Logs,
Non-Global Logs,
and
Non-Global Non-Logs
in Dijet Observables

arXiv:1105.4628 [Today!]

Christopher Lee
MIT

in collaboration with
Andrew Hornig, Iain Stewart,
Jon Walsh and **Saba Zuberi**

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May 25, 2011

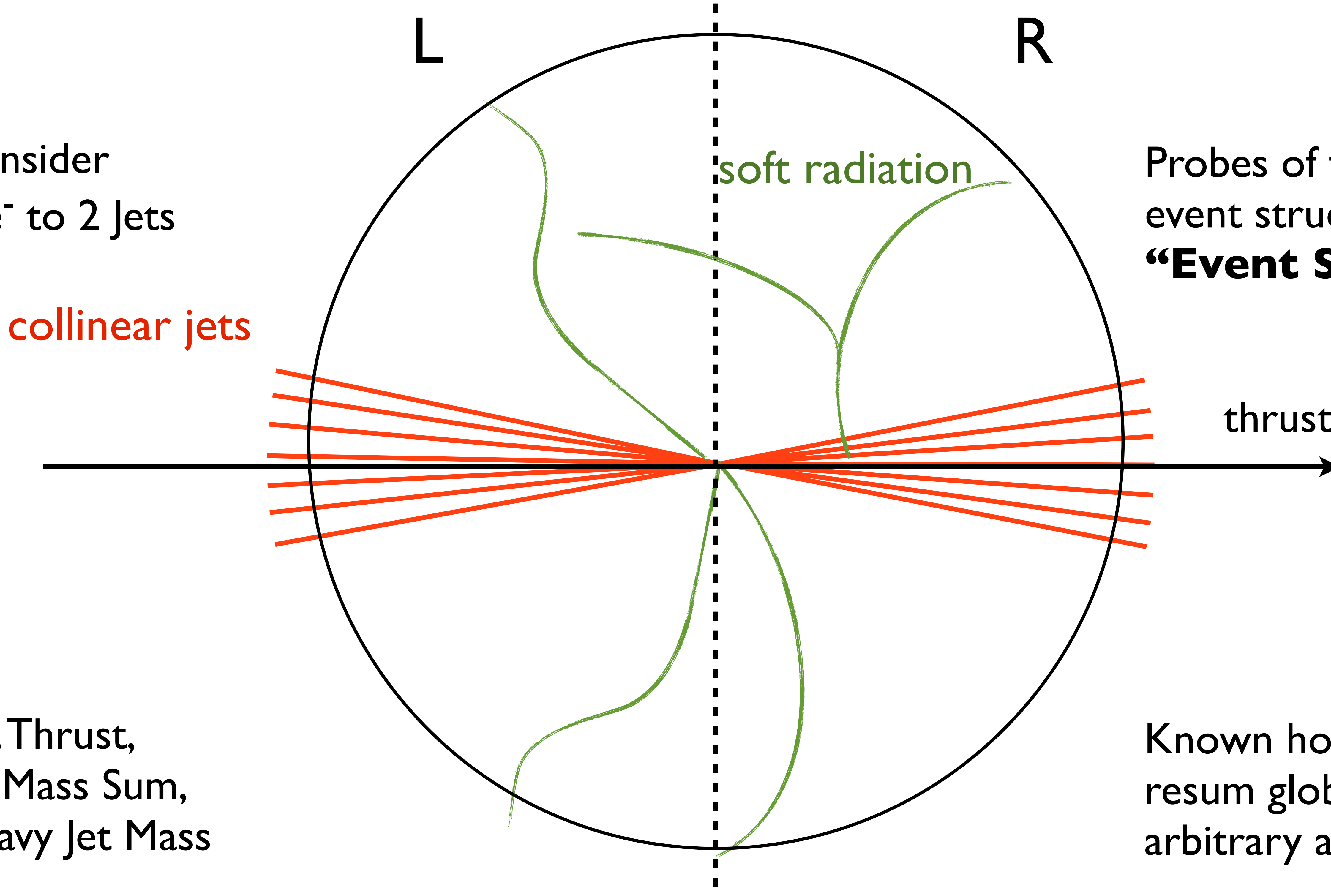
The Next 25 Minutes

- Global and Non-Global Measures of Jettiness
- Factorization and (Global) Log Resummation, EFT
- The Two-Loop Dijet Soft Function: Non-global structures
  - Recent literature:
    - EFT calculation of leading NGL
      CL, Hornig, Stewart, Walsh, Zuberi
      [SCET 2011 Workshop, March 2011]
    - $\mathcal{O}(\alpha_s^2)$ Momentum-Space Dijet Soft Function including constant
      Kelley, Schabinger, Schwartz, Zhu
      [1105.3676]
    - $\mathcal{O}(\alpha_s^2)$ Thrust Soft Function
      Monni, Gehrmann, and Luisoni
      [1105.4560]
    - $\mathcal{O}(\alpha_s^2)$ Momentum- and Position-Space Dijet Soft Function
      Hornig, CL, Stewart, Walsh, Zuberi
      [1105.4628]
- Ultimate Goal: EFT/RGE-based resummation of NGLs

Global Measures of Dijet Structure

Consider
e^+e^- to 2 Jets

Probes of full
event structure
“Event Shapes”



e.g. Thrust,
Jet Mass Sum,
Heavy Jet Mass

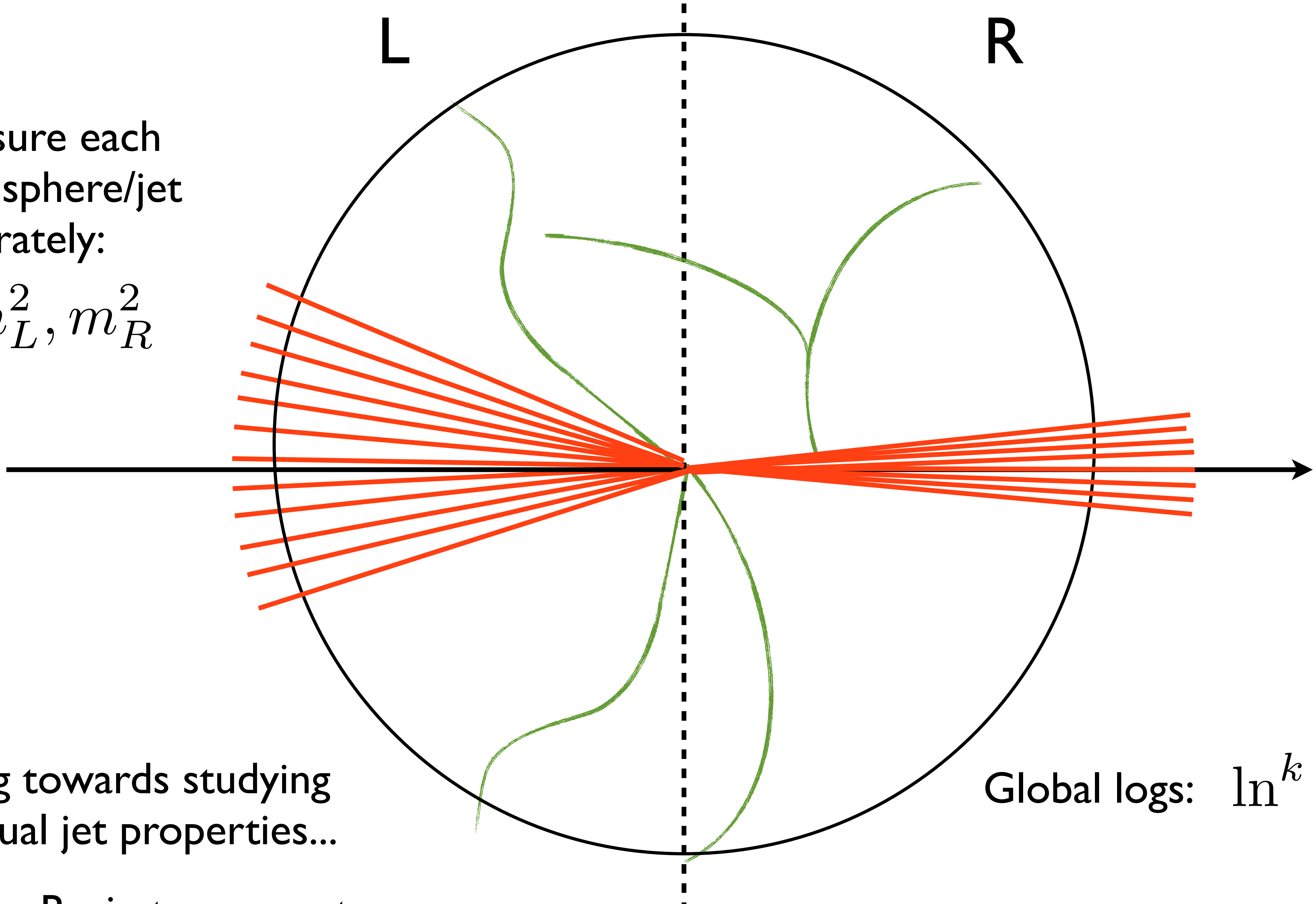
Known how to
resum global logs to
arbitrary accuracy

$$\tau = 1 - \frac{|\mathbf{p}_L| + |\mathbf{p}_R|}{Q} \qquad m^2 = m_L^2 + m_R^2 \qquad m_H^2 = \max\{m_L^2, m_R^2\}$$

Non-Global Measures of Dijet Structure

Measure each hemisphere/jet separately:

$$m_L^2, m_R^2$$



Moving towards studying individual jet properties...

Begin to encounter so-called "non-global" logs

Global logs: $\ln^k \frac{m_{L,R}}{Q}$

Non-global logs: $\ln^n \frac{m_L}{m_R}$

Non-Global Measures of Dijet Structure

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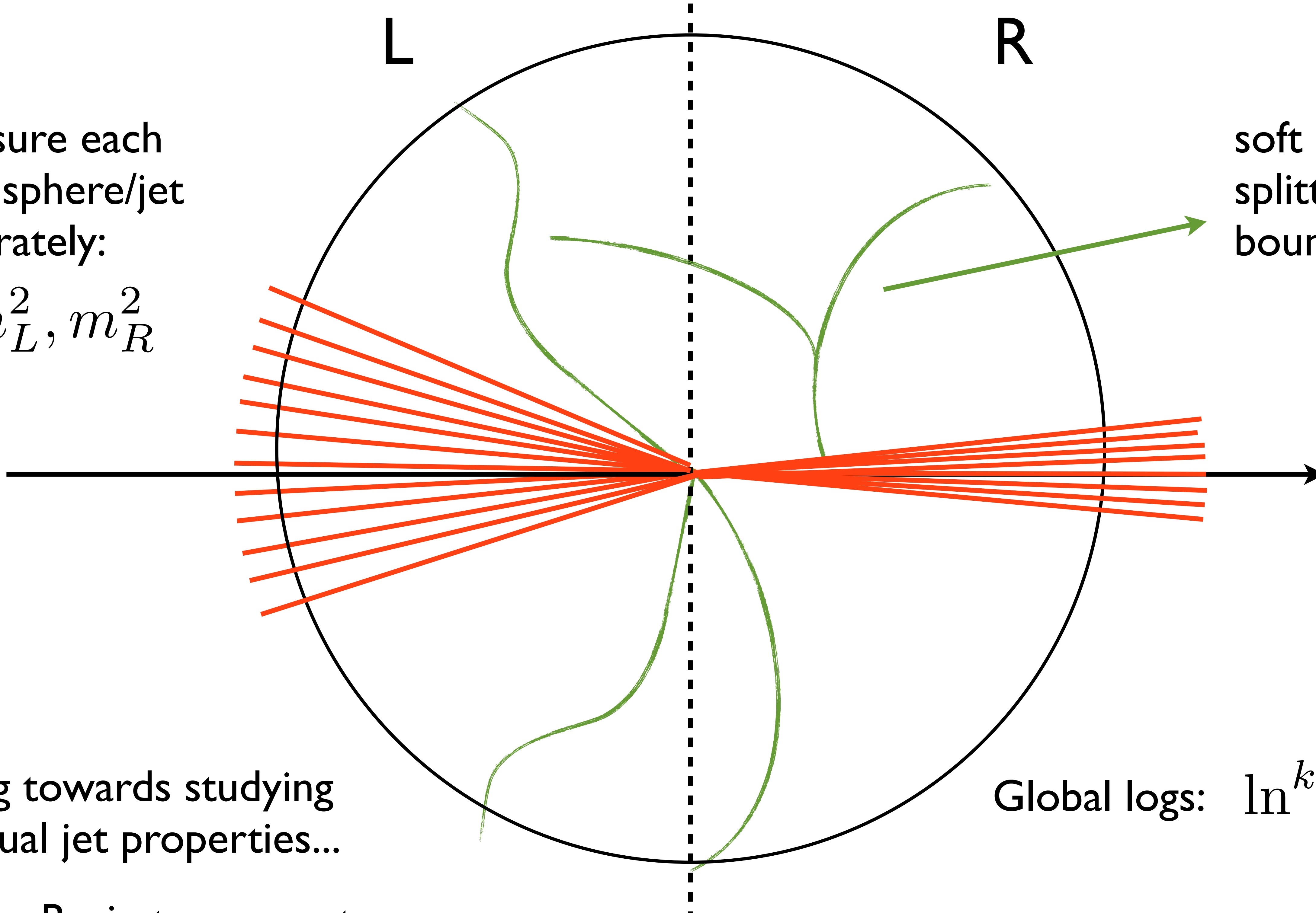


soft radiation splitting across boundary

Moving towards studying individual jet properties...

Begin to encounter so-called "non-global" logs

Global logs: $\ln^k \dfrac{m_{L,R}}{Q}$

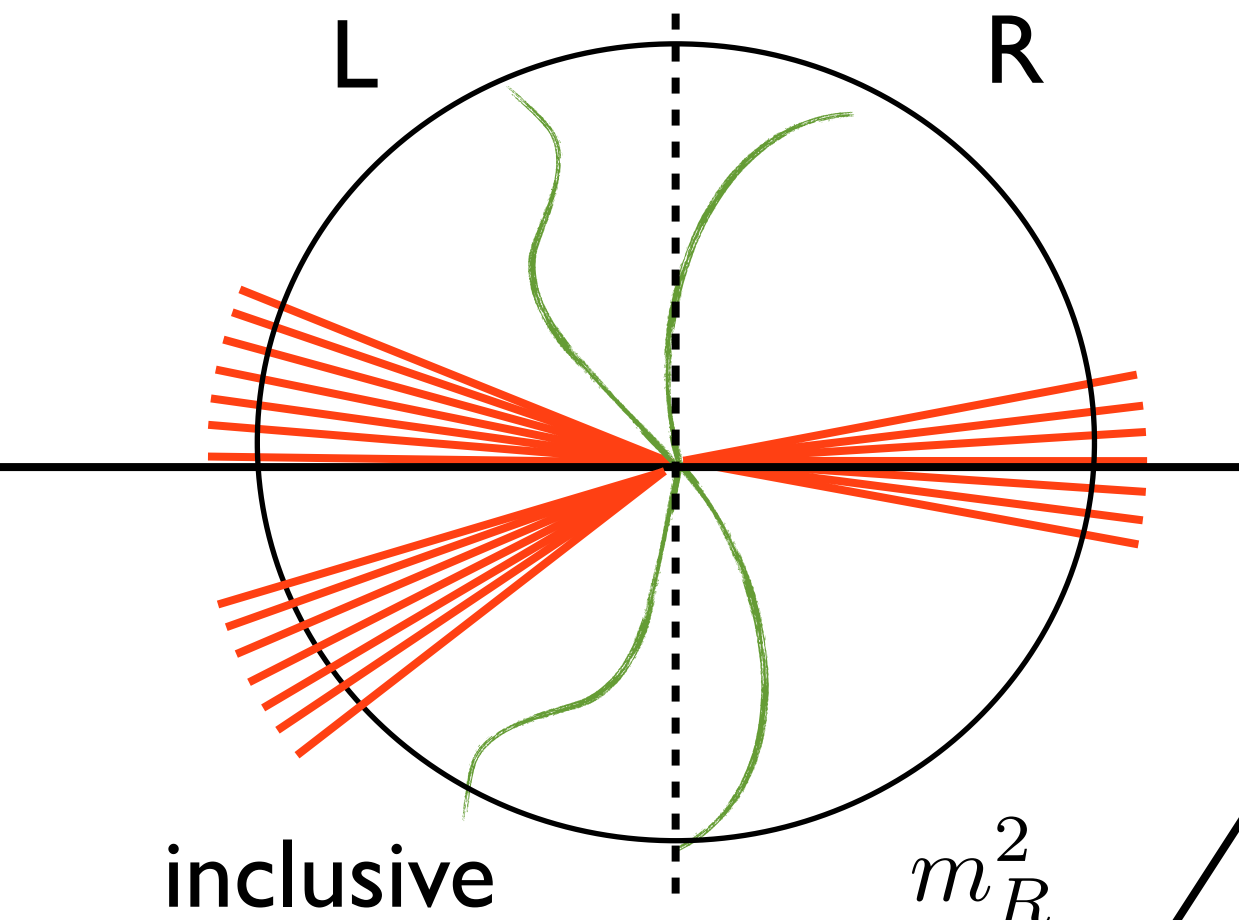
Non-global logs: $\ln^n \dfrac{m_L}{m_R}$

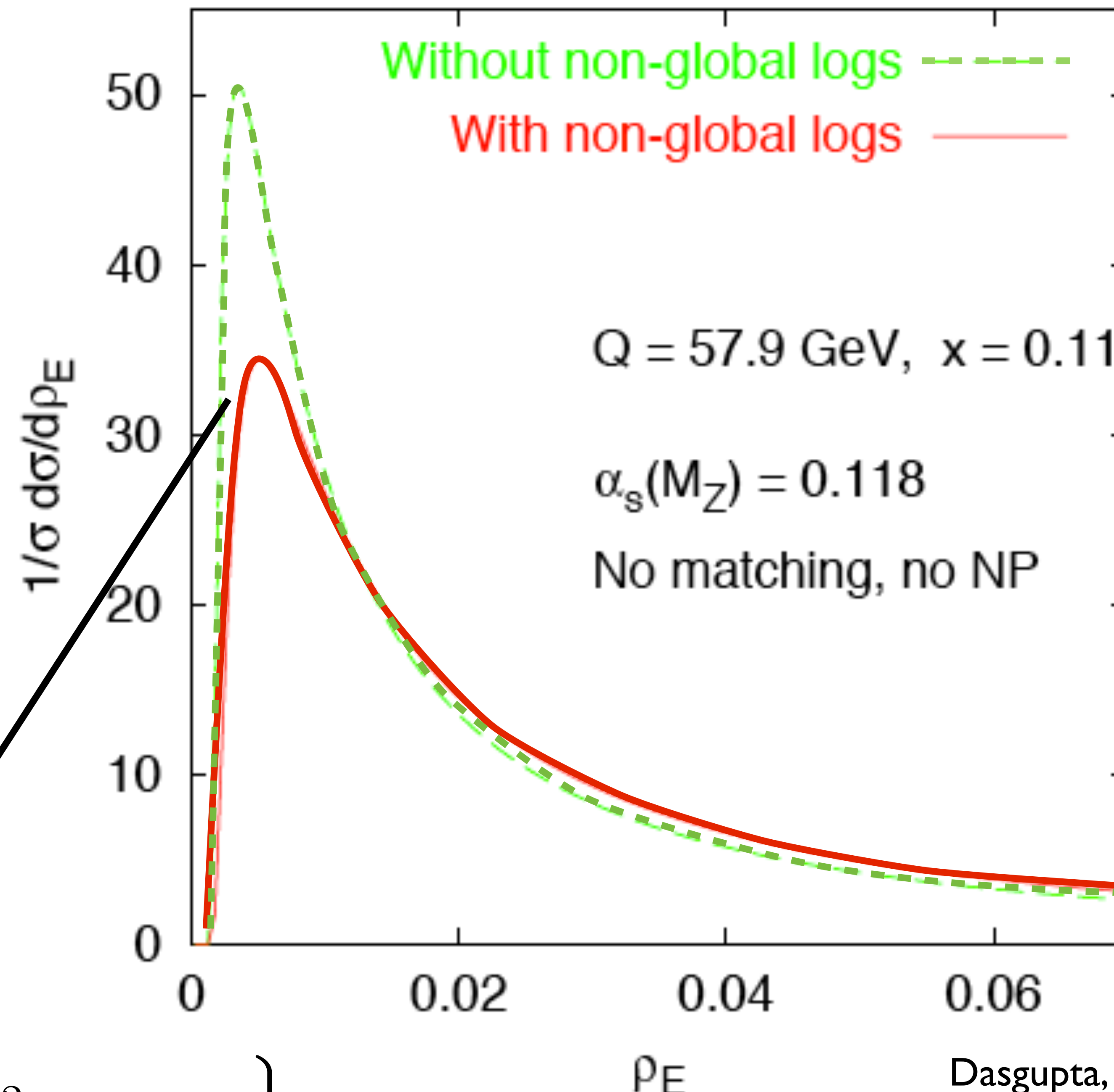
Non-Global Logs

single hemisphere mass cumulant:

$$\Sigma(\rho_R) = \int_0^{Q^2\rho_R} dm_R^2 \int_0^{\infty} dm_L^2 \frac{d\sigma}{dm_L^2\, dm_R^2}$$

Dasgupta, Salam (2001, 2002)





hemisphere mass in DIS

Dasgupta, Salam (2002)

NGLs multiply green (global) curve by

$$\times \left\{ 1 - \left(\frac{\alpha_s}{2\pi}\right)^2 C_F C_A \frac{\pi^2}{3} \ln^2 \rho_R + \cdots \right\}$$

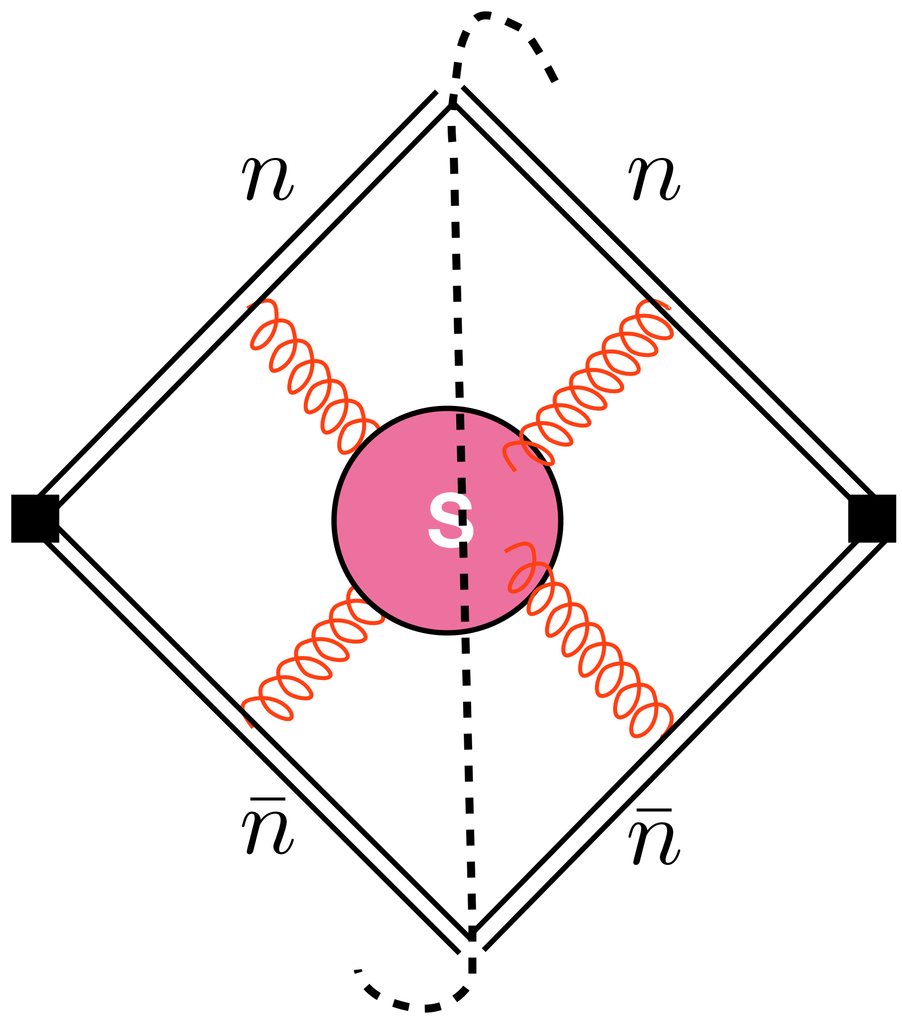
Dijet Factorization Theorem

$$\frac{d\sigma}{dm_1^2 dm_2^2} = \sigma^0_{q\bar{q}} H(Q;\mu) \int dm_1^2\, dm_2^2 J(m_1^2 - Ql^+;\mu) J(m_2^2 - Ql^-;\mu) S(l^+, l^-;\mu)$$

Fourier transform

$$\tilde{\sigma}(y_1, y_2) = \int dm_1^2 dm_2^2 e^{-im_1^2 y_1 - im_2^2 y_2} \frac{d\sigma}{dm_1^2 dm_2^2} = \sigma^0_{q\bar{q}} H(Q;\mu) J(y_1;\mu) J(y_2;\mu) S(Qy_1, Qy_2;\mu)$$

The Dijet Soft Function:



same* soft function appears in pp beam thrust/0-jettiness

* up to constant term

$$S(\ell_1, \ell_2) = \frac{1}{N_C} \sum_{X_S} \left| \langle X_S | T[Y_n^\dagger Y_{\bar{n}}] | 0 \rangle \right|^2 \delta\left(\ell_1 - \sum_{i \in L} \bar{n} \cdot k_i\right) \delta\left(\ell_2 - \sum_{i \in R} n \cdot k_i\right)$$

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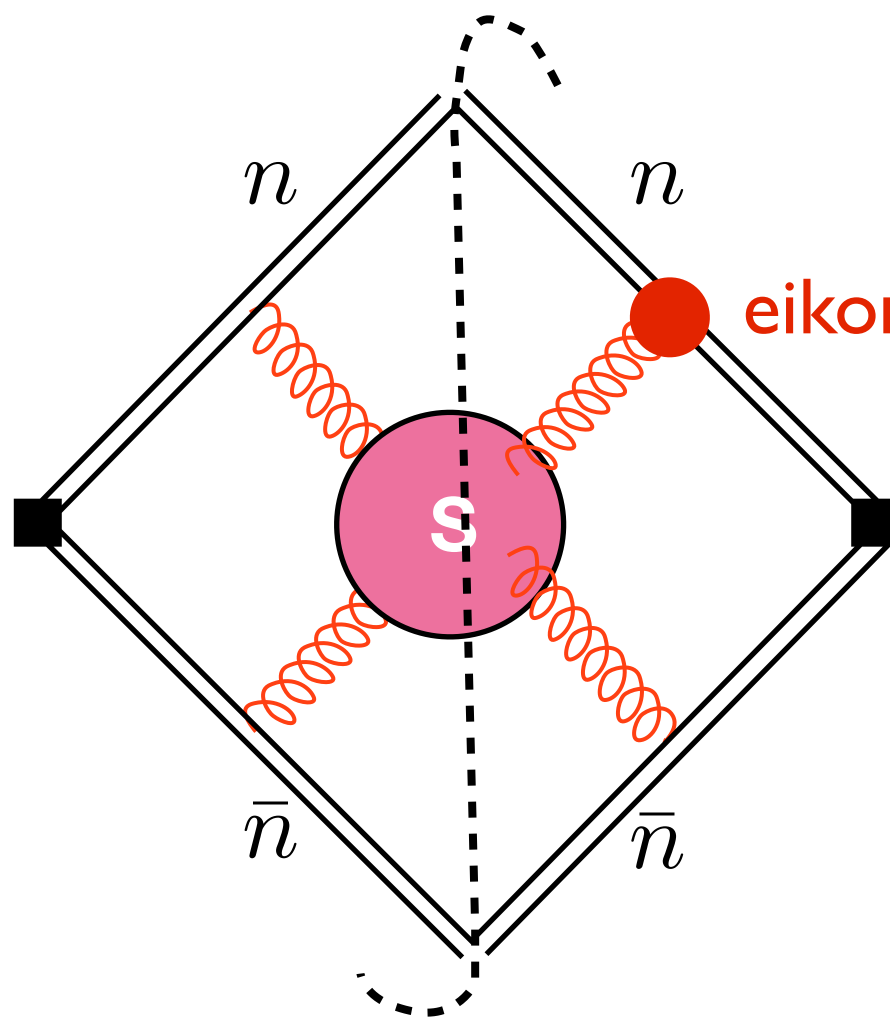

eikonal vertices $\dfrac{gn^\mu T^A}{n \cdot k}$

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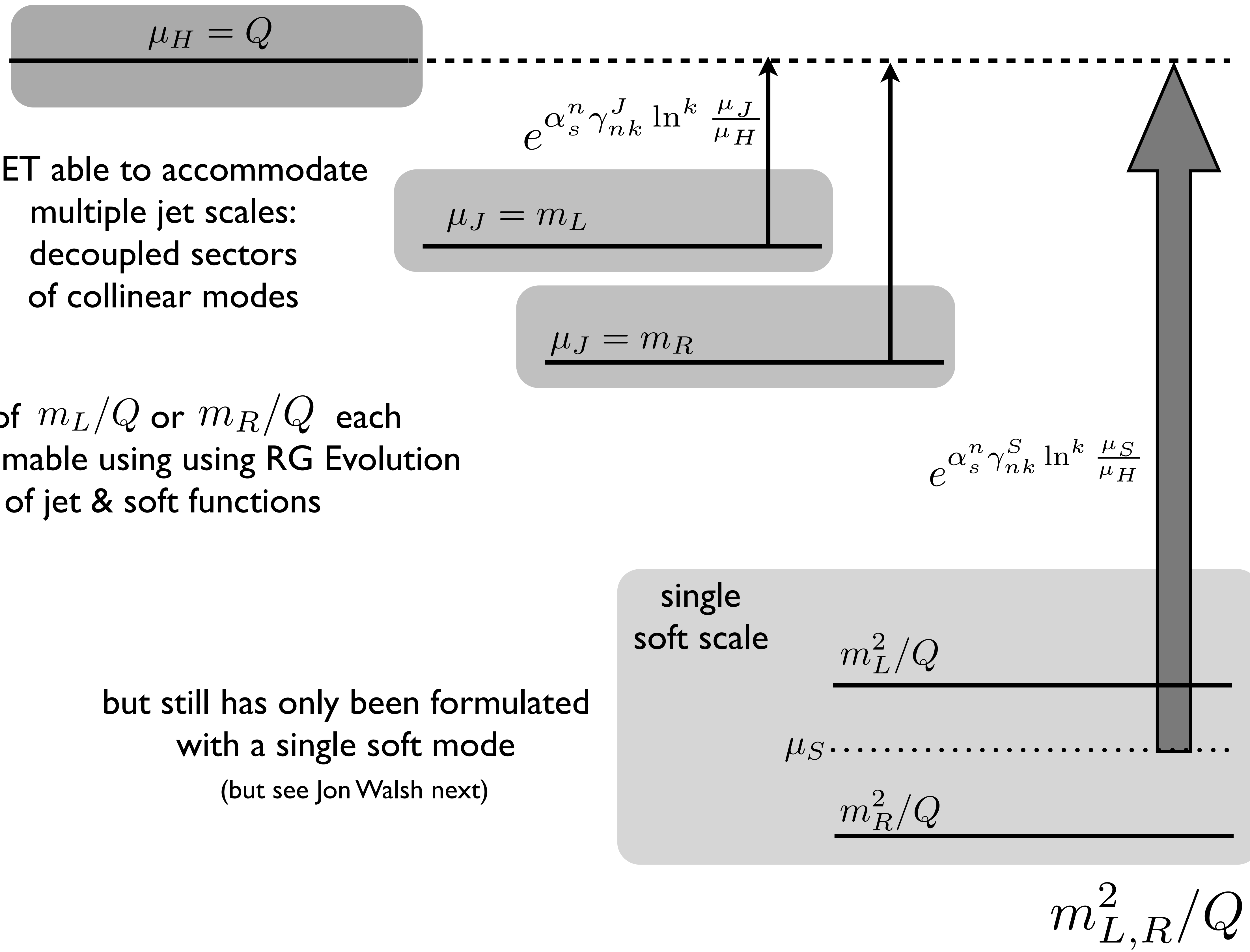
Effective Field Theory

SCET: Bauer, Fleming, Luke, Pirjol, Stewart (2000-2001)



SCET able to accommodate multiple jet scales: decoupled sectors of collinear modes

logs of m_L/Q or m_R/Q each each summable using using RG Evolution of jet & soft functions

but still has only been formulated with a single soft mode

(but see Jon Walsh next)

Effective Field Theory

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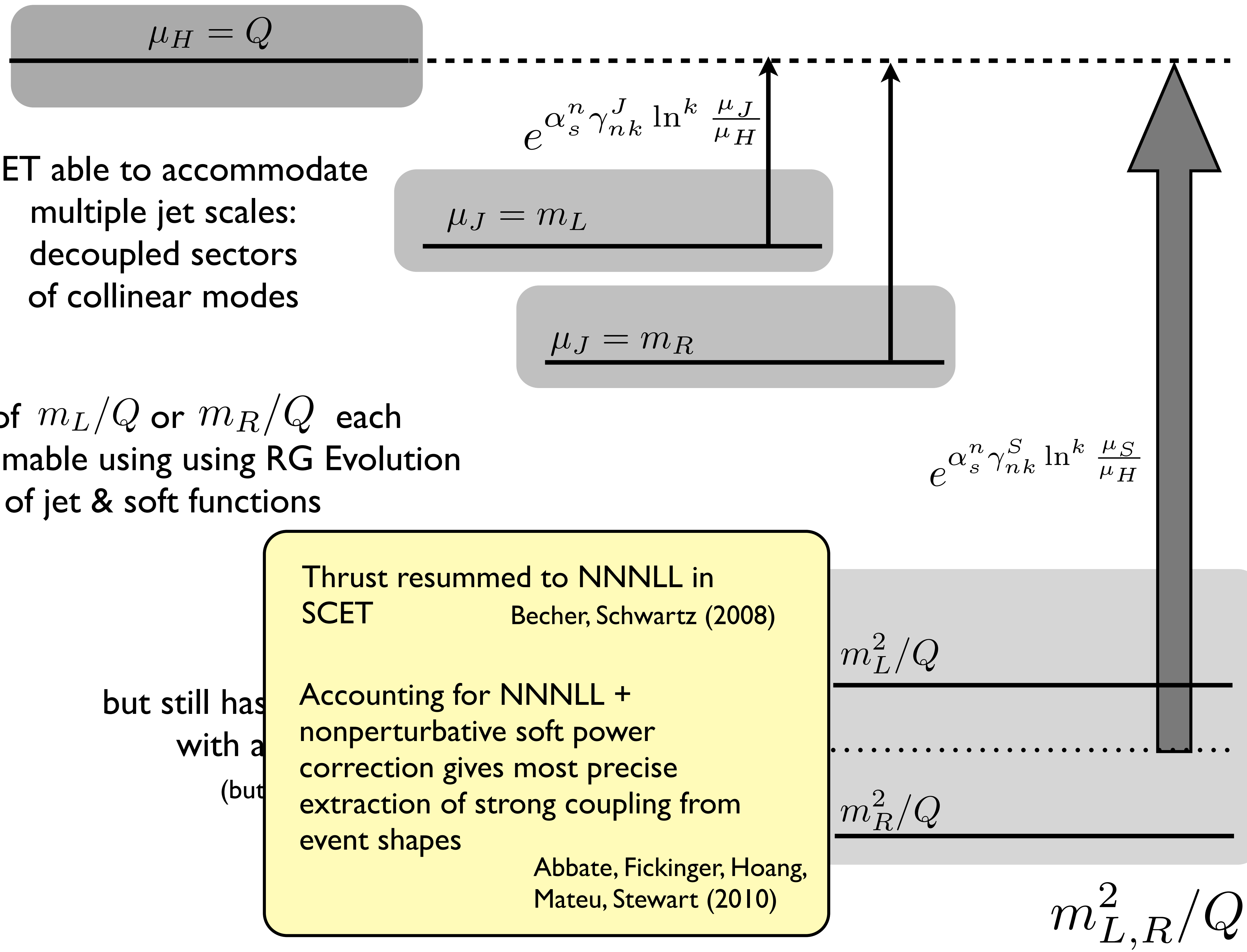

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but still has

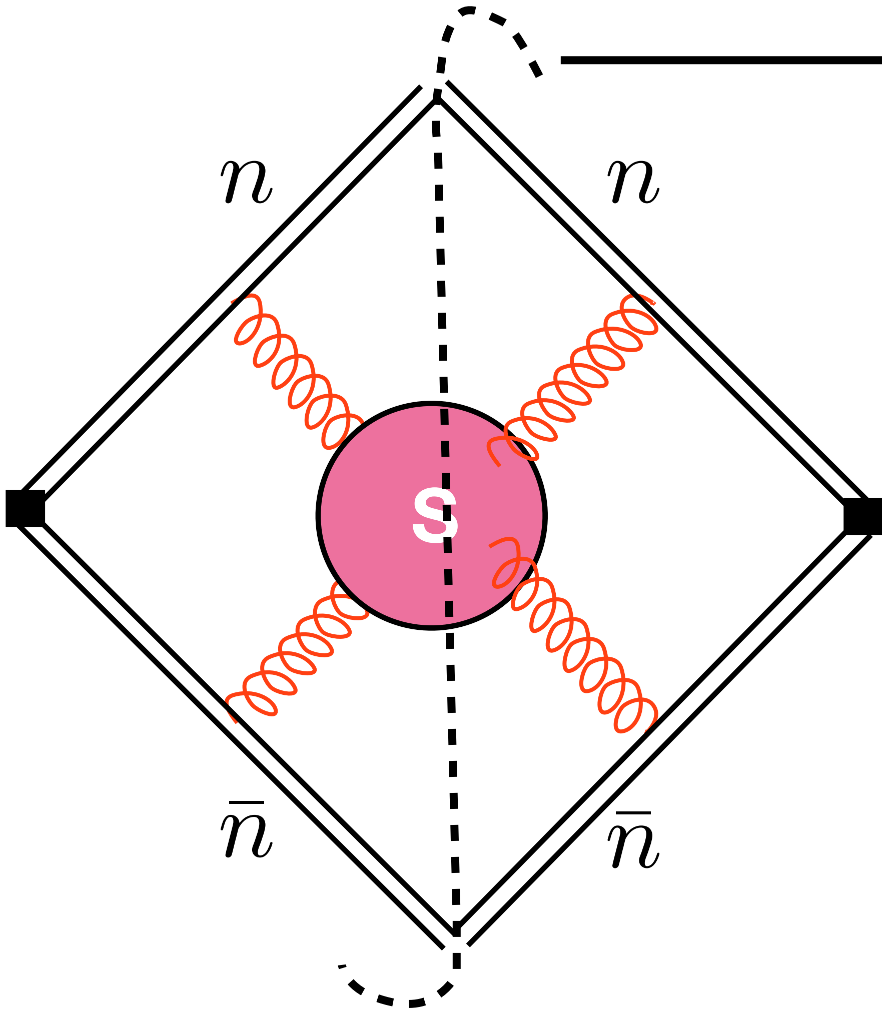
with a

(but

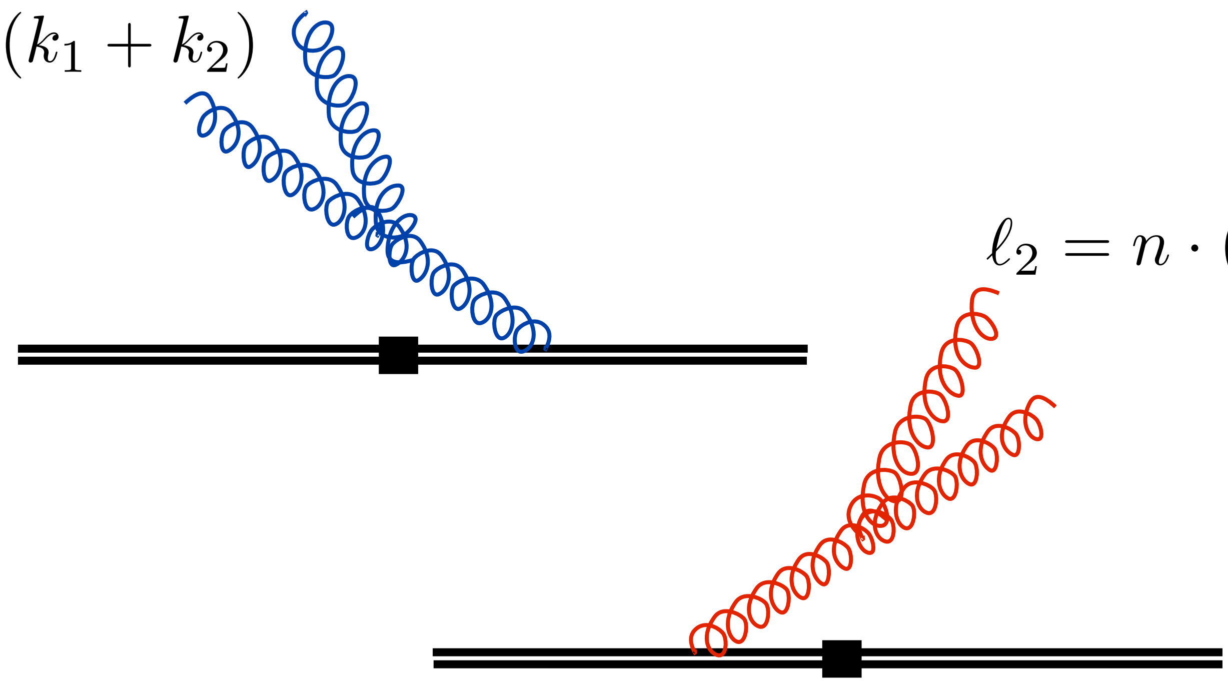
Thrust resummed to NNNLL in SCET — Becher, Schwartz (2008)

Accounting for NNNLL + nonperturbative soft power correction gives most precise extraction of strong coupling from event shapes — Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

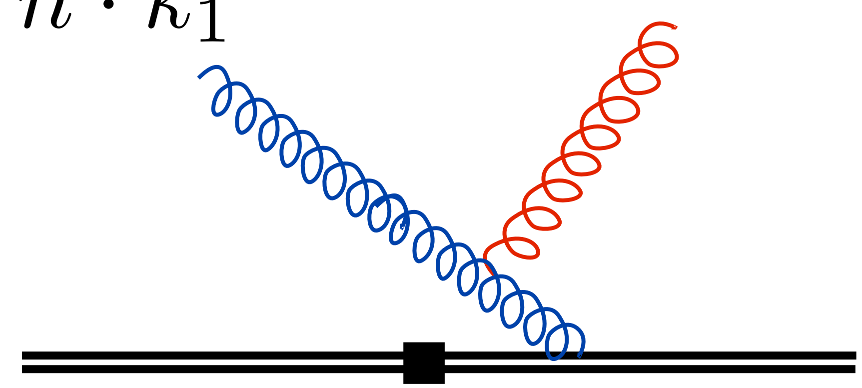
Calculating the Dijet Soft Function at $\mathcal{O}(\alpha_s^2)$



Possible 2-particle final states along cut:



“Same hemisphere”

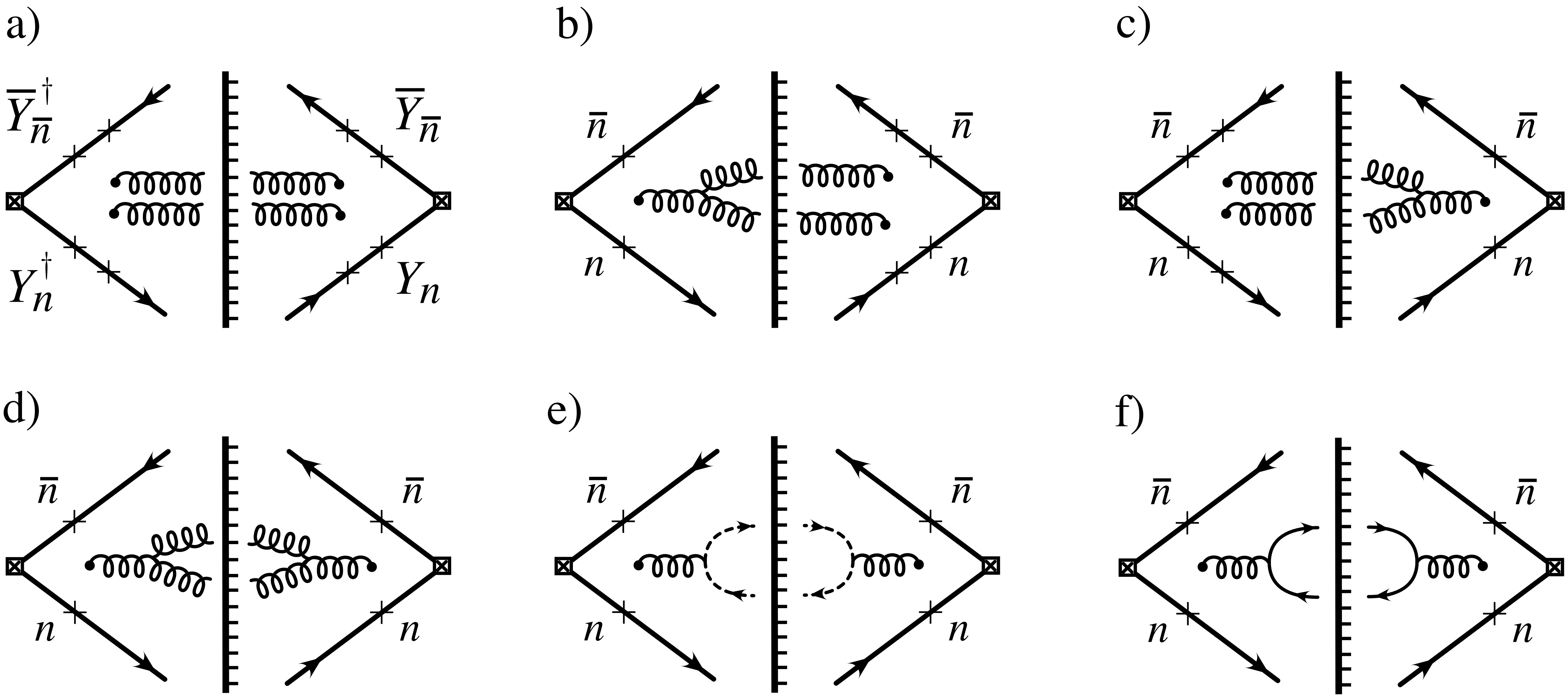


“Opposite hemisphere”

$$S(\ell_1,\ell_2) = \frac{1}{N_C}\sum_{X_S}\left|\langle X_S|T[Y_n^\dagger Y_{\bar{n}}]|0\rangle\right|^2 \times \delta\left(\ell_1 - \sum_{i\in L}\bar{n}\cdot k_i\right)\delta\left(\ell_2 - \sum_{i\in R} n\cdot k_i\right)$$

Contains all non-global structure, can deduce same hemisphere contributions (up to constant) from it + RG invariance + IR finiteness ← “Opposite hemisphere”

Calculating the Dijet Soft Function at $\mathcal{O}(\alpha_s^2)$



$$C_F^2 : \quad \mathrm{a}$$

$$C_F C_A : \quad \mathrm{a,b,c,d,e}$$

$$C_F T_R n_f : \quad \mathrm{f}$$

Known Properties and a Conjecture

Knowns:

Symmetry: $$\widetilde{S}(x_1, x_2; \mu) = \widetilde{S}(x_2, x_1; \mu)$$

Exponentiation: $$\widetilde{S}(x_1, x_2; \mu) = e^{K(x_1, x_2; \mu)} e^{T(x_1, x_2)}$$

RG Evolution: $$\widetilde{K}(x_1, x_2; \mu) = \sum_{n,k} \gamma_{n,k} \alpha_s^n \left[\ln^k(\mu i x_1) + \ln^k(\mu i x_2)\right]$$

(sums global logs)

known to two (three) loops
enough for NNNLL

RG-independent part to 1-loop: $$\widetilde{T}(x_1, x_2) = -\frac{\alpha_s C_F}{4\pi}\pi^2 + 2\frac{\alpha_s^2}{(4\pi)^2} t_2(x_1/x_2)$$

Conjecture:

Hoang, Kluth (2008)

$$t_2(x_1/x_2) = s_2^{[2]} \ln^2(x_1/x_2) + s_2$$

Reasons: No single log of x_1/x_2 due to symmetry
Only logs due to expectation based on event shape distributions

Note: Constant can be extracted numerically from EVENT2 thrust predictions

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Non-global structure

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The Result!

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$
\begin{aligned}
\frac{1}{2} t_2(x_1/x_2) = & - C_F C_A \frac{\pi^2}{3} \ \ln^2\left(\frac{x_1}{x_2}\right) \\
& + \ln\left(\frac{x_1/x_2 + x_2/x_1}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \left(\frac{6 - 4\pi^2}{9}\right)\right] \\
& + C_F T_R n_f \left[F_Q\left(\frac{x_1}{x_2}\right) + F_Q\left(\frac{x_2}{x_1}\right) - 2F_Q(1)\right] + C_F C_A \left[F_N\left(\frac{x_1}{x_2}\right) + F_N\left(\frac{x_2}{x_1}\right) - 2F_N(1)\right] \\
& + C_F^2 \frac{\pi^4}{8} + \frac{1}{2} C_F C_A s_2^{[C_F C_A]} + \frac{1}{2} C_F T_R n_f s_2^{[n_f]}
\end{aligned}
$$

The Result!

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$\frac{1}{2}t_2(x_1/x_2) = -C_F C_A \frac{\pi^2}{3}\ \ln^2\left(\frac{x_1}{x_2}\right)$$ **Double NGL**

$$+\ln\left(\frac{x_1/x_2 + x_2/x_1}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \left(\frac{6-4\pi^2}{9}\right)\right]$$

$$+ C_F T_R n_f \left[F_Q\left(\frac{x_1}{x_2}\right) + F_Q\left(\frac{x_2}{x_1}\right) - 2F_Q(1)\right] + C_F C_A \left[F_N\left(\frac{x_1}{x_2}\right) + F_N\left(\frac{x_2}{x_1}\right) - 2F_N(1)\right]$$

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The Result!

Hornig, CL, Stewart, Walsh, Zuberi
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$$\frac{1}{2} t_2(x_1/x_2) = -C_F C_A \frac{\pi^2}{3} \ln^2\left(\frac{x_1}{x_2}\right)$$

Double NGL

Single NGL!
(symmetric!)

$$+ \ln\left(\frac{x_1/x_2 + x_2/x_1}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \left(\frac{6 - 4\pi^2}{9}\right)\right]$$

$$+ C_F T_R n_f \left[F_Q\left(\frac{x_1}{x_2}\right) + F_Q\left(\frac{x_2}{x_1}\right) - 2F_Q(1)\right] + C_F C_A \left[F_N\left(\frac{x_1}{x_2}\right) + F_N\left(\frac{x_2}{x_1}\right) - 2F_N(1)\right]$$

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The Result!

Hornig, CL, Stewart, Walsh, Zuberi
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$$+ C_F T_R n_f \left[F_Q\left(\frac{x_1}{x_2}\right) + F_Q\left(\frac{x_2}{x_1}\right) - 2F_Q(1)\right] + C_F C_A \left[F_N\left(\frac{x_1}{x_2}\right) + F_N\left(\frac{x_2}{x_1}\right) - 2F_N(1)\right]$$
$$+ C_F^2 \frac{\pi^4}{8} + \frac{1}{2}C_F C_A s_2^{[C_F C_A]} + \frac{1}{2}C_F T_R n_f s_2^{[n_f]}$$

Non-Global Non-Logs:

$$F_Q(b) = \frac{2\ln b}{3(b-1)} - \frac{b\ \ln^2 b}{3(b-1)^2} - \frac{(3-2\pi^2)}{9}\ \ln\left(b+\frac{1}{b}\right) + \frac{2}{3}\ln^2 b\ \ln(1-b) + \frac{8}{3}\ln b\ \mathrm{Li}_2(b) - 4\,\mathrm{Li}_3(b),$$

$$F_N(b) = -\frac{\pi^4}{36} - \frac{\ln b}{3(b-1)} + \frac{b\ \ln^2 b}{6(b-1)^2} + \frac{(3-11\pi^2+18\zeta_3)}{18}\ \ln\left(b+\frac{1}{b}\right) - \frac{11}{6}\ln^2 b\ \ln(1-b) + \frac{\ln^4 b}{24}$$
$$- \frac{\pi^2}{3}\,\mathrm{Li}_2(1-b) + \left[\mathrm{Li}_2(1-b)\right]^2 - \frac{22}{3}\ln b\ \mathrm{Li}_2(b) + 2\ln b\,\mathrm{Li}_3(1-b) + 11\,\mathrm{Li}_3(b)$$

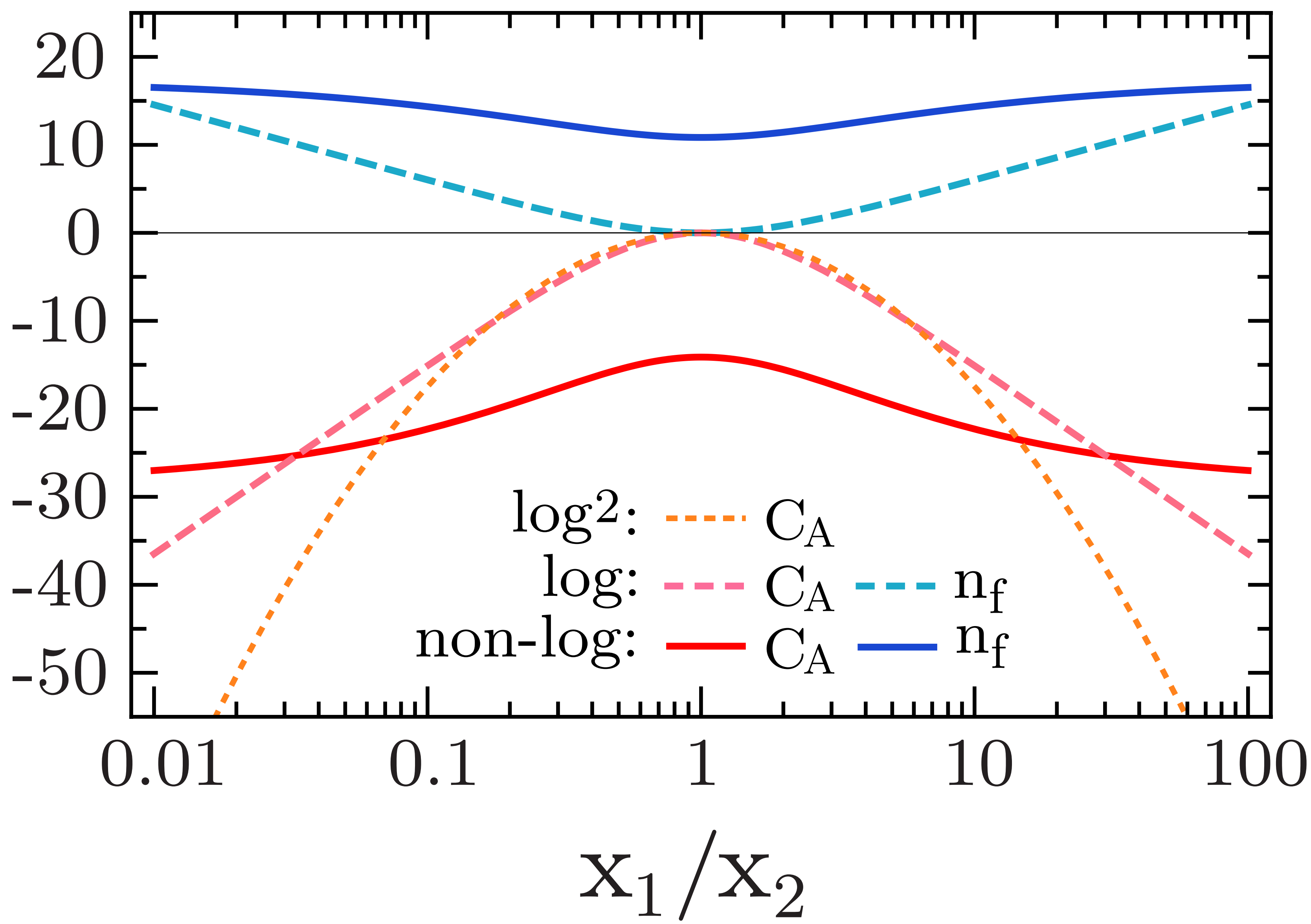
$$b \equiv x_1/x_2$$

The Result!

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$\frac{1}{2}t_2(x_1/x_2) = \text{Double NGL} + \text{Single NGL} + \text{Non-Logs*}$$

* including constants



Position space result shows directly what terms must be added to HK ansatz
and is required to achieve (global log) resummation of many dijet observables analytically

Momentum Space Result

Hornig, CL, Stewart, Walsh, Zuberi [1105.4628]

agrees exactly numerically with Kelley, Schabinger, Schwartz, Zhu [1105.3676]

For the double cumulant $\mathcal{S}_c(\ell_1^c, \ell_2^c; \mu) = \int^{\ell_1^c} d\ell_1 \int^{\ell_2^c} d\ell_2 \, S(\ell_1, \ell_2; \mu)$

$$
\begin{aligned}
\frac{1}{2} t_2^c(\ell_1^c, \ell_2^c, \mu) = \theta(\ell_1^c)\theta(\ell_2^c) \Bigg\{ & -\frac{\pi^2}{3} C_F C_A \ln^2\left(\frac{\ell_1^c}{\ell_2^c}\right) \\
& + \ln\left(\frac{\ell_1^c/\ell_2^c + \ell_2^c/\ell_1^c}{2}\right) \left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \frac{6 - 4\pi^2}{9} \right] \\
& + C_F C_A \left[f_N\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_N\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_N(1) \right] + C_F T_R n_f \left[f_Q\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_Q\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_Q(1) \right] \\
& + C_F^2 \frac{\pi^4}{8} + \frac{1}{2} C_F C_A s_{2\rho}^{[C_F C_A]} + \frac{1}{2} C_F T_R n_f s_{2\rho}^{[n_f]} \Bigg\}
\end{aligned}
$$

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$$\frac{1}{2} t_2^c(\ell_1^c, \ell_2^c, \mu) = \theta(\ell_1^c)\theta(\ell_2^c) \bigg\{ -\frac{\pi^2}{3} C_F C_A \ln^2\left(\frac{\ell_1^c}{\ell_2^c}\right)$$

Double NGL

$$+ \ln\left(\frac{\ell_1^c/\ell_2^c + \ell_2^c/\ell_1^c}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \frac{6 - 4\pi^2}{9}\right]$$

$$+ C_F C_A\left[f_N\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_N\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_N(1)\right] + C_F T_R n_f\left[f_Q\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_Q\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_Q(1)\right]$$

$$+ C_F^2 \frac{\pi^4}{8} + \frac{1}{2} C_F C_A s_{2\rho}^{[C_F C_A]} + \frac{1}{2} C_F T_R n_f s_{2\rho}^{[n_f]} \bigg\}$$

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$$\frac{1}{2} t_2^c(\ell_1^c, \ell_2^c, \mu) = \theta(\ell_1^c)\theta(\ell_2^c) \Bigg\{ -\frac{\pi^2}{3} C_F C_A \ln^2\left(\frac{\ell_1^c}{\ell_2^c}\right)$$

Double NGL

Single NGL

$$+ \ln\left(\frac{\ell_1^c/\ell_2^c + \ell_2^c/\ell_1^c}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \frac{6 - 4\pi^2}{9}\right]$$

$$+ C_F C_A \left[f_N\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_N\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_N(1)\right] + C_F T_R n_f \left[f_Q\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_Q\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_Q(1)\right]$$

$$+ C_F^2 \frac{\pi^4}{8} + \frac{1}{2} C_F C_A s_{2\rho}^{[C_F C_A]} + \frac{1}{2} C_F T_R n_f s_{2\rho}^{[n_f]} \Bigg\}$$

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Double NGL

Single NGL

$$
\begin{aligned}
\frac{1}{2} t_2^c(\ell_1^c, \ell_2^c, \mu) = \theta(\ell_1^c)\theta(\ell_2^c)\Bigg\{ & -\frac{\pi^2}{3} C_F C_A \ln^2\left(\frac{\ell_1^c}{\ell_2^c}\right) \\
& + \ln\left(\frac{\ell_1^c/\ell_2^c + \ell_2^c/\ell_1^c}{2}\right)\left[C_F C_A \frac{11\pi^2 - 3 - 18\zeta_3}{9} + C_F T_R n_f \frac{6 - 4\pi^2}{9}\right] \\
& + C_F C_A\left[f_N\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_N\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_N(1)\right] + C_F T_R n_f\left[f_Q\left(\frac{\ell_1^c}{\ell_2^c}\right) + f_Q\left(\frac{\ell_2^c}{\ell_1^c}\right) - 2 f_Q(1)\right] \\
& + C_F^2 \frac{\pi^4}{8} + \frac{1}{2} C_F C_A s_{2\rho}^{[C_F C_A]} + \frac{1}{2} C_F T_R n_f s_{2\rho}^{[n_f]}\Bigg\}
\end{aligned}
$$

Non-Global Non-Logs:

$$
f_Q(a) \equiv \left(\frac{2\pi^2}{9} - \frac{2}{3(a+1)}\right)\ln a - \frac{4}{3}\ln a\,\mathrm{Li}_2(-a) + 4\,\mathrm{Li}_3(-a) - \frac{1}{9}(3 - 2\pi^2)\ln\left(a + \frac{1}{a}\right),
$$

$$
\begin{aligned}
f_N(a) \equiv & -4\mathrm{Li}_4\left(\frac{1}{a+1}\right) - 11\,\mathrm{Li}_3(-a) + 2\mathrm{Li}_3\left(\frac{1}{a+1}\right)\ln\left[\frac{a}{(a+1)^2}\right] \\
& + \mathrm{Li}_2\left(\frac{1}{a+1}\right)\left\{\pi^2 - \ln^2(a+1) - \frac{1}{2}\ln a \ln\left[\frac{a}{(a+1)^2}\right] + \frac{11}{3}\ln a\right\} \\
& + \frac{1}{24}\left\{22\ln\left[\frac{a}{(a+1)^2}\right] - 6\ln\left(1 + \frac{1}{a}\right)\ln(1+a) + \pi^2\right\}\ln^2 a - \frac{(a-1)\ln a}{6(a+1)} \\
& + \frac{5\pi^2}{12}\ln\left(1 + \frac{1}{a}\right)\ln(1+a) - \frac{11\pi^4}{180}
\end{aligned}
$$

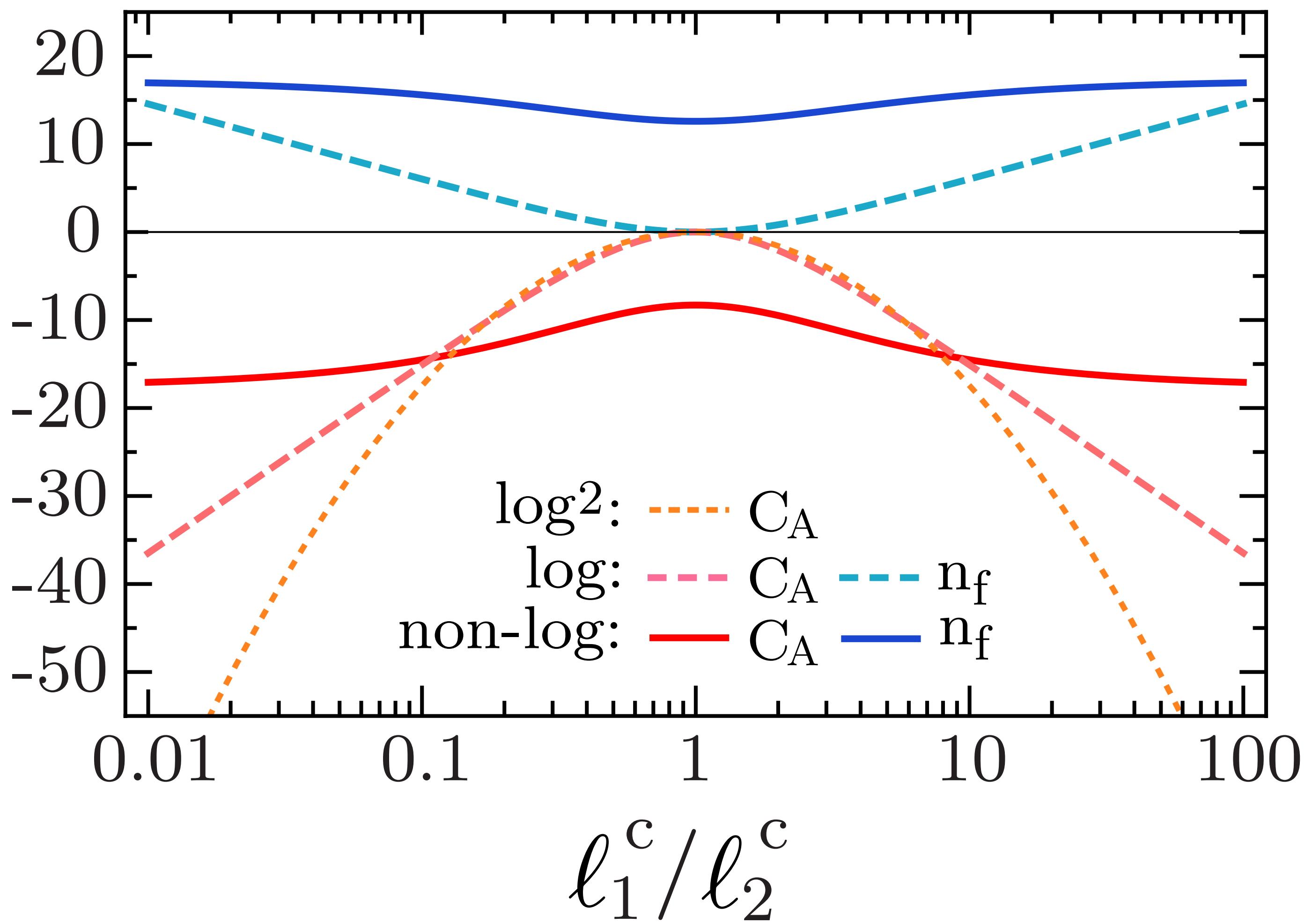
$$
a \equiv \ell_1^c/\ell_2^c
$$

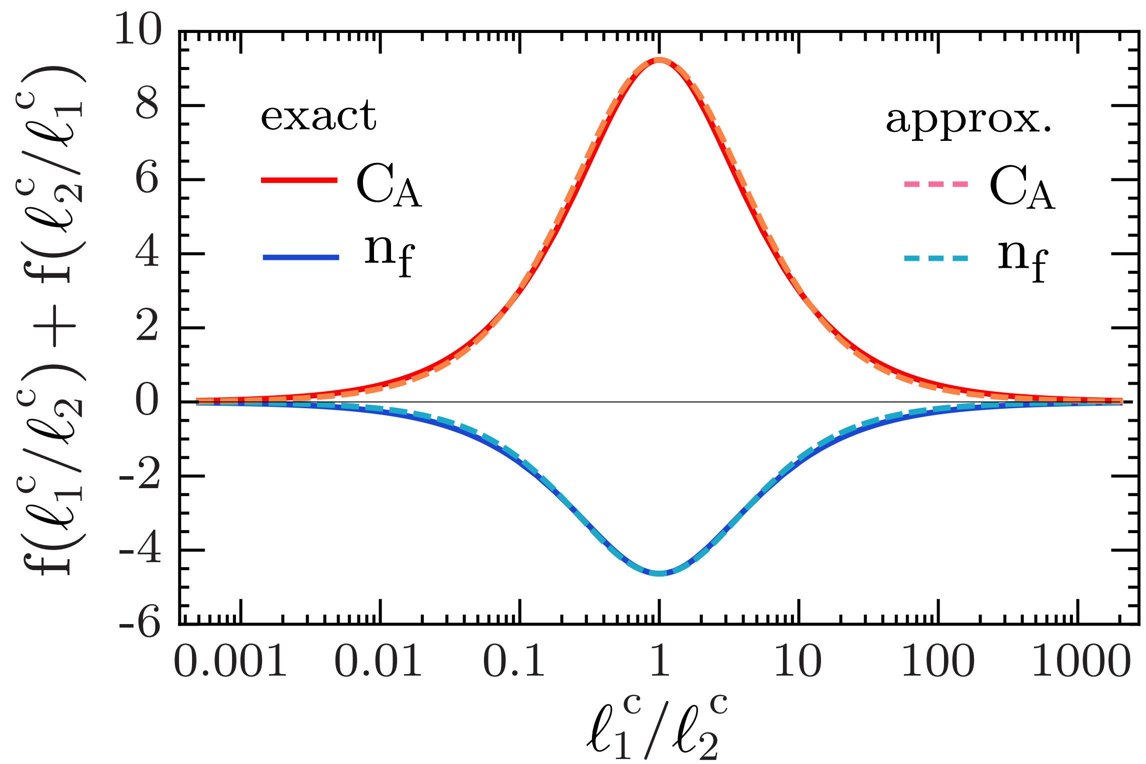
Non-Global Structures in Momentum Space

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$\frac{1}{2} t_2(\ell_1^c/\ell_2^c) =$$ Double NGL + Single NGL + Non-Logs*

* including constants





After isolating the single NGL $\sim \ln\left(\frac{\ell_1^c}{\ell_2^c} + \frac{\ell_2^c}{\ell_1^c}\right)$

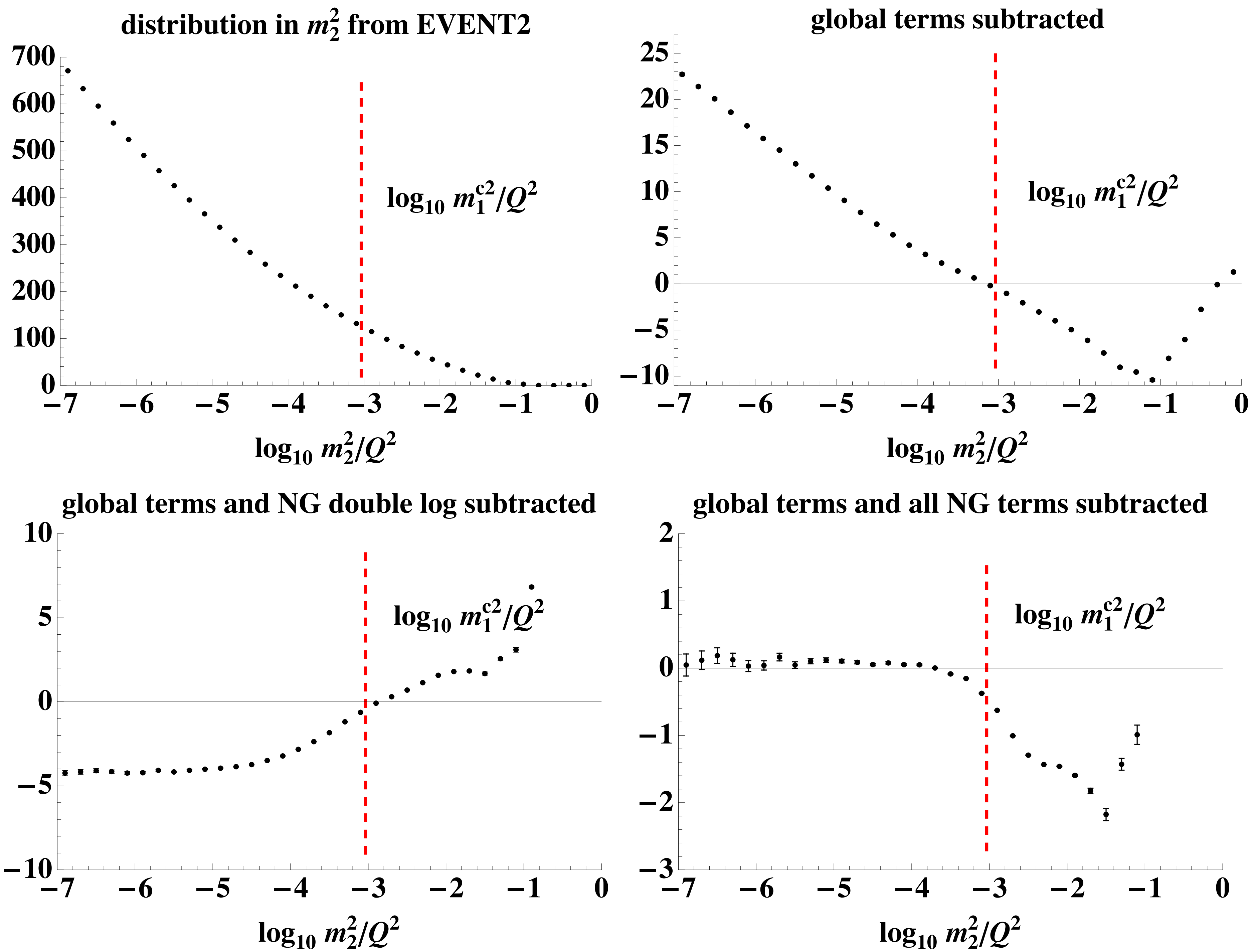
one can notice the non-constant part of the non-log function is closely approximated by:

$$f\left(\frac{\ell_1^c}{\ell_2^c}\right) + f\left(\frac{\ell_2^c}{\ell_1^c}\right) = 2f(1)\frac{4\ell_1^c/\ell_2^c}{(1+\ell_1^c/\ell_2^c)^2}$$

Cross-Check with EVENT2

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$\frac{d\sigma}{dm_2^2}(m_1^{c2}) = \int_0^{m_1^{c2}} dm_1^2 \frac{d^2\sigma}{dm_1^2 dm_2^2}$$



C_F C_A
color structure

Numerical $\mathcal{O}(\alpha_s^2)$ prediction from EVENT2

Catani, Seymour

1.82 trillion events
2 CPU years

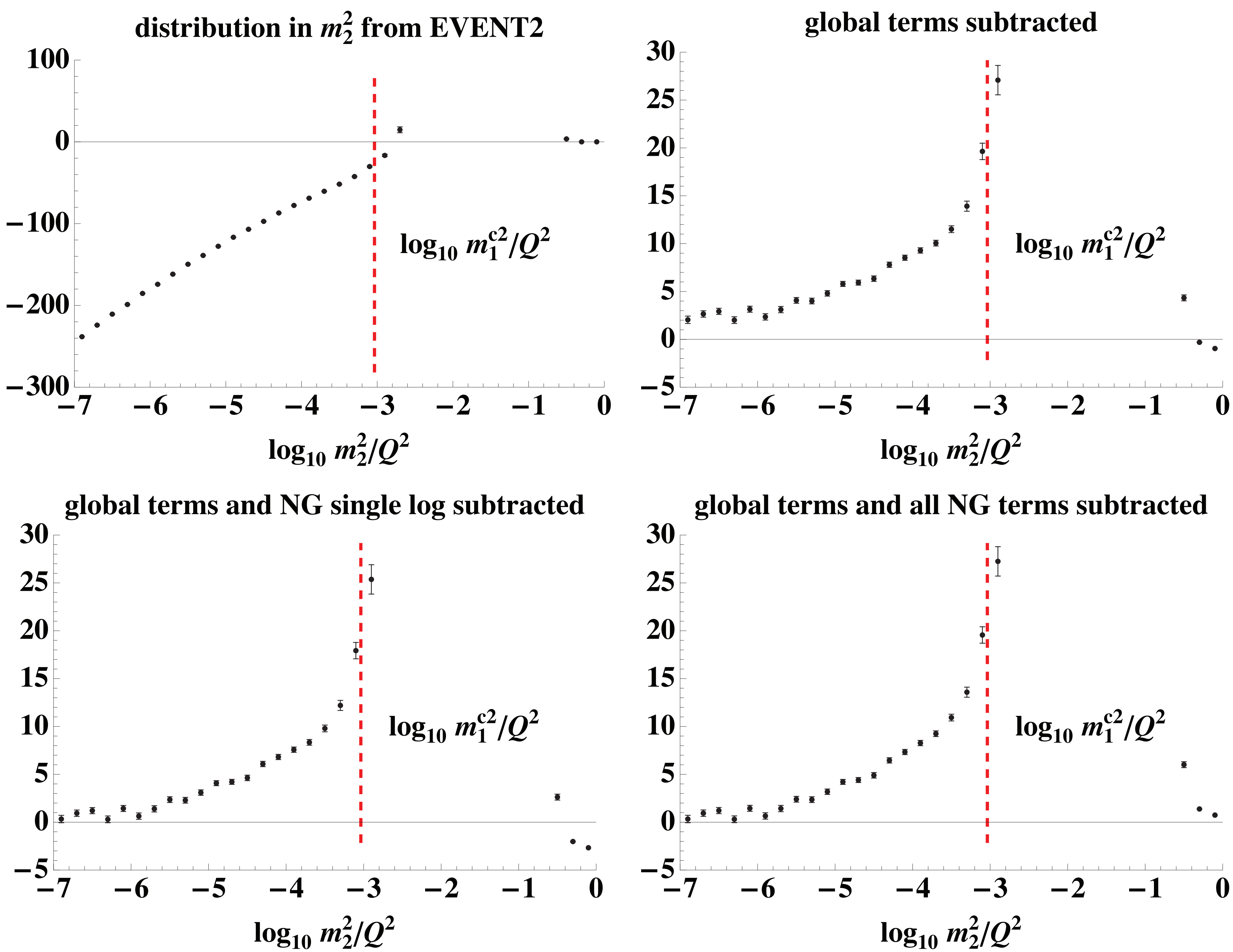
CUTOFF = 10^{-15}

NPOW1=NPOW2 = 3

Cross-Check with EVENT2

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

$$\frac{d\sigma}{dm_2^2}(m_1^{c2}) = \int_0^{m_1^{c2}} dm_1^2 \frac{d^2\sigma}{dm_1^2 dm_2^2}$$



C_F T_R n_f color structure

Numerical $\mathcal{O}(\alpha_s^2)$ prediction from EVENT2

Catani, Seymour

253 billion events
3 CPU months

CUTOFF = 10^{-15}

NPOW1=NPOW2 = 4

Cross-Check with Event Shapes

Hornig, CL, Stewart, Walsh, Zuberi
[1105.4628]

Heavy Jet Mass:

$$\Sigma_H^{NG}(\rho) = \frac{\alpha_s^2}{8\pi^2}\theta(\rho)\int_0^\pi \frac{d\theta}{\pi} t_2(e^{i\theta}) = \frac{\alpha_s^2}{8\pi^2}\theta(\rho)\left[C_F C_A s_{2\rho}^{[C_F C_A]} + C_F T_R n_f s_{2\rho}^{[n_f]}\right]$$

Thrust:

$$\Sigma_H^{\tau}(\tau) = \frac{\alpha_s^2}{8\pi^2}\theta(\tau) t_2(1) = \frac{\alpha_s^2}{8\pi^2}\theta(\tau)\left[C_F C_A s_2^{[C_F C_A]} + C_F T_R n_f s_2^{[n_f]}\right]$$

Our analytic results predict:

$$s_{2\rho}^{[n_f]} - s_2^{[n_f]} = 4\zeta_3 - \frac{4}{3} = 3.4749\,,$$

$$s_{2\rho}^{[C_F C_A]} - s_2^{[C_F C_A]} = \frac{2}{3} + \frac{19\pi^4}{45} + \frac{2\pi^2}{3}\ln^2 2 - \frac{2}{3}\ln^4 2 - 16\,\mathrm{Li}_4\left(\frac{1}{2}\right) - 11\zeta_3 - 14\zeta_3 \ln 2 = 11.6352$$

While EVENT2 predicts:

$$s_{2\rho}^{[n_f]} - s_2^{[n_f]} = 3.33 \pm 0.14\,,$$

$$s_{2\rho}^{[C_F C_A]} - s_2^{[C_F C_A]} = 11.54 \pm 0.47$$

Abbate, Hoang, Mateu, Schwartz, Stewart (2011)

Kelley, Schabinger, Schwartz, Zhu [1105.3676] and Monni, Gehrmann, and Luisoni [1105.4560] now calculate s_2 analytically.

EVENT2 $s_{2\rho}$ minus analytic s_2 predicts:

$$s_{2\rho}^{[n_f]} - s_2^{[n_f]} = 3.46^{+0.08}_{-0.05}\,,$$

$$s_{2\rho}^{[C_F C_A]} - s_2^{[C_F C_A]} = 11.55 \pm 0.46$$

New Opportunities

- Understanding origin of fixed order NGLs in effective field theory opens door to RGE-based method to resum them
    - cf. nonlinear evolution equation, solution currently only known numerically in large-N_C limit.
- When NGLs are not large, our new results allow analytic resummation of global logs in dijet observables to NNNLL accuracy.
- Dijet soft function directly applicable to beam thrust or 0-jettiness in hadron collisions
- NGLs will appear in multijet/subjet observables, jet cross sections with jet energy vetoes, etc.

cf. Banfi, Dasgupta, Khelifa-Kerfa, Marzani (2010)
Rubin (2010): NGLs in Filtered Jet Algorithms

- Calculation and resummation of global and non-global logs bring us into the realm of precision jet physics.