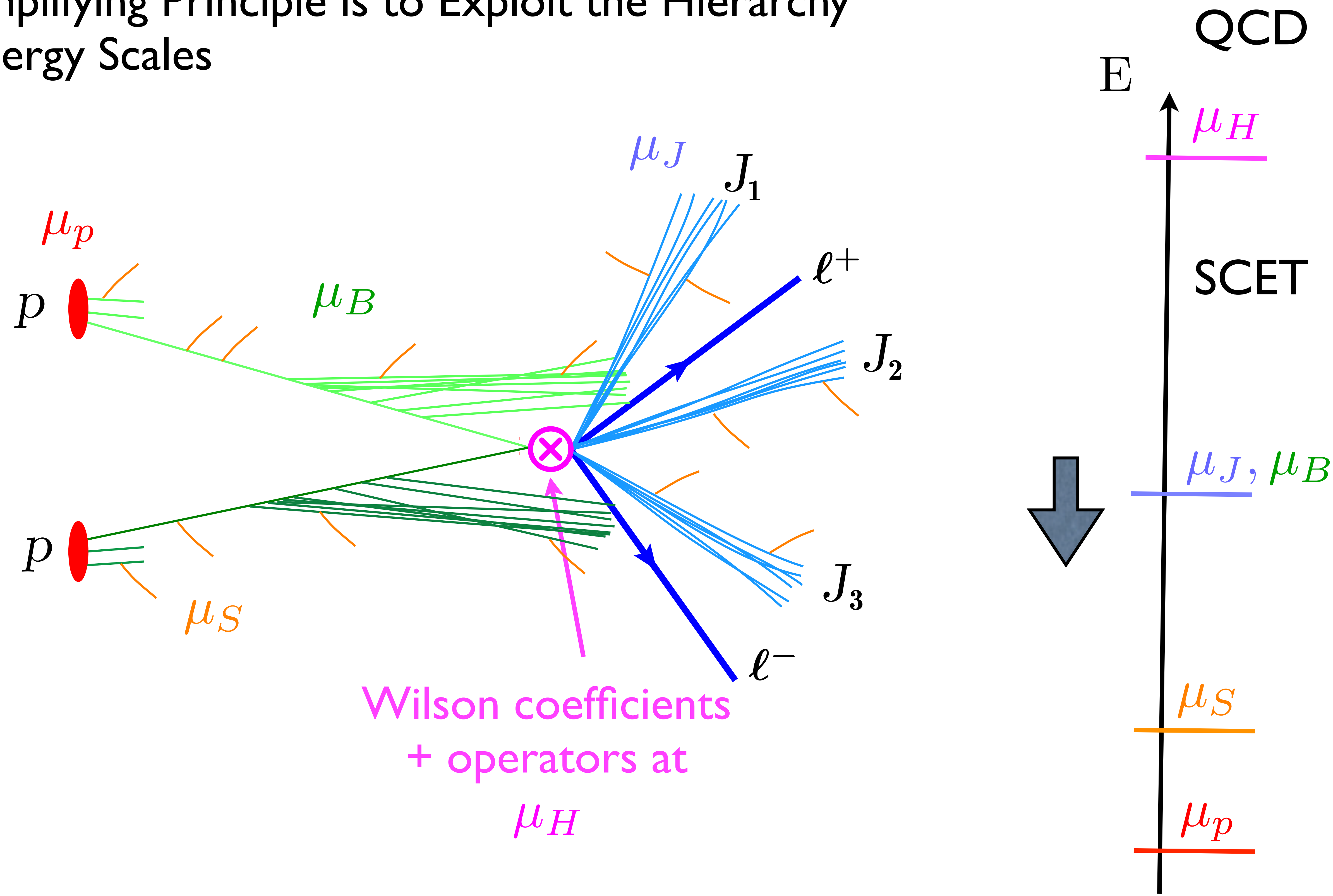
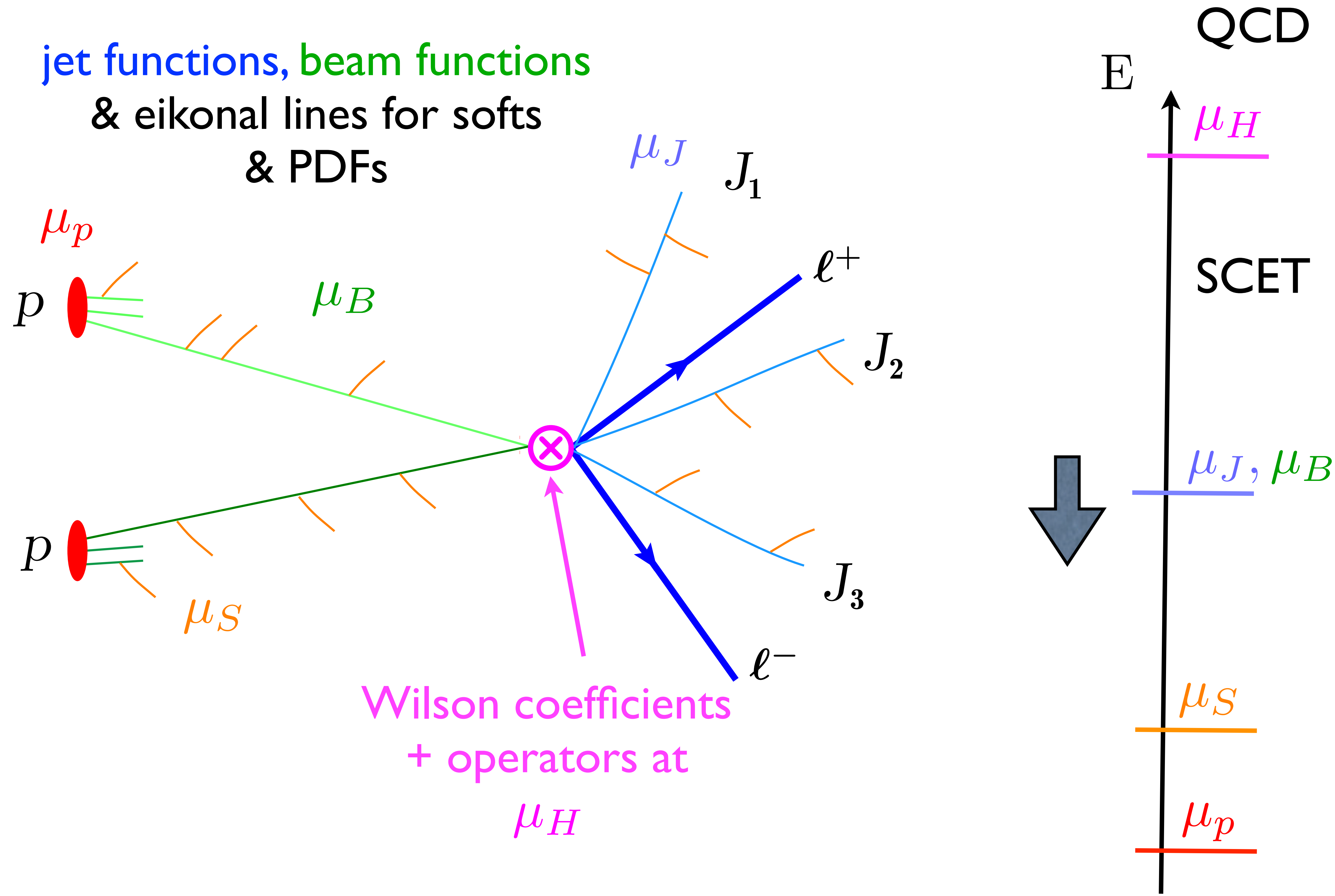
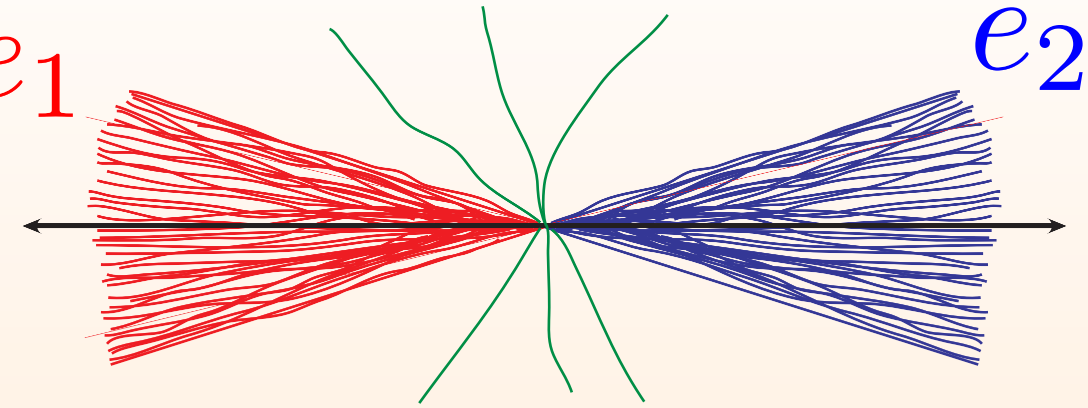




Factorization: $$d\sigma = f_{a,b} \otimes \mathcal{I}_{a,b} \otimes H \otimes \prod_i J_i \otimes S$$

Λ_{QCD} $\quad \mu_B \quad \mu_H \quad \mu_J \quad \mu_S$

Factorization Friendly Observables



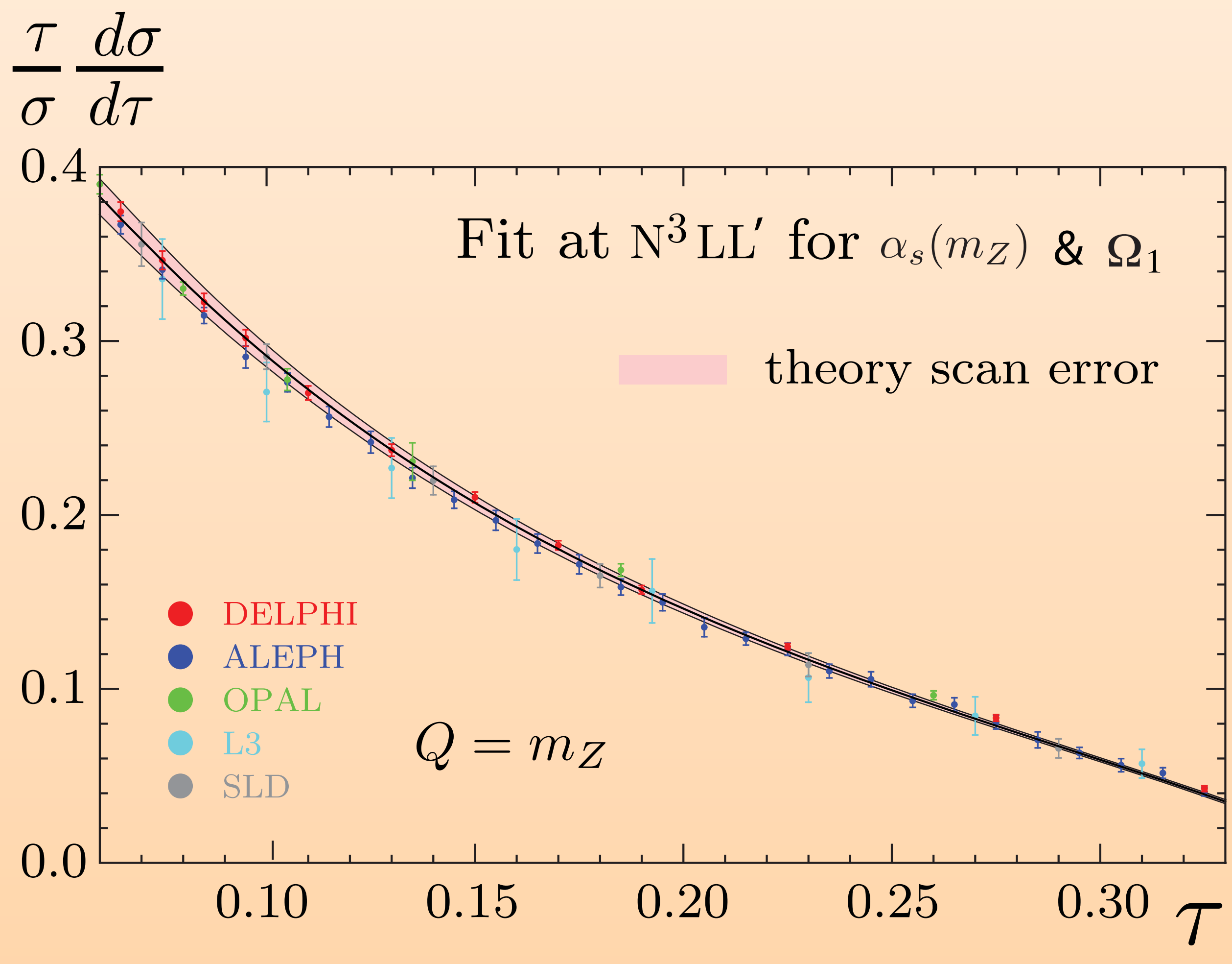
eg. $e^+e^- \to 2$ jets

dijet event shape

$$e = e_1 + e_2 + e_s$$

$$\frac{d\sigma}{de} = H(Q) \int de_1 de_2 de_s\, J(e_1) J(e_2) S(e_s) \delta(e - e_1 - e_2 - e_3)$$

eg. thrust



$\mathrm{N^3LL} + \mathcal{O}(\alpha_s^3)$

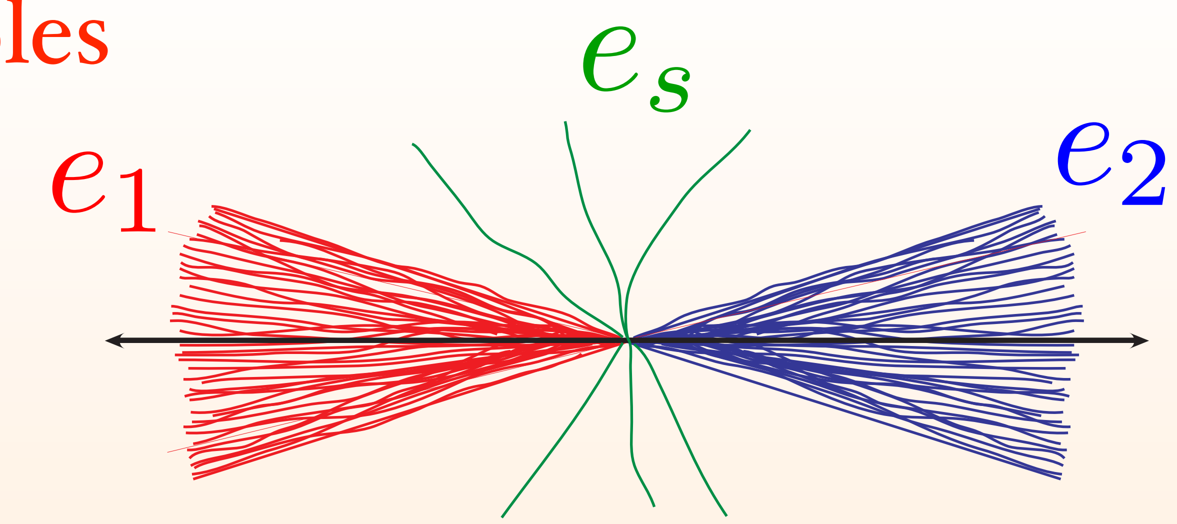
Gehrmann et al. & Weinzierl

Becher & Schwartz

Abbate, Fickinger, Hoang, Mateu, I.S.

global fit with power corrections

Factorization Friendly Observables



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Not as friendly for resummation:

soft radiation grouped by jet algorithms

Procedures that introduce multiple jet or soft scales

see eg. Ellis, Hornig, Lee, Vermilion, Walsh;
Banfi, Dasgupta, Khelifa-Kerfa, Marzani; Kelley, Schwartz, Zhu

N-Jettiness $\mathcal{T}_N$

$pp \to \text{jets},\ pp \to W/Z + \text{jets},\ \ldots$

consider an inclusive N-jet sample with jet energies E_i &
directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

$$q_i^\mu = E_i(1, \hat{n}_i)$$

$$q_a^\mu = \frac{1}{2} x_a\, E_{\rm cm}(1, \hat{z})\,,$$

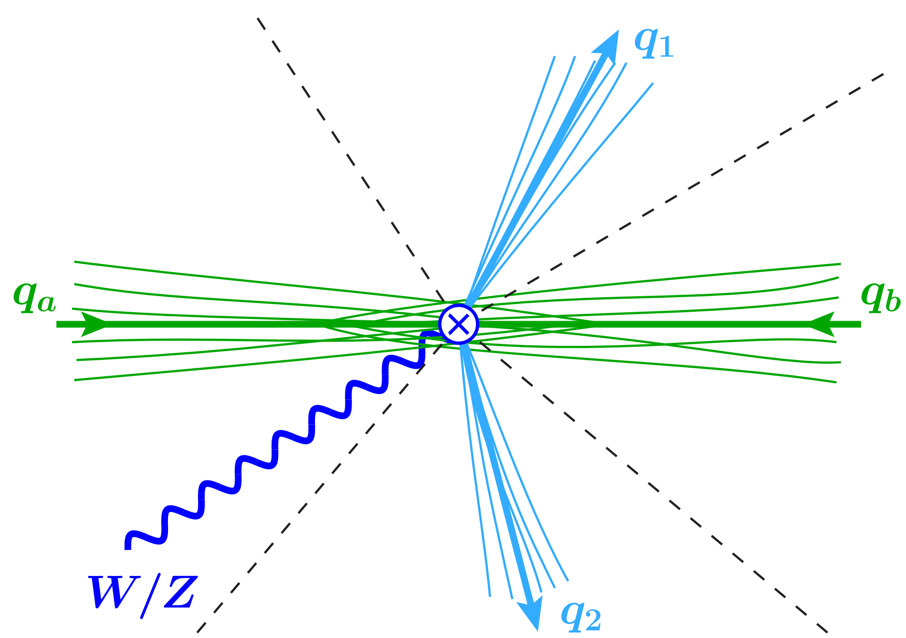
$$q_b^\mu = \frac{1}{2} x_b\, E_{\rm cm}(1, -\hat{z})$$

$$x_a x_b = \frac{Q^2}{E_{\rm cm}^2} = \frac{(q_1 + \ldots + q_N + q)^2}{E_{\rm cm}^2}$$

$$\ln \frac{x_a}{x_b} = Y = \ldots$$

(set $x_a = x_b = 1$ for cases with MET)

measure $\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$



- $d_{a,b}(p_k)$, $d_j(p_k)$: Distance of particle k to beam and jet directions
- Divides phase space into N jet regions and 2 beam regions

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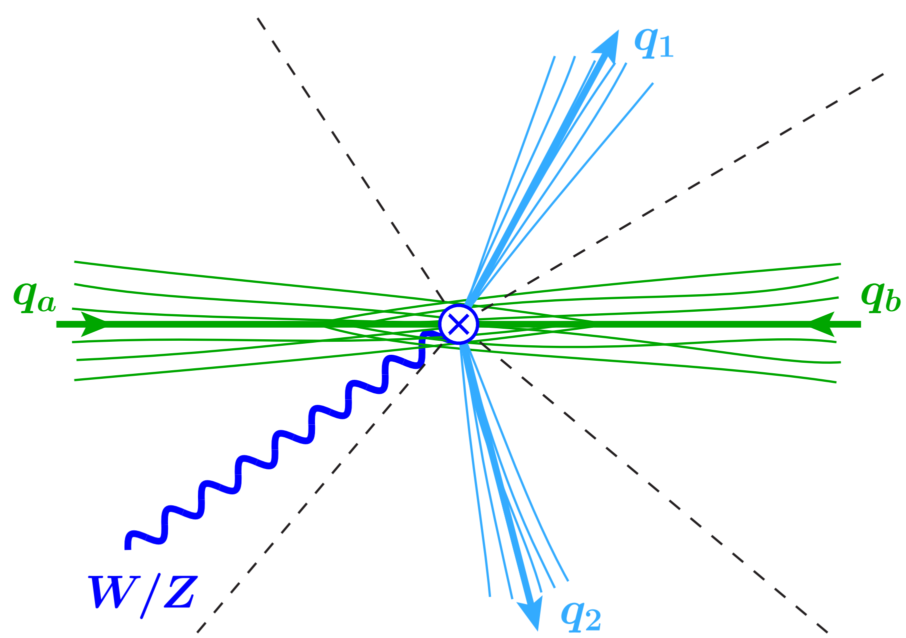
measure $$\mathcal{T}_N = \sum_k \min\Big\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \Big\}$$



- Here Q_j determines the measure
- Small $\mathcal{T}_N$ constrains us to N-jets (one added scale)

$$\mathcal{T}_N^{\rm alg.1} = \mathcal{T}_N^{\rm alg.2} + \mathcal{O}[(\mathcal{T}_N^{\rm alg.2})^2]$$

Large $\mathcal{T}_N$ has >N jets

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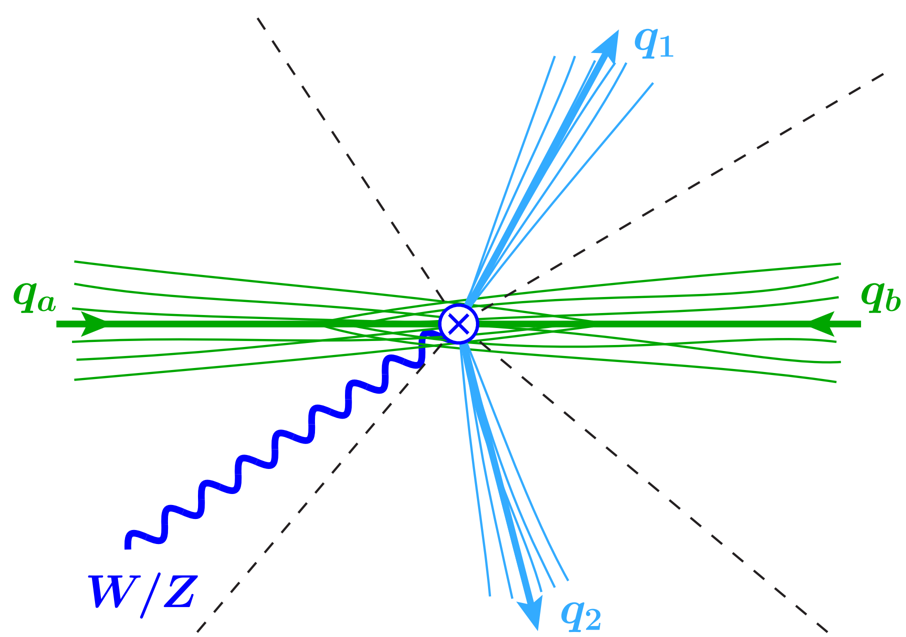


"make it a true event shape"

- Determine q_i by minimization

  For $Q_i = |\vec{q}_{iT}|$, $\vec{p}^{\,i}_{\rm jet} = \sum_{k\in i} \vec{p}_k$

- Extension to N-subjettiness

Thaler, Van Tilburg

[Jesse's talk]

N-Jettiness Factorization

$$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$

$$\mathcal{T}_N = \Big(\sum_{k \in \text{soft}} \min_m \left\{ \frac{2q_m \cdot p_k}{Q_m} \right\} \Big) + \sum_{j=a,b,1,\ldots,N} \Big(\sum_{k \in \text{coll}_j} \frac{2q_j \cdot p_k}{Q_j} \Big)$$

Only soft particles get a nontrivial grouping. Jet boundaries are determined by the q_m

collinear particles all grouped with their q_j

(more later)

N-Jettiness & Jet Masses

$$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$

$$\mathcal{T}_N = \mathcal{T}_a + \mathcal{T}_b + \mathcal{T}_1 + \cdots + \mathcal{T}_N$$

$$\mathcal{T}_N^j = \sum_{k \in j} |\vec{p}_{kT}|\, d_j(p_k)$$

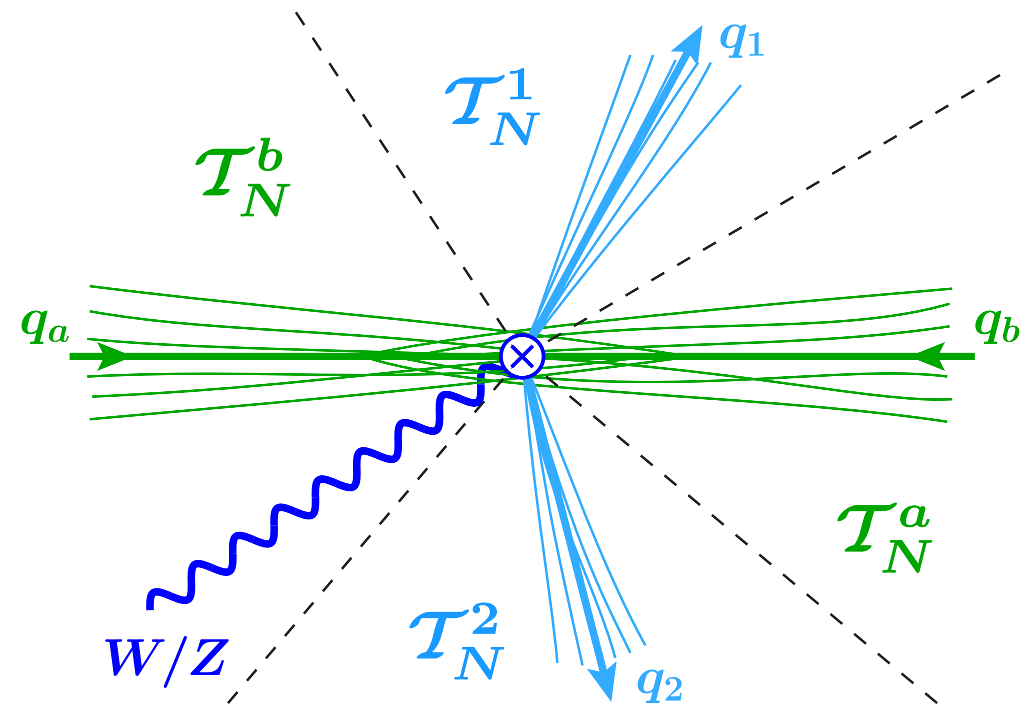
Can measure: $$\frac{d\sigma}{d\mathcal{T}_a d\mathcal{T}_b d\mathcal{T}_1 \cdots d\mathcal{T}_N}$$

with jet axes aligned

These are Jet Masses:

$$M_J^2 = P_J^2 = P_J^- P_J^+ = Q_i \mathcal{T}_N^i$$

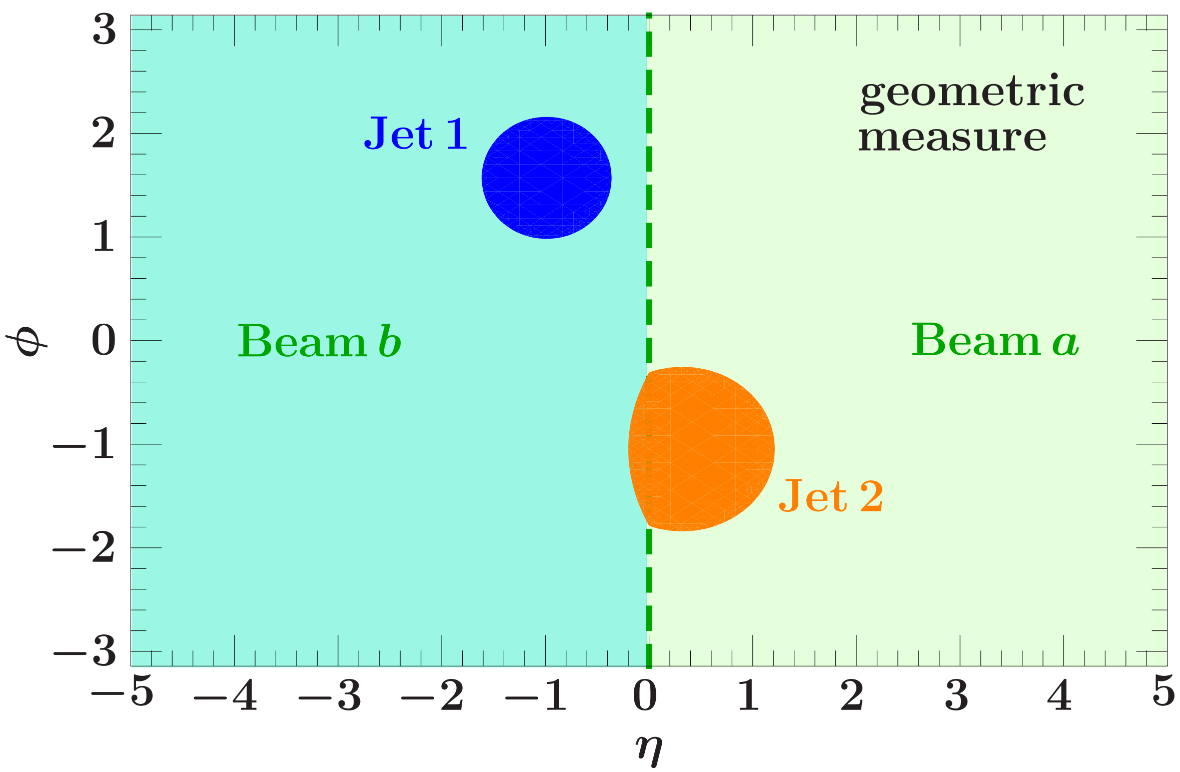
So one can study the masses of jets! (or subjets!)

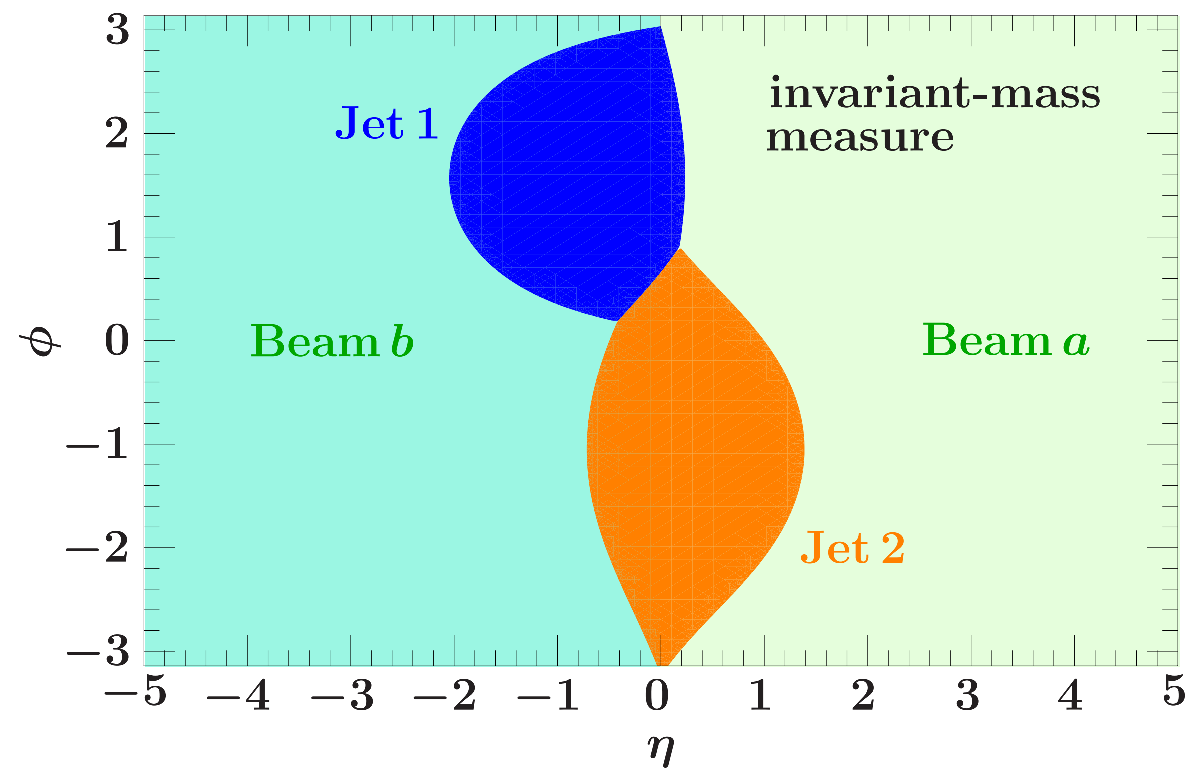


Jet definition:

N-jettiness divides particles into jet and beam regions

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$





$$d_{a,b}(p_k) = e^{\mp\eta_k}$$

$$d_j(p_k) = 2\cosh\Delta\eta_{jk} - 2\cos\Delta\phi_{jk}$$
$$\approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

$$d_i(p_k) = \frac{2q_i \cdot p_k}{Q|\vec{p}_{kT}|}$$

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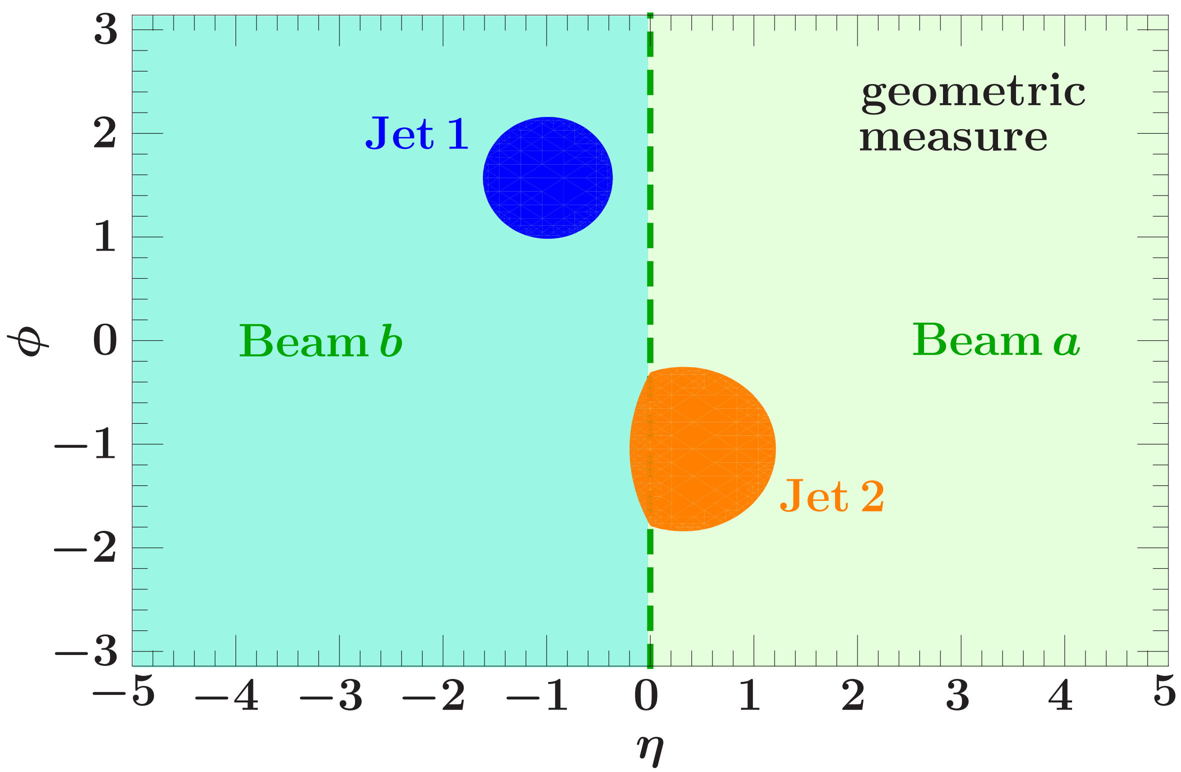

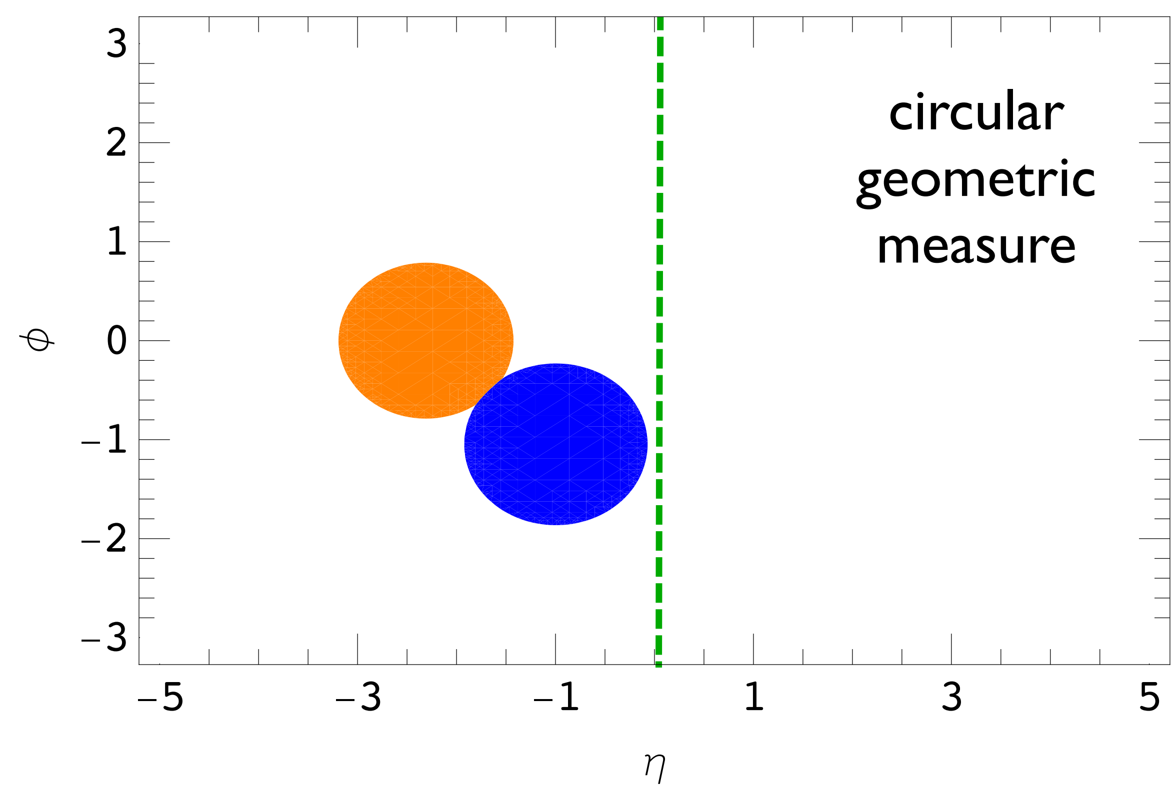

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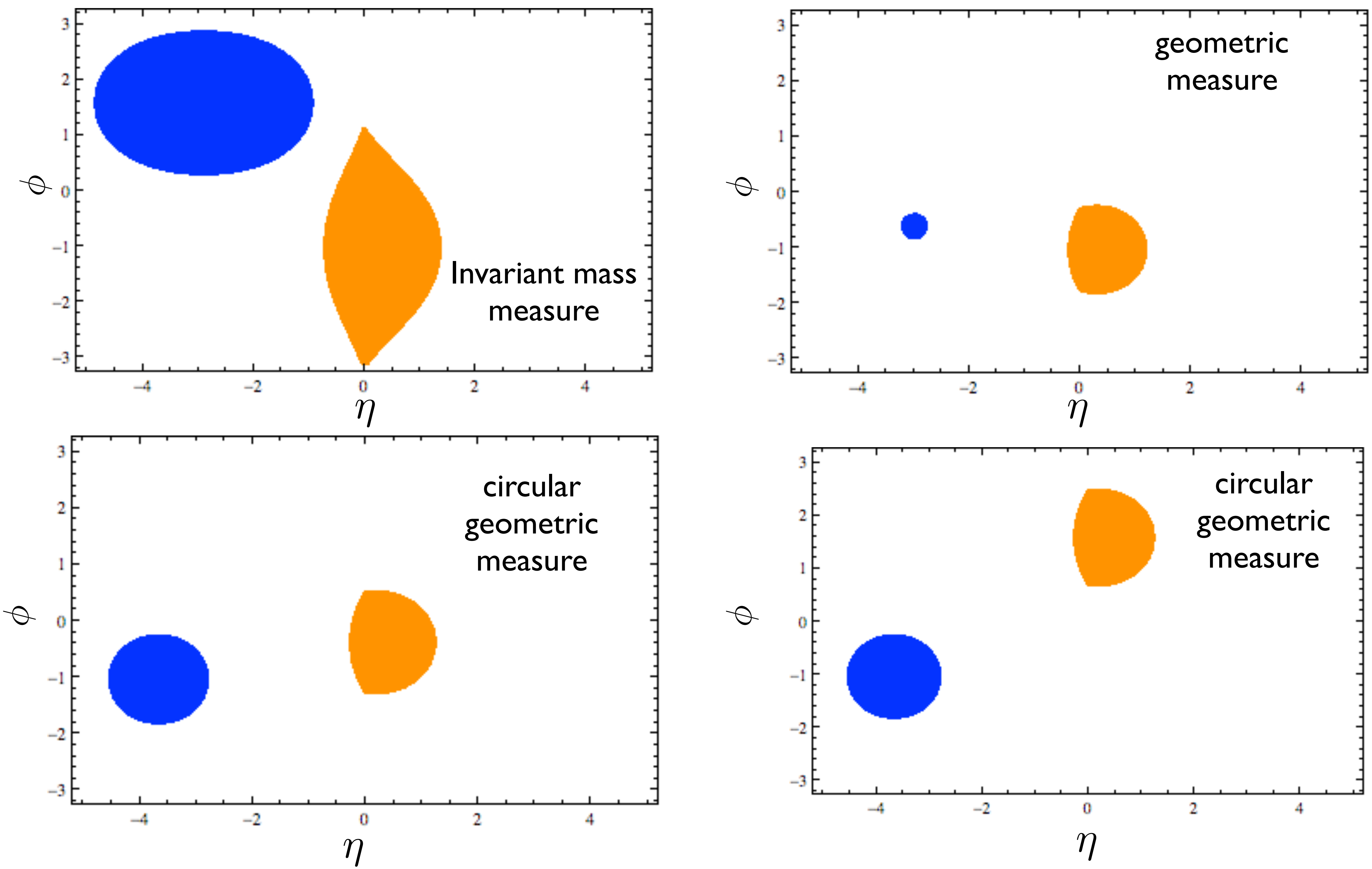
$$\approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

$$d_{a,b}(p_k) = \text{same}$$

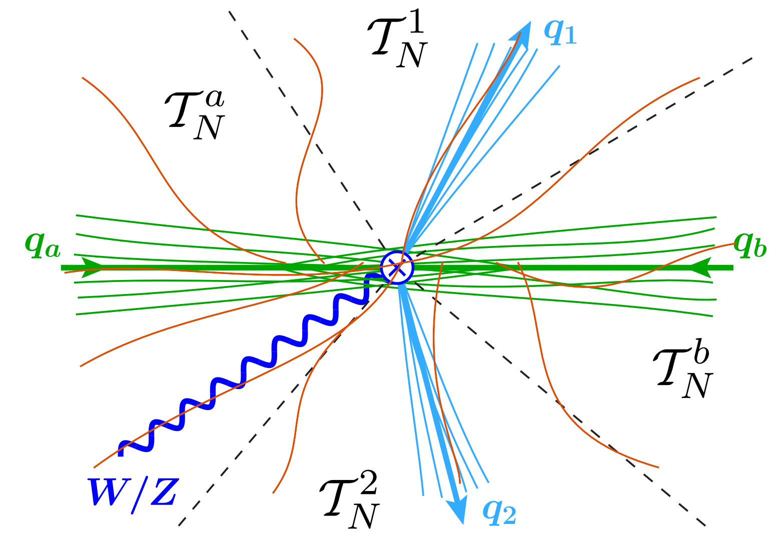
$$d_j(p_k) = (\text{same})/\cosh\Delta\eta_{jk}$$

Jets treatment of soft radiation depends on the distance measure

$$\hat{q}_i^\mu \equiv \frac{q_i^\mu}{Q_i}, \quad \mathcal{T}_N \equiv \sum_k \min_i \{2\hat{q}_i \cdot p_k\}$$



N-Jettiness Factorization Formula



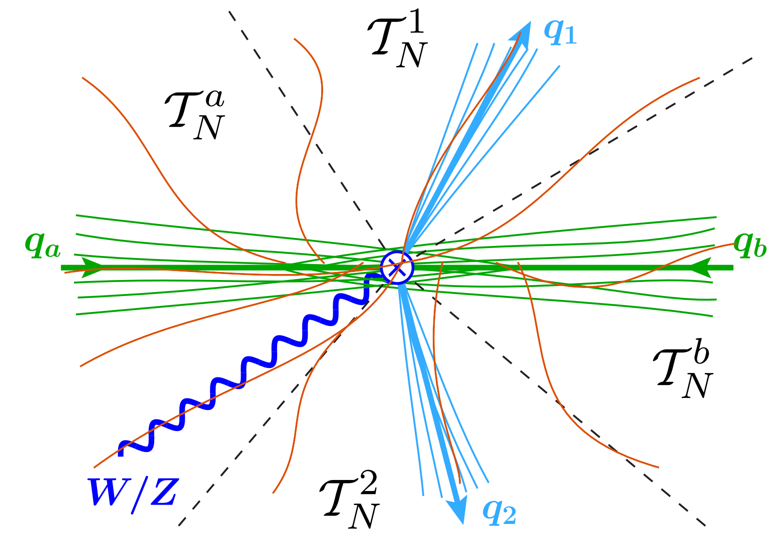
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N^a\,\mathrm{d}\mathcal{T}_N^b\cdots\mathrm{d}\mathcal{T}_N^N} = \int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}(\text{phase space})$$

$$\times \sum_\kappa \int \mathrm{d}t_a\, B_{\kappa_a}(t_a, x_a) \int \mathrm{d}t_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^N \int \mathrm{d}s_J\, J_{\kappa_J}(s_J)$$

$$\times\ \mathrm{tr}\Big[H_N^\kappa(\{q_i\cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i\cdot\hat{q}_j\}\Big)\Big]$$

$$\times\Big[1 + \mathcal{O}(\mathcal{T}_N^j)\Big]$$

N-Jettiness Factorization Formula



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hard virtual corrections
$2 \to N + q$

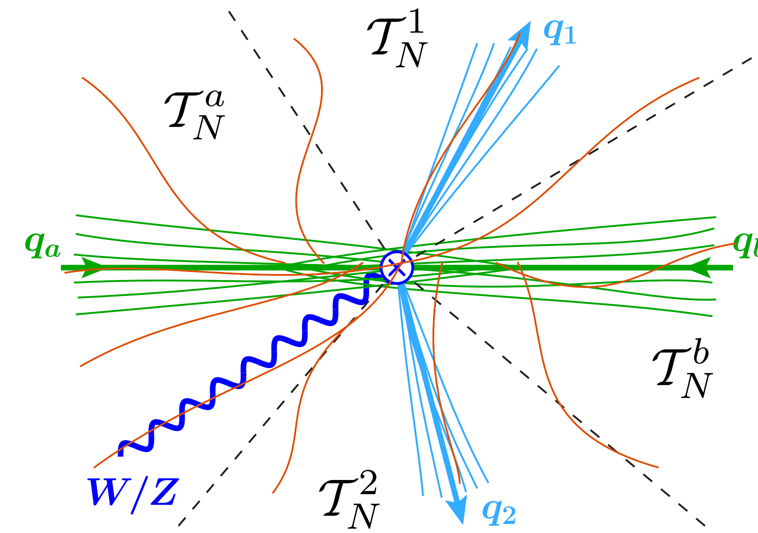
beam function
$B_\kappa = \mathcal{I}_{\kappa\kappa'} \otimes f_{\kappa'}$

N-jettiness soft function

jet function known to $\mathcal{O}(\alpha_s^2)$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

N-Jettiness Factorization Formula



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$$q_i\cdot q_j = (Q_i Q_j)(\hat{q}_i\cdot\hat{q}_j)$$

Assumptions needed to sum logs with this formula:

1) $\mathcal{T}_i \sim \mathcal{T}_j$ ($\mathcal{T}_i \ll \mathcal{T}_j$ gives non-global logs of Dasgupta & Salam)

[Chris Lee's talk]

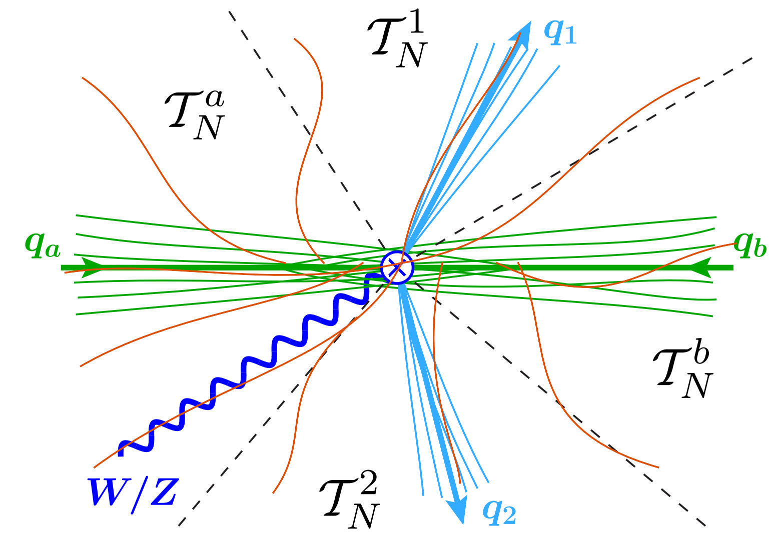
2) $\hat{q}_i\cdot\hat{q}_j \gg \mathcal{T}_i/Q_i$ (jets merge, "Ninja" limit)

jets are well separated

[Jon Walsh's talk]

3) $Q_i \sim Q_j$

N-Jettiness Factorization Formula



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A couple of interesting projections

One Central Jet's Mass

$$\frac{d\sigma}{d\mathcal{T}}(Q_i, R, \ldots) = \int_0^{Q_a R/2} d\mathcal{T}_a \int_0^{Q_b R/2} d\mathcal{T}_b \left[\int d\mathcal{T}_1 \prod_{j\geq 2} \int_0^{Q_j R/2} d\mathcal{T}_j \right] \frac{d\sigma}{d\mathcal{T}_a d\mathcal{T}_b d\mathcal{T}_1 \cdots d\mathcal{T}_N} \delta(\mathcal{T} - \mathcal{T}_1)$$

A Central Jet "Thrust"

$$\frac{d\sigma}{d\mathcal{T}}(Q_i, R, \ldots) = \int_0^{Q_a R/2} d\mathcal{T}_a \int_0^{Q_b R/2} d\mathcal{T}_b \left[\int \prod_j d\mathcal{T}_j \right] \frac{d\sigma}{d\mathcal{T}_a d\mathcal{T}_b d\mathcal{T}_1 \cdots d\mathcal{T}_N} \delta\Big(\mathcal{T} - \frac{1}{N} \sum_j \mathcal{T}_j\Big)$$

where $m_J^2 = Q_J \mathcal{T}$

eg. Higgs Jet Veto

Berger, Marcantonini, IS, Tackmann, Waalewijn

Higgs + 0 jets

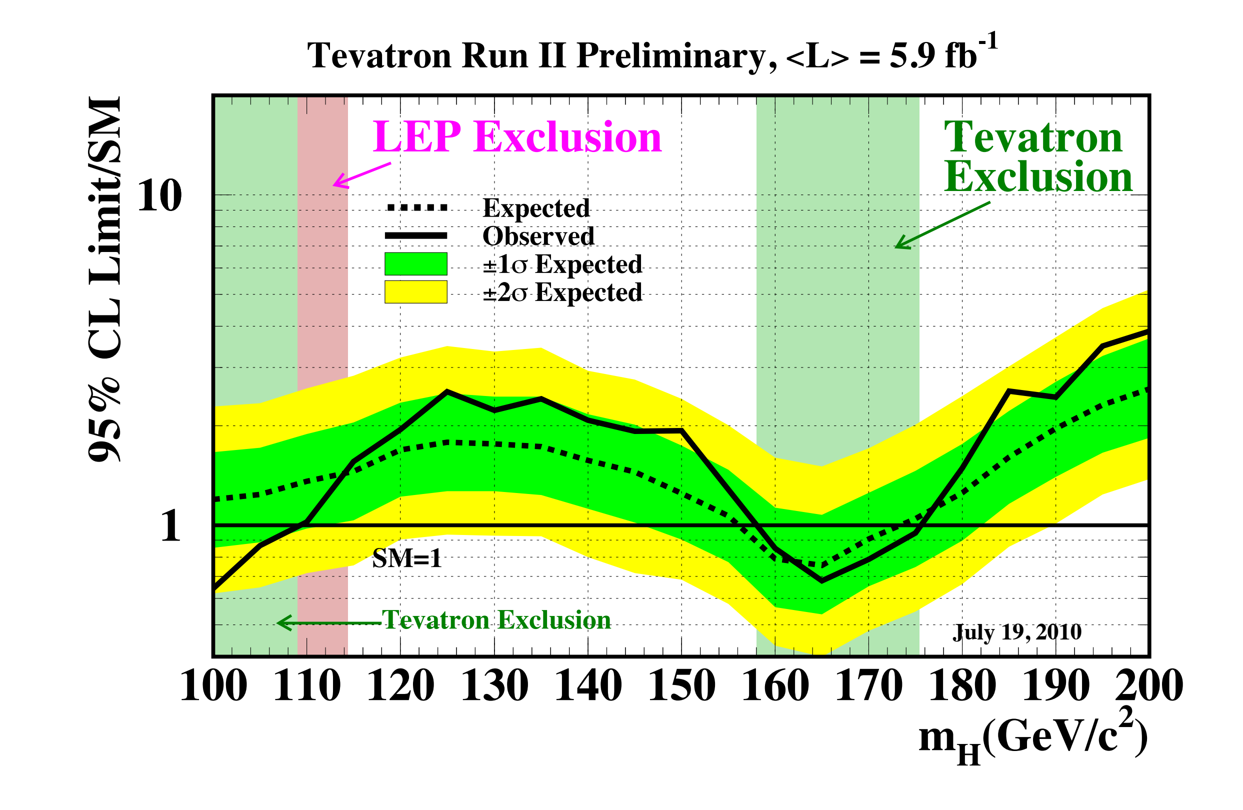
$gg \to H \to WW \to \ell\bar{\nu}\bar{\ell}\nu$

- Strong discovery potential at the LHC for $m_H \gtrsim 130\,\mathrm{GeV}$

$$pp \to H \to WW \to \mu^+\nu_\mu e^-\bar{\nu}_e$$

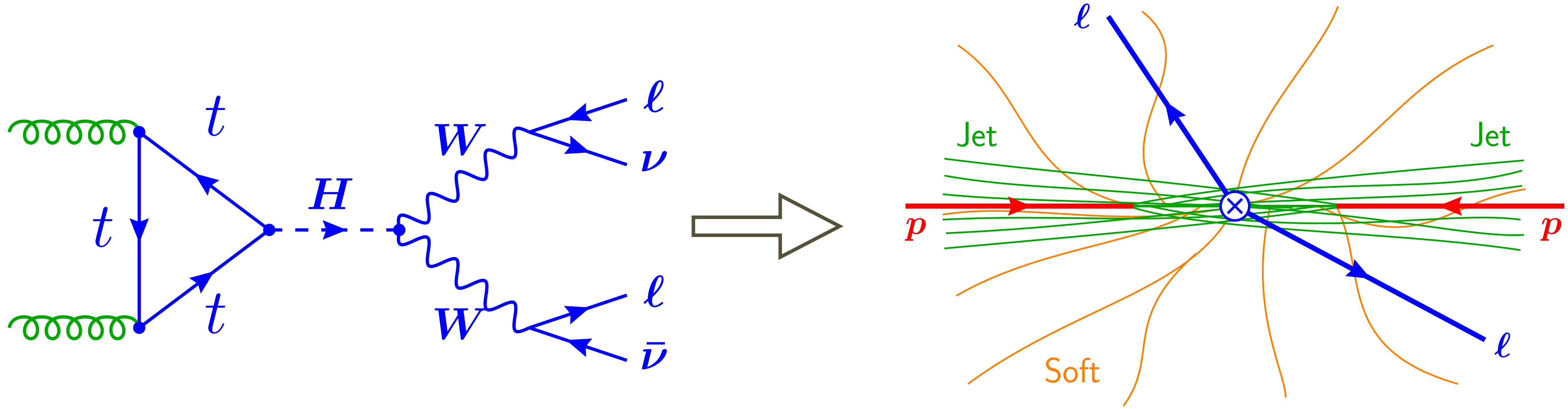
- dominant channel in Tevatron search

$$p\bar{p} \to H \to WW \to \mu^+\nu_\mu e^-\bar{\nu}_e$$



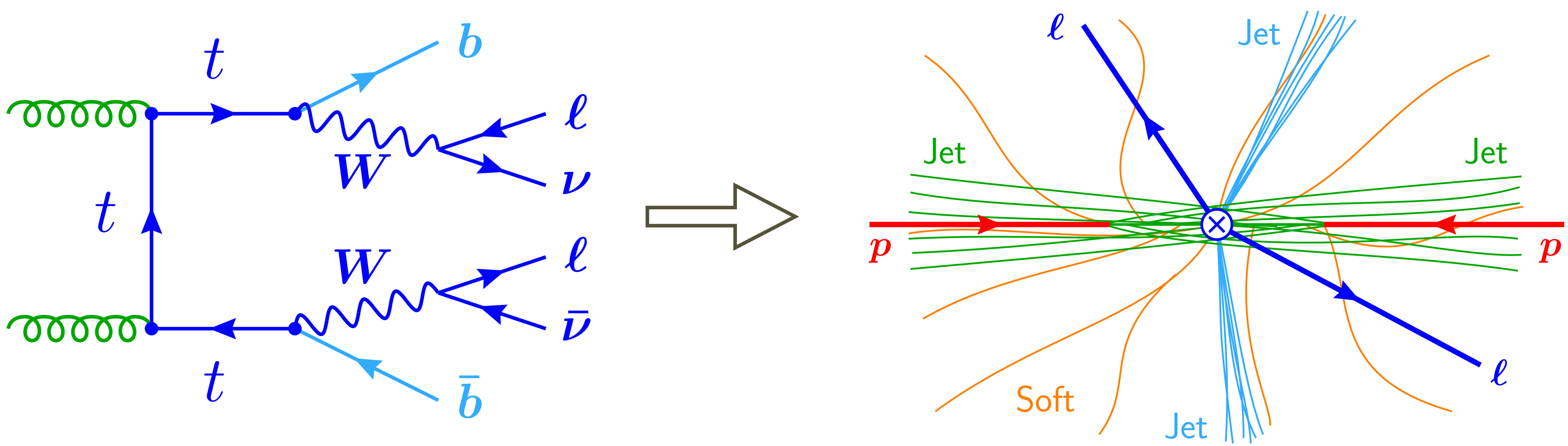
Large Background from Top Decays

1



to

40



$\Rightarrow$ Veto events with central jets, measure $pp \to H(\to WW) + 0$ jets

(Sensitivity dominated by 0-jet sample)

Jet Vetoes

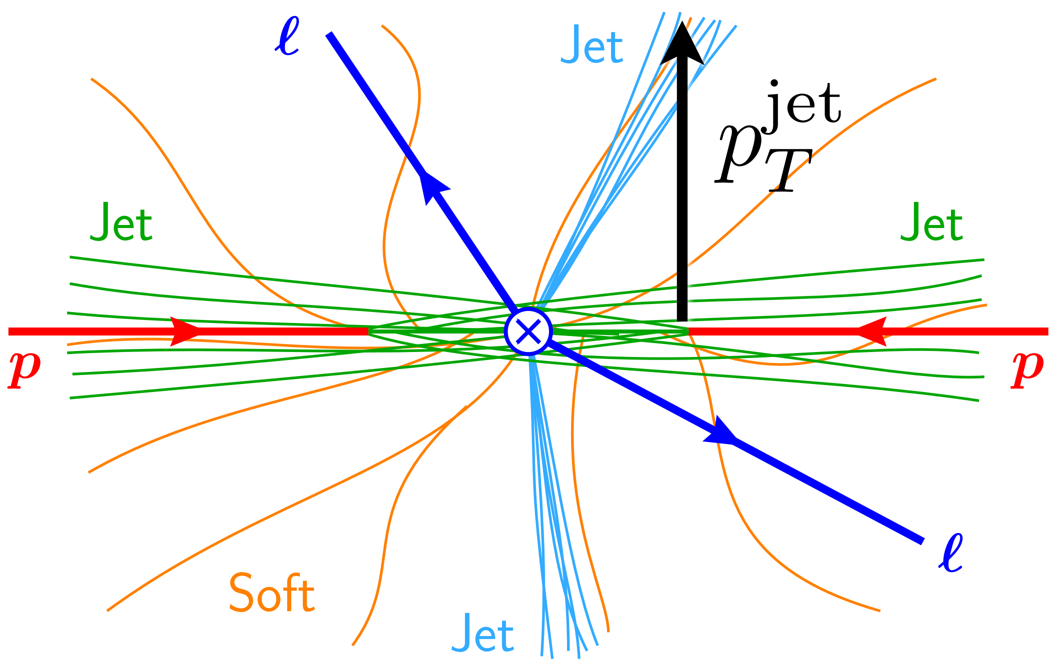
Conventional: Jet Algorithm

- Search for jets and require $p_T^{\rm jet} < p_T^{\rm cut}$

Tevatron: $p_T^{\rm cut} \simeq 20\,{\rm GeV}$

LHC: $p_T^{\rm cut} \simeq 25\,{\rm GeV}$

- Complicated phase-space restrictions



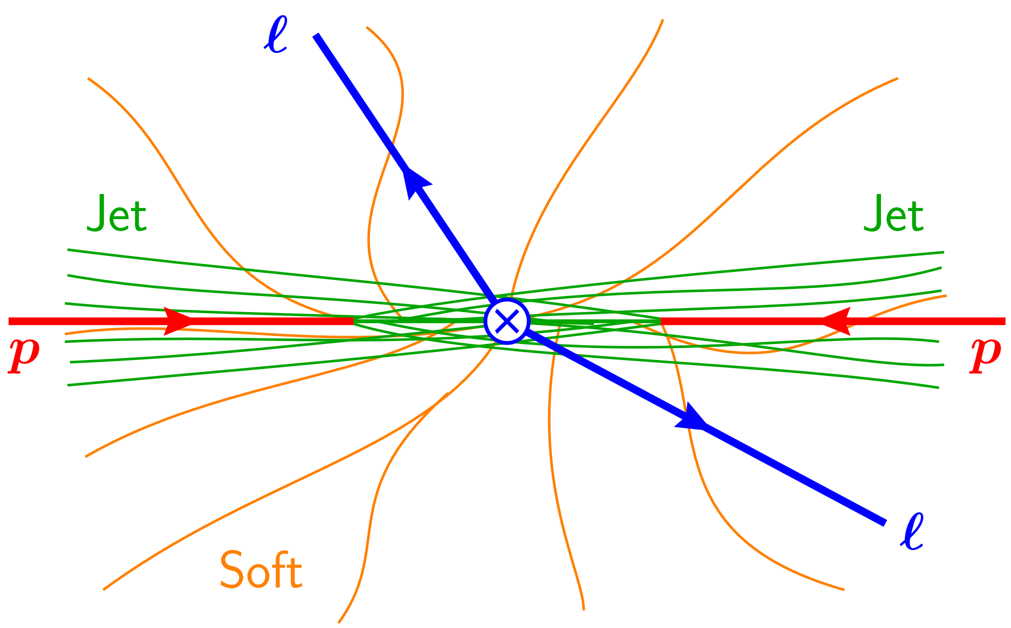
Alternative: Event Shape

- Measure beam thrust for each event

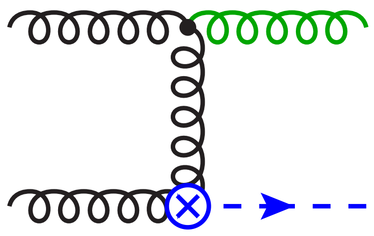
$$\mathcal{T}_{\rm cm} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

and require $\mathcal{T}_{\rm cm} < \mathcal{T}_{\rm cm}^{\rm cut}$

- Nice for higher order calculations



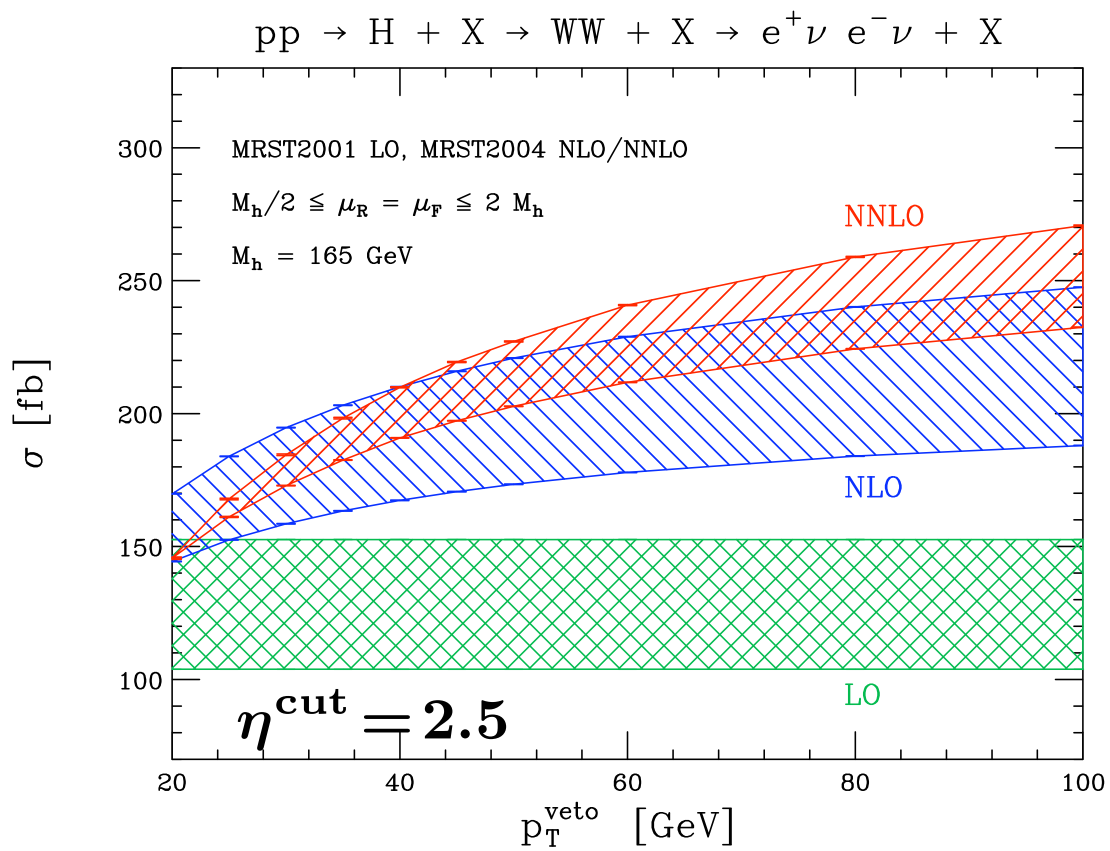
Jet veto restricts ISR, gives double logs



$L = \log$

$$
\begin{array}{llllll}
 & \text{LO} & \text{NLO} & \text{NNLO} & & \\
\sigma_{\text{0-jet}} = & 1 & + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \dots \\
 & & + \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \dots \\
 & & + \alpha_s n_1(p_T^{\text{cut}}) & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \dots \\
 & & & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \dots \\
 & & & + \alpha_s^2 n_2(p_T^{\text{cut}}) & + \alpha_s^3 L^2 & + \dots \\
 & & & & + \alpha_s^3 L & + \dots \\
 & & & & + \alpha_s^3 & + \dots
\end{array}
$$

Fixed Order to NNLO



FEHiP, HNNLO: Numerical fully differential NNLO cross section for $gg \to H$

[Anastasiou, Melnikov, Petriello; Grazzini]

Jet veto restricts ISR, gives double logs

Fixed Order to NNLO

Current recipe being used by experiments [Anastasiou et al., arXiv:0905.3529]

- Common scale variation for jet bins, e.g. for the Tevatron

$$\frac{\Delta\sigma}{\sigma} = \underbrace{66.5\% \times \binom{+5\%}{-9\%}}_{\text{0 jets}} + \underbrace{28.6\% \times \binom{+24\%}{-22\%}}_{\text{1 jet}} + \underbrace{4.9\% \times \binom{+78\%}{-41\%}}_{\geq \text{2 jets}} = \binom{+14\%}{-14\%}$$

Problem:

$$\sigma_{\text{total}} = 1 + \alpha_s + \alpha_s^2 + \cdots$$

$$\sigma_{\geq 1}(p_T^{\text{cut}}) = \alpha_s(L^2 + L) + \alpha_s^2(L^4 + L^3 + L^2 + L) + \cdots$$

$$\begin{aligned} \sigma_0(p_T^{\text{cut}}) &= \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}}) \\ &= \left[1 + \alpha_s + \alpha_s^2 + \cdots\right] - \left[\alpha_s(L^2 + L) + \alpha_s^2(L^4 + \cdots) + \cdots\right] \end{aligned}$$

- perturbative series have different structures and are not related
- small uncertainties are result of cancellation of two large corrections

Jet veto restricts ISR, gives double logs

Fixed Order to NNLO

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Proposed Fixed Order Solution

[Tackmann, ...]

- The *inclusive* jet cross sections are considered uncorrelated

$$\sigma_{\text{total}},\ \sigma_{\geq 1},\ \sigma_{\geq 2} \qquad \text{for scale variation}$$

- The covariance matrix for the *exclusive* jet cross sections follows from

$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}\,, \qquad \sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}\,, \qquad \sigma_{\geq 2}$$

Jet veto restricts ISR, gives double logs

Fixed Order to NNLO

Current recipe being used by experiments [Anastasiou et al., arXiv:0905.3529]

- Common scale variation for jet bins, e.g. for the Tevatron

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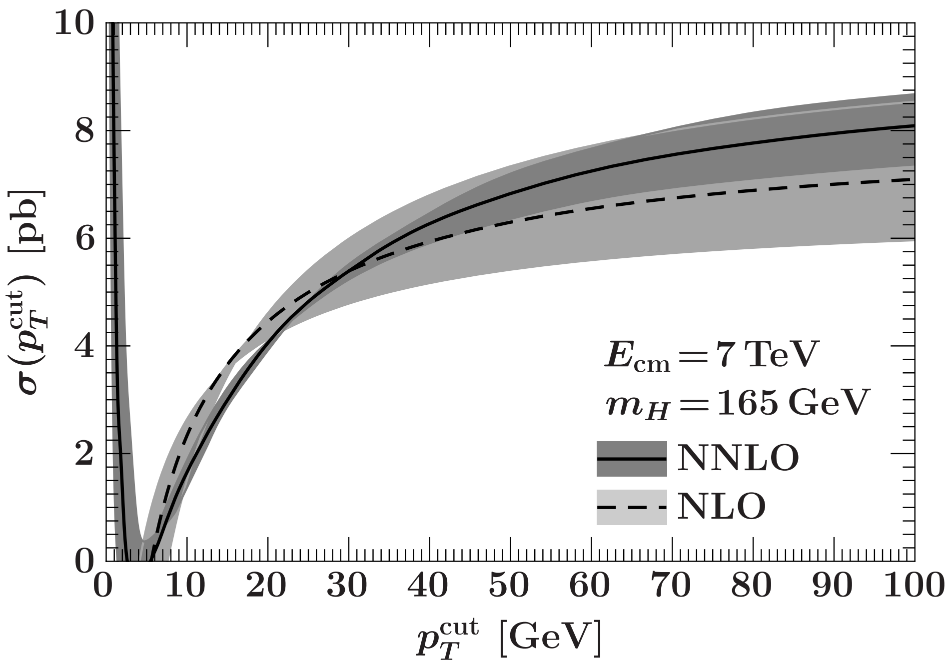
Proposed Fixed Order Solution

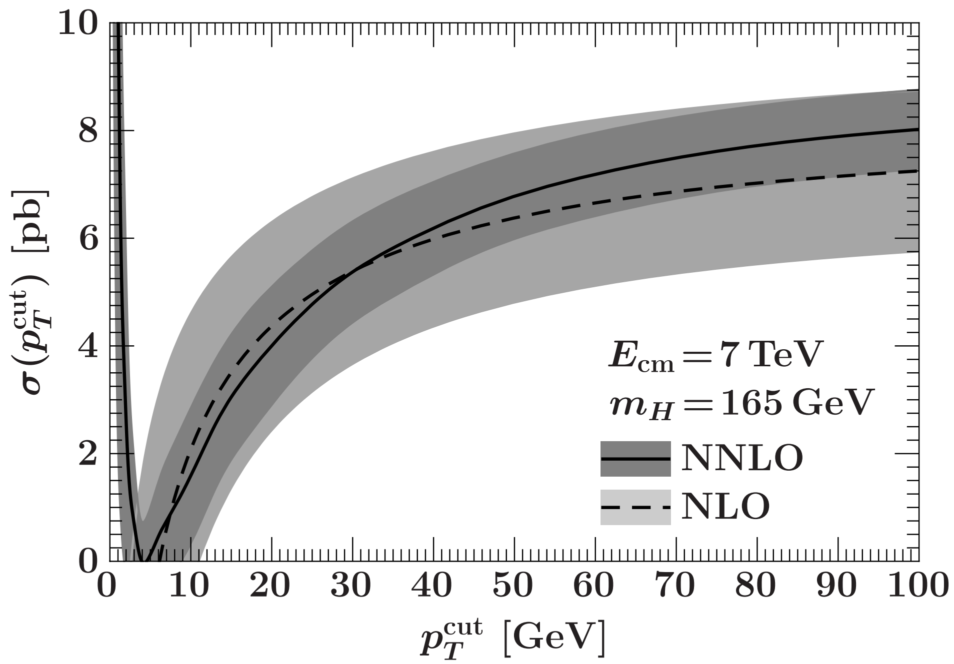
[Tackmann, ...]

Using naive scale variation for σ_0

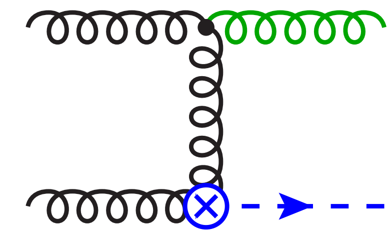


$\Rightarrow$

Using above procedure for σ_0



Jet veto restricts ISR, gives double logs using “beam thrust” or “0-jettiness”

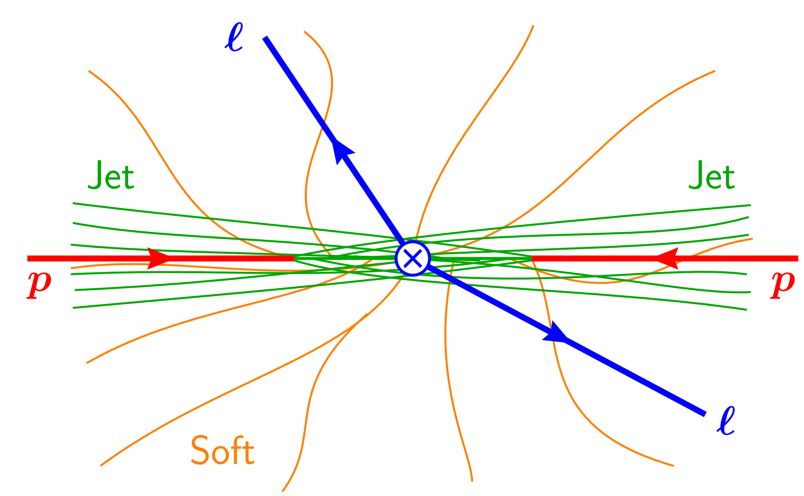


$$
\begin{array}{lcllll}
 & \text{LO} & \text{NLO} & \text{NNLO} & & \\
\sigma_{0\text{-jet}} = & 1 & + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \dots \quad \text{LL} \\
 & & + \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \dots \quad \text{NLL} \\
 & & + \alpha_s n_1(p_T^{\text{cut}}) & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \dots \\
 & & & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \dots \quad \text{NNLL} \\
 & & & + \alpha_s^2 n_2(p_T^{\text{cut}}) & + \alpha_s^3 L^2 & + \dots \\
 & & & & + \alpha_s^3 L & + \dots \\
 & & & & + \alpha_s^3 & + \dots
\end{array}
$$

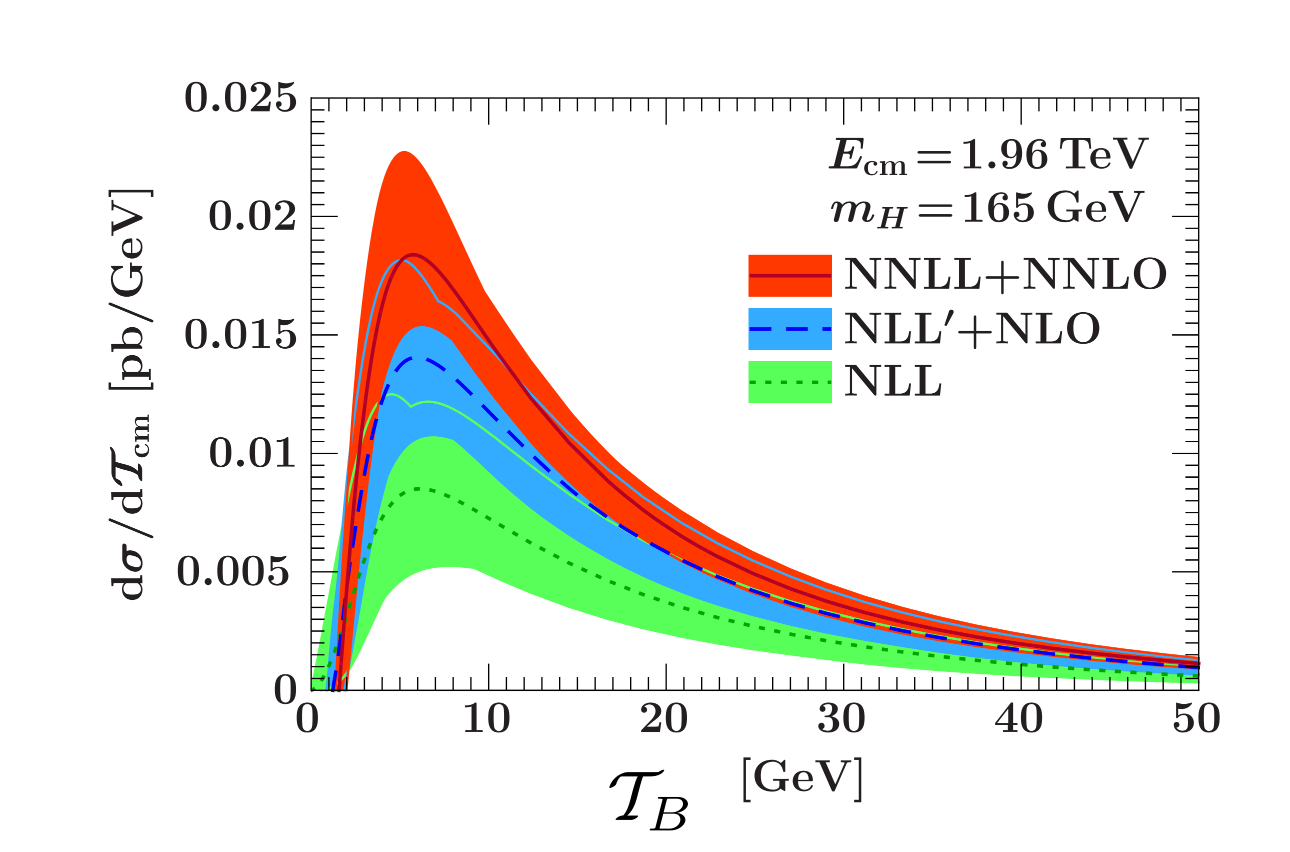
Our calculation:

NNLL + NNLO

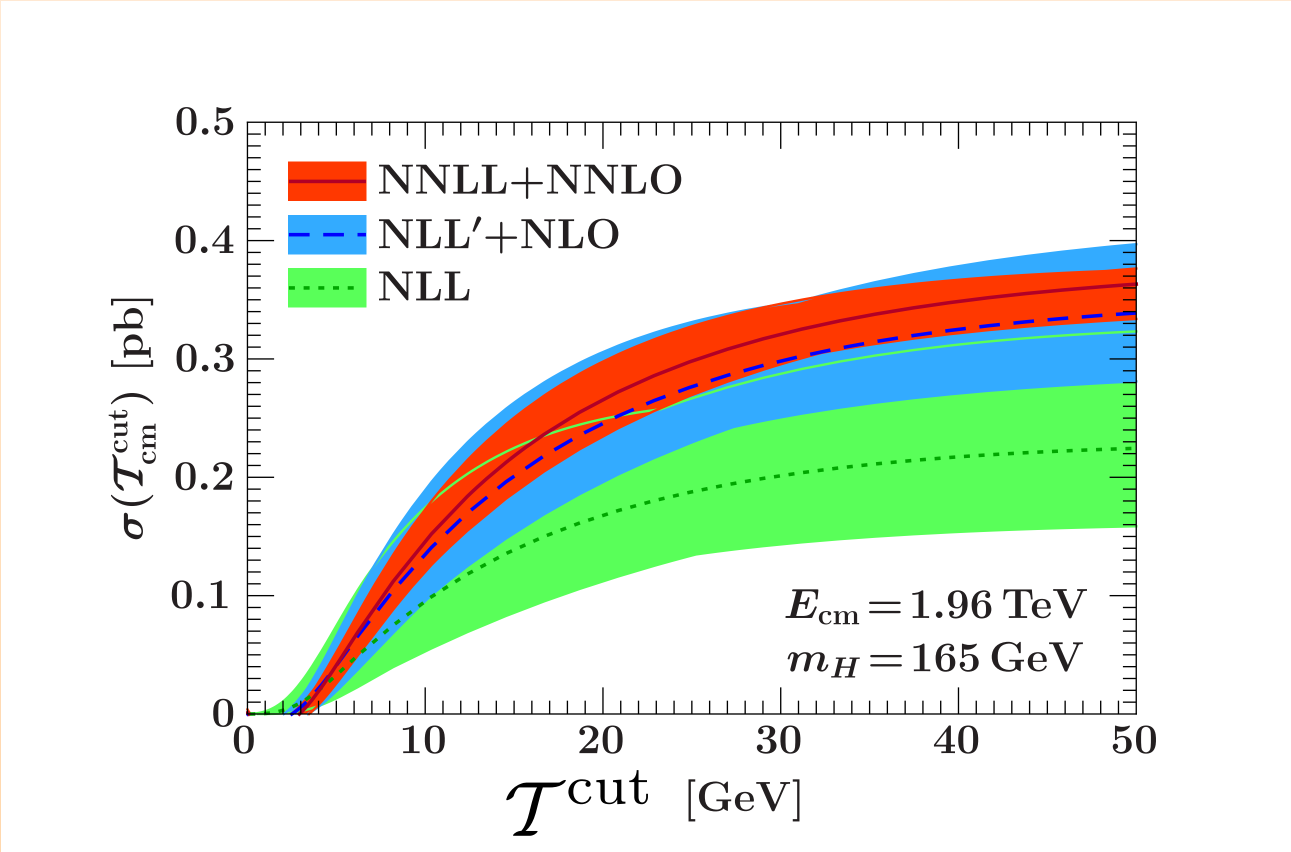
two orders of summation beyond LL shower programs



Berger, Marcantonini, IS, Tackmann, Waalewijn



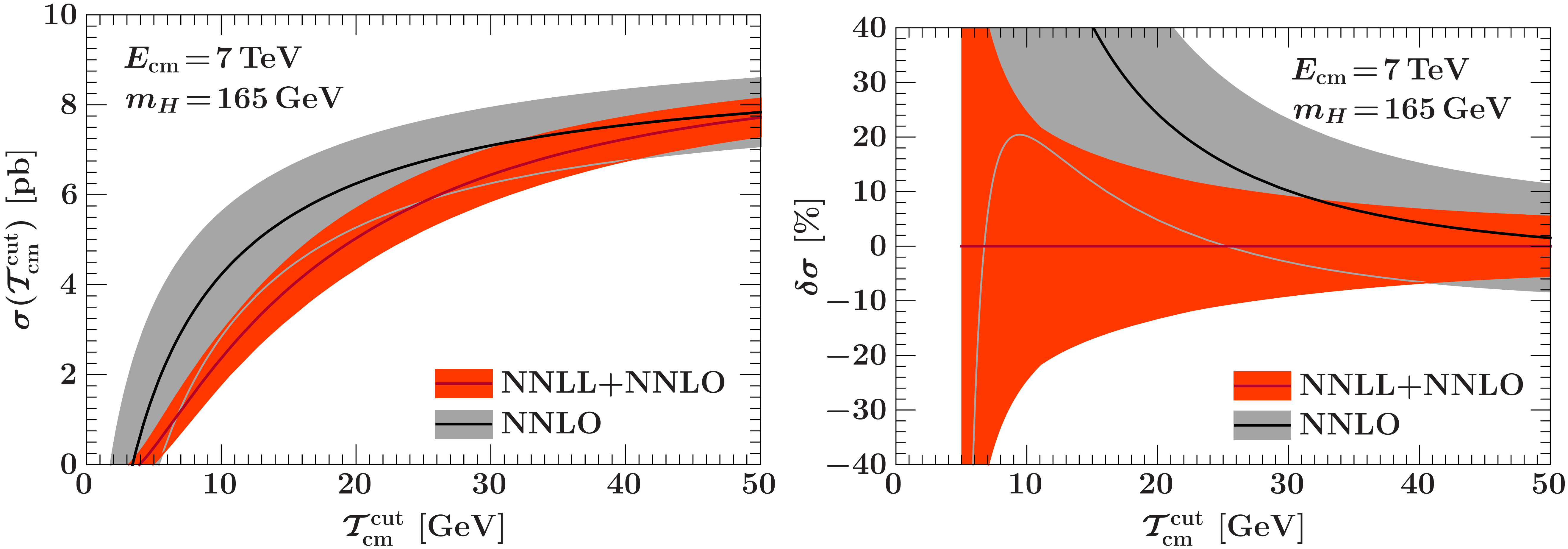
- two orders of summation beyond LL shower programs
- logs are large
- theory error bands from varying μ_i



- NNLO underestimates size of errors by factor of two

scale uncertainty at NNLL+NNLO is 10-20%

NNLO: using inclusive jet cross sections & correlation matrix



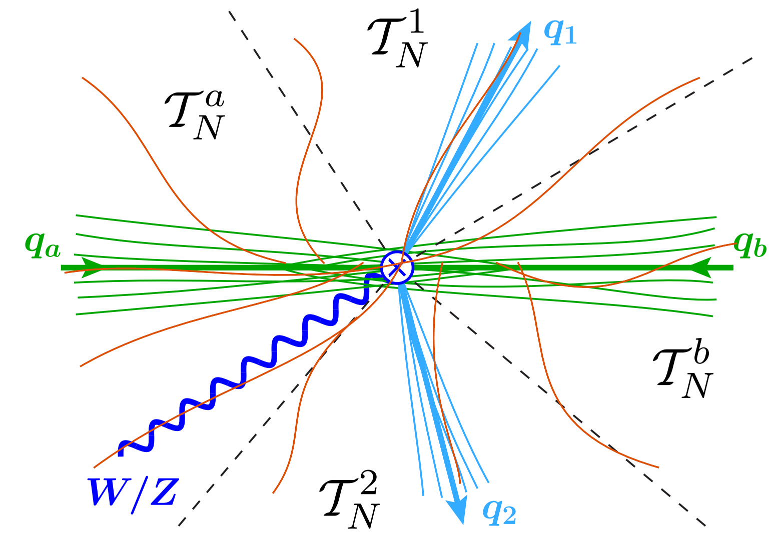
cut	order	$\delta\sigma_{\text{total}}$	$\delta\sigma_{\geq 1}$	$\delta\sigma_0$
$\mathcal{T}_{cm}^{cut} = 20\,\text{GeV}$	NNLO	8.5%	28%	16%
$\mathcal{T}_{cm}^{cut} = 20\,\text{GeV}$	NNLL+NNLO	5.2%	21%	13%

- NNLO uncertainties now consistent with those from NNLL+NNLO resummation
  - increased theory errors will impact Higgs bound

Jet Mass

Jouttenus, IS, Tackmann, Waalewijn

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^N \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times \operatorname{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

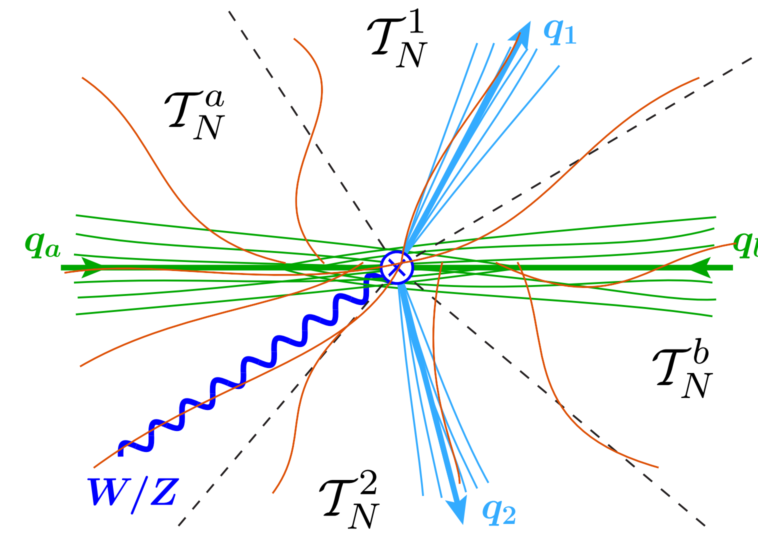
Pieces needed for NNLL are now all in hand:

- Three Loop Cusp Anom. Dim, Two Loop Non Cusp.
  (Note: Beam function has same Logs as Jet Function)
- One Loop Hard functions: when available in QCD literature
  (only part that restricts N)
- Jet & Beam Functions at one loop
- N-jet Soft function

Jouttenus, IS, Tackmann, Waalewijn

also: Bauer, Hornig, Dunn

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

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$$\times \operatorname{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

With assumptions: $\mathcal{T}_i \sim \mathcal{T}_j$, $\hat{q}_i \cdot \hat{q}_j \gg \mathcal{T}_i / Q_i$, $Q_i \sim Q_j$

Can explore angular dependence,
R dependence,
Q_i dependence

Have Color / Kinematic info. Can look at jet mass in samples with various amounts of quarks vs. gluons.

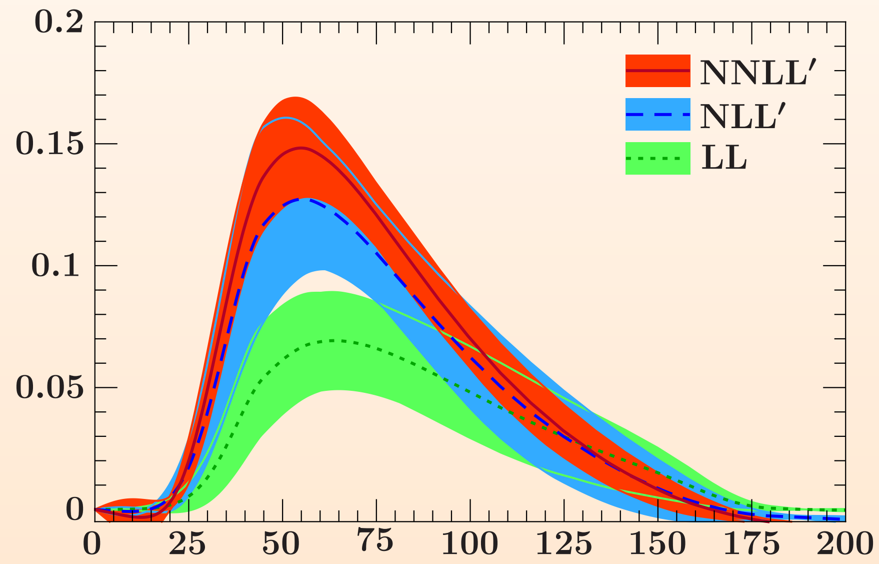
Unfortunately we did not quite get final results in time for the workshop ...

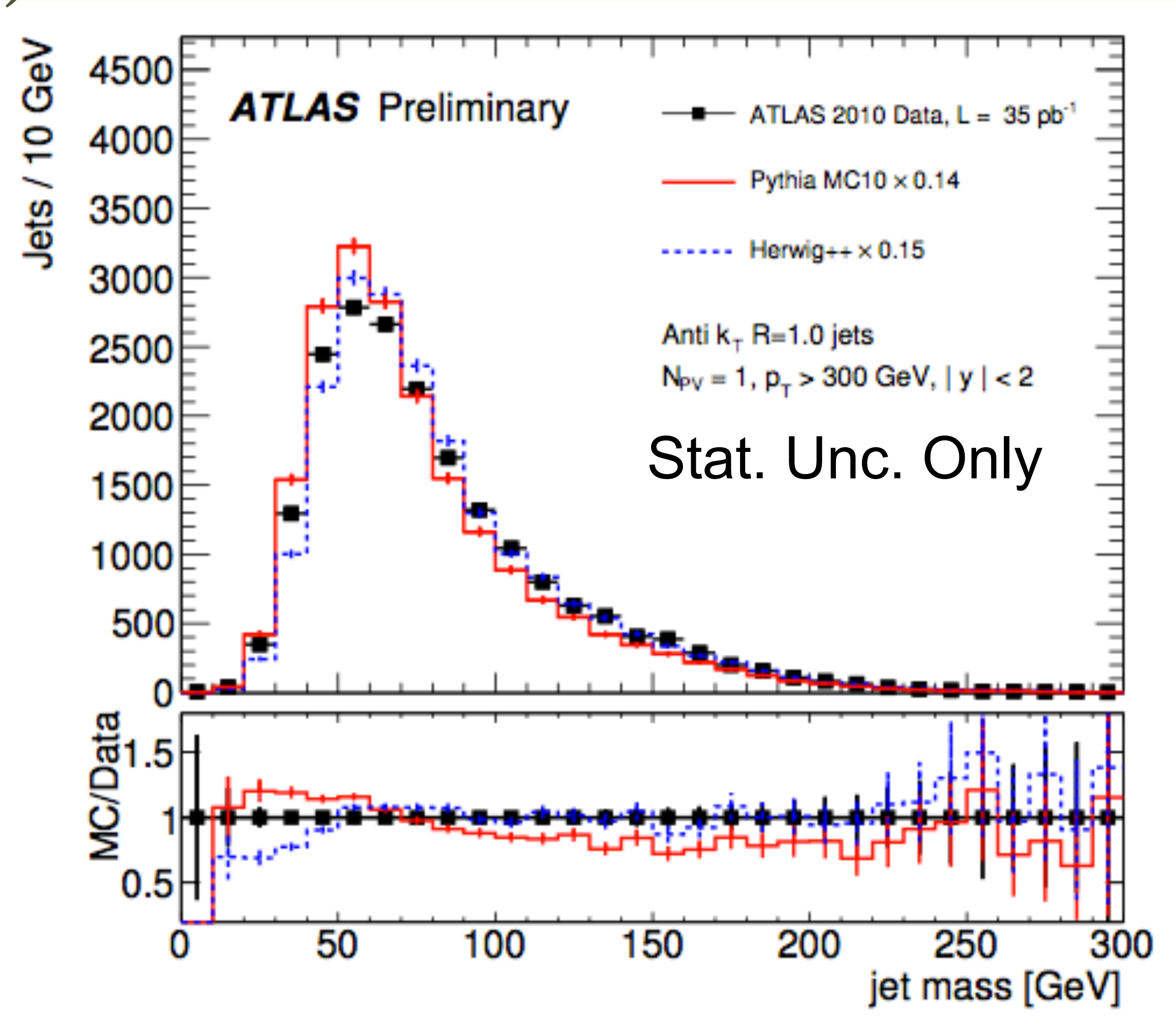
Jouttenus, IS, Tackmann, Waalewijn

Beam-Jet Mass (glue ie. Higgs production)

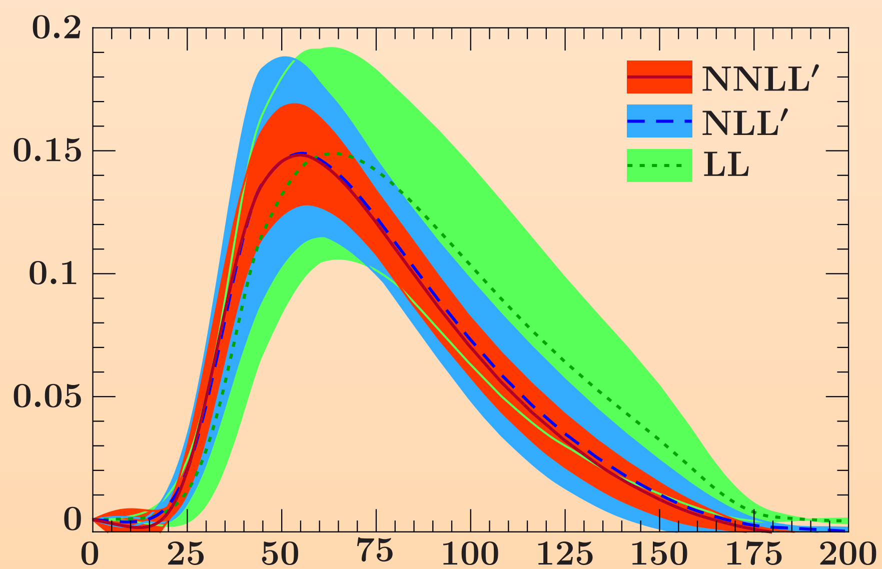
[Adam Davison's talk here]

$m_J^2 = Q_J \mathcal{T}$

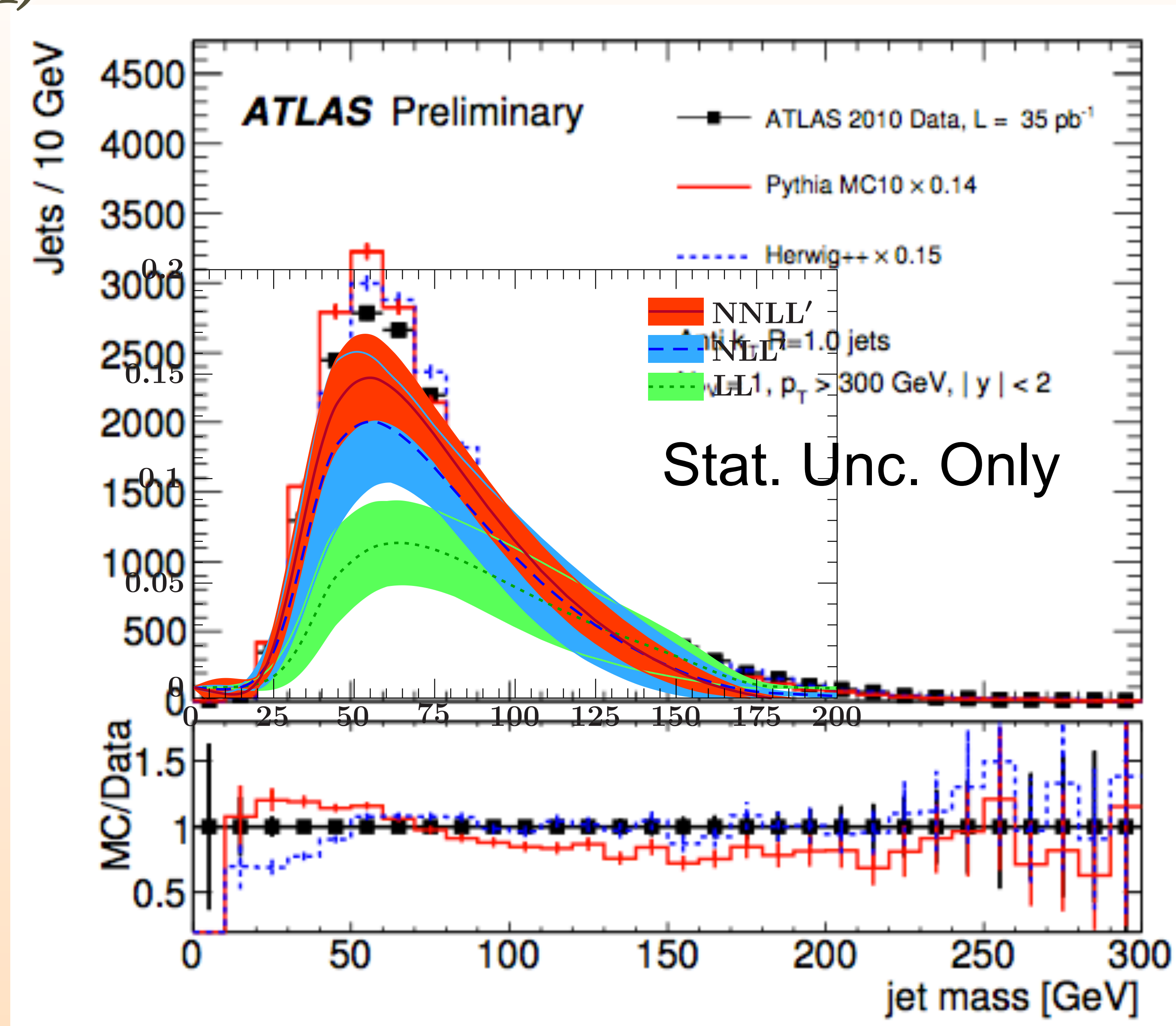




looking only at the shape:



Beam-Jet Mass
(glue ie. Higgs production)



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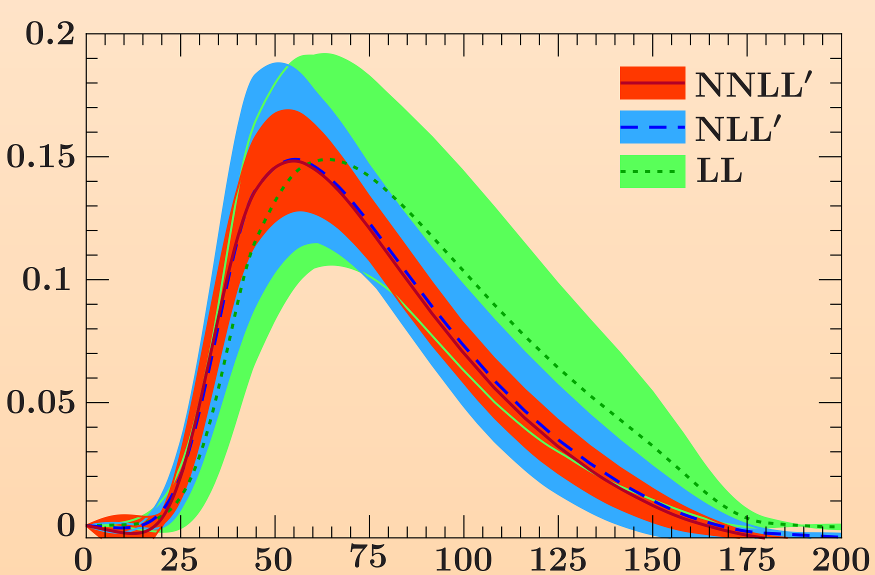

The End