

# The substructure of jets

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- This talk is not a review of analysis methods that use jet substructure.
- Rather it is about the structure in nature that these methods try to take advantage of.
- I present mostly information that everybody knows.
- That seems appropriate for the start of the workshop.

# Signal and background

- We seek new physics signals in a standard model background.
- In Eugene OR, there are many background creatures.



- There are signal creatures that look a somewhat like background creatures, but there are very few of them.

- A signal creature.

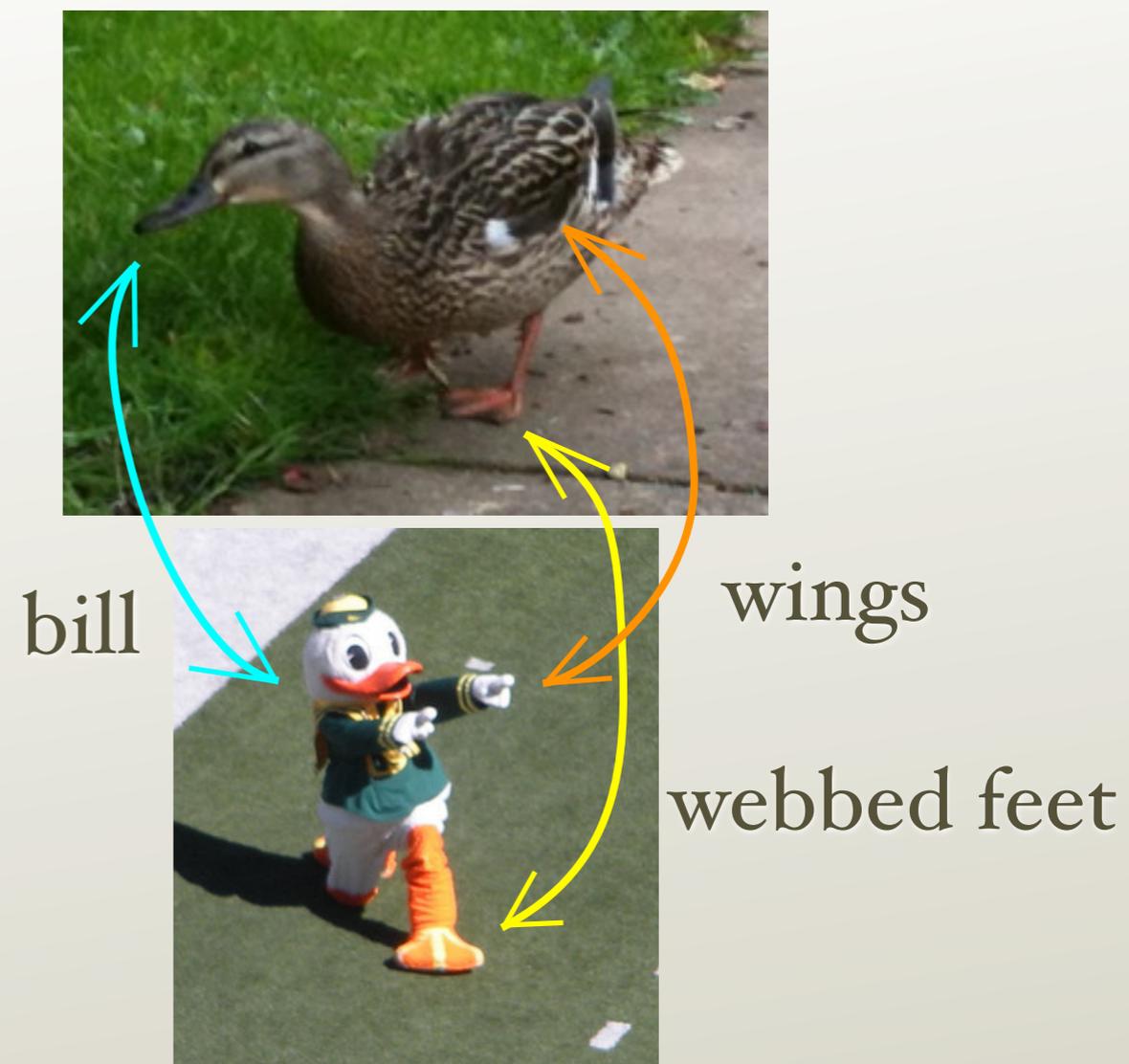


- There are many features in common between signal and background creatures.

- We need to find the differences. (Eg. one of these can't swim.)

- Unfortunately, background creatures come with a range of mutations that make them sometimes look like a signal creatures.

- Thus we look need a statistical method to tell if there are signal creatures.

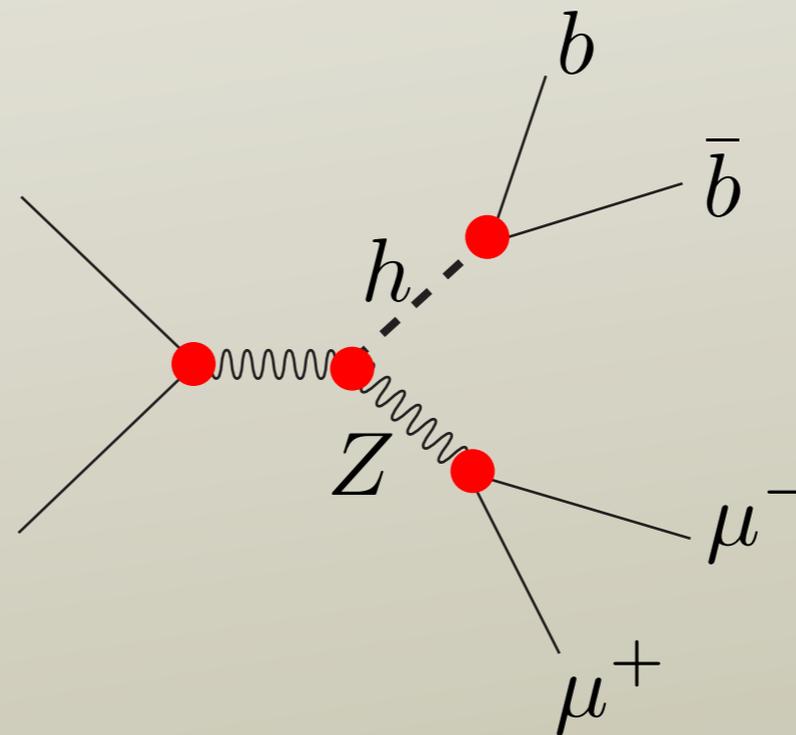


# Signals and backgrounds at the LHC

# Signal events

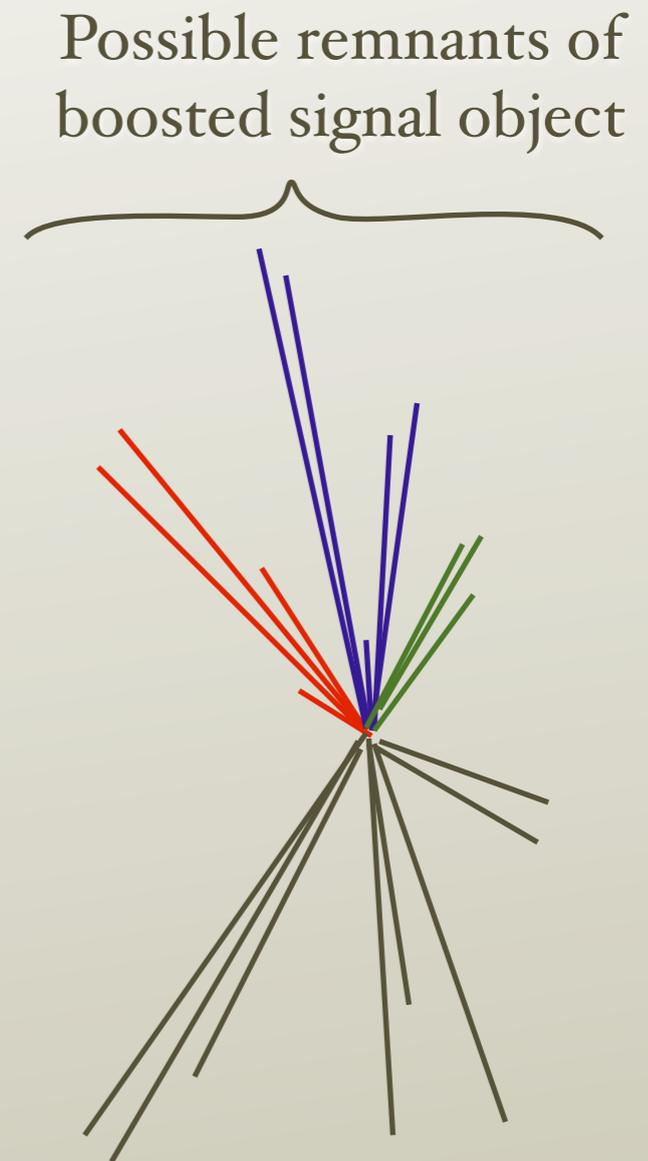
- I take a signal event to be one in which one or more new heavy particles is created and decays.
- Sometimes there is a chain of decays.
- Also, top quarks could be part of both signal and background events for some signals.

A Higgs boson  
signal event



# Signal and background with jets

- Both signal and background events have jets.
- There are several techniques for separating signal from background using the structure of the jets.
- This is especially effective for highly boosted heavy objects.

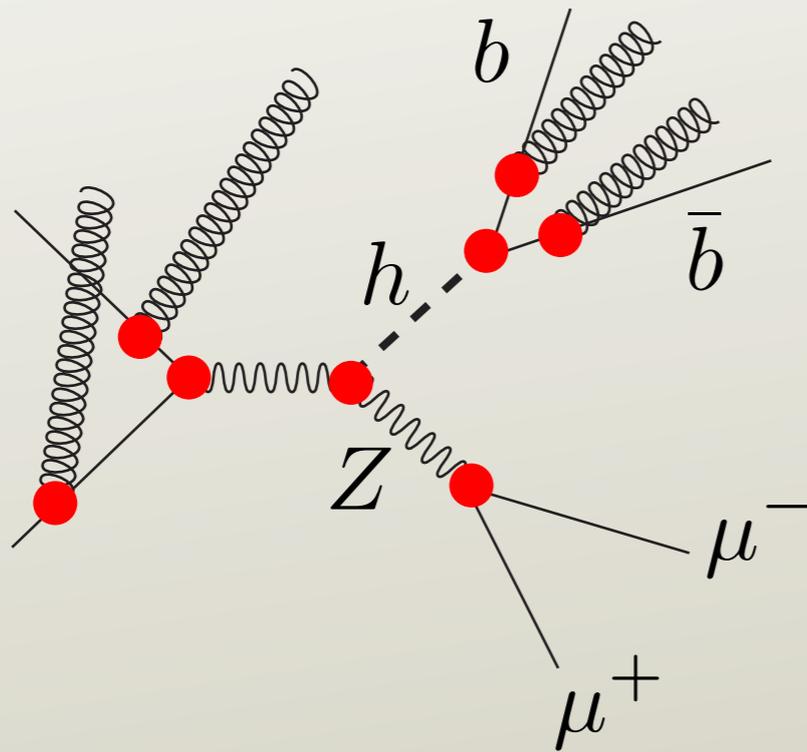


# What gauge field theory has to say

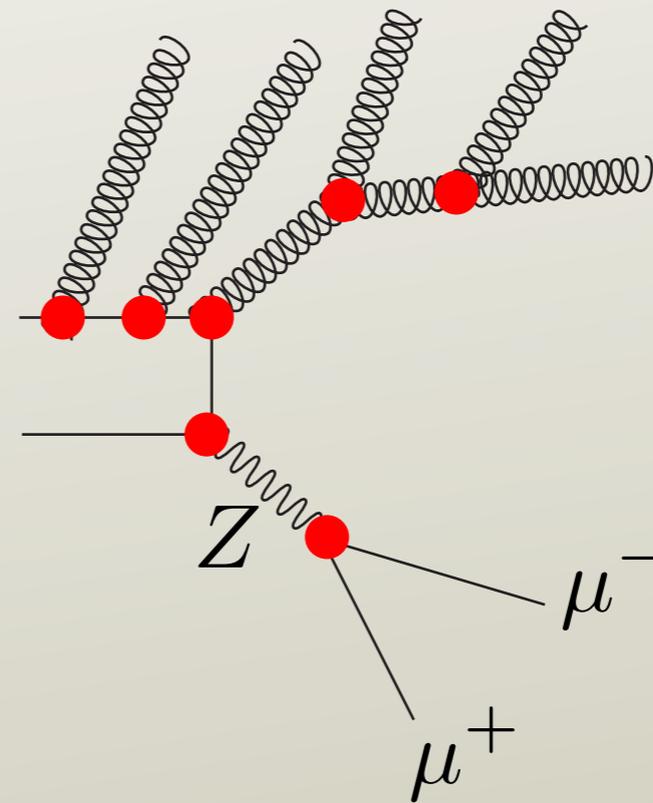
- One can use field theory to understand the characteristics of background events and the characteristics of signal events (with heavy particle decays).
- Schemes for selecting signal events depend on using the similarities and differences in signal and background events.
- I will try to outline the characteristics that can be used.

# Initial state radiation

- One immediate similarity is that both signal and background events have initial state radiation.



A Higgs signal event

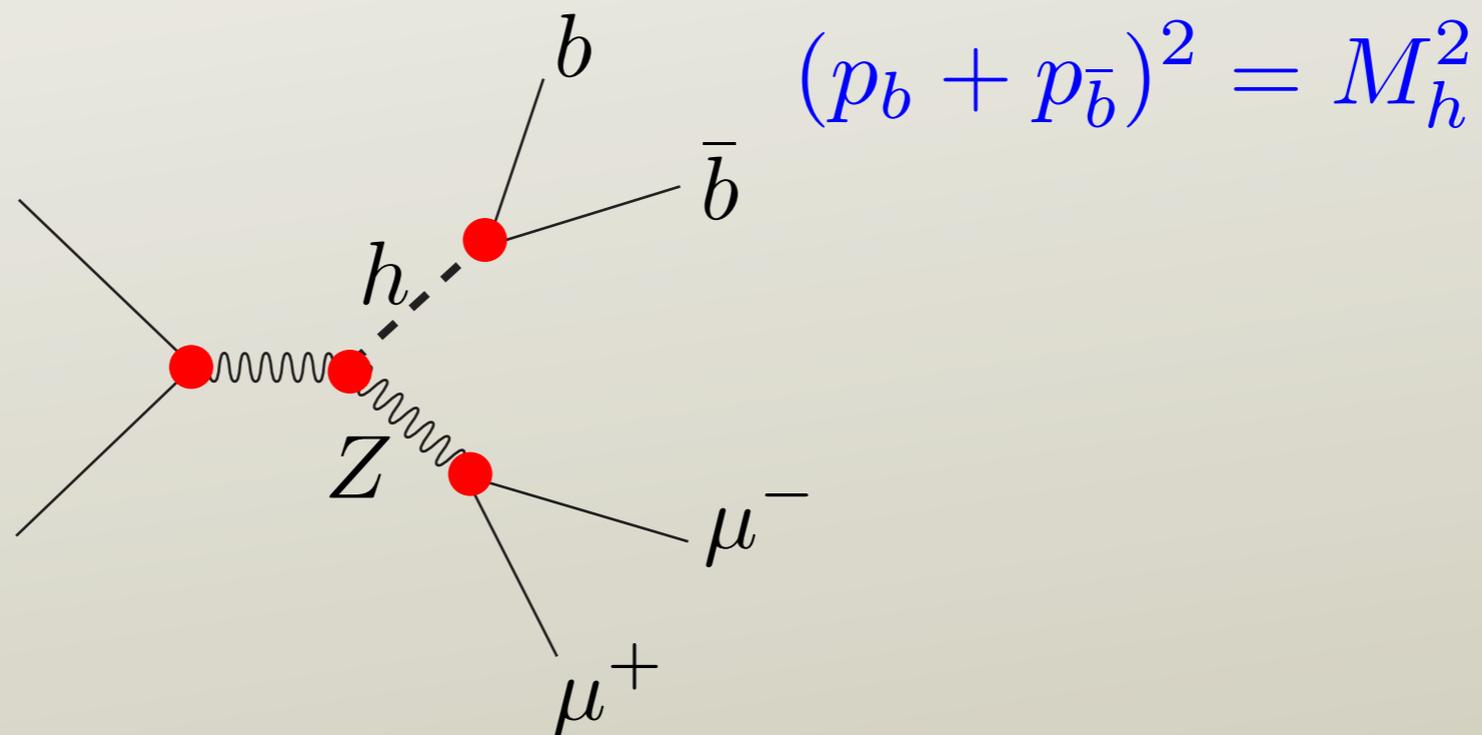


A similar background event

- This makes the signal/background separation difficult.
- More on this later.

# Mass bumps

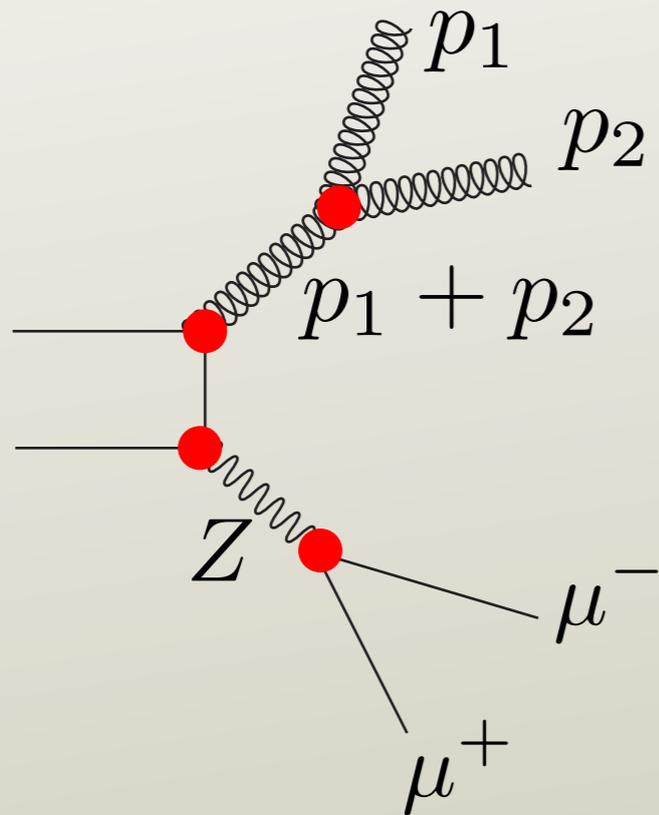
- One key feature of signal events is that the decay products of a heavy particle have a fixed mass.



- The theoretical uncertainty of this is  $M_h \Gamma_h$ .
- The practical uncertainty is much larger.

# Mass in QCD splittings

- QCD splittings have a very different distribution as a function mass.



- I will use a scaled virtuality as a “hardness variable”:

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

- $Q_0$  is the momentum of the outgoing partons in the hard process.

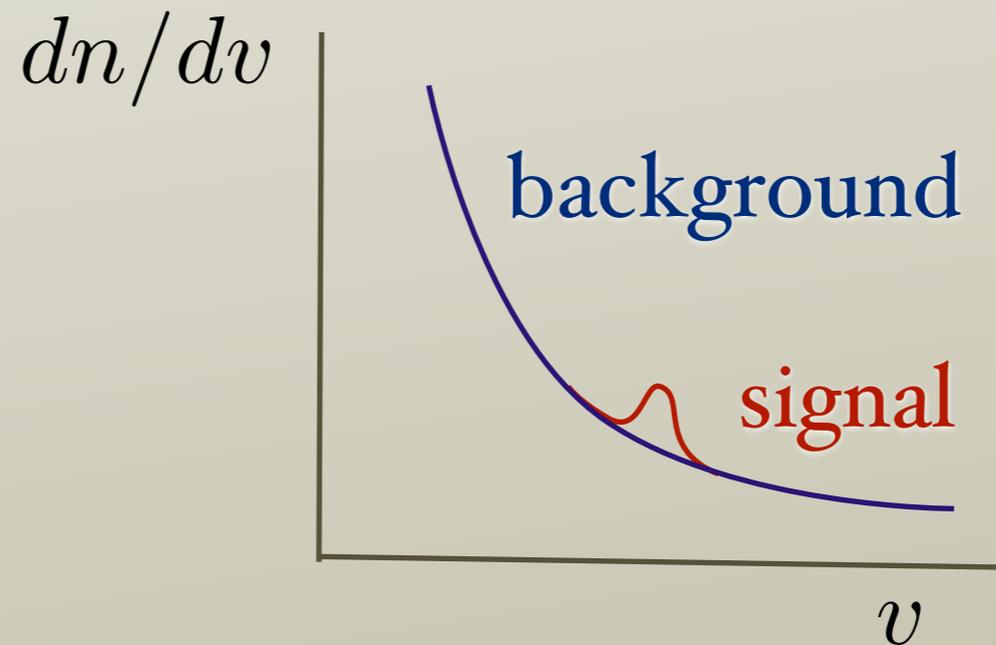
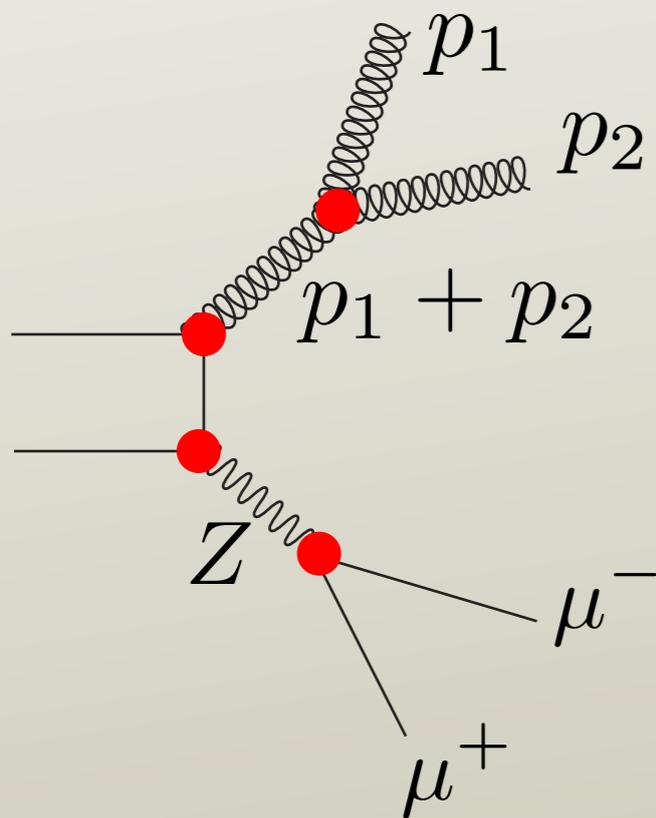
# QCD tree graph results

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

- Differential probability is

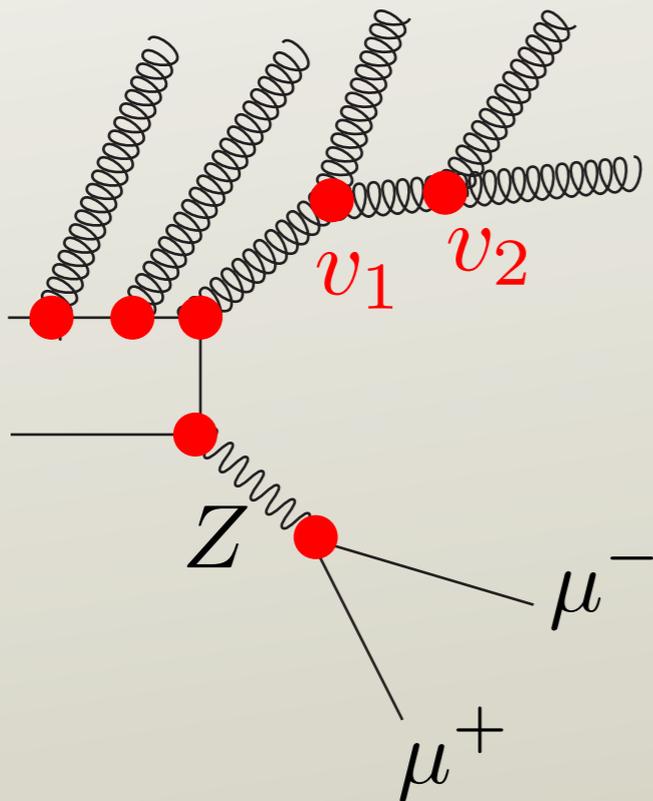
$$dn \propto \frac{dv}{v} \alpha_s \log(v/\tilde{v})$$

- The log (and  $\tilde{v}$ ) comes from  $\int dz$ .



# Nested splittings

- The same structure can be (approximately) iterated.

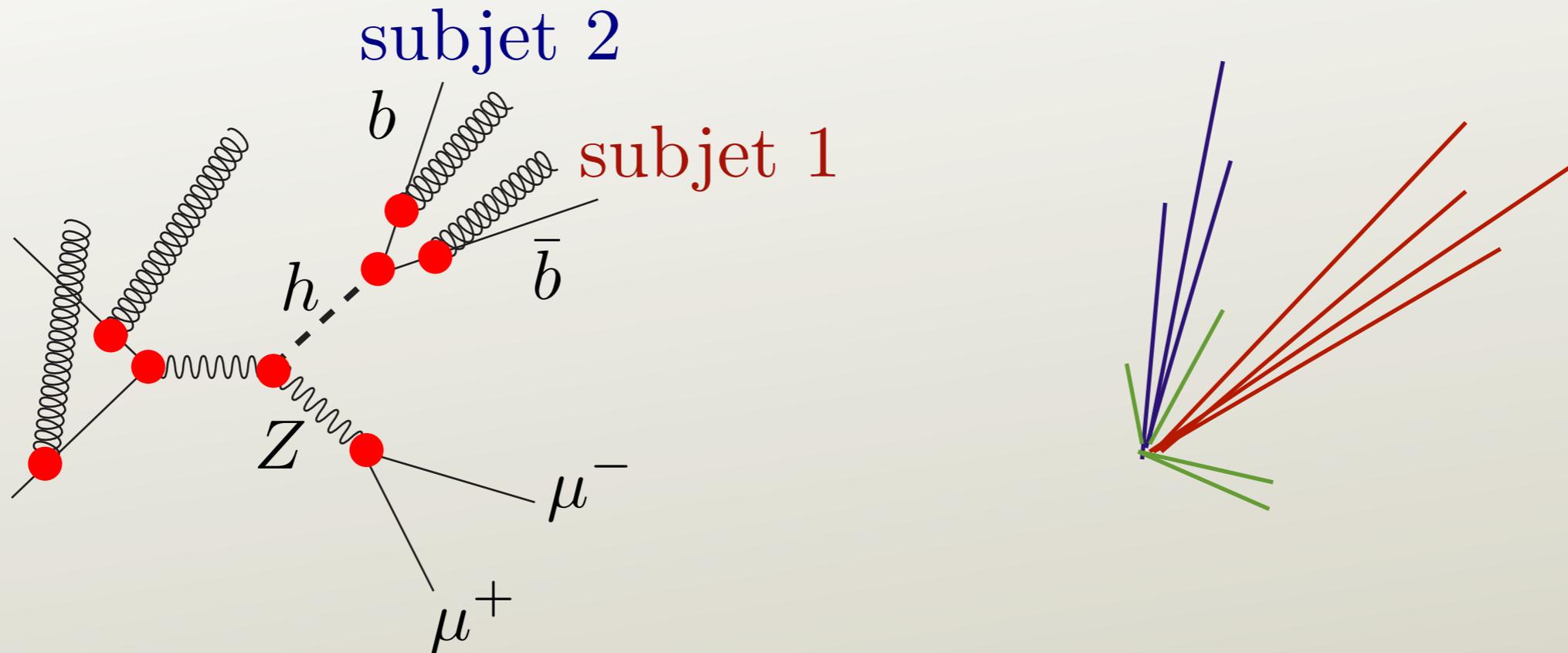


- $v_2 < v_1$ .

- Set  $v_2 = 0$  in splitting 1.

$$dn = \frac{dv_1}{v_1} \alpha_s \log(v_1/\tilde{v}_1) \times \frac{dv_2}{v_2} \alpha_s \log(v_1/\tilde{v}_2) \theta(v_2 < v_1)$$

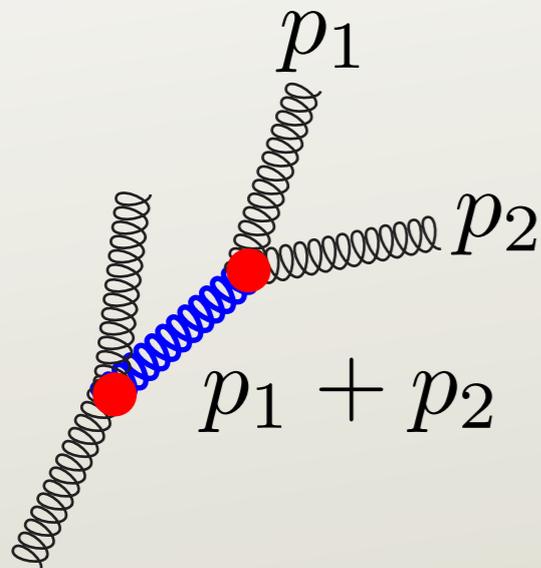
# An application of this



- A Higgs boson will likely decay to two low mass subjets, plus soft gluons.
- Try to find the low mass subjets.
- A “mass drop” condition can be part of this.  
(Butterworth, Davison, Rubin & Salam.)
- Measure the mass of subjet pair.

# Sudakov factor

- A better approximation for  $dn$  includes a Sudakov factor.



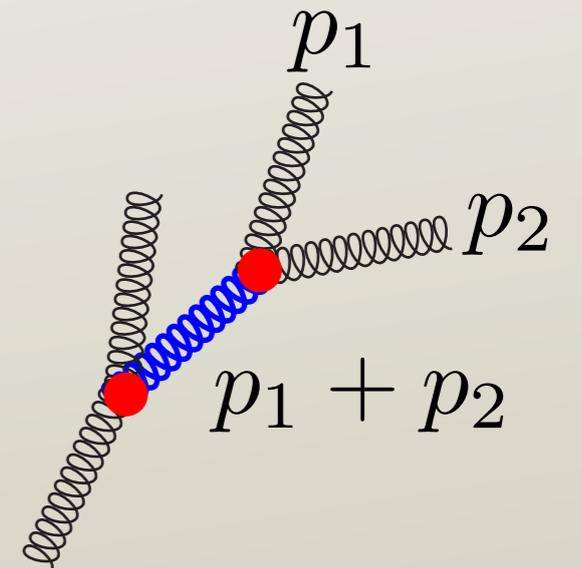
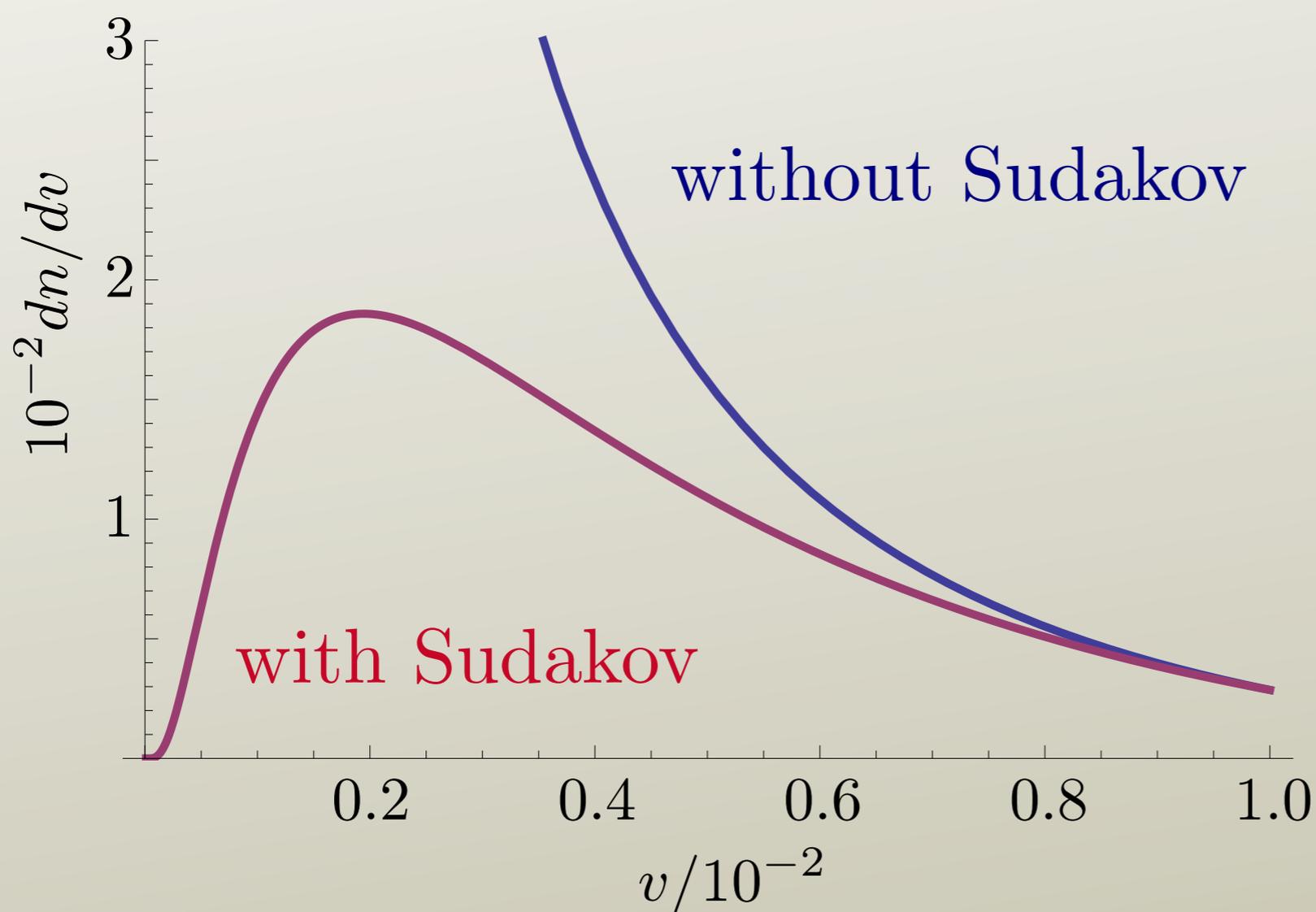
$$dn \propto \frac{dv}{v} \alpha_s \log(v/\tilde{v})$$

$$\exp \left( - \int_v^{v_1} \frac{d\bar{v}}{\bar{v}} \alpha_s \log(\bar{v}/\tilde{v}) \right)$$

- We think of a renormalization group approach with a running resolution scale.
- The Sudakov factor accounts for virtual graphs and unresolved real parton emissions.

# Example of effect of Sudakov

$$dn \propto \frac{dv}{v} \alpha_s \log(v/\tilde{v}) \exp\left(-\int_v^{v_1} \frac{d\bar{v}}{\bar{v}} \alpha_s \log(\bar{v}/\tilde{v})\right)$$



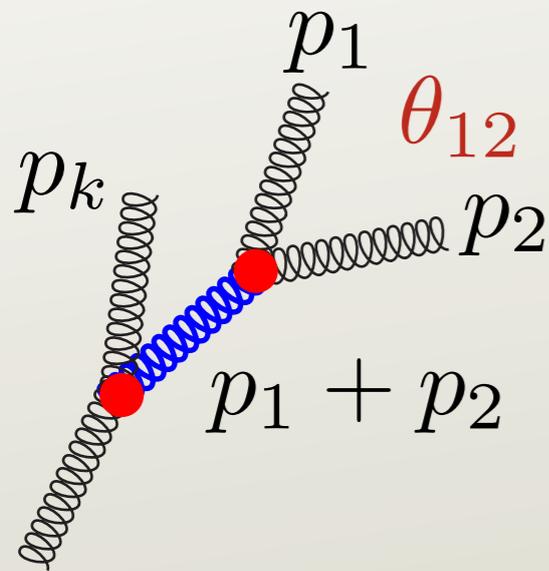
$$Q_0^0 = 1000 \text{ GeV}$$

$$|\vec{p}_1 + \vec{p}_2| = 100 \text{ GeV}$$

$$(\vec{p}_1 + \vec{p}_2)_{\text{max}}^2 = 31.6 \text{ GeV}$$

$$\theta_{\text{max}} = 0.3$$

# Distribution in angles



- For a given  $v$ , what is the distribution in the angle  $\theta_{12}$  between  $\vec{p}_1$  and  $\vec{p}_2$ ?

- Kinematically,

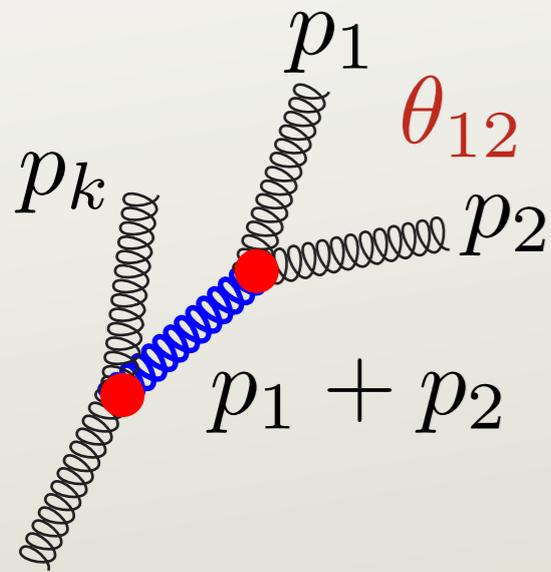
$$\theta_{12}^2 > v \frac{4Q_0^2}{(p_1 + p_2) \cdot Q_0}$$

- For larger, but not too large,  $\theta_{12}$ ,

$$v = \frac{(p_1 + p_2)^2}{(p_1 + p_2) \cdot Q_0}$$

$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2}$$

# Large angles



- Large  $\theta_{12}$  corresponds to  $|\vec{p}_1| \ll |\vec{p}_2|$ .
- (Or gluon 2 could be the soft one.)
- For emission of soft gluon 1, interference with emission from parton  $k$  is important.
- Parton  $k$  is the color connected partner.

- With interference,

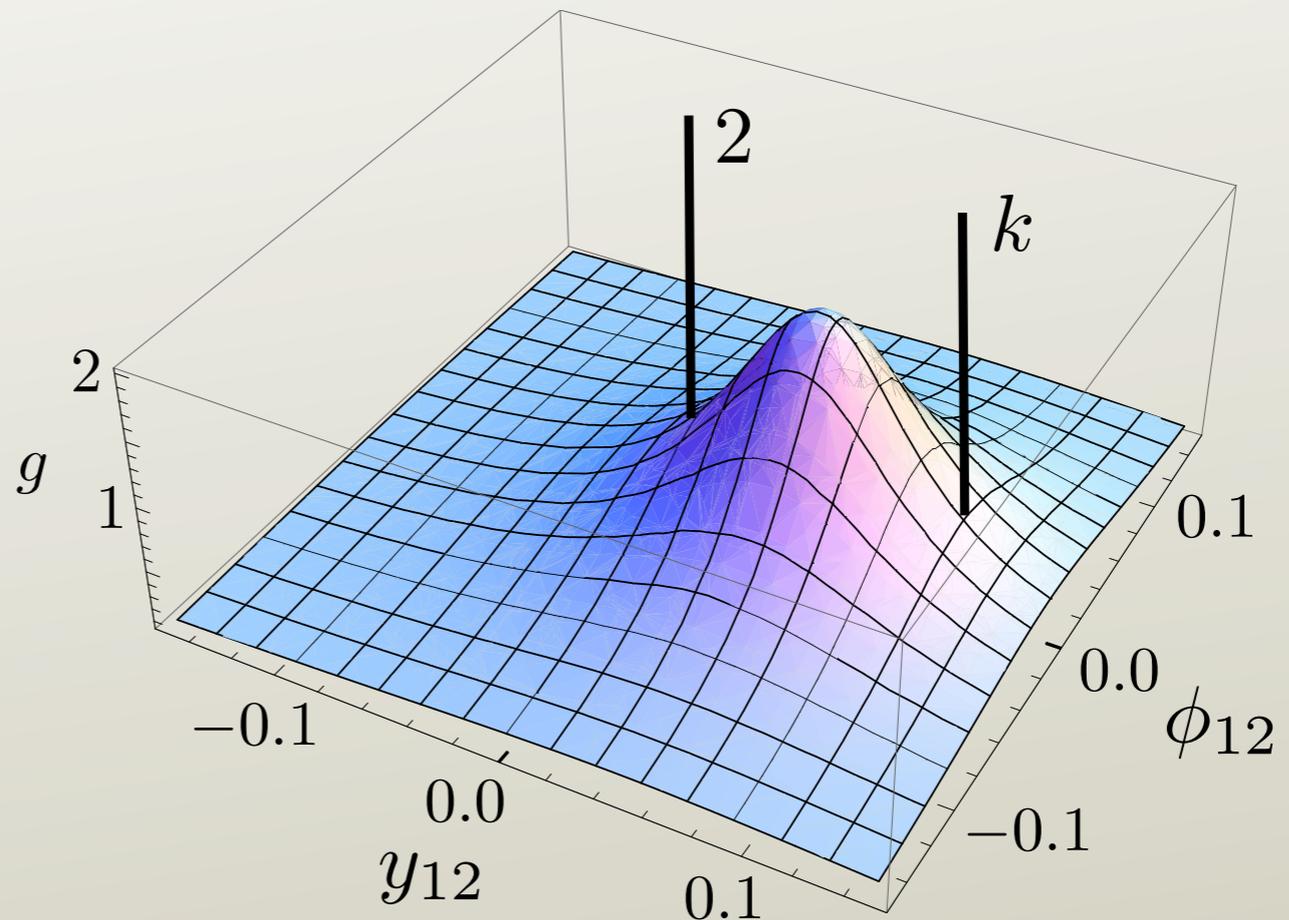
$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2} g(\theta_{12}^2)$$

$$g(\theta_{12}^2) = \frac{\theta_{2k}^2}{\theta_{12}^2 + \theta_{1k}^2}$$

# Effect of the angle factor

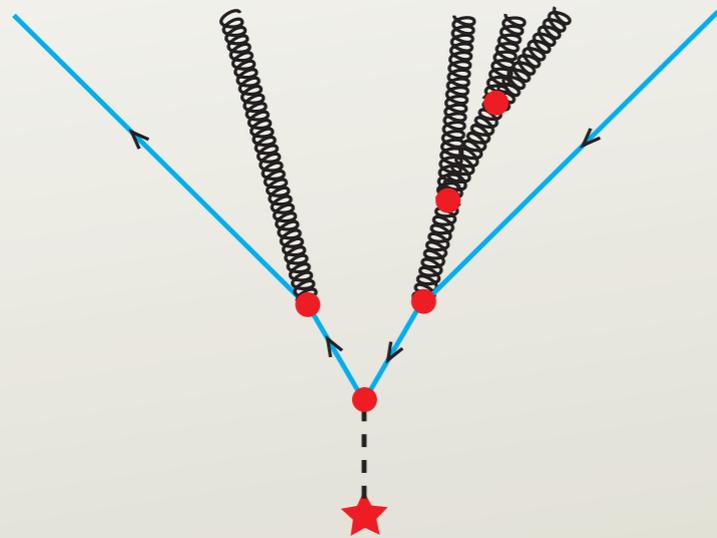
$$dn \propto \frac{d\theta_{12}^2}{\theta_{12}^2} g(\theta_{12}^2)$$

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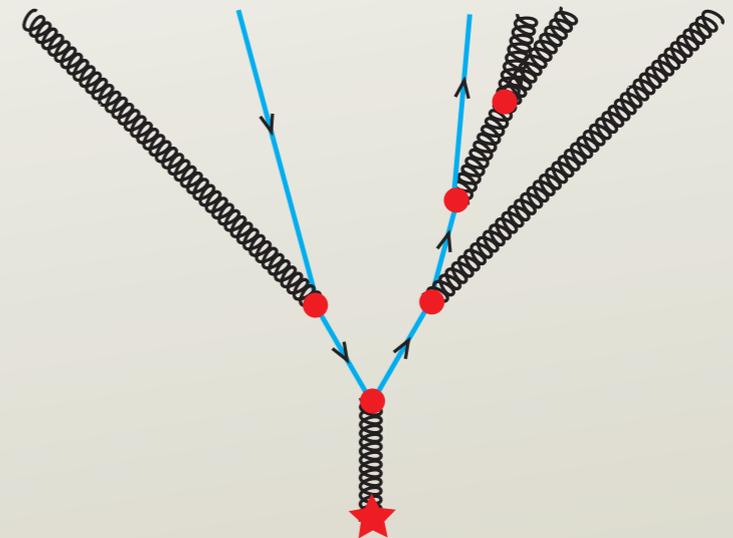
- Emission with  $\theta_{12} \gg \theta_{2k}$  is suppressed.
- Emission between partons 2 and  $k$  is enhanced.

# An application of this



After  $H \rightarrow b + \bar{b}$

Soft gluons between  $b$  and  $\bar{b}$



After  $g \rightarrow b + \bar{b}$

Soft gluons away from  $b$  and  $\bar{b}$

- Cf. “pull” (Gallicchio & Schwartz).

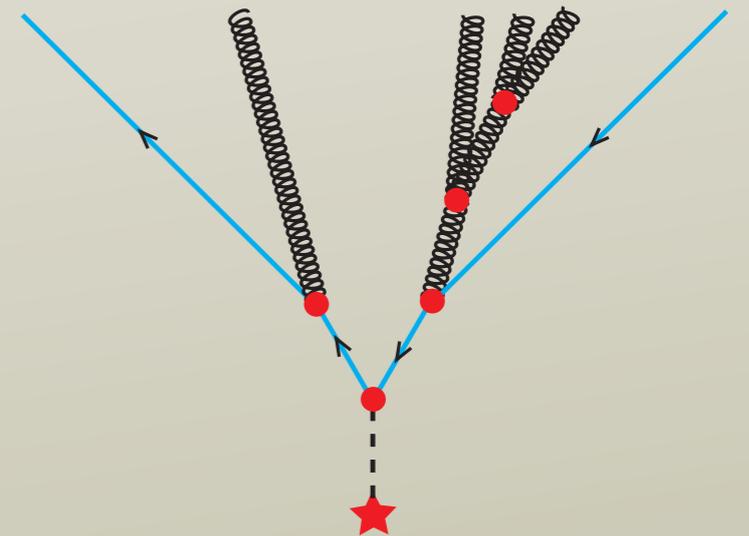
# Gluons want to be soft

- Large angles  $\implies$  small  $z$ :

$$z(1-z) = \frac{v}{\theta^2} \frac{Q_0^2}{(p_1 + p_2) \cdot Q_0}$$

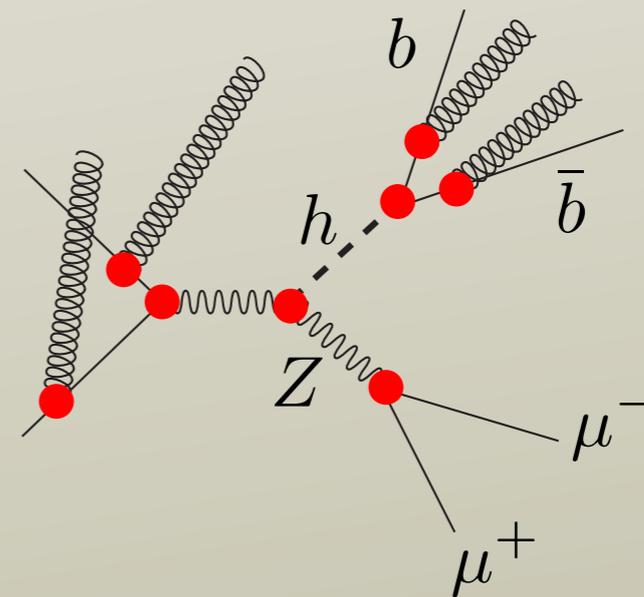
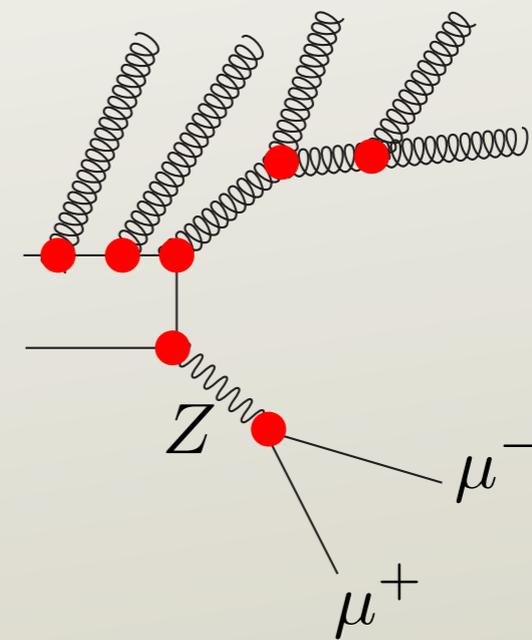
- So we expect lots of soft gluon radiation.

- However, soft gluon radiation is limited by “angular ordering” from  $g(\theta)$ .



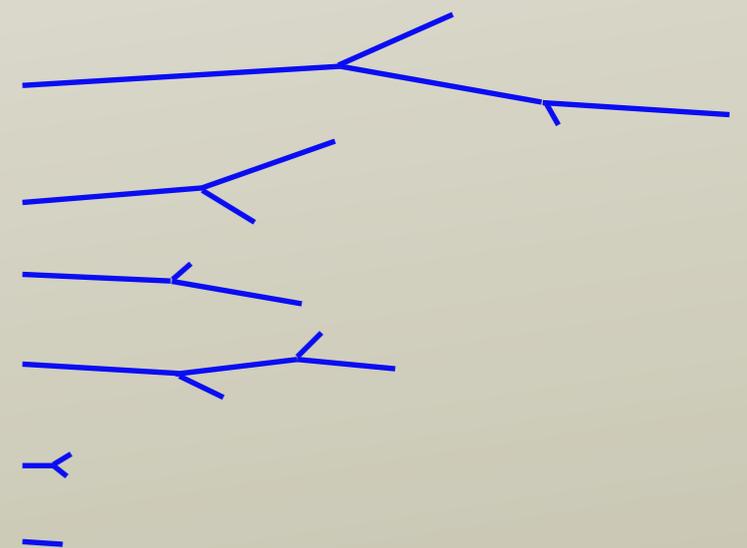
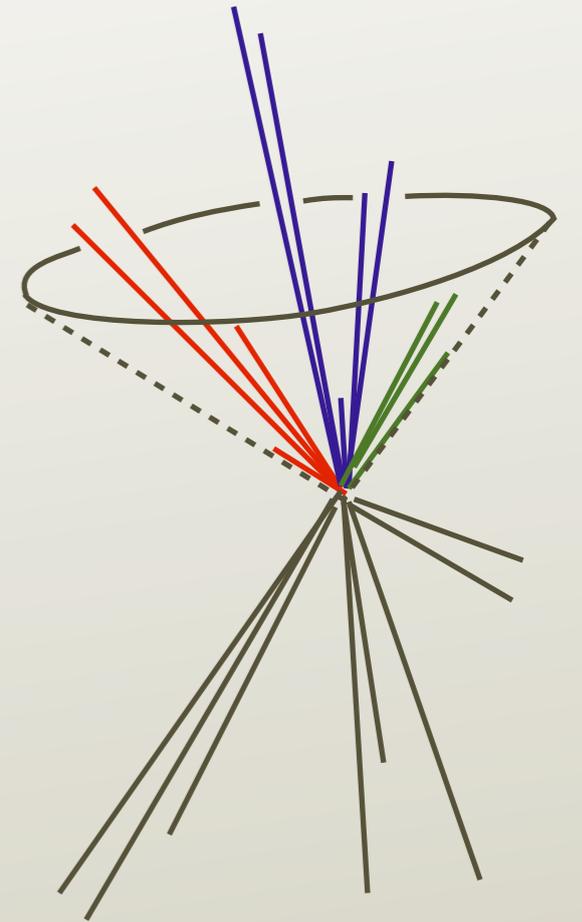
# More soft gluons come from initial state radiation

- There is initial state radiation in both signal and background events.
- It can come at central rapidities.
- It is largely rather low transverse momentum.



# An application

- Look at all of the hadrons or calorimeter clusters in an angular region that might contain a boosted heavy particle.
- The  $k_T$  jet algorithm can put the starting clusters together into jets in something like a shower history.
- Use a fairly small angular size parameter.
- The smallest  $P_T$  jets are likely from initial state radiation.
- Throw them out. (Krohn, Thaler & Wang).



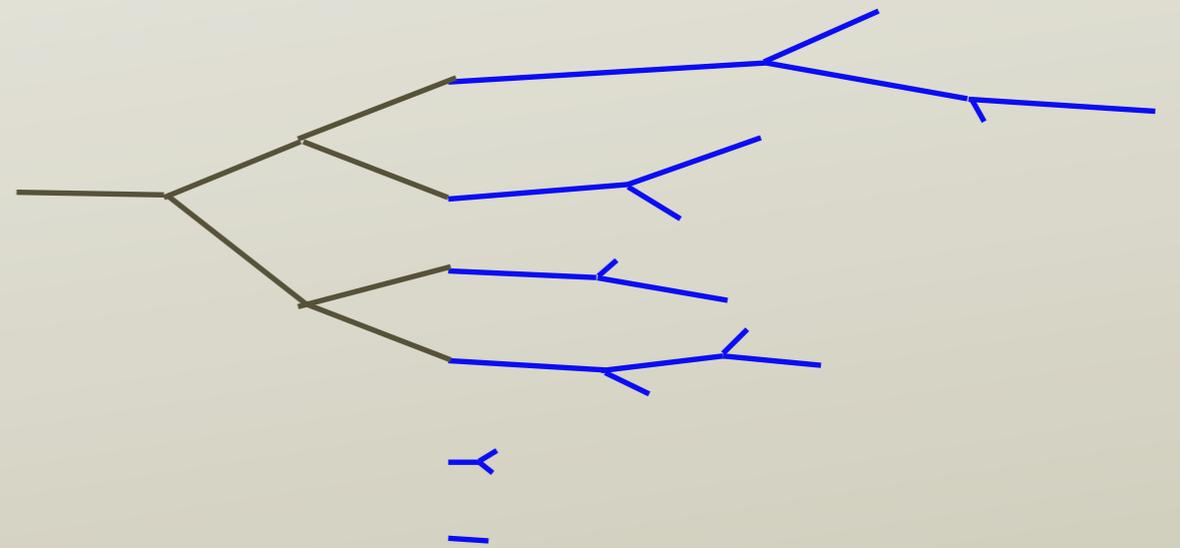
# Another application

- It's hard to tell where soft gluons belong, so try to get rid of them.
- For instance, “pruning” joins protojets starting at small angular separations. But if

$$z = \frac{\min(p_{T,i}, p_{T,j})}{|\vec{p}_{T,i} + \vec{p}_{T,j}|}$$

is too small, one throws the softer protojet away.

(Ellis, Vermilion & Walsh)



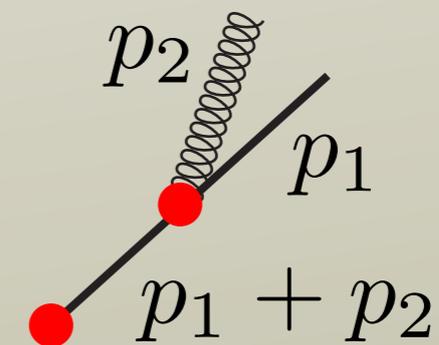
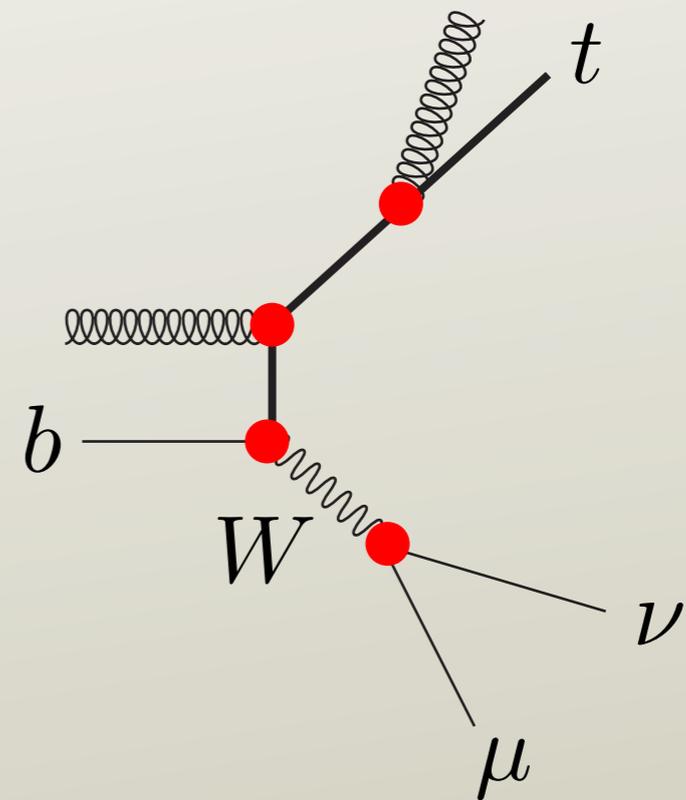
# Heavy particles with color

- A heavy particle with color will radiate gluons.
- There is singularity in  $|\mathcal{M}|^2$ .

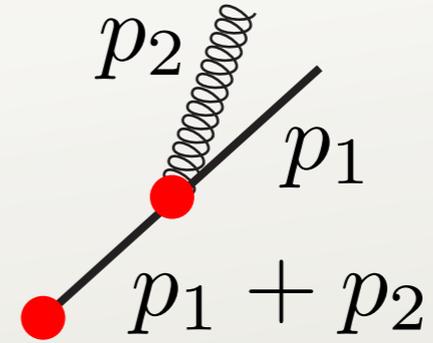
$$dn \propto \frac{dv}{v} \alpha_s \log(\theta_{\min}/\theta_{\max})$$

where the hardness  $v$  is now

$$v = \frac{(p_1 + p_2)^2 - M^2}{(p_1 + p_2) \cdot Q_0}$$



- The collinear singularity is cut off:

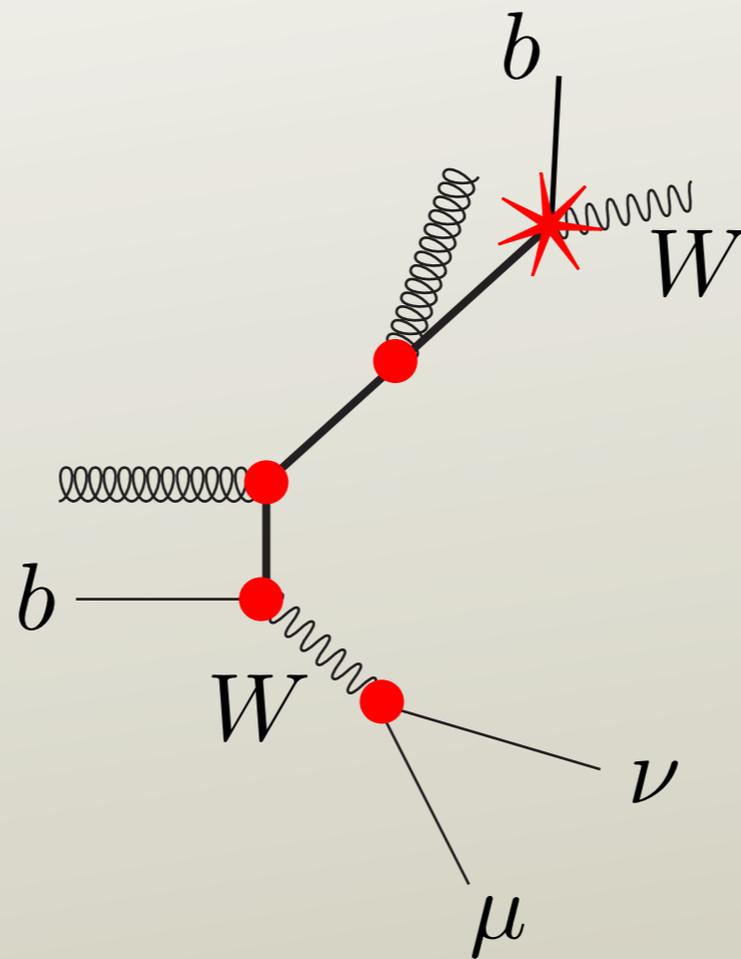


$$\theta_{\min}^2 \sim \min \left( \frac{M^2 Q_0^2}{((p_1 + p_2) \cdot Q_0)^2}, v \frac{4Q_0^2}{(p_1 + p_2) \cdot Q_0} \right)$$

- There are no emissions that take longer than the particle lifetime:

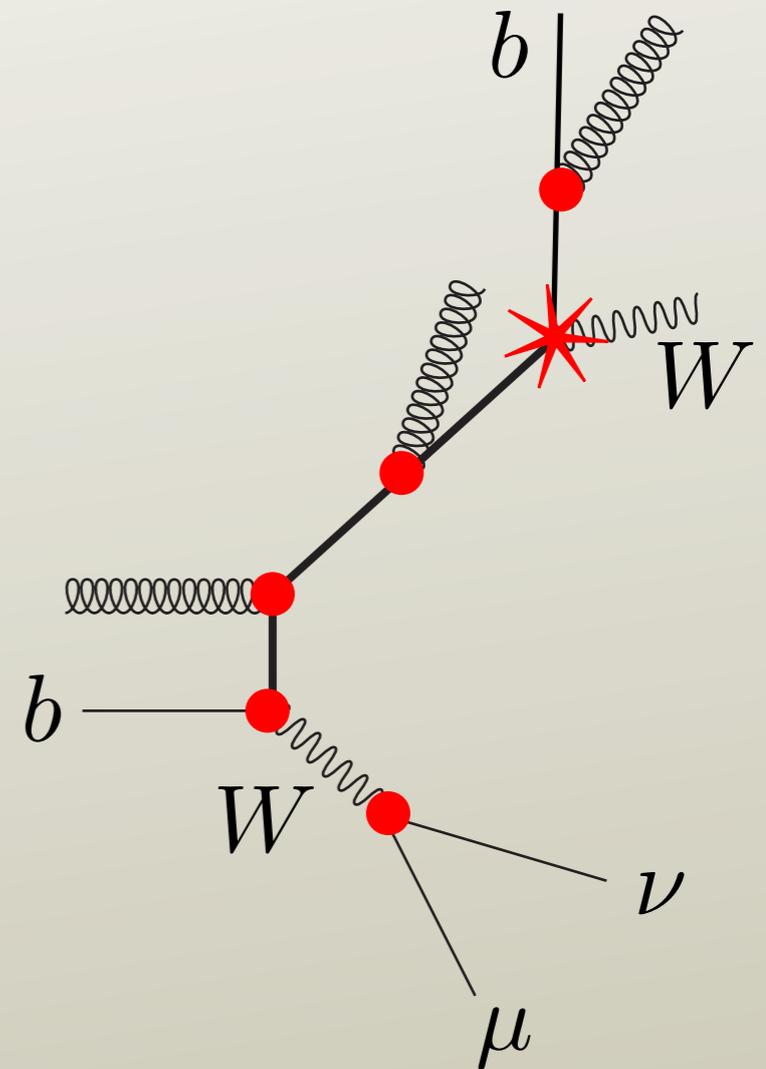
$$v > \frac{M\Gamma}{(p_1 + p_2) \cdot Q_0}$$

# Heavy particles decay



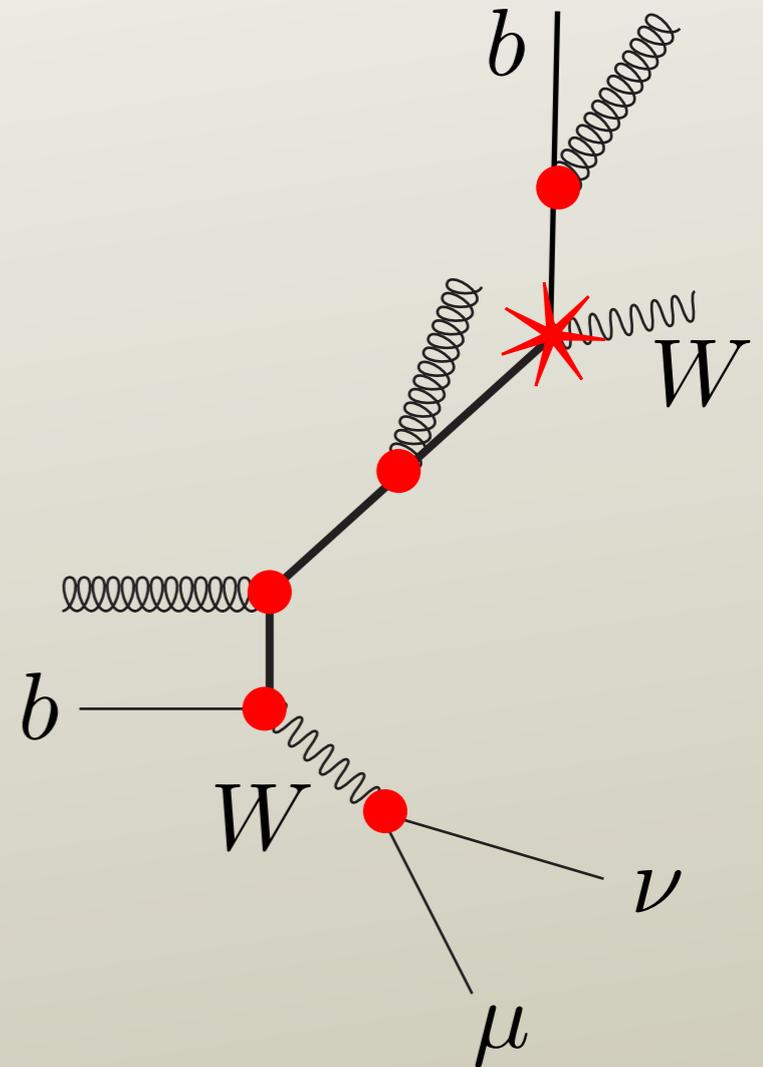
# Daughters radiate

- In  $t \rightarrow b + W$ , a lot of energy is released.
- Color charge is accelerated.
- Gluons are radiated.
- The first radiation is from a color dipole of
  - The final state  $b$ -quark;
  - The “initial state”  $t$ -quark;



# Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state  $b$ -quark;
  - The “initial state”  $t$ -quark;
- Direction of  $g$  is likely collinear with the  $b$ .
- Direction of  $g$  is likely within angle  $M/|\vec{P}_t|$  of the top.
- If the top is highly boosted, this is a narrow cone.





# Daughters radiate

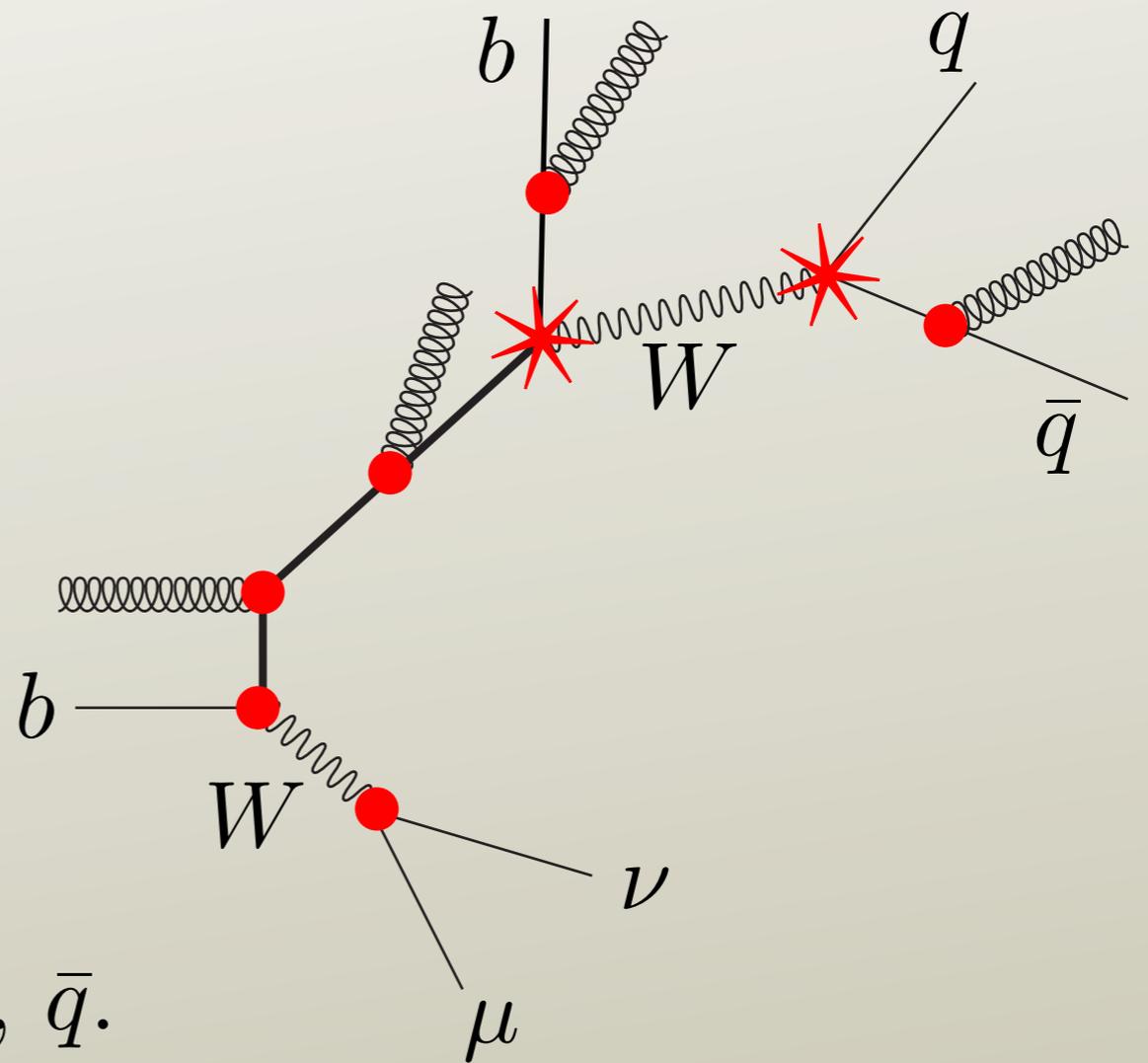
- In  $W \rightarrow q + \bar{q}$ , a lot of energy is released.

- Color charge is accelerated.

- Gluons are radiated.

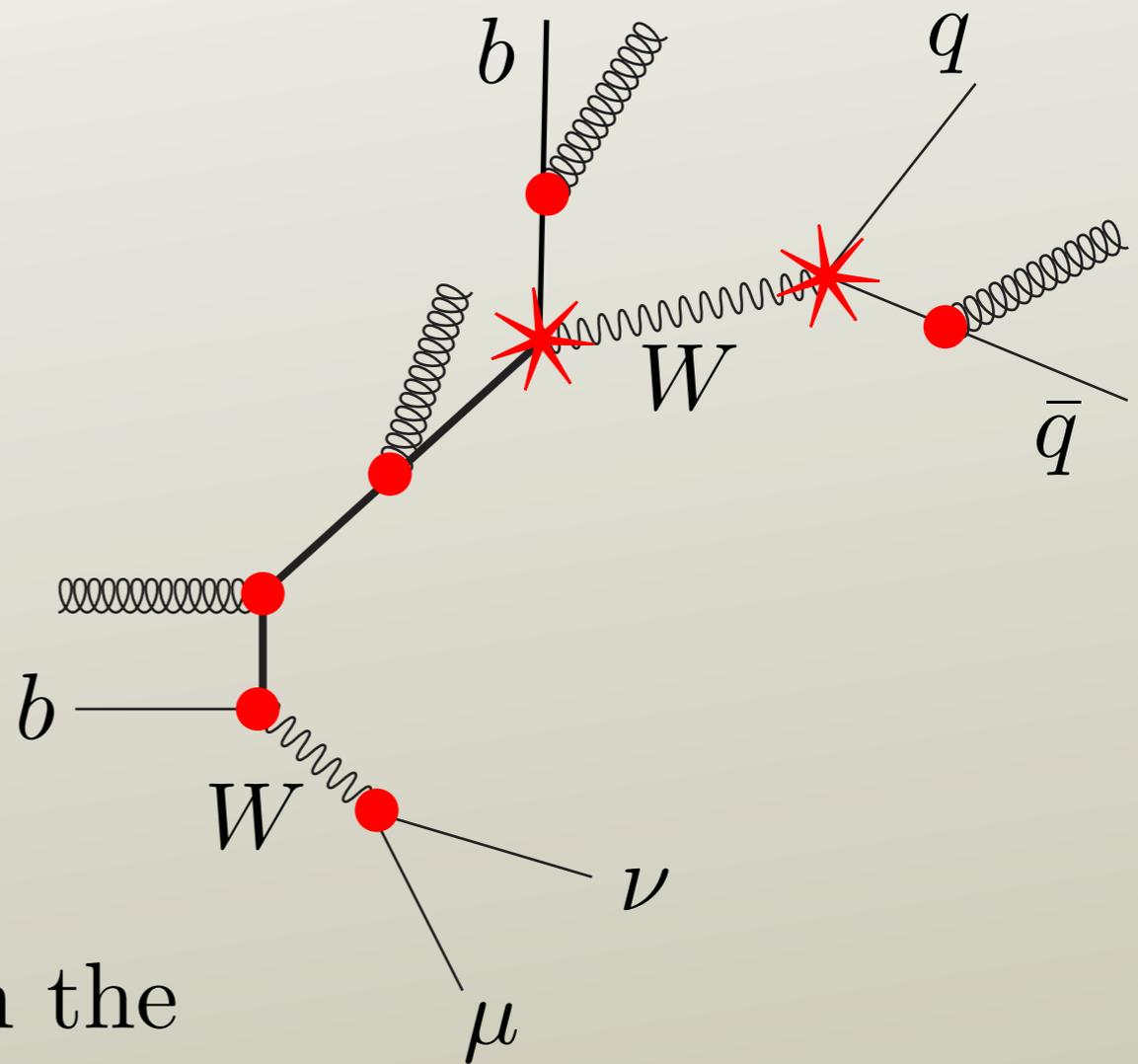
- The first radiation is from a color dipole of

- The final state quark,  $q$ ;
- The final state anti-quark,  $\bar{q}$ .



# Where does the radiation go?

- The first radiation is from a color dipole of
  - The final state quark,  $q$ ;
  - The final state anti-quark,  $\bar{q}$ .
- Direction of  $g$  is likely collinear with the  $q$  or  $\bar{q}$ .
- In any case, it is likely between the  $q$  and  $\bar{q}$ .

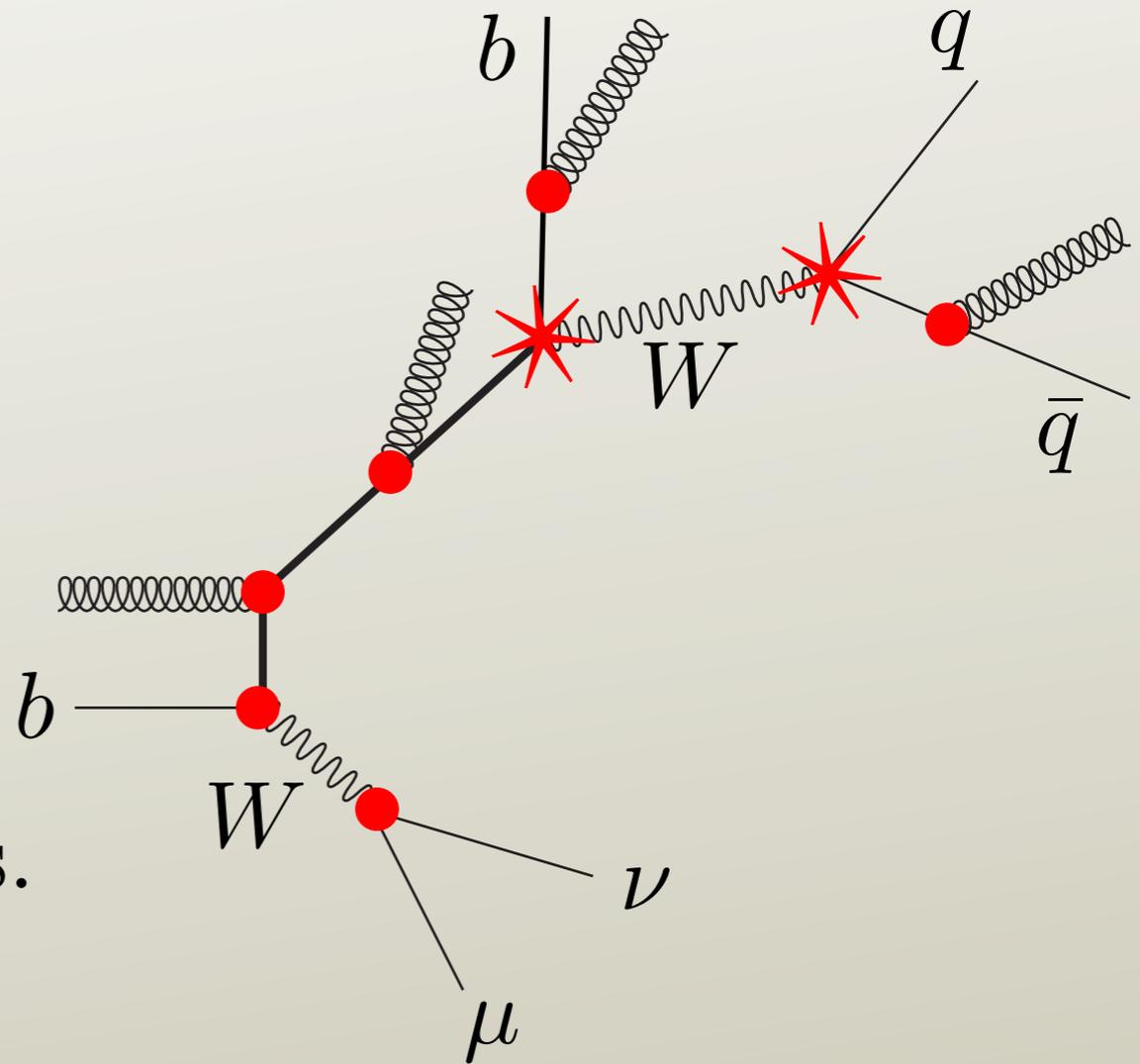


# Very soft radiation

- For very soft emissions,

$$v < \frac{M_t \Gamma_t}{p_t \cdot Q_0} \sim \frac{M_W \Gamma_W}{p_W \cdot Q_0}$$

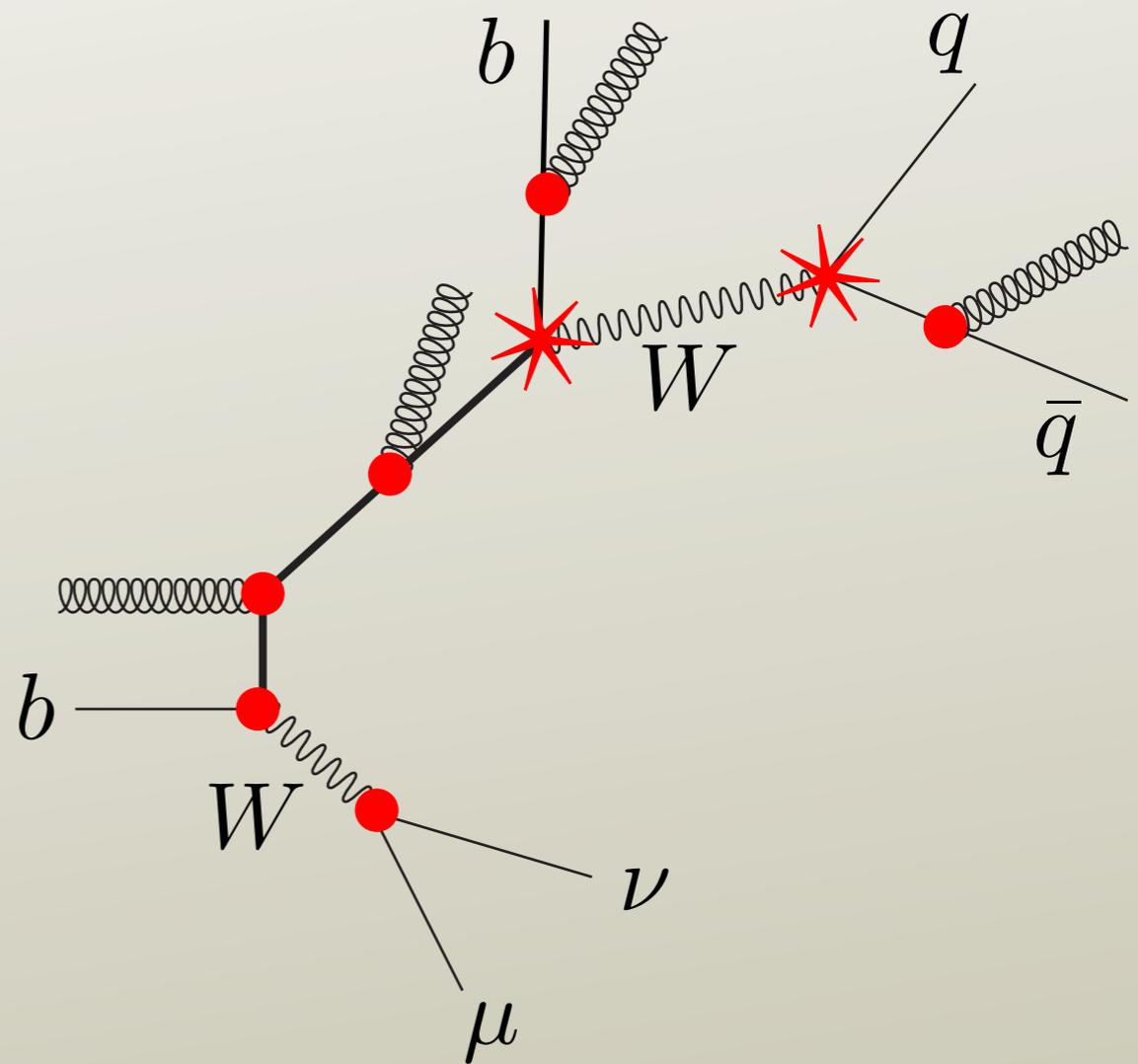
there is quantum interference between emissions from any two (color connected) particles.



- Probably the structure of these emissions is too complicated to be useful for separating signal from background.

# Comments

- Since we know something about where the collinear or soft radiation goes in a heavy particle decay, sequential heavy particle decays should have good signatures.
- We do have the problem of contamination from initial state radiation.
- Having highly boosted heavy objects helps with this.



# Summary

- With a model for sequential heavy particle decays and knowledge of the structure of radiation in gauge theories, one can build filters for separating background events from signal events.

