

SUPERWEAK EXTENSION OF THE STANDARD MODEL

based on

arXiv:1812.11189 (*Symmetry*), 1911.07082 (*PRD*), 2104.11248 (*JCAP*), 2104.14571 (*PRD*), 2105.13360 (*J.Phys.G*), 2204.07100 (*PRD*), 2301.07961 (*JHEP*), 2301.06621 (PRD), 2305.11931 (PRDL), 2402.14786 (submitted) with S. Iwamoto, T.J. Kärkkäinen, I. Nándori, Z. Péli, K. Seller, Zs. Szép

V4 workshop, 12 March, 2024

OUTLINE

- 1. Motivation: status of particle physics
	- **Energy frontier**
	- Cosmology & intensity frontiers
- 2. Superweak U(1)*z* extension of SM (SWSM)
- 3. Neutrino masses and dark matter candidate
- 4. Vacuum stability and scalar sector constraints
- 5. Contribution to M_W and gauge sector constraints
- 6. Conclusions
- *7. Appendix:*
	- *Muon anomalous magnetic moment*
	- Constraints from non-standard interactions

Status of particle physics: energy frontier

Colliders: SM describes final states of particle collisions precisely [\[CMS public](https://cms-results.web.cern.ch/cms-results/public-results/publications/SMP/index.html)]

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[CMS [preprint](http://www.apple.com)] $pp \rightarrow X$ (= new Higgs boson) $\rightarrow e^{\pm} \mu^{\mp}$

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Status of particle physics: cosmic and intensity frontiers

- **Universe at large scale described precisely by cosmological SM:** Λ CDM ($\Omega_{\rm m}$ = 0.3)
- **Neutrino flavours oscillate**
- **Existing baryon asymmetry cannot be explained by CP** asymmetry in SM
- **Inflation of the early, accelerated expansion of the present** Universe [<https://pdg.lbl.gov>]

Established observations require physics beyond SM, but do not suggest rich BSM physics

Phenomenological approach to new physics

Can we explain these observations, but not more, by the same (simple) model?

Before proceeding: a word on the muon anomalous magnetic moment

We are certain that there is new physics beyond the SM

HVP from lattice

 $\mathcal{\mathcal{\mathcal{\mathcal{S}}}}$

 $1a_C$

Final word" on a will tell how BSM should affect the value of \overline{a} "Final word" on *aμ* will tell how BSM should affect the muon g-2

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Resolve discrepancy between theory predictions

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Resolve discrepancy between theory predictions

Until then

everything else is speculation

Muon anomalous magnetic moment: complying with lattice result

- New physics should have a small (smaller then EW) contribution to *aμ*
- **May constrain the available parameter space, but unlikely to** exclude a model compatible with ElectroWeak Precision Observables (EWPOs)

Extension of SM: three alternatives with different strength and weaknesses

- **Effective field theory, such as SMEFT: general but highly** complex (2499 dim 6 operators), focuses on new physics at high scales
- Simplified models, such as dark photon, extended scalar sector or right-handed neutrinos: "easily accessible" phenomenology, but focus on specific aspect of new physics, so cannot explain all BSM phenomena
- UV complete extension with potential of explaining BSM phenomena within a single model such as SuperWeak extension of the Standard Model: SWSM

Particle content of SM

Particle content of SWSM (take-home picture)

Superweak extension of SM (SWSM)

- **Symmetry of the Lagrangian: local** $G = G_{SM} \times U(1)$ _z with $G_{SM} = SU(3)$ _c $\times SU(2)$ _L $\times U(1)$ _Y
- renormalizable gauge theory, including all dim 4 operators allowed by G

Superweak extension of SM (SWSM)

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- renormalizable gauge theory, including all dim 4 operators allowed by G
- *z*-charges fixed by requirement of
	- **gauge and gravity anomaly cancellation and**
	- **gauge invariant Yukawa terms for neutrino mass** generation

Mixing in the neutral gauge sector

^µ fields [19]. The

mixing coupling *gyz* parametrizes the kinetic mixing between the *B^µ* and *B*⁰

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\begin{pmatrix}\nB_{\mu} \\
W_{\mu}^3 \\
B_{\mu}'\n\end{pmatrix} = \begin{pmatrix}\nc_W & -s_W & 0 \\
s_W & c_W & 0 \\
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0 & c_Z & -s_Z \\
0 & s_Z & c_Z\n\end{pmatrix} \begin{pmatrix}\nA_{\mu} \\
Z_{\mu} \\
Z_{\mu}'\n\end{pmatrix} \qquad\nC_X = \cos \theta_X
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where θ_W is the weak mixing angle & θ_Z is the $Z-Z'$ mixing, implicitly: $\tan(2\theta_Z) = -2\kappa \left(\left(1 - \kappa^2 - \tau^2 \right) \right)$, with κ and τ effective couplings, functions of the Lagrangian couplings s*^W* = *gy r* Lagrangian coapinigs

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The expressions for the neutral gauge boson masses are somewhat cumbersome, but exists a nice, compact generalization of the SM mass-relation formula: M_W^2 c_W^2 $= c_Z^2 M_Z^2 + s_Z^2 M_Z^2$ $\left(M_W\right)$ 1 2 *g*L*v* $\frac{1}{2}$ so a particular contraction of the Medicine of Belling and *gauge wesen midded and sering*. $\boldsymbol{\ell}$

[Zoltán Péli and ZT, arXiv: [2305.11931\]](https://link.aps.org/doi/10.1103/PhysRevD.108.L031704)

Scalars in the SWSM \mathbb{Z} $\frac{1}{2}$ $\overline{\mathbf{d}}$ \mathbf{i} <u>ار</u> p Scalars in the SWSM 3 + 1 - 1 + 1 formations. The gauge invariant Lagrangian of the scalar fields is

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⇤

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in addition to the usual BEH-field that is an *SU*(2)L-doublet

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⇤

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$$
V(\phi, \chi) = V_0 - \mu_{\phi}^2 |\phi|^2 - \mu_{\chi}^2 |\chi|^2 + (|\phi|^2, |\chi|^2) \left(\frac{\lambda_{\phi}}{\frac{\lambda}{2}} \frac{\frac{\lambda}{2}}{\lambda_{\chi}}\right) \left(|\phi|^2\right)
$$

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, (2.12)

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Mixing in the scalar sector

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5 new parameters:

• in gauge sector: $\{g_z \text{ and } g_{yz}\}$ or $\{\kappa \text{ and } \tau\}$ or $\{\theta_Z \text{ and } M_{Z'}\}$ • in scalar sector: $\{\mu_{\chi}^2, \lambda_{\chi} \text{ and } \lambda\}$ or $\{W, \lambda_{\chi} \text{ and } \lambda\}$ or $\{M_{S}, \theta_{S} \text{ and } \lambda\}$

After SSB neutrino mass terms appear *<u>El 33D H</u>* <mark>ass term</mark> *w* + *s*⁰ + i *v d i* $\frac{1}{2}$

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M*^D* M*^N*

• Quantum corrections to active neutrinos are not dangerous <u>[Iwamoto et al, arXiv[:2104.14571\]](https://arxiv.org/abs/2104.14571)</u> @ 15711 A *.* (II.34) **ctr** active neutrinos are not dangerous n **a** *re* not dangerous arXiv:<u>2104.14571]</u>

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Expected consequences (take-home messages)

■ Dirac and Majorana neutrino mass terms are generated by the SSB of the scalar fields, providing the origin of neutrino masses and oscillations [Iwamoto, Kärkäinnen, Péli, ZT, arXiv[:2104.14571](https://inspirehep.net/literature/1861571); Kärkkäinen and ZT, arXiv:[2105.13360\]](https://arxiv.org/abs/2105.13360)

The lightest new particle is a natural and viable candidate for WIMP dark
[Seller, Iwamoto and ZT, arXiv:2104.11248] [Seller, Iwamoto and ZT, arXiv:[2104.11248\]](https://arxiv.org/abs/2104.11248)

Diagonalization of neutrino mass terms leads to the PMNS matrix, which in turn can be th[e source o](https://inspirehep.net/files/df159f25b65e70e2344eb1acc92c6048)f lepto-baryogenesis
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■ The second scalar together with the established BEH field can stabilize the vacuum and be related to the accelerated expansion now and inflation in the early universe [Péli, Nándori and ZT, arXiv: 1911.07082; Péli and ZT, arXiv: [2204.07100](https://inspirehep.net/literature/2067427)]

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Dark matter candidate

- **DM** exists, but known evidence is based solely on the gravitational effect of the dark matter on the luminous astronomical objects and on the Hubble-expansion of the Universe
- **Example 18 Assume that the DM has particle origin**

Dark matter candidate

- **DM** exists, but known evidence is based solely on the gravitational effect of the dark matter on the luminous astronomical objects and on the Hubble-expansion of the Universe
- **E** Assume that the DM has particle origin
- **Only chance to observe such a particle if it interacts** with the SM particles, which needs a portal In the superweak model the vector boson portal *Z'* with the lightest sterile neutrino ν_4 as dark matter candidate is a natural scenario (Higgs portal exists, but negligible)

Parameter space for the freeze-out scenario of dark matter production in the SWSM

SM+SM \rightarrow Z' \rightarrow DM+DM can dominate the integral in the rate It is essential for the SWSM DM candidate that the resonance in
Experimental constraints

- Anomalous magnetic moment of electron and muon
	- Z' couples to leptons modifying the magnetic moment
	- Constraints on $(g 2)$ translate to upper bounds on the coupling $g_z(M_{Z^\prime})$
- NA64 search for missing energy events
	- Strict upper bounds on $g_z(M_{Z^\prime})$ for any U(1) extension (dark photons)
- Supernova constraints based on SN1987A
	- Constraints are based on comparing observed and calculated neutrino fluxes
- Big Bang Nucleosynthesis provides constraints on new particles
	- New particles should have negligible effects during BBN
	- **Meson production can be dangerous close to BBN**
- Further constraints are due to CMB, solar cooling, beam dump experiments etc.

Cosmological constraints on the freeze-out scenario of dark matter production in the SWSM

25

Expected consequences (take-home messages)

- Dirac and Majorana neutrino mass terms are generated by the SSB of the scalar fields, providing the origin of neutrino masses and oscillations [Iwamoto, Kärkäinnen, Péli, ZT, arXiv[:2104.14571](https://inspirehep.net/literature/1861571); Kärkkäinen and ZT, arXiv:[2105.13360\]](https://arxiv.org/abs/2105.13360)
- The lightest new particle is a natural and viable candidate for WIMP dark
[Seller, Iwamoto and ZT, arXiv:2104.11248] [Seller, Iwamoto and ZT, arXiv:[2104.11248\]](https://arxiv.org/abs/2104.11248)
- Diagonalization of neutrino mass terms leads to the PMNS matrix, which \Box in turn can be th[e source o](https://inspirehep.net/files/df159f25b65e70e2344eb1acc92c6048)f lepto-baryogenesis [Seller, Szép, ZT, arXiv:[2301.07961](https://inspirehep.net/files/df159f25b65e70e2344eb1acc92c6048) and under investigation]
- The second scalar together with the established BEH field can stabilize the vacuum and be related to the accelerated expansion now and inflation in the early universe [Péli, Nándori and ZT, arXiv: 1911.07082; Péli and ZT, arXiv: [2204.07100](https://inspirehep.net/literature/2067427)]

Prerequisite: Phase-transitions in the SWSM

[Seller, Szép, ZT, arXiv[:2301.07961](https://inspirehep.net/files/df159f25b65e70e2344eb1acc92c6048)]

Prerequisite: phase-transition temperatures in the SWSM

U(1)_z is broken earlier than SU(2)_LxU(1)_Y

[Seller, Szép, ZT, arXiv[:2301.07961](https://inspirehep.net/files/df159f25b65e70e2344eb1acc92c6048)]

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SWSM has the potential of explaining all known results beyond the SM

Main questions

Is there a non-empty region of the parameter space where all these promises are fulfilled?

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Can we predict any new phenomenon observable by present or future experiments?

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Present focus:

Is there a non-empty region of the parameter space where all these promises are fulfilled?

Can we predict any new phenomenon observable by present or future experiments?

Important test

Once the allowed region of the parameter space for fulfilling the expectations is understood

the observation of the *Z'* or *S* in the allowed region

Experimental constraints in the scalar sector from direct searches and M_W

 $M_s > M_h$: [Zoltán Péli and ZT, arXiv: <u>2204.07100</u>] $y_x = 0$: scalar sector decouples $0.8₁$ \therefore $\delta M_W = -15$ MeV ... $\delta M_W = -15$ MeV $y_x(M_t) = 0.$ $y_x(M_t) = 0.$ 0.6 0.6 $\lambda(M_{\rm t})=0.2$ $\lambda(M_{\rm t})=0.1$ $|\sin(\theta_S)|$ $\sin(\theta_S)|$ 0.4 0.4 0.2 0.2 Ω Ω 800 400 200 400 600 1000 200 600 800 1000 M_s [GeV] M_s [GeV] 35 Experimental constraints in the scalar sector from direct searches and M_W

 $M_s > M_h$: [Zoltán Péli and ZT, arXiv: <u>2204.07100</u>]

Mw is measured and computed precisely (with per myriad precision)

[<https://pdg.lbl.gov>] 37

Prediction of M_W in the SWSM

Can be determined from the decay width of the muon:

$$
M_W^2 = \frac{\cos^2 \theta_Z M_Z^2 + \sin^2 \theta_Z M_Z^2}{2} \left[1 + \sqrt{1 - \frac{4\pi \alpha / (\sqrt{2}G_F)}{\cos^2 \theta_Z M_Z^2 + \sin^2 \theta_Z M_Z^2}} \frac{1}{1 - \Delta r_{SM} - (\Delta r_{BSM}^{(1)} + \Delta r_{BSM}^{(2)})} \right]
$$

- Valid in MS п
- θ ^{*Z*} is the $Z Z'$ mixing angle \Box
- Δr_{SM} collects the SM quantum corrections (known completely at two loops and partially at three loops)
- $\Delta r_{BSM}^{(1)}$ collects the formally SM quantum corrections but with BSM loops
- $\Delta r_{BSM}^{(2)}$ collects the BSM corrections to $M_{Z'}$ and θ_Z π

Prediction of M_W in the SWSM

Case (i) full one-loop corrections Case (ii) corrections without Δ $r_{BSM}^{(2)}$

Prediction of M_W in the SWSM

Case (i) full one-loop corrections Case (ii) corrections without Δ $r_{BSM}^{(2)}$

Experimental constraints in the gauge sector from direct searches and EWPOs

Gauge sector parameters: g_z , g_{yz} (= ϵg_y), tan β , z_{ϕ} , z_N Not all independent: exclusion bounds depend on either \Box $(\sin \theta_Z, M_{Z'}, x)$ or $(g_z z_N, M_{Z'}, x)$

where

$$
x = \frac{z_{\phi} - \frac{1}{2} \frac{g_{yz}}{g_z}}{z_N}
$$

and z_N is the z charge of the right-handed neutrino

General U(1)_z anomaly free charge assignment sixth column gives a particular realization of the *U*(1)*^Z* charges, motivated below, and the

and scalar fields of the complete model. The charges *y^j* denote the eigenvalue of *Y/*2, with

Experimental constraints in the gauge sector from direct searches and EWPOs

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- Most stringent limits emerge in direct searches \Box
	- for small masses ($\xi = M_{Z^\prime}/M_Z \ll 1$): from NA64 search for dark photon
	- *for large masses (ξ >> 1): from LHC search for <i>Z'* п
	- difficult to distinguish from *Z* for intermediate masses \Box best limits from LEP (not discussed here)

Experimental constraints in the gauge sector from direct searches and EWPOs: SWSM region

Experimental constraints in the gauge sector from direct searches and EWPOs

Conclusions

- Established observations require physics beyond SM, but do not suggest rich n BSM physics
- U(1)*z* superweak extension has the potential of explaining all known results beyond the SM
- Neutrino masses are generated by SSB at tree level
- One-loop corrections to the tree-level neutrino mass matrix computed and n found to be small (below 1%o) in the parameter space relevant in the SWSM
- Lightest sterile neutrino is a candidate DM particle in the \blacksquare [10,50] MeV mass range for freeze-out mechanism with resonant enhancement **→** predicts an approximate mass relation between vector boson and lightest sterile neutrino
- In the scalar sector we find non-empty parameter space for $M_s > M_h$
- Contributions to EWPOs (e.g. M_W , lepton g-2) are negligible in the superweak п region and a systematic exploration of the parameter space is ongoing

the end

Appendix

Status of the muon anomalous magnetic **moment: experiment** tiplied by things

Run-1/2/3 3.70730082(75)

The muon g-2 has been a smoking gun for new physics for many years, more recently:

as described in the text.

^g Also at Shanghai Key Laboratory for Particle Physics

Status of the muon anomalous magnetic moment: experiment

- **The muon g-2 has been a smoking gun for new** physics for many years
- **The most precise experimental value is from FNAL** (2023) : **E...** equivalent to a bathroom scale sensitive to a single eyelash: $a_\mu =$ *g* − 2 2 $= 116 592 055(24) \cdot 10^{-11}$ (0.20 ppm) Experimental result Newly announced result at Fermilab

Status of the muon anomalous magnetic moment: experiment vs. theory

Status of the muon anomalous magnetic moment: experiment vs. theory

[BMW compilation]

Status of the muon anomalous magnetic moment: theory with R-ratio

The muon g-2 has been a smoking gun for new physics for many years, but tension already in earlier

data used for theory prediction:

 σ (e⁺e[−] → π ⁺ π [−]) cross section in this energy range gives more than 50% to total HVP contribution to *aμ*

Status of the muon anomalous magnetic moment: theory with R-ratio

New CMD3 data show a ~15 unit increase in central value and 4.4σ tension with old average:

 σ (e⁺e[−] → π ⁺ π [−]) cross section in this energy range gives more than 50% to total HVP contribution to *aμ*

Status of the muon anomalous magnetic moment: window observable

- restrict correlation window to [0.4,1.0] fm:
	- two orders of magnitude easier (less CPU, less manpower needed) lattice vs. R-ratio: 4.9σ tension: Tension in the window of t

Non-standard interactions and the SWSM [Timo J. Kärkäinen and ZT, arXiv: [2301.06621\]](https://doi.org/10.1103/PhysRevD.107.115020)

$$
\mathcal{O}_{6a} = \frac{C_{6a}}{\Lambda^2} (\overline{L} \gamma^{\mu} P_{L} L) (\overline{f} \gamma_{\mu} P_{X} f)
$$

where Λ is the scale of new physics, can be as low as few MeV, which can be probed in

Coherent Elastic Neutrino-Nucleus Scattering (CEνNS)

56 Standard parametrization of NSI: where $\varepsilon_{\ell,\ell'}^{J,\Lambda}$, $\alpha + \frac{1}{2}$ if $q^2 \gg M^2$, "light NSI" , "heavy NSI", $\mathscr{L}_{\text{NSI}} = -2\sqrt{2}G_F$ *f*,*X*=±,*ℓ*,*ℓ*′ *εf*,*^X* $\frac{f_{\mathcal{X}}f_{\mathcal{X}}(\bar{\nu}_{\ell}\gamma^{\mu}P_{L}\nu_{\ell^{\prime}})(\bar{f}\gamma_{\mu}P_{X}f)}{f}$ *εf*,*^X ℓ*,*ℓ*′ α + 1 *q*2 if $q^2 \gg M^2$ *εf*,*^X ℓ*,*ℓ*′ $\alpha - \frac{1}{\sqrt{2}}$ *M*2 if $q^2 \ll M^2$ for a mediator of mass *M*

Status of the muon anomalous magnetic moment: lattice vs. R-ratio

Lattice: $a_{\mu}^{\rm HVP@LO} = 707.5(2.3)_{\rm stat}(5.0)_{\rm sys}[5.5]_{\rm tot}$ HVP from lattice

~15 units above the R-ratio white paper value (a 2.1σ tension) μ
The unite above the P ratio white paper value (a 2.1

Non-standard interactions and the SWSM

assume $M = 50$ MeV, which is

- light in CHARM or NuTEV $q^2 = O((20 \text{ GeV})^2)$
- heavy in neutrino oscillation experiments $q^2 \approx 0$ • but $q^2 \approx M^2$ in CE_VNS

We can still apply the NSI formalism using the full propagator with q^2 being the characteristic momentum transfer squared

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- We can still apply the NSI formalism using the full propagator with q^2 being the characteristic momentum transfer squared
- Can be used to [Timo J. Kärkäinen and ZT, arXiv: [2301.06621\]](https://doi.org/10.1103/PhysRevD.107.115020)
	- Constrain the parameter space of SWSM
	- Predict relations between NSI couplings assuming SWSM \Box
Non-standard interactions and the SWSM

Ligh anargy $\overline{ }$ $\begin{cases} \varepsilon \\ \varepsilon \end{cases}$ ϵ_{ee}^m ϵ_{e}^m ϵ_{e}^l $\epsilon_{e\mu}^{m*}$ $\epsilon_{\mu\mu}^{m}$ ϵ_{μ}^{r} $\mathcal{E}_{\mathbf{r}}^{\mathbf{m}\star}$ $\mathcal{E}_{\mathbf{\mu}\tau}^{\mathbf{m}\star}$ $\mathcal{E}_{\tau}^{\mathbf{r}}$ $\overline{}$ r
n
n
n $\overline{}$ $\begin{array}{c} \hline \end{array}$ $\varepsilon_{\bm{e}}$ 0 0 $0 \quad \varepsilon_{\mu} \quad 0$ 0 0 ε_{τ} $\overline{}$ \int $\overline{}$ $\overline{}$ *Á* 0 0 0 *Á* 0 0 0 *Á* $\overline{}$ \int $\mu-\tau$ symmetry Flavour-conserving Flavour-universal **CLFV** decays **V** V **No** ν oscillation $\sqrt{2\pi}$ $CE\nu$ NS $\sqrt{ }$ ν scattering \vert maybe maybe maybe maybe **High-energy theory enforces texture for NSI matrix:** <u>SWSM</u> \overline{a} *¸¸* ⁼ ^v ² $\varepsilon_{\ell\ell}^{\bm{m}} = \varepsilon_{\ell\ell}^{\bm{e}} + 2\varepsilon_{\ell\ell}^{\bm{u}} + \varepsilon_{\ell\ell}^{\bm{d}}$ $\overbrace{\hspace{2.5cm}}=0$ $+$ N_n N_e $(\varepsilon^u_{\ell\ell} + 2\varepsilon^d_{\ell\ell})$ e≠ and p contributions vanish due to (y*,* z) charge Sum over the fermions f = e*,* u*,* d and chiralities X = L*,* R. 4 in operavithe any opforces to vture for NSI In-energy theory emorces texture for NSI matrix. $\overline{}$ $\sqrt{ }$ $\overline{\mathcal{C}}$ ε_{ee}^m $\varepsilon_{e\mu}^m$ $\varepsilon_{e\tau}^m$ $\begin{array}{ccc} \varepsilon^{\textbf{\textit{m}}*} & \varepsilon^{\textbf{\textit{m}}}_{\mu\mu} & \varepsilon^{\textbf{\textit{m}}}_{\mu\tau} \end{array}$ $\epsilon_{e\tau}^{\dot{m}*}$ $\epsilon_{\mu\tau}^{\dot{m}*}$ $\epsilon_{\tau\tau}^{\dot{m}}$ $\sum_{i=1}^{n}$ $\overline{}$ $\begin{bmatrix} 0 & \varepsilon_\mu & 0 \end{bmatrix}$ *(ε* 0 0) | *d* ϵ **0** *<i>d <i>e f l e f i e f i i f i f i f* neutrino-nucleon scattering (CLFV decays⁾ ν oscillation $CE\nu$ NS - - - - SM, tree $\frac{1}{\sqrt{2}}$ gpZF^Z (*|***q***|* ²)+gnNFN(*|***q***[|]* ν scattering maybe maybe maybe maybe

 S_{max} limits on NICI constrain the parameters of Existing limits on NSI constrain the parameters of the high-energy theory f ing limite on NCL constrain the parameters of **8 / 19**

Non-standard interactions and the SWSM: preferred regions of the parameters

Non-standard interactions and the SWSM: preferred regions of the parameters

Particle model

New fields: 3 right-handed neutrinos $\nu_{_{\mathbf{R}}}^{J}$, a new scalar χ , and new U(1) $_{\mathsf{z}}$ gauge boson $\nu^{f}_{\rm R'}$ a new scalar χ *B*′ \mathbf{N} consider the usual three fermion \mathbf{r}_R , a new search λ , and new extraction on \mathbf{r}_R

fermion fields (Weyl spinors): right-handed Dirac neutrino in each family of the new integration of the new integration of the notation of the n
<u>Family of notation</u> in the notation of the notation of the notation of the new seconduction of the notation

$$
\psi_{q,1}^f = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_{L} \qquad \psi_{q,2}^f = U_R^f, \qquad \psi_{q,3}^f = D_R^f
$$

$$
\psi_{l,1}^f = \begin{pmatrix} \nu^f \\ \ell^f \end{pmatrix}_{L} \qquad \psi_{l,2}^f = \nu_R^f, \qquad \psi_{l,3}^f = \ell_R^f
$$

Particle model \Box

comprehensive. We also recall some of the conventions that are different in SARAH and the conventions that are
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$$

with extended U(1) part of the covariant derivative: **for the and field included by and for the covariant denvalue.** In the left and R denote the left and R denote the left and R denote the left and R denveloped and R denveloped and R denveloped and R denote the left and R d the co \overline{v} \overline{a} *F* \overline{b} *R* \overline{b} 4 *Fabive:*

$$
\mathcal{D}_{\mu}^{\text{U}(1)} = -\mathrm{i}(y g_y B_{\mu} + z g_z B'_{\mu})
$$

⇣

y z⌘

*g*ˆ*yy g*ˆ*yz*

*B*ˆ*µ*

Particle model one-loop corrections to neutrino masses, we recall the details relevant to such computations, with Feynman rules in the **R**ules of R \Box

comprehensive. We also recall some of the conventions that are different in SARAH and the conventions that are
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New fields: 3 right-handed neutrinos $\nu_{_{\mathbf{R}}}^{J}$, a new scalar χ , and new U(1) $_{\mathsf{z}}$ gauge boson $\nu^{f}_{\rm R'}$ a new scalar χ *B*′ \mathbf{N} consider the usual three fermion \mathbf{r}_R , a new search λ , and new extraction on \mathbf{r}_R inew fields: 3 right-handed heutrinos $\nu_{\rm R}^{\rm g}$, a new scalar χ , and new U(T) $_{\rm Z}$ may compute those more discussed in $R^{\rm g}$

fermion fields (Weyl spinors): right-handed Dirac neutrino in each farmino fields (We vi soinnes).

$$
\psi_{q,1}^f = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_{\mathcal{L}} \qquad \psi_{q,2}^f = U^f_{\mathcal{R}}, \qquad \psi_{q,3}^f = D^f_{\mathcal{R}}
$$

$$
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$$

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$$
\mathcal{D}_{\mu}^{\text{U}(1)}=-\text{i}(yg_yB_{\mu}+zg_zB'_{\mu})
$$

= i(*ygyB^µ* + *zgzB*⁰

the new U(1) kinetic term includes kinetic mixing: **hypercharge U(1)** and the new U(1) \blacksquare the new U(1) kinetic term includes kinetic mixing: where *B^µ* is the U(1)*^y* gauge field. However, equivalently, we can choose the basis—the con $v_{\rm eff}$ in SARAH—in which the gauge-field strengths do not mix, which the coupling are given by α

*^D*U(1)

$$
\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F^{\prime \mu\nu} F^{\prime}_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F^{\prime}_{\mu\nu}
$$
⁶²

⇣

µ)

*g*ˆ*yy g*ˆ*yz*

*B*ˆ*µ*

y z⌘

Kinetic mixing invariant, kinetic mixing invariant, kinetic mixing is allowed between the gauge fields belonging to the gauge field

*g*ˆ*yy g*ˆ*yz*

kinetic mixing:

$$
\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F^{\prime \mu\nu} F^{\prime}_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F^{\prime}_{\mu\nu}
$$

g^y ⌘*g*⁰

z

cosa cosa cosa cos

Kinetic mixing invariant, kinetic mixing invariant, kinetic mixing is allowed between the gauge fields belonging to the gauge field new scalar , and the U(1)*^z* gauge boson *B*⁰ groups are gauge in the gauge fields between the gauge fields belonging to the gauge fields belonging

*g*ˆ*yy g*ˆ*yz*

kinetic mixing:

$$
\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F^{\prime \mu\nu} F^{\prime}_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F^{\prime}_{\mu\nu}
$$

g^y ⌘*g*⁰

0

z

0

1

cosa cosa cosa cos

1

0

The particle content of the standard model is extended by $3 R,$ right-handed by $3 R,$ $3 R,$ $4 R,$ and $5 R,$ and $6 R,$ and $7 R,$ and

hypercharge U(1)*^y* and the new U(1)*^z* gauge symmetries, whose strength is measured by ✏ in

covariant derivative:

$$
\mathcal{D}_{\mu}^{\mathrm{U}(1)}=-\mathrm{i}(yg_yB_{\mu}+zg_zB'_{\mu})
$$

Kinetic mixing invariant, kinetic mixing invariant, kinetic mixing is allowed between the gauge fields belonging to the gauge field new scalar , and the U(1)*^z* gauge boson *B*⁰ groups are gauge in the gauge fields between the gauge fields belonging to the gauge fields belonging $\overline{1}$ *X* $\overline{1}$ *R*

kinetic mixing:

c mixing:
$$
\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F^{\prime \mu\nu} F^{\prime}_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F^{\prime}_{\mu\nu}
$$

g^y ⌘*g*⁰

0

z

0

4

1

cosa cosa cosa cos

1

0

covariant derivative:
 $\mathcal{D}^{U(1)}$ and \mathcal{D}' are P' and P'

$$
\mathcal{D}_{\mu}^{\mathrm{U}(1)}=-\mathrm{i}(yg_yB_{\mu}+zg_zB'_{\mu})
$$

where **B***I***₂** is the U(1)^{*y*} can choose basis s. t.: by a 2 coupling matrix in the covariant matrix in the covariant of the covariant of the covariant of the covar
Line of the covariant density of the covariant derivative contribution of the covariant of the covariant of the

*g*ˆ*yy g*ˆ*yz*

$$
D_\mu^{{\rm U}(1)} = -{\rm i}\left(y\left| z\right\rangle \left(\begin{matrix} \hat{g}_{yy} \;\; \hat{g}_{yz} \\ \hat{g}_{zy} \;\; \hat{g}_{zz} \end{matrix} \right) \left(\begin{matrix} \hat{B}_\mu \\ \hat{B}_\mu' \end{matrix} \right)
$$

The particle content of the standard model is extended by $3 R,$ right-handed by $3 R,$ $3 R,$ $4 R,$ and $5 R,$ and $6 R,$ and $7 R,$ and

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F ^µ⌫*F*⁰

kinetic mixing: *L* $\ddot{\bullet}$ *^F ^µ*⌫*Fµ*⌫ ¹ *^D*U(1) = i(*ygyB^µ* + *zgzB*⁰

✏*gy/gz*. In this paper, it will be convenient to use the kinetic mixing representation defined by

new scalar , and the U(1)*^z* gauge boson *B*⁰

c mixing:
$$
\mathcal{L} \supset -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F^{\prime \mu\nu} F^{\prime}_{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F^{\prime}_{\mu\nu}
$$

. As the field strength tensors of the U(1) gauge

g^y ⌘*g*⁰

z

0

4

1

cosa cosa cosa cos

(II.1).

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 \blacksquare covariant derivative:
 $\mathcal{D}^{U(1)}$ and \mathcal{D}'

$$
\mathcal{D}_{\mu}^{\text{U}(1)} = -\mathrm{i}(y g_y B_{\mu} + z g_z B'_{\mu})
$$

where **B***I***₂** is the U(1)^{*y*} can choose basis s. t.: **Example 2** or equivalently can choose basis s. t.: by a 2 _m 2 _{cou}pling matrix in the covariant derivative matrix in the covariant

4

*F*0*µ*⌫*F*⁰

$$
D_\mu^{{\rm U}(1)} = -{\rm i}\left(y\left| z\right\rangle \left(\begin{matrix} \hat{g}_{yy} \;\; \hat{g}_{yz} \\ \hat{g}_{zy} \;\; \hat{g}_{zz} \end{matrix} \right) \left(\begin{matrix} \hat{B}_\mu \\ \hat{B}_\mu' \end{matrix} \right)
$$

The particle content of the standard model is extended by $3 R,$ right-handed by $3 R,$ $3 R,$ $4 R,$ and $5 R,$ and $6 R,$ and $7 R,$ and

hypercharge U(1)*^y* and the new U(1)*^z* gauge symmetries, whose strength is measured by ✏ in

 and can parametrize the coupling matrix s.t.: ⇣ *y z*⌘ a^{\dagger} *g*ˆ*yy g*ˆ*yz* $\ddot{}$ an parametrize the coupling matrix s.t.: $\hspace{1.5cm}$ where *y* and *z* are the U(1)*^y* and U(1)*^z* charges. We can parametrize the coupling matrix as

*g*ˆ*yy g*ˆ*yz*

4

$$
\hat{\mathbf{g}} = \begin{pmatrix} \hat{g}_{yy} & \hat{g}_{yz} \\ \hat{g}_{zy} & \hat{g}_{zz} \end{pmatrix} = \begin{pmatrix} g_y & -\eta g_z' \\ 0 & g_z' \end{pmatrix} \begin{pmatrix} \cos \epsilon' & \sin \epsilon' \\ -\sin \epsilon' & \cos \epsilon' \end{pmatrix} \text{ with } \frac{g_z'}{\eta} = \frac{g_z}{\sqrt{1 - \epsilon^2}} \\ \eta = \frac{\epsilon g_y}{g_z}.
$$

c ovariant derivative: *D*^{neut.} ⊃ − i($Q_A A_\mu + Q_Z Z_\mu + Q_Z Z_\mu'$)

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Exercise couplings:

$$
\bullet \ \mathcal{Q}_A = (T_3 + y) | e | \equiv \mathcal{Q}_A^{\rm SM}
$$

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Exercise couplings:

 $\mathcal{Q}_A = (T_3 + y) |e| \equiv \mathcal{Q}_A^{\text{SM}}$ $\mathcal{Q}_Z = (T_3 \cos^2 \theta_W - y \sin^2 \theta_W) g_{Z^0} \cos \theta_Z - (z - \eta y) g_z \sin \theta_Z$ $\mathcal{Q}_Z^{\textsf{SM}}$

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Exercise couplings:

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c ovariant derivative: *D*^{neut.} ⊃ − i($Q_A A_\mu + Q_Z Z_\mu + Q_Z Z_\mu'$)

Exercise couplings:

- $\mathcal{Q}_A = (T_3 + y) |e| \equiv \mathcal{Q}_A^{\text{SM}}$ $\mathcal{Q}_Z = (T_3 \cos^2 \theta_W - y \sin^2 \theta_W) g_{Z^0} \cos \theta_Z - (z - \eta y) g_z \sin \theta_Z$ $\mathcal{Q}_Z^{\textsf{SM}}$ Φ $\mathcal{Q}_{Z'}$ = ($T_3 \cos^2 \theta_W - y \sin^2 \theta_W$)*g_z*⁰ sin $\theta_Z + (z - \eta y)g_z \cos \theta_Z$
- Z Z' mixing is small, the weak neutral current is only modified at order $O(g_z^2/g_{Z^0}^2)$

Gauge coupling, $g_{\overline{z}}$:

in order to avoid SM precision constraints $O(g_z/g_{Z0}) \ll 1$

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- Vacuum expectation value of *χ* singlet, *w*:
	- in the gauge sector rather use the mass of *Z'* & assume that $M_{Z'} \ll M_{Z'}$

- Gauge coupling, $g_{\overline{z}}$:
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*g*3

Vacuum expectation value of *χ* singlet, *w*:

in the gauge sector rather use the mass of *Z'* & assume that $M_{Z'} \ll M_Z$ $Z - Z'$ mixing angle, $\theta_{\vec{Z}}$: $\tan(2\theta_Z) =$ 4*ζϕgz* $\frac{\varphi}{\varphi} + \mathcal{O}$ *g*3 *z* $\left(\frac{2}{3}, \frac{3}{20}\right)$ $\ll 1$

- Gauge coupling, $g_{\overline{z}}$:
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$$
\tan(2\theta_Z) = \frac{4\zeta_{\phi}g_z}{g_{Z^0}} + \mathcal{O}\left(\frac{g_z^3}{g_{Z^0}^3}\right) \ll 1
$$

 \bullet U(1)_{*z*} \otimes U(1)_{*z*} gauge mixing parameter, η : its value can be determined from RGE: $0 \le \eta \le 0.66$

- Gauge coupling, $g_{\overline{z}}$:
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\tan(2\theta_Z) = \frac{4\zeta_\phi g_z}{g_{Z^0}} + \mathcal{O}\left(\frac{g_z^3}{g_{Z^0}^3}\right) \ll 1
$$

 \bullet U(1)_{*z*} \otimes U(1)_{*z*} gauge mixing parameter, η : its value can be determined from RGE: $0 \le \eta \le 0.66$ **Masses of sterile neutrinos:**

assume N_1 to be light (keV-MeV scale), while $M_{2,3} = O(M_{Z^0})$

Production of DM in freeze-out scenario

We consider $M_1 = O(10) \text{ MeV} \Rightarrow$ decoupling happens at $T_{\text{dec}} = O(1) \text{ MeV}$ At this temperature electrons and SM neutrinos are abundant, negligible amounts of heavier fermions

Production of DM in freeze-out scenario

- We consider $M_1 = O(10)$ MeV \Rightarrow decoupling happens at $T_{\text{dec}} = O(1) \text{ MeV}$ At this temperature electrons and SM neutrinos are abundant, negligible amounts of heavier fermions
- Relevant cross section for the production process

Resonant production of DM

Need to increase $\langle \sigma v_{\text{Mol}} \rangle$ **without increasing** g_z **(excluded** ϵ experimentally): exploit resonant production ($2M_1 \lesssim M_{Z'}$)

Resonant production of DM

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$$
\langle \sigma v_{\text{Mol}} \rangle = (\dots) \int_{4M_1^2}^{\infty} ds \underbrace{\frac{(\dots)}{(s - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}}_{\text{strongly peaked around } s = M_{Z'}^2} \times K_1 \left(\frac{\sqrt{s}}{T} \right)
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Resonant production of DM

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$$

the Bessel function K_1 vanishes exponentially at large arguments

 $T_{\text{dec}} \approx 0.1 M_1$, hence $K_1(10 M_{Z'}/M_1)$ can be small at the $\text{resonance } s = M_{Z'}^2$ depending on the ratio $M_{Z'}/M_1$

Resonant amplification: example

calculated within the SWSM for $M_1 = 10 \,\text{MeV}$ & $M_{Z} = 30 \,\text{MeV}$

Masses of the neutral gauge bosons again *^z* sin ✓*^Z* = *M^Z* sin ✓^G and *wg*⁰ *^z* cos ✓*^Z* = *M^Z*⁰ cos ✓^G *,* (II.29) *^Z*⁰ = *v*² $\frac{1}{2}$

can also be expressed with chiral couplings: **z** sin and be expressed with critical coapinitys.

^L`

1

⌫*c*

$$
M_Z^2 = \frac{v^2 e^2}{\cos^2 \theta_G} \left(C_{Z\nu\nu}^L - C_{Z\nu\nu}^R \right)^2
$$

$$
M_{Z'}^2 = \frac{v^2 e^2}{\sin^2 \theta_G} \Big(C_{Z'\nu\nu}^L - C_{Z'\nu\nu}^R\Big)^2
$$

^R Y*^N* ⌫*R* + *L^L ^c* Y⌫ ⌫*^R* + h.c. (II.31)

which are crucial for checking gauge independence are crucial for checking gauge independence

Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R}

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recall: Γ_V^μ $\frac{\mu}{V\bar{f}f} = -{\rm i}e\gamma^{\mu}(C_{V\bar{f}f}^R P_R + C_{V\bar{f}f}^L P_L)$

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sin ✓*^Z* = sgn ()

Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R} **E. E. Gauge Boson – Neutral current coup**

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 ij

recall: Γ_V^μ which reads on the basis of propagating mass eigenstates as $\frac{\mu}{V\bar{f}f} = -{\rm i}e\gamma^{\mu}(C_{V\bar{f}f}^R P_R + C_{V\bar{f}f}^L P_L)$ $\frac{1 - Vl}{l}$ $\mathbf{E}^{\prime\prime}$ $\frac{v}{\sqrt{2}}$ t_n $\Gamma^{\mu}_{V\nu_i\nu_j} = -{\rm i} e \gamma^{\mu}$ $\overline{1}$ $\Gamma_{V\nu\nu}^{L}P_{L}+\Gamma_{V\nu\nu}^{R}P_{R}$ $\overline{}$

i
Li

^z) sin ✓*^Z* cos ✓^W

¹ ¹ ² ⌧ ²

sin ✓*^Z* = sgn ()

Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R} **E. E. Gauge Boson – neutral current coup**

As the neutral currents are written in terms of flavour eigenstates, the interactions between

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*g*L

recall: Γ_V^μ which reads on the basis of propagating mass eigenstates as recall: $\Gamma^{\mu}_{V\bar{f}f} = -ie\gamma^{\mu} (C^{R}_{V\bar{f}f}P_R + C^{L}_{V\bar{f}f}P_L)$ where Z^L *z*^{*Z*} $\left(\frac{\textbf{I} + V \nu \nu^{\textbf{I}} L + \textbf{I} \nu \nu^{\textbf{I}} R}{\textbf{I} \cdot \textbf{I} \cdot \textbf{I}}\right)_{ij}$ $\boldsymbol{\Gamma}_{V\nu\nu}^{L}=C_{V\nu\nu}^{L}\mathbf{U}_{L}^{\dagger}\mathbf{U}_{L}-C_{V\nu\nu}^{R}\mathbf{U}_{R}^{T}\mathbf{U}_{R}^{*}$ $\mathbf{E}^{\prime\prime}$ $\frac{v}{\sqrt{2}}$ t_n $\Gamma^{\mu}_{V\nu_i\nu_j} = -{\rm i} e \gamma^{\mu}$ $\overline{1}$ $\Gamma_{V\nu\nu}^{L}P_{L}+\Gamma_{V\nu\nu}^{R}P_{R}$ $\overline{}$ ij which reads *µ V* ⌫*i*⌫*^j* ⁼ i*e^µ L ^V* ⌫⌫*P^L* + *^R ^V* ⌫⌫*P^R* ⌘ \overline{U} \overline{R}

¹ ¹ ² ⌧ ²

Useful relations of these matrices are collected in Appendix A.

where

sin ✓*^Z* = sgn ()

Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R} **E. E. Gauge Boson – neutral current coup**

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Li $\frac{1 - VL}{R + T}$ $\mathbf{E}^{\prime\prime}$ $\frac{v}{\sqrt{2}}$ t_n $\Gamma^{\mu}_{V\nu_i\nu_j} = -{\rm i} e \gamma^{\mu}$ $\overline{1}$ $\Gamma_{V\nu\nu}^{L}P_{L}+\Gamma_{V\nu\nu}^{R}P_{R}$ $\overline{}$ ij which reads *µ V* ⌫*i*⌫*^j* ⁼ i*e^µ L ^V* ⌫⌫*P^L* + *^R ^V* ⌫⌫*P^R* ⌘ $\Gamma^{\mu}_{V \nu \nu \nu} = -ie \gamma^{\mu} \left(\Gamma^{L}_{V \nu \nu} P_{L} + \Gamma^{R}_{V \nu \nu} P_{R} \right)$ VJJ provide the scalar and Goldstone bosons in Eq. (II.32) provide interactions in Eq. (between the neutrinos. The neutrinos and the neutrinos. The same structure with small dif-term small dif-term i
The same structure with small dif-term in the same structure with small dif-term in the same structure with sm

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sin ✓*^Z* = sgn ()

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where
$$
\Gamma_{V\nu\nu}^{L} = C_{V\nu\nu}^{L} \mathbf{U}_{L}^{\dagger} \mathbf{U}_{L} - C_{V\nu\nu}^{R} \mathbf{U}_{R}^{T} \mathbf{U}_{R}^{*}
$$

⌘(Z*S*)*k*¹

and also:
$$
\Gamma_{S_k/\sigma_k \nu_i \nu_j} = \left(\Gamma_{S_k/\sigma_k \nu\nu}^L P_L + \Gamma_{S_k/\sigma_k \nu\nu}^R P_R\right)_{ij}
$$

(Z*S*)*k*²

Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R} **E. E. Gauge Boson – neutral current coup** F. Scalar bosson – neutrino and Goldstone boson – neutrino T terms containing the scalar and G is the scalar and G interactions in Eq. (II.32) provide interactions in

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/ 1 $\mu_i \nu_j = -1e^{\gamma} \gamma$ ^c $\left(\frac{\mathbf{I}}{V} \gamma \nu \nu^T \right)$,
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between those and the neutrinos. These interactions have the same structure with small dif-

Useful relations of these matrices are collected in Appendix A.

where where

sin ✓*^Z* = sgn ()

for both *V* = *Z* and *V* = *Z*⁰

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where
$$
\mathbf{\Gamma}_{V\nu}^{L} = C_{V\nu\nu}^{L} \mathbf{U}_{L}^{T} \mathbf{U}_{L} - C_{V\nu\nu}^{R} \mathbf{U}_{R}^{T} \mathbf{U}_{R}^{*}
$$

⌘(Z*S*)*k*¹

(ZG)*k*²

and also:
$$
\Gamma_{S_k/\sigma_k \nu_i \nu_j} = \left(\Gamma_{S_k/\sigma_k \nu\nu}^L P_L + \Gamma_{S_k/\sigma_k \nu\nu}^R P_R\right)_{ij}
$$

$$
\Gamma_{S_k \nu\nu}^L = -i \left[\left(M U_L^{\dagger} U_L + U_L^T U_L^* M\right) \frac{(Z_S)_{k1}}{v} + U_R^{\dagger} M_N U_R^* \frac{(Z_S)_{k2}}{w} \right]
$$

$$
\Gamma_{\sigma_k \nu\nu}^L = -\left[\left(M U_L^{\dagger} U_L + U_L^T U_L^* M\right) \frac{(Z_S)_{k1}}{v} + U_R^{\dagger} M_N U_R^* \frac{(Z_S)_{k2}}{w} \right]
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Neutral current couplings on mass basis $\overline{\mathsf{V}}$ current plin *,* \mathbb{R} **E. E. Gauge Boson – neutral current coup** F. Scalar bosson – neutrino and Goldstone boson – neutrino T terms containing the scalar and G is the scalar and G interactions in Eq. (II.32) provide interactions in ferences. For the propagating scalar states *S^k* or *^k* (*k* = 1 denoting *h* or the Goldstone boson belonging to *Z* and *k* = 2 referring to *s* or the Goldstone boson belonging to *Z*⁰

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¹ ¹ ² ⌧ ²

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recall: Γ_V^μ which reads on the basis of propagating mass eigenstates as where and also: $\Gamma_{S_k/\sigma_k \nu_i \nu_j} = \left(\Gamma_{S_k/\sigma_k \nu \nu'}^L\right)$ 70 recall: $\Gamma^{\mu}_{V\bar{f}f} = -ie\gamma^{\mu} (C^{R}_{V\bar{f}f}P_R + C^{L}_{V\bar{f}f}P_L)$ where \overline{L} $\overline{$ i
Li *, ex*⁺ = *r* $\frac{1 + V\nu\nu + R}{i j}$
 R $I T T T^*$ $\boldsymbol{\Gamma}_{V\nu\nu}^{L}=C_{V\nu\nu}^{L}\mathbf{U}_{L}^{\dagger}\mathbf{U}_{L}-C_{V\nu\nu}^{R}\mathbf{U}_{R}^{T}\mathbf{U}_{R}^{*}$ $\Gamma_{V \nu\nu}^{R} = -0$ $\overline{ }$ $E_{\nu\nu}^T \mathbf{U}_L^T \mathbf{U}_L^* + C_V^R$ g_L $\boldsymbol{\Gamma}_{V\nu\nu}^{R} = -C_{V\nu\nu}^{L}\mathbf{U}_{L}^{T}\mathbf{U}_{L}^{*} + C_{V\nu\nu}^{R}\mathbf{U}_{R}^{\dagger}\mathbf{U}_{R} = -\left(\boldsymbol{\Gamma}_{V\nu\nu}^{L}\right)^{*}$ $\left[\frac{\mathbf{I}_{\mathcal{S}_k \nu \nu}}{\mathbf{I}_{\mathcal{S}_k \nu \nu}}\right] = -\left[\frac{\mathbf{I}_{\mathcal{S}_k \nu} \mathbf{I}_{\mathcal{S}_k \nu \nu}}{\mathbf{I}_{\mathcal{S}_k \nu \nu}} + \mathbf{U}_{\mathcal{R}} \mathbf{I}_{\mathcal{S}_k \nu \nu} \mathbf{I}_{\mathcal{S}_k \nu \nu \nu} \right] \mathbf{I}_{\mathcal{S}_k \nu \nu \nu \nu \nu}$ 5 $\mathbf{E}^{\prime\prime}$ $\frac{v}{\sqrt{2}}$ t_n $\Gamma^{\mu}_{V\nu_i\nu_j} = -{\rm i} e \gamma^{\mu}$ $\overline{1}$ $\Gamma_{V\nu\nu}^{L}P_{L}+\Gamma_{V\nu\nu}^{R}P_{R}$ $\overline{}$ ij As the neutral currents are written in terms of flavour eigenstates, the interactions between which reads *µ V* ⌫*i*⌫*^j* ⁼ i*e^µ L ^V* ⌫⌫*P^L* + *^R ^V* ⌫⌫*P^R* ⌘ $\Gamma^{\mu}_{V \nu \nu \nu} = -ie \gamma^{\mu} \left(\Gamma^{L}_{V \nu \nu} P_{L} + \Gamma^{R}_{V \nu \nu} P_{R} \right)$ \overline{R} and $\mathbf{\Gamma}_{V\nu\nu}^{R} = -C_{V\nu\nu}^{L}\mathbf{U}_{L}^{T}\mathbf{U}_{L}^{*} + C_{V\nu\nu}^{R}\mathbf{U}_{R}^{\dagger}\mathbf{U}_{R} = -R_{V}\mathbf{\Gamma}_{V}\mathbf{\Gamma}_{V}\mathbf{\Gamma}_{V}^{T}\mathbf{U}_{R}^{*}$ $\sqrt{2}$ $\left.\Gamma_{V\nu\nu}^{L}\right)^{*}$ \int also: \int $\Gamma_{S_k/\sigma_k \nu_i \nu_j} =$ $\begin{array}{cc} \mathbb{C} & \mathbb{C} & \mathbb{C} \end{array}$ so $\begin{array}{cc} w & \mathbb{C} & \mathbb{C} \mathbb{C}^{R} \end{array}$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ bosons in $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ provide interactions in $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ VJJ provide the scalar and Goldstone bosons in Eq. (II.32) provide interactions in Eq. (between the neutrinos. The neutrinos and the neutrinos. The same structure with small dif-term small dif-term i
The same structure with small dif-term in the same structure with small dif-term in the same structure with sm \mathbf{R} $\mathbf{$ $\boldsymbol{\tau}$ interactions can be decomposed in the decomposed into left and right chiral terms of $\boldsymbol{\tau}$ $\sqrt{2}$ $\Gamma^L_{S_k/\sigma_k}$ $_{\nu\nu}P_L+\Gamma^R_{S_k/\sigma_k}$ $_{\nu\nu}P_R$ $\overline{}$ ij $\Gamma_{S_k\nu\nu}^{\nu} = -i \left[\left(\mathbf{M} \mathbf{U}_L^{\dagger} \mathbf{U}_L + \mathbf{U}_L^{\dagger} \mathbf{U}_L^* \mathbf{M} \right) \frac{\partial S_{\nu}^{\dagger}}{\partial t} + \mathbf{U}_R^{\dagger} \mathbf{M}_N \mathbf{U}_R^* \frac{\partial S_{\nu}^{\dagger}}{\partial t} \right]_{\mathbf{R}^R}$ $\Gamma^L = -\left[(\mathbf{M}\mathbf{H}^\dagger \mathbf{H}_\mathbf{F} + \mathbf{H}^\dagger \mathbf{T} + \mathbf{M}) \frac{(\mathbf{Z}_{\mathrm{G}})_{k1}}{(\mathbf{Z}_{\mathrm{G}})_{k1}} + \mathbf{H}^\dagger \mathbf{M}_\mathbf{M} \mathbf{H}^\dagger \right]^* \frac{(\mathbf{Z}_{\mathrm{G}})_{k2}}{(\mathbf{Z}_{\mathrm{G}})_{k2}}$ ⇣ ⌘(Z*S*)*k*¹ (Z*S*)*k*² The terms containing the scalar and Goldstone bosons in Eq. (II.32) provide interactions which reads on the basis of propagating mass boson belonging to *Z* and *k* and *k* and *k* or the Goldstone boson boson boson belonging to *z*00 $\mathbf{I}_{V \nu_i \nu_j} = -\mathrm{i} e \gamma^{\mu} \left(\mathbf{I}_{V \nu \nu} \right)$ ⇣ Γ ^{*V*}*V*^{*w*} = C ^{*V*}*V*^{*V*}*V*^{*L*} *R* $\frac{1}{2}$ *Ck* $\frac{1}{2}$ $\mathbf{\Gamma}^L_{V\nu\nu} = C^L_{V\nu\nu} \mathbf{U}^\dagger_L \mathbf{U}_L - C^R_V$ where the matrix L contain both the mixing matrix of and also: $\mathbf{L}_{S_k/\sigma_k \nu_i \nu_j} = (\mathbf{L}_{S_k/\sigma_k \nu \nu}^2 P_L +$ $\mathbf{\Gamma}^L_{S_k \nu \nu} = -\mathrm{i}$ \lceil $\mathbf{M}\mathbf{U}_{L}^{\dagger}\mathbf{U}_{L} + \mathbf{U}_{L}^T\mathbf{U}_{L}^*\mathbf{M}$ $\setminus (\mathbf{Z}_S)_{k1}$ *v* $+ \, \mathbf U_R^{\intercal} \mathbf M_N \mathbf U_R^*$ $(\mathbf{Z}_S)_{k2}$ *w* $\overline{1}$ ⇣ ⌘(ZG)*k*¹ (ZG)*k*² I $\Gamma^\mu_{\nu \bar{\varepsilon} s} = -ie\gamma^\mu (C^R_{\nu \bar{\varepsilon} s} P_R + C^L_{\nu \bar{\varepsilon} s} P_L)$ which reade on the basis of propagating mass interactions can be decomposed into left and right chiral terms し
/ 1 $\mu_i \nu_j = -1e^{\gamma} \gamma$ ^c $\left(\frac{\mathbf{I}}{V} \gamma \nu \nu^T \right)$,
L *ij* $L_{V\nu\nu} - C_{V\nu\nu} C_L C_L$ contain both the mixing matrix of the mixing matri $\mathbf{I}_{V\nu\nu}^{\mathbf{w}} = -C_{V\nu\nu}^{\mathbf{w}} \mathbf{U}_{L}^{\mathbf{t}} \mathbf{U}_{L} +$ ⇣ $\mathbf{L}_{S_k/\sigma_k} \nu_i \nu_j =$ $\Gamma^L_{S, \; / \sigma_{1,1}}$ *v* $\mathcal{L}^{\mathcal{I}} L \perp \mathcal{S}_k /$ ϵ ₂ *w* $\overline{\mathbf{D}}$ $\int_{-R}^{R} S_k/\sigma_k \nu_l \nu^L L \pm S_k/\sigma_k \nu^L I_L$ $\Gamma^L_{\sigma_k \nu\nu} = \lceil$ $\mathbf{M}\mathbf{U}_{L}^{\dagger}\mathbf{U}_{L} + \mathbf{U}_{L}^{T}\mathbf{U}_{L}^{\ast}\mathbf{M}$ $\setminus (\mathbf{Z}_\text{G})_{k1}$ *v* $+ \, \mathbf U_R^{\intercal} \mathbf M_N \mathbf U_R^*$ $(\mathbf{Z}_\text{G})_{k2}$ *w* $\overline{1}$ \int *E^{<i>i***}** \int *i* $\big($ \sqrt{V} *Ff*^{*P*}*R* + C ^{*V*}*Ff*^{*P*}*L*) where the matrix L _R contain both the mixing matrix of the mixing matrix $\Gamma^{\mu}_{\nu} = -i e \gamma^{\mu} \left(\Gamma^{L}_{\nu} P_{\nu} + \Gamma^{R}_{\nu} P_{\nu} \right)$ $\frac{1}{L}$ $U_{L} = C_{V\nu\nu}^{L} \mathbf{U}_{L}^{\dagger} \mathbf{U}_{L} - C_{\mathbf{U}}^{L}$ R ¹ $R \sim R$ $\frac{1}{R}$ ⇣ $\frac{dk}{\theta_k}$ $S_k/\sigma_k\nu\nu$ *v* $\frac{1}{\sqrt{2}}$ and the right conjugation, $\frac{w}{\prod_{k}^{R}} \Gamma_{S_k/\sigma_k}^R v \nu = \overline{a}$ $\left[\mathbf{\Gamma}^L_{S_k/\sigma_k}$ $_{\nu\nu}\right)^*$

Neutrino mass matrix at one-loop order

calculation is simple conceptually

Neutrino mass matrix at one-loop order *^M*² ¹ (III.5)

*m*³

ln *^m*²

a

6

calculation is simple conceptually self energy can be decomposed as Self energy can be decomposed as

to the scalar-neutrino coupling *^L*

 $\mathbf{i}\mathbf{\Sigma}(p) = \mathbf{A}_L(p^2) p P_L + \mathbf{A}_R(p^2) p P_R + \mathbf{B}_L(p^2) P_L + \mathbf{B}_R(p^2) P_R \; ,$

Neutrino mass matrix at one-loop order *^M*² ¹ (III.5)

6

of dimension mass and with summation running over all neutrinos.

calculation is simple conceptually self energy can be decomposed as Self energy can be decomposed as nergy can be decomposed as

 $\mathrm{i}\mathbf{\Sigma}(p)=\mathbf{A}_{L}(p^{2})pP_{L}+\mathbf{A}_{R}(p^{2})pP_{R}+\mathbf{B}_{L}(p^{2})P_{L}+\mathbf{B}_{R}(p^{2})P_{R} \; ,$

and using the decomposition of $\delta M_\tau = H^*H$

takes contributions from

to the scalar-neutrino coupling *^L*

to the scalar-neutrino coupling *^L*

bution. Top right: scalar contribution. Bottom left: neutral gauge boson contribution. Bottom right:

 ℓ_a^{\mp} *a*

*m*³

ln *^m*²

a

with Feynman rules given in the Appendix T_a order are shown in Fig. 1. There are also tadpole contributions to B*L*(0). Those are proportional with Feynman rules given in the Appendix **contributions** ν_a
Neutrino mass matrix at one-loop order *^M*² ¹ (III.5)

6

of dimension mass and with summation running over all neutrinos.

calculation is simple conceptually self energy can be decomposed as Self energy can be decomposed as nergy can be decomposed as

 $\mathrm{i}\mathbf{\Sigma}(p)=\mathbf{A}_{L}(p^{2})pP_{L}+\mathbf{A}_{R}(p^{2})pP_{R}+\mathbf{B}_{L}(p^{2})P_{L}+\mathbf{B}_{R}(p^{2})P_{R} \; ,$

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Neutrino mass matrix at one-loop order

calculation involves "miracles" technically

Neutrino mass matrix at one-loop order **diagonal diagonal di**
and the main diagonal diagon where *variation*

(`² *M*²

XXX

µ

^P^L = i ^Z ^d*^d*`

^V)(`² ⇠*^V M*²

p/ /` + *m^a*

ga.

Pµ⌫(`*, M*²

`² *M*²

^L (*M^V , {ma}*; ⇠*^V*)

⇣ *B^V*

calculation involves "miracles" technically neutral vectors – with notation $m_{\ell}^{(n)} = diag\left(\frac{m_1^n}{\ell^2 - m_1^2}, \ldots, \frac{m_6^n}{\ell^2 - m_6^2}\right)$: $\delta {\bf M}_L^V = {\rm i} e^2$ $\sqrt{2}$ $C_{V\nu\nu}^L - C_{V\nu\nu}^R \bigg)^2 \, \int \! \frac{{\rm d}^d \ell}{(2\pi)^2}$ $\frac{d}{(2\pi)^d}\mathbf{U}^*_L$ *L* $\int d\, {\bf m}_\ell^{(1)}$ $\ell^2 - M_V^2$ $+$ $\mathbf{m}_{\ell}^{(3)}$ M_V^2 $\sqrt{1}$ $\ell^2 - \xi_V M_V^2$ $-\frac{1}{\ell^2}$ $\ell^2 - M_V^2$ $\int \frac{\mathrm{d}^d \ell}{\left(2\pi\right)d} \mathbf{U}_L^* \left[\frac{d}{\ell^2 - M^2} + \frac{\mathbf{m}_{\ell}^{(3)}}{M^2} \left(\frac{1}{\ell^2 - \xi \cdot M^2} - \frac{1}{\ell^2 - M^2} \right) \right] \mathbf{U}_L^{\dagger}$ (III.11) $\mathbf{m}_{\ell}^{(n)} = \text{diag} \left(\frac{m_1^n}{\ell^2 - \ell^2} \right)$ 1 $\ell^2 - m_1^2$ *,...,* m_6^n $\ell^2 - m_6^2$ ◆ *,* (III.10) $J(\angle n)^{\infty}$ [ℓ

Neutrino mass matrix at one-loop order **diagonal diagonal di**
and the main diagonal diagon $\mathcal{M} = \mathcal{M} \cup \mathcal{M} = \mathcal$ 1 where *variation*

(`² *M*²

XXX

µ

v

^P^L = i ^Z ^d*^d*`

^V)(`² ⇠*^V M*²

`² ⇠*^V M*²

`² ⇠*^V M*²

V

p/ /` + *m^a*

ga.

Pµ⌫(`*, M*²

`² *M*²

^L (*M^V , {ma}*; ⇠*^V*)

(2⇡)*d*U⇤

⇣ *B^V*

calculation involves "miracles" technically neutral vectors – with notation $m_{\ell}^{(n)} = diag\left(\frac{m_1^n}{\ell^2 - m_1^2}, \ldots, \frac{m_6^n}{\ell^2 - m_6^2}\right)$: $\delta {\bf M}_L^V = {\rm i} e^2$ $\sqrt{2}$ $C_{V\nu\nu}^L - C_{V\nu\nu}^R \bigg)^2 \, \int \! \frac{{\rm d}^d \ell}{(2\pi)^2}$ $\frac{d}{(2\pi)^d}\mathbf{U}^*_L$ *L* $\int d\, {\bf m}_\ell^{(1)}$ $\ell^2 - M_V^2$ $+$ $\mathbf{m}_{\ell}^{(3)}$ M_V^2 $\sqrt{1}$ $\ell^2 - \xi_V M_V^2$ $-\frac{1}{\ell^2}$ $\ell^2 - M_V^2$ $\bigcap \mathbf{U}^{\dagger}_L$ (III.11) D. Contributions with scalar bosons in the loop $\sum_{\mathbf{r}} f(\mathbf{r}) = \mathbf{r} \mathbf{r$ contribution is the scalar boson $\int (2\pi)^{d-1} \left[\ell^2 - M_V^2 \right]^{1/2} M_V^2 \left[\ell^2 - \xi_V M_V^2 \right]^{1/2}$ $\mathbf{m}_{\ell}^{(n)} = \text{diag} \left(\frac{m_1^n}{\ell^2 - \ell^2} \right)$ 1 $\ell^2 - m_1^2$ *,...,* m_6^n $\ell^2 - m_6^2$ ◆ *,* (III.10) α and α α \mathbf{m}_{ℓ} in α $J(\angle n)^{\infty}$ [ℓ

scalars: Eq. (III.14):

$$
\delta \mathbf{M}_L^{S_k} = \mathrm{i} \int \! \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathbf{U}_L^* \mathbf{M} \mathbf{m}_{\ell}^{(1)} \mathbf{M} \mathbf{U}_L^{\dagger} \left(\frac{(\mathbf{Z}_S)_{k1}}{v} \right)^2 \frac{1}{\ell^2 - M_{S_k}^2}
$$

Neutrino mass matrix at one-loop order **diagonal diagonal di**
and the main diagonal diagon $\mathcal{M} = \mathcal{M} \cup \mathcal{M} = \mathcal$ m(3) The contribution of the neutral Goldstone boson *^V* (*V* = 1 means the Goldstone boson 1 where *variation*

(`² *M*²

XXX

µ

v

^P^L = i ^Z ^d*^d*`

^V)(`² ⇠*^V M*²

`² ⇠*^V M*²

`² ⇠*^V M*²

.

V

p/ /` + *m^a*

ga.

Pµ⌫(`*, M*²

`² *M*²

^L (*M^V , {ma}*; ⇠*^V*)

(2⇡)*d*U⇤

⇣ *B^V*

= i ✓(Z*S*)*k*¹

E. The complete one-loop mass correction

calculation involves "miracles" technically neutral vectors – with notation $m_{\ell}^{(n)} = diag\left(\frac{m_1^n}{\ell^2 - m_1^2}, \ldots, \frac{m_6^n}{\ell^2 - m_6^2}\right)$: $\delta {\bf M}_L^V = {\rm i} e^2$ $\sqrt{2}$ $C_{V\nu\nu}^L - C_{V\nu\nu}^R \bigg)^2 \, \int \! \frac{{\rm d}^d \ell}{(2\pi)^2}$ $\frac{d}{(2\pi)^d}\mathbf{U}^*_L$ *L* $\int d\, {\bf m}_\ell^{(1)}$ $\ell^2 - M_V^2$ $+$ $\mathbf{m}_{\ell}^{(3)}$ M_V^2 $\sqrt{1}$ $\ell^2 - \xi_V M_V^2$ $-\frac{1}{\ell^2}$ $\ell^2 - M_V^2$ $\bigcap \mathbf{U}^{\dagger}_L$ (III.11) *m*_{*B*} *MP P*_{*M*} *(<i>m***₂)** *m***₂** $\sum_{\mathbf{r}} f(\mathbf{r}) = \mathbf{r} \mathbf{r$ contribution is the scalar boson $\int (2\pi)^{d-1} \left[\ell^2 - M_V^2 \right]^{1/2} M_V^2 \left[\ell^2 - \xi_V M_V^2 \right]^{1/2}$ **ZOU HIVOIVES** (2⇡)*^d* miracies technicali \overline{a} $\binom{n}{k}$ $=$ $\text{diag}\left(\frac{\overline{\rho}}{\overline{\rho}}\right)$ m_1^n $\mathbf{m}_{\ell}^{(n)} = \mathrm{diag} \left(\frac{m_1^n}{\ell^2 - m_1^2}, \ldots, \frac{m_6^n}{\ell^2 - m_2^2} \right) \, ,$ $\delta M^V = i e^2 (C^L - C^R)$ $\int \frac{d^a \ell}{\ell} I I^* \frac{d}{\ell} \frac{d n_{\ell}^{(1)}}{d \ell} + \frac{m_{\ell}^{(2)}}{d \ell} \left(\frac{1}{\ell} \right)$ Z d*^d*` 1 \limsup 1 $\ell^2 - m_1^2$ *,...,* m_6^n $\ell^2 - m_6^2$ ◆ *,* (III.10) $J(\angle n)^{\infty}$ [ℓ

scalars: Eq. (III.14):

$$
\delta \mathbf{M}_{L}^{S_{k}} = \mathrm{i} \int \frac{\mathrm{d}^{d} \ell}{(2\pi)^{d}} \mathbf{U}_{L}^{*} \mathbf{M} \mathbf{m}_{\ell}^{(1)} \mathbf{M} \mathbf{U}_{L}^{\dagger} \left(\frac{(\mathbf{Z}_{S})_{k1}}{v}\right)^{2} \frac{1}{\ell^{2} - M_{S_{k}}^{2}}
$$

Goldstones:

$$
\delta \mathbf{M}_L^{\sigma_V} = -\mathrm{i} \int\!\frac{\mathrm{d}^d\ell}{(2\pi)^d} \mathbf{U}_L^* \mathbf{M} \mathbf{m}_{\ell}^{(1)} \mathbf{M} \mathbf{U}_L^{\dagger} \left(\frac{(\mathbf{Z}_\mathrm{G})_{V1}}{v}\right)^2 \frac{1}{\ell^2 - \xi_V M_V^2}
$$

Neutrino mass matrix at one-loop order **diagonal diagonal di**
and the main diagonal diagon $\mathcal{M} = \mathcal{M} \cup \mathcal{M} = \mathcal$ 1 N (2⇡)*^d a*=1 *^V* ⌫*i*⌫*^a* λ α *v det*

(`² *M*²

XXX

µ

v

^P^L = i ^Z ^d*^d*`

^V)(`² ⇠*^V M*²

`² ⇠*^V M*²

`² ⇠*^V M*²

V

p/ /` + *m^a*

ga.

Pµ⌫(`*, M*²

`² *M*²

^L (*M^V , {ma}*; ⇠*^V*)

(2⇡)*d*U⇤

⇣ *B^V*

calculation involves "miracles" technically neutral vectors – with notation $m_{\ell}^{(n)} = diag\left(\frac{m_1^n}{\ell^2 - m_1^2}, \ldots, \frac{m_6^n}{\ell^2 - m_6^2}\right)$: $\delta {\bf M}_L^V = {\rm i} e^2$ $\sqrt{2}$ $C_{V\nu\nu}^L - C_{V\nu\nu}^R \bigg)^2 \, \int \! \frac{{\rm d}^d \ell}{(2\pi)^2}$ $\frac{d}{(2\pi)^d}\mathbf{U}^*_L$ *L* $\int d\, {\bf m}_\ell^{(1)}$ $\ell^2 - M_V^2$ $+$ $\mathbf{m}_{\ell}^{(3)}$ M_V^2 $\sqrt{1}$ $\ell^2 - \xi_V M_V^2$ $-\frac{1}{\ell^2}$ $\ell^2 - M_V^2$ $\bigcap \mathbf{U}^{\dagger}_L$ (III.11) D. Contributions with scalar bosons in the loop $\sum_{\mathbf{r}} f(\mathbf{r}) = \mathbf{r} \mathbf{r$ contribution is the scalar boson $\int (2\pi)^{d-1} \left[\ell^2 - M_V^2 \right]^{1/2} M_V^2 \left[\ell^2 - \xi_V M_V^2 \right]^{1/2}$ $\mathbf{m}_{\ell}^{(n)} = \text{diag} \left(\frac{m_1^n}{\ell^2 - \ell^2} \right)$ 1 $\ell^2 - m_1^2$ *,...,* m_6^n $\ell^2 - m_6^2$ \mathbf{v} **do** \mathbf{v} **m** m_0^n **m** m_0^n **m** m_0^n **m** m_0^n **n** n **n** *,* (III.10) α and α α \mathbf{m}_{ℓ} in α $J(\angle n)^{\infty}$ [ℓ <u>Laiculation</u> a*lves* m *racies* tec *L* \overline{a} S_{S} and S_{S} and P_{S} a $\delta N_{L} = 1e \left(C_{V\nu\nu} - C_{V\nu\nu} \right)$ $\int \frac{1}{(2\pi)^{d}} \mathbf{C}_{L} \left[\frac{1}{l^{2} - M_{V}^{2}} + \frac{1}{M_{V}^{2}} \right]$

scalars: Eq. (III.14):

$$
\delta \mathbf{M}_{L}^{S_{k}} = \mathrm{i} \int \frac{\mathrm{d}^{d} \ell}{(2\pi)^{d}} \mathbf{U}_{L}^{*} \mathbf{M} \mathbf{m}_{\ell}^{(1)} \mathbf{M} \mathbf{U}_{L}^{\dagger} \left(\frac{(\mathbf{Z}_{S})_{k1}}{v}\right)^{2} \frac{1}{\ell^{2} - M_{S_{k}}^{2}}
$$

Goldstones:

1eS:
\n
$$
\delta M_L^{\sigma_V} = -ie^2 \Big(C_{V\nu\nu}^L - C_{V\nu\nu}^R \Big)^2 \int \frac{d^d \ell}{(2\pi)^d} U_L^* \frac{m_{\ell}^{(3)}}{M_V^2} U_L^{\dagger} \frac{1}{\ell^2 - \xi_V M_V^2}
$$

E. The complete of the complete one-loop mass cancel

. (III.15)

Numerical estimates

of the boson in the loop m_{loop} , assuming m_1 ^{tree} = 0.01 eV, m₄tree = 30 keV, m₅tree \approx m₆tree = 2.5 GeV, and normal neutrino mass hierarchy and normal neutrino mass hierarchy Eigenvalues of the matrix F as a function of the mass

74 content of the model consists of a new complex scalar field and three right-handed neutrinos eigenvalues can be large, but coupling suppression tames the relative The neutrino masses are generated by Dirac and Majorana type Yukawa terms, which after correction to the tree-level mass below percent level

Scanning couplings for vacuum stability: allowed region of scalar couplings at 2 loops

Scanning couplings for vacuum stability: allowed region of scalar couplings at 2 loops

For given input $\{\lambda_{\phi}(m_t), \lambda_{\chi}(m_t), \lambda(m_t), y_{\chi}(m_t)\}$ $\text{Check } w^{(1)}(m_t) > 0$ (VEV of 2nd scalar exists)

Scanning couplings for vacuum stability: allowed region of scalar couplings at 2 loops

For given input $\{\lambda_{\phi}(m_t), \lambda_{\chi}(m_t), \lambda(m_t), y_{\chi}(m_t)\}$ $\text{Check } w^{(1)}(m_t) > 0$ (VEV of 2nd scalar exists) **Run RGE and check** stability \Box

perturbativity \Box

