

LHC jet masses and resummation

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- Introduction
- Towards all-orders predictions for jet masses
- Non-global logs
- The role of the jet definition
- Still open questions

- Jet masses significant variables in studies of boosted objects and jet substructure. Important role in LHC phenomenology.
- One important feature is potentially large logarithms $\alpha_s \ln^2 \frac{P_t^2}{M^2}$. LHC opens up **large phase space**.
- Have seen measurements of inclusive jet distributions and comparisons solely to event generators.
- What about **analytical resummation** of the large logarithms involved? Analytical handles on jet substructure?

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- What about **analytical resummation** of the large logarithms involved? Analytical handles on jet substructure?

Some work has been carried out.

- **Leading order** estimate by Almeida et.al

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T dm_J} \sim \alpha_s(p_T) \frac{4C_J}{\pi m_J} \log\left(\frac{Rp_T}{m_J}\right) + \dots$$

L. Almeida et.al,2010

Already compared to Tevatron data! Above result is straightforward to improve. If the logarithm dominates we need to resum, if not use NLOJET++.

- Work involving resummation within the context of SCET for e^+e^- annihilation but aiming at extension to hadron colliders. Jet angularities with a jet veto e.g.

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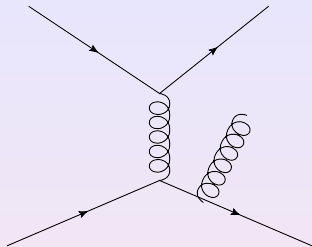
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Leading order calculation

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Consider $q(p_1) + q'(p_2) = q(p_3) + q'(p_4)$ with $p_3 = p_J$.

$$p_1 = \frac{\sqrt{s}}{2} x_1 (1, 0, 0, 1)$$

$$p_2 = \frac{\sqrt{s}}{2} x_2 (1, 0, 0, -1)$$

$$p_3 = p_t (\cosh y_1, 1, 0, \sinh y_1)$$

$$p_4 = p_t (\cosh y_2, -1, 0, \sinh y_2)$$

$$k = k_t (\cosh \eta, \cos \phi, \sin \phi, \sinh \eta)$$



$$\Sigma(\rho, p_t) = -\frac{\alpha_s}{2\pi} \sum_{(ij)} C_{ij} \int \frac{dk_t}{k_t} d\eta \frac{d\phi}{2\pi} \times \\ \times \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \Theta\left(\frac{M_j^2}{p_t^2} - \rho\right) \Theta_{\text{in}}(k)$$

Results for dipole involving triggered jet will contain leading double logarithms:

$$W_{23} = W_{13} = \frac{1}{2} \ln^2 \frac{R^2}{\rho} + \ln \frac{1}{\rho} \left(\frac{R^2}{4} + \frac{R^4}{288} + \mathcal{O}(R^6) \right)$$

$$W_{34} = \frac{1}{2} \ln^2 \frac{R^2}{\rho} + \ln \frac{1}{\rho} \left(\frac{R^4}{16} + \mathcal{O}(R^6) \right)$$

Other dipoles will produce single logarithms and terms regular in R wide-angle soft contributions:

$$W_{12} = \ln \frac{1}{\rho} (R^2)$$

$$W_{24} = W_{14} = \ln \frac{1}{\rho} \left(\frac{R^2}{4} + \frac{R^4}{32} \right)$$

Overall one gets

$$\Sigma(\rho) \sim 1 - C_F \frac{\alpha_s}{2\pi} \left[\ln^2 \frac{R^2}{\rho} + 2 \ln \frac{1}{\rho} \left(\frac{R^2}{2} + \frac{5R^4}{144} + \dots \right) \right] - \frac{\alpha_s}{\pi} \frac{1}{N} \ln \frac{1}{\rho} \left[R^2 - \frac{R^4}{24} + \dots \right]$$

Note series in R will converge rapidly for smaller R values such as $R = 0.4$ and $R = 0.6$. May proceed in a small R approximation. Above result is ok in **any** algorithm. The leading double-log term is the one used by Almeida et.al.

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Exponentiation and all-orders

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The leading log behaviour we have computed straightforwardly exponentiates. The finite R terms also exponentiate but with matrix valued coefficients – ignore for simplicity.

Include collinear emission and running coupling

$$\Sigma^{ind} \left(\frac{R^2}{\rho}, p_t \right) = \frac{\exp \left[-\mathcal{R}_\rho - \gamma_E \mathcal{R}'_\rho \right]}{\Gamma \left(1 + \mathcal{R}'_\rho \right)}.$$

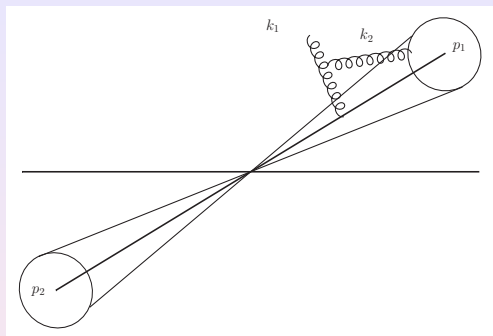
$$\mathcal{R}_\rho = \frac{C_F}{\pi} \int \frac{dk_t^2}{k_t^2} \alpha_s(k_t) \mathcal{F}(k_t^2),$$

$$\begin{aligned} \mathcal{F}(k_t^2) = & \ln \left(\frac{p_t R e^{-\frac{3}{2}}}{2k_t} \right) \Theta \left(\frac{R p_t}{2} - k_t \right) \Theta \left(\frac{k_t^2}{p_t^2} - \frac{\rho}{4} \right) \\ & + \ln \left(\frac{2R k_t}{\rho p_t} \right) \Theta \left(\frac{\rho}{4} - \frac{k_t^2}{p_t^2} \right) \Theta \left(\frac{k_t^2}{p_t^2} - \frac{\rho^2}{4R^2} \right), \end{aligned}$$

Soft wide-angle emissions beyond one-loop

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The fun starts at two-loop level. Looking in the interior of jet corresponds to **non-globalness**. Can do explicit calculation with 2 soft energy ordered gluons $\omega_1 \gg \omega_2$.

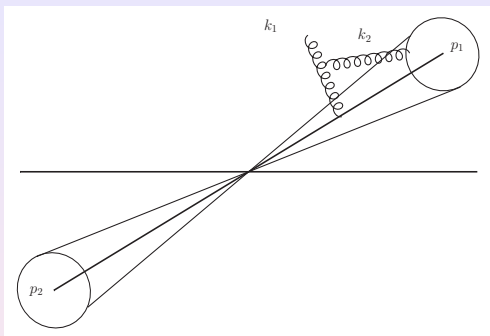
Configuration would *cancel* to our accuracy in global observables.

MD and Salam, 2001, 2002

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MD and Salam, 2001, 2002

One can ask what happens in the limit of a small jet $R \rightarrow 0$.

$$p_1 = E(1, 0, 0, 1) \quad (1)$$

$$p_2 = E(1, 0, 0, -1)$$

$$k_1 = \omega_1(1, \sin \theta_1, 0, \cos \theta_1)$$

$$k_2 = \omega_2(1, \sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2)$$

$$\Omega = \frac{2}{(\cos \theta_2 - \cos \theta_1)(1 - \cos \theta_1)(1 + \cos \theta_2)}.$$

Energy ordering \rightarrow logarithmic enhancement $\alpha_s^2 \ln^2 \frac{1}{\rho}$

$$S_2 = -2C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \int_{1-R^2/2}^1 d \cos \theta_2 \int_{-1+R^2/2}^{1-R^2/2} d \cos \theta_1$$
$$\ln^2 \frac{\rho E}{4E_0(1 - \cos \theta_2)} \Theta \left(1 - \frac{\rho E}{4E_0(1 - \cos \theta_2)}\right) \Omega.$$

Integrating over angles

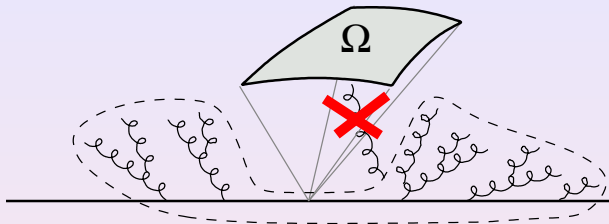
$$S_2 = -C_F C_A \left(\frac{\alpha_s}{2\pi} \right)^2 \frac{\pi^2}{3} \ln^2 \frac{2E_0 R^2}{\rho E}.$$

Relevant single-log term. Same coefficient as for hemisphere masses upto corrections $\mathcal{O}(R^2)$. Effect comes from edge of jet. E_0 is a veto scale.

Resummation

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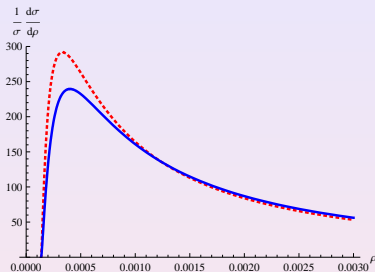
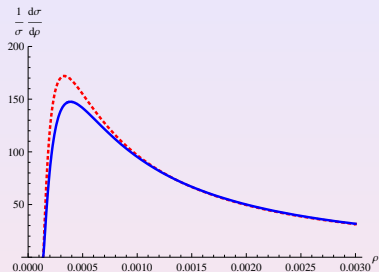
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Only in leading N_c limit. Can use the hemisphere result computed numerically via dipole evolution.

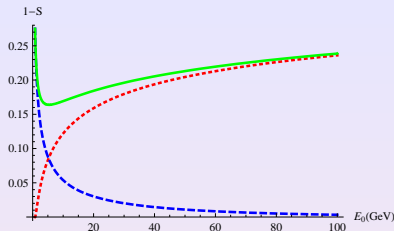
$$S(t) = \exp \left(-C_F C_A \frac{\pi^2}{3} \left(\frac{1 + (at)^2}{1 + (bt)^c} \right) t^2 \right),$$

where $a = 0.85C_A$, $b = 0.86C_A$, $c = 1.33$ and $t \sim \alpha_s L$.



Plots are for $p_t = 250$ GeV and $E_0 = 15$ GeV and $E_0 = 60$ GeV

Banfi, MD ,Khelifa Kerfa, Marzani, 2010



Also non-global factors from edges of recoiling (unrestricted) jets.

$$t_{\text{measured}} = -\frac{1}{4\pi\beta_0} \ln \left(1 - \beta_0 \alpha_s(E_0) \ln \frac{2E_0 R^2}{E_\rho} \right),$$

$$t_{\text{unmeasured}} = -\frac{1}{4\pi\beta_0} \ln \left(1 - \beta_0 \alpha_s(E/2) \ln \frac{E}{2E_0} \right).$$

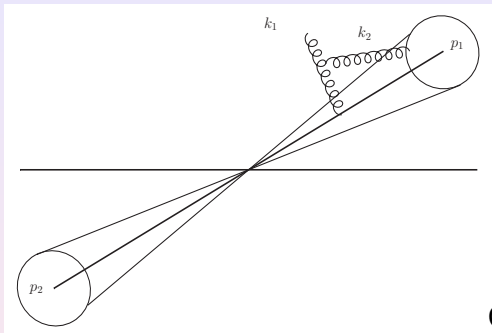
Playing with veto scale doesn't help. Need to restrict emissions **everywhere** like for global event shape variables. 20 to 30 percent effect of NG logs on peak height.

Banfi, Salam, Zanderighi, 2004, 2010.

Jets in other algorithms

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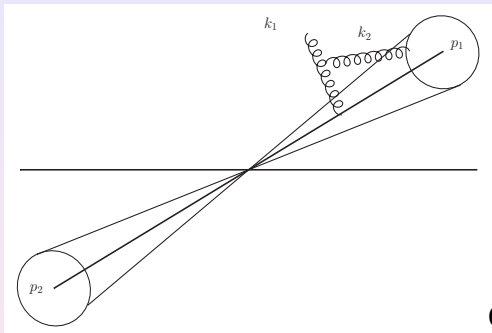
Consider jet masses in algorithms like C-A where self clustering of soft gluons is important. Non-global logs will be affected by clustering. Using a large R value may eliminate them.

Appleby and Seymour 2002

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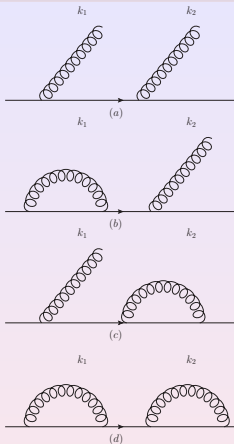
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Further complications

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Clustering spoils the naive gluon exponentiation ! Impacts real virtual cancellations.

Generates additional terms.

Uncancelled contributions appear. E.g configuration (b) contributes a new term to distribution when k_1 is out and k_2 in.

Clustering Logs

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$$\begin{aligned}\frac{d}{d\rho} \Sigma_2^{\text{cluster}} &= -4C_F^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{\rho} \ln \frac{1}{\rho} \int \frac{d\phi}{\pi} \ln^2(2 \cos \phi) \\ &\quad \Theta\left(\cos \phi - \frac{1}{2}\right) \\ &= -0.728 C_F^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{\rho} \ln \frac{1}{\rho}.\end{aligned}$$

These terms are independent of R at small R and need to be retained. They exponentiate but computing the exponent is tough!

Delenda, Appleby, Banfi, MD2006

- There is a lot of scope for resummation in the context of substructure variables.
- Unfortunately many of the most interesting variables are non-global.
- In long term NG logs need to be better understood.
- In the short term one can focus on those ingredients that are important for a phenomenological description. Need for systematic approximations.