EUROSTRINGS 2024

STRONG COUPLING EXPANSION OF DETERMINANT OBSERVABLES IN SUPERSYMMETRIC GAUGE THEORIES

Zoltan Bajnok[1]
Bercel Boldis[1,2]
Gregory P. Korchemsky[3]

Based on:
[Bajnok,Boldis,Korchemsky'24]

[1]HUN-REN Wigner Research Centre for Physics
[2] Department of Theoretical Physics, Budapest University of Technology and Economics
[3]Université Paris Saclay

2024

HUN REN | Wigner   BME PHYSICS

The project supported by the **Doctoral Excellence Fellowship Programme** (DCEP) is funded by the National Research Development and Innovation Fund of the Ministry of Culture and the Budapest University of Technology and Economics, under a grant agreement with the National Research, Development and Innovation Office.

THEORY CHALLENGES

cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Funded by the European Union

EuroStrings
2024
Southampton

AdS/CFT correspondence

AdS/CFT correspondence:

- CFT on the flat boundary
- Strings in the bulk

$$Z_{str}\left[\Phi\big|_{\partial AdS} = J\right] = Z_{CFT}[J]$$

String observables $\leftrightarrow$ CFT_d observables

Our case:

- IIB superstrings on $AdS_5 \times S_5$
- $\mathcal{N} = 4$ SYM on the boundary
- Gauge symmetry: $SU(N_c)$

Parameters:

- $\lambda = 4\pi^2 T^2$
- $\frac{1}{N_c} = \frac{g_{str}}{4\pi^2 T^2}$

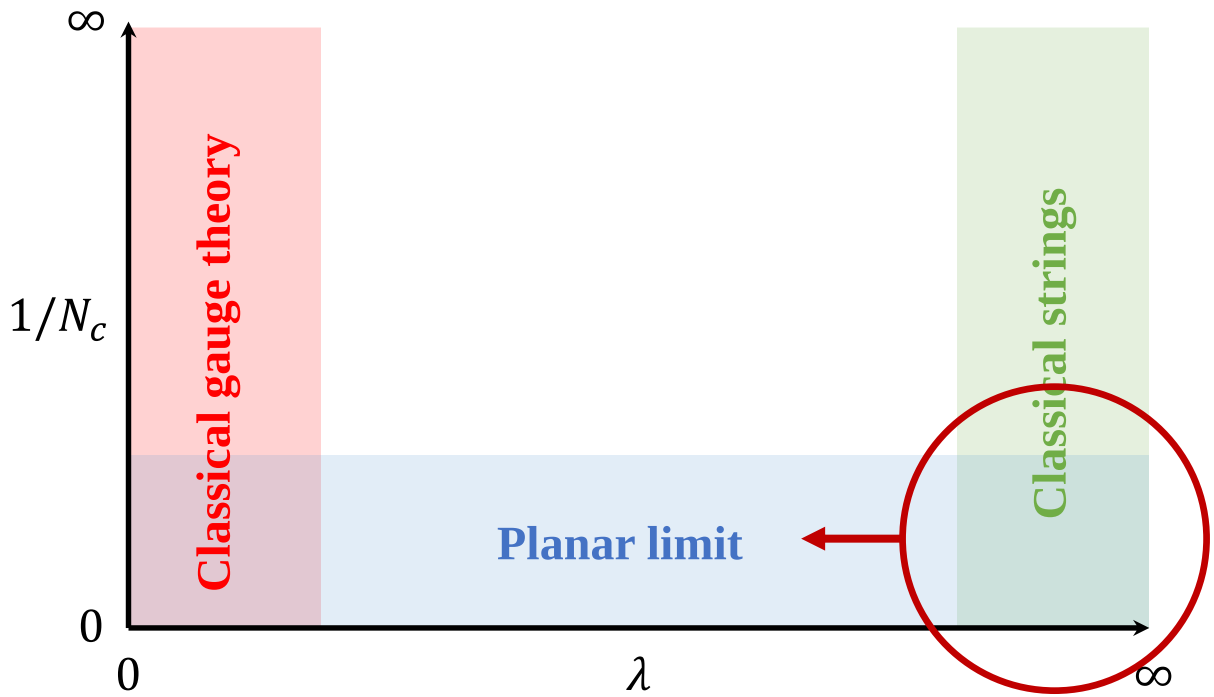
Planar limit ($N_c \to \infty$)
$\updownarrow$
Weakly coupled strings ($g_{st} \to 0$)

Planar limit:

- $N_c \to \infty$
- t'Hooft coupling: $\lambda = g_{YM}^2 N_c$ is constant

Strong coupling expansion: $\lambda \to \infty$ or $g_{YM} \to \infty$

AdS/CFT correspondence

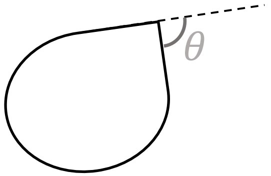


Determinant observables

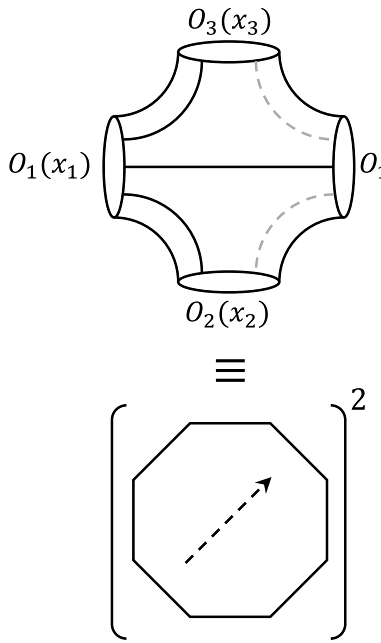
Examples:

1. **Planar $\mathcal{N} = 4$ SYM:**

- **Cusp anomalous dimension:** [Polyakov'80]
  - Renormalization of a Wilson loop with a cusp
  - $\langle Tr\, \mathcal{P}\, exp\{i \int_C dx \cdot A(x)\}\rangle \sim \Lambda^{\Gamma_{cusp}(\theta)}$

$C =$ 

- **Octagon form factor:** [Coronado'18]
  - Four-point correlation function:
    $$G_4 = \langle O_1(x_1) O_2(x_2) O_3(x_3) O_1(x_4)\rangle$$
  - O_1, O_2, O_3: half-BPS operators
  - Dual to scattering amplitude of four closed strings



2. **Planar equivalent $\mathcal{N}$ =2 SYM:** [Pestun'12]

- **Free energy**

What do they have in common?

Determinant observables

Determinant observables:

- Observable $\mathcal{F}(g)$:

$$e^{\mathcal{F}(g)} = \det_{1\le n,m<\infty}\left(\delta_{nm} - K_{nm}(g)\right)$$

- Where $g = \frac{\sqrt{\lambda}}{4\pi}$
- The semi-infinite matrix is: $K_{nm}(g) = \int_0^\infty dx \psi_n(x)\chi\left(\frac{\sqrt{x}}{2g}\right)\psi_m(x)$
- The functions $\psi_n(x)$ are: $\psi_n(x) = (-1)^n\sqrt{2n+\ell+1}J_{2n+\ell+1}(\sqrt{x})/\sqrt{x}$
- Observables are specified by the symbol $\chi(x)$ and the number ℓ!

Observable	$\boldsymbol{\chi(x)}$	$\boldsymbol{\ell}$
$\mathcal{N}=4$ cusp	$1-\coth x/2$	0
$\mathcal{N}=4$ octagon	$1/\cosh^2 x/2$	0
$\mathcal{N}=2$ free energy	$-1/\sinh^2 x/2$	1

Determinant observables

Symbol in general:

- Observables are specified by the symbol $\chi(x)$ and the number ℓ!
- Define: $1-\chi(x)=bx^{2\beta}\phi(x)\phi(-x)$
- Where: $\phi(x)=\prod_{n=1}^{\infty}\frac{1-\frac{ix}{2\pi x_n}}{1-\frac{ix}{2\pi y_n}}$
- Observables are specified by:
  - Numbers β,ℓ
  - Locations of zeros x_n
  - Locations of poles y_n

$$\phi_{cusp}(x)=\sqrt{\pi}\frac{\Gamma\left(1-\frac{ix}{2\pi}\right)}{\Gamma\left(\frac{1}{2}-\frac{ix}{2\pi}\right)}$$

Observable	ℓ	$\boldsymbol{\beta}$	$\boldsymbol{\phi}$	$\boldsymbol{x_n}$	$\boldsymbol{y_n}$
$\mathcal{N}=4$ cusp	0	$-1/2$	ϕ_{cusp}	$n-1/2$	n
$\mathcal{N}=4$ octagon	0	1	ϕ_{cusp}^{-2}	n	$n-1/2$
$\mathcal{N}=2$ free energy	1	-1	ϕ_{cusp}^{2}	n	$n-1/2$

Strong coupling expansion

Perturbative coefficients:

- $\mathcal{F}(g) = \boxed{-gA_0 + \frac{1}{2}A_1^2 \log g + B + \sum_{k=1}^{\infty} \frac{A_{k+1}}{2k(k+1)} g^{-k}} + \boxed{\Delta f(g)}$, as $g \to \infty$
- Perturbative coefficients: [Belitsky,Korchemsky'20]

Perturbative coefficient	Value
A_0	$2I_0$
A_1^2	$2\ell\beta + \beta^2$
A_2	$-(4\ell_\beta^2 - 1)/4\, I_1$
A_3	$-3(4\ell_\beta^2 - 1)/16\, I_1^2$
…	

$\ell_\beta = \ell + \beta$

- Moments: $I_n = \int_0^\infty \frac{dx}{\pi} \frac{(x^{-1}\partial_x)^n}{(2n-1)!!} x\partial_x \log\big(1 - \chi(x)\big)$
- A_n grow factorially at large n! → **Non-perturbative terms ($\Delta f(g)$)!**

$$\boxed{\Delta f(g) = \sum_{n\geq 1} (g e^{-8\pi g x_1})^n \left(A_1^{(n)} + \sum_{k\geq 1} \frac{A_{k+1}^{(n)}}{2k(k+1)} g^{-k} \right)}$$

Non-perturbative corrections

Aim: non-perturbative coefficients

Truncated Bessel operator:

- Define: $K_{nm}(g) = \int_0^\infty dx \psi_n(x) \chi\left(\frac{\sqrt{x}}{2g}\right) \psi_m(x) \equiv \langle \psi_n | \boldsymbol{K}_\chi | \psi_m \rangle$
- Integral operator: $\boldsymbol{K}_\chi f(x) = \int_0^\infty K(x,z) \chi\left(\frac{\sqrt{y}}{2g}\right) f(x)$

Truncated Bessel kernel

- Where: $K(x,y) = \sum_{n\geq 1} \psi_n(x) \psi_n(y)$
- Then the observable: $\mathcal{F}(g) = \log \det(1 - \boldsymbol{K}_\chi)$ **Fredholm determinant**

Non-perturbative corrections

Auxiliary function:

- Define: $$q(x,g) = \langle x| \frac{1}{1-\boldsymbol{K}_\chi} |\phi_0\rangle$$
- Such that: $\langle x|\phi_0\rangle = J_\ell(\sqrt{x})$
- **$\boldsymbol{q(x,g)}$ is an entire function!**

1. Integral equation:

- $q(x)$ satisfies: $$\int_0^\infty dx\big(1-\chi(x)\big)J_{2n+\ell-1}(2gx)q(x,g) = 0$$
- Solution: $$q(x,g) = \frac{e^{2igx}}{\Phi(-x)}h(x,g) + (-1)^\ell \frac{e^{-2igx}}{\Phi(x)}h(-x,g)$$
- With: $h(x,g) = \sum_{n\geq 0} e^{-8\pi g x_1 n} \sum_{k\geq 0} h_k^{(n)}(x) g^{-k-\frac{1}{2}}$
- **Gives an ansatz for $\boldsymbol{q(x)}$**

Non-perturbative corrections

2. Differential equation:

- Connects the observable $\mathcal{F}(g)$ with the function $q(x)$

$$\left(\left(g\partial_g\right)^2 + 4g^2x^2 - \ell^2 + 2g^2\partial_g^2\mathcal{F}(g)\right)q(x,g) = 0$$

- **The functions $h_k^{(n)}$ can be expressed in terms of the coefficients $A_k^{(n)}$!**

3. Integral relation:

- Observable $\mathcal{F}(g)$ and $q(x)$ are also related by:

$$\partial_g\mathcal{F}(g) = -\frac{1}{2}\int_0^\infty dx\partial_x\chi(x)q^2(x)\partial_x\partial_g q(x)$$

- **The functions $A_k^{(n)}$ can be expressed in terms of the coefficients $h_k^{(n)}$!**

Non-perturbative corrections

- Solution for : $q(x,g) = \frac{e^{2igx}}{\Phi(-x)} h(x,g) + (-1)^{\ell} \frac{e^{-2igx}}{\Phi(x)} h(-x,g)$
- With: $h(x,g) = \sum_{n\geq 0} e^{-8\pi g x_1 n} \sum_{k\geq 0} h_k^{(n)}(x) g^{-k-\frac{1}{2}}$

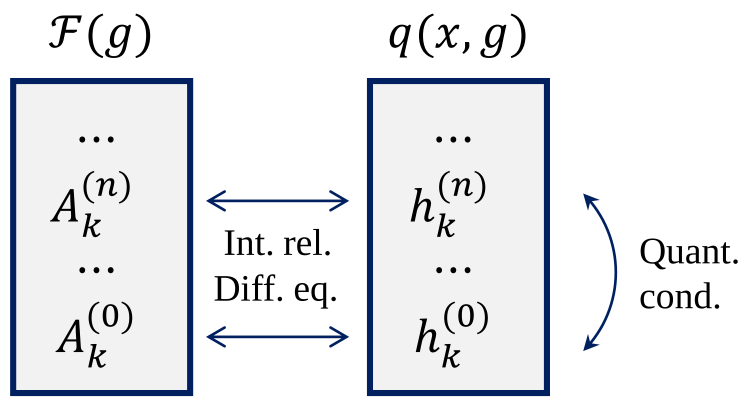
4. Quantization condition:

- $q(x,g)$ is an entire function
- Therefore:

Quantization condition

$$\lim_{x\to -2\pi i x_n} \big(1-\chi(x)\big) q(x,g) = 0$$

- **Connects the functions $h_k^{(n)}$ with different n's!**
- $A_k^{(n)}$ can be expressed in terms of $A_k \equiv A_k^{(0)}$!
- Whole expansion of $\mathcal{F}(g)$ can be determined!



Method

Integral equations:

$$\int_0^\infty dx(1-\chi(x))J_{2n+\ell-1}(2gx)q(x,g)=0$$

- Give an **ansatz for $\boldsymbol{q(x)}$** in the form of a transseries.

$\boldsymbol{q(x)}$ and the observable:

$$\partial_g\mathcal{F}(g)=-\frac{1}{2}\int_0^\infty dx\partial_x\chi(x)q^2(x)\partial_x\partial_g q(x)$$

- Connects the observable to the function $q(x)$.

Differential equation:

$$\left((g\partial_g)^2+4g^2x^2-\ell^2+2g^2\partial_g^2\mathcal{F}(g)\right)q(x,g)=0$$

- Coefficents $\boldsymbol{A_k^{(n)}}$ **enter the function $\boldsymbol{q(x)}$**.

Quantization condition:

$$\lim_{x\to-2\pi i x_n}(1-\chi(x))q(x,g)=0$$

- **Connects different levels of exponential corrections** via the analytic structure of $\chi(x)$.

Whole transseries of $\boldsymbol{\mathcal{F}(g)}$ can be determined in an efficient way!

Results

Cusp anomalous dimension:

- $\mathcal{F}(g) = \frac{3}{8} \log \cosh 2\pi g - \frac{1}{8} \log \frac{\sinh 2\pi g}{2\pi g}$
- Agrees with the known result [Beccaria,Korchemsky,Tseytlin'23]

Octagon form factor:

- $\Delta f(g) = \frac{i\pi g'}{4} e^{-8\pi g} \left(1 - \frac{7}{4(4\pi g')} - \frac{63}{32(4\pi g')^2} + \cdots\right) + \\ + \frac{(\pi g')^2}{32} e^{-16\pi g} \left(1 + \frac{81i-14}{4(4\pi g')} + \frac{-1431i-24}{32(4\pi g')^2} + \cdots\right)$
- Where $g' = g + \log 2 / \pi$

$\mathcal{N}$ =2 SYM:

- $\Delta f(g) = 2i\pi g'' e^{-4\pi g} \left(1 + \frac{16 \log 2 + 1}{2(4\pi g'')} + \frac{96 \log 2 - 15}{8(4\pi g'')^2} + \cdots\right) + \\ + 2(\pi g'')^2 e^{-8\pi g} \left(1 + \frac{16 \log 2 + 1}{(4\pi g'')} + \frac{64 \log^2 2 + 32 \log 2 - 3}{(4\pi g'')^2} + \cdots\right)$
- Where $g'' = g - \log 2 / \pi$

Summary

Results:

- Observables in planar $\mathcal{N} = 4$ and $\mathcal{N}$ =2 SYM
- Strong coupling expansion
- Systematic method to determine non-perturbative contributions
- Numerical verification
- Resurgence properties checked

Outlook:

- Further observables in supersymmetric gauge theories
- Other limits of the octagon
- Application to other models

[Bajnok,Boldis,Korchemsky'24]

THANK YOU
FOR YOUR ATTENTION!

EuroStrings
2024
Southampton

Summary

Results:

- Observables in planar $\mathcal{N} = 4$ and $\mathcal{N} = 2$ SYM
- Strong coupling expansion
- Systematic method to determine non-perturbative contributions
- Numerical verification
- Resurgence properties checked

Outlook:

- Further observables in supersymmetric gauge theories
- Other limits of the octagon
- Application to other models

[Bajnok,Boldis,Korchemsky'24]