AdS/CFT and integrability

how to use 2D integrability in 4D quantum gauge theories

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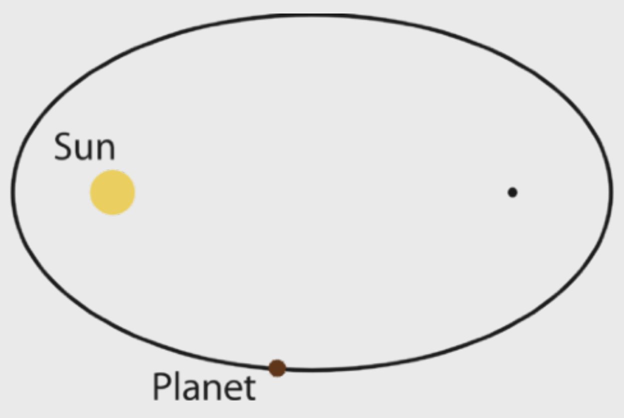
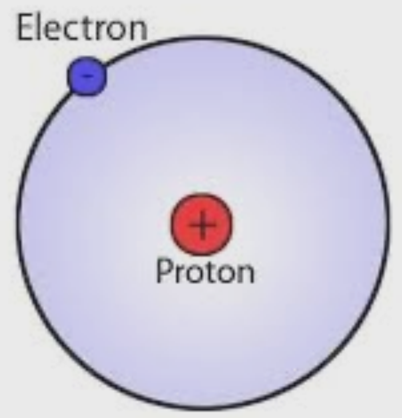
Exactly soluble problems in physics

Kepler's problem

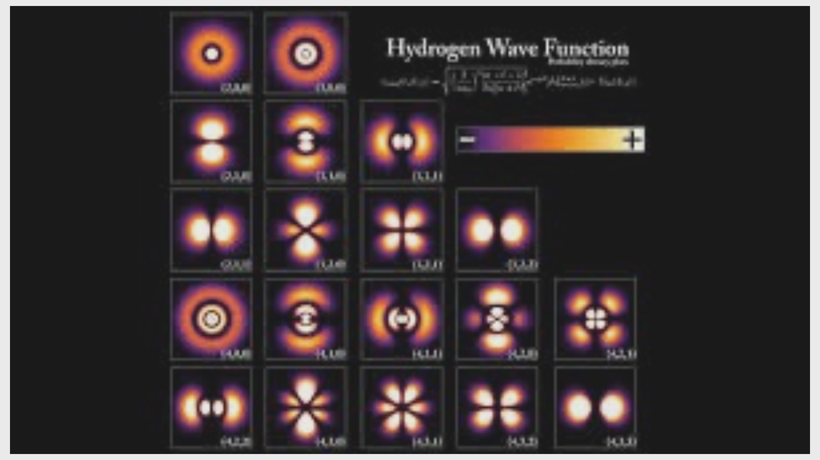


Newtonian mechanics

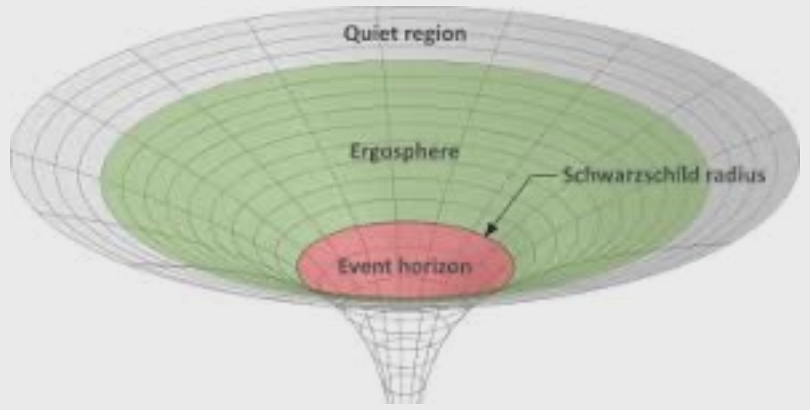
Hydrogen atom



Quantum mechanics

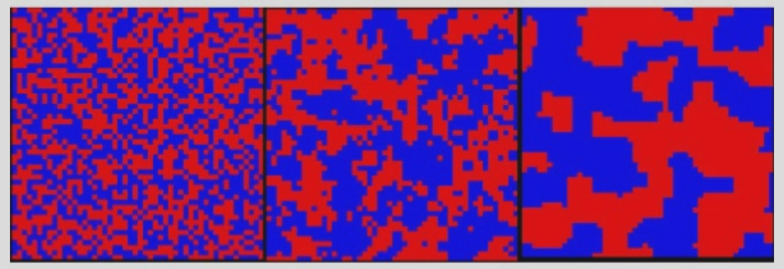


Schwarzschild solution



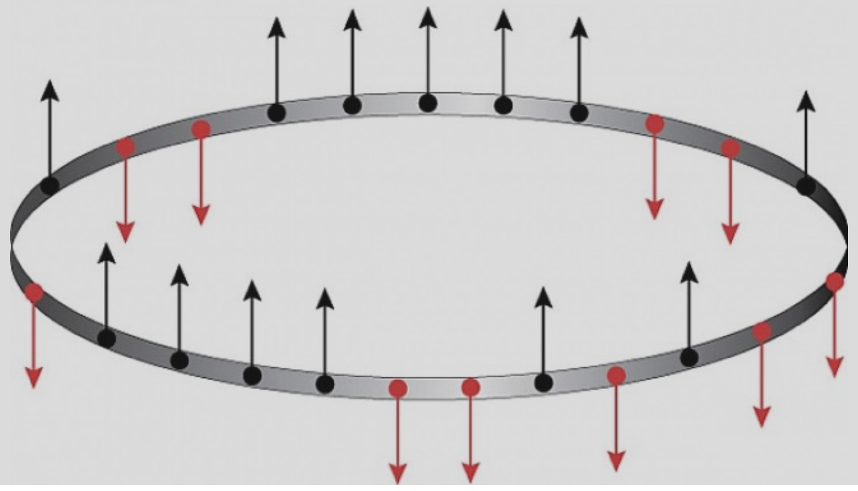
Testing general relativity

Ising model

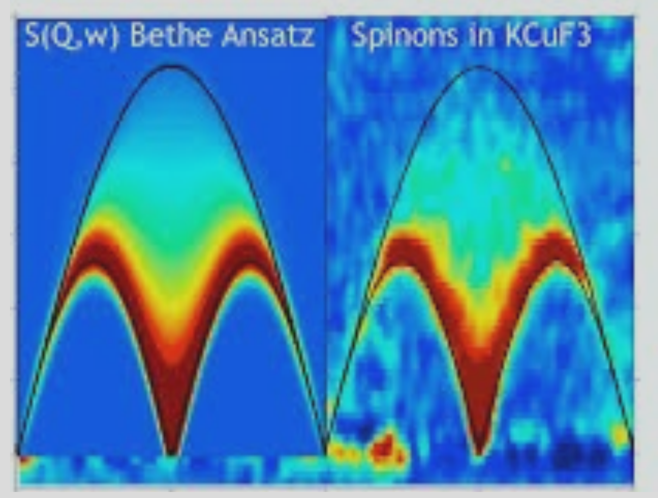


Second order phase transition
2D conformal field theories
statistical field theories

Heisenberg spin chain
Bethe Ansatz



Integrable spinchains
strongly anizotrop solidstate systems
cold atoms

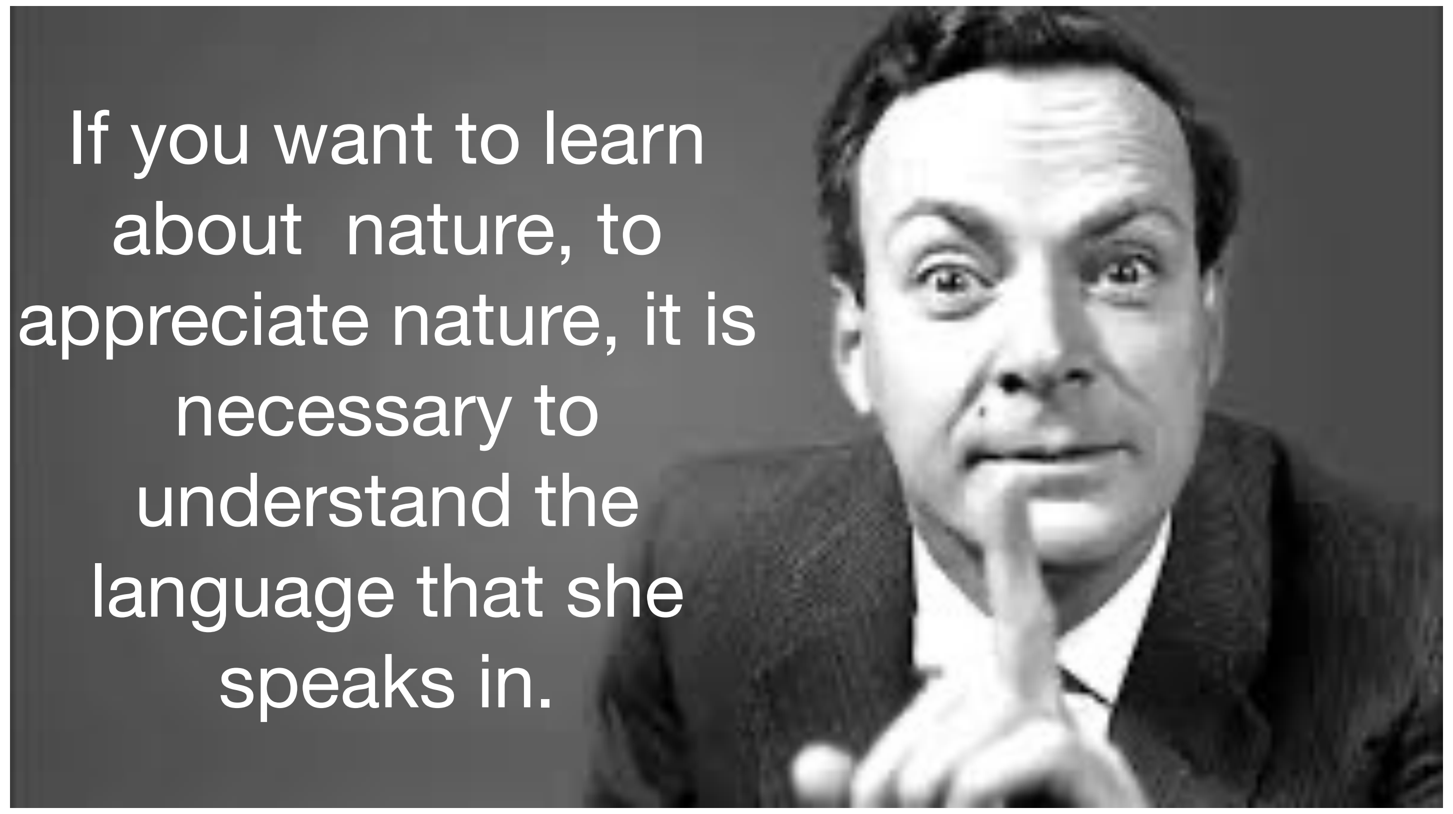


Maximally symmetric
3+1 D gauge theory

potential applications in QCD and
in the standard model

The language of nature is gauge theory

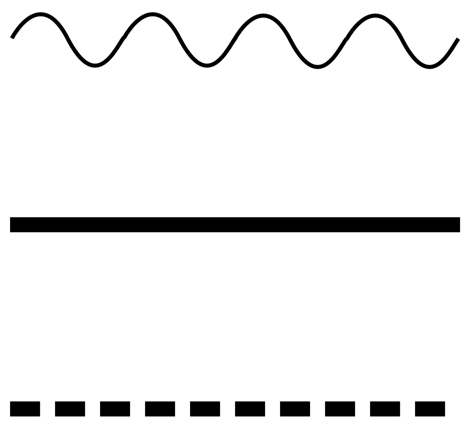
[Feynman]



Gauge group G

compact Lie group **U(1)** **SU(N)** **N=2,3,…**

particle content

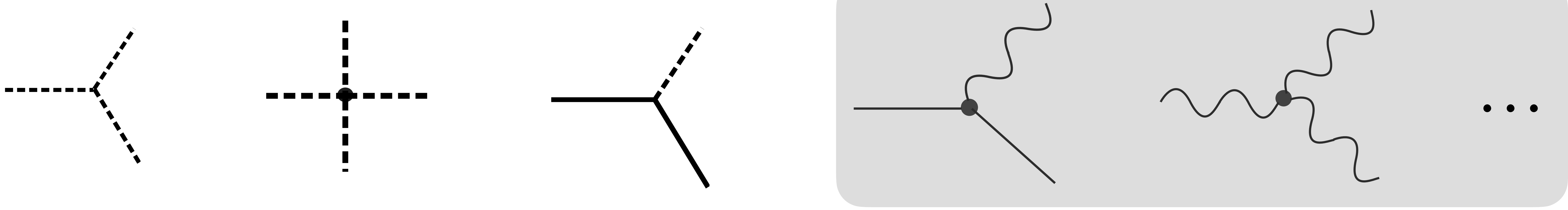


gauge boson A

matter particles: fermions Ψ form representations of G

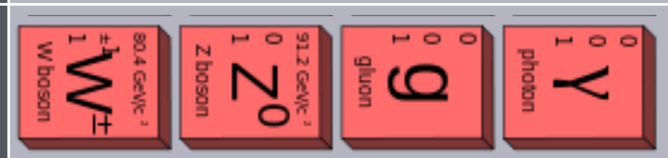
scalar boson Φ

interactions



Φ^3 Φ^4 $\Psi\Phi\Psi$ $\Psi A\Psi$ fixed by gauge symmetry

Fundamental interactions of nature

Lie group	Vector boson adjoint	Fermion	scalar	
U(1)	1	electron positron	0	electrodynamics
U(1)xSU(2)	1+3	6 quark 3 neutrino electron,muon,tau	2 Higgs	electroweak
U(1)xSU(2)xSU(3)	 1+3+8		2 Higgs	standard model
SU(N)	$N^2 - 1$	$8(N^2 - 1)$	$6(N^2 - 1)$	all adjoint, most symmetric 4D gauge theory [Brink,Schwartz]

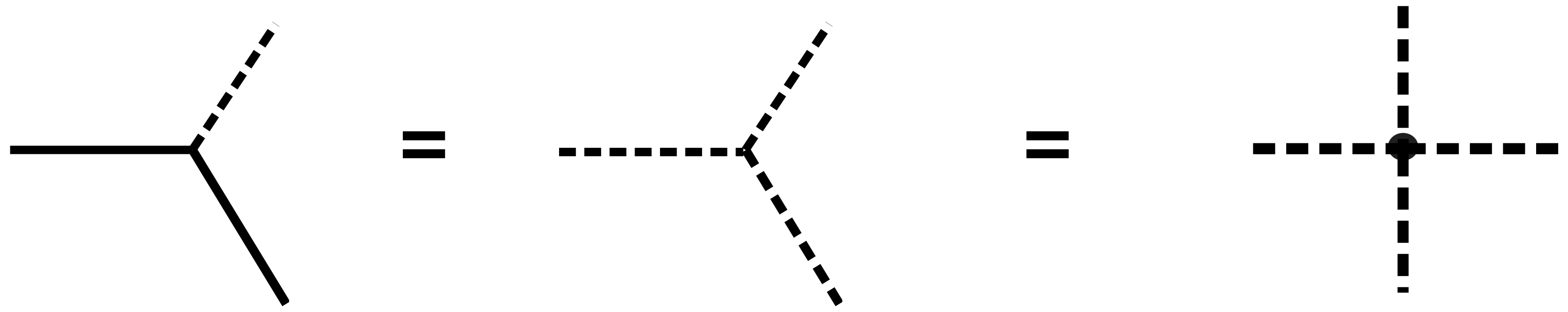
*gravity is a classical gauge theory
but there is no quantum gravity

Maximally symmetric gauge theory

gauge group SU(N)
all adjoint $N^2 - 1$

1 gauge boson 8 fermion 6 scalar

Interactions



$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

Maximally supersymmetric gauge theory

$so(6) \otimes so(2,4) \subset psu(2,2|4)$

$Q_{1,2,3,4}$ $\nearrow$ $\Psi^{ij}_{1,2,3,4}$ $\searrow$ $Q_{1,2,3,4}$

A^{ij}_{μ} $su(4) = so(6)$ $\circlearrowleft$ $\Phi^{ij}_{1,2,3,4,5,6}$

$Q^{\dagger}_{1,2,3,4}$ $\searrow$ $\overline{\Psi}^{ij}_{1,2,3,4}$ $\nearrow$ $Q^{\dagger}_{1,2,3,4}$

global symmetry $\otimes$ conformal symmetry

Perturbation theory $E = E_0 + \lambda E_1 + \ldots + \frac{1}{N}(1 + \ldots)$

't Hooft coupling

$\lambda = g_{YM}^2 N$ $\frac{1}{N}$

Standard model

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{Dirac}} + \mathcal{L}_{\mathrm{mass}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{gauge}/\psi} \; . \quad (1)$$

Here,

$$\mathcal{L}_{\mathrm{Dirac}} = i\bar{e}^i_{\mathrm{L}} \not{\partial} e^i_{\mathrm{L}} + i\bar{\nu}^i_{\mathrm{L}} \not{\partial} \nu^i_{\mathrm{L}} + i\bar{e}^i_{\mathrm{R}} \not{\partial} e^i_{\mathrm{R}} + i\bar{u}^i_{\mathrm{L}} \not{\partial} u^i_{\mathrm{L}} + i\bar{d}^i_{\mathrm{L}} \not{\partial} d^i_{\mathrm{L}} + i\bar{u}^i_{\mathrm{R}} \not{\partial} u^i_{\mathrm{R}} + i\bar{d}^i_{\mathrm{R}} \not{\partial} d^i_{\mathrm{R}} \; ; \quad (2)$$

$$\mathcal{L}_{\mathrm{mass}} = -v \left(\lambda^i_e \bar{e}^i_{\mathrm{L}} e^i_{\mathrm{R}} + \lambda^i_u \bar{u}^i_{\mathrm{L}} u^i_{\mathrm{R}} + \lambda^i_d \bar{d}^i_{\mathrm{L}} d^i_{\mathrm{R}} + \mathrm{h.c.} \right) - M^2_{\mathrm{W}} W^+_{\mu} W^{-\mu} - \frac{M^2_{\mathrm{W}}}{2\cos^2\theta_W} Z_{\mu} Z^{\mu} \; ; \quad (3)$$

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4} (G^a_{\mu\nu})^2 - \frac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} \; , \quad (4)$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc} A^b_\mu A^c_\nu \\ W^{\pm}_{\mu\nu} &= \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \; , \quad (5) \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = \; & i g_2 \cos\theta_{\mathrm{W}} \left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu \right) \partial^\mu Z^\nu + W^+_{\mu\nu} W^{-\mu} Z^\nu - W^-_{\mu\nu} W^{+\mu} Z^\nu \right] \\ & + ie \left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu \right) \partial^\mu A^\nu + W^+_{\mu\nu} W^{-\mu} A^\nu - W^-_{\mu\nu} W^{+\mu} A^\nu \right] \\ & + g_2^2 \cos^2\theta_{\mathrm{W}} \left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu} Z_\nu Z^\nu \right) \\ & + g_2^2 \left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu} A_\nu A^\nu \right) \\ & + g_2 e \cos\theta_{\mathrm{W}} \left[W^+_\mu W^-_\nu \left(Z^\mu A^\nu + Z^\nu A^\mu \right) - 2 W^+_\mu W^{-\mu} Z_\nu A^\nu \right] \\ & + \frac{1}{2} g_2^2 \left(W^+_\mu W^-_\nu \right) \left(W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu} \right) \; ; \quad (6) \end{aligned}$$

and

$$\mathcal{L}_{\mathrm{gauge}/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2 \left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z_\mu J^\mu_Z \right) - e A_\mu J^\mu_A \; , \quad (7)$$

where

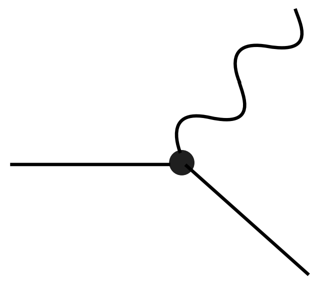
$$\begin{aligned} J^{\mu a}_{(3)} &= \bar{u}^i \gamma^\mu T^a_{(3)} u^i + \bar{d}^i \gamma^\mu T^a_{(3)} d^i \\ J^\mu_{W^+} &= \frac{1}{\sqrt{2}} \left(\bar{\nu}^i_{\mathrm{L}} \gamma^\mu e^i_{\mathrm{L}} + V^{ij} \bar{u}^i_{\mathrm{L}} \gamma^\mu d^j_{\mathrm{L}} \right) \\ J^\mu_{W^-} &= (J^\mu_{W^+})^* \\ J^\mu_Z &= \frac{1}{\cos\theta_{\mathrm{W}}} \left[\frac{1}{2} \bar{\nu}^i_{\mathrm{L}} \gamma^\mu \nu^i_{\mathrm{L}} + \left(-\frac{1}{2} + \sin^2\theta_{\mathrm{W}} \right) \bar{e}^i_{\mathrm{L}} \gamma^\mu e^i_{\mathrm{L}} + (\sin^2\theta_{\mathrm{W}}) \bar{e}^i_{\mathrm{R}} \gamma^\mu e^i_{\mathrm{R}} \right. \\ & \quad + \left(\frac{1}{2} - \frac{2}{3} \sin^2\theta_{\mathrm{W}} \right) \bar{u}^i_{\mathrm{L}} \gamma^\mu u^i_{\mathrm{L}} + \left(-\frac{2}{3} \sin^2\theta_{\mathrm{W}} \right) \bar{u}^i_{\mathrm{R}} \gamma^\mu u^i_{\mathrm{R}} \\ & \quad \left. + \left(-\frac{1}{2} + \frac{1}{3} \sin^2\theta_{\mathrm{W}} \right) \bar{d}^i_{\mathrm{L}} \gamma^\mu d^i_{\mathrm{L}} + \left(\frac{1}{3} \sin^2\theta_{\mathrm{W}} \right) \bar{d}^i_{\mathrm{R}} \gamma^\mu d^i_{\mathrm{R}} \right] \\ J^\mu_A &= (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i \; . \quad (8) \end{aligned}$$

Perturbation theory

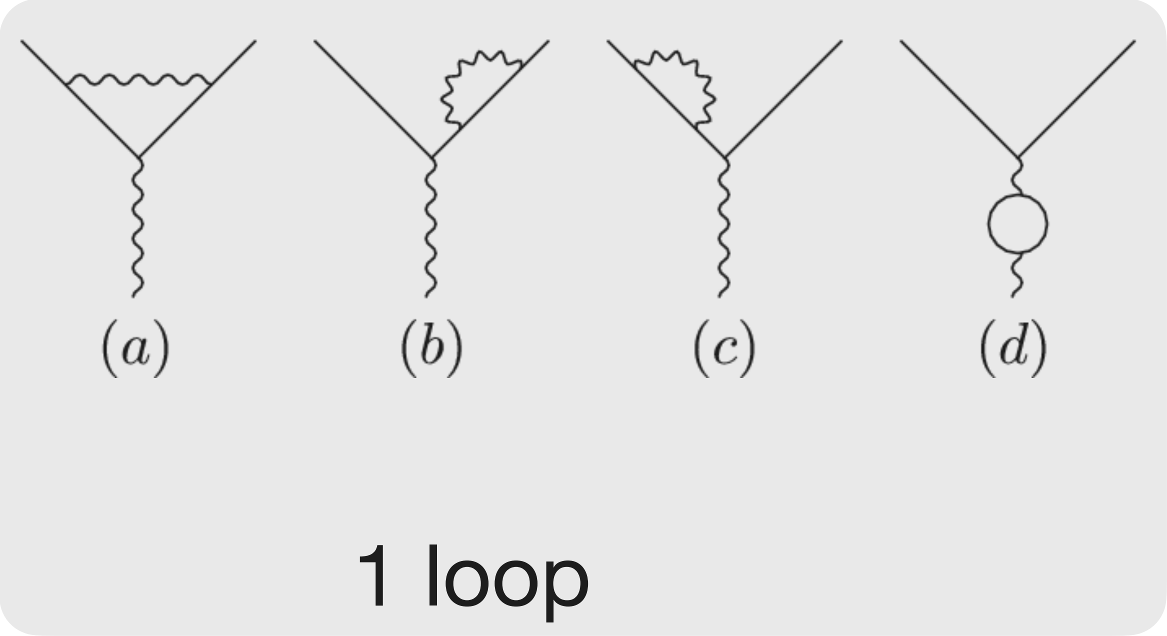
Quantum electrodynamics

Magnetic moment of the electron

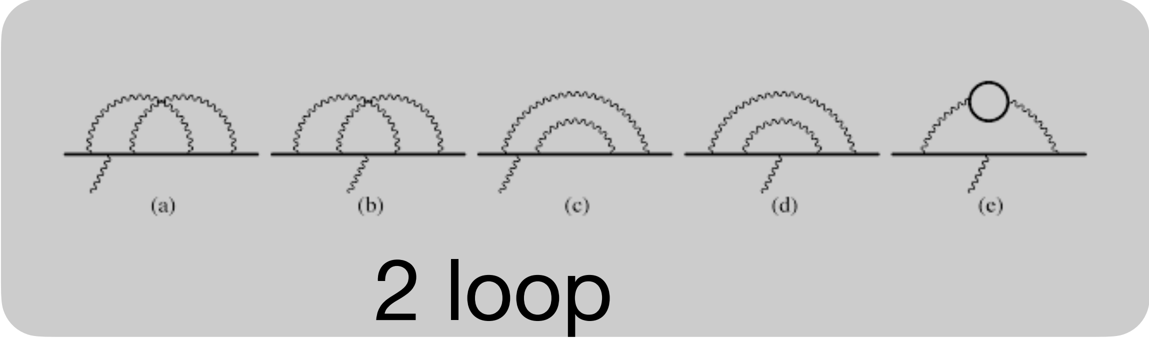
agrees with experiment for 10 digits



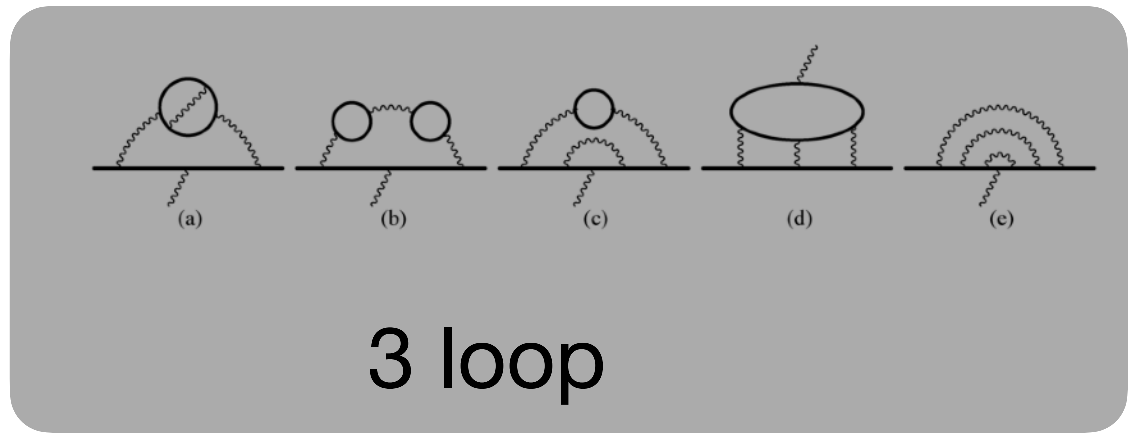
$+\frac{1}{1000}$



$+10^{-6}$

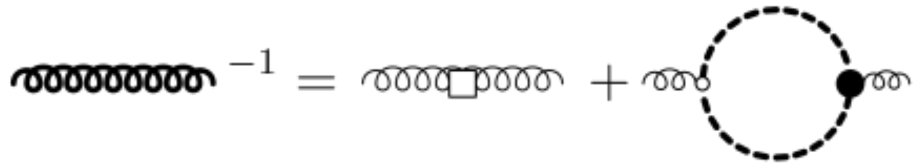


$+10^{-9}$

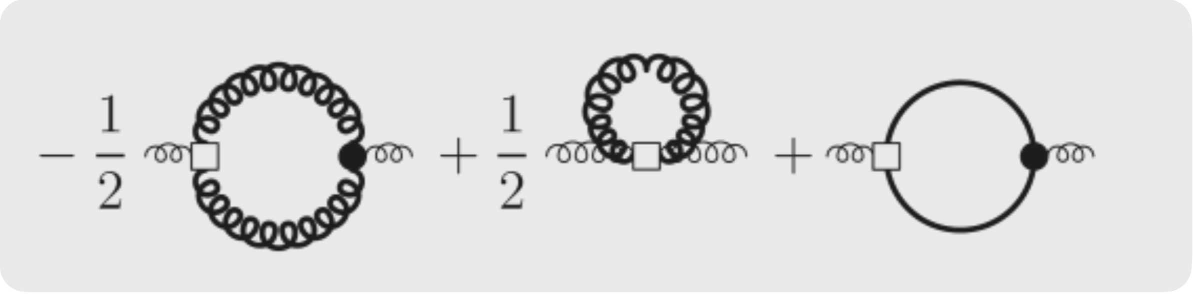


QCD: strong interaction

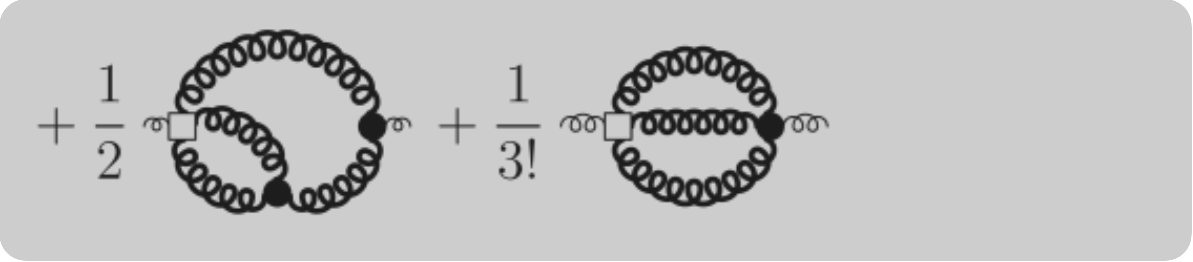
gluon propagator



$+1$



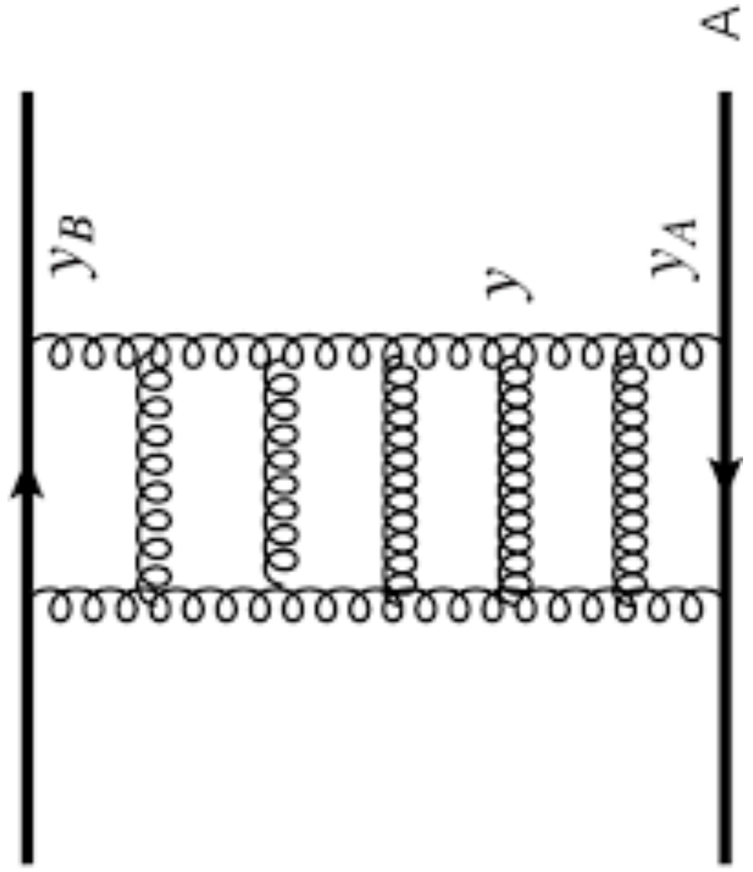
$+1$



Masses of the mesons

mass of the glueball?

Millenium prize: 1 million dollar

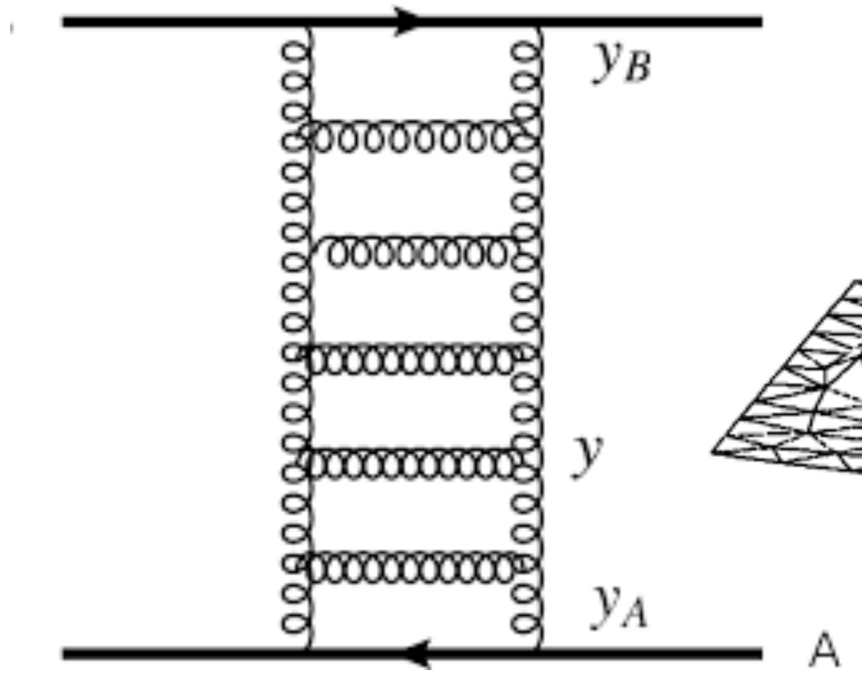


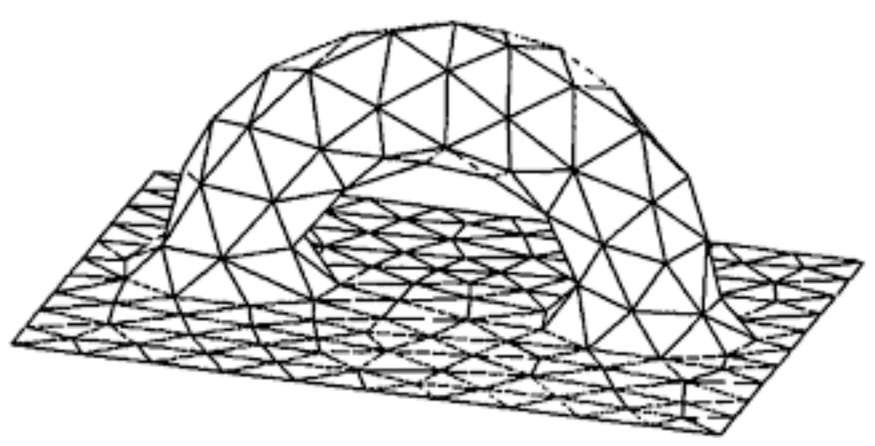
Strong coupling limit

quark-antiquark interaction

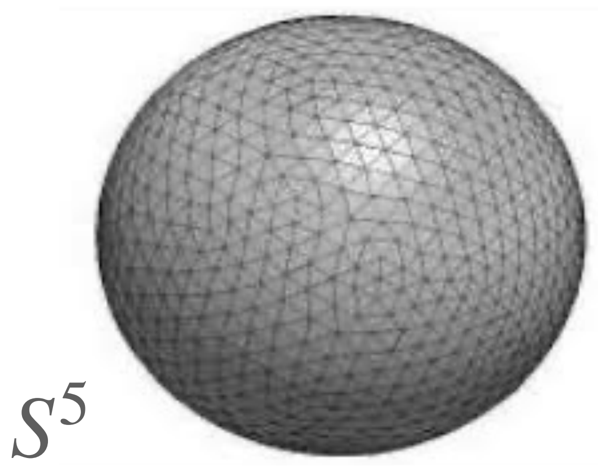
Very dense Feynmann graphs draw a surface

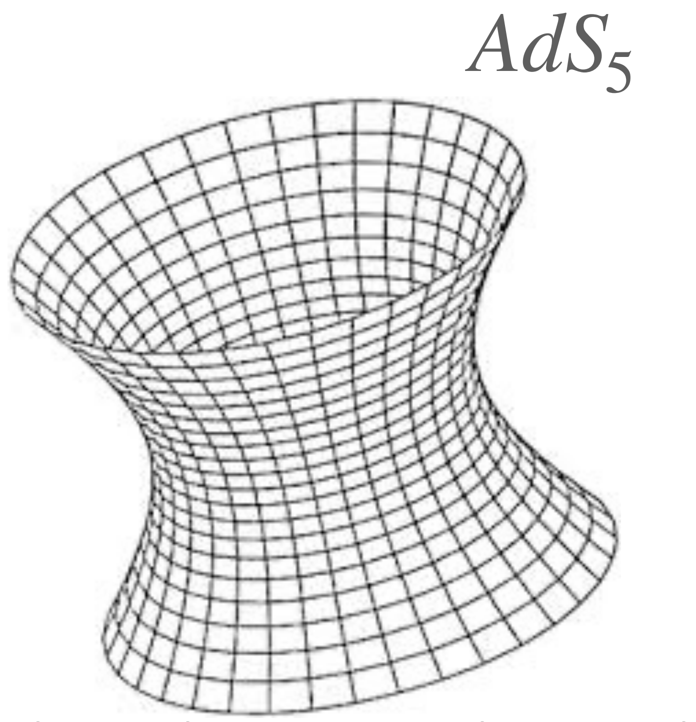
['t Hooft]





$$X_1^2 + X_2^2 + \cdots + X_6^2 = R^2$$

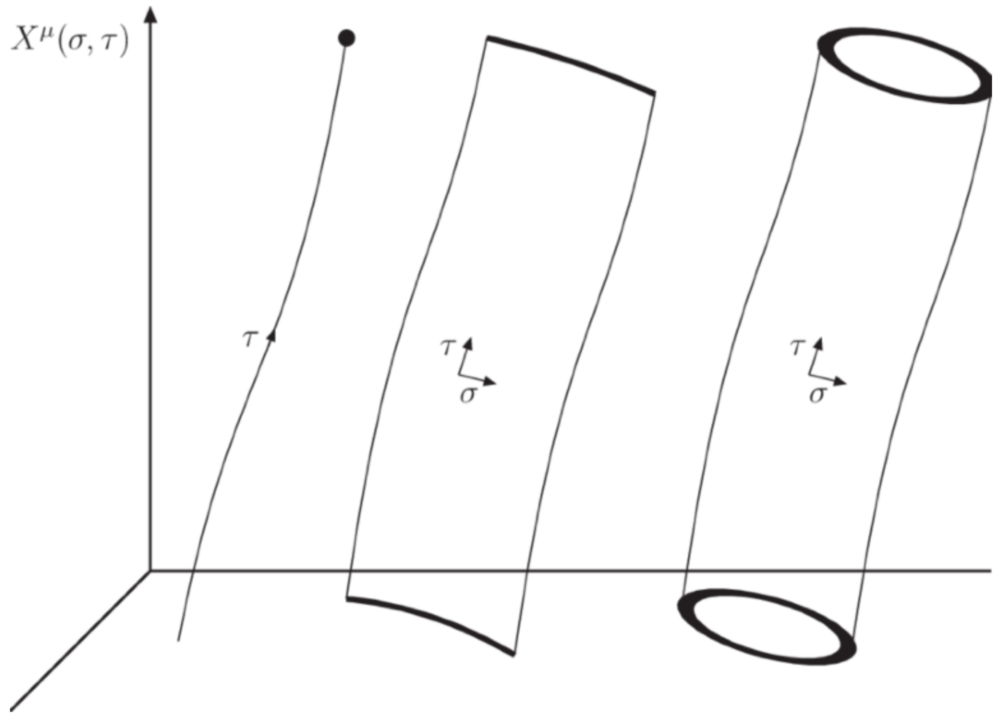




$$-Y_1^2 - Y_2^2 + \cdots + Y_5^2 = -R^2$$

The world sheet is swept by a string

string action $\sqrt{\lambda} \int d\tau d\sigma \, \mathrm{Area}(\tau, \sigma)$



Positive Curvature Negative Curvature Flat Curvature

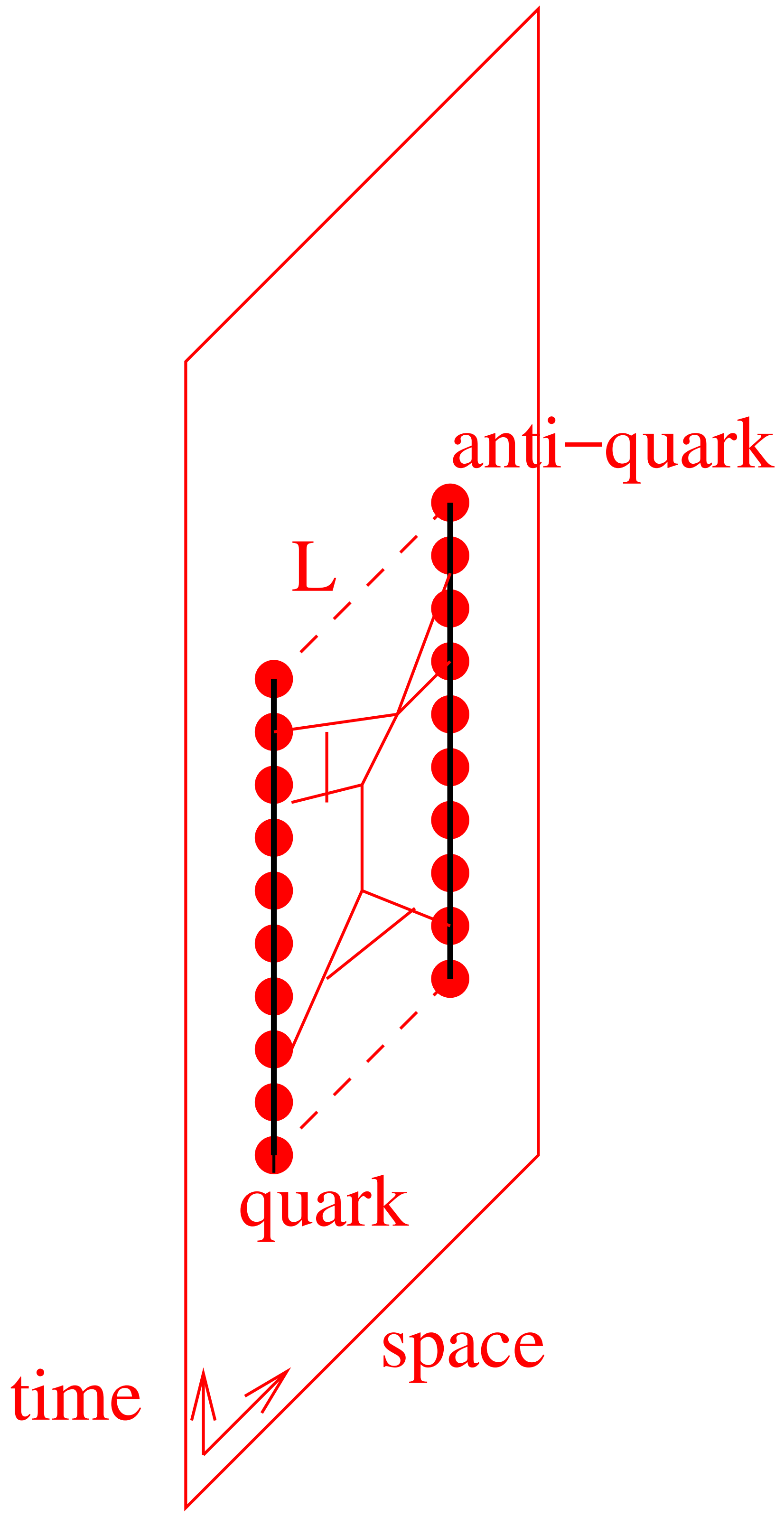
semiclassical expansion

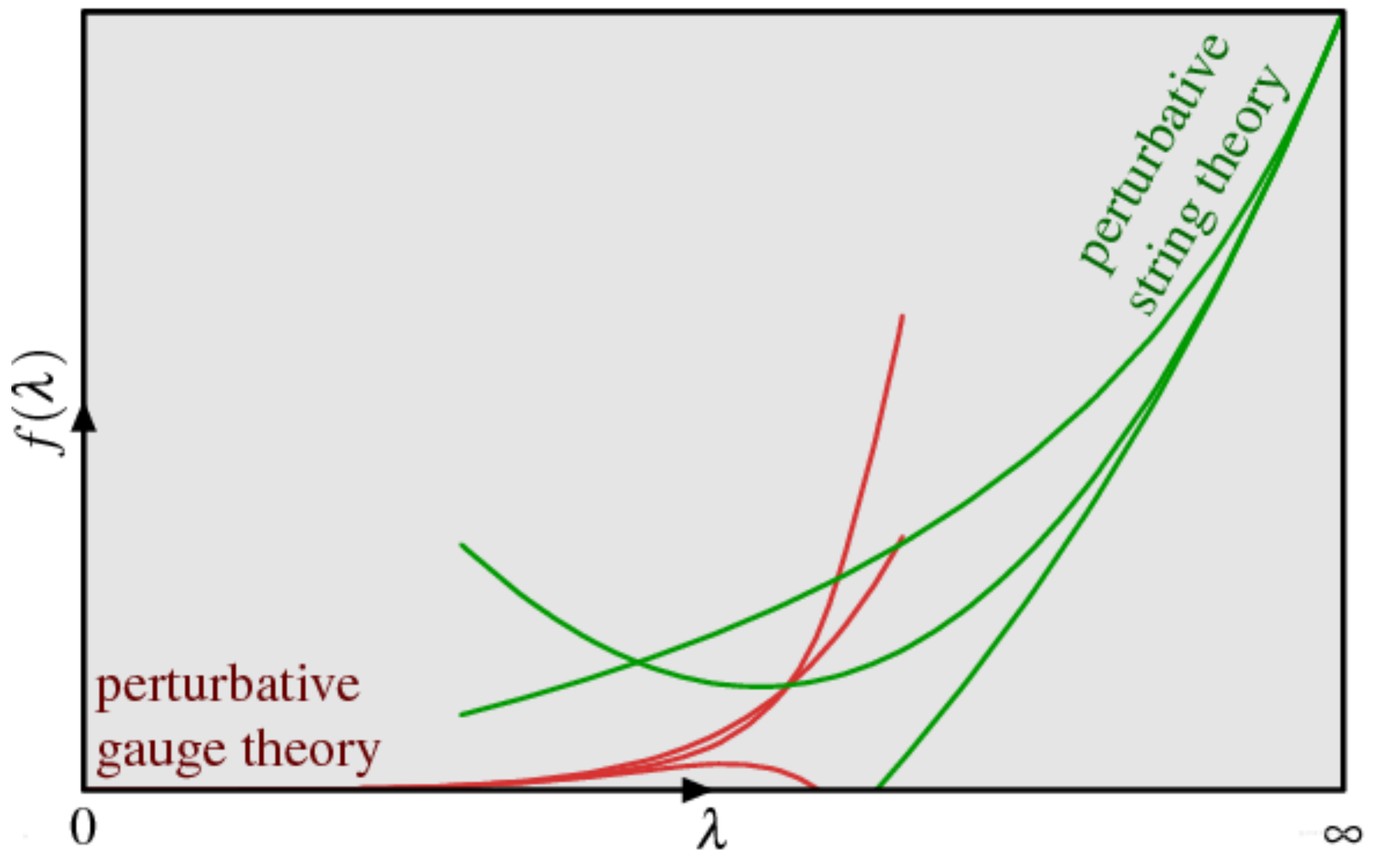
$$E = E_\infty + \frac{1}{\sqrt{\lambda}} E_{\frac{1}{2}} + \dots$$

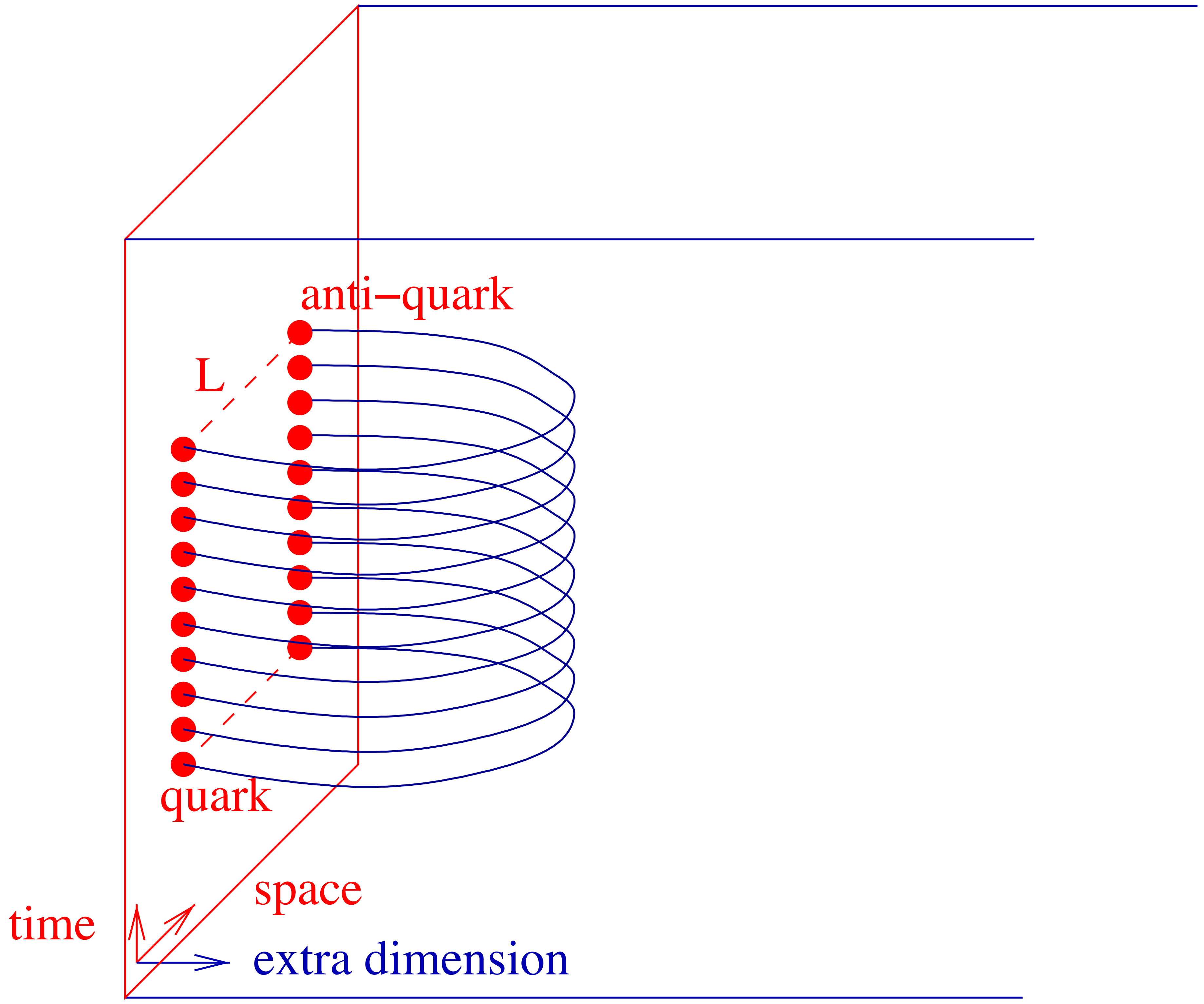
AdS/CFT duality conjecture [Maldacena]

3+1 D maximally symmetric gauge theory

9+1 D string theory on $AdS_5 \times S^5$







$$V(L) = \frac{-\lambda}{4\pi L} + \ldots$$

loop expansion

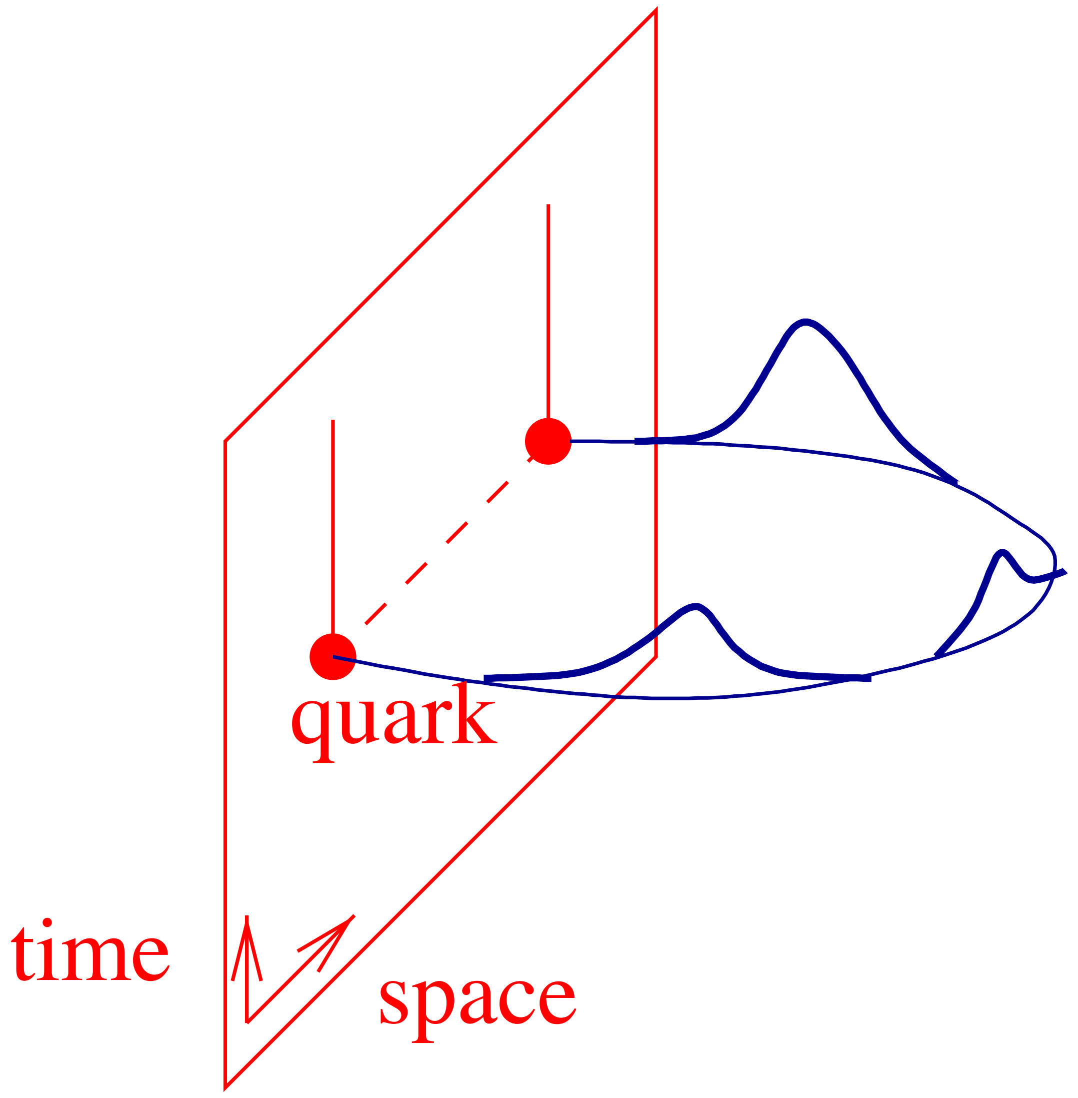
$$V(L) = \frac{f(\lambda)}{L}$$

$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}\left(1 - \frac{1.3359}{\sqrt{\lambda}} + \ldots\right)$$

minimal AdS surface

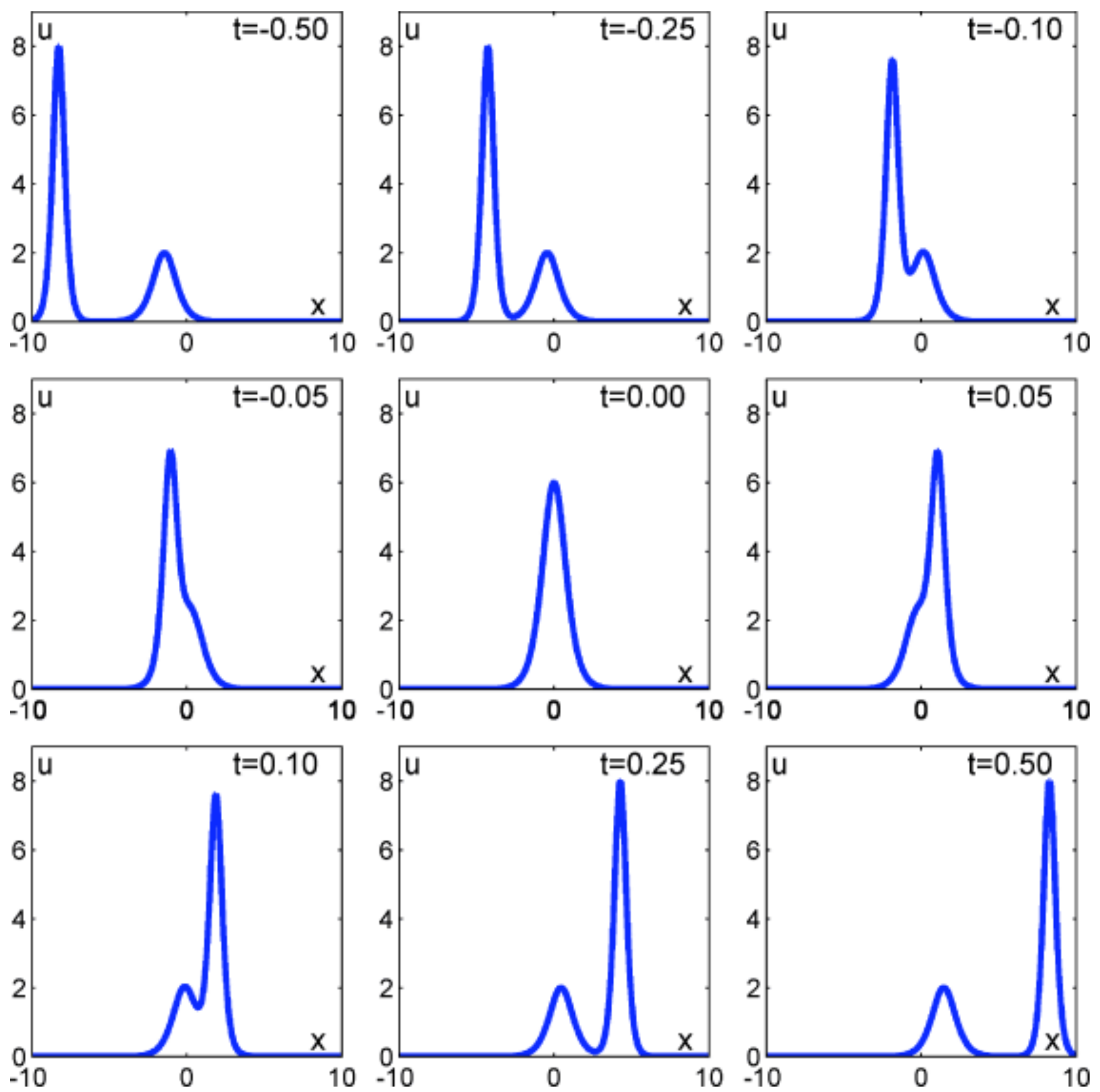
quadratic fluctuations

2D integrable description



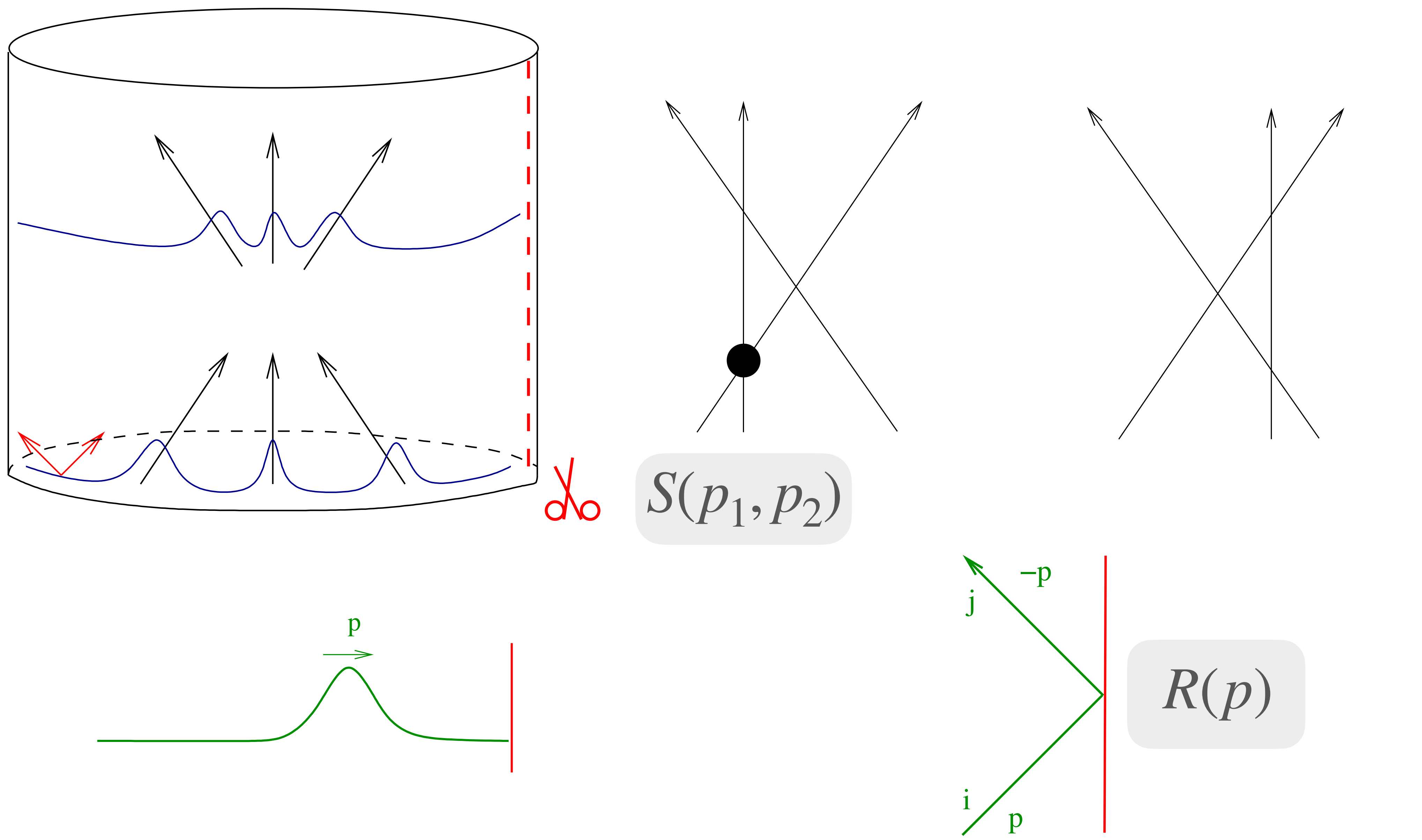
1+1 D string theory is integrable

no particle production



[Bajnok,Palla,Takacs]

spacetime diagram



scattering, reflection, dispersion relation exact calculation

$$E(p) = \sqrt{1 + \lambda \sin^2 \frac{p}{2}}$$

[Correa,Maldacena,Sever][Drukker]

[Bajnok,Balog et al]

ϵ :egzakt TBA integrálegyenlet

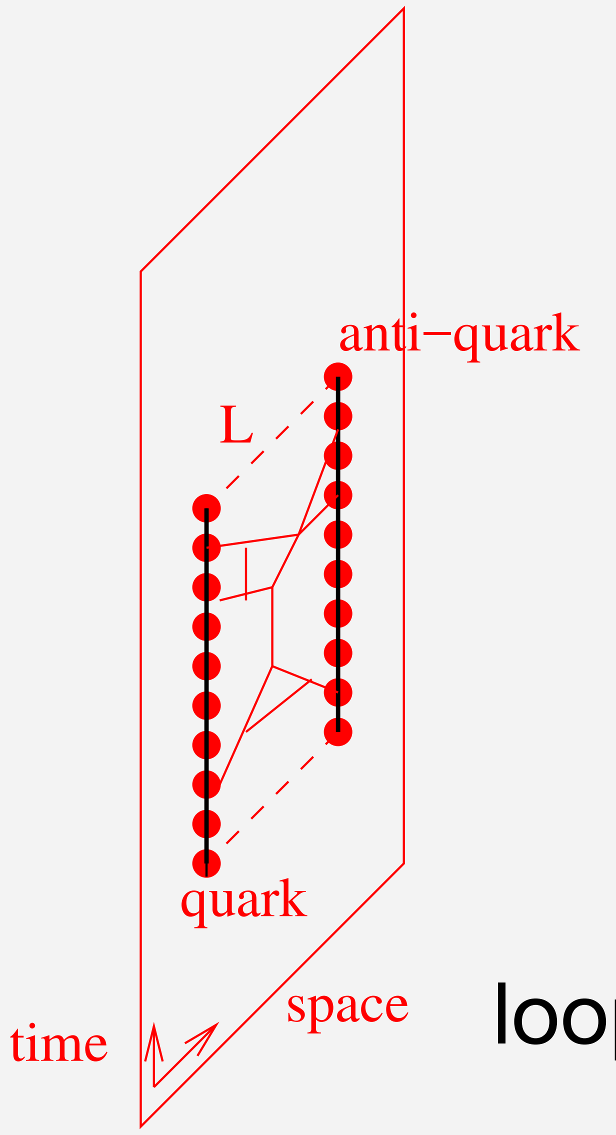
Quark antiquark potential = 1+1 D Casimir effect

ground-state energy on the strip

$$E_0 = \int dp \log(1 - R(p)\bar{R}(p)e^{-\epsilon})$$

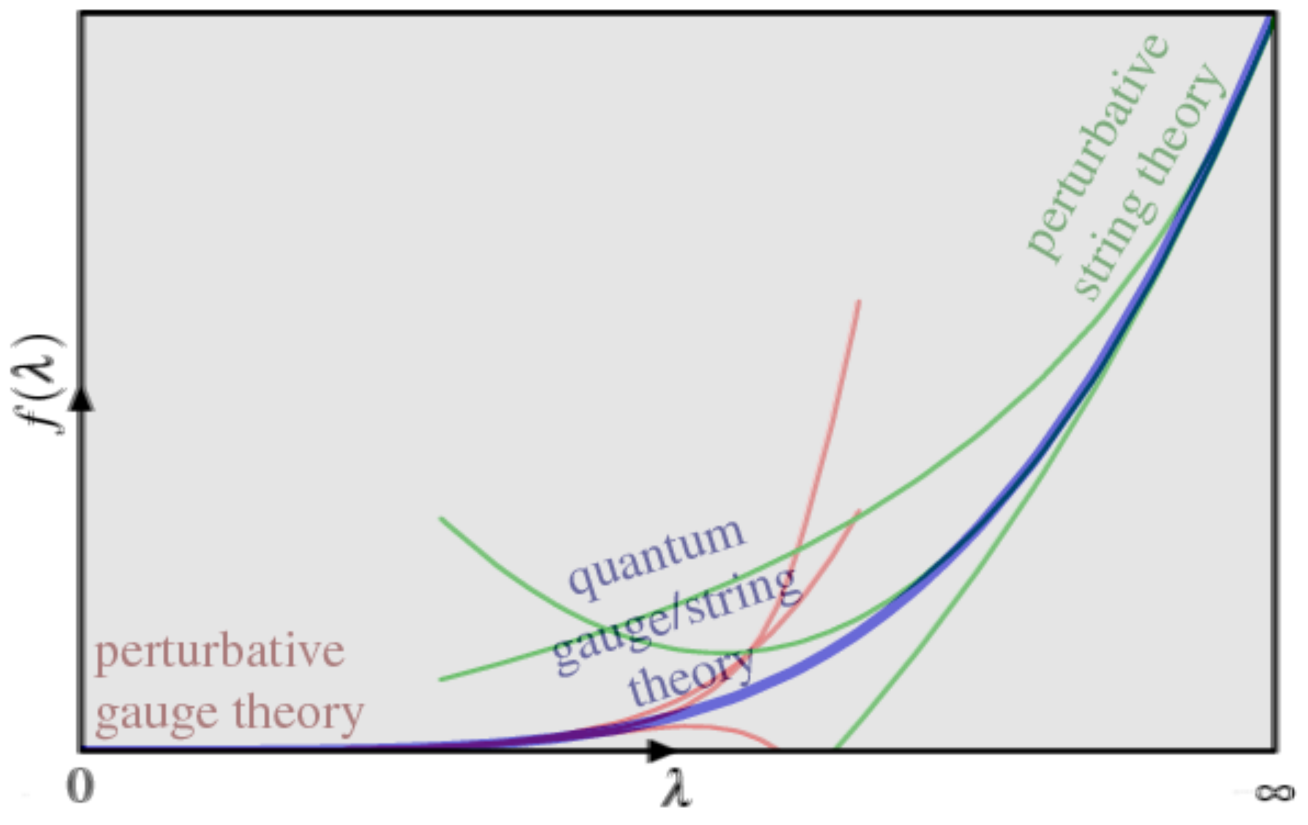
Triality

3+1 D gauge theory

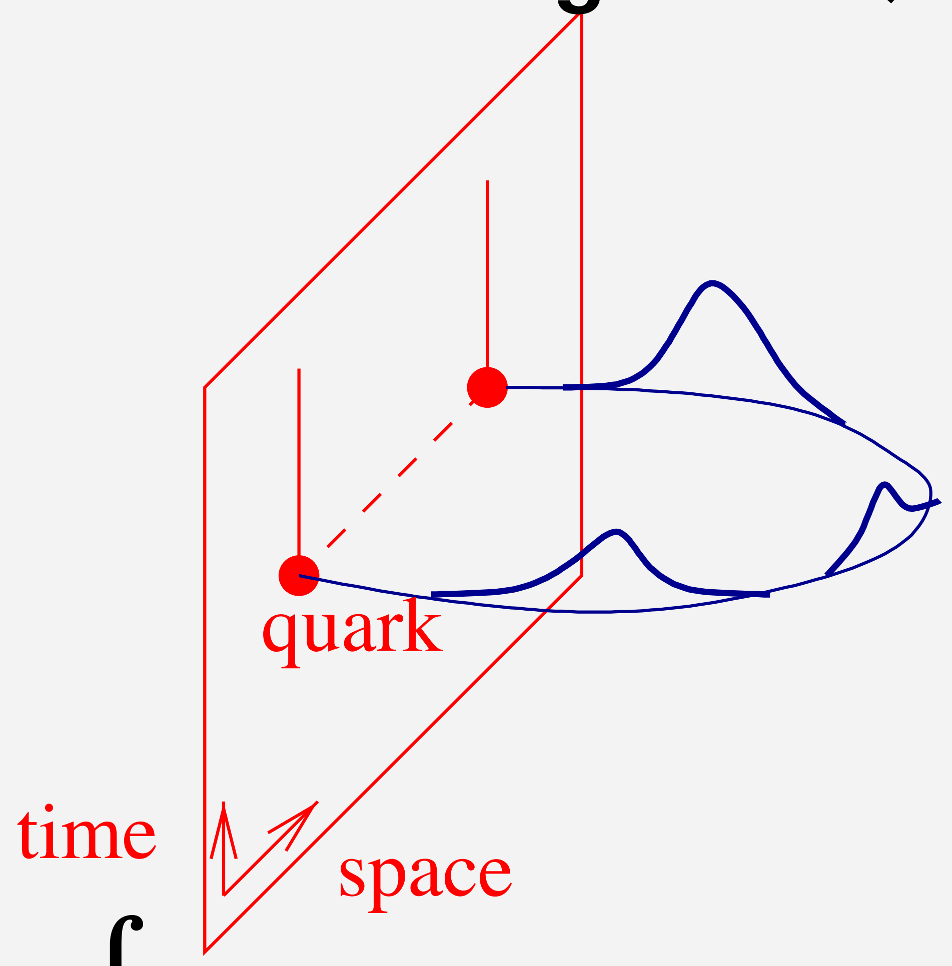


loop expansion

$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$



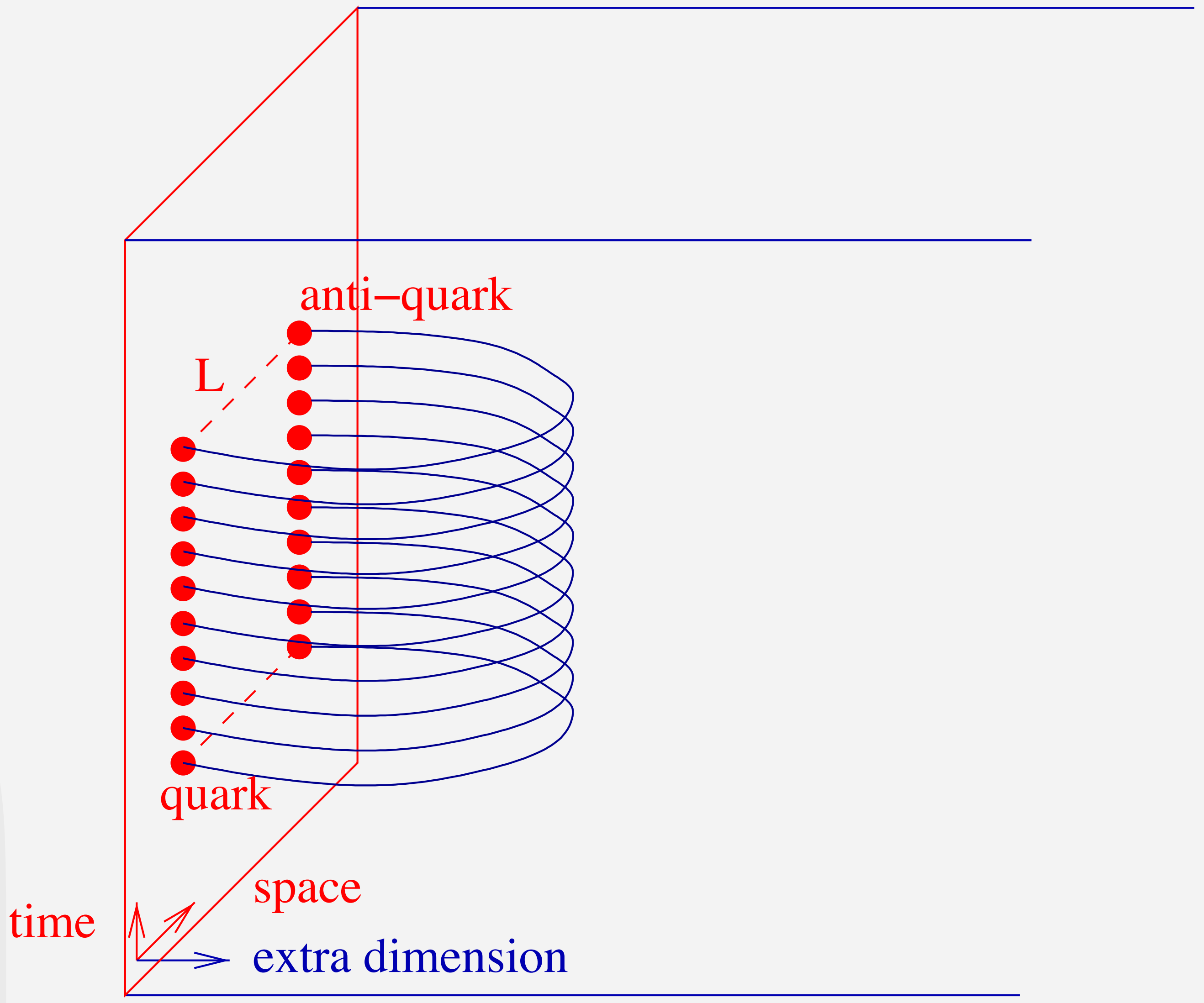
1+1 D integrable QFT



$$V(L) = L^{-1}\int dp\,\log(1 - R(p)\bar{R}(p)e^{-\epsilon(p)})$$

Exact groundstate energy

1+9 D string theory



$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots)$$

minimal surface fluctuations

Spectral problem

3+1 D gauge theory

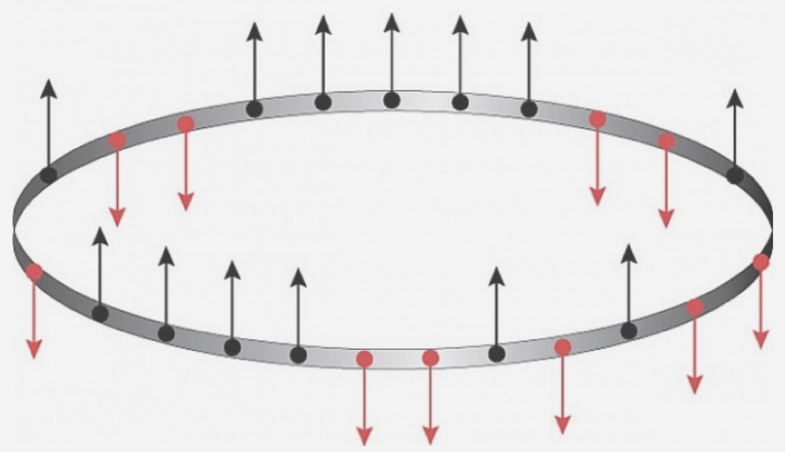
Conformal Field Theory

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \mathrm{Tr}(Z^J)$$

$$Z = \Phi_1 + i\Phi_2$$
$$X = \Phi_3 + i\Phi_4$$

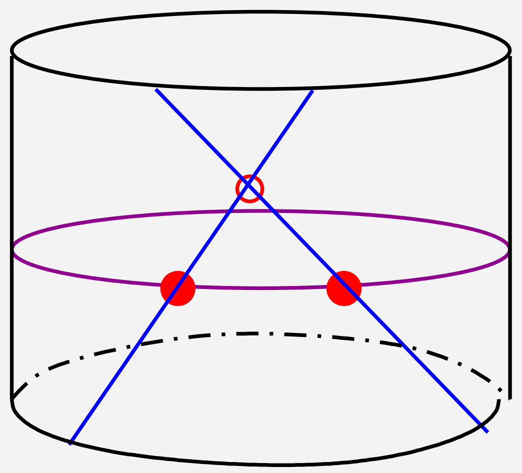
$$\mathcal{O} = \mathrm{Tr}(Z^{J-M}X^M) \qquad |\uparrow\uparrow . \downarrow\downarrow\rangle$$



Integrable spin chain

1+1 D integrable QFT

Finite size spectrum of multiparticle states



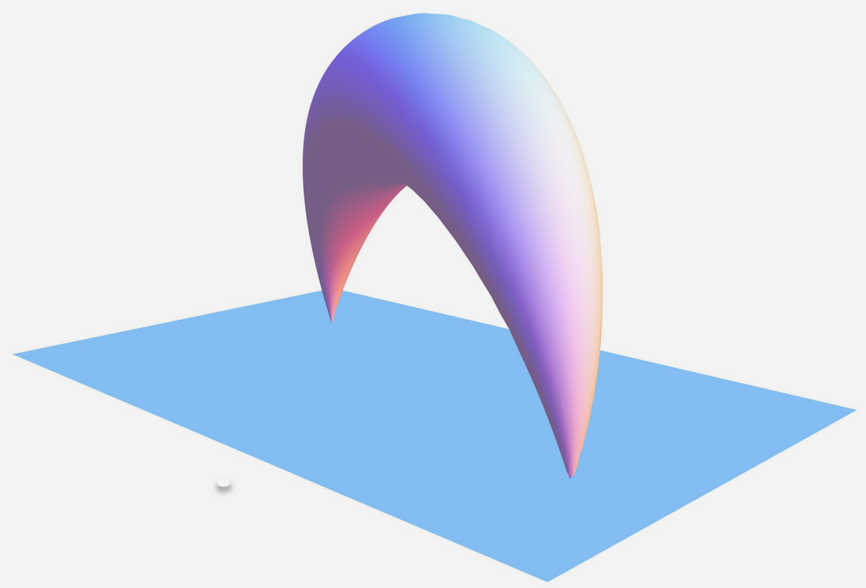
$$E(J) = 2E(p) + \delta E(p)$$

Bethe ansatz $\quad e^{ipJ}S(p,-p) = 1$

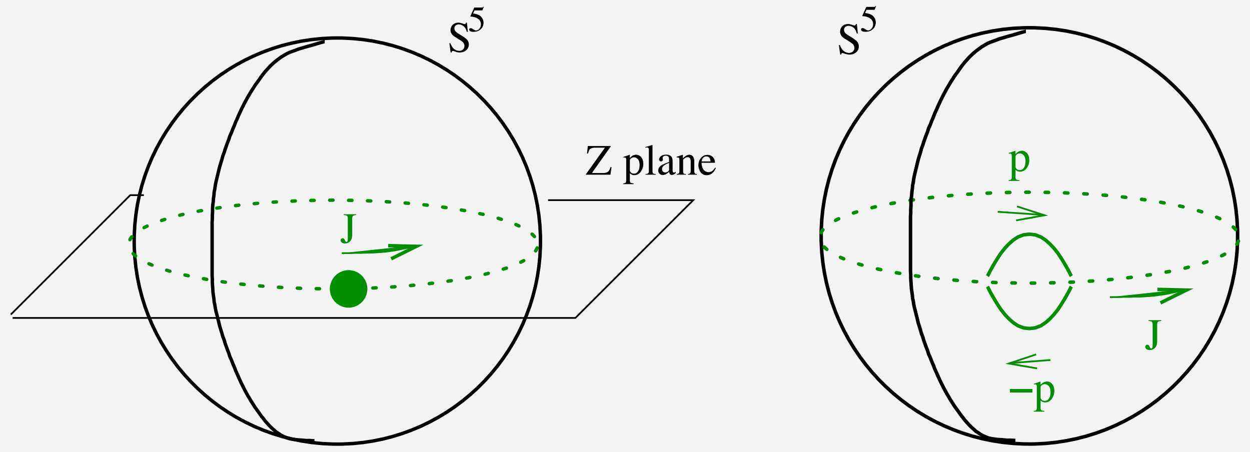
vacuum polarisation [Bajnok,Janik]

$$\delta E(p) = \int dq S(q,p)S(q,-p)e^{-\epsilon(q)}$$

9+1 D string theory



Spinning strings' energy



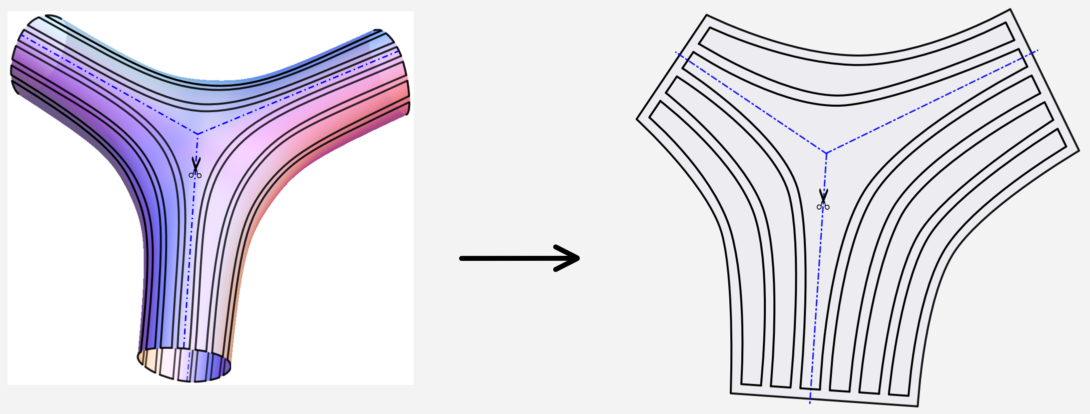
String interactions

3+1 D gauge theory

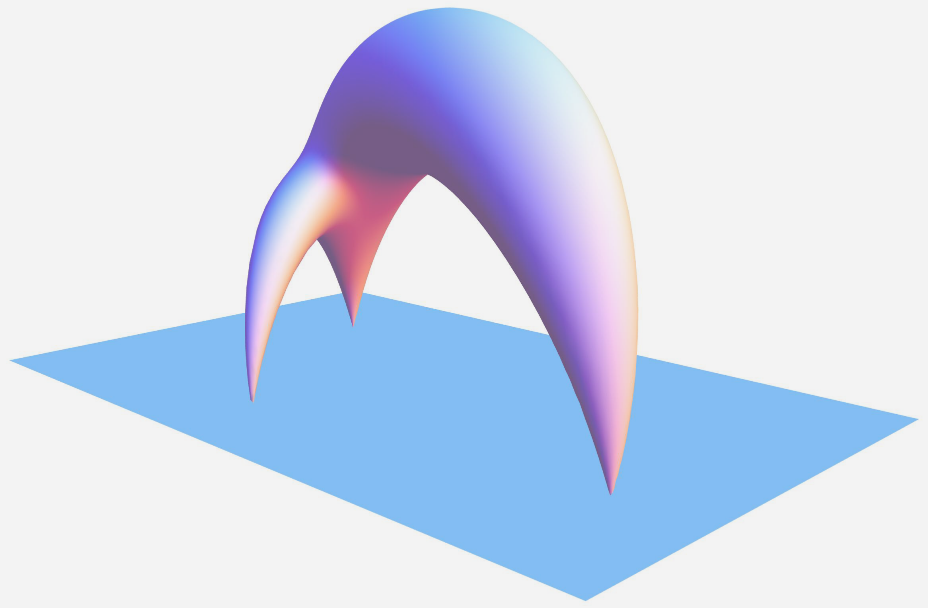
3-point function

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = C_{ijk}(\lambda)$$

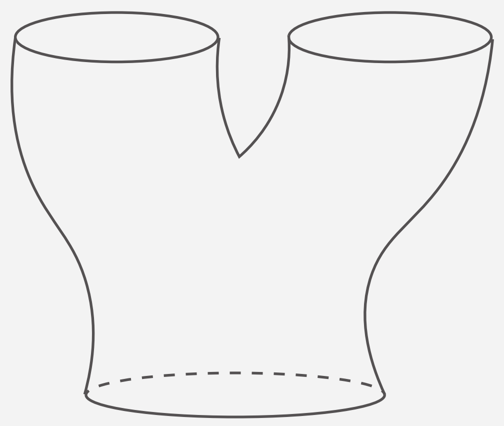
spin-chain overlaps



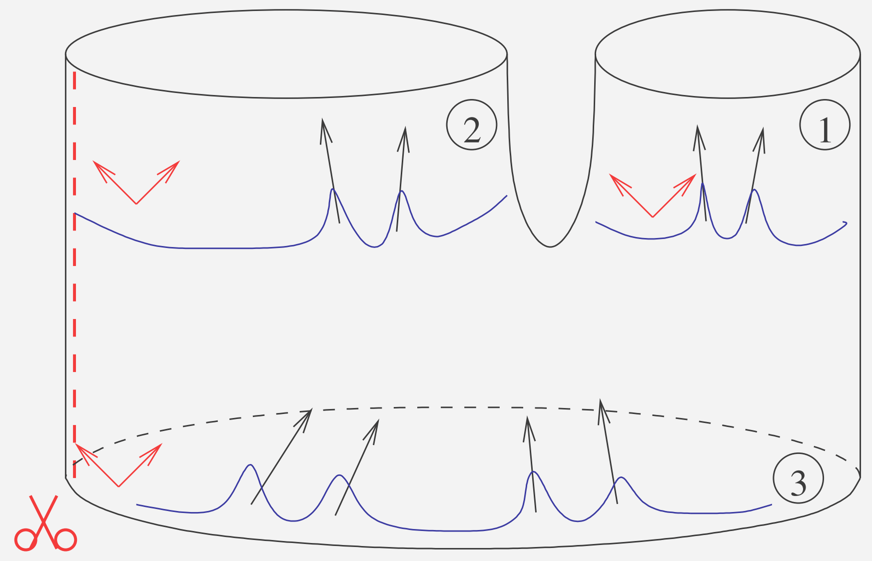
9+1 D string theory



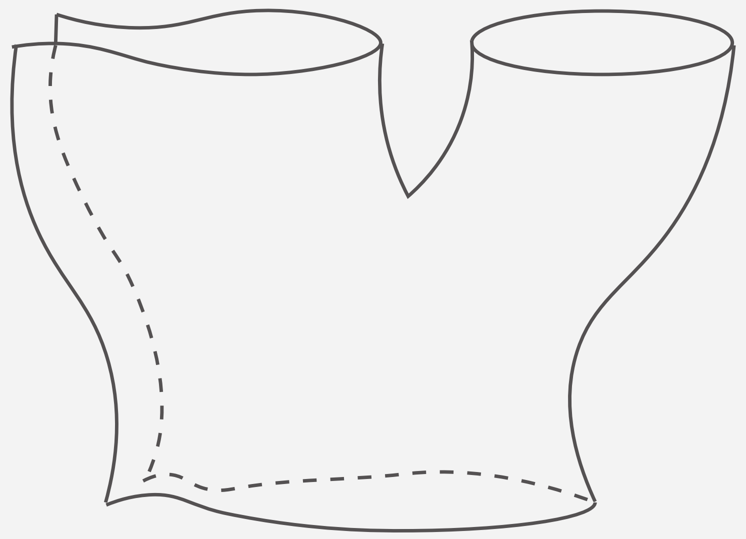
string interaction vertex



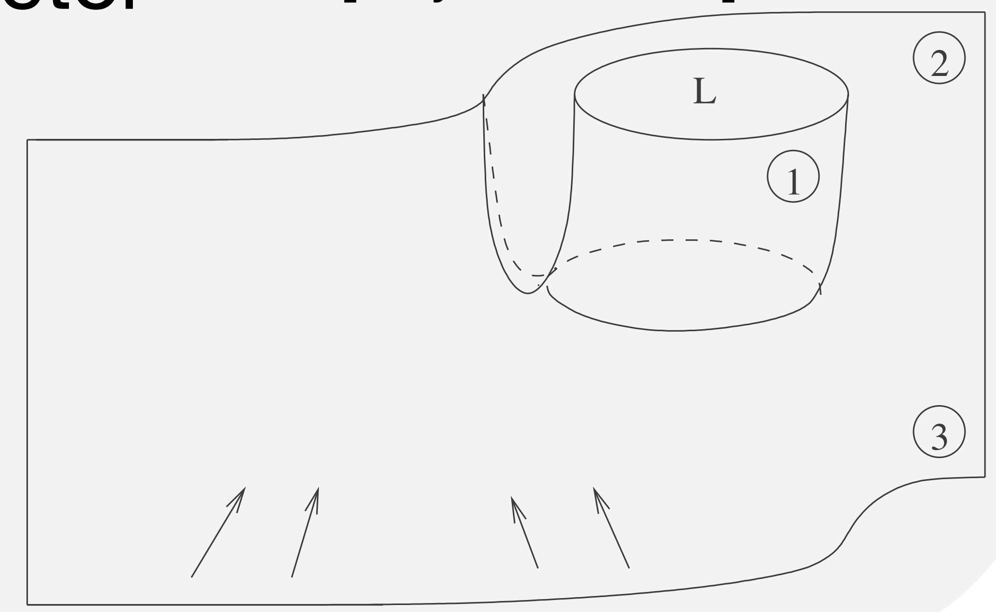
1+1 D integrable QFT



finite size form factor

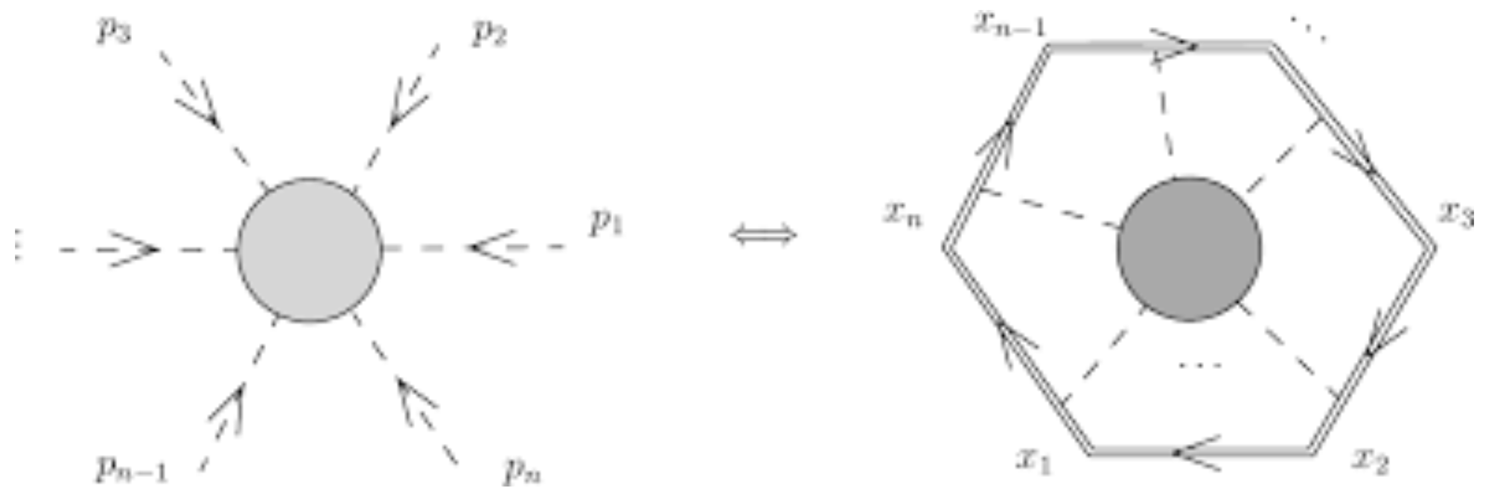


[Bajnok,Janik]

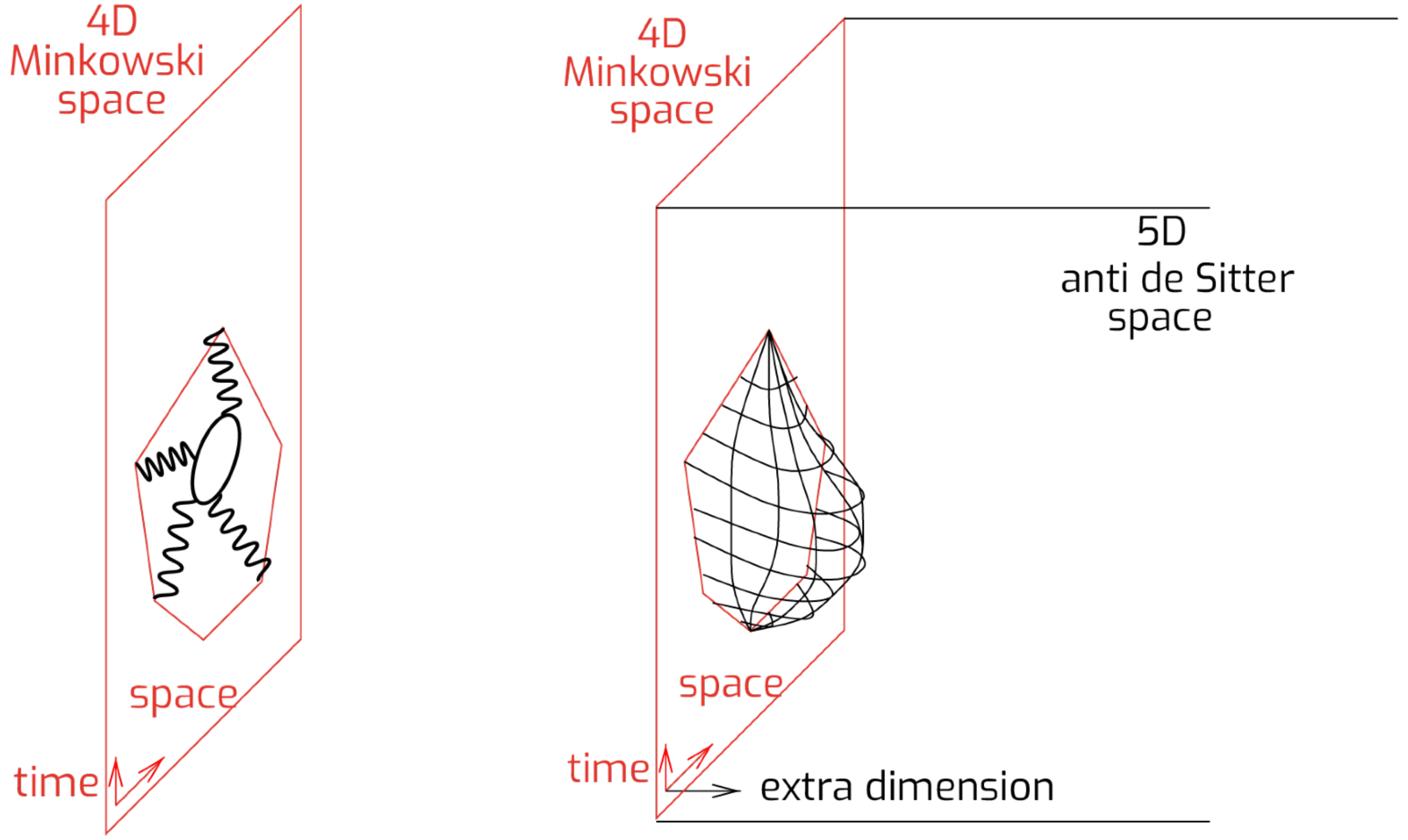


Other observables

gluon scattering amplitudes = light-like Wilson lines



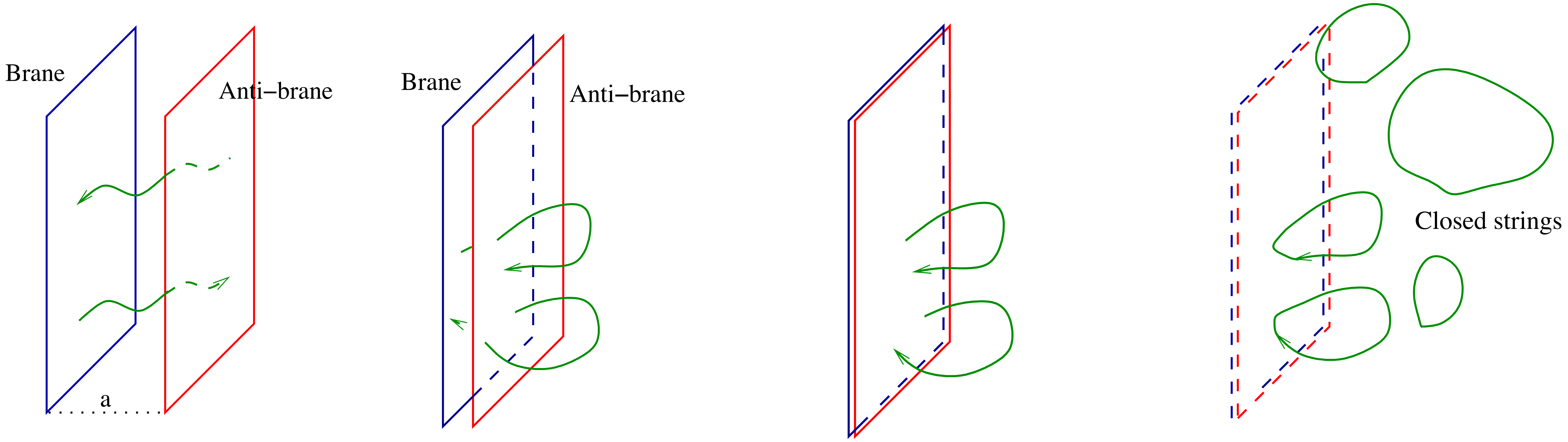
[Bajnok,Balog,Ito,Satoh,Toth]



Tachyon condensation

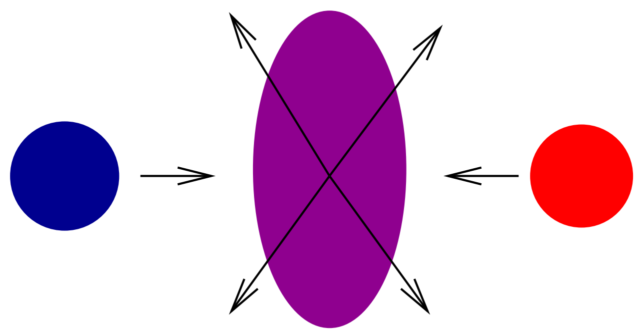
$$\mathcal{O} = \mathrm{Det}(Z\ldots ZXZ\ldots ZX\ldots Z)$$

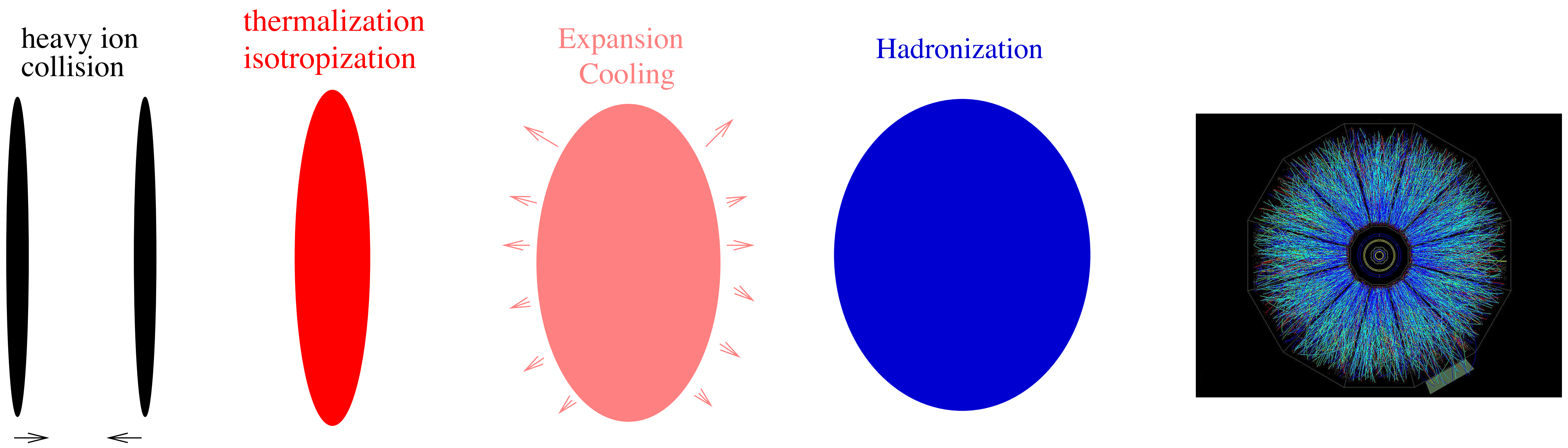
[Bajnok,Drukker et al.]

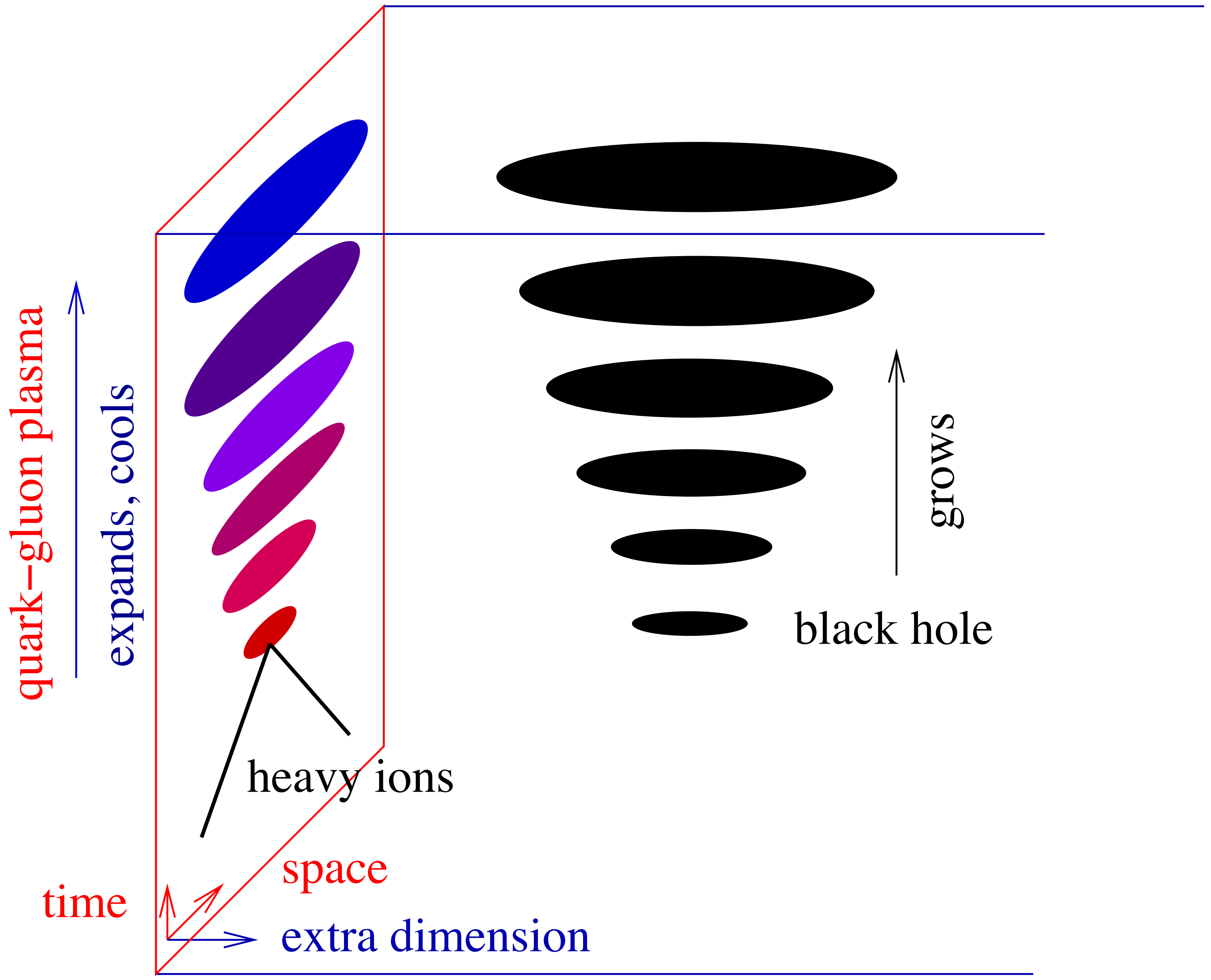


Quark-gluon plasma

heavy ion collision







Gauge theory at finite temperature

low energy string theory = gravitation

Relativistic hydrodynamics

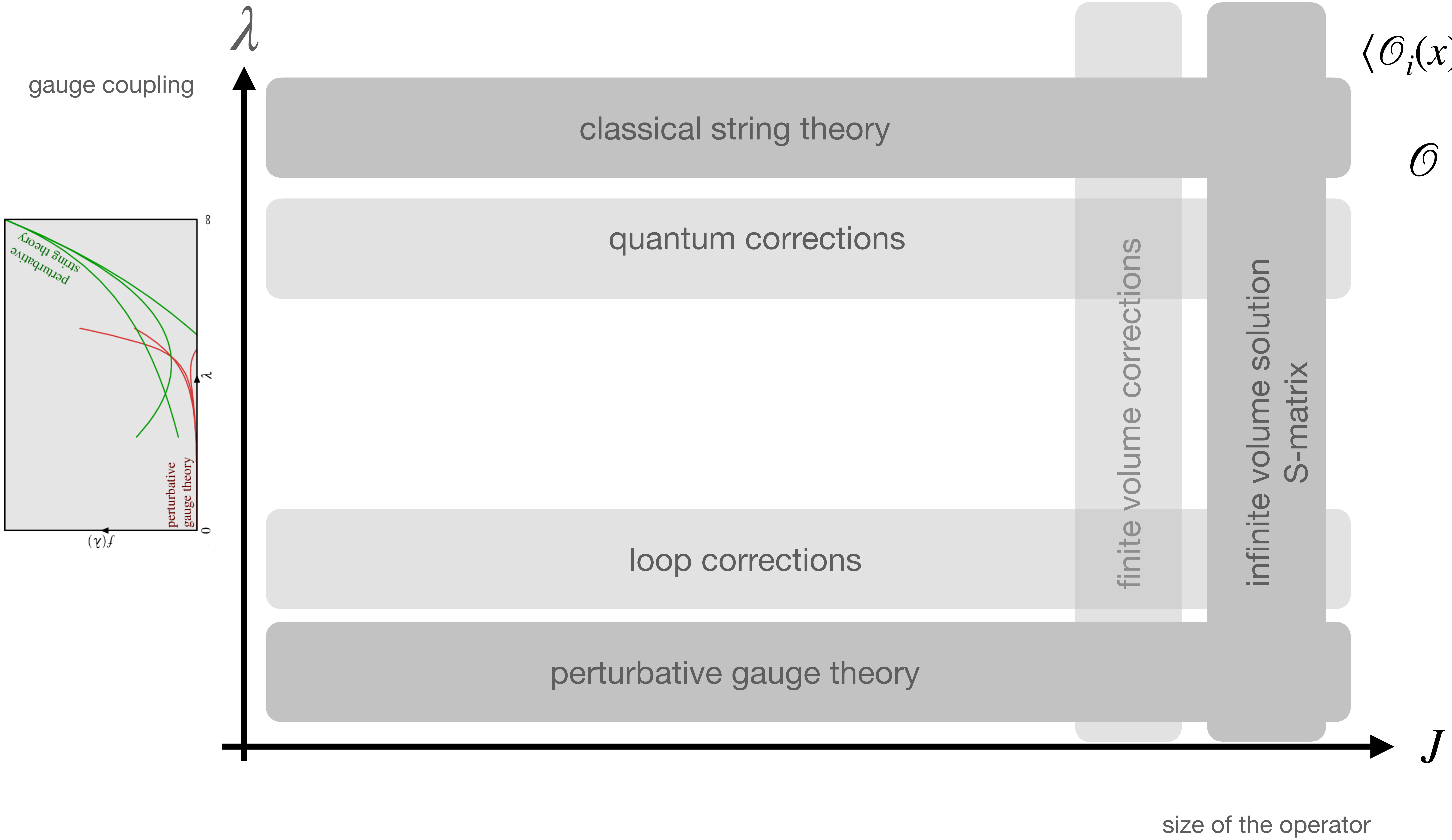
Einstein equations

perfect fluid $\frac{\eta}{s} = \frac{1}{4\pi}$

growing black hole [Janik et al]

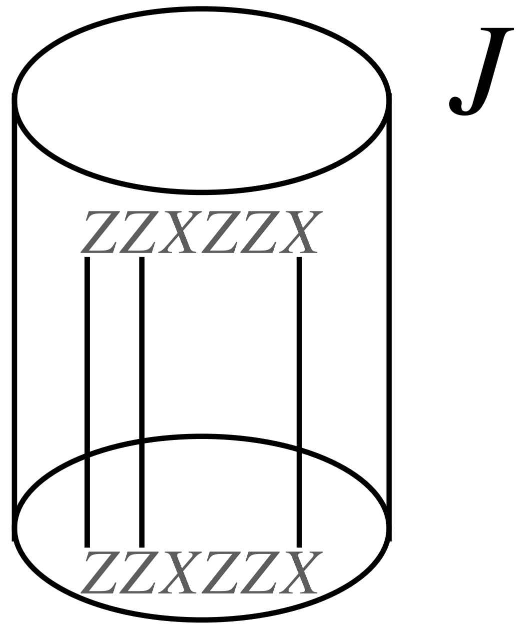
Qualitatively similar phenomena in accelerators in the heavy ion collision

Spectral problem: how integrability works



$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \mathrm{Tr}(Z^{J-M}X^M)$$



How integrability works: diagonal case

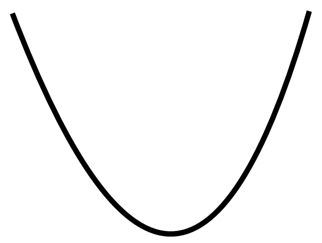
UV description

IR description

1+1 d scalar: sinh-Gordon theory

$$\mathscr{L} = \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}(\partial_x\varphi)^2 - \frac{m^2}{b^2}\left(\cosh b\varphi - 1\right)$$

[Dorey] [Bajnok]



LSZ reductions formula

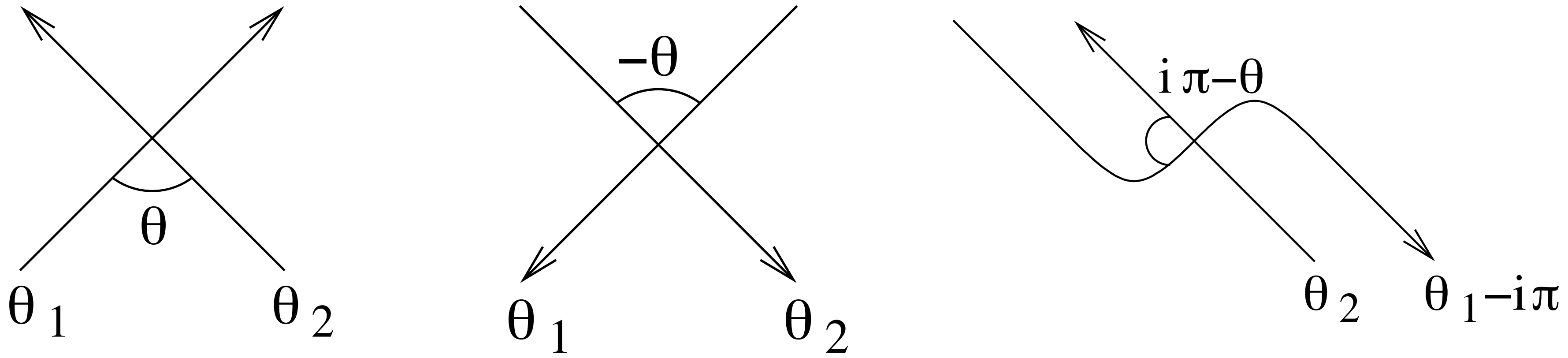
$$\mathscr{D}_j = i\int d^2x_j e^{ip_jx - i\omega_j t}\left\{\partial_t^2 - \partial_x^2 + m^2\right\}$$

$$\langle p_1', p_2' \,|\, p_1, p_2\rangle = \bar{\mathscr{D}}_1'\bar{\mathscr{D}}_2'\mathscr{D}_1\mathscr{D}_2\langle 0\,|\,T(\varphi(1)\varphi(2)\varphi(3)\varphi(4))\,|\,0\rangle = S(\theta_1 - \theta_2)$$

S-matrix perturbatively

$$S(\theta) = 1 - \frac{ib^2}{4\sinh\theta} - \frac{b^4(\theta(\pi/\sinh\theta - i))}{32\pi\sinh\theta} + \frac{ib^6(\pi/\sinh\theta - i)^2}{256\pi^2\sinh\theta} + O\left(b^8\right)$$

all perturbative orders: unitarity and crossing symmetry

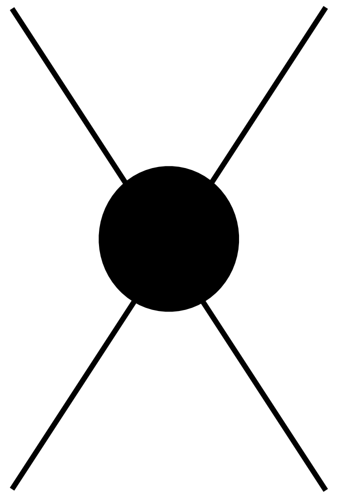


$$S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$$

one massive particle

$$p = m\sinh\theta$$

∞ many conserved charges
factorised scattering



unitarity, crossing symmetry

$$S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$$

$$S(\theta) = \frac{\sinh\theta - i\sin\pi a}{\sinh\theta + i\sin\pi a}$$

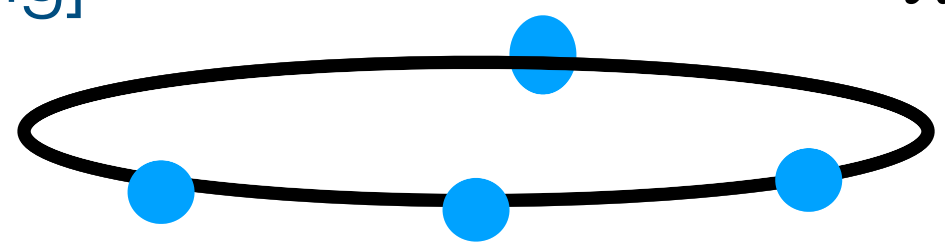
$$a = \frac{b^2}{8\pi + b^2}$$

Large volume spectrum: diagonal case

multiparticle state on the circle
momentum quantization

$$p = m \sinh \theta$$

[Bethe] [Yang]

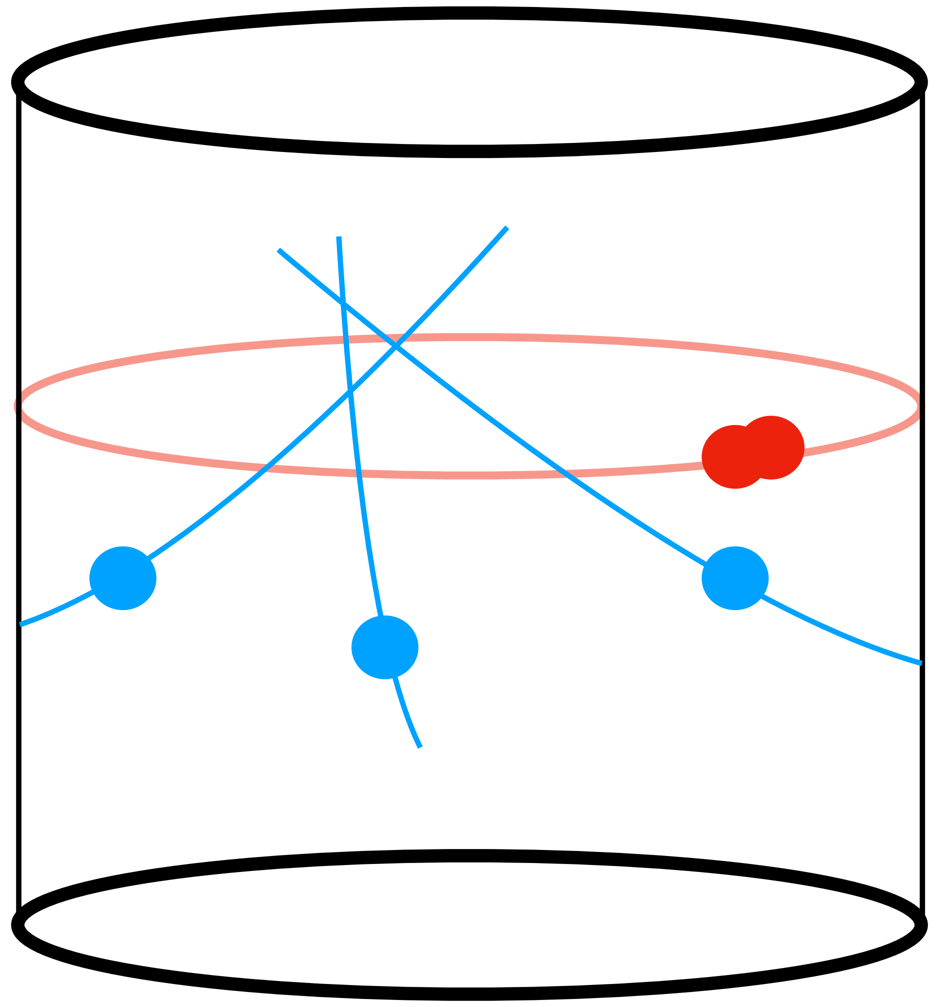


$$x \equiv x + L$$

Periodicity of wave function $$e^{imL\sinh\theta_j} \prod_{k:k\neq j} S(\theta_j - \theta_k) = 1 \qquad \Phi_j = mL\sinh\theta_j - i \sum_{k:k\neq j} \log S(\theta_j - \theta_k) = 2\pi n_j$$

Energy $$E(\{\theta\}) = \sum_k m\cosh\theta_k$$

Finite size corrections



direct corrections
to energy

$$\delta E(\{\theta\}) = \int \frac{du}{2\pi} \prod_i S\left(u + \frac{i}{2\pi} - \theta_i\right) e^{-mL\cosh u}$$

corrections
to momentum quantization

$$\delta\Phi(\{\theta\}) = \int \frac{du}{2\pi} \prod_i S'\left(u + \frac{i}{2\pi} - \theta_i\right) e^{-mL\cosh u}$$

all virtual effects can be summed up by the Thermodynamic Bethe Ansatz

How integrability works: non-diagonal case

UV description

1+1 d scalar: sine-Gordon theory

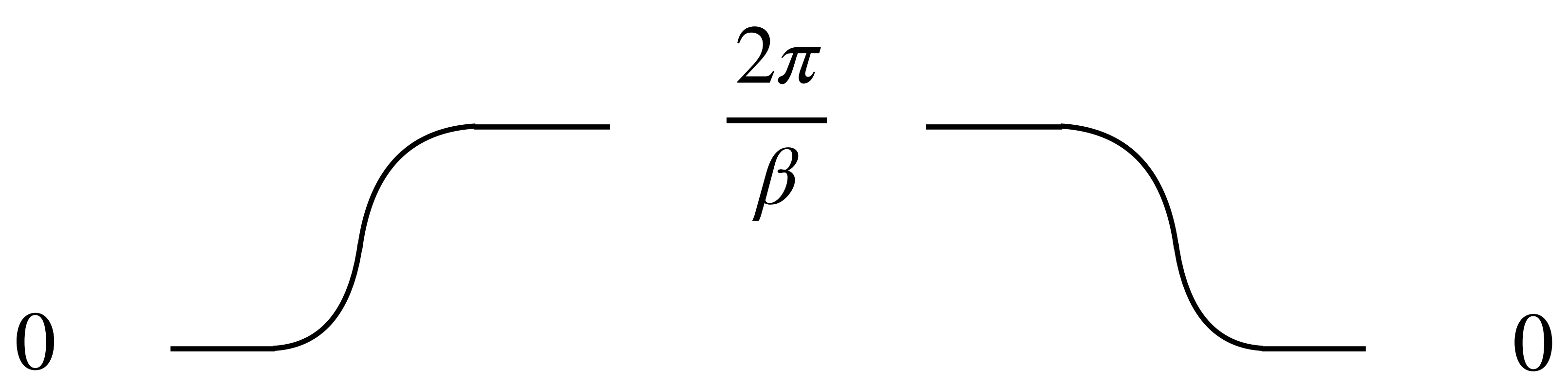
IR description

$$\mathcal{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 + \frac{m^2}{\beta^2}\left(\cos\beta\varphi - 1\right)$$

analytic continuation

$$b \to i\beta$$

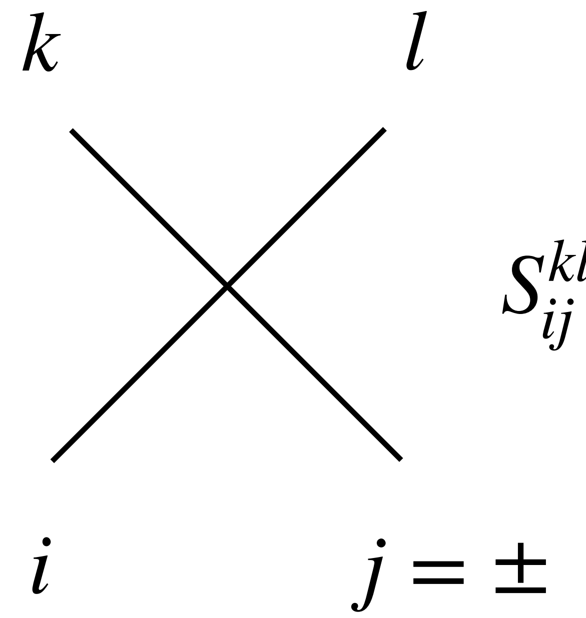
topological excitations: soliton and anti-soliton



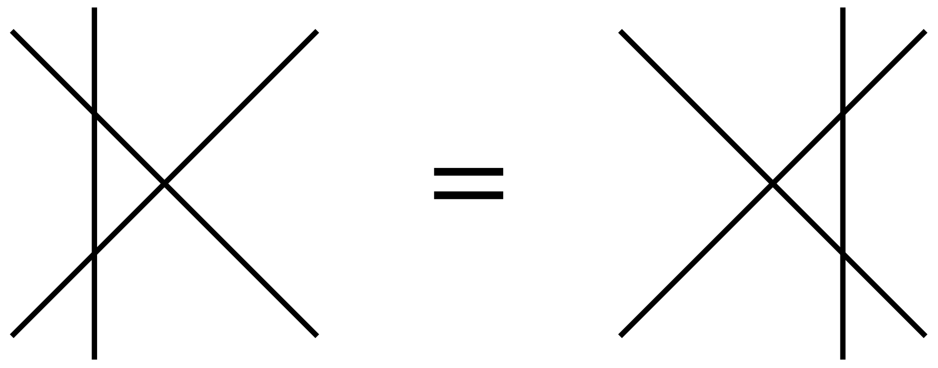
classical scatterings: time delays

quantum symmetry $U_q(sl_2)$

[Felder, LeClair]



$$S_{ij}^{kl}(\theta) = \mathcal{S}$$



YBE, symmetry

$$S(\theta) = S_{XXZ} S_0(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} S_0(\theta)$$

doublet of massive particles

$$p = m \sinh\theta$$

unitarity, crossing symmetry

$$\mathcal{S}(\theta) = \mathcal{S}(-\theta)^{-1} = \mathcal{C}\mathcal{S}(i\pi - \theta)\mathcal{C}^{-1}$$

∞ many conserved charges
factorised scattering

$$b = \frac{\sin(i\lambda\theta)}{\sin(\lambda(\pi + i\theta))}$$

$$c = \frac{-\sin\lambda\pi}{\sin(\lambda\pi + i\theta)}$$

[Zamolodchikov, Zamolodchikov]

How integrability works: AdS5/CFT4

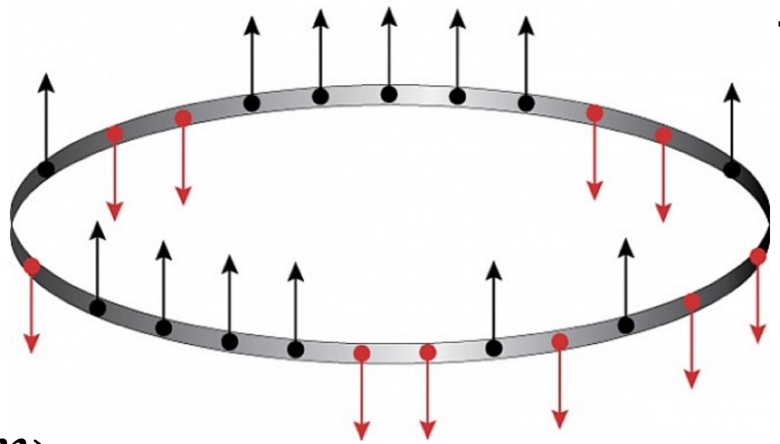
UV description

1+1 d scalar: sine-Gordon theory

IR description

$$\frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2-\frac{1}{2}(D\Phi)^2+i\bar{\Psi}\not{D}\Psi+V\right]$$

magnonic excitations



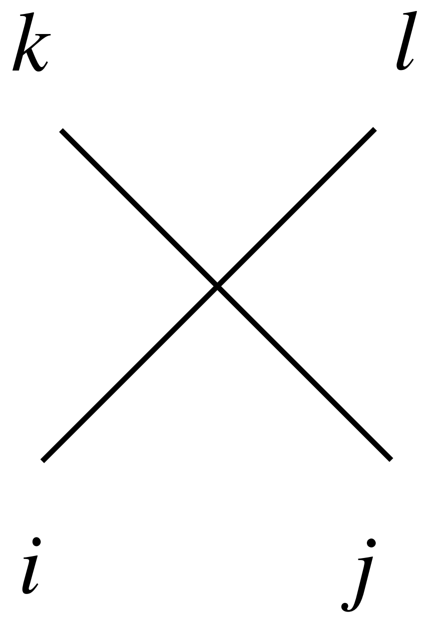
$$\mathcal{O}=\mathrm{Tr}(Z^j\Psi Z^k\bar{\Psi}Z^l XYDZ^m)$$

$$\sum_{n,m} e^{i(pn+qm)}\mathrm{Tr}(\underbrace{Z\ldots ZX}_{n}\underbrace{Z\ldots ZX}_{n+m}Z\ldots Z)+S(p,q)e^{i(qn+pm)}()$$

8 boson + 8 fermion particles

quantum symmetry $psu(2|2)_c\otimes psu(2|2)_{\dot{c}}$

dispersion relation $E(p)=\sqrt{1+\lambda\sin^2(p/2)}$



$S_{ij}^{kl}(\theta)=\mathcal{S}$

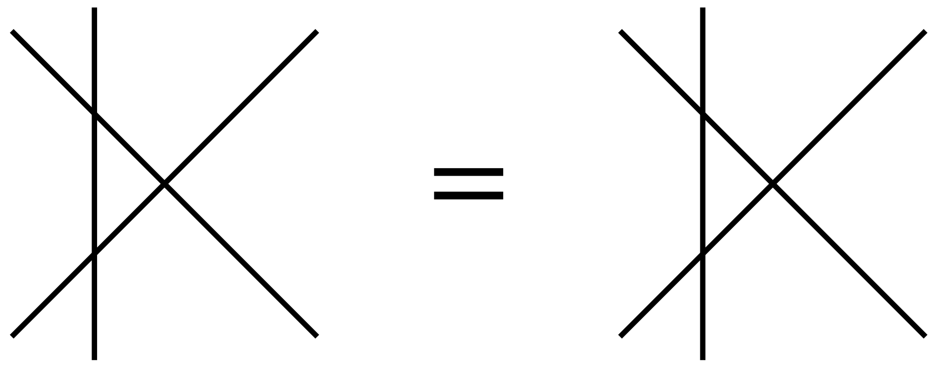
massive particles

$E(p)=\sqrt{1+\lambda\sin^2(p/2)}$

$$\begin{pmatrix}b_1\\b_2\\f_1\\f_2\end{pmatrix}\otimes\begin{pmatrix}\dot{b}_1\\\dot{b}_2\\\dot{f}_1\\\dot{f}_2\end{pmatrix}$$

unitarity, crossing symmetry

$$\mathcal{S}(p_1,p_2)=\mathcal{S}(p_2,p_1)^{-1}=\mathscr{C}\mathcal{S}(p(z_1),p(z_2+\omega))\mathscr{C}^{-1}$$



∞ many conserved charges
factorised scattering

YBE, symmetry

$$\mathcal{S}=\mathcal{S}_{Hubbard}\otimes\mathcal{S}_{Hubbard}\,S_0$$

[Beisert]

$\mathcal{S}_{Hubbard}$ **16x16 sparse matrix**

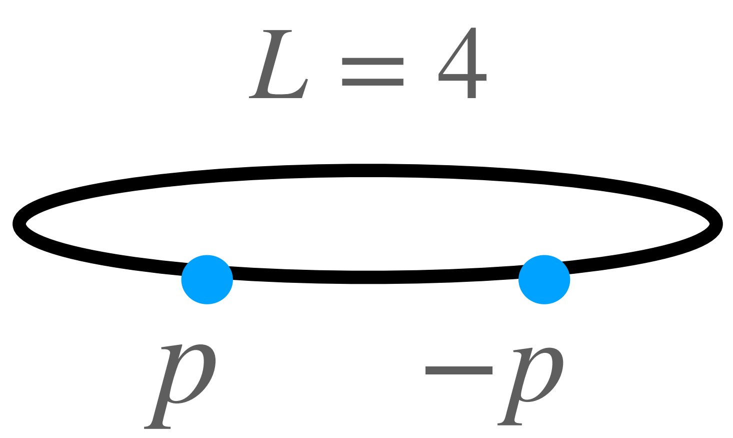
The Konishi dimension

Konishi is the simplest gauge invariant non-protected single trace operator

$$\mathcal{O} = \mathrm{Tr}(\Phi_i^2) \sim \mathrm{Tr}(\mathrm{ZZXX} + \dots) \sim \mathrm{Tr}(\mathrm{D}^2\mathrm{Z}^2)$$

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

asymptotic Bethe Ansatz



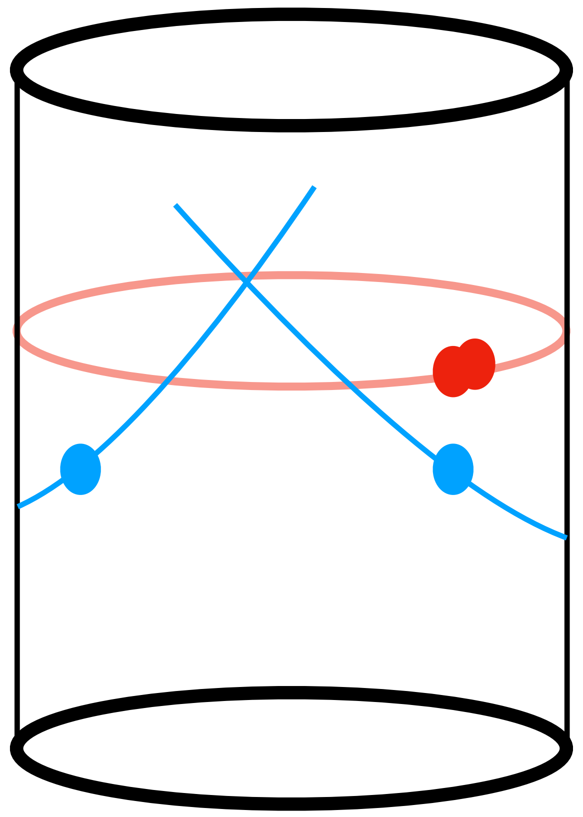
$$e^{ipL}S(p,-p) = 1 \qquad p = \frac{2\pi}{3} - \frac{\sqrt{3}}{16}\lambda + \frac{9\sqrt{3}}{512}\lambda^2 + \frac{9\sqrt{3}(1+\zeta(3))}{512}\lambda^6 + \dots$$

$$\Delta(\lambda) = 2\sqrt{1+\lambda\sin^2(p/2)} = 4 + \frac{3}{4}\lambda - \frac{3}{16}\lambda^2 + \frac{21}{256}\lambda^3 - \frac{2820+288\zeta(3)}{16384}\lambda^4$$

vacuum polarisation effects

$$\delta E(\{\theta\}) = \sum_Q \int \frac{u}{2\pi} \prod_i S_Q(\tilde{u},p)e^{-\tilde{E}(u)L}$$

[Bajnok, Janik]



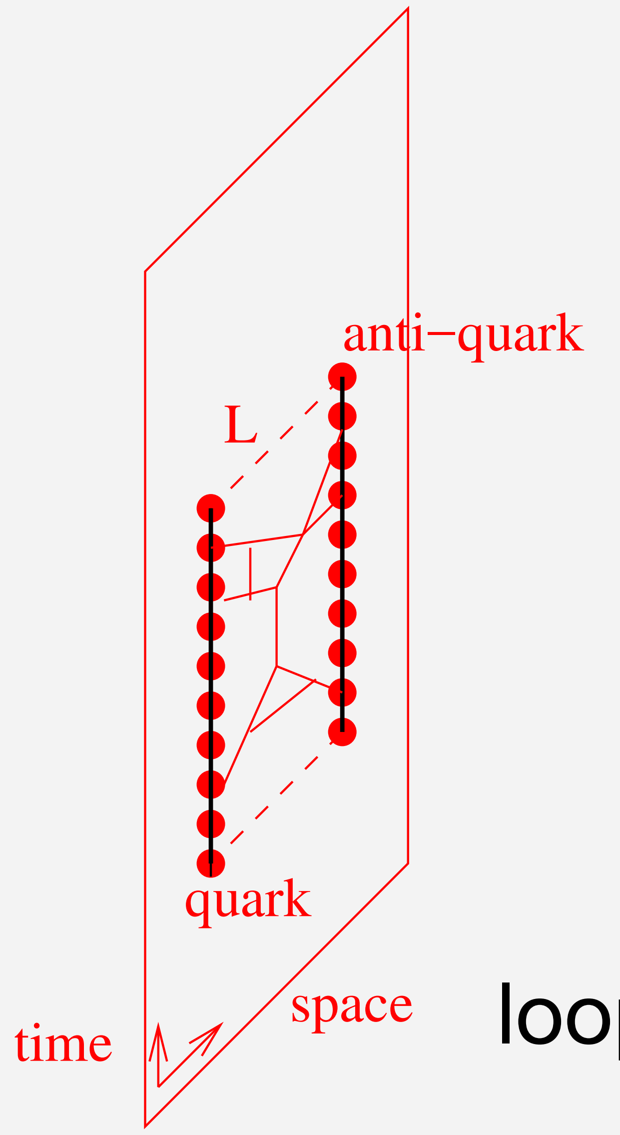
$$\frac{324+864\zeta(3)-1440\zeta(5)}{16384}\lambda^4$$

agrees with 4 loop gauge theory calculations

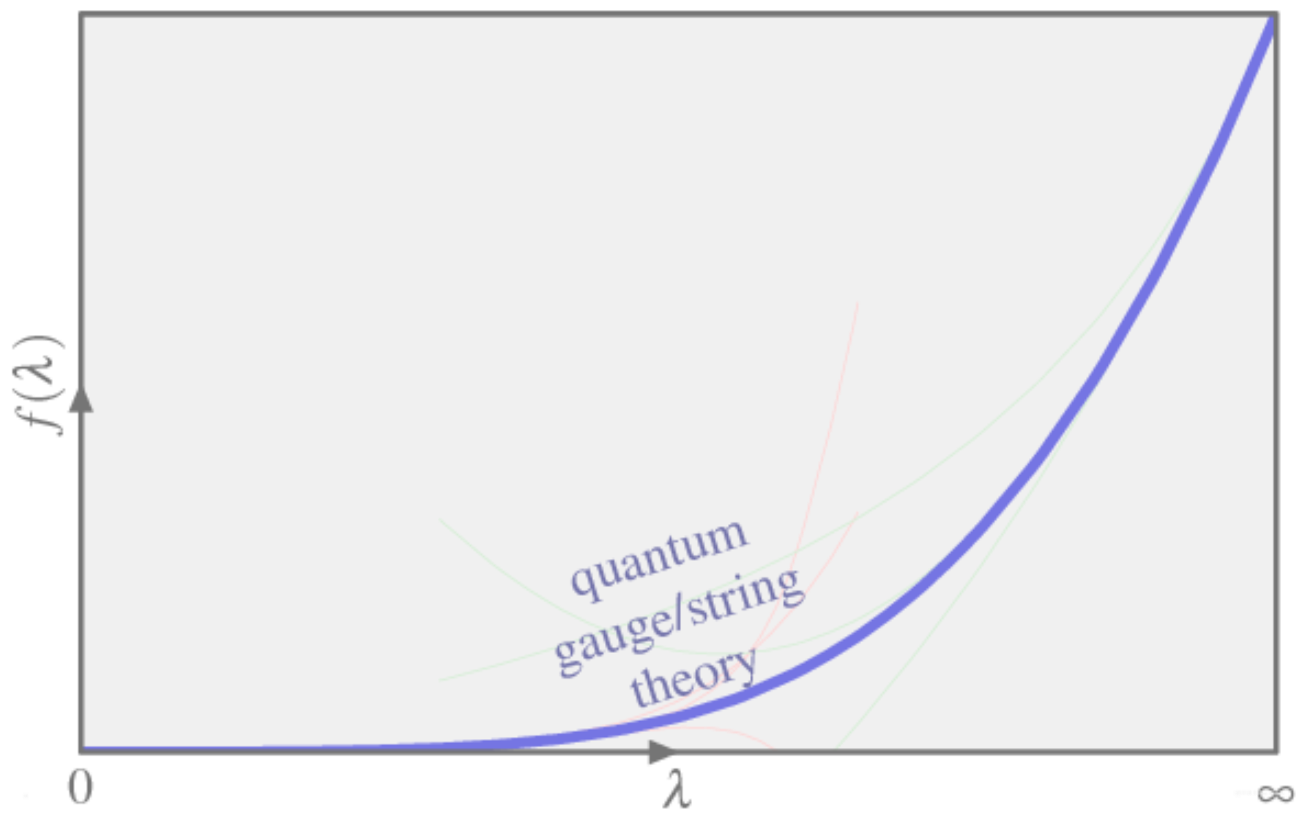
[Fiamberti et al]

Conclusion

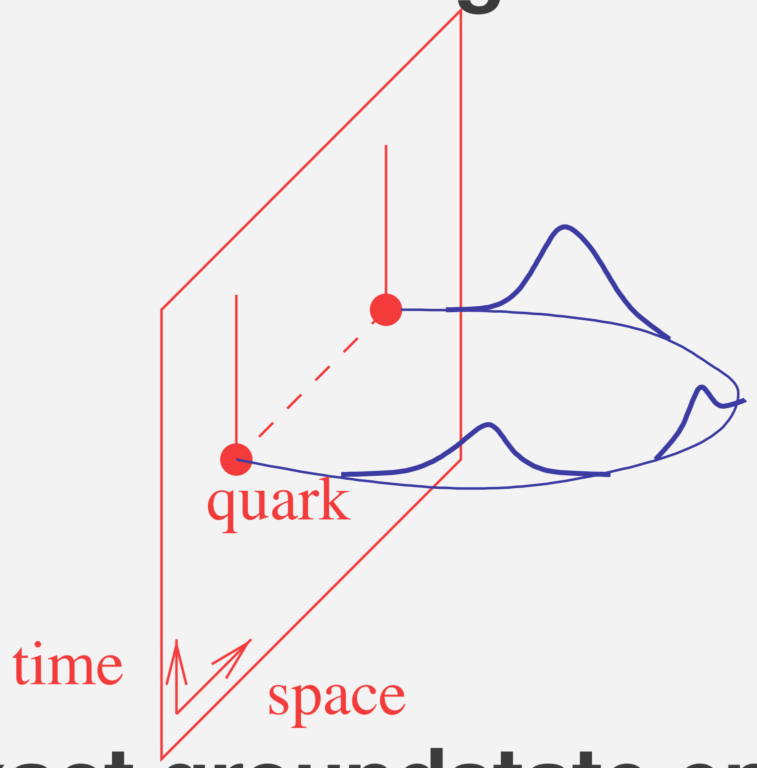
3+1 D gauge theory



loop expansion

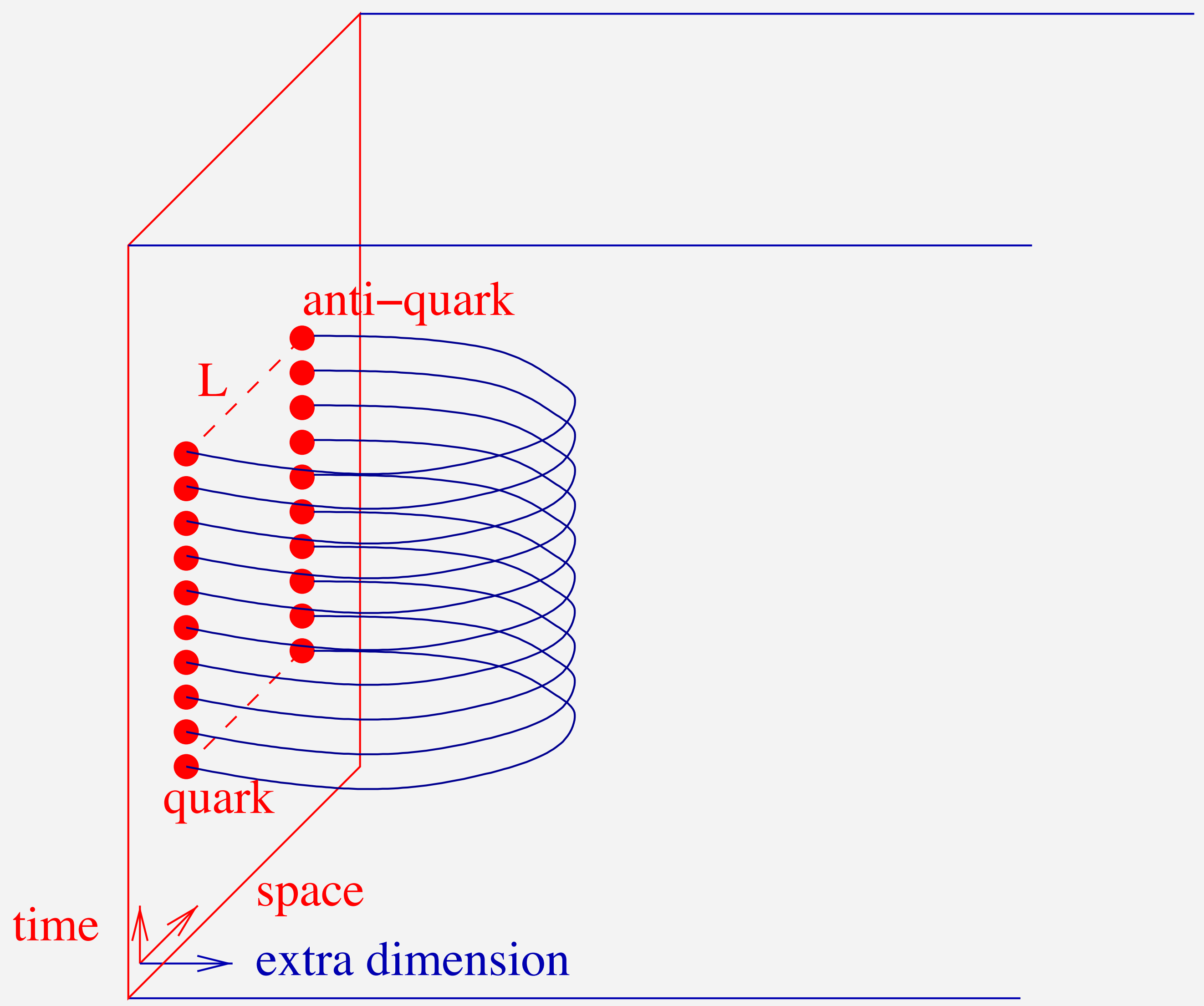


1+1 D integrable QFT



Exact groundstate energy

1+9 D string theory



minimal surface fluctuations

Heroic efforts to extend to many more observables, including

correlations function, 1,2,3,4,…, gluon scattering amplitudes, …, heavy ion collisions