Can Rotation Solve the Hubble Puzzle?[1]

Theory and Experiment in High Energy Physics, V4-HEP Workshop

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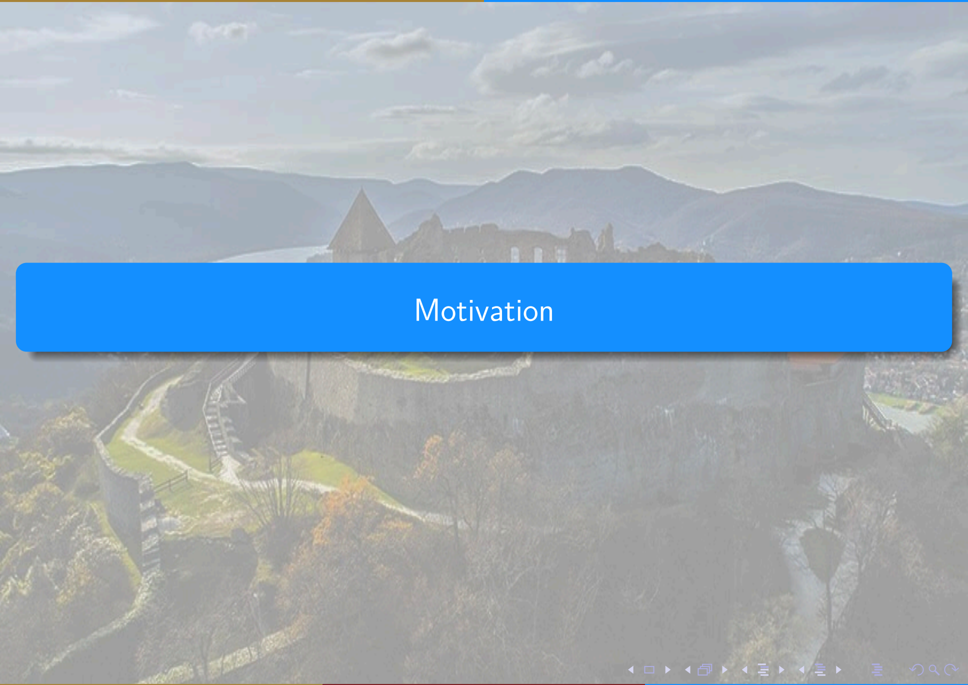
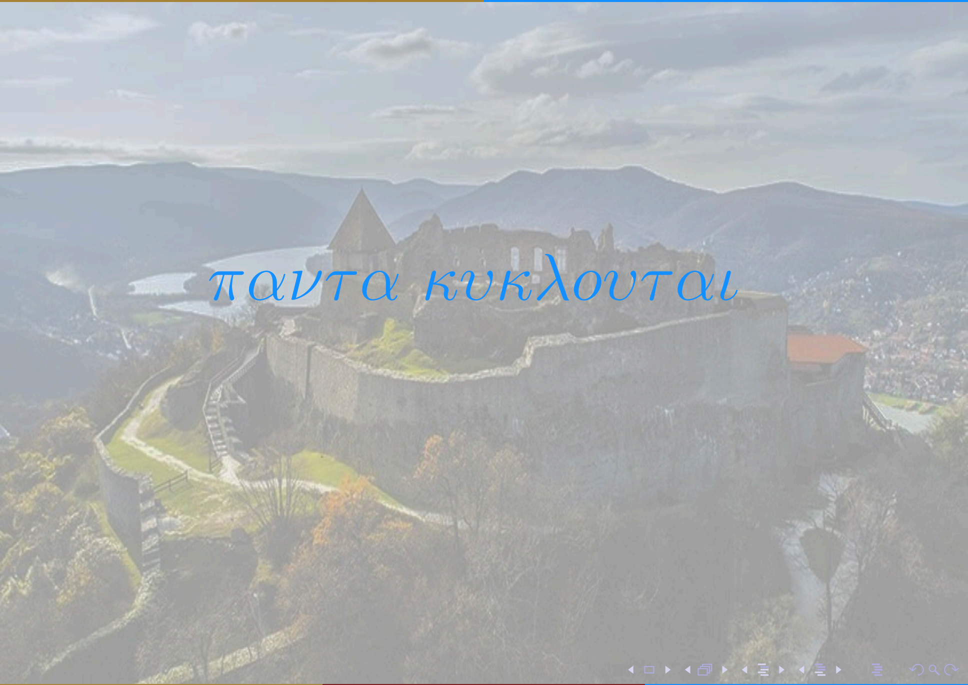
[1]OTKA K135515, 2021-4.1.2-NEMZ KI-2024-0003, WSCLAB



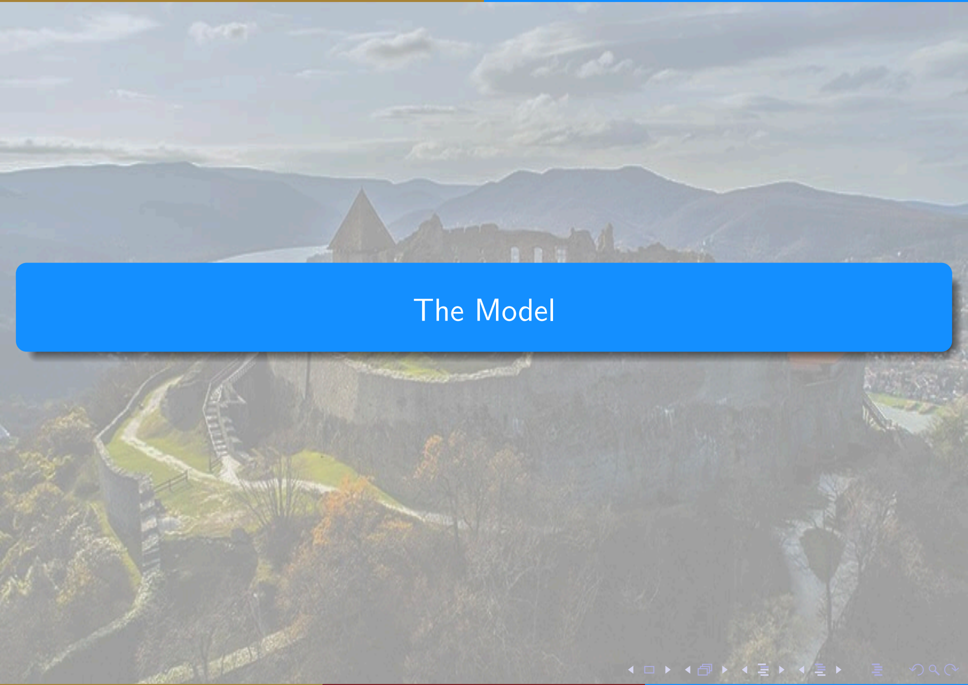
Motivation

Motivation

- **The Hubble tension**, the inconsistency of the late and early time measurements of the universe' s expansion rate
- The discrepancy has been established in a wide range of data sets and reached a 5σ **significance** between cepheid-calibrated local super-novae and cosmic microwave background (CMB) measurements
- $H_{\mathrm{CMB}} = 67.4 \pm 0.5\,\mathrm{kms^{-1}Mpc^{-1}}$ [Planck2018 results, A&A **641**, A6 (2020)]
- $H_{\mathrm{SNe}} = 73.04 \pm 1.07\,\mathrm{kms^{-1}Mpc^{-1}}$ [The Astrophys. Jour. Lett. **934**, L7 (2022)]
- All objects within our Universe rotate. Moreover, BHs, spherically symmetric object with horizons, display near maximal rotation.
- The idea that **everything revolves** naturally extends to the whole universe



$\pi\alpha\nu\tau\alpha\ \kappa\upsilon\kappa\lambda o\upsilon\tau\alpha\iota$



The Model

The Model

- We present a **dark fluid model** described as a non-relativistic and self-gravitating fluid.
- We studied these coupled **non-linear differential equation** systems using self-similar time-dependent solutions
- Our main goal of this research is to find **scaling solutions** of the gravitational fields, which can be good candidates to describe the evolution of the Universe or collapse of compact astrophysical objects.

Euler-Poisson Equation

Continuity Equation:

$$\partial_t \rho + \mathrm{div}(\rho \boldsymbol{u}) = 0$$

Euler Equation:

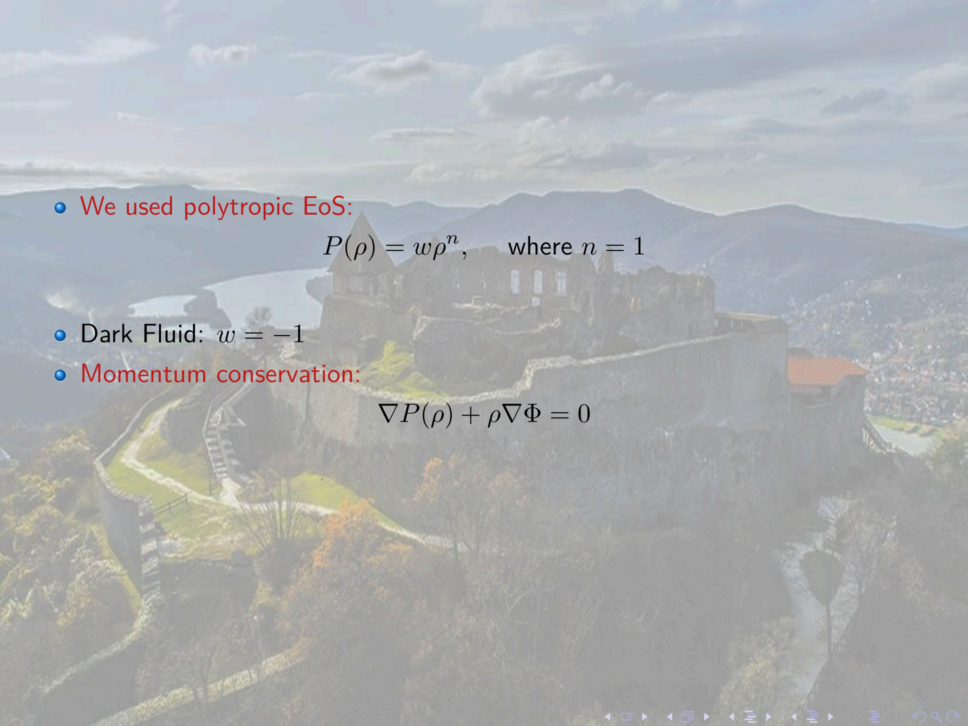
$$\partial_t(\rho \boldsymbol{u}) + \mathrm{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) = -\nabla P(\rho) - \rho \nabla \Phi + \rho \boldsymbol{g}^*$$

Poisson Equation:

$$\nabla^2 \Phi = 4\pi G \rho.$$

Equation of Stae (EoS)

$$P = P(\rho) \rightarrow (\mathrm{Baritropic})$$



- We used polytropic EoS:

$$P(\rho) = w\rho^n, \quad \text{where } n = 1$$

- Dark Fluid: $w = -1$
- Momentum conservation:

$$\nabla P(\rho) + \rho \nabla \Phi = 0$$

Rotation, Spherical symmetry

- Effective rotating term:

$$\rho \boldsymbol{g} = \frac{\rho \sin\theta \omega^2 r}{t^2} \quad \omega : \text{angular velocity}$$

- Rotation is slow! $\Rightarrow$ Asymptotic spherically symmetry
- Spherical Symmetry:

$$\partial_t \rho + (\partial_r \rho) u + (\partial_r u) \rho + \frac{2u\rho}{r} = 0,$$

$$\partial_t u + (u \partial_r) u = -\frac{1}{\rho} \partial_r P - \nabla \Phi + \frac{\sin\theta \omega^2 r}{t^2},$$

$$\Delta \Phi = 4\pi G \rho \ .$$

$$P = P(\rho) \ .$$

Self-Similarity

- Self-similarity in 1D $\Rightarrow$ Sedov – Taylor *ansatz*
  G. I. Taylor, British Report RC-210, June 27, (1941)
  IF Barna, MA Pocsai, GG Barnaföldi Mathematics 10 (18), 3220 (2022)

$$u(r,t) = t^{-\alpha} f\left(\frac{r}{t^{\beta}}\right) \quad \rho(r,t) = t^{-\gamma} g\left(\frac{r}{t^{\beta}}\right)$$

$$\Phi(r,t) = t^{-\delta} h\left(\frac{r}{t^{\beta}}\right),$$

- (f, g, h) shape-functions only depend on $\zeta = rt^{-\beta}$
- Similarity exponents: $\alpha, \beta, \gamma, \delta$
- The β describes the rate of spread of the spatial distribution
- Other exponents describe the rate of decay of the intensity of the corresponding field

Self-Similar equation

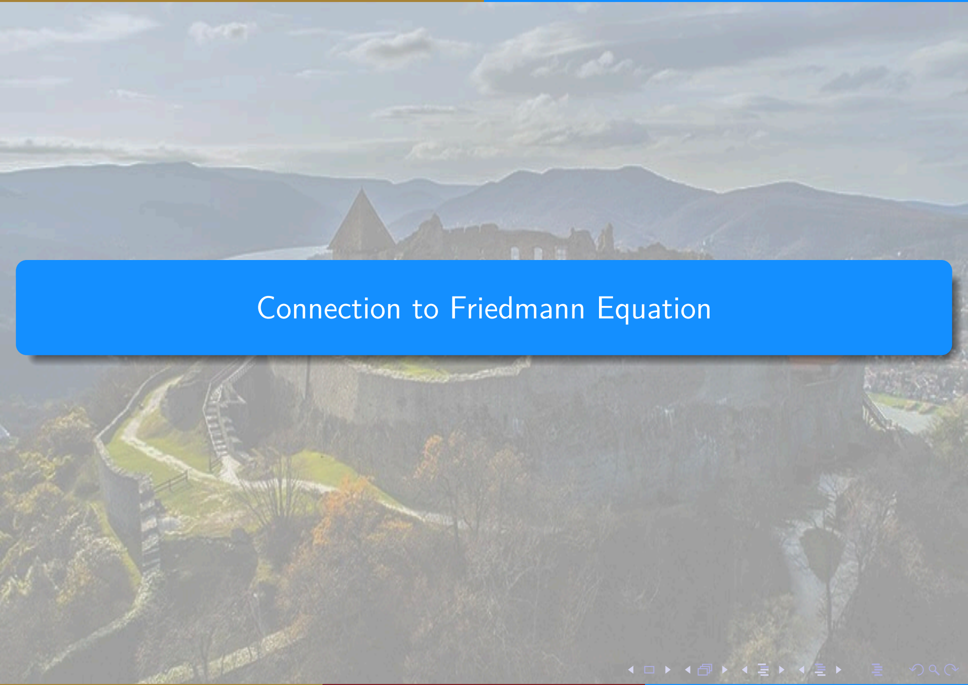
- Self-Similarity: PDE reduce to ODE
- Depend only on ζ self-similar variable
- Algebraic equation system for the exponents
  $\Rightarrow \alpha = 0,\ \beta = 1,\ \gamma = 2$, and $\delta = 0$

$$-\zeta g'(\zeta) + f'(\zeta)g(\zeta) + f(\zeta)g'(\zeta) + \frac{2f(\zeta)g(\zeta)}{\zeta} = 0,$$

$$-\zeta^2 f'(\zeta) + \zeta f'(\zeta)f(\zeta) + \frac{wg'(\zeta)}{g(\zeta)} = -h'(\zeta)\zeta + \omega^2 \sin\theta\zeta^2,$$

$$h'(\zeta) + h''(\zeta)\zeta = g(\zeta)4\pi G\zeta\ .$$

- **Solve this numerically!**



Connection to Friedmann Equation

Newtonian Friedmann Equation

- We are introducing a well-known scale-factor $a(t)$ which contains all of the temporal changes
- Relative distances in time: $R(t) = a(t)l$
- $\Omega(t) \subset \mathbb{R}^3$ is a sphere with radius $R(t)$ and $r \in (0, R(t))$

Mass

$$M(t) = \int_{\Omega(t)} \rho(R(t), t) dV = 4\pi \int_0^r \rho(R(t), t) R(t)^2 dR(t)$$

Mass Conservation

$$\frac{d}{dt} M(t) = 4\pi \frac{d}{dt} \int \rho(a(t)l, t) a^3(t) l^2 dl \stackrel{!}{=} 0$$

Self-similarity and Friedmann

First Friedmann Equation

$$\frac{\frac{d}{dt}[\rho(a(t)l,t)]}{\rho(a(t)l,t)} = -3\frac{\dot{a}(t)}{a(t)}$$

Kinematic Condition

$$\frac{d}{dt}R(t) = u(R(t),t) \Rightarrow \frac{\frac{d}{dt}\left[t^{-\gamma}g(R(t),t)\right]}{g(R(t),t)} = -3\frac{t^{-\alpha}f(R(t),t)}{R(t)}$$

Power series in the similarity variable

$$\rho(r,t) \sim t^{-\gamma} \sum_n^\infty \rho_n \zeta^n \text{ and } u(r,t) \sim t^{-\alpha} \sum_n^\infty u_n \zeta^n$$

In the relevant space and time scale

- $\rho(r,t) \sim t^{-\gamma} A \zeta^\kappa$, where $\kappa \in \mathbb{R}^+$

We assume, that

$\rho(r,t) \sim t^{-\gamma} A \zeta^\kappa$, and $u(r,t) \sim t^{-\alpha} \sum_n^8 u_n \zeta^n$

Non-rotating case: $\omega \to 0$ limit

Non-rotating:

$$u(r,t) \sim t^{-\alpha} \left(u_1 \zeta^1 + u_2 \zeta^2 \right)$$

Assumptions

Summarizing this,

Non-Rotating:	**Rotating:**
$\rho(r,t) \sim t^{-\gamma} A \zeta^{\kappa}$	$\rho(r,t) \sim t^{-\gamma} A \zeta^{\kappa}$
$u(r,t) \sim u_1 \zeta + u_2 \zeta^2$	$u(r,t) \sim t^{-\alpha} \sum_{k=0}^{8} \tilde{u}_k \zeta^k$

- Non-autonomous first-order non-linear differential equation

$$\kappa \dot{R}(t) + 3u_2 t^{-(\alpha+2\beta)} [R(t)]^2 - \frac{1}{t}[\gamma + \kappa\beta] R(t) + 3u_1 R(t) t^{-(\alpha+\beta)} = 0$$

Rotating

For the non-rotating case, the differential equation is

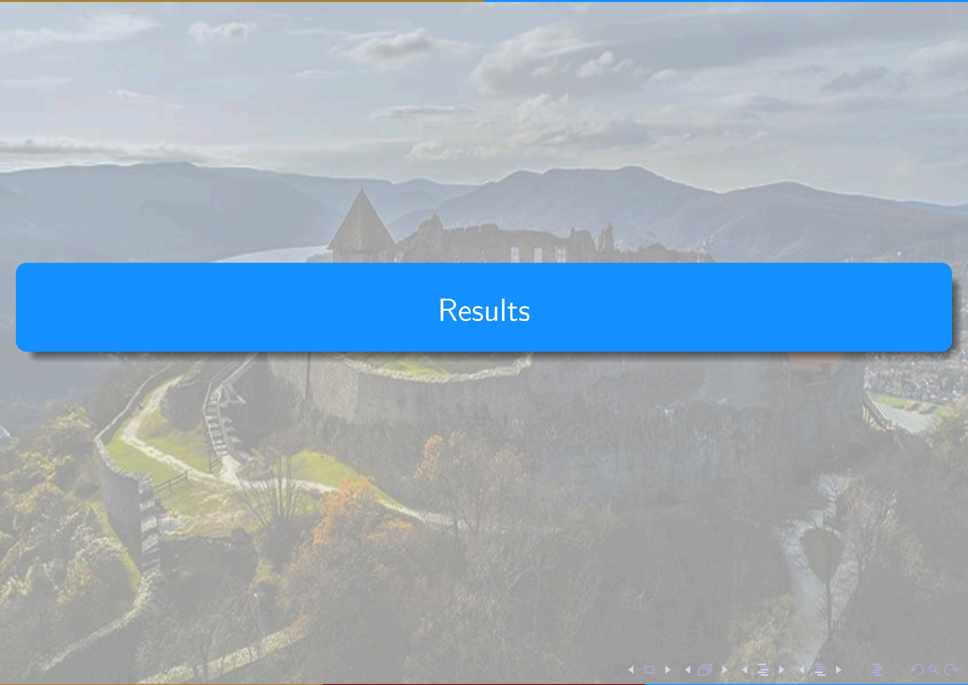
$$\kappa \dot{R}(t) - \frac{1}{t}[\gamma + \kappa\beta]R(t) + 3t^{-\alpha}\sum_{k=0}^{8}\tilde{u}_k\left(\frac{R(t)}{t^{\beta}}\right)^k = 0. \tag{1}$$

- It cannot be solved explicitly
- Hubble's law of expansion to determine the C_1 integration constant

$$\left.\frac{\dot{a}(t)}{a(t)}\right|_{t=t_0} = H_0, \text{ if } \quad a(t_0) = 1 \tag{2}$$

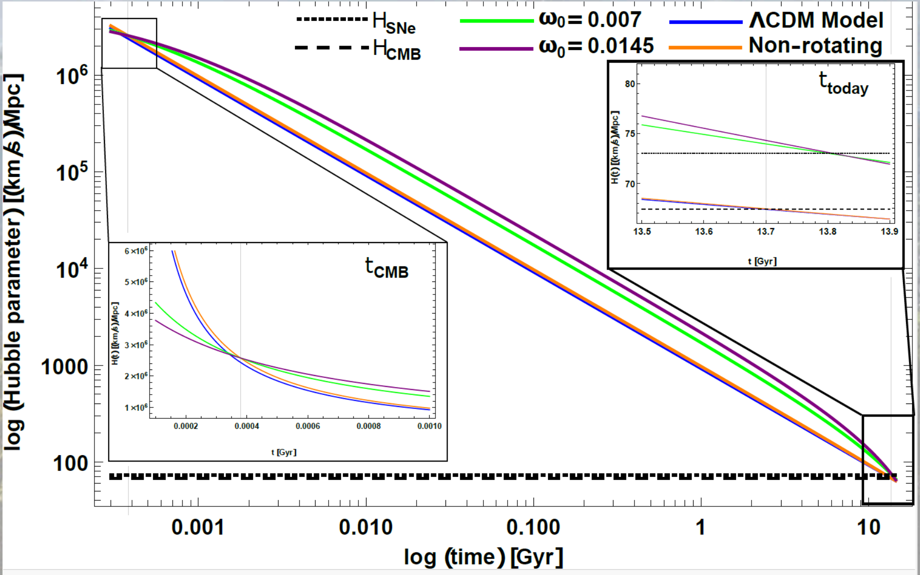
where $H_0 = 66.6^{+4.1}_{-3.3}$ km/s/Mpc[1] is the experimental value of the Hubble-constant.

[1] Kelly, P. L. et al. (2023) Science doi:10.1126/science.abh1322

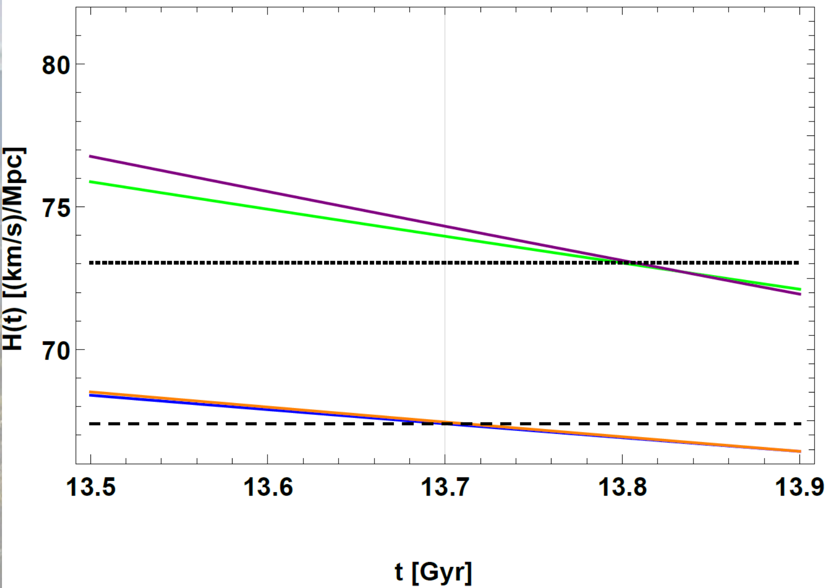


Results

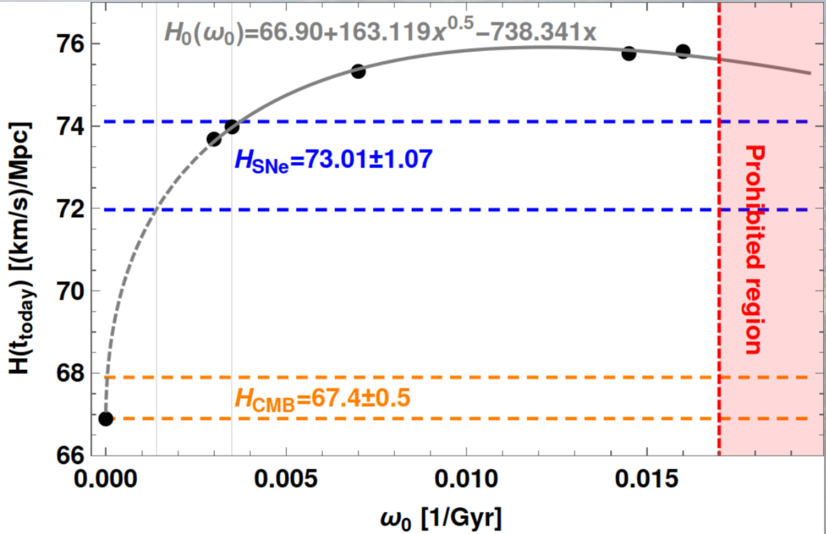
Evolution of the Universe



Let's take a closer look



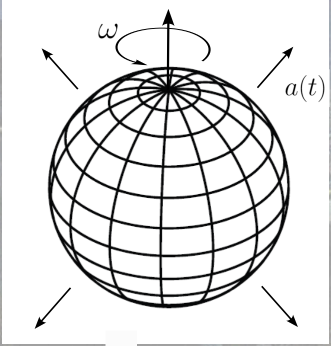
Angular Parameter Dependence



Results

- Different initial rotations result in different H_0 values today, but all solutions converge to zero at the asymptotic limit, $t \to \infty$.
- The evolution of the Hubble parameter with $\omega(t) \to 0$ limit approaches the non-rotating model.
- As a test, we compare our model with the standard ΛCDM result $a(t) = (3H_{\text{CBM};0}t/2)^{2/3}\,\Omega_m^{1/3}$, with the value of $\Omega_m = 0.3089$.)

Schematic figure of the reference frame



Results

- Numerical extrapolation for the Hubble constant, H_0 with $\omega_0 = 0.002^{+0.001}_{-0.001}$ GYr^{-1} predicts a value today comparable to the measured by H_{SNe}.
- The present day ω_0 rotation corresponds to an initial, $\omega(t_{\mathrm{CMB}}) = 0.015 - 0.04$ GYr^{-1}, where H_{CMB} is measured at $t_{\mathrm{CMB}} = 380$ kyr.
- We require that the speeds remain below the speed of light within the observable horizon, hence $\omega \lesssim H$. Taking $H(a) \sim a^{-3/2}$, we estimate ω_0 today as,

$$\omega_0 \lesssim H_0 a^{1/2}(t_{\mathrm{today}}) \simeq 2 \times 10^{-3} \mathrm{Gyr}^{-1}. \tag{3}$$

Most remarkably, the allowed maximal rotation is approximately the same as the one required to solve the Hubble Puzzle.

THANK YOU!