Probing hadron-quark mixed phase in twin stars using f-modes

arXiv:2309.08775

David Álvarez Castillo

Institute of Nuclear Physics PAS
Cracow, Poland

UANL
UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

CSSF
INCUBATOR OF SCIENTIFIC EXCELLENCE

Uniwersytet Wrocławski

RESEARCH UNIVERSITY
EXCELLENCE INITIATIVE

Theory and Experiment in High Energy Physics
V4-HEP workshop @ Wigner Research Centre for Physics
Budapest, Hungary
14 of March 2024

dalvarez@ifj.edu.pl

CREDO
THE QUEST FOR THE UNEXPECTED

PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

compstar

Outline

- A brief introduction to the physics of compact stars.
- The neutron star twins scenario and the role of pasta phases.
- Study of f-modes in compact stars and their observation with third-generation gravitational wave detectors.

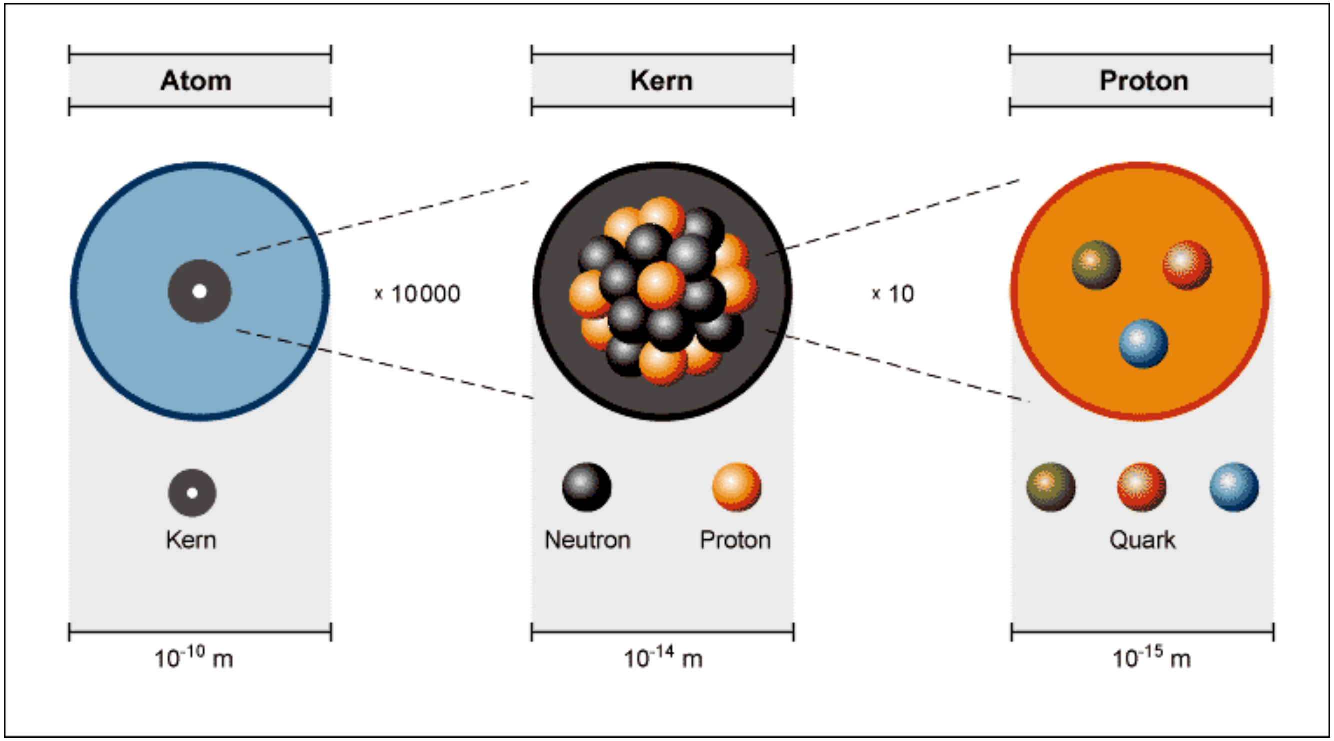
Motivation

- New channels of multi-messenger observations like gravitational radiation from merger events of binary systems of compact stars or radio and X-ray signals from isolated pulsars allow to study their most basic structural properties like mass, radius, compactness, cooling rates and compressibility of their matter.

- Nuclear measurement and experiments have narrowed the Equation of State (EoS) uncertainty in the lowest to intermediate density range.

- Violent, transient energetic emissions are associated not only with the strong magnetic fields and extreme gravity in the proximity of NS but with explosive, evolutionary stages often triggered by mass accretion from companion stars. It is expected that are f-modes are excited in many of these astrophysical processes.

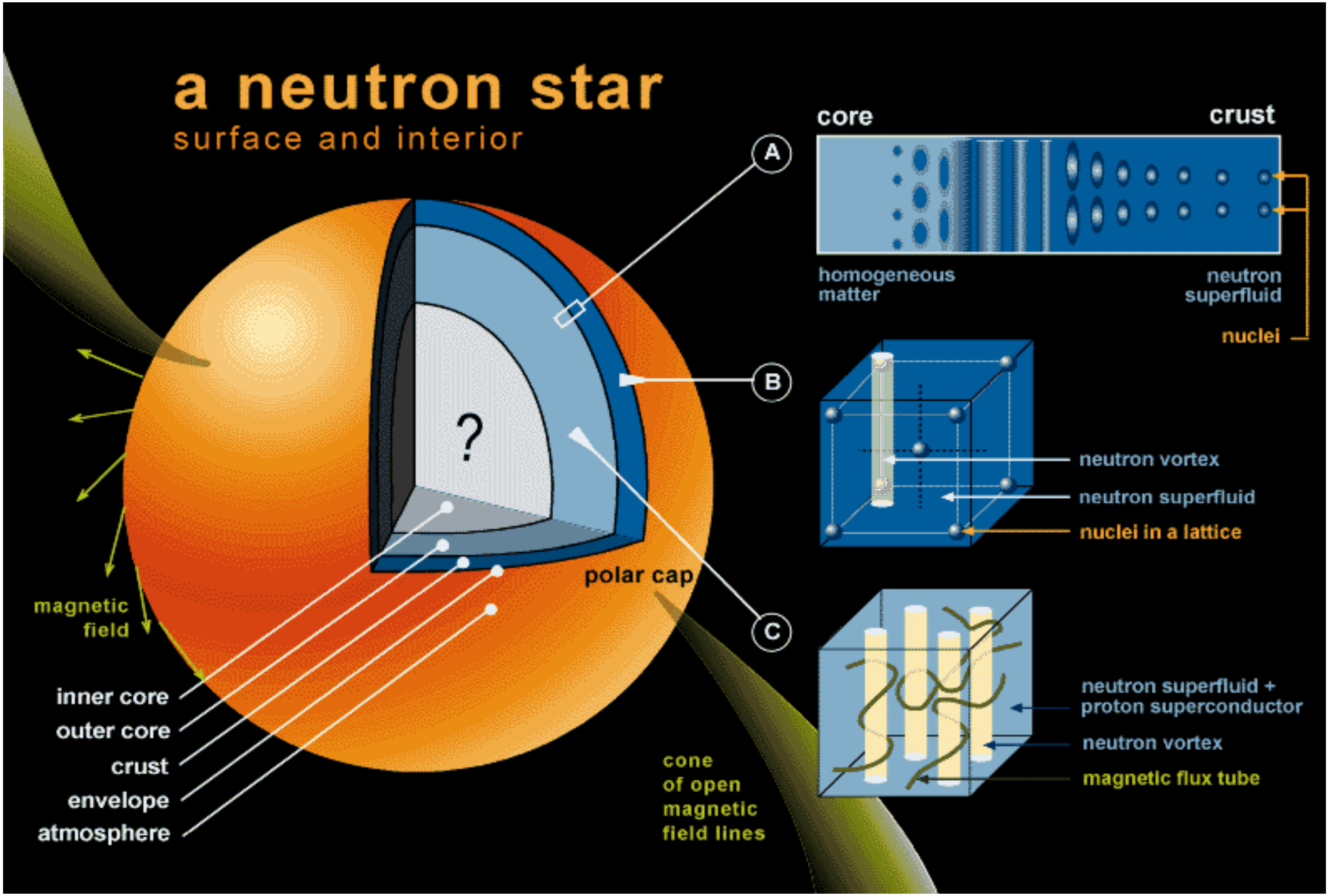
Superdense objects – what is inside?



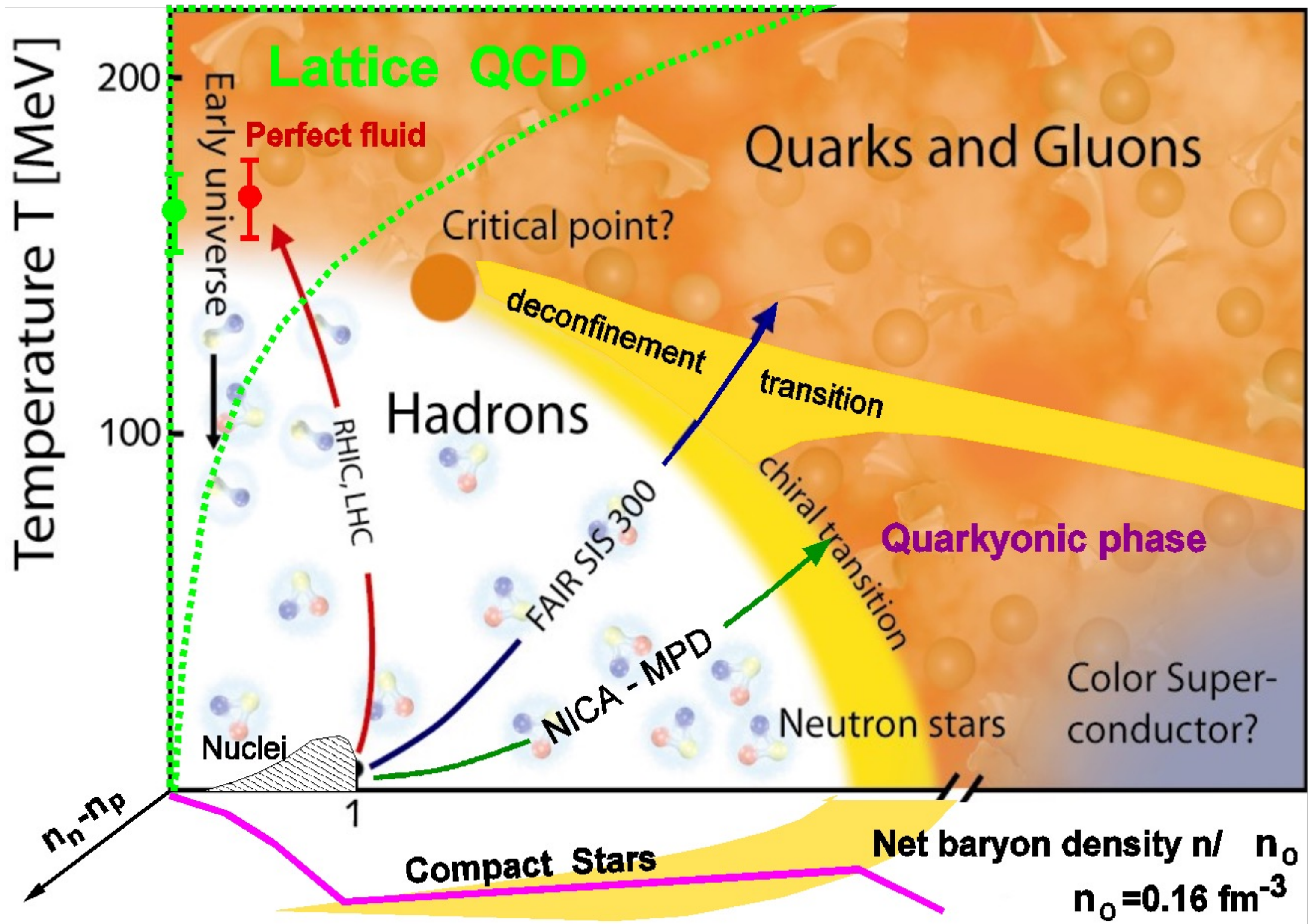
Nucleus, A nucleons: R_A=1.2 10^{-13} cm $A^{1/3}$; ρ_0=A1.67 10^{-24} g/(4π/3 R_A^3)= 2.3 10^{14} g/cm^3

Neutron star: R= 10 km; ρ= 2 Mo/(4π/3 R^3) = 4 10^{33} g/(4 10^{18} cm^3)= 10^{15} g/cm^3 = 4 ρ_0

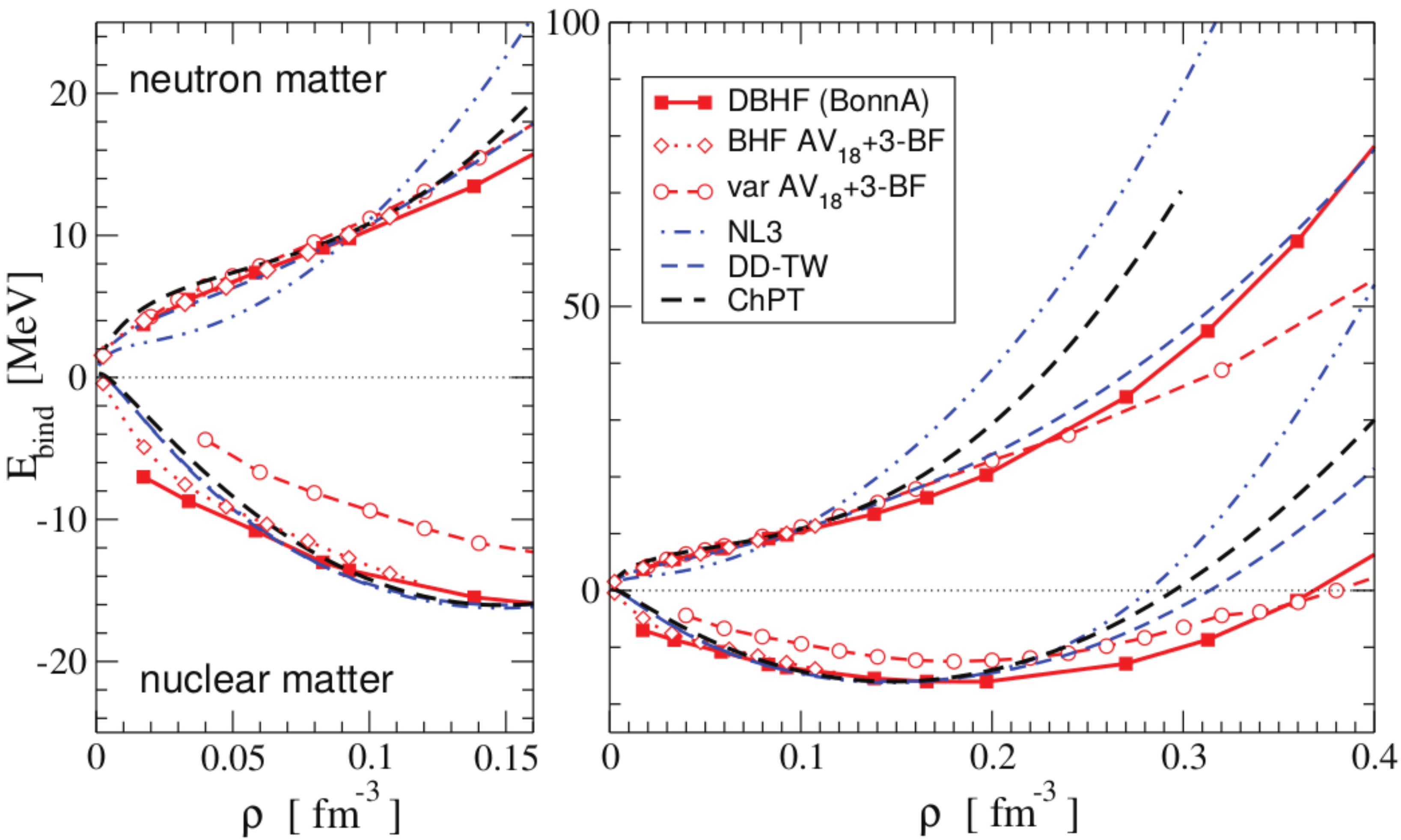
Superdense objects – what is inside?



Critical Endpoint in QCD



Nuclear Matter



C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\mathrm{tot}}(n,\{x_i\}) &= E_{\mathrm{b}}(n,x_p)+E_{\mathrm{lep}}(n,x_e,x_\mu)\ , \\ E_{\mathrm{b}}(n,x_p) &= E_0(n)+S(n,x_p) \\ E_{\mathrm{lep}}(n,x_e,x_\mu) &= E_e(n,x_e)+E_\mu(n,x_\mu)\ , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n, x_p) = (1 - 2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n,x_l) = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2}\left[\sqrt{1+z_l^2}\left(1+\frac{z_l^2}{2}\right) - \frac{z_l^4}{2}\mathrm{Arsinh}\left(\frac{1}{z_l}\right)\right]\ ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$E_l(n,x_l)\big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4}\left(3\pi^2 n\right)^{1/3} x_l^{4/3}\ .$$

Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i, j = n, p, e, \mu \ ,$$

where $\varepsilon_i = n \ E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x) E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

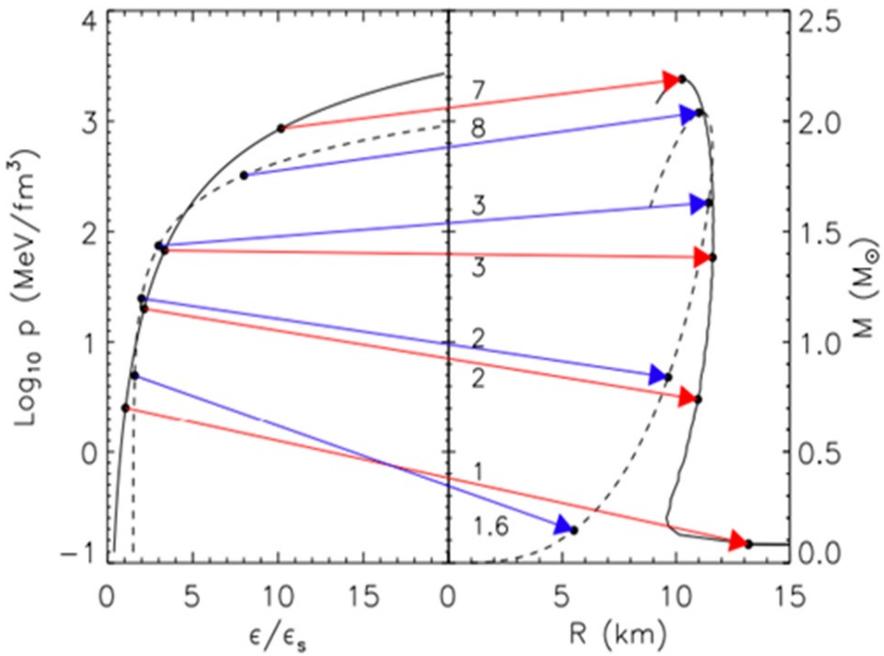
$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\rm tot}}{\partial n} \right)$.

Compact Star Sequences

(M-R ⇔ EoS)



James Lattimer,
Annu. Rev. Nucl. Part. Sci.
62, 485 (2012),
arXiv:1305.3510

- TOV Equations

$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

- Equation of State (EoS)

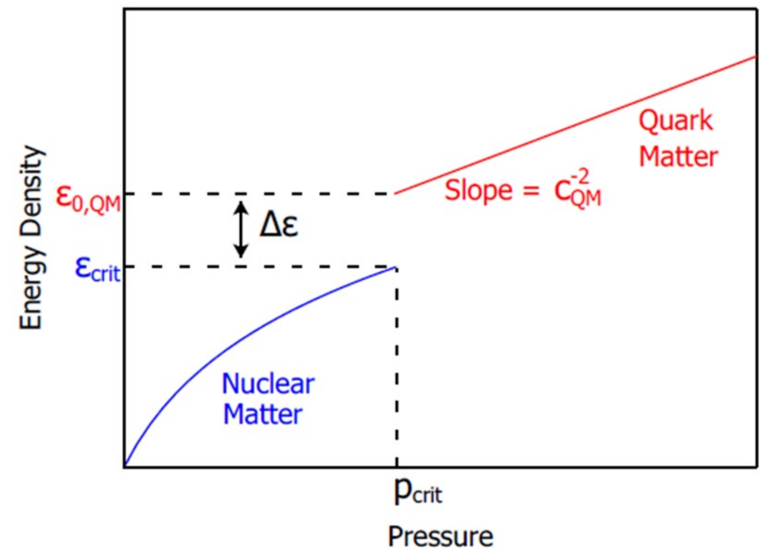
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon \qquad p(\varepsilon)$$

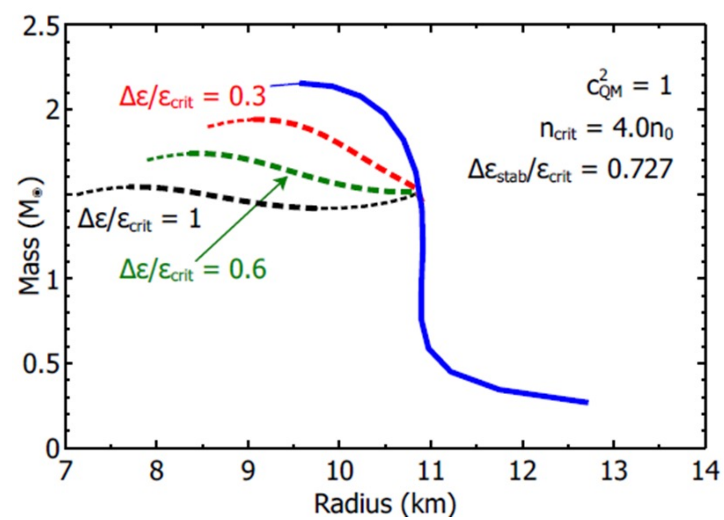
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the "latent heat" (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → "**third family of CS**".

- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)
arxiv:1302.4732



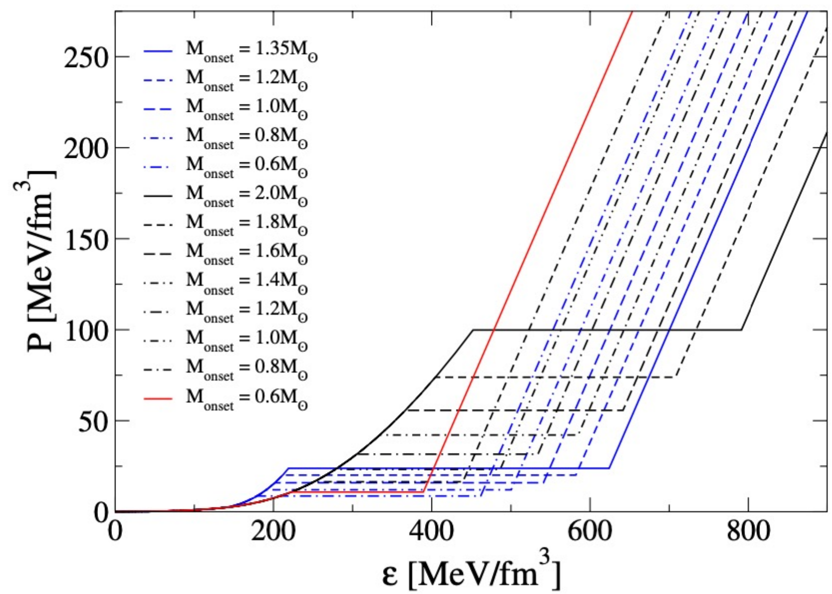


Compact Star Mass Twins

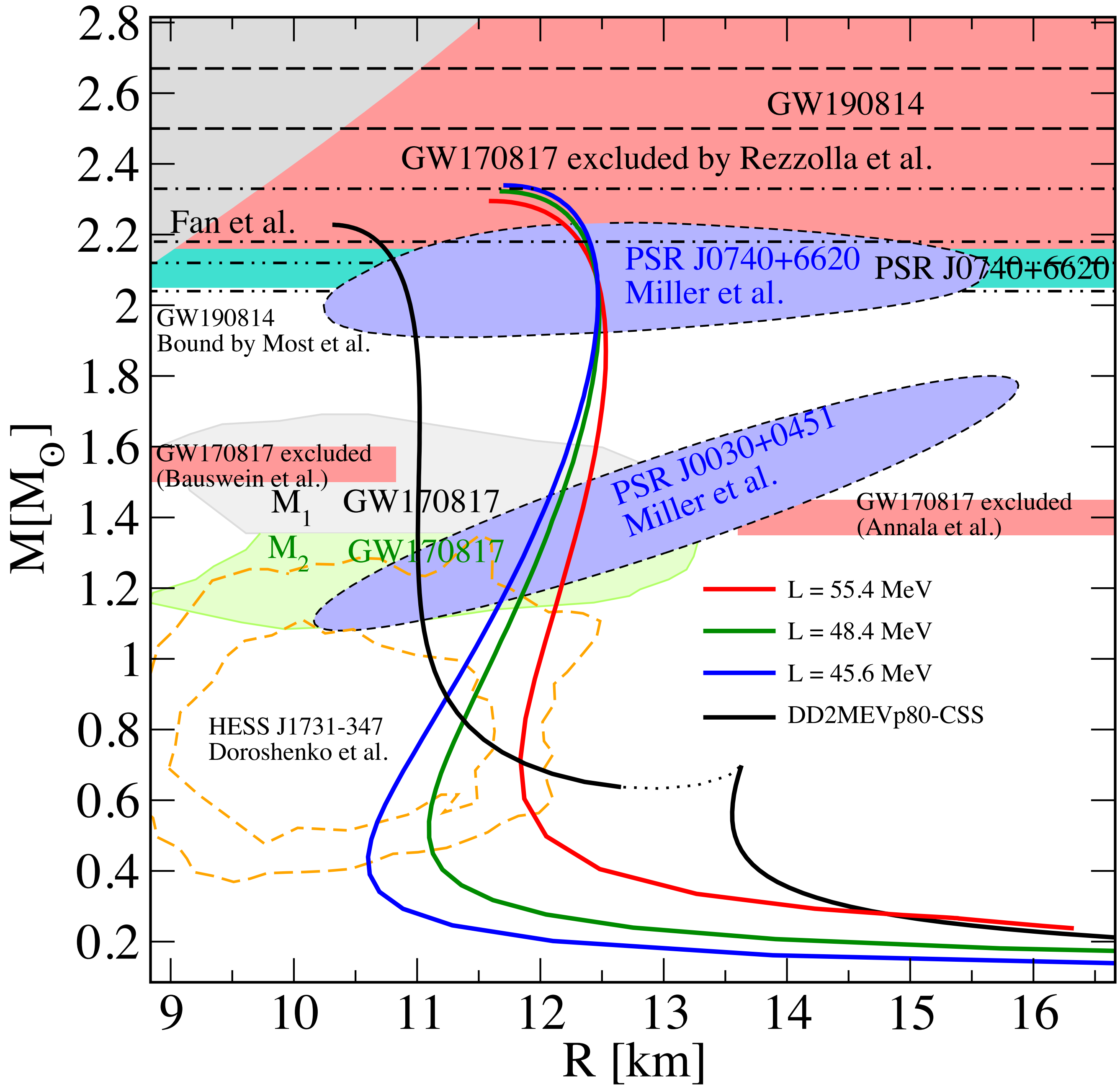
$$\varepsilon(p) = \begin{cases} \varepsilon_{\rm NM}(p) & p < p_{\rm trans} \\ \varepsilon_{\rm NM}(p_{\rm trans}) + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm trans}) & p > p_{\rm trans} \end{cases}$$

Model	M_{onset} $[\mathrm{M}_\odot]$	n_{trans} $[1/\mathrm{fm}^3]$	ε_{trans} $[\mathrm{MeV/fm}^3]$	p_{trans} $[\mathrm{MeV/fm}^3]$	$\Delta\varepsilon$ $[\mathrm{MeV/fm}^3]$	c_{QM} $[c]$
DD2p80	0.7	0.193	185.223	10.3131	268.573	0.9

$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}}$$



D. Alvarez-Castillo - Astron. Nachr. 1–6 (2021)



RMF models: arXiv:2307.02979, Phys. Rev. C 108, 054010 (2023)

Piecewise polytrope EoS

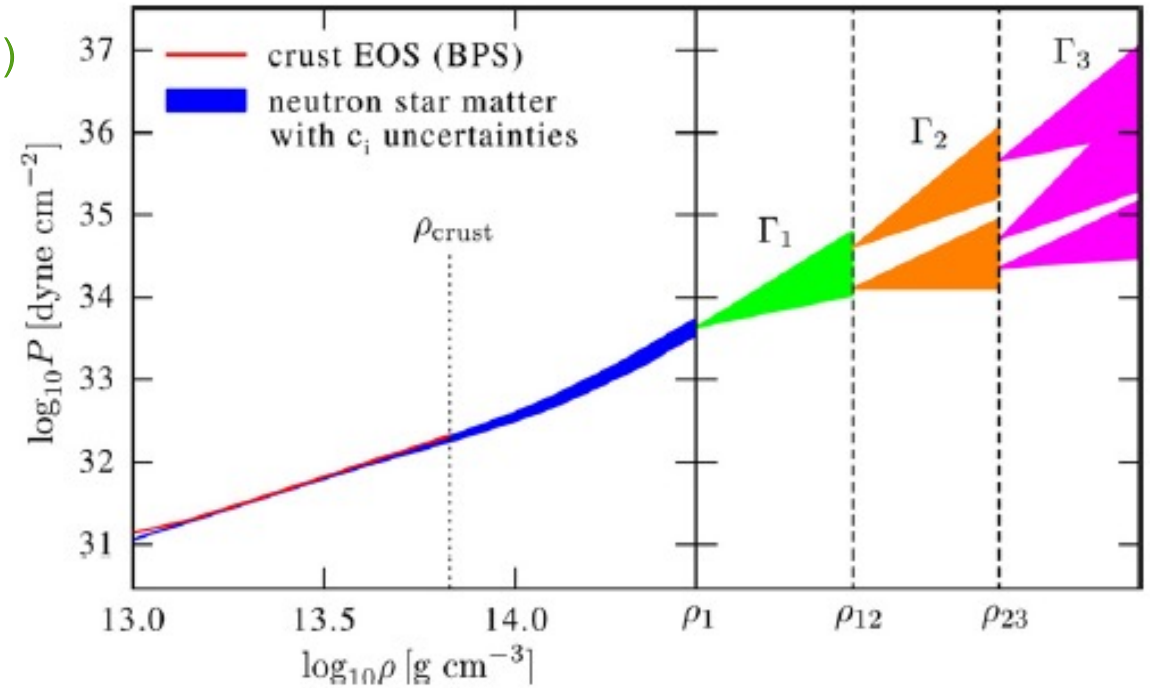
Hebeler et al., ApJ 773, 11 (2013)

$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$i = 1 \ : \ n_1 \le n \le n_{12}$$
$$i = 2 \ : \ n_{12} \le n \le n_{23}$$
$$i = 3 \ : \ n \ge n_{23} \ ,$$

Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\rm crit}$$



$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn},$$

$$\varepsilon(n)/n = \int dn \, \frac{P(n)}{n^2} = \int dn \, \kappa n^{\Gamma-2} = \frac{\kappa \, n^{\Gamma-1}}{\Gamma-1} + C,$$

$$\mu(n) = \frac{P(n)+\varepsilon(n)}{n} = \frac{\kappa \, \Gamma}{\Gamma-1} n^{\Gamma-1} + m_0,$$

$$n(\mu) = \left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{1/(\Gamma-1)}$$

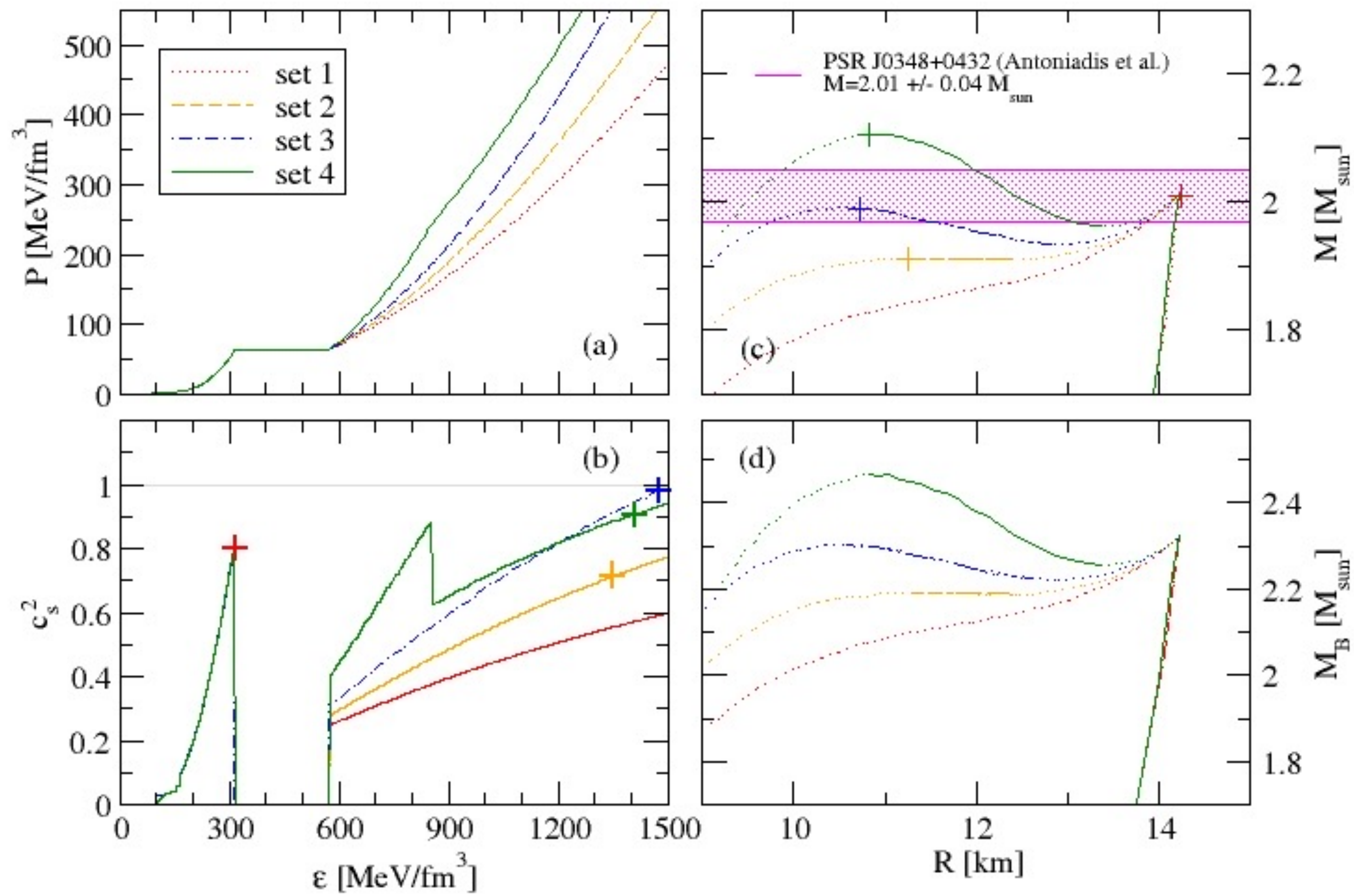
$$P(\mu) = \kappa \left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}$$

Maxwell construction:

$$P_1(\mu_{\rm crit}) = P_3(\mu_{\rm crit}) = P_{\rm crit}$$
$$\mu_{\rm crit} = \mu_1(n_{12}) = \mu_3(n_{23})$$

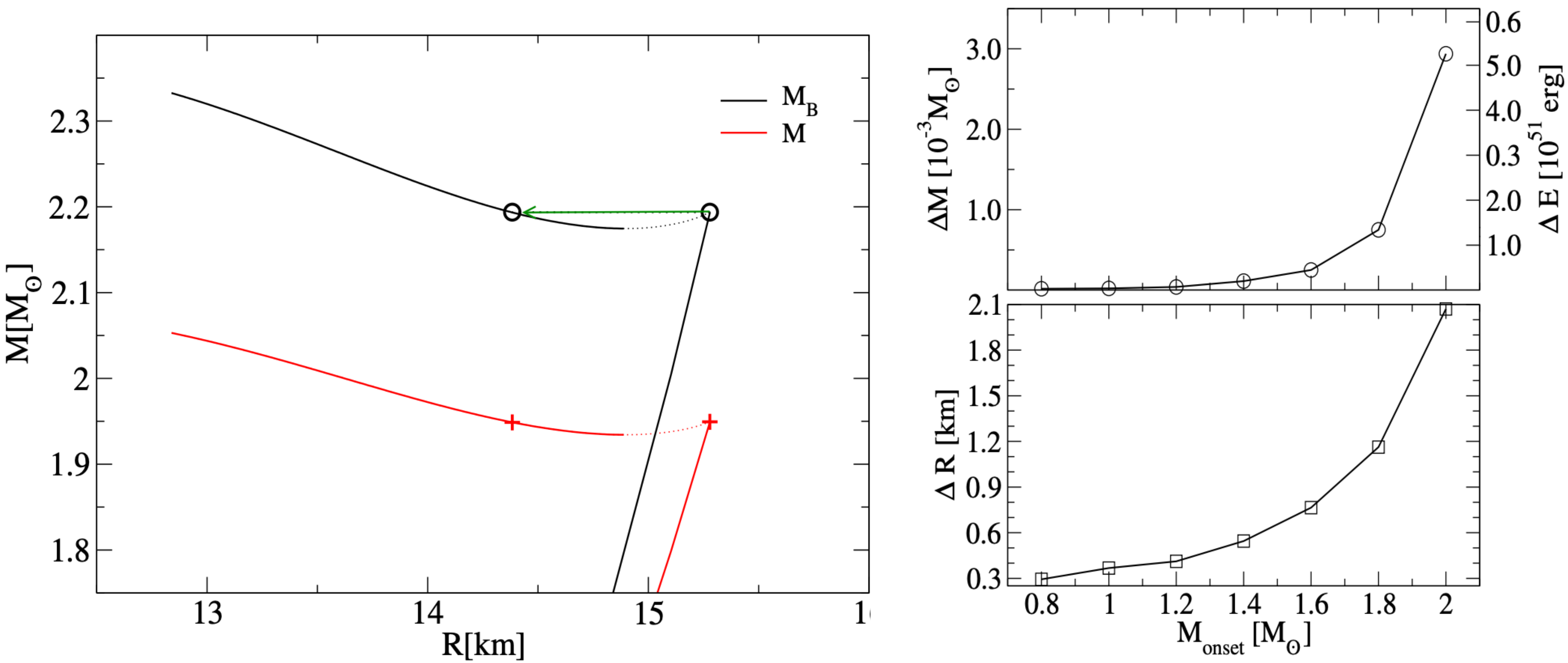
Seidov criterion for instability: $\frac{\Delta\varepsilon}{\varepsilon_{\rm crit}} \ge \frac{1}{2} + \frac{3}{3}\frac{P_{\rm crit}}{\varepsilon_{\rm crit}}$

Compact Star Twins



Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

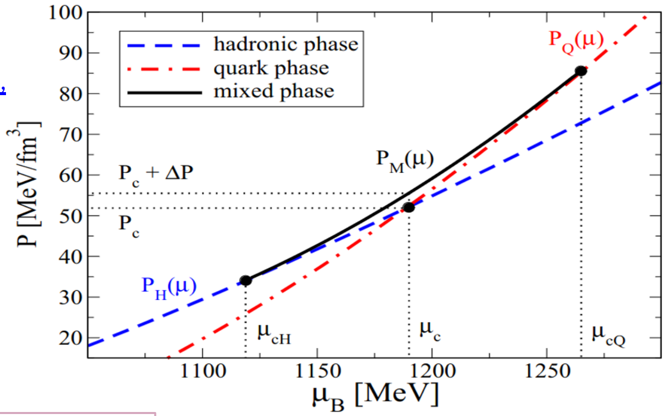
Mass Twins – Energy Released



DD2MEV-CSS EoS, D. A-C, Astronomischen Nachrichten (2021) 1–6, arXiv: 2011.11145

The EoS Model : Phenomenological Description

➢ A. Ayriyan , H. Grigorian, EPJ Web of Conferences. p. 03003,(2018),
➢ A. Ayriyan, N. Bastian, D. Blaschke, H. Grigorian, K. Maslov, D. N. Voskresensky, PRC 97, 045802 (2018), ,
➢ V. Abgaryan,D. Alvarez-Castillo, A. Ayriyan, D. Blaschke,H. Grigorian, Universe, 4, 94 (2018).



★ Surface tension effect leads to existence of pasta phases.
★ A parabolic interpolation method used to construct the mix ph

$$p(\mu) = \begin{cases} p^H(\mu), & \mu \le \mu_{cH}, \\ P^M(\mu) = \alpha_2(\mu-\mu_c)^2 + \alpha_1(\mu-\mu_c) + P_c + \Delta P, & \mu_{cH} \le \mu \le \mu_{cQ}, \\ p^Q(\mu), & \mu \ge \mu_{cQ} \end{cases}$$

$\alpha_1, \alpha_2, \mu_{cH}, \mu_{cH}$ → Determined from the continuity of pressure and its derivative.

★ Mix Phase is parametrized by $\Delta p = \Delta P/P_c$.
★ $\Delta p = 0$: Maxwell Construction.

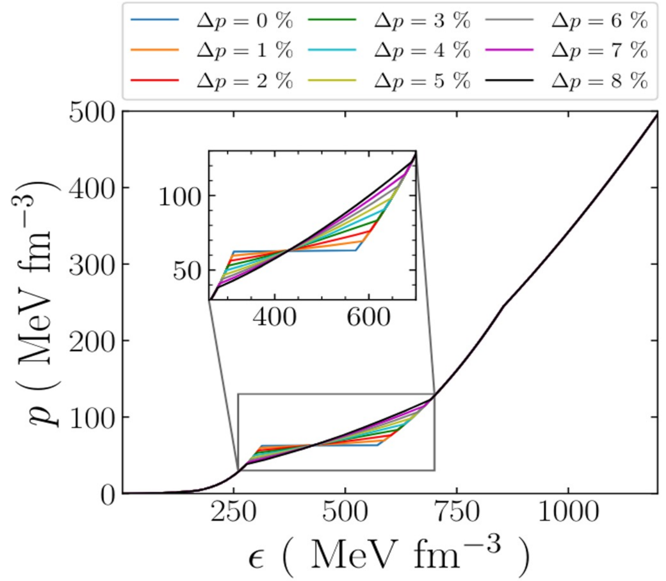
ACB4 Parametrization:

➢ D. E. Alvarez-Castillo., D. Blaschke, PRC, 96, 045809,(2017) ,
➢ V. Paschalidis, K. Yagi, D. Alvarez-Castillo, D Blaschke, A Sedrakian, PRD, 97, 084038, (2018).

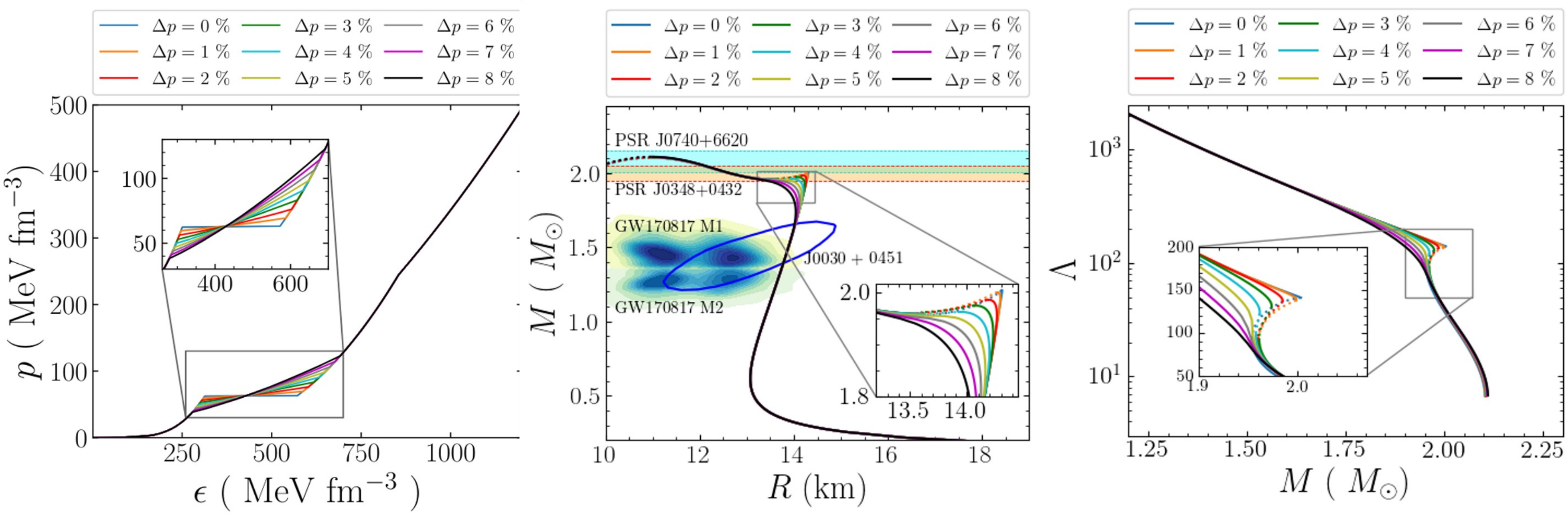
$$P(n) = \kappa_i \left(\frac{n}{n_0}\right)^{\Gamma_i}, \quad n_i < n < n_{i+1}, \; i = 1, ..4$$

$$P(\mu) = \kappa_i \left[(\mu - m_{0,i})\frac{\Gamma_i - 1}{\kappa_i \Gamma_i}\right]^{\frac{\Gamma_i}{(\Gamma_i - 1)}}$$

i	Γ_i	κ_i [MeV fm^{-3}]	n_i [fm^{-3}]	$m_{0,i}$ [MeV]
1	4.921	2.1680	0.1650	939.56
2	0.0	63.178	0.3174	939.56
3	4.00	0.5075	0.5344	1031.2
4	2.80	3.2401	0.7500	958.55

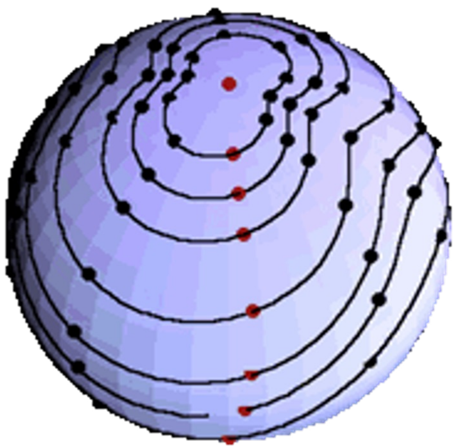


Stellar Properties



- ❖ The second and third family merge to form a single branch for Δp > 4% .
- ❖ Precise measurement of *M-R* required for detection of twin star.
- ❖ The jump ΔΛ (if any) can be measured ~15 % (< 90% CI) with next-generation GW detectors ([P. Landry & K. Chakravarti , arXiv:2212.09733 ,2022]).

Gravitational Waves from NS

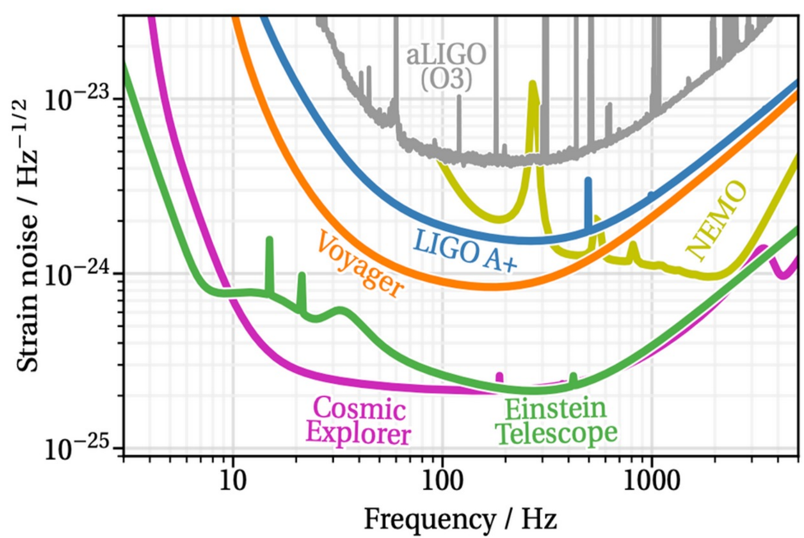


Credit: C. Hanna and B. Owen

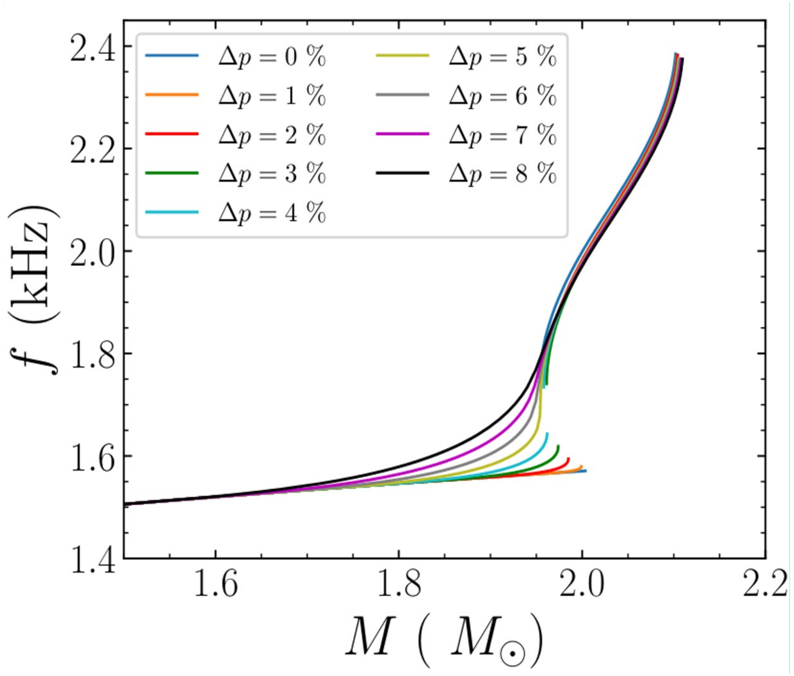
Credit: CERN/Indico

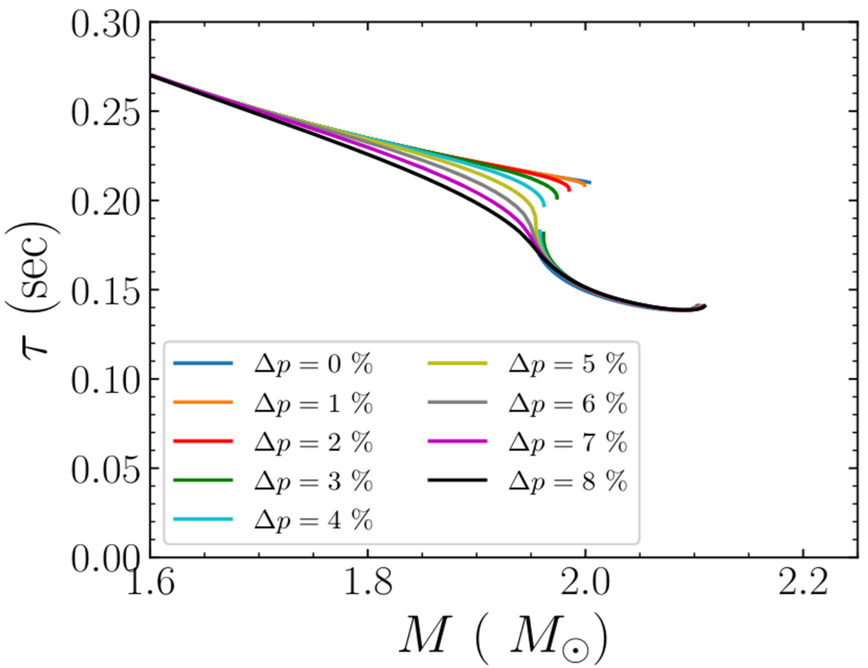
PC: cosmicexplorer.org/sensitivity

- Non-radial QNMs raised from time varying quadrupole deformations are source of GWs.
  - fundamental (f) mode,
    - no node, probe for mean density,(1 kHz < f < 3kHz)
  - pressure (p) mode,
    - Sound speed, (5 kHz < f < 10kHz)
  - gravity (g) mode,
    - (50 Hz < f < 500 Hz)
- R-mode, for rotating stars only.
  - Viscosity, (0.5 kHz < f < 2kHz)
- Space-time (w) mode.
  - 5 kHz< f



f-mode characteristics





- f-mode characteristics are obtained within ***General relativistic*** formalism.
- Sudden increase (decrease) in the frequency (damping time) observed with appearance of twin star.
- Detections of f-mode GWs from compact stars with known mass may reveal the presence of twin stars.
- Simultaneous measurement of *M-f* (from binary system) can be used to comment on twin stars.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775

Asteroseismology and Universal Relations

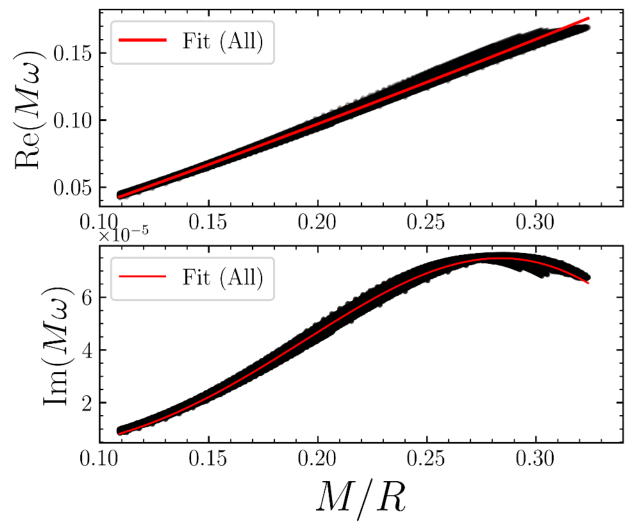
- URs among f-mode characteristics (f, τ_f or ω=2πf+1/τ_f) and NS observables.

$$f(\mathrm{kHz}) = a_r + b_r\sqrt{\frac{M}{R^3}}$$

Empirical relations (EOS dependent)

$$\mathrm{Re}(M\omega) = a_0 + a_1\left(\frac{M}{R}\right) + a_2\left(\frac{M}{R}\right)^2$$

$$\mathrm{Im}(M\omega) = b_0\left(\frac{M}{R}\right)^4 + b_1\left(\frac{M}{R}\right)^5 + b_2\left(\frac{M}{R}\right)^6$$



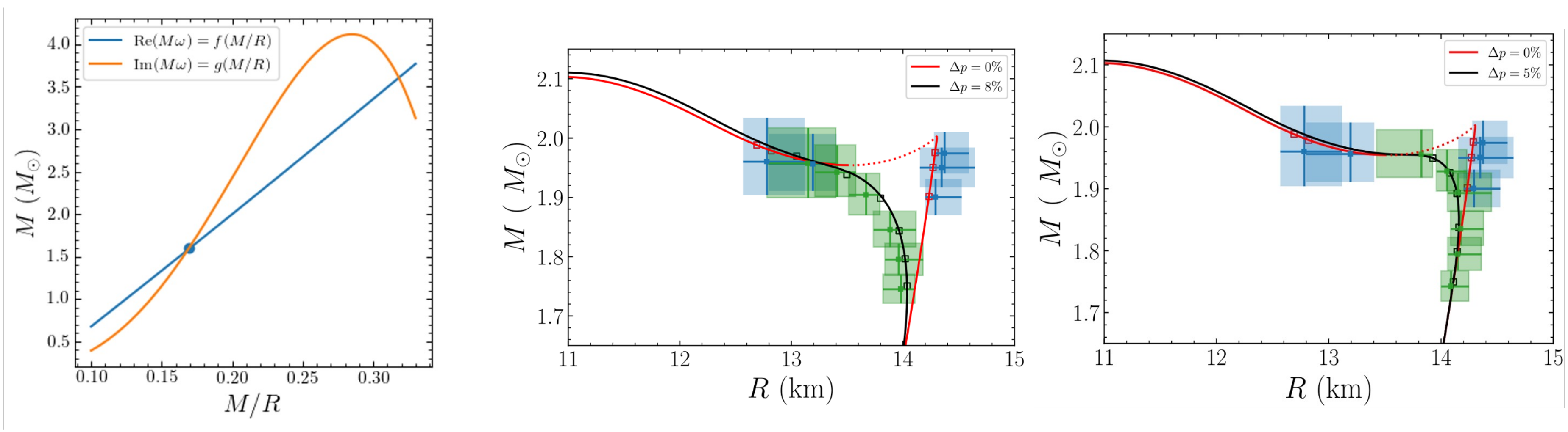
	Re(Mω)		Im(Mω)
a_0	-0.027±9 × 10^{-5}	b_0	(9.81 ± 0.004) × 10^{-2}
a_1	0.610 ± 0.0015	b_1	(−4.444 ± 0.003) × 10^{-1}
a_2	0.049±0.002	b_2	(4.91 ± 0.0045) × 10^{-1}

- Scaled Universal relations are more useful.
- The URs can be used for EoS inference.
- URs involving tidal deformability have also been examined.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775

Compact star observables from f-mode observations: the role of UR Uncertainty

- ❖ Determination under the assumption that *f*, *τ* are measured precisely.
- ❖ Errors on UR results uncertainties on *M*-*R*.



★ The presence of the twins maybe confirmed with exact measurement of *f*, *and τ* .

★ The unstable branch of Δp = 0% can be distinguished from the connecting stable branch of Δp =8%.

★ Differentiating among Δp =0% and Δp =5% is more challenging.

<u>David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775</u>

Inclusion of Observational Uncertainties

- F-mode being excited during pulsar glitches. All the energy radiated through GW.
- The burst waveform is modelled as an exponentially damped oscillation.

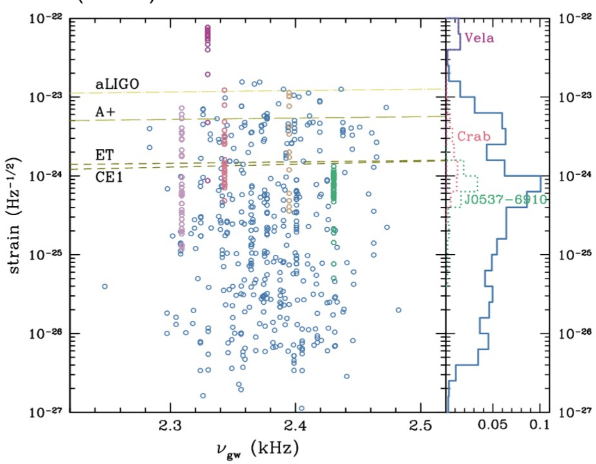
$$h(t) = h_0 \exp\left(-t/\tau_f\right) \sin\left(2\pi\nu_f t\right), \ \ t > 0$$

(B.J. Owen,2010, Ho et al. 2020)

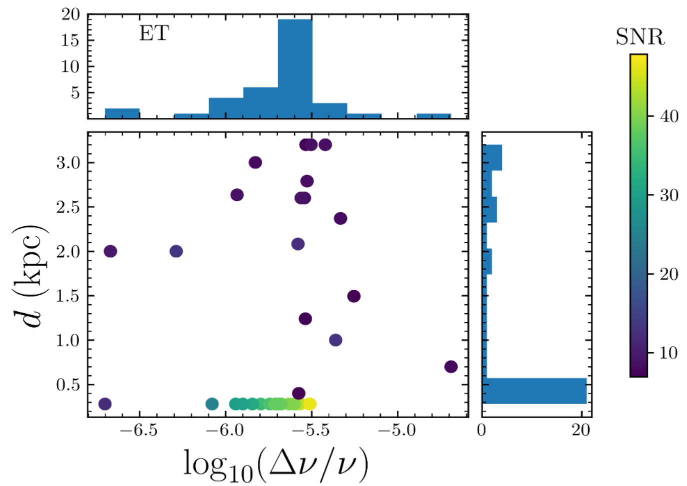
$$h_0 = 4.85\times 10^{-17}\sqrt{\frac{E_{\mathrm{gw}}}{M_{\odot}c^2}}\sqrt{\frac{0.1\mathrm{sec}}{\tau_f}}\frac{1\mathrm{kpc}}{d}\left(\frac{1\mathrm{kHz}}{\nu_f}\right)$$

f-modes GW

$$E_{\mathrm{gw}} = E_{\mathrm{glitch}} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu}\right)$$



Ho et al ,PRD 101, 103009 (2020)



B. K. Pradhan, D. Pathak, and D. Chatterjee, *ApJ* 956 38, (2023)

- B. Abbott et al.,LVC, *ApJ* **874** 163, 2019.
- R. Abbott et al.,LVK, PhRvD, 104, 122004, 2021.
- R. Abbott, et al., LVK, arXiv:2210.10931, 2022.
- R. Abbott, et al., LVK, arXiv:2203.12038, 2022.
- D. Lopez et al., PhRvD, 106, 103037, 2022

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775

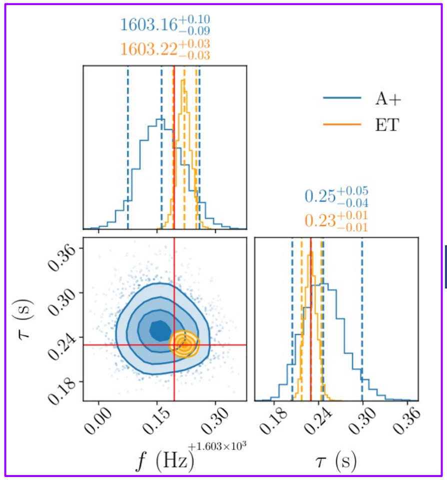
Inclusion of Observational Uncertainties

- Parameter Estimation for GW signal parameters are carried out using **Bilby.**
- Priors are kept,
  - logUniform in E_{gw} .
  - $\nu_f \in$ U[800,3500] Hz.
  - $\tau_f \in$ U[0.05,0.7] s.
  - Distance is fixed.

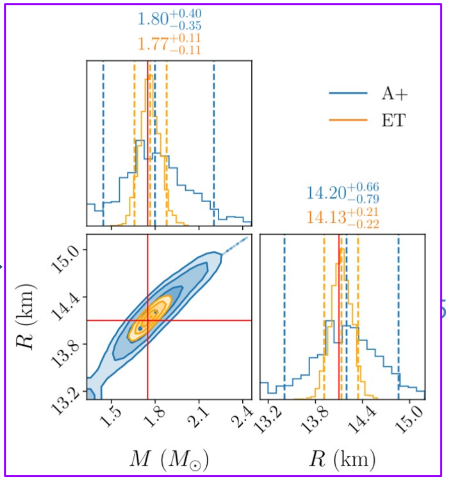
$$h(t) = h_0 \exp\left(-t/\tau_f\right) \sin\left(2\pi\nu_f t\right), \ t > 0$$

$$h_0 = 4.85\times10^{-17}\sqrt{\frac{E_{\rm gw}}{M_\odot c^2}}\sqrt{\frac{0.1{\rm sec}}{\tau_f}}\frac{1{\rm kpc}}{d}\left(\frac{1{\rm kHz}}{\nu_f}\right)$$

- Frequency can be measured accurately in A+ and ET.
- Damping time can have error ~20-50% in A+ and ~5-15% in ET.
- M-R posterior is obtained using UR.
- Within a 90% CI, M can be measured to ~6% in ET.
- Within a 90% CI, R can be measured to ~2% in ET.
- Error on M,R are large in A+.



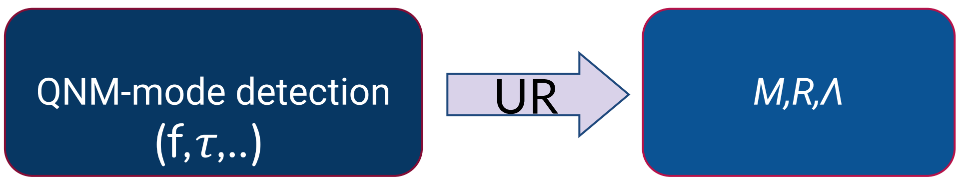
UR

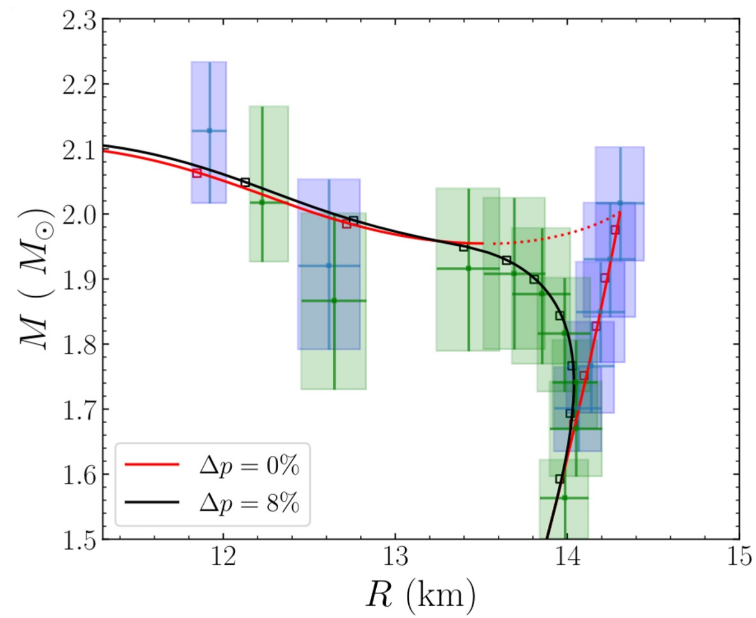


David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775

Inclusion of Observational Uncertainties

- Glitching pulsars data taken from the Jodrell Bank Glitch catalogue.
- Spin frequency, distance (d) and sky position to each pulsar are assign from ATNF Pulsar Catalogue.
- Consider few random mass configurations with an assumed EOS model .
- Then f-mode frequency, damping time, moment of inertia to pulsars from the assume EoS model.





- The measurement of *R* from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins. However, we have more observations at low masses.
- Differentiating the nature of Δp is more challenging.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee - arXiv:2309.08775

Outlook

- Multi-messenger astronomy and collider experiments will continue probing the properties of dense matter.
- Bayesian Analysis and Machine Learning methods are useful for estimation of unknown physical parameters.
- f-mode oscillation of hybrid stars and twin stars involving the “pasta phase” has been investigated.
- Re-examination of the asteroseismology problem considering the twin stars.
- Precise f-mode measurement provides suitable scenario for twin star detection.

Outlook

- f-mode GW detection with next-generation GW offers a promising scenario for confirming the existence of the twin stars.

- Distinguishing the nature of hadron-quark crossover phase transition requires further studies. Interestingly, accretion onto neutron stars can potentially excite fundamental modes in connection with neutrino emissions allowing for probing for phase transitions, see arXiv:2311.15992.

- Consideration of the effect of rotation and magnetic fields can potentially improve this study.

- A detailed Bayesian study is in progress to constrain the pasta phase parameters from f-mode/binary observation.

symmetry

Submit to Special Issue

Submit Abstract to Special Issue

Review for *Symmetry*

Propose a Special Issue

Journal Menu

- *Symmetry* Home
- Aims & Scope
- Editorial Board
- Reviewer Board
- Topical Advisory Panel
- Instructions for Authors
- **Special Issues**
- Topics
- Sections & Collections
- Article Processing Charge
- Indexing & Archiving
- Editor's Choice Articles
- Most Cited & Viewed
- Journal Statistics
- Journal History
- Journal Awards

The Equation of State of Compact Stars

- **Print Special Issue Flyer**
- **Special Issue Editors**
- **Special Issue Information**
- **Keywords**
- **Published Papers**

A special issue of ***Symmetry*** (ISSN 2073-8994). This special issue belongs to the section "**Physics**".

Deadline for manuscript submissions: **30 April 2024** | Viewed by 693

Share This Special Issue

Special Issue Editor

Dr. David Edwin Alvarez Castillo **E-Mail** **Website**
Guest Editor
Institute of Nuclear Physics Polish Academy of Sciences, 31-342 Krakow, Poland
Interests: compact stars; dense matter; equation of state

The Equation of State of Compact Stars

Gracias