RF mushroom cavity design for High Temperature Superconductor (HTS) material test

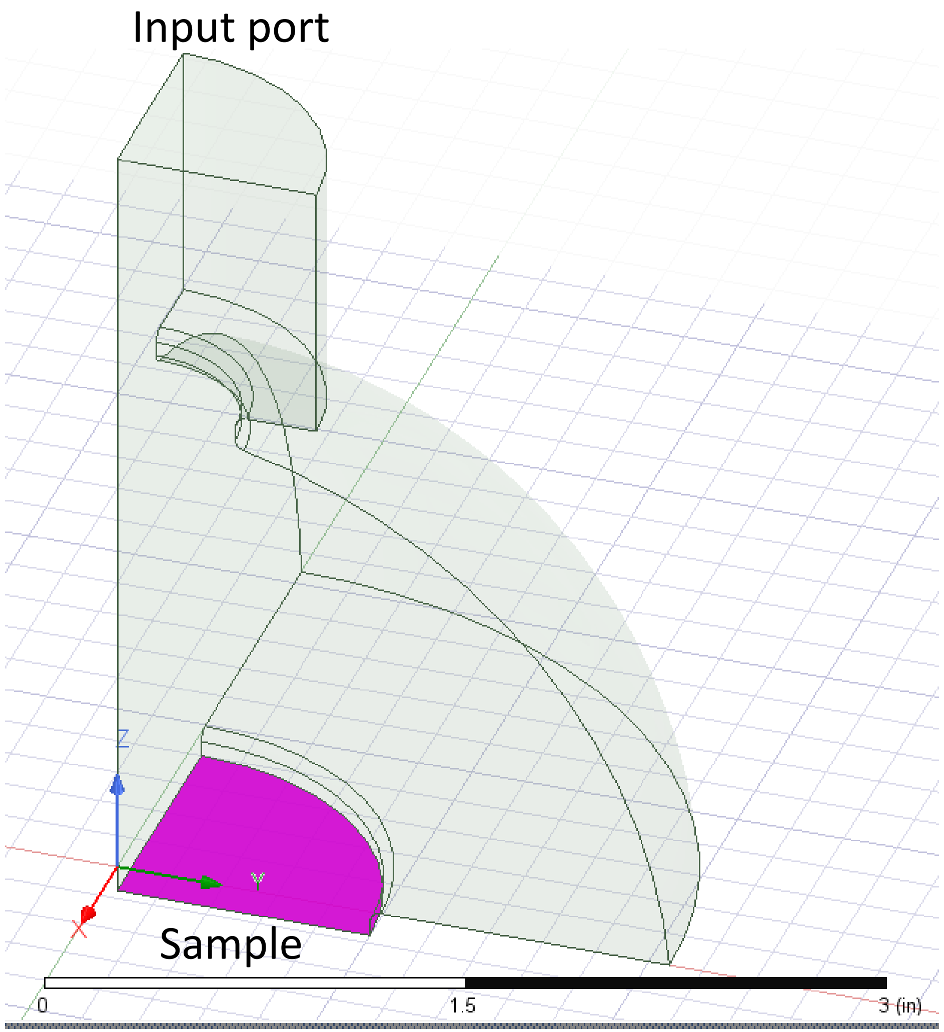
P. Martinez-Reviriego[1], J. Golm[3], W. Wuensch[3], D. Esperante[1,2], B. Gimeno[1], C. Blanch[1], M. Boronat[1], N. Fuster-Martínez[1], D. Gonzalez-Iglesias[1], P. Martín-Luna[1], E. Martinez[1], L. Pedraza[1], J. Reina[1], A. Menendez[1] and J. Fuster[1]

1 Instituto de Física Corpuscular (IFIC), CSIC-University of Valencia, Parque Científico, C/ Catedrático José Beltrán, 2 46980 Paterna (Valencia)

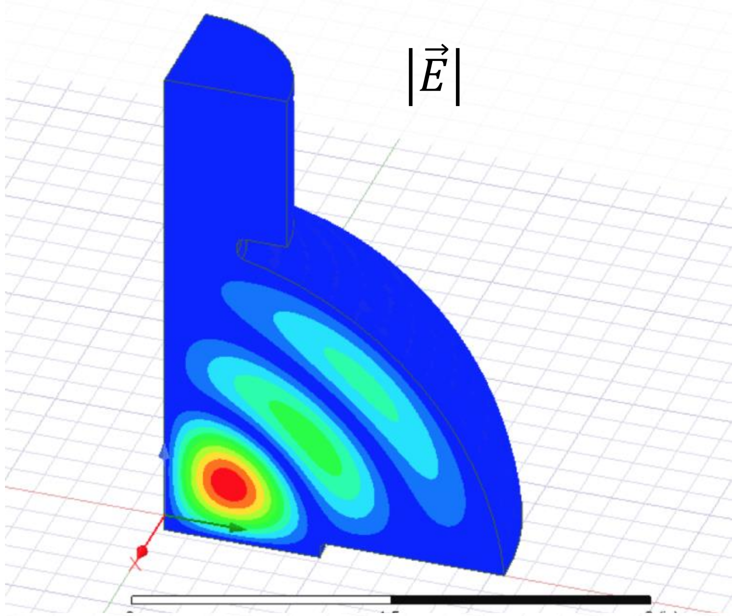
2 Electronics Engineering Department, University of València, 46100 Burjassot, Spain

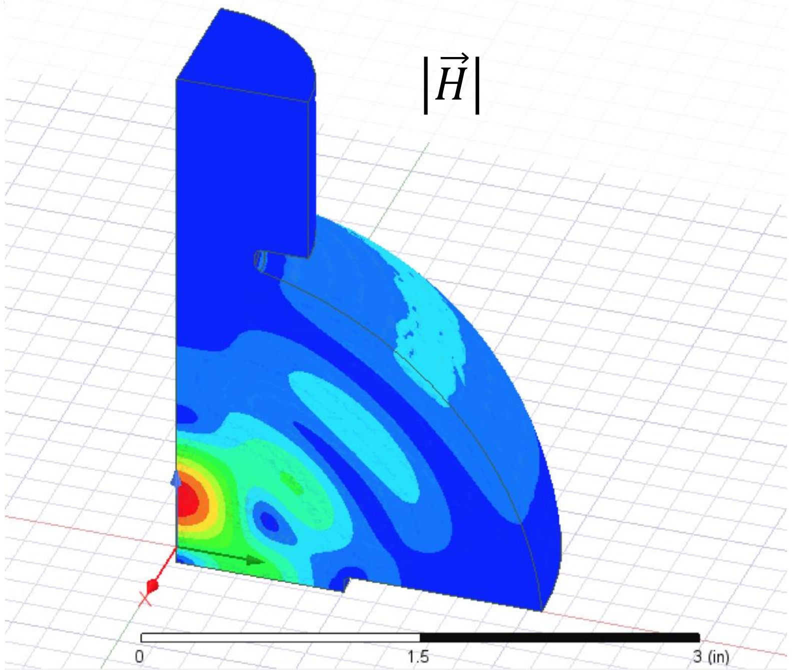
3 CERN, 01631 Meyrin, Switzerland

SLAC cavity



$f = 11.3995$ GHz





TE_{320}-like mode in a "semispherical" cavity

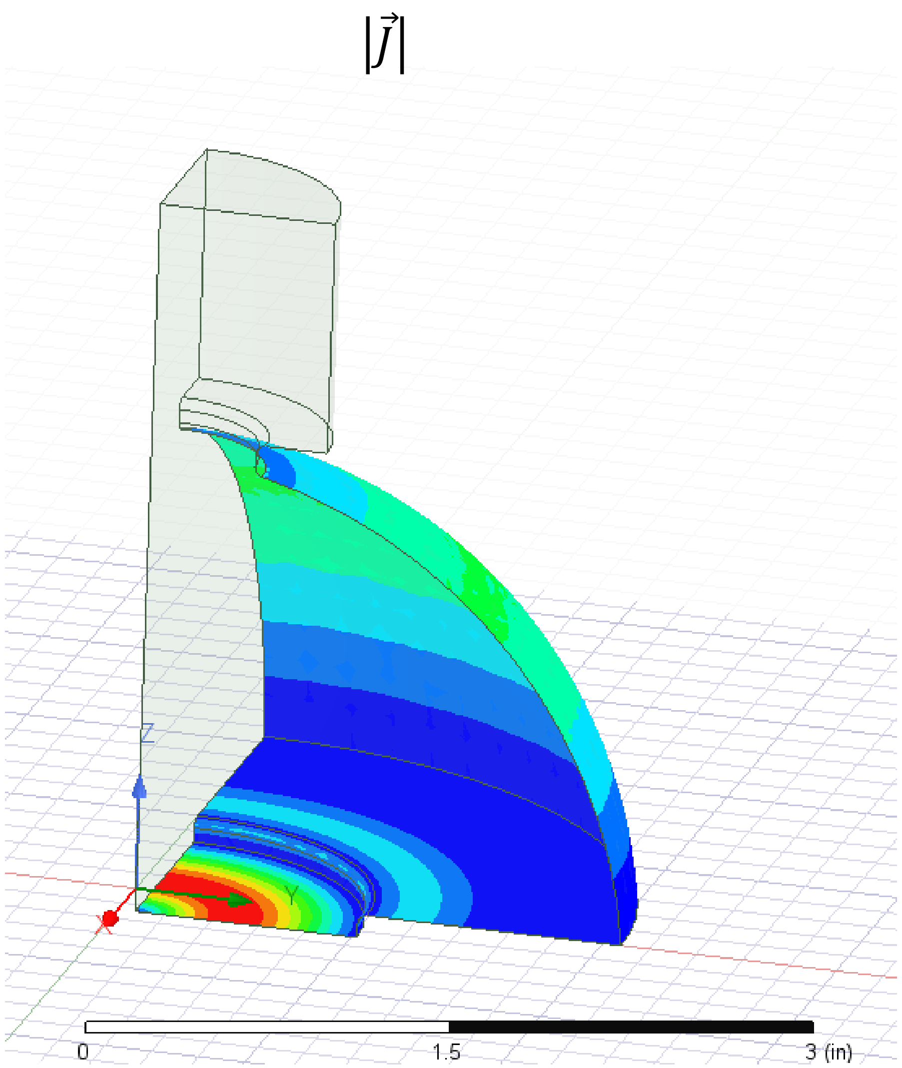
Test samples under high magnetic fields:

- High-Gradient material
- High temperature superconductor

Test properties:

- No Electric field
- High Magnetic field
- No electric current on the edge

SLAC cavity



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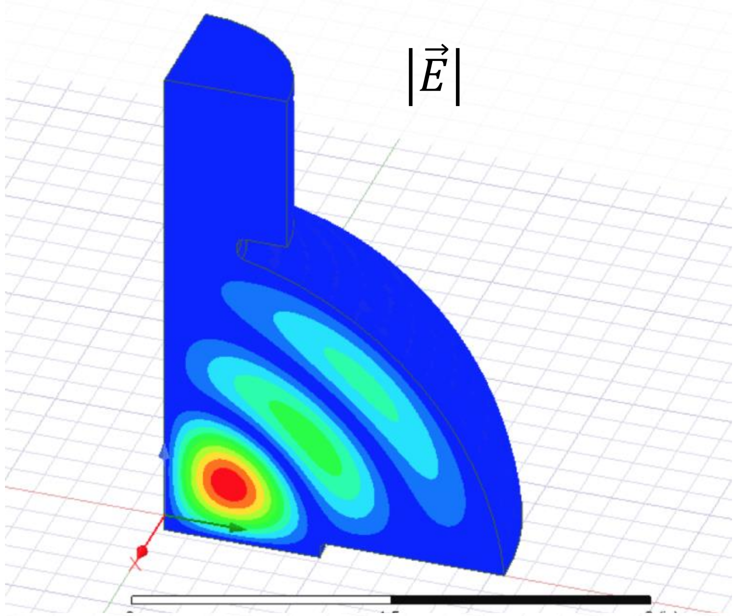

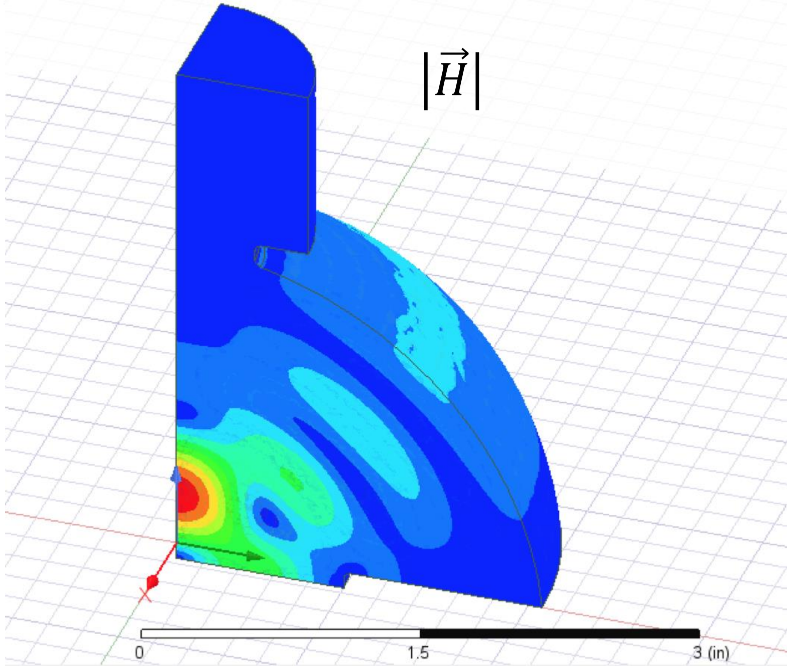

TE_{320}-like mode in a “semispherical” cavity

Test samples under high magnetic fields:

- High-Gradient material
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Test properties:

- No Electric field
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- No electric current on the edge

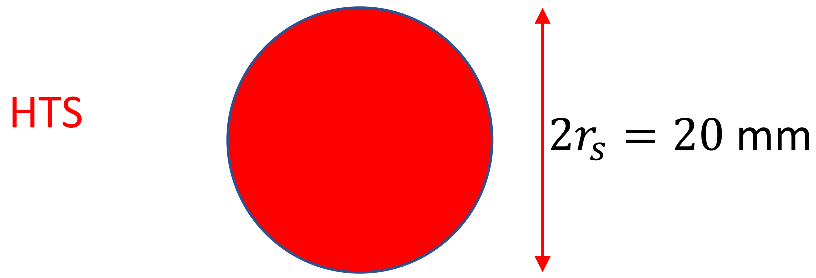
Solution approaches

Challenge:

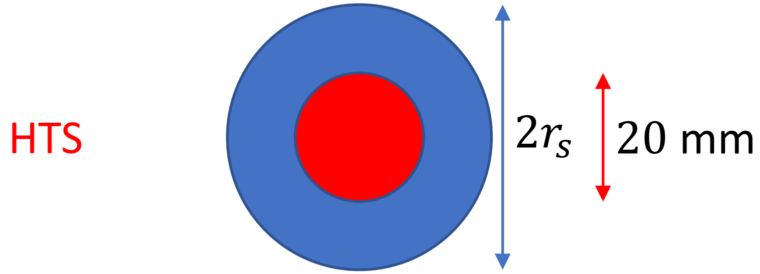
- Radius $= 0.95$ in ~ 24 mm
  HTS sample radius $= 10$ mm
- Same set up -> Same RF frequency

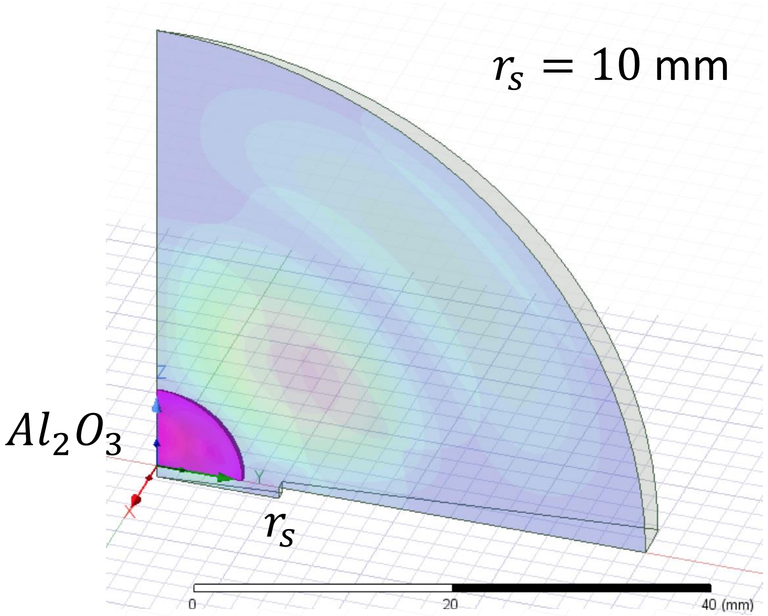
Solutions:

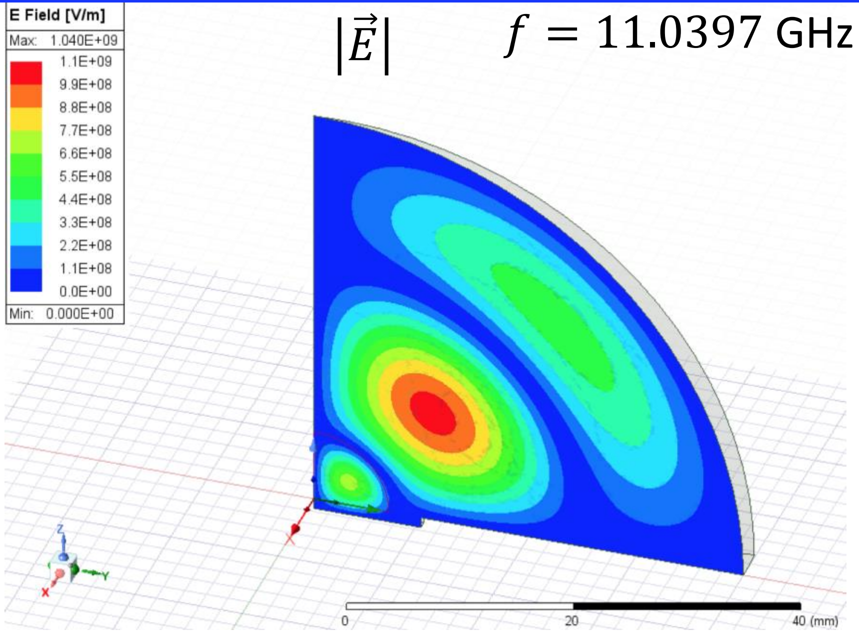
- [ ] Design a new cavity using dielectric.

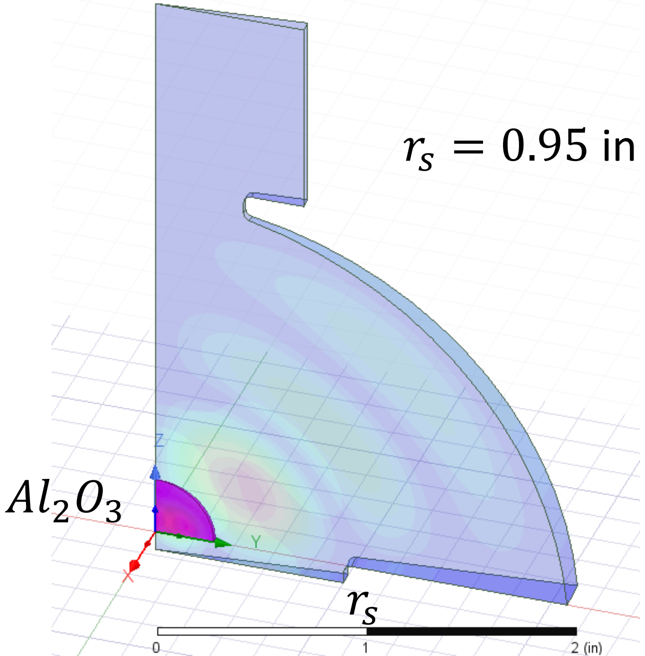


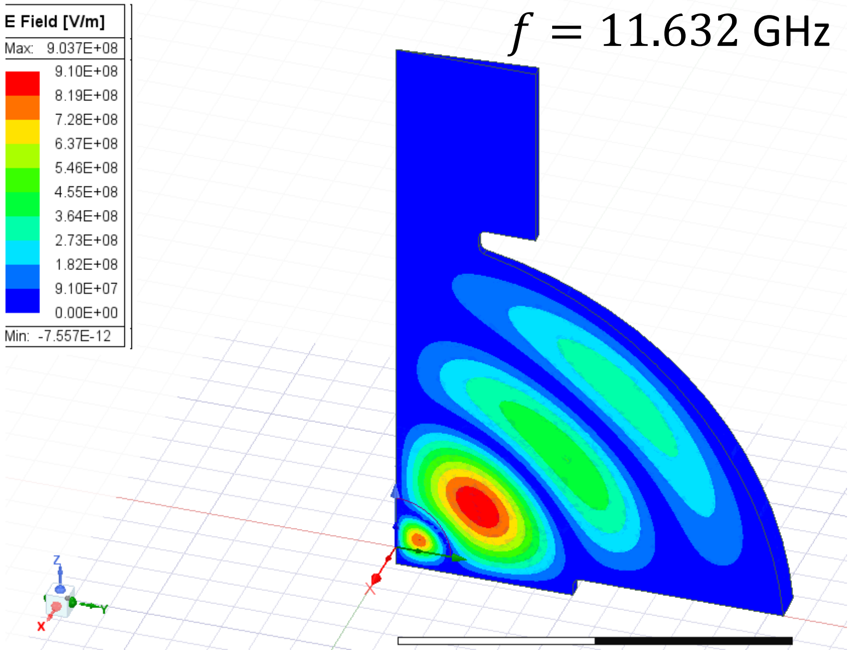
- [ ] Next higher order mode with SLAC cavity using dielectric.







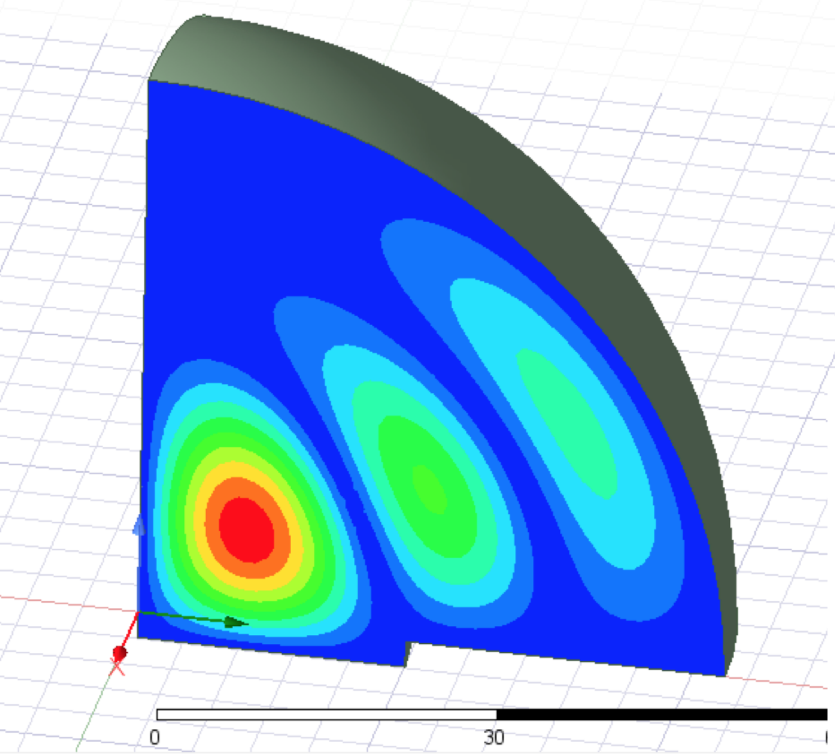




Resonant cavity: analytical study

TE_{320}-like mode in a "semispherical" cavity

$|\vec{E}|$



$$E_\phi = E_0 j_2(k_{32} r) L_2^{0\prime}(\cos\theta)$$

Zeros of $j_2(x)$

5.76346	9.09501	12.3229	15.5146

$$TE, \frac{\partial}{\partial\phi} = 0 \text{ (axial symmetry)}$$

$$E_\phi(r,\theta) = E_0 j_n(k_{mn} r) \frac{d}{d\theta} L_n^0(\cos\theta)$$

$$\omega_{m,n} = c k_{mn} = c\left(\frac{r_{mn}}{R}\right)$$

$$\vec{H} = H_r \hat{r} + H_\theta \hat{\theta}$$

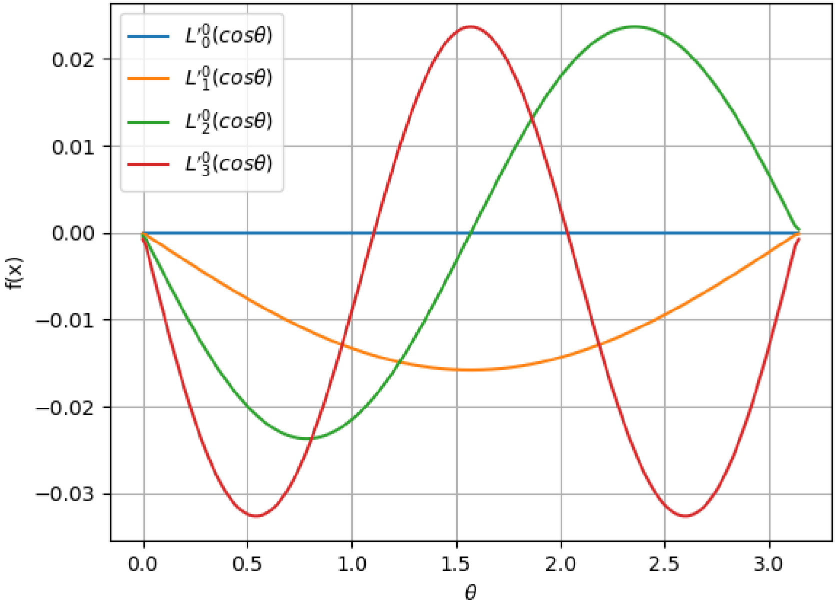
$$\vec{E} = E_\phi \hat{\phi}$$

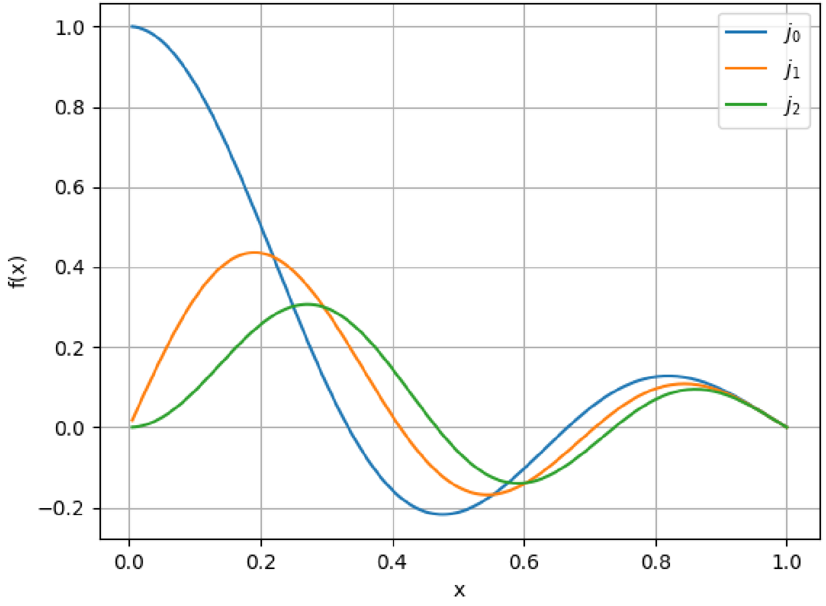
$L_n^0(x)$: Legendre polynomial

$j_n(r)$: spherical Bessel function

r_{mn}: n-th zero of the m-th spherical Bessel function

$$j_n(r) = \sqrt{\frac{\pi}{2r}} J_{n+\frac{1}{2}}(r)$$



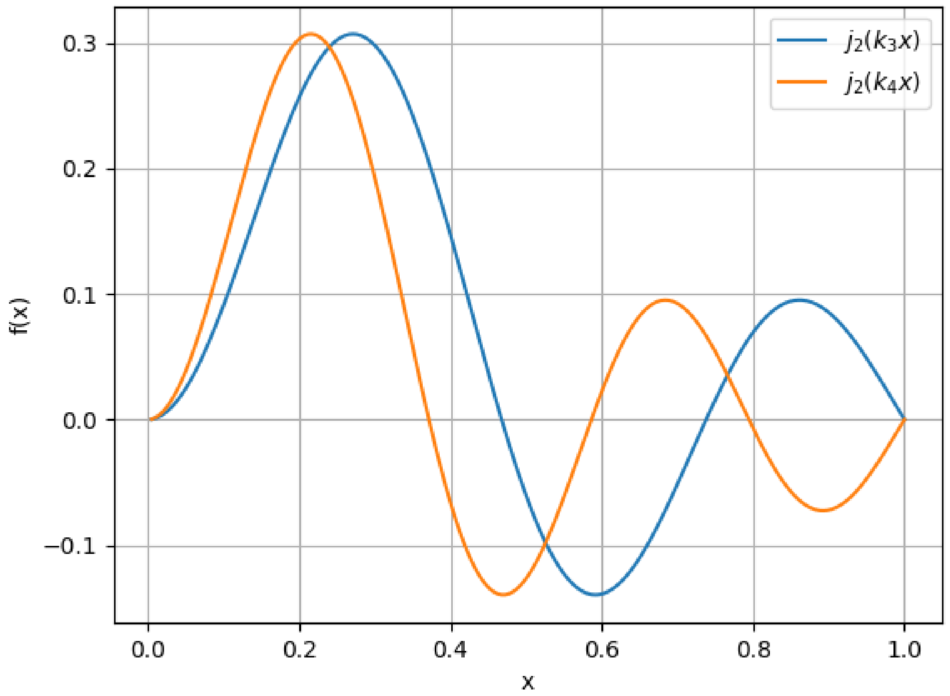


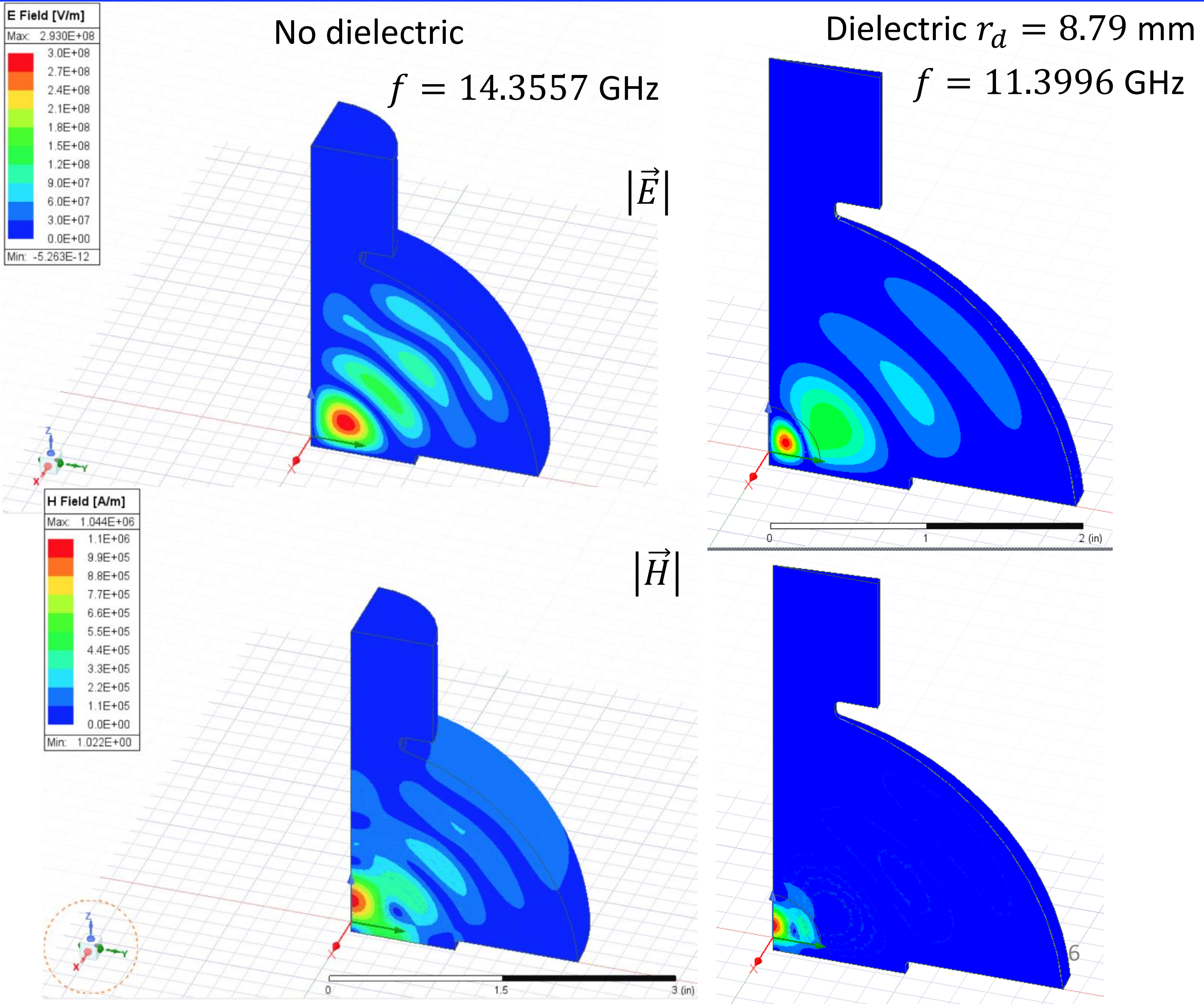
Dielectric resonant cavity

$$\omega_{m,n} = c\left(\frac{r_{mn}}{R}\right)$$

$$\boldsymbol{E_\phi \propto j_2(k_{42}r)}$$

λ_{12}	0.37*R
λ_{22}	0.215*R
λ_{32}	0.208*R
λ_{42}	0.207*R

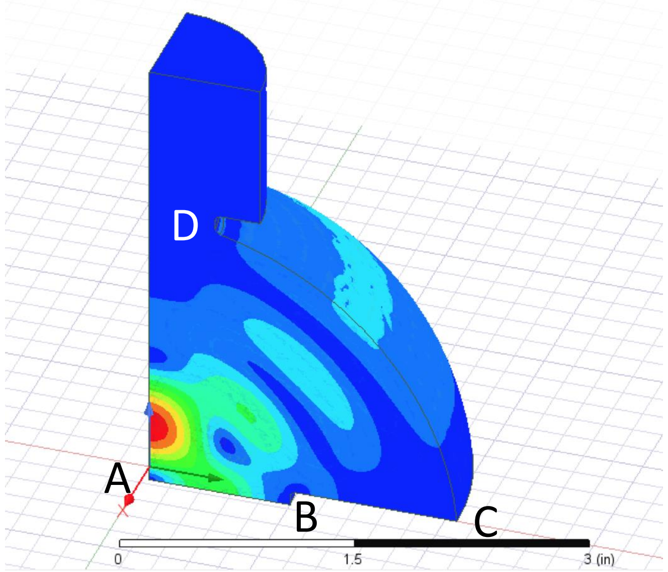




Dielectric resonant cavity

$E_\phi \propto j_2(k_{32} r)$

$f = 11.3995$ GHz



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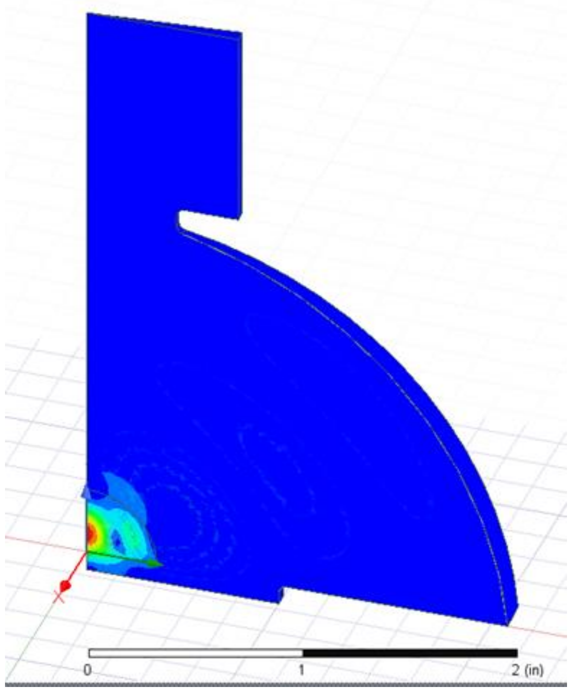
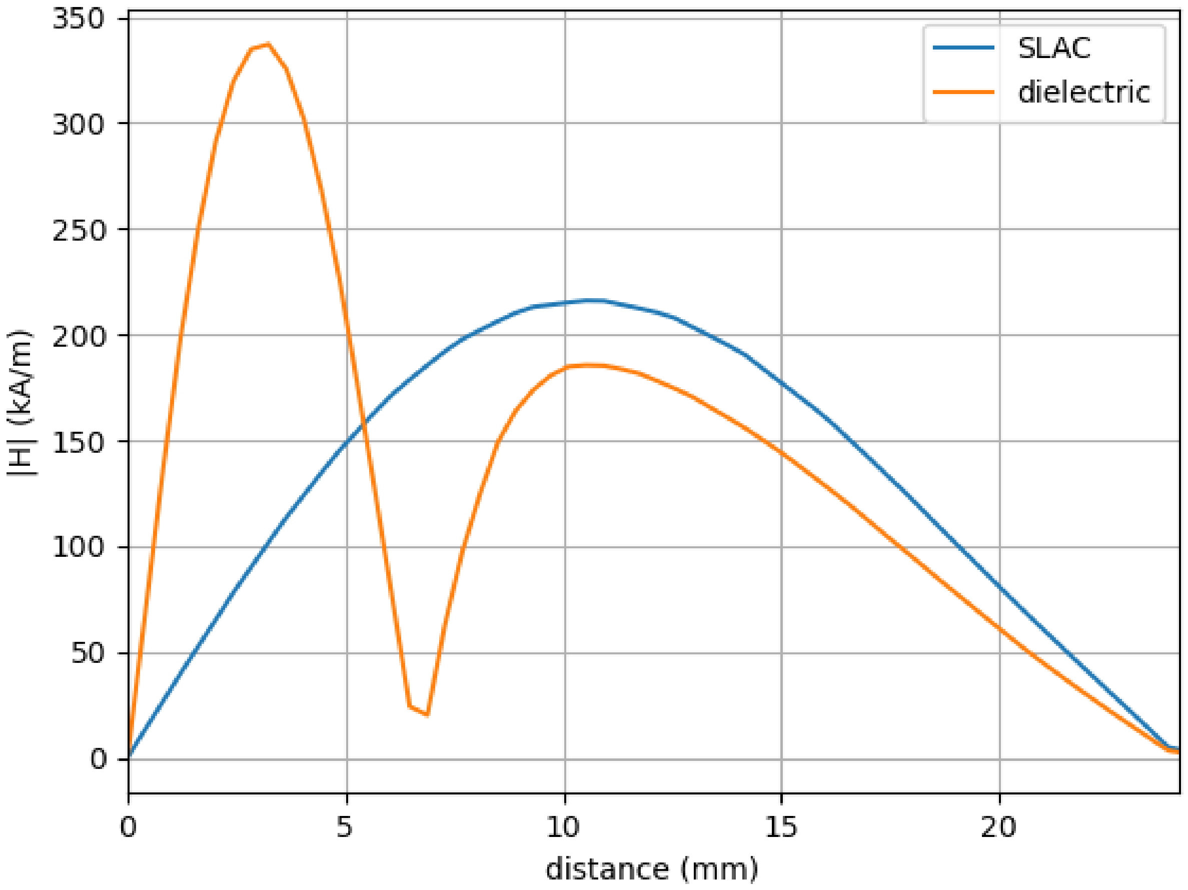


Surface magnetic field along $\overline{AD}$

- Field normalization: $W = 1$ J in the full cavity



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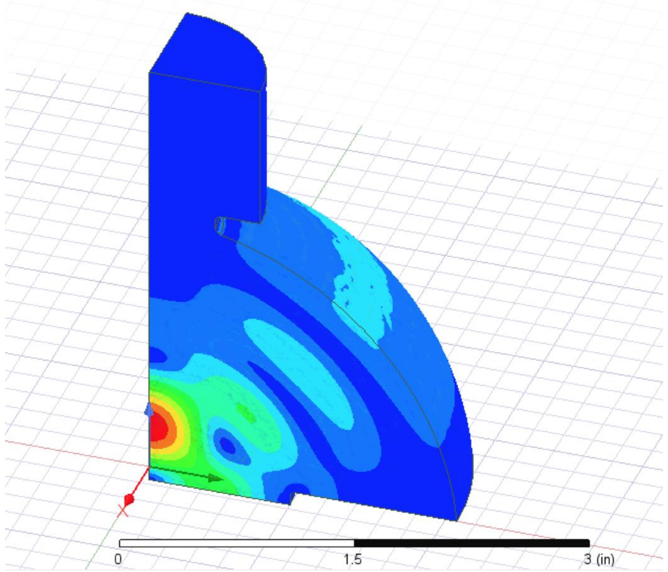

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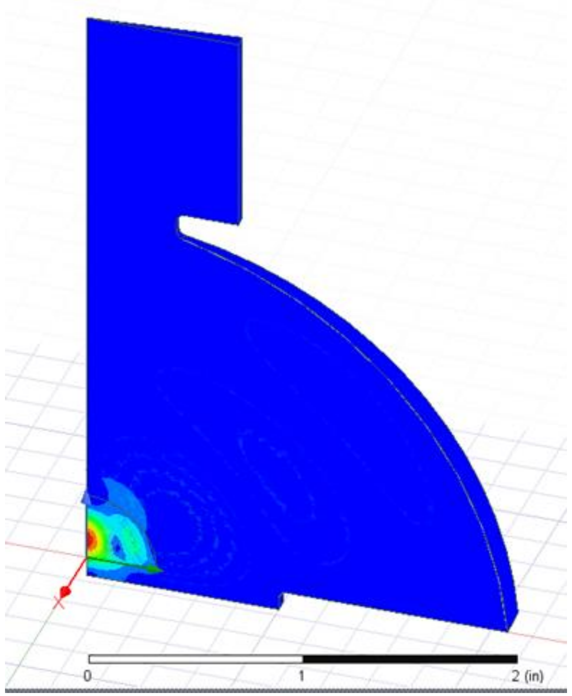
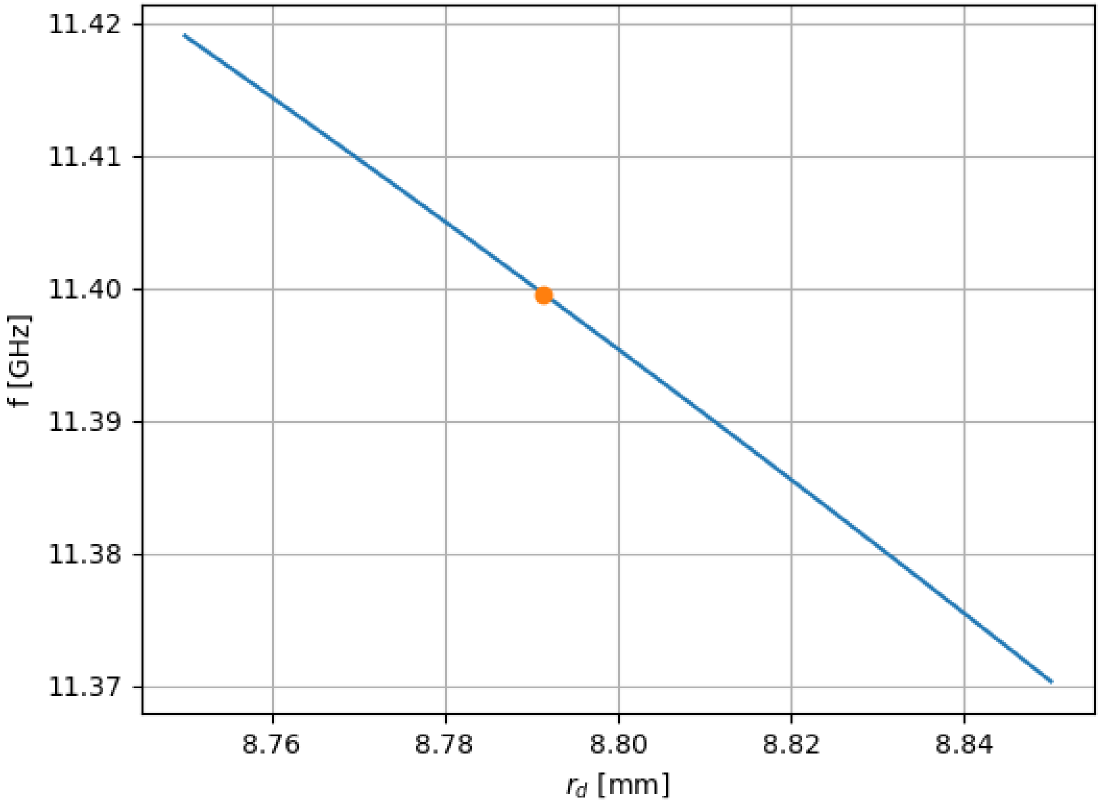


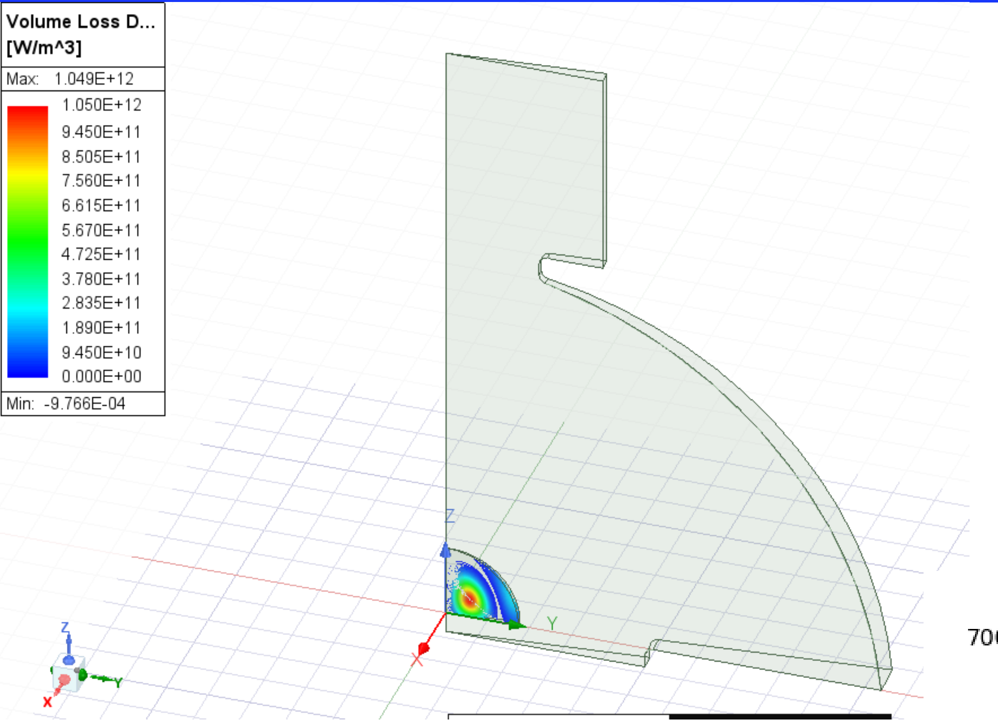
Tolerance study



$$\frac{df}{dr_d} = -482 \left[\frac{GHz}{m}\right] = -482 \left[\frac{kHz}{\mu m}\right]$$

How is the cavity tuned?

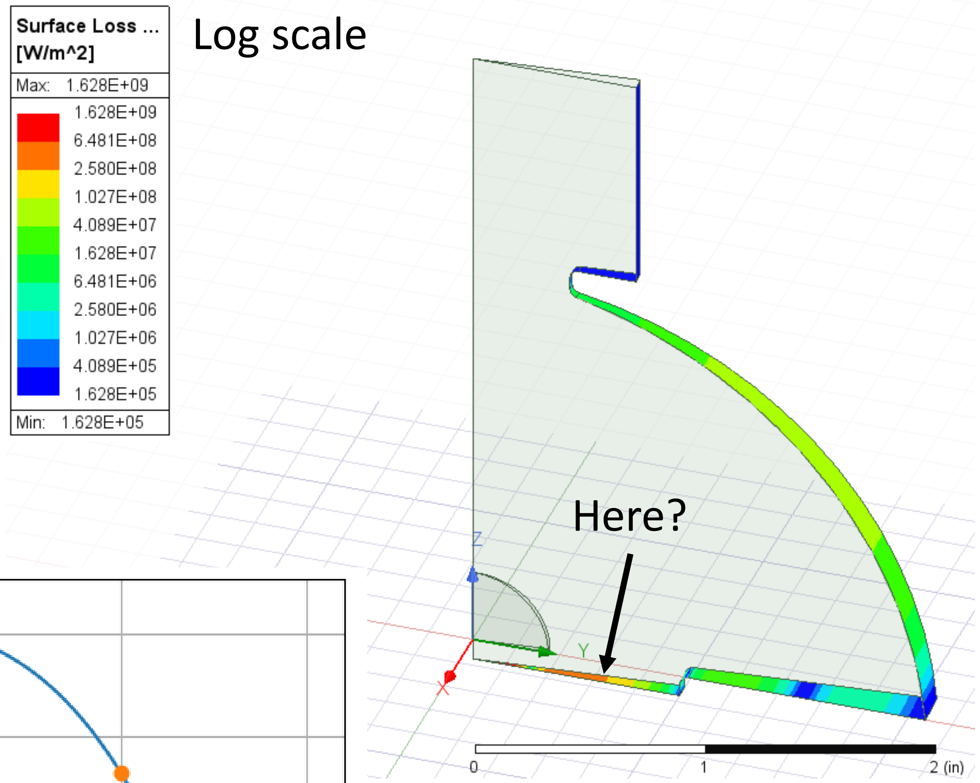
Dielectric resonant cavity



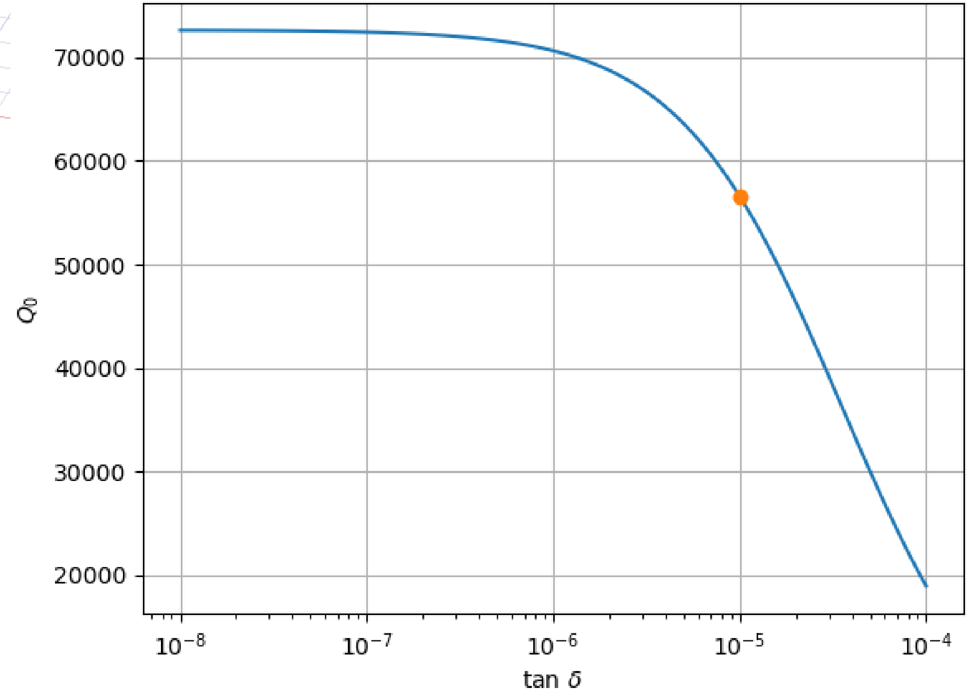
$$f = 11.3996 \text{ GHz}$$

$$Q_0 = \omega \frac{W}{P} = \frac{\omega W}{P_c + P_d}$$

Log scale



$$P_d = \frac{1}{2}\omega \tan\delta\, \epsilon_0 \epsilon_r \int |\boldsymbol{E}|^2 dV$$



$$P_c = \frac{R_s}{2} \int |\boldsymbol{n} \times \boldsymbol{H}|^2 dS$$

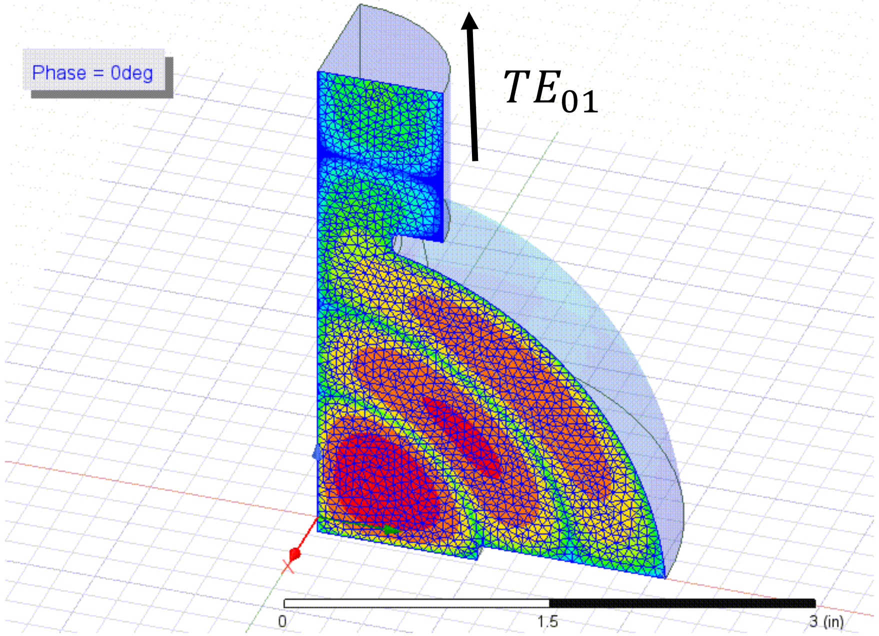
Dielectric resonant cavity

$E_\phi \propto j_2(k_{32}r)$ $f = 11.3995$ GHz

Material	Q
Copper	50021
External port	111069

$$\begin{cases} Q_0 = 50021 \\ Q_{ext} = 111069 \end{cases}$$

$\beta = 0.45 \rightarrow$ Undercoupled



$$\beta = \frac{P_{ext}}{P_d} = \frac{Q_0}{Q_{ext}}$$

$$S_{11} = \frac{1-\beta}{1+\beta}$$

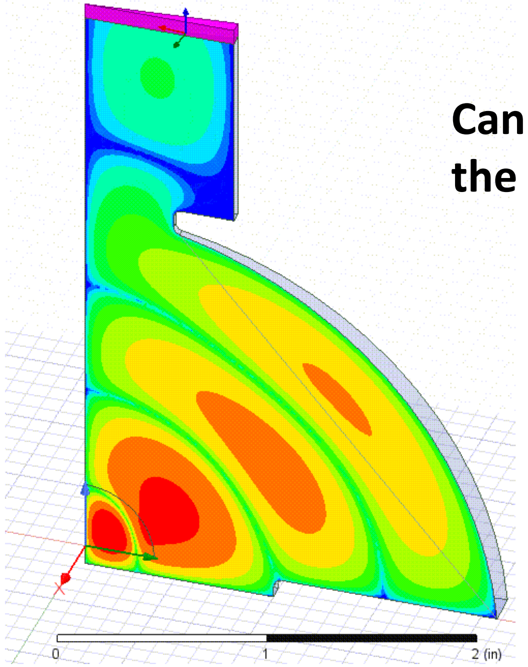
$E_\phi \propto j_2(k_{42}r)$ $f = 11.3996$ GHz

$\tan\delta = 10^{-5}$

Material	Q
Copper	72659
Dielectric	256309
External port	331331

$$\begin{cases} Q_0 = 56611 \\ Q_{ext} = 331331 \end{cases}$$

$\beta = 0.17 \rightarrow$ Heavily undercoupled



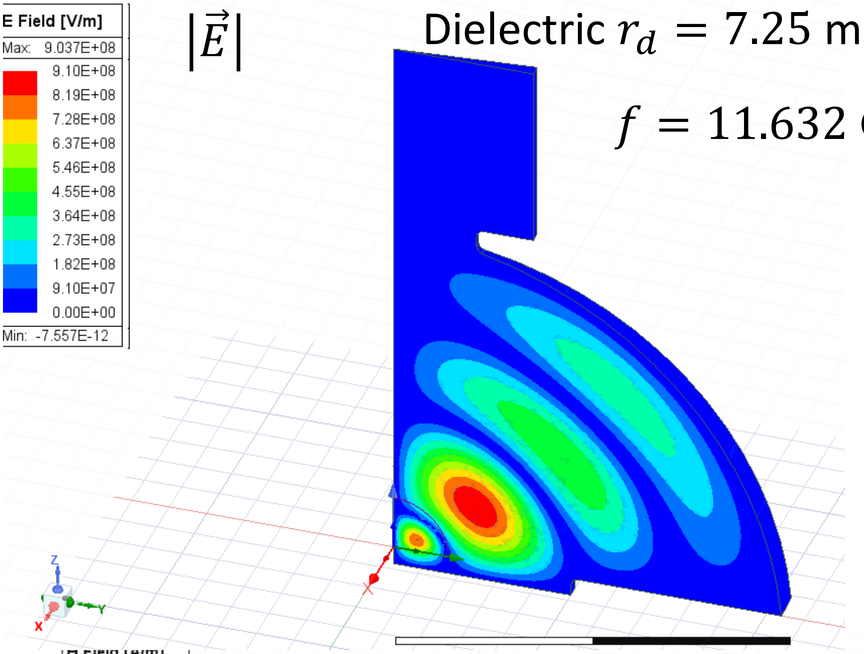
Can we tune the coupling?

Dielectric resonant cavity

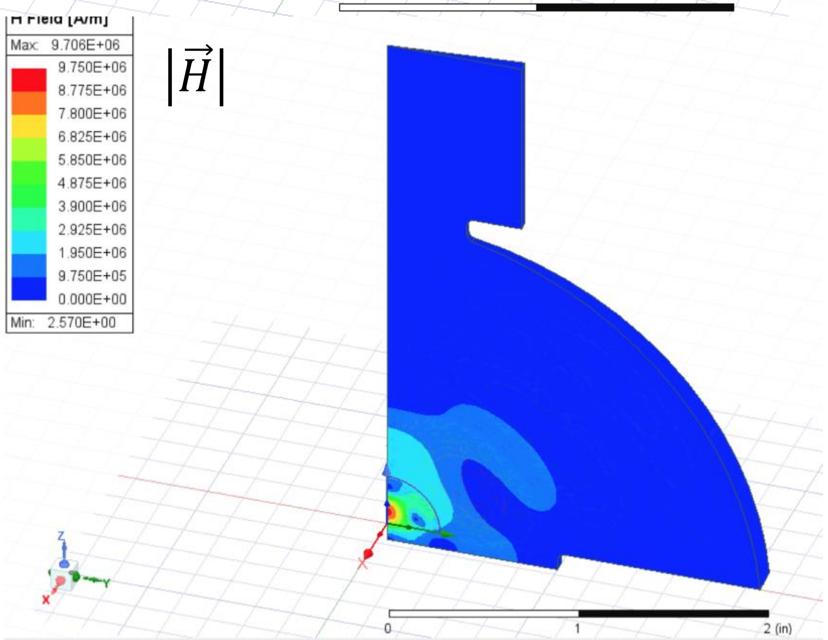
$|\vec{E}|$

Dielectric $r_d = 7.25$ mm

$f = 11.632$ GHz



$|\vec{H}|$



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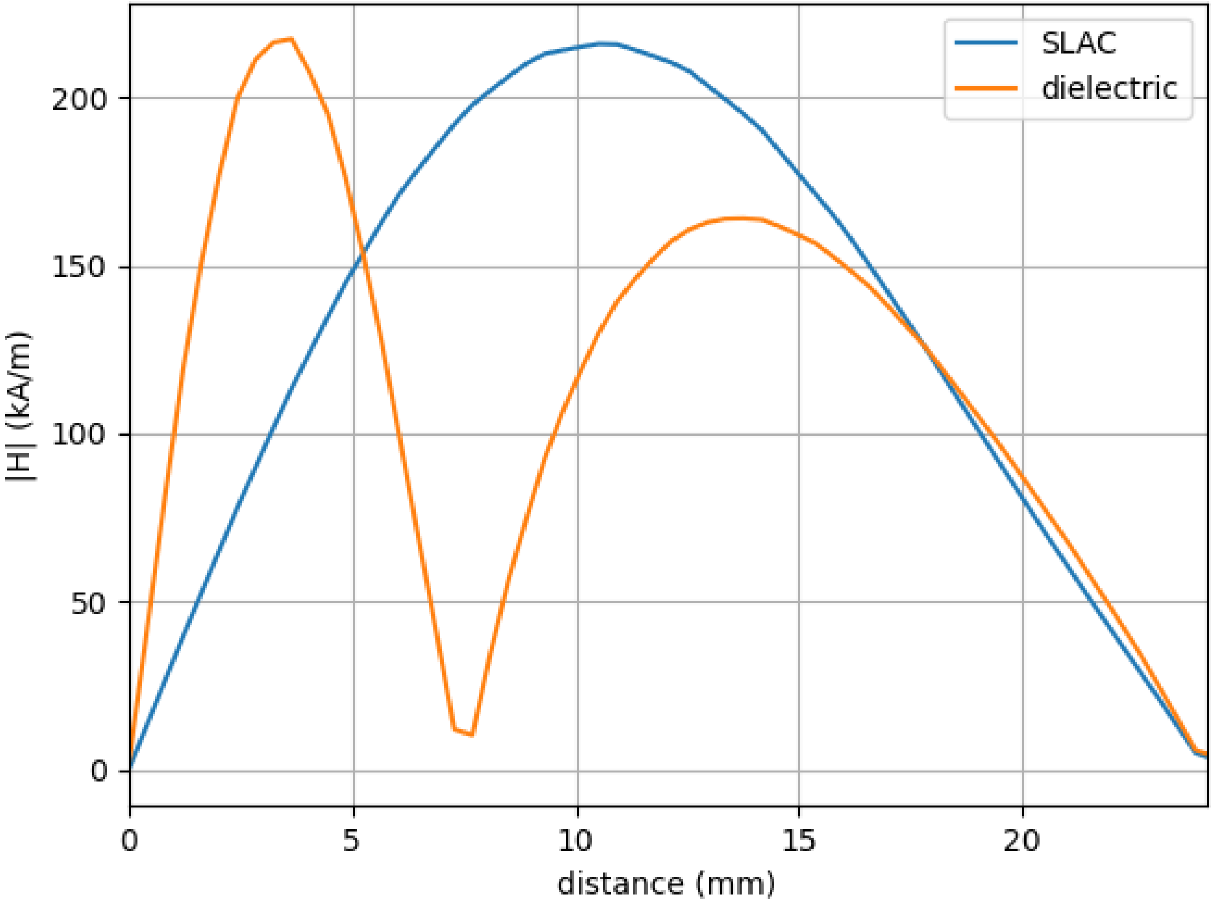


Dielectric resonant cavity

Without dielectric

$E_\phi \propto j_2(k_{32}r)$ $f = 11.3995$ GHz

Material	Q
Copper	50021
External port	111069

$\begin{cases} Q_0 = 50021 \\ Q_{ext} = 111069 \end{cases}$ $\beta = 0.45 \rightarrow$ Undercoupled

New

$E_\phi \propto j_2(k_{42}r)$ $f = 11.632$ GHz

$\tan\delta = 10^{-5}$

Material	Q
Copper	80275
Dielectric	120480
External port	97731

$\begin{cases} Q_0 = 48176 \\ Q_{ext} = 97731 \end{cases}$ $\beta = 0.49 \rightarrow$ Undercoupled

Previous

$f = 11.3996$ GHz

$\tan\delta = 10^{-5}$

Material	Q
Copper	72659
Dielectric	256309
External port	331331

$\begin{cases} Q_0 = 56611 \\ Q_{ext} = 331331 \end{cases}$ $\beta = 0.17 \rightarrow$ Heavily undercoupled

Back up

Dielectric resonant cavity

Anisotropic behaviour:

- Sapphire (crytalline form of Al_2O_3) has anisotropic behaviour in ϵ_r and $\tan\delta$

  $\epsilon_r \parallel c = 11.5$ at $10^3 - 10^9$ Hz, $\tan\delta \parallel c = 8.6 \times 10^{-5}$ at 10^{10} Hz and 25ºC

  $\epsilon_r \perp c = 9.3$ at $10^3 - 10^9$ Hz, $\tan\delta \perp c = 3 \times 10^{-5}$ at 10^{10} Hz and 25ºC

  https://rayotek.com/PDF/Sapphire-Properties-Data-Sheet.pdf

- Amorphous alumina is isotropic.

  $\epsilon_r = 9.6 - 10, \tan\delta = 10^{-4} - 10^{-5}$

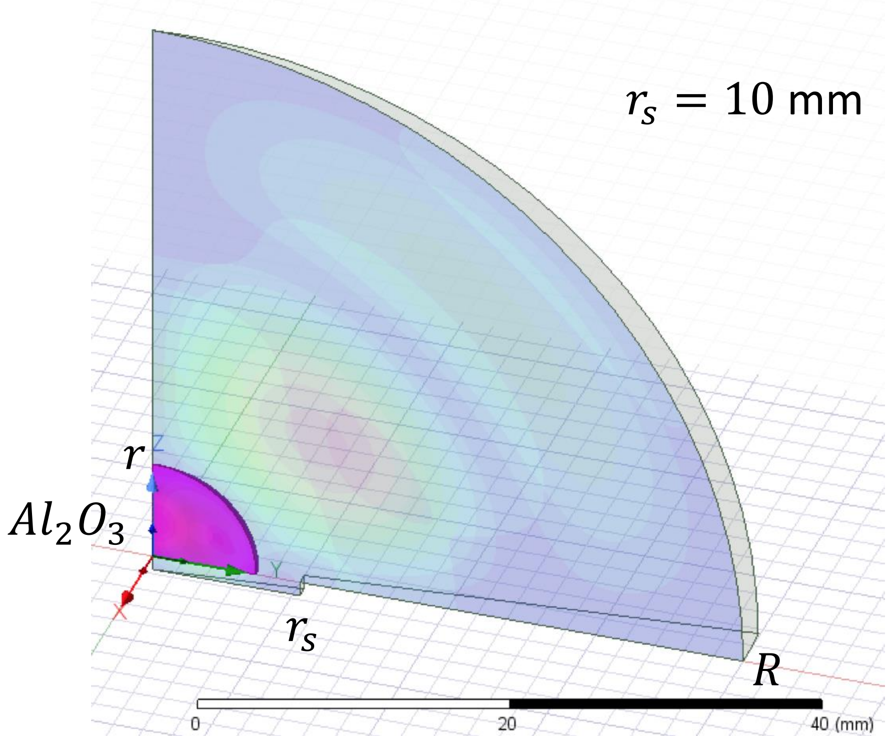
  https://thermalsupport.com/wp-content/uploads/2018/06/Sapphal%C2%AE-U-High-purity-99.9-Translucent-Alumina.pdf
  https://www.industrial-ceramicparts.com/sale-13785342-high-purity-99-9-aluminum-oxide-ceramics-260mpa-tensile-strength.html

Solution I

$$\omega_{m,n} = c\left(\frac{r_{mn}}{R}\right) \qquad \boldsymbol{E_\phi \propto j_2(k_{32} r)}$$

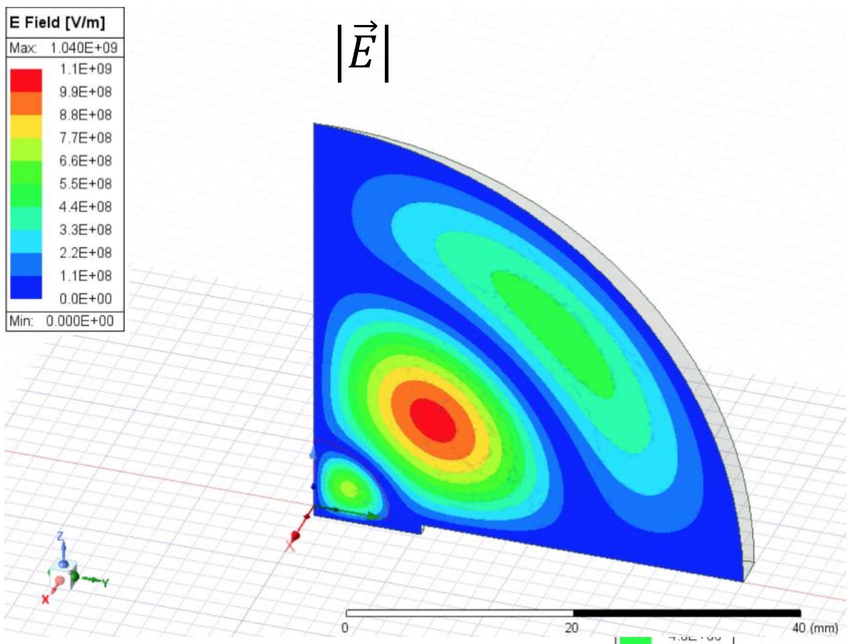
λ_{12}	0.46*R
λ_{22}	0.27*R
λ_{32}	0.27*R

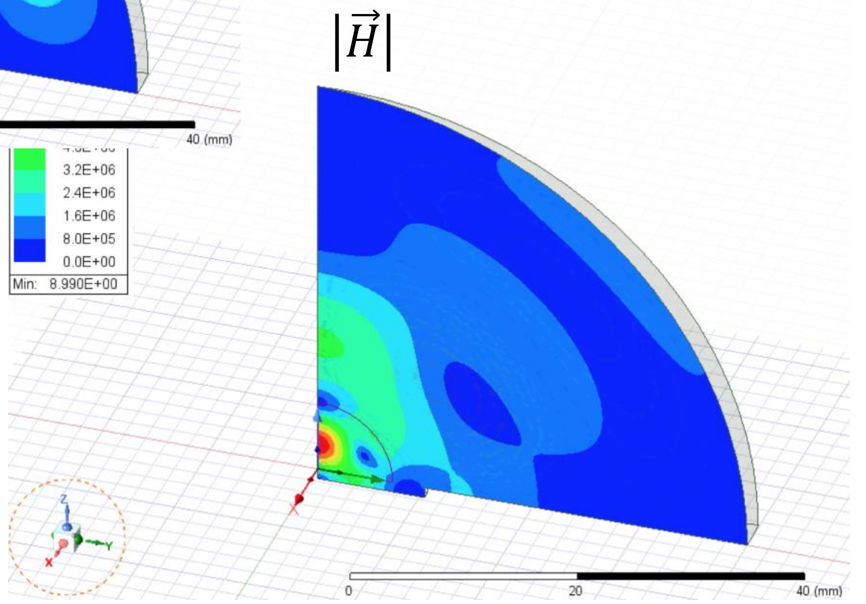
$f = 11.0397$ GHz $\quad r = 7$ mm $\quad R = 40$ mm



$$\lambda_d = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

λ_0 must be the same to keep frequency constant





Solution I

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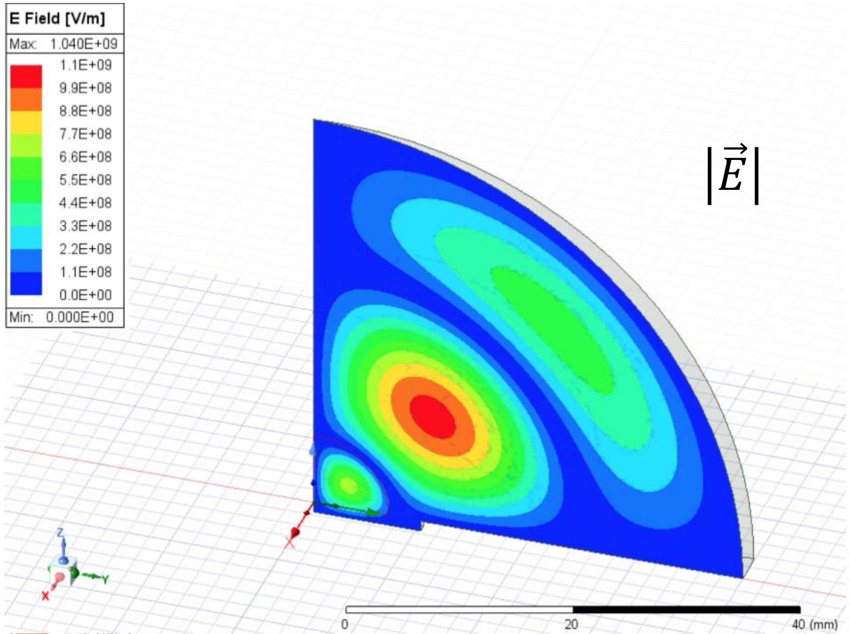
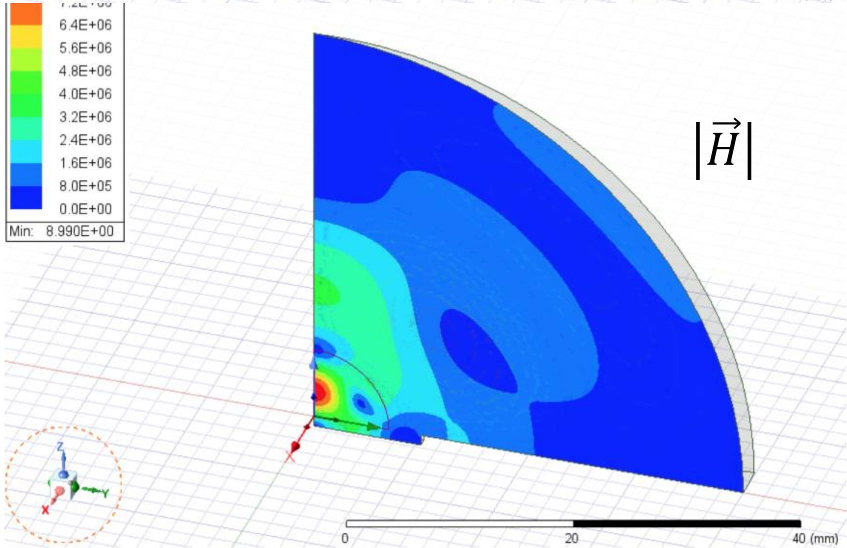
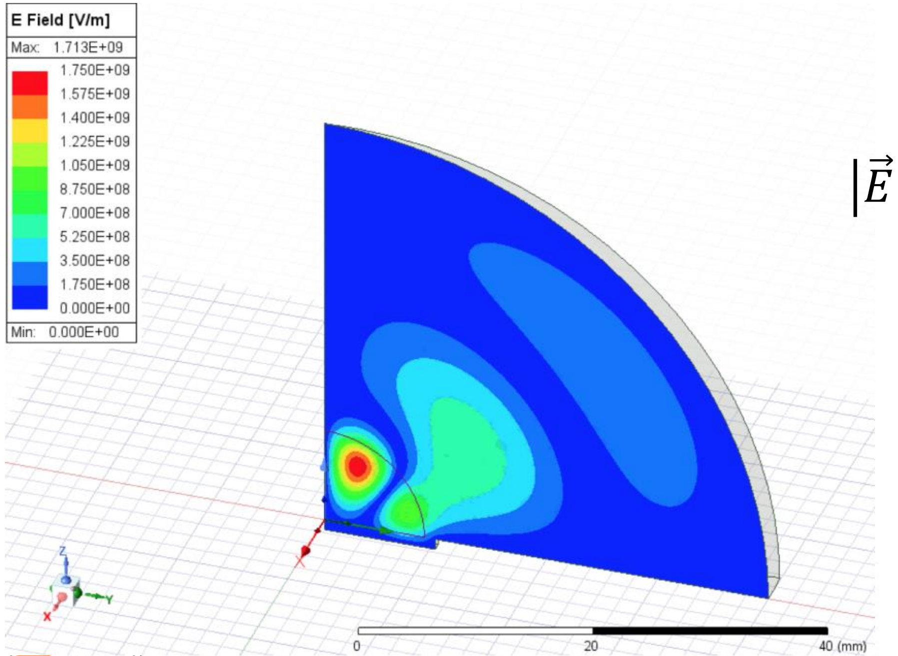
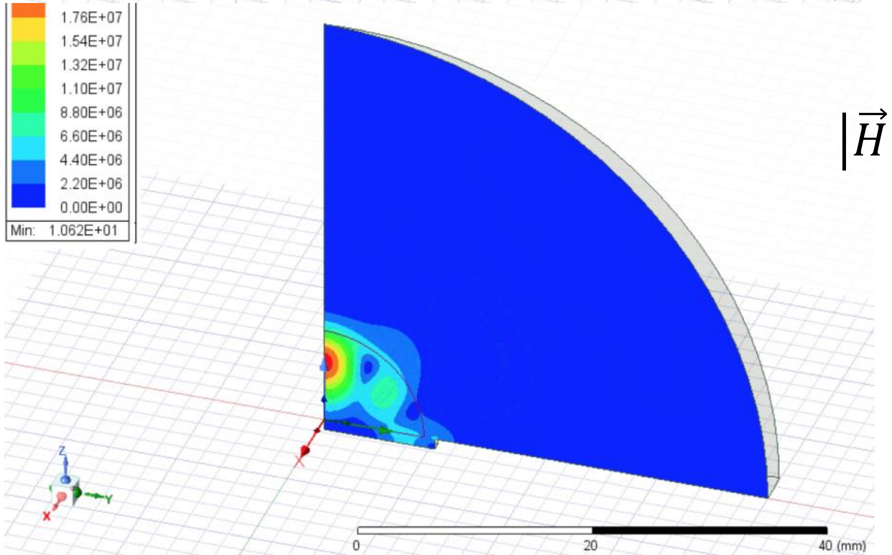




$f = 11.128$ GHz $r = 9$ mm $R = 40$ mm



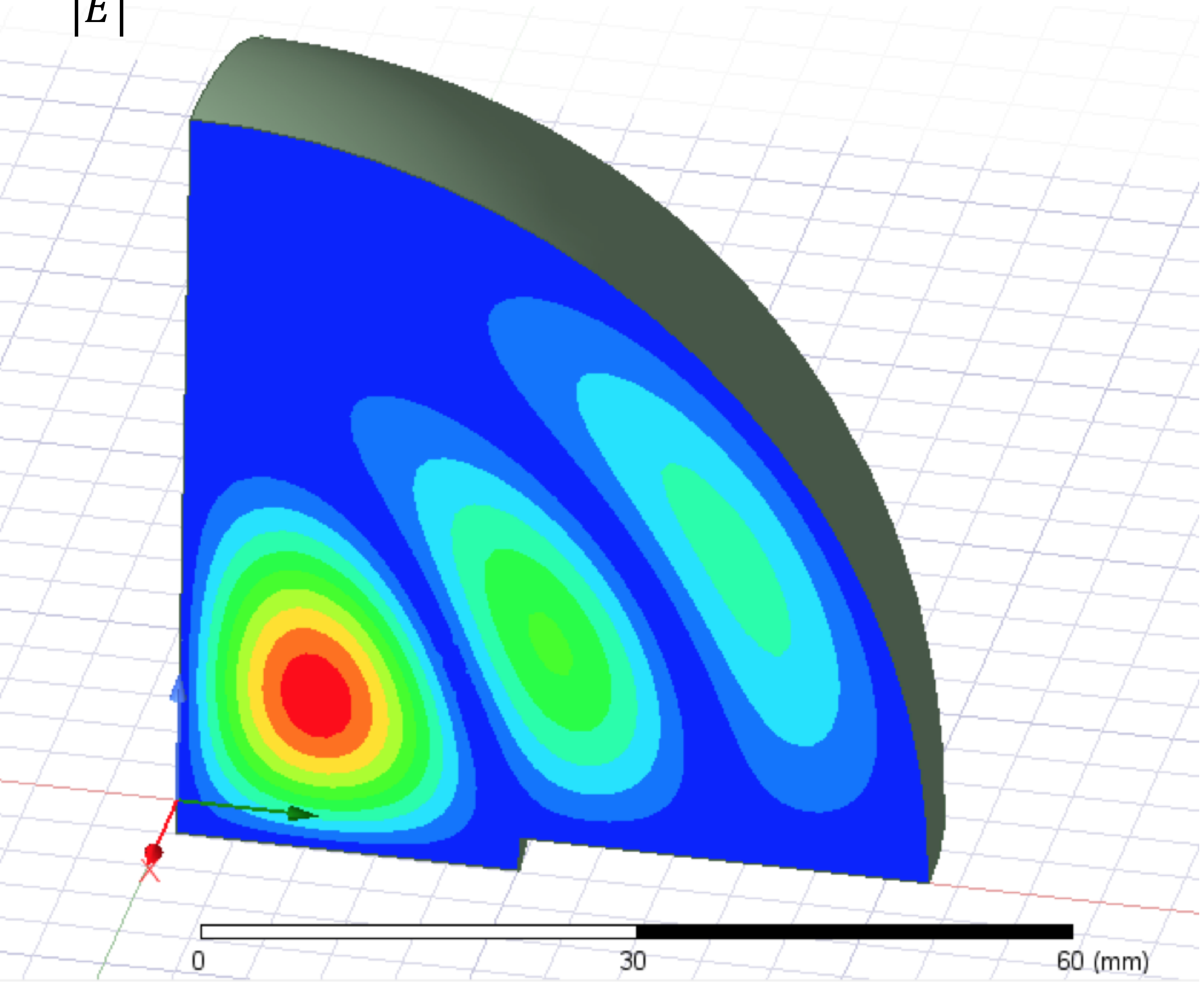


Resonant cavity

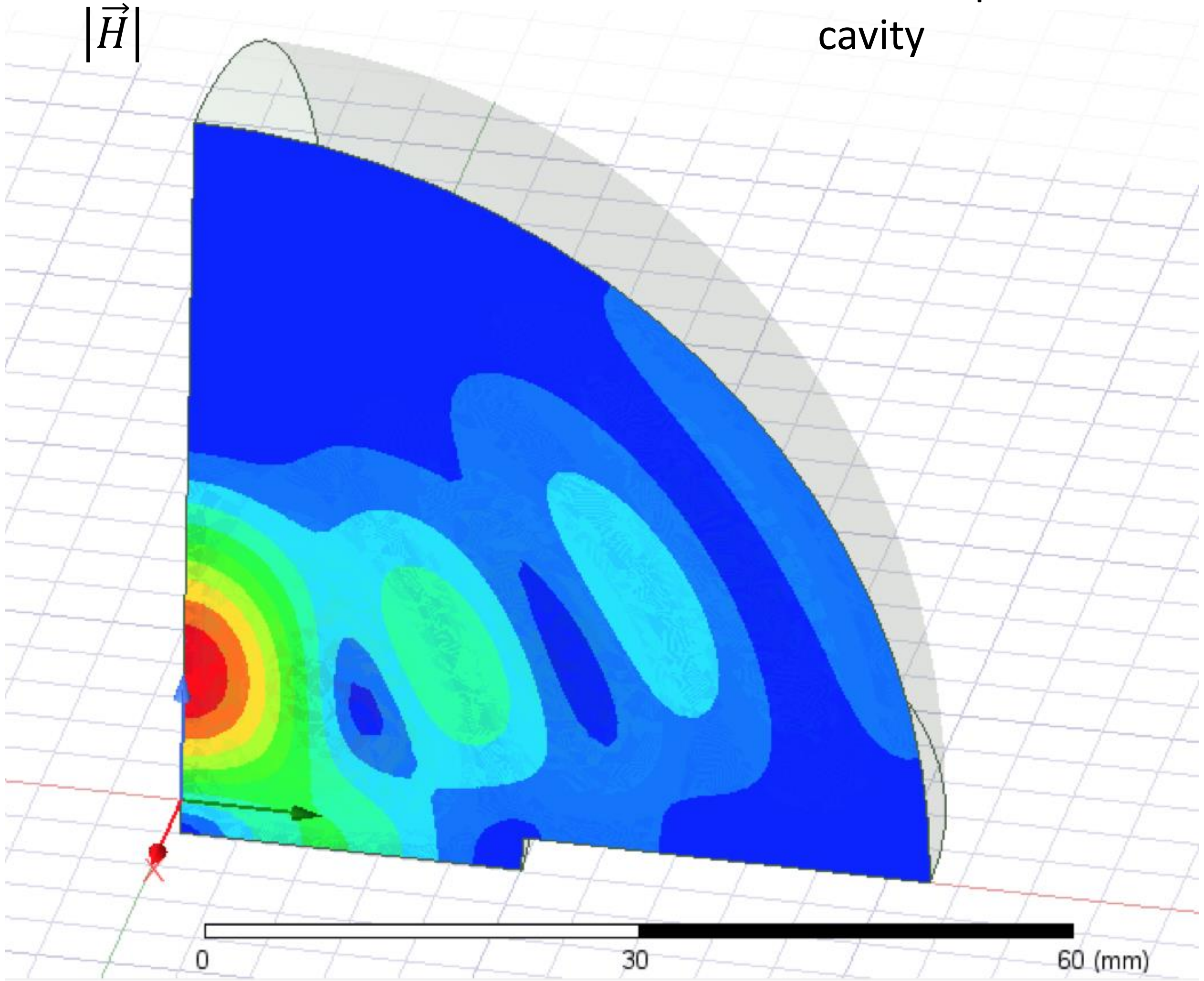
$f\ = 10.8501$ GHz

TE_{320}-like mode in a “semispherical” cavity

$|\vec{E}|$



$|\vec{H}|$



Dielectric resonant cavity

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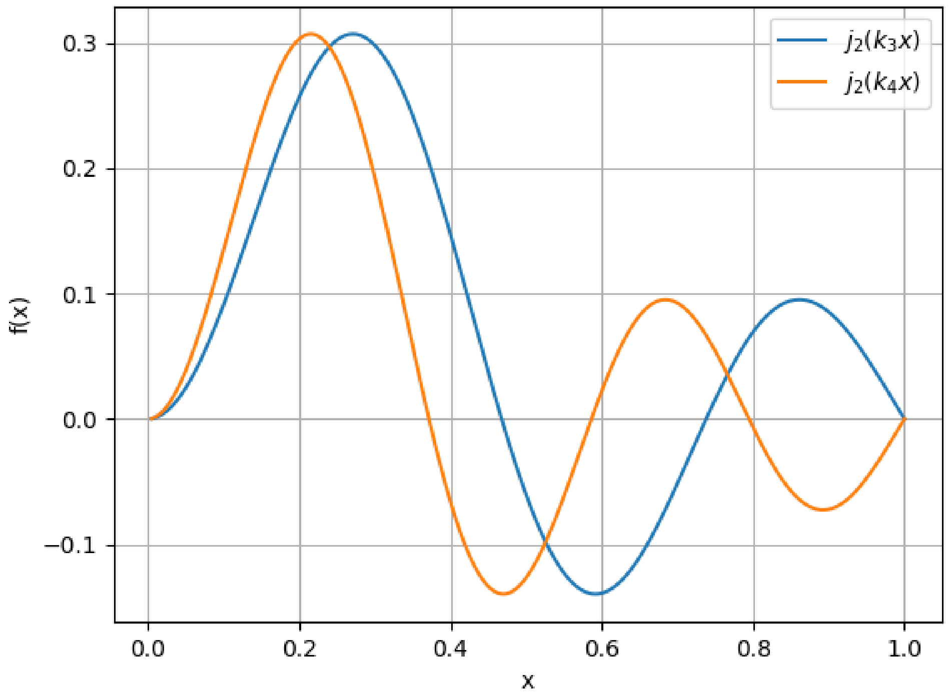

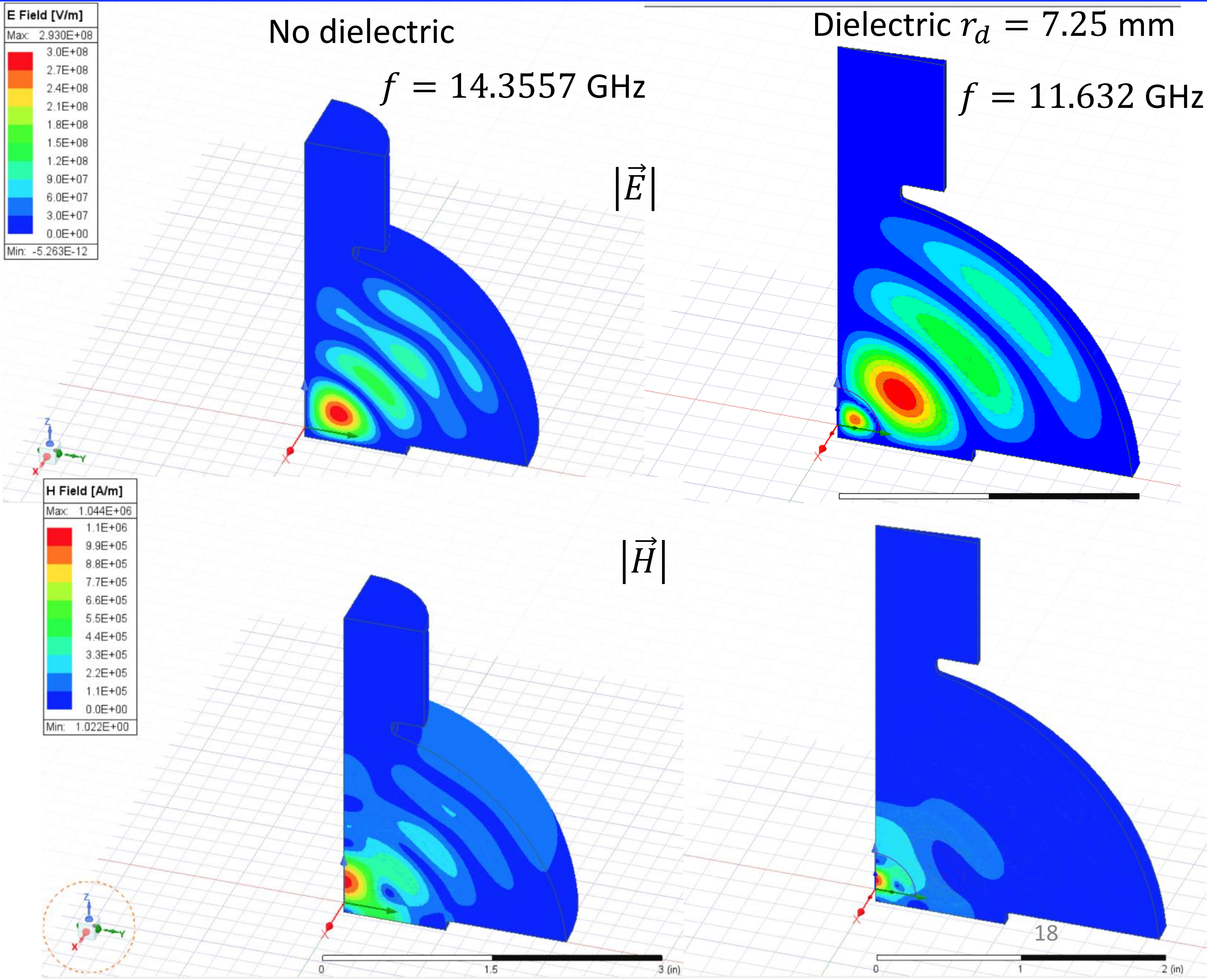

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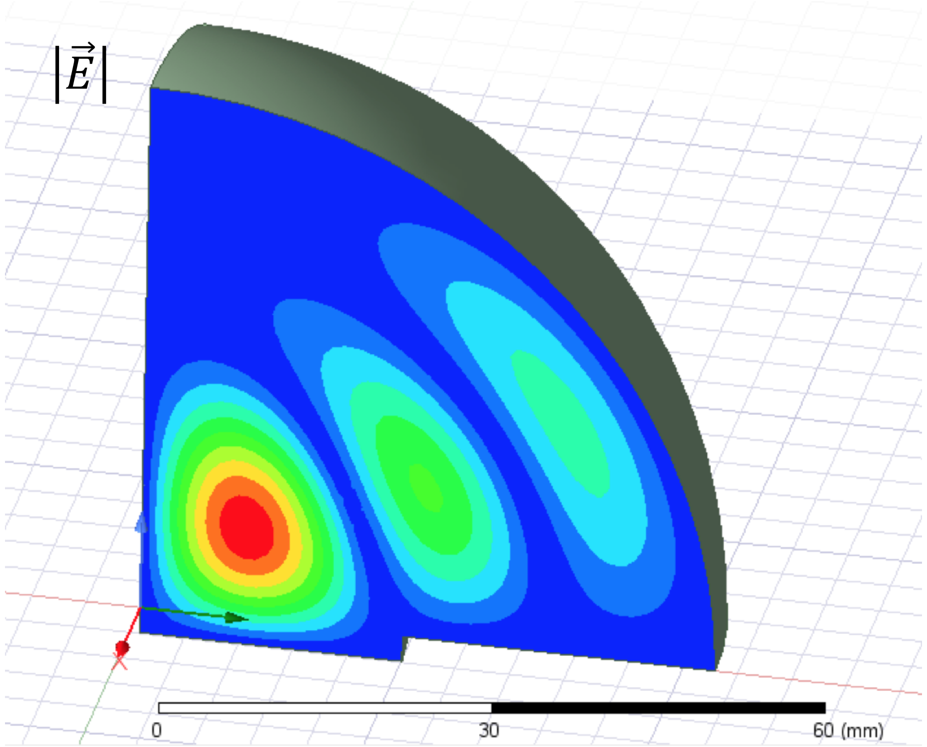
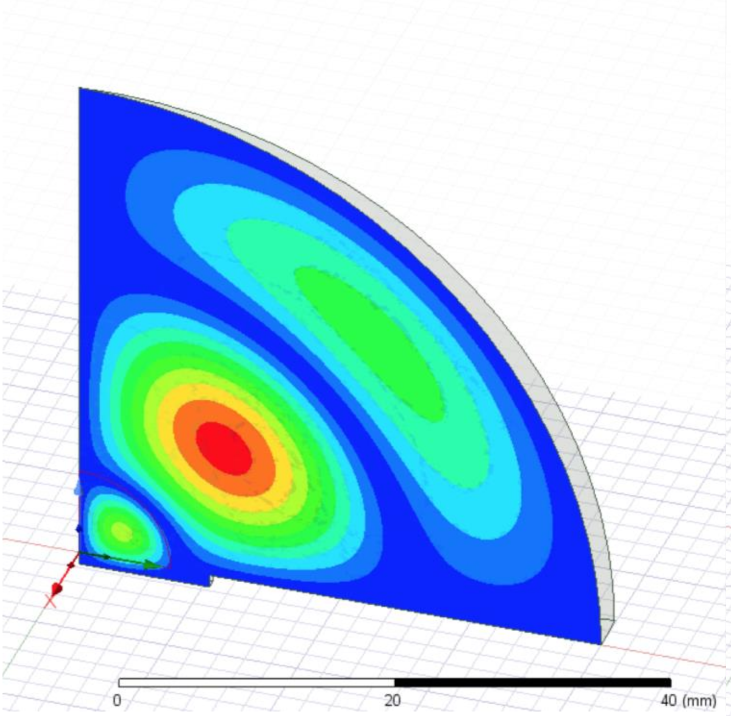
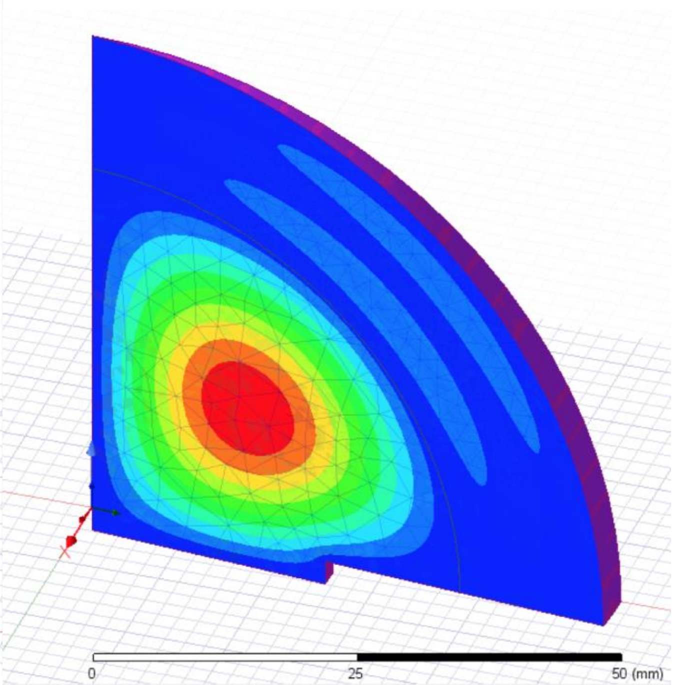


Reduce first lambda with dielectric



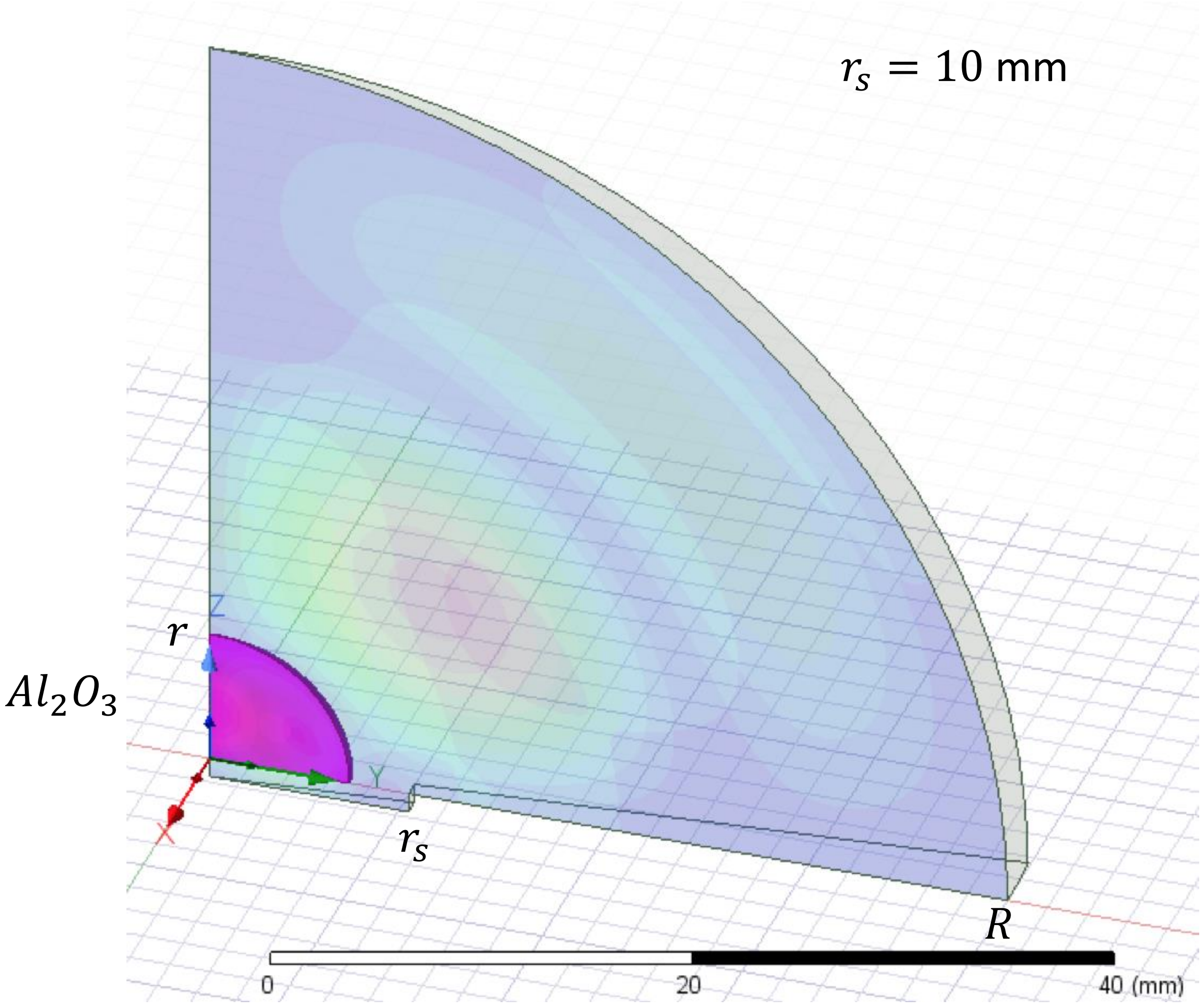
Reduce 2nd and 3rd lambda with dielectric

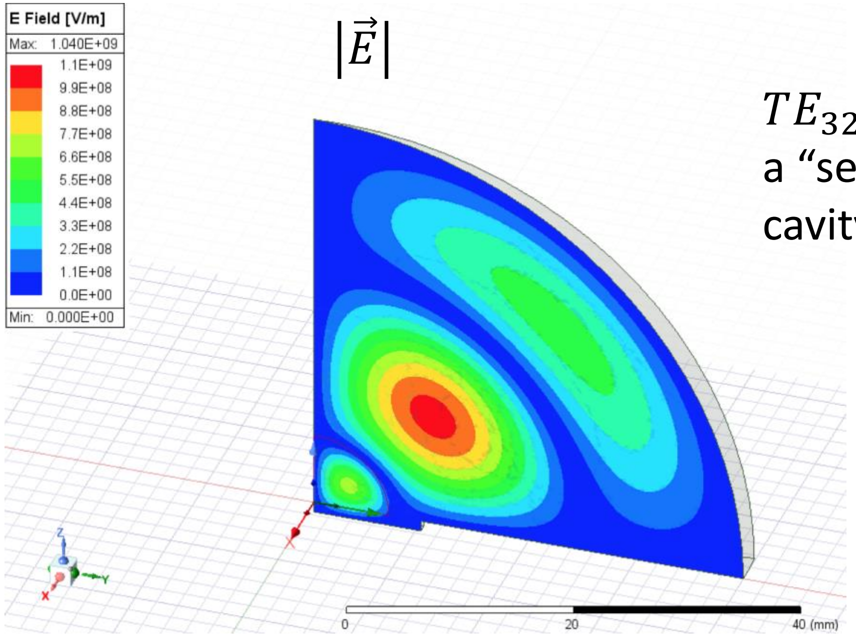


Dielectric resonant cavity I

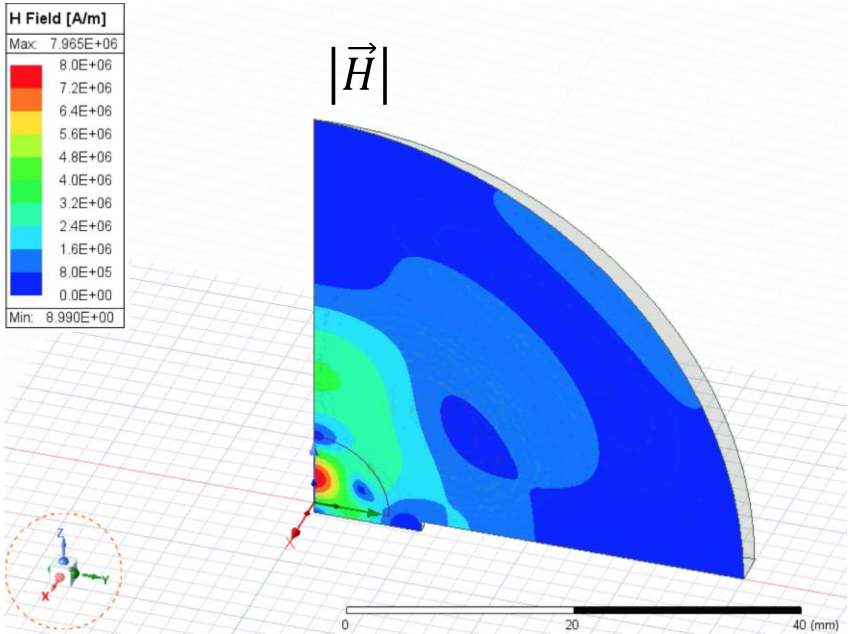
$f = 11.0397$ GHz $r = 7$ mm $R = 40$ mm

$r_s = 10$ mm





TE_{320}-like mode in a "semispherical" cavity

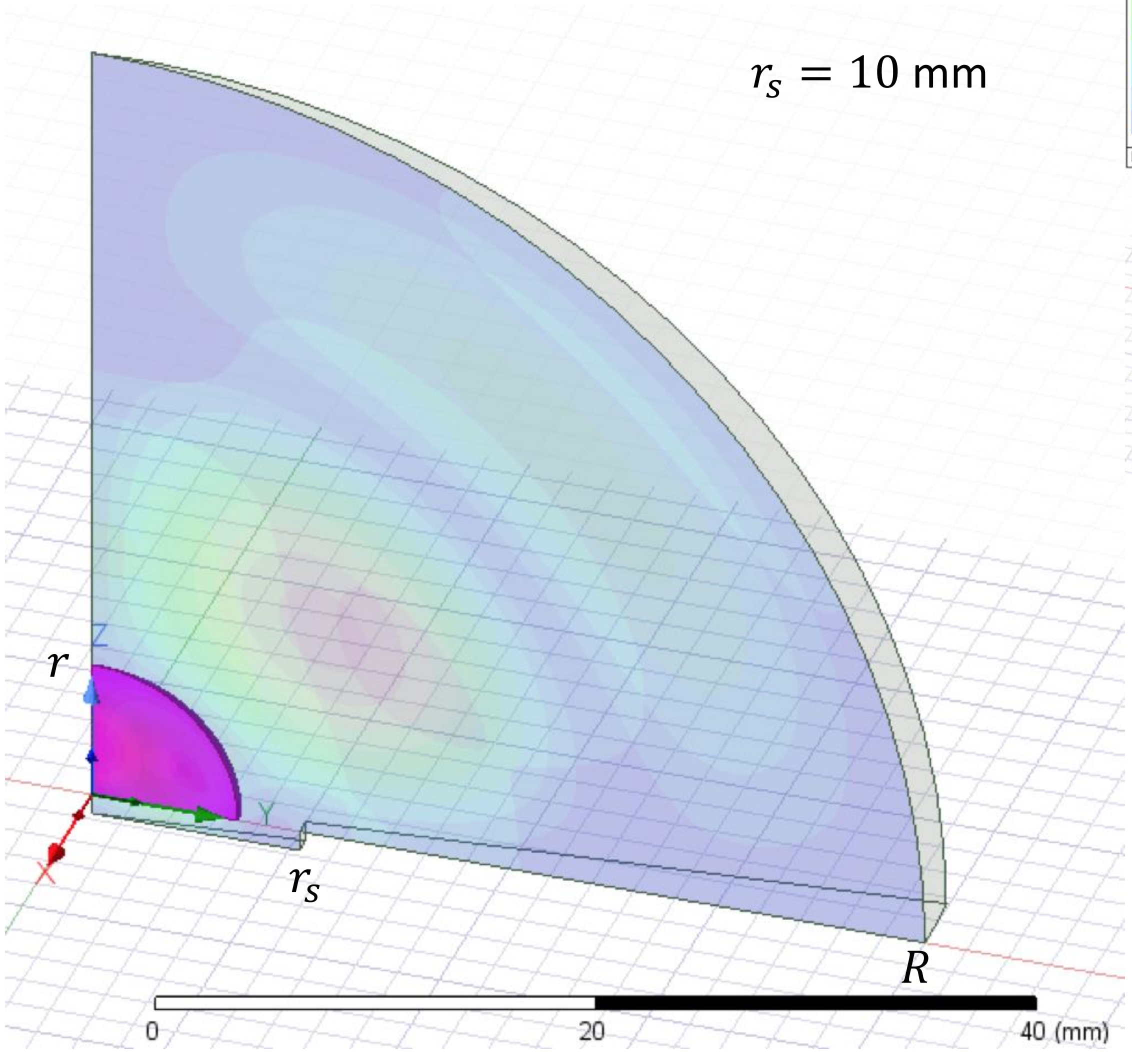


Dielectric resonant cavity I

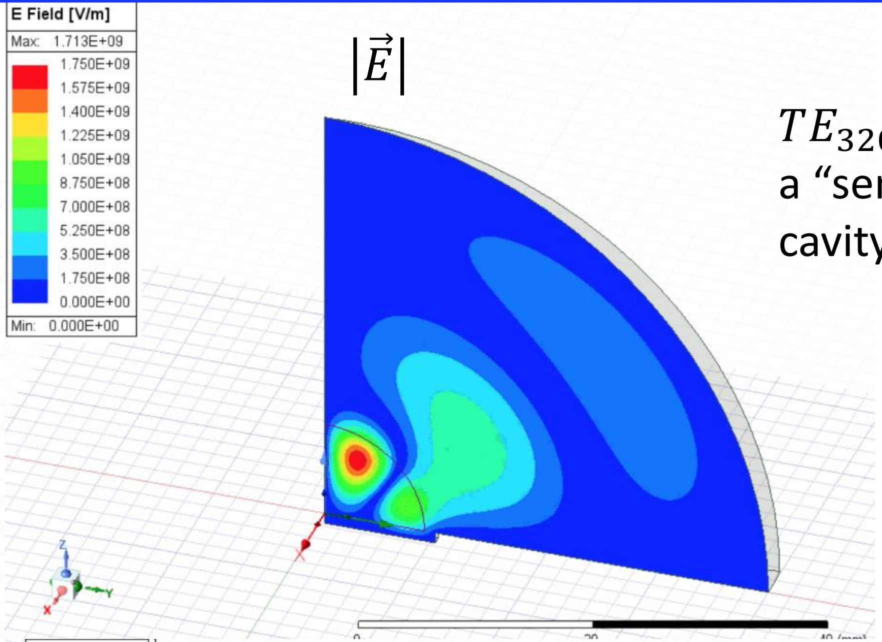
$f = 11.128$ GHz $\quad r = 9$ mm $\quad R = 40$ mm

$r_s = 10$ mm

Al_2O_3

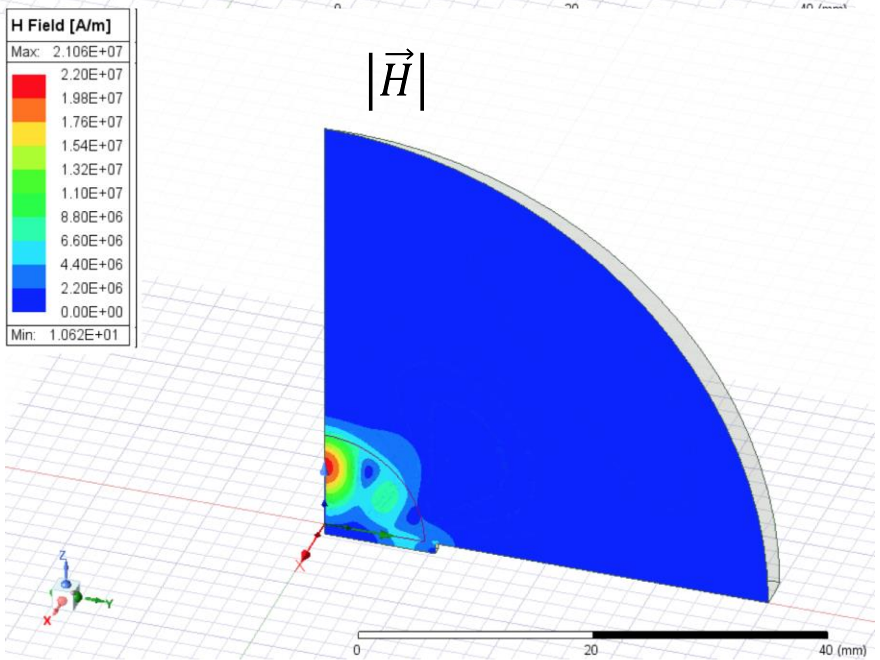


$|\vec{E}|$



TE_{320}-like mode in a “semispherical” cavity

$|\vec{H}|$

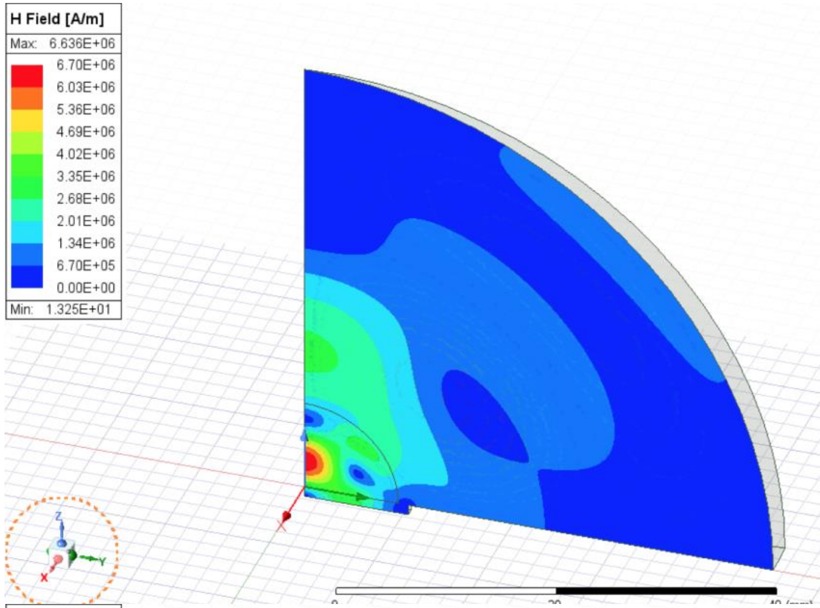


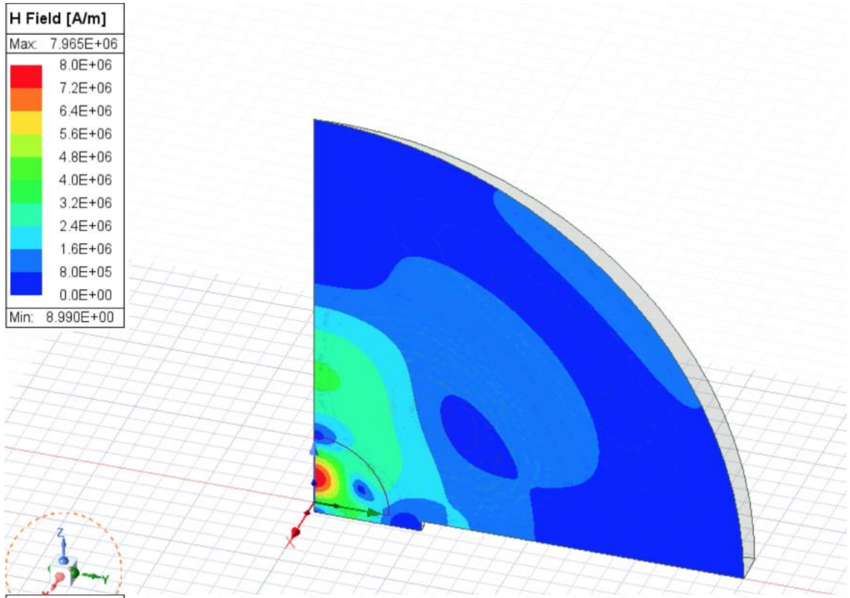
Dielectric resonant cavity I

$R = 45$ mm $r = 9$ mm $f = 9.81$ GHz

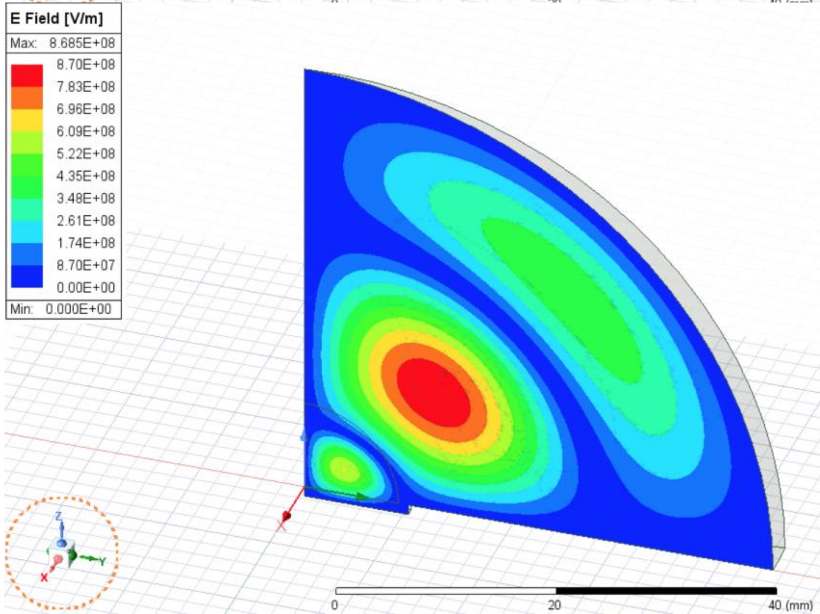
$R = 40$ mm $r = 7$ mm $f = 11.0397$ GHz

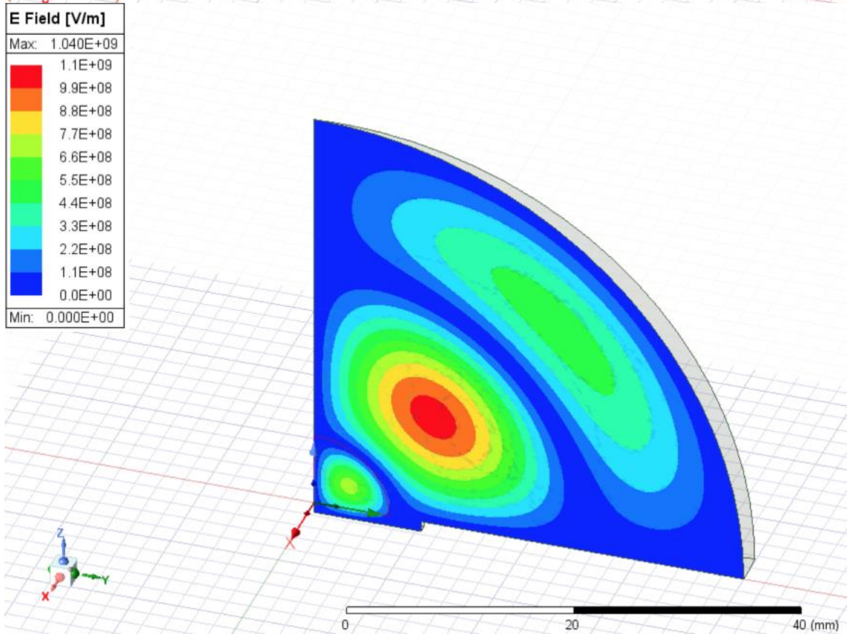
$|\vec{H}|$





$|\vec{E}|$





Dielectric resonant cavity I

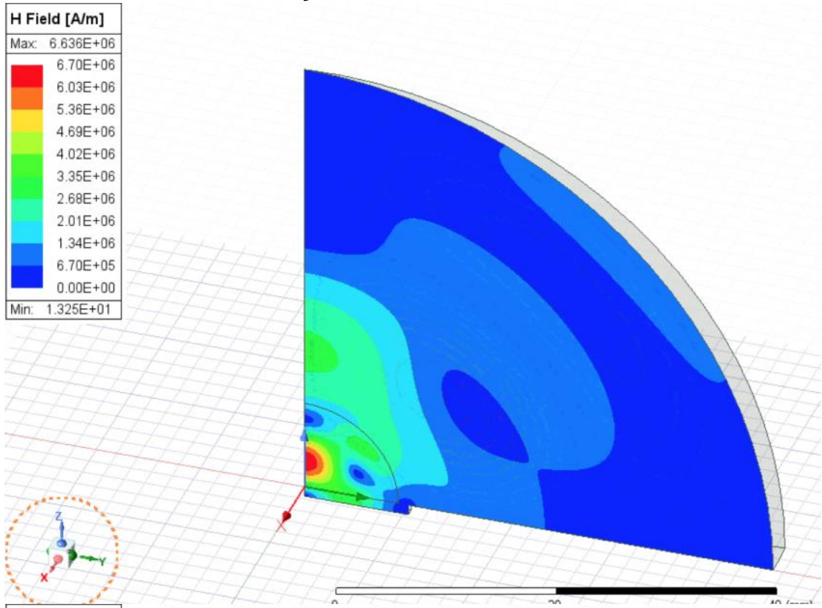
$R = 45$ mm

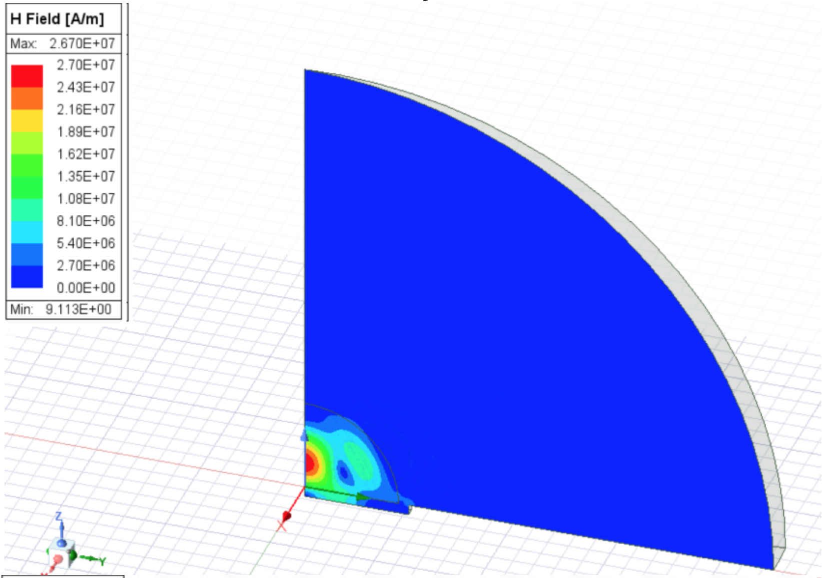
$r = 9$ mm

$f = 9.81$ GHz

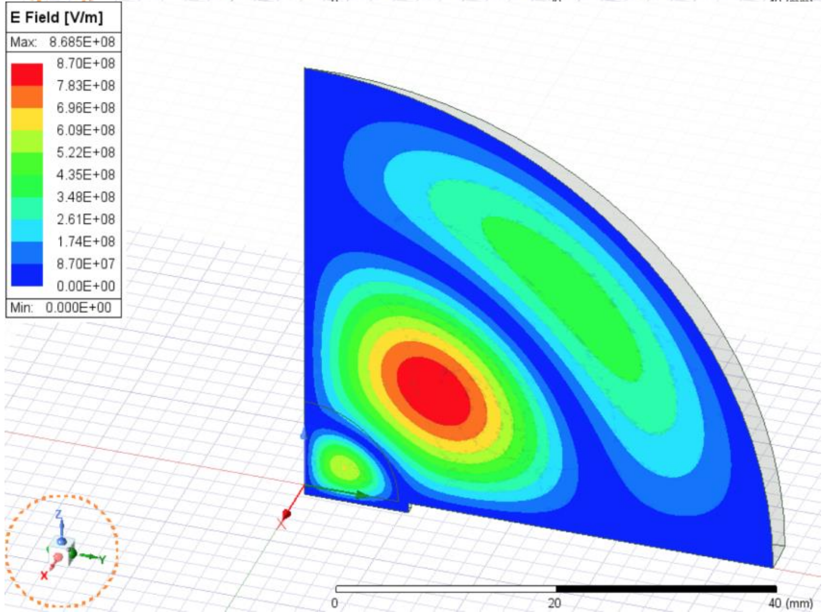
$f = 12.056$ GHz

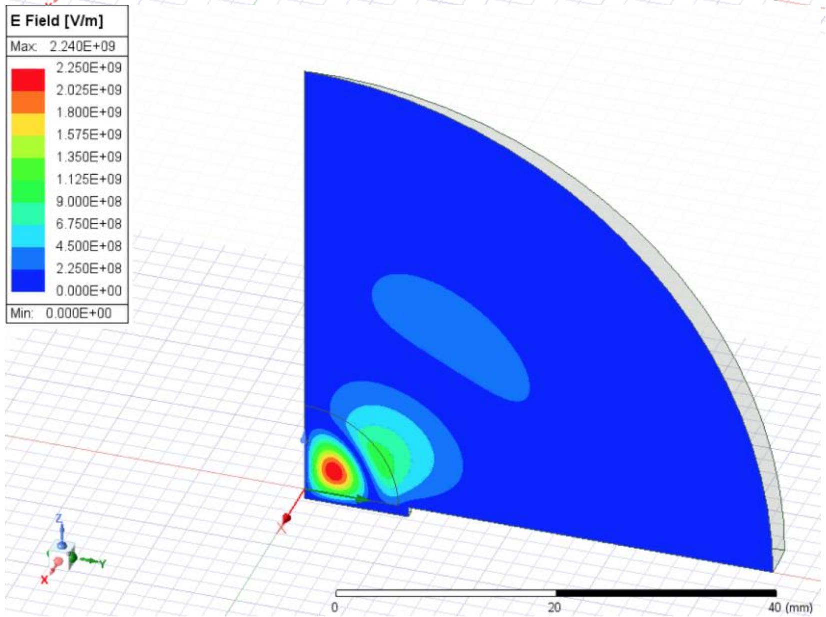
$|\vec{H}|$





$|\vec{E}|$

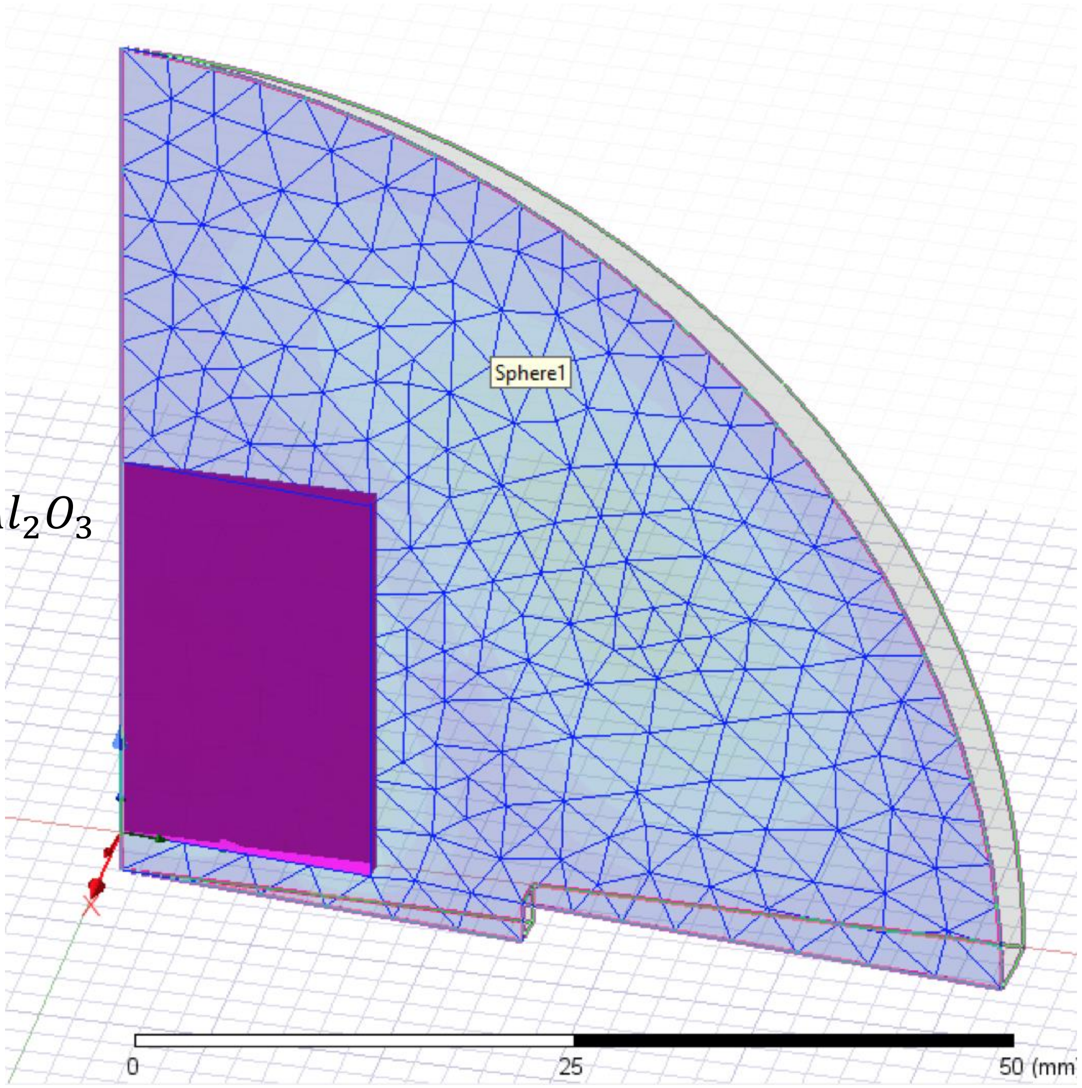


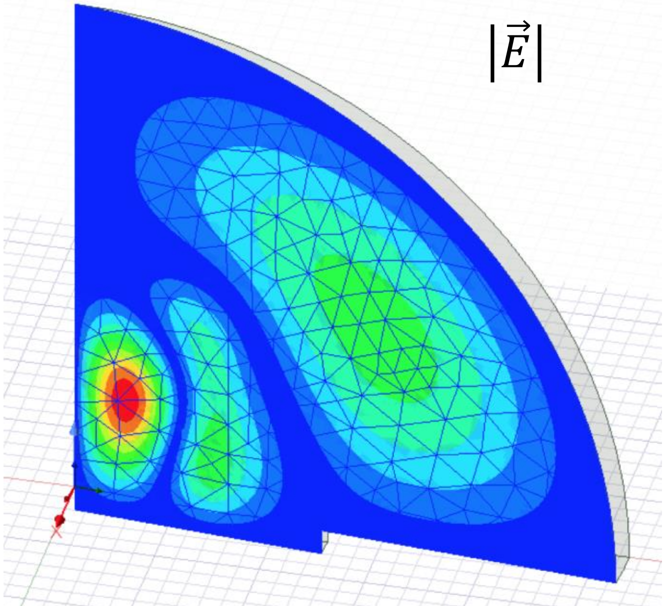


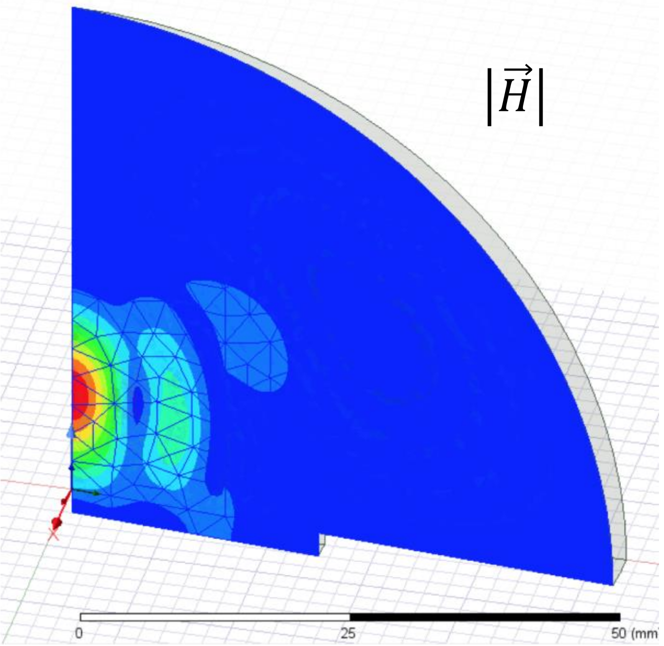
Dielectric resonant cavity I

$f = 6.26137$ GHz

Al_2O_3







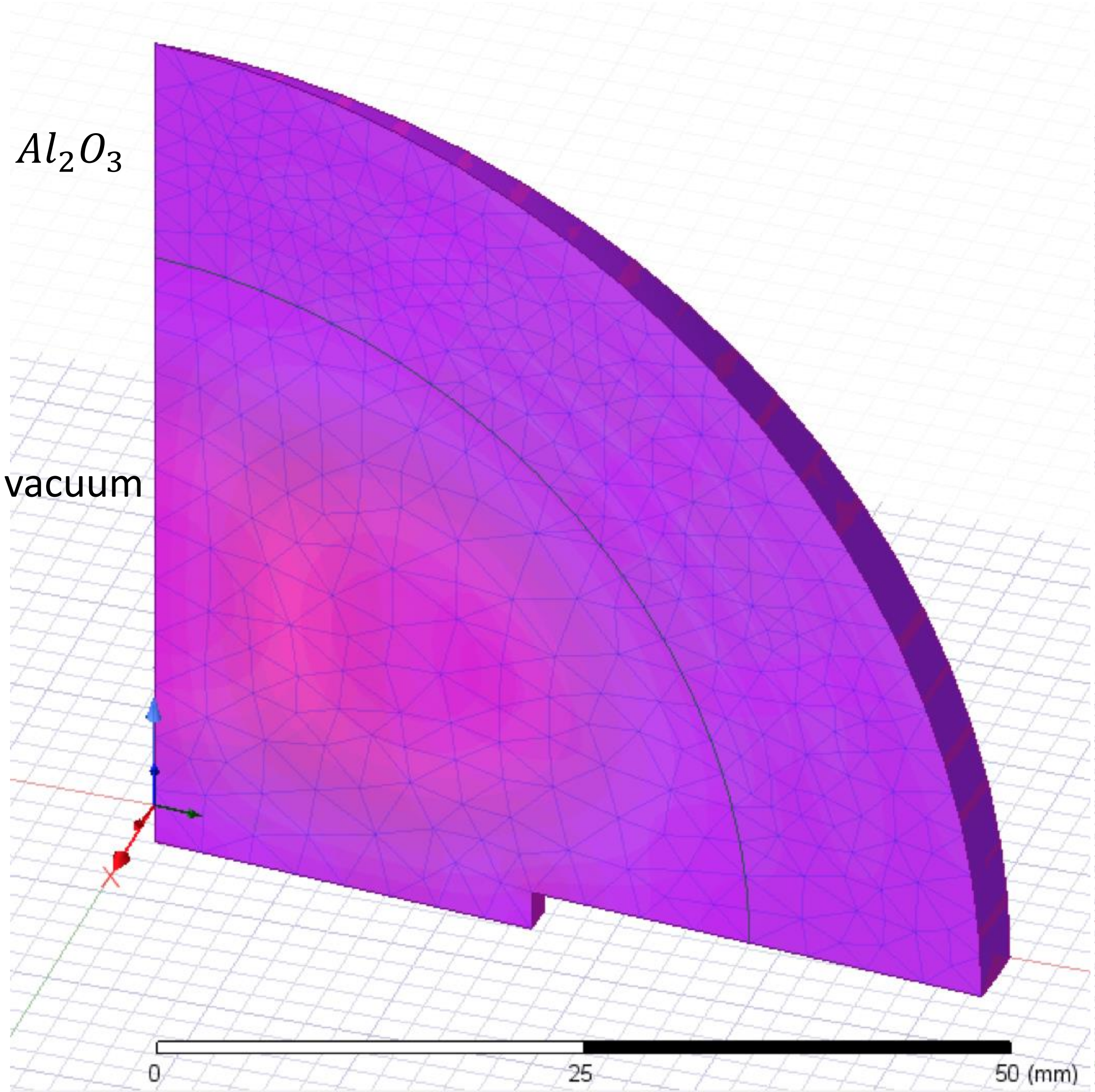
TE_{320}-like mode in a “semispherical” cavity

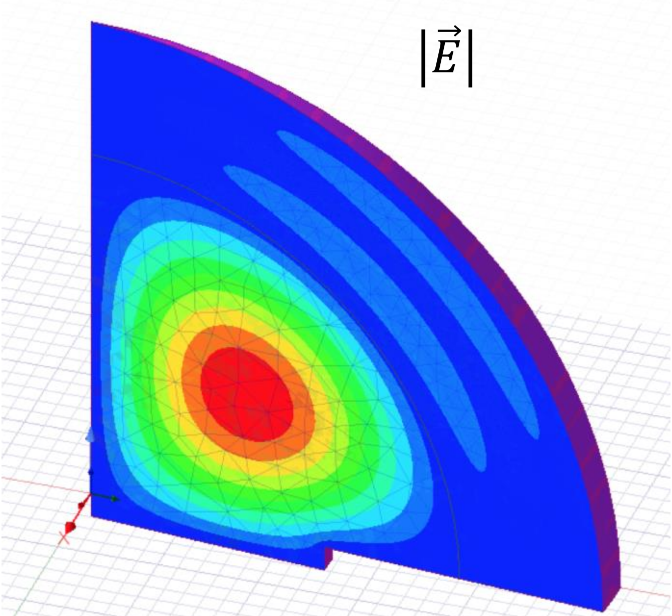
Dielectric resonant cavity II

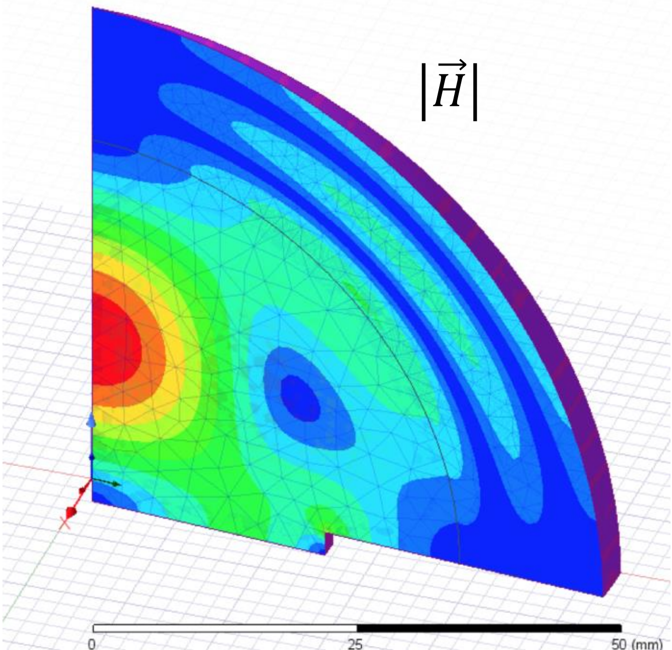
$f = 6.8949$ GHz

Al_2O_3

vacuum







TE_{320}-like mode in a “semispherical” cavity

Dielectric resonant cavity

$$\omega_{m,n} = c\left(\frac{r_{mn}}{R}\right)$$

$$E_\phi = E_0 j_2(k_{42} r) L_2^0(\cos\theta)$$

$f = 13.6795$ GHz

λ_{12}	0.37*R
λ_{22}	0.215*R
λ_{32}	0.208*R
λ_{42}	0.207*R