Functional QCD and the QCD phase structure

PhD School XQCD 2024

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Universität Heidelberg & ExtreMe Matter Institute

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GEFÖRDERT VOM
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STRUCTURES CLUSTER OF EXCELLENCE

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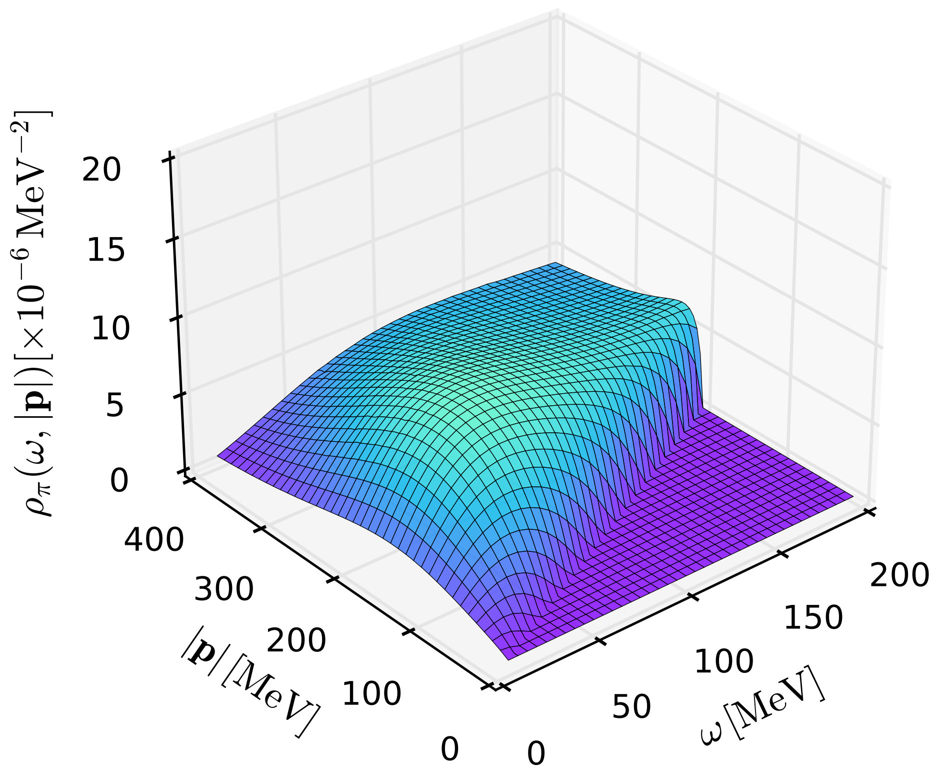
EMMI

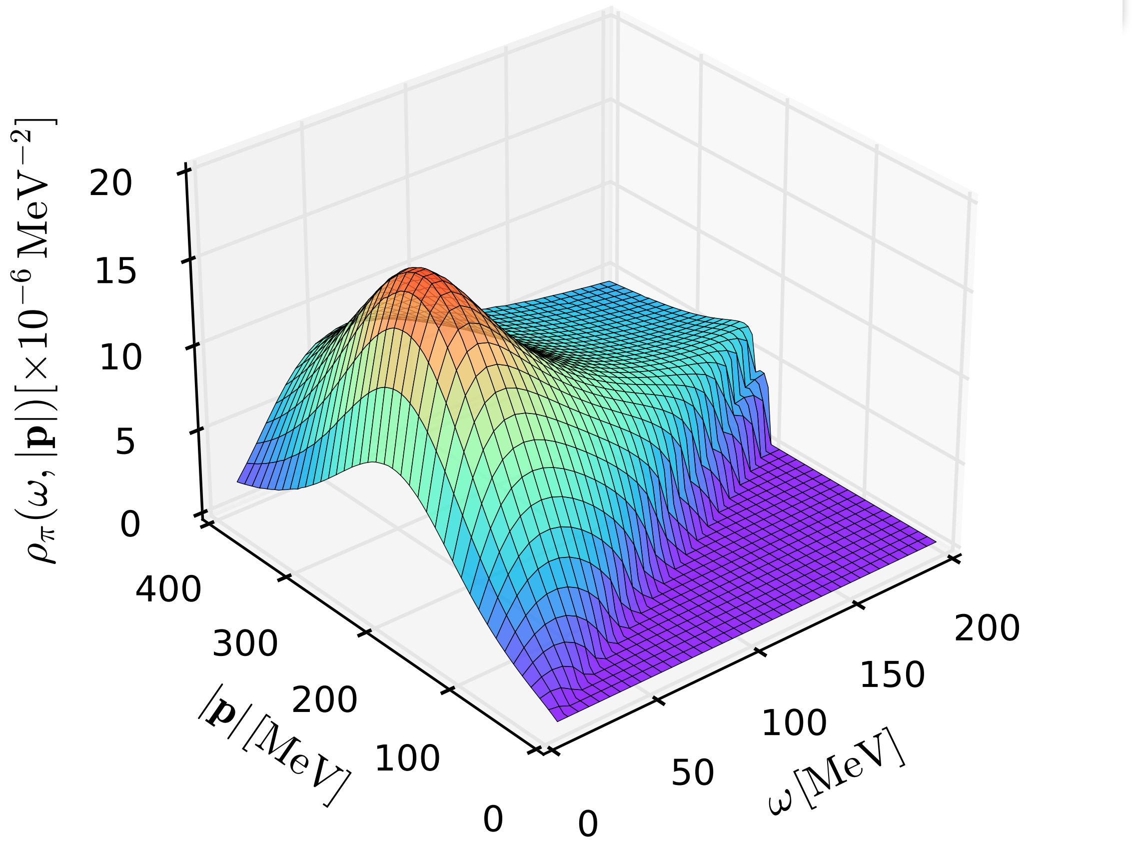
Phase structure of QCD

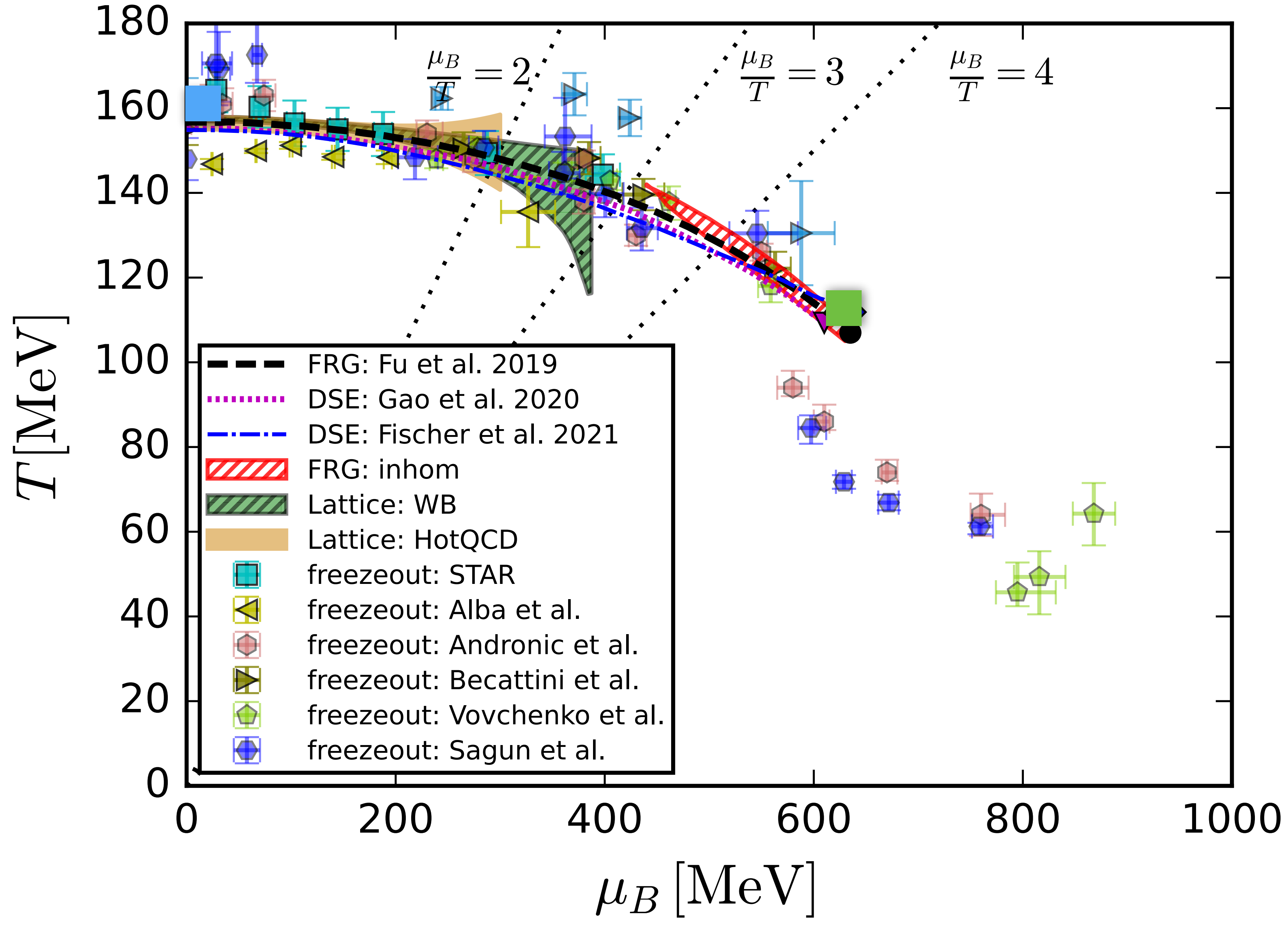
Spectral functions & the moat

T=160 MeV & μ_B =0



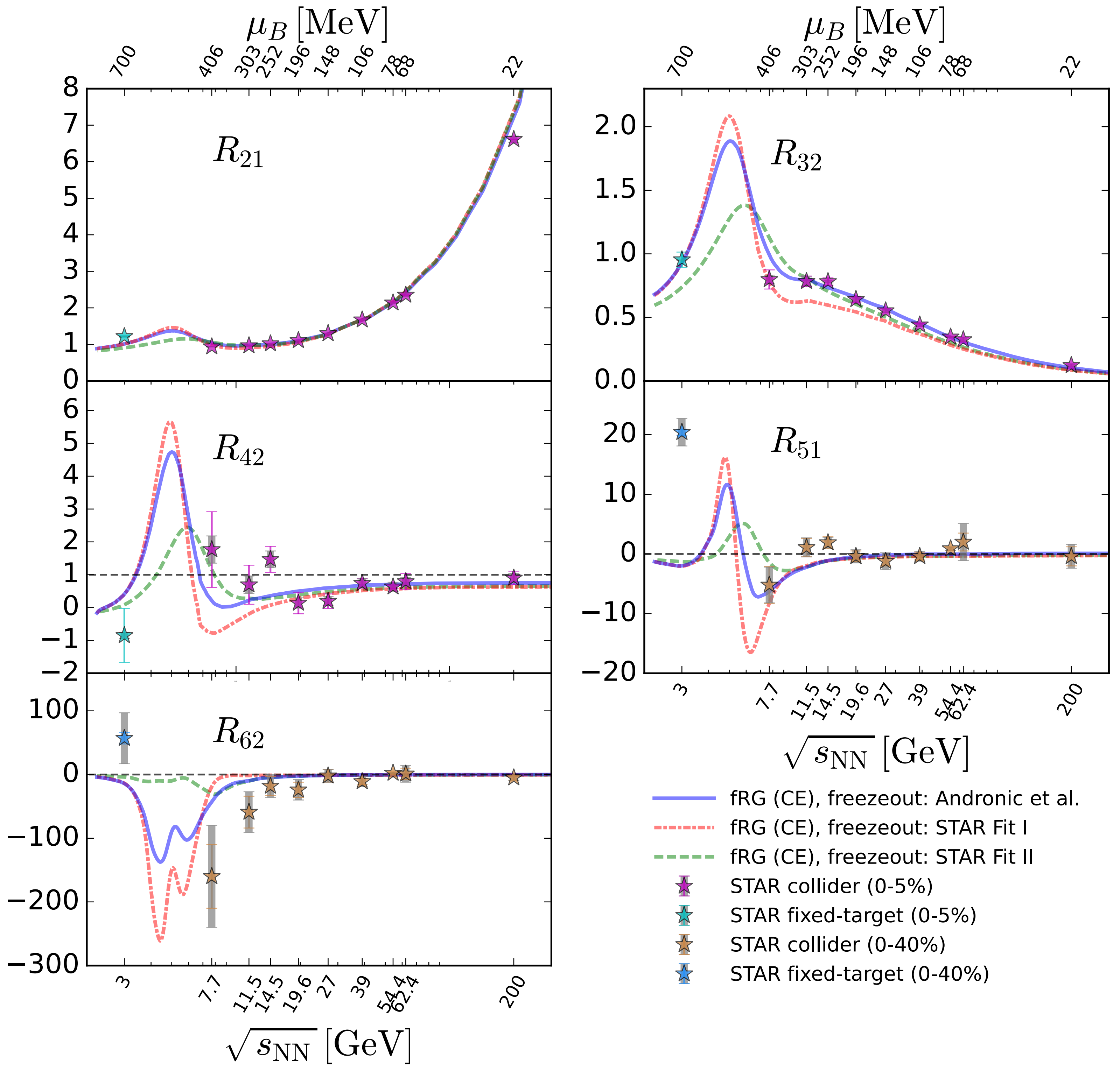
T=114 MeV & μ_B =630 MeV





Ripples of the critical end point

baryon & proton number fluctuations



Phase structure of QCD

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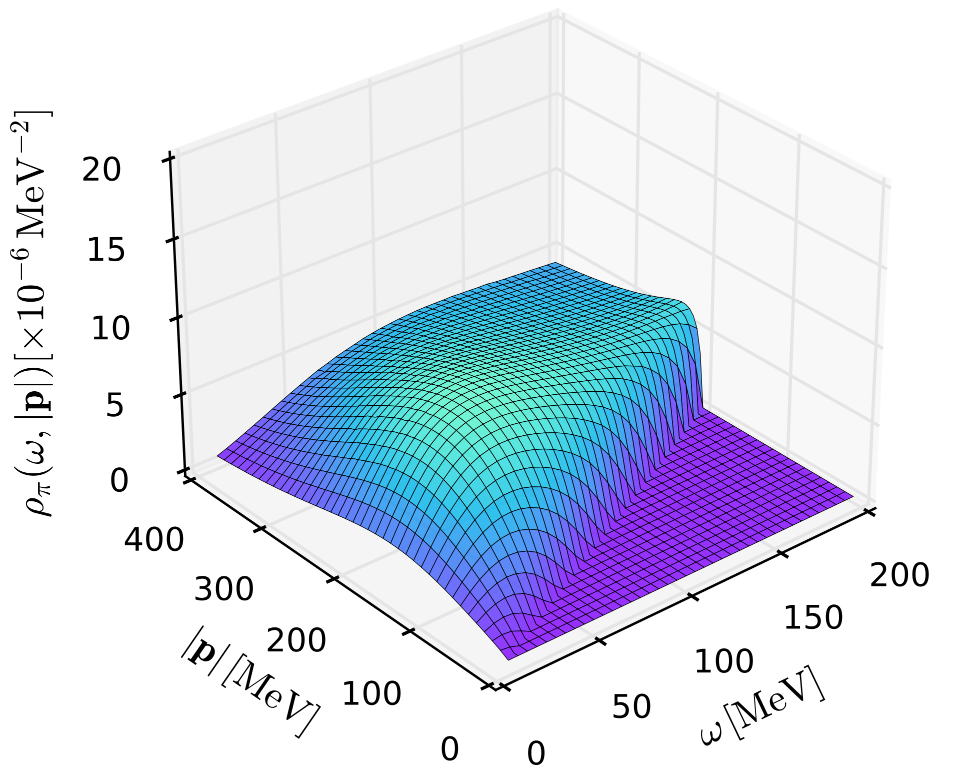

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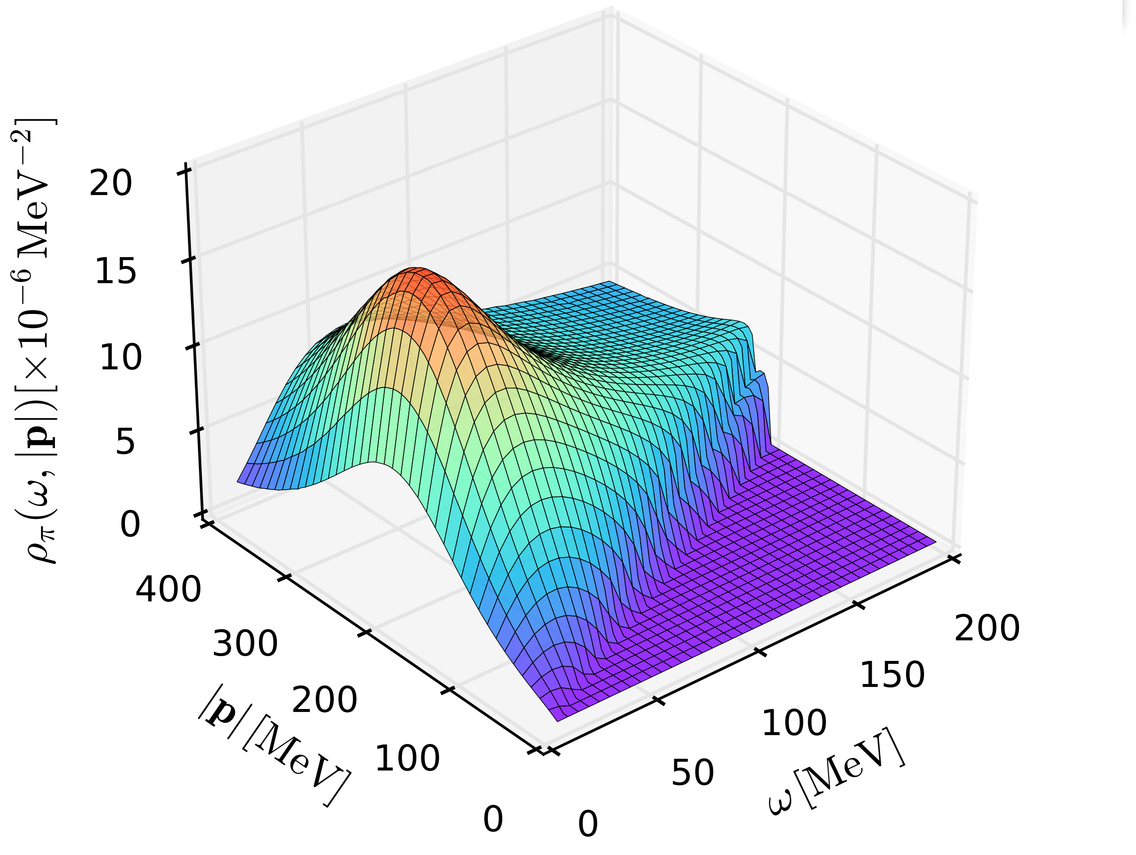

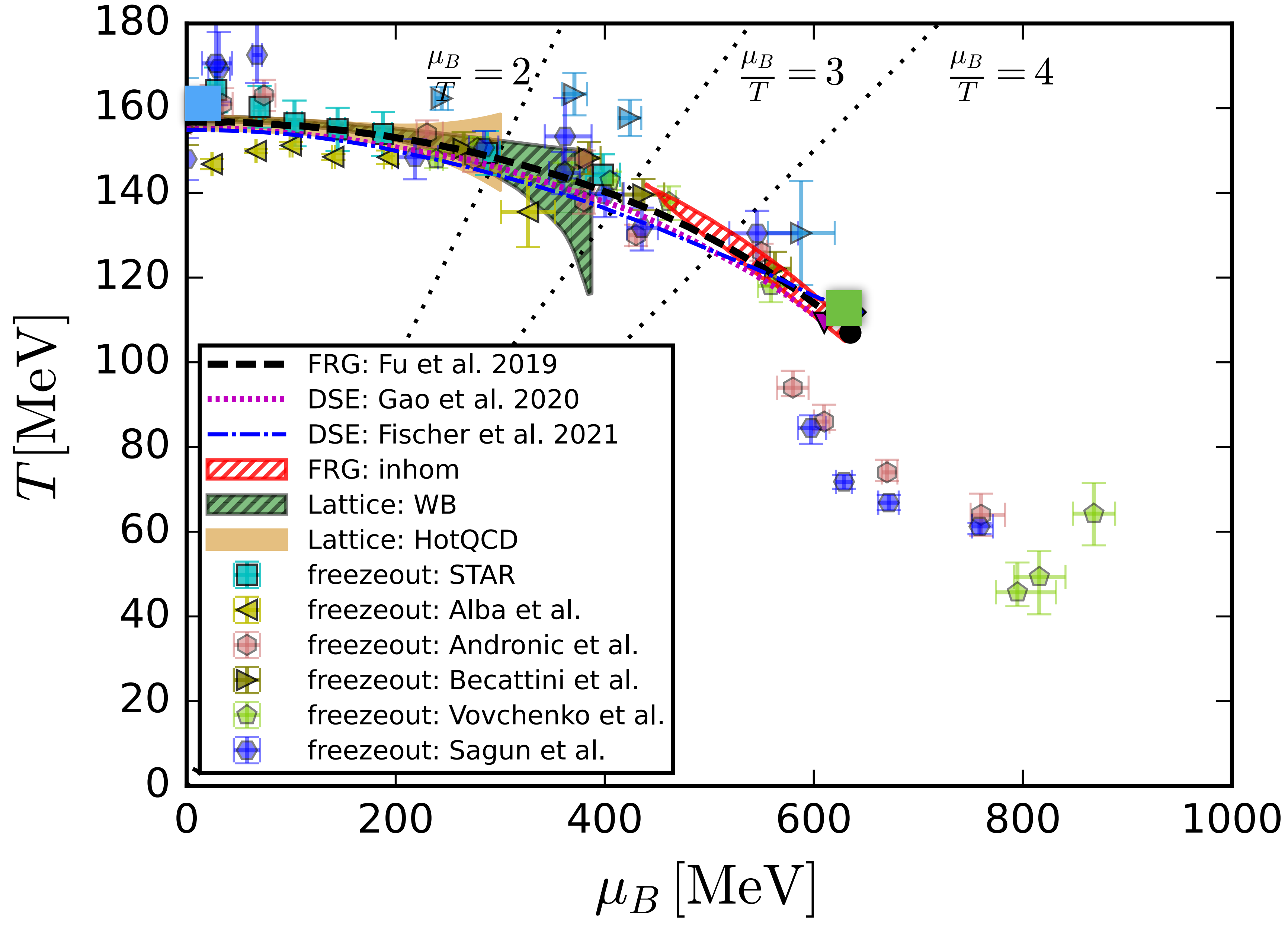

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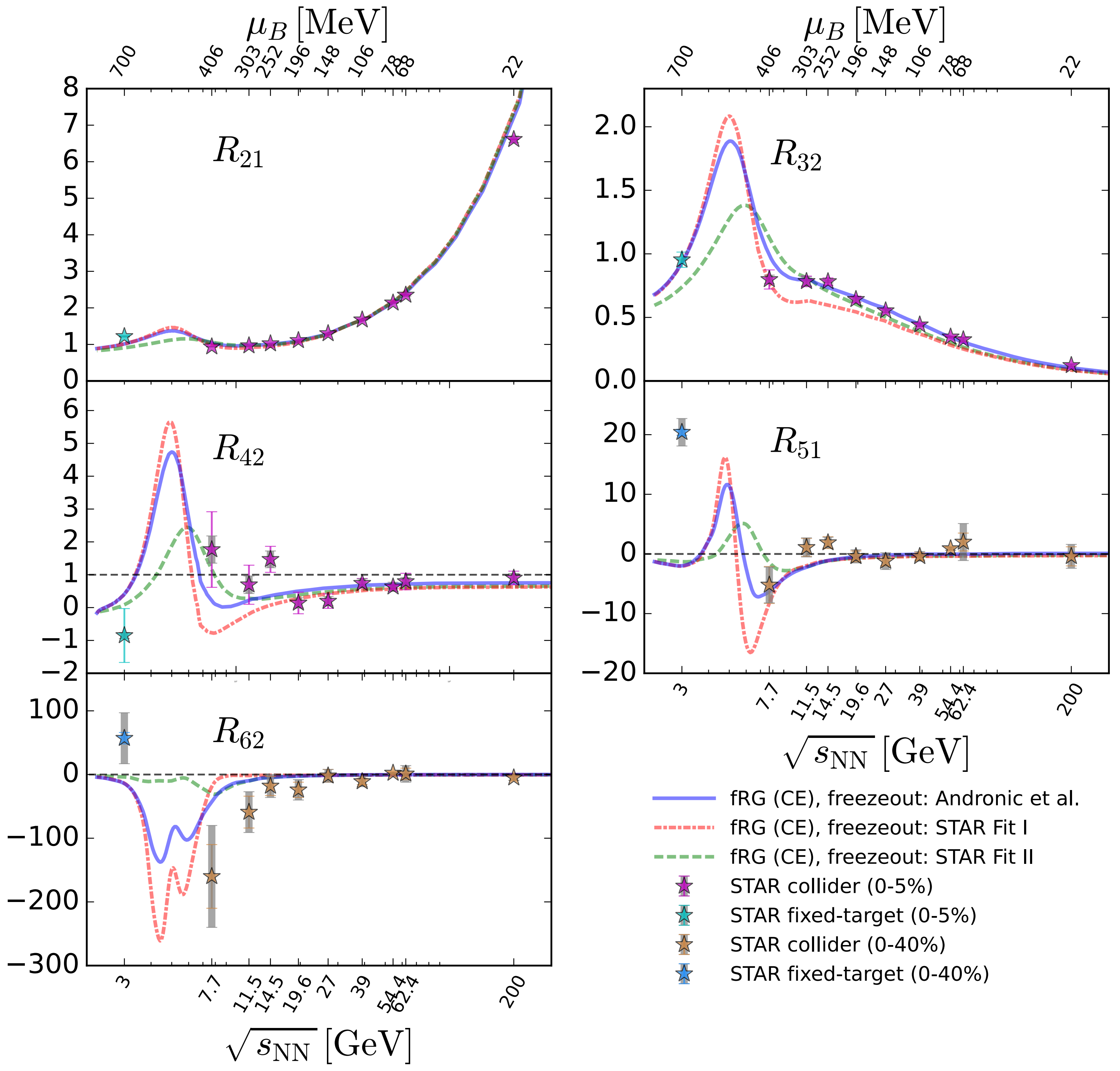

How to compute

&

How to judge

Material

Topical reviews

Collection of reviews & lecture notes on the FRG & DSE

Structure of the FRG: Aspects of the FRG

JMP, Annals Phys. 322 (2007) 2831-2915

The nonperturbative functional renormalization group and its applications

Dupuis et al, Phys.Rep. 910 (2021) 1-114

QCD at finite temperature and density within the fRG approach: An overview

Fu, Commun.Theor.Phys. 74 (2022) 9, 097304

Outline

- (I) Functional Renormalisation group

- (II) Functional QCD and the QCD phase structure

(I) Functional Renormalisation Group for QCD

- **Introduction to the functional renormalisation group**
  - Derivation of the flow equation
  - Spontaneous symmetry breaking
  - Systematic error control & optimisation

- **Functional flows for QCD**
  - Flows for correlation functions & chiral symmetry breaking
  - Getting dynamical: emergent hadrons & diquarks
  - Dynamical hadronisation at work

(II) Functional QCD and the QCD phase structure

- **QCD at finite temperature and density**
  - **Benchmarks in the vacuum**
  - **Correlation functions at finite temperature**
  - **Polyakov loop from functional approaches**

- **QCD phase structure**
  - **Locating the QCD phase boundary and the critical end point**
  - **Fluctuations of conserved charges: Ripples of the critical end point**

(I) Functional Renormalisation Group for QCD

- **Introduction to the functional renormalisation group**
  - Derivation of the flow equation
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- **Functional flows for QCD**
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Introduction to the functional renormalisation group

Derivation of the flow equation

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi]+\int_x J\varphi}$$

partition function

$$\langle \varphi \rangle_J = \phi$$

$$S[\varphi] = \frac{1}{2} \int_x \left[\partial_\mu \varphi \partial_\mu \varphi + \mathrm{m}^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 \right]$$

classical action

zero-dimensional example: 'Functional' flows for integrals

Functional Renormalisation Group

Generating functional Z

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$$\langle \varphi \rangle_J = \phi$$

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi} \, e^{-S[\hat{\varphi}+\phi] + \int_x \hat{\varphi} \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

free energy

$$\varphi = \hat{\varphi} + \phi$$

$$\langle \hat{\varphi} \rangle_{\frac{\delta\Gamma}{\delta\phi}} = 0$$

$$J = \frac{\delta\Gamma}{\delta\phi}$$

Functional Renormalisation Group

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$$J = \frac{\delta \Gamma}{\delta \phi}$$

$$\Gamma[\phi] = \sup_J \left(\int_x J \cdot \phi - \log Z[J] \right)$$

Legendre transform

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi]+\int_x \, J\varphi}$$

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$$J = \frac{\delta\Gamma}{\delta\phi}$$

Dyson-Schwinger equation

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

quantum equation of motion

Functional Renormalisation Group

Dyson-Schwinger equation

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi} + \phi]}{\delta \phi(x)} \right\rangle$$

Diagrammatics

$$S[\phi] = \frac{1}{2} \int_x \left[\partial_\mu \phi \partial_\mu \phi + \mathrm{m}^2 \phi^2 + \frac{\lambda}{4} \phi^4 \right]$$

Functional Renormalisation Group

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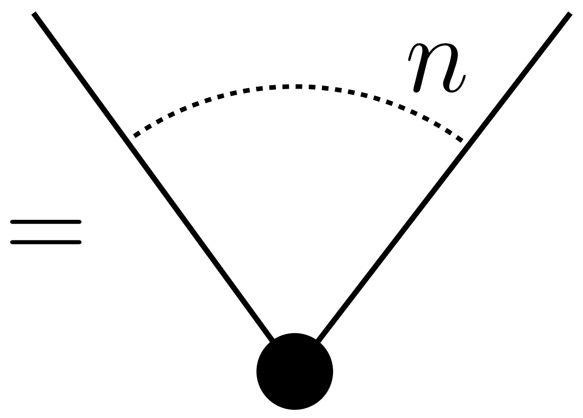
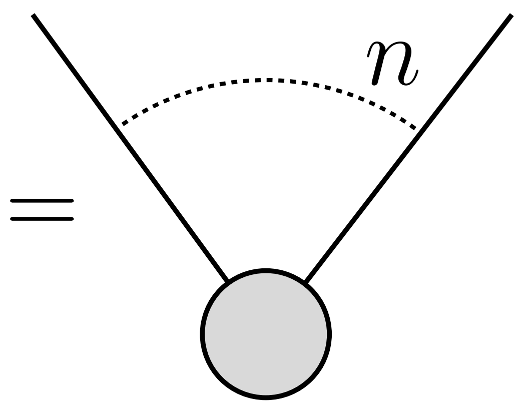
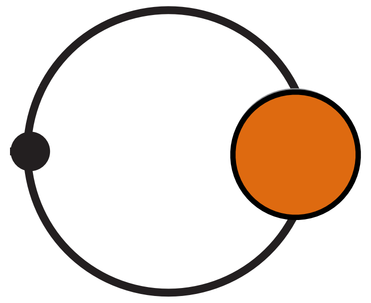
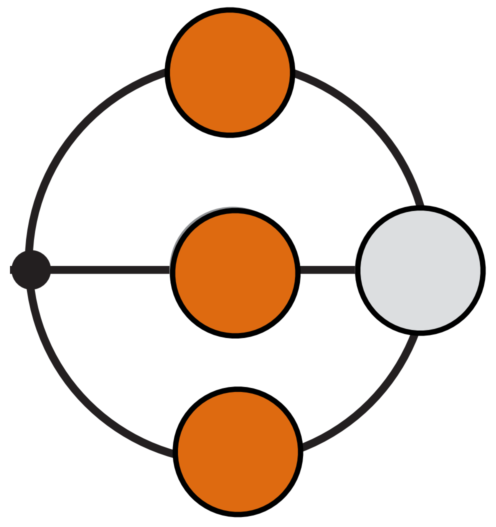
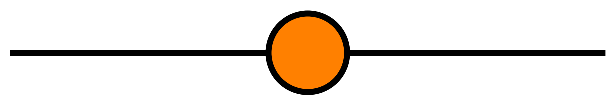
$S^{(n)} =$ 

$\Gamma^{(n)} =$ 

$\frac{\lambda}{2} \langle [\hat{\varphi}(x) + \phi(x)]^3 \rangle = \frac{\lambda}{2} \phi^3(x) + \frac{3\lambda}{2} \phi(x)$  $- \quad \frac{\lambda}{2}$ 

$G =$  $= \langle \hat{\varphi}(x) \hat{\varphi}(y) \rangle_c$

Functional Renormalisation Group

Dyson-Schwinger equation

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

Diagrammatics

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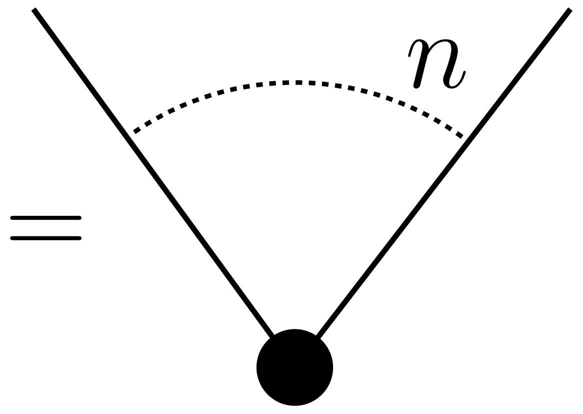
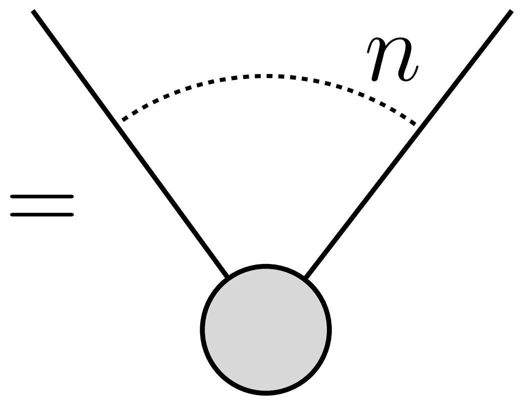
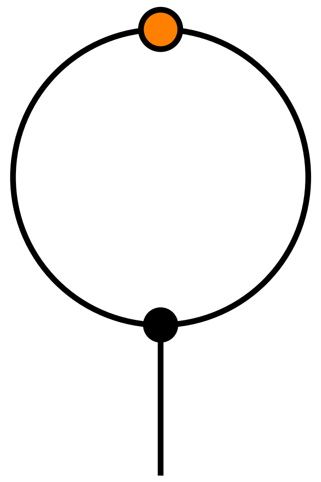
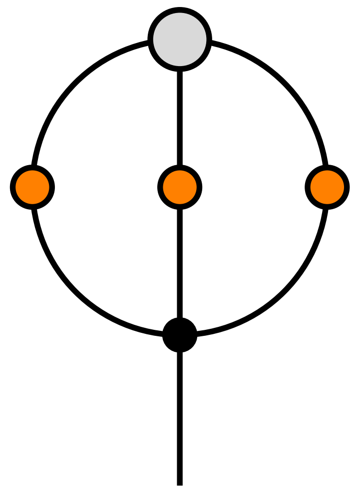
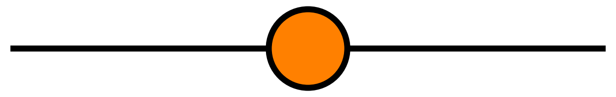
$S^{(n)} =$ 

$\Gamma^{(n)} =$ 

$\frac{\delta\Gamma}{\delta\phi} = \frac{\delta S}{\delta\phi} + \frac{1}{2}$  $-\frac{1}{6}$ 

$G =$  $= \langle\hat{\varphi}(x)\hat{\varphi}(y)\rangle_c$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi]+\int_x \hat{\varphi}\, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

No quantum fluctuations

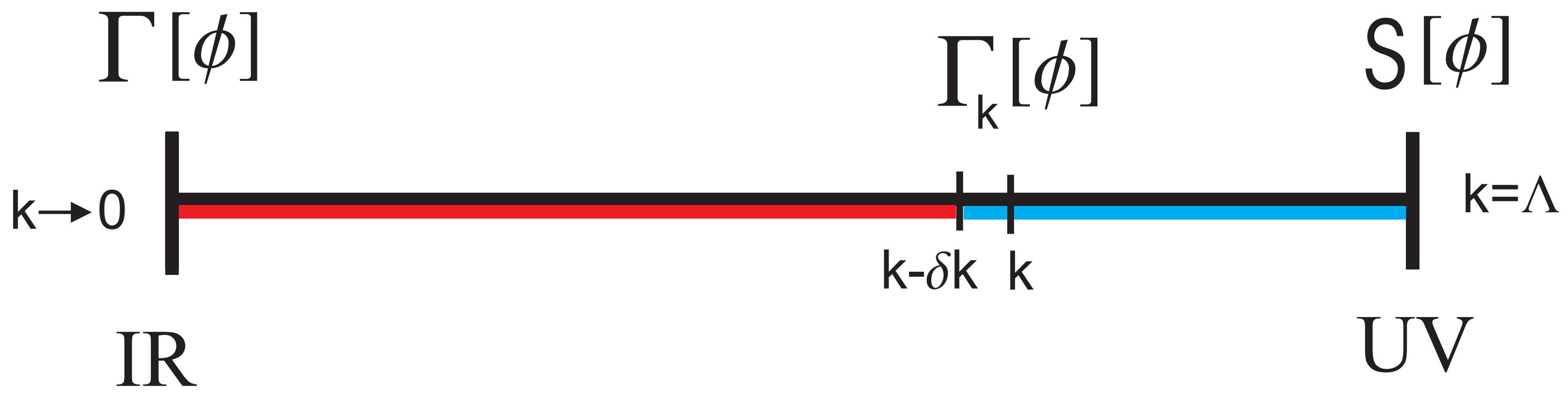
$$\Gamma[\phi] = -\log e^{-S[\phi]} = S[\phi]$$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi]+\int_x \hat{\varphi}\, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

UV quantum fluctuations up to $p^2 \approx k^2$



Functional Renormalisation Group

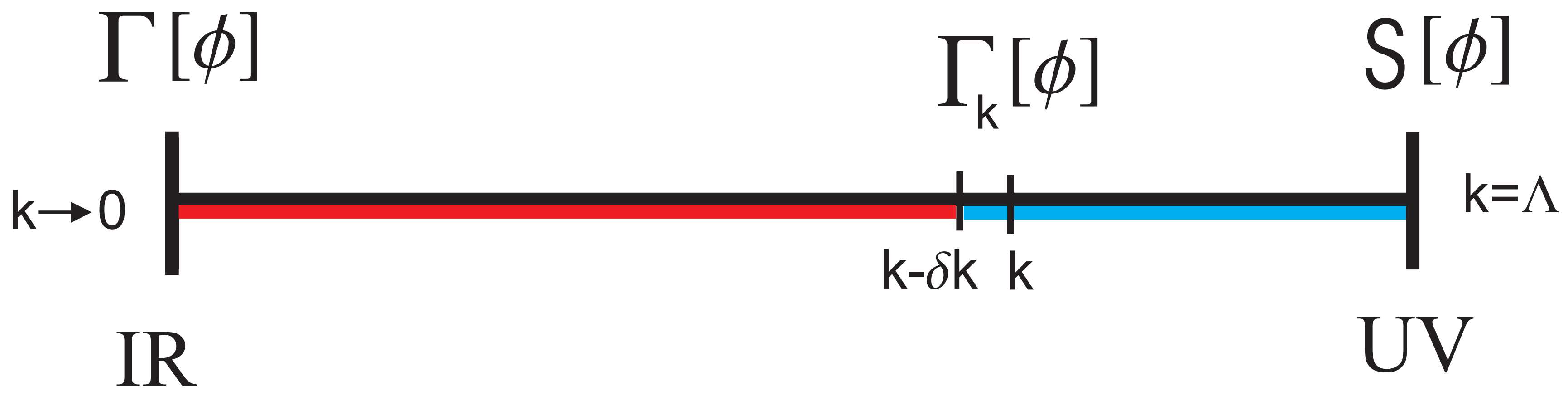
Effective action Γ

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\phi+\hat{\varphi}] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2) \hat{\varphi}(-p) + \int_x \hat{\varphi}(x) \frac{\delta \Gamma_k[\phi]}{\delta \phi(x)}}$$

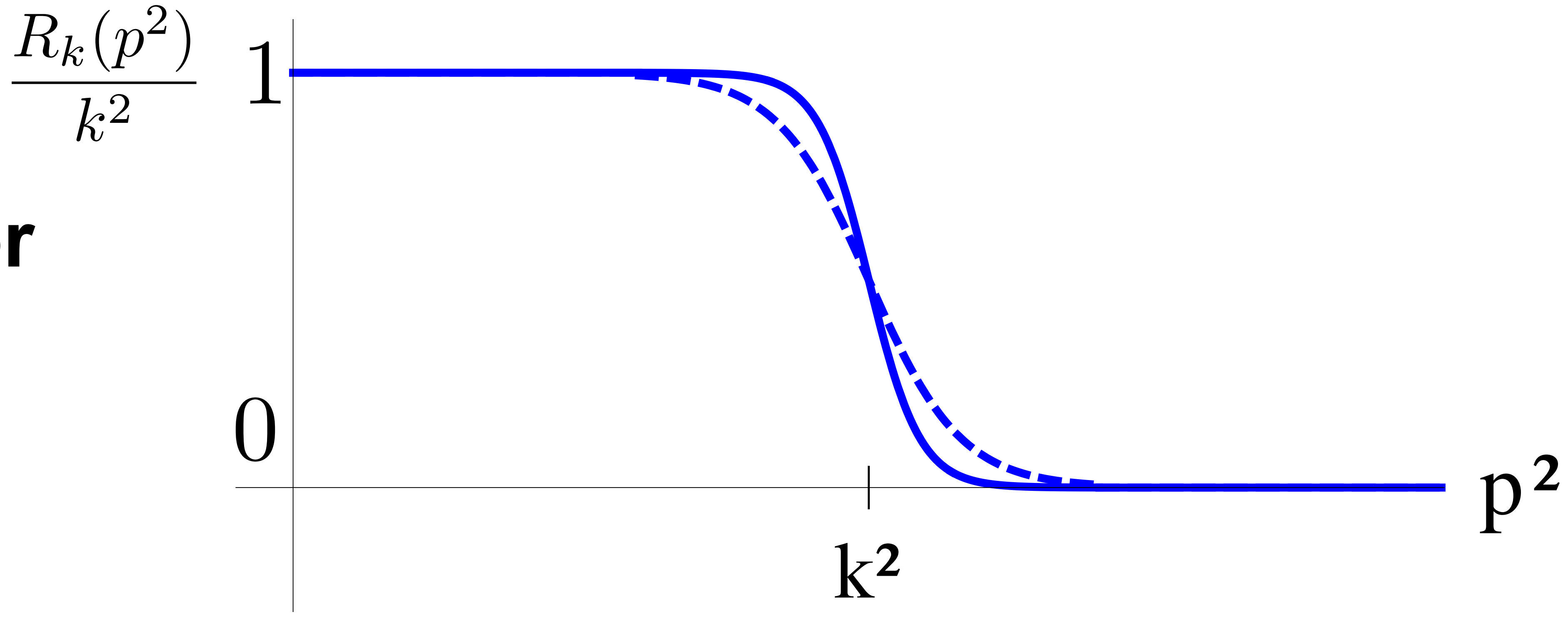
DSE

$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta \phi(x)} \right\rangle$$

UV quantum fluctuations up to $p^2 \approx k^2$



Regulator



Functional Renormalisation Group

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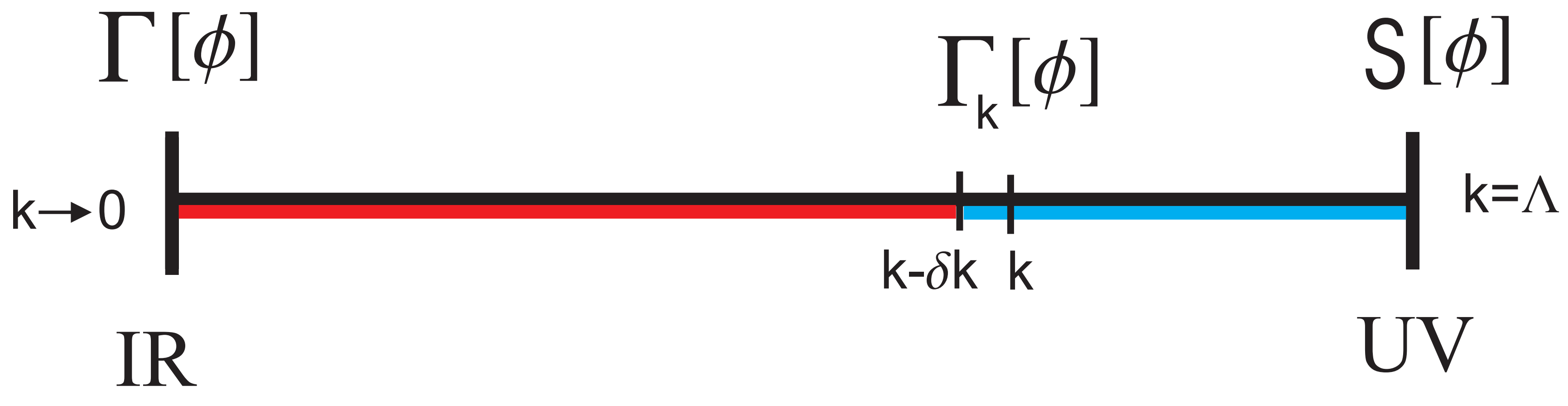

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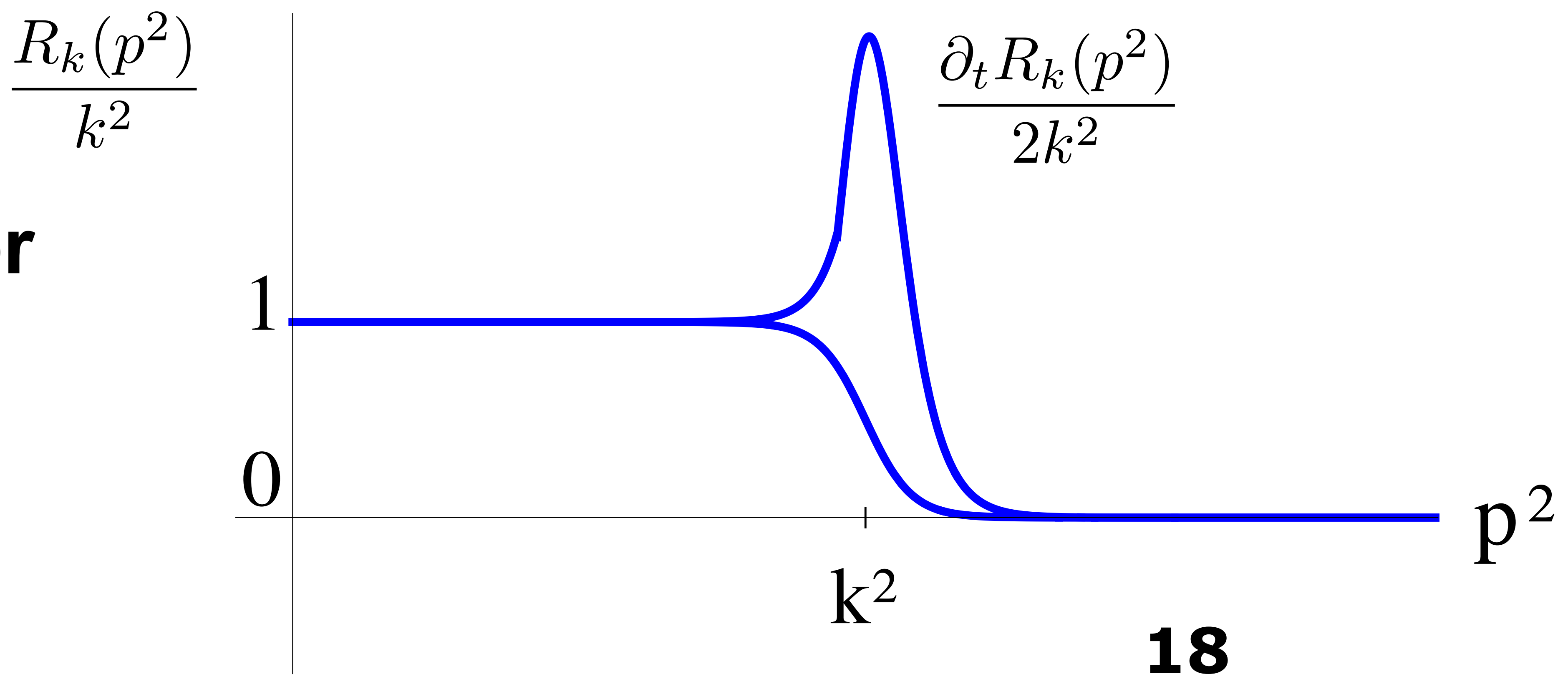

$t = \log \frac{k}{\Lambda}$

Functional Renormalisation Group

Effective action Γ

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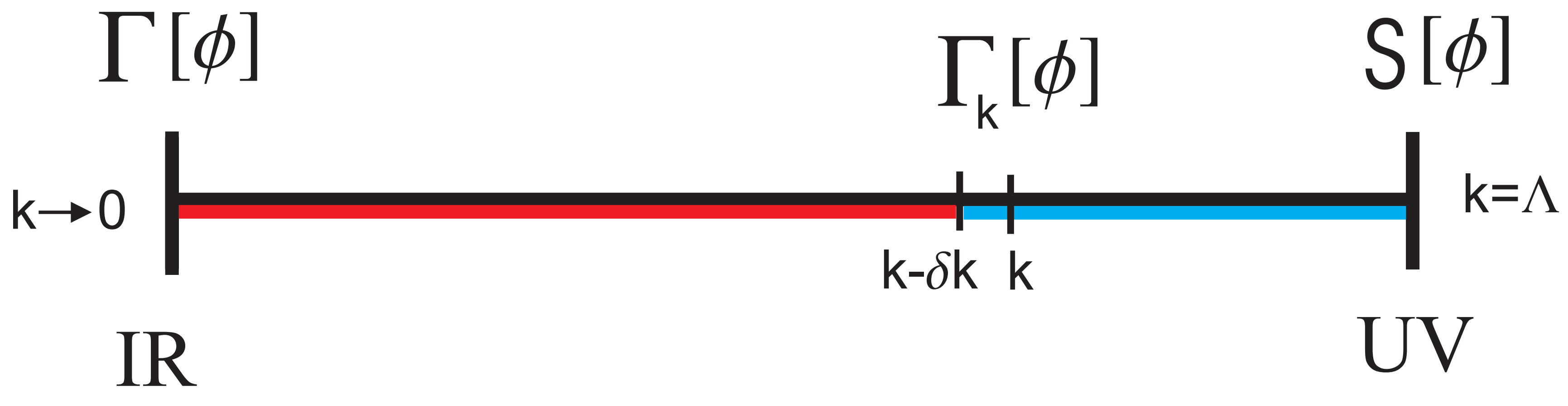

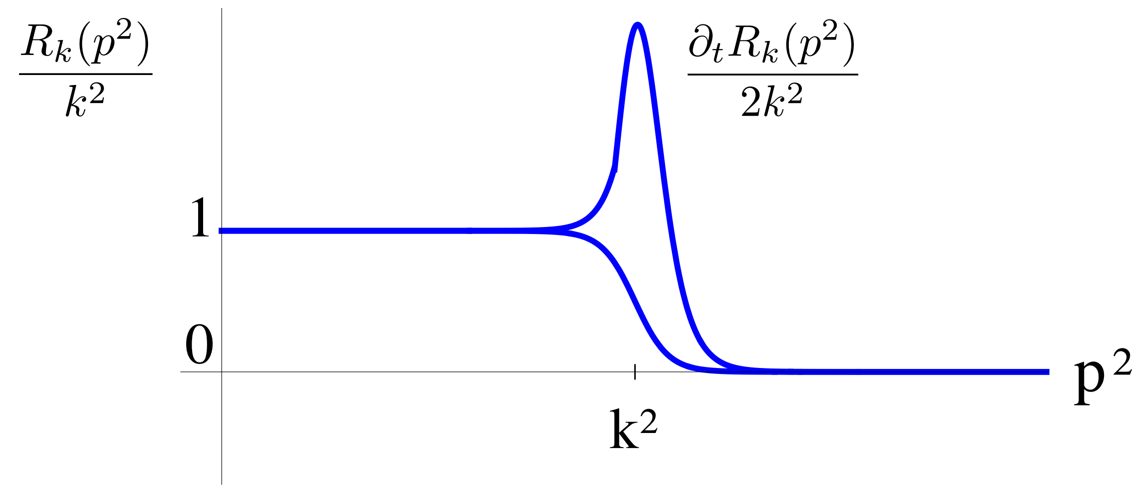

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \langle \hat{\varphi}(p)\hat{\varphi}(-p)\rangle\, \partial_t R_k(p^2)$$

$$t = \log\frac{k}{\Lambda}$$

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \, \langle \hat{\varphi}(p) \hat{\varphi}(-p) \rangle \, \partial_t R_k(p^2)$$

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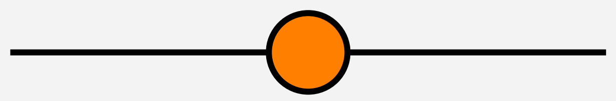
Propagator

$G =$  $= \langle \hat{\varphi}(x) \hat{\varphi}(y) \rangle_c$

$$G^{-1}[\phi] = \Gamma_k^{(2)}[\phi] + R_k$$

Functional Renormalisation Group

Flow

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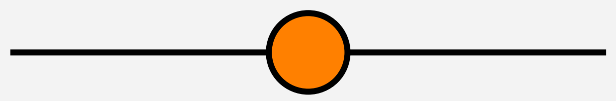
$G =$  $= \langle \hat{\varphi}(x) \hat{\varphi}(y) \rangle_c$

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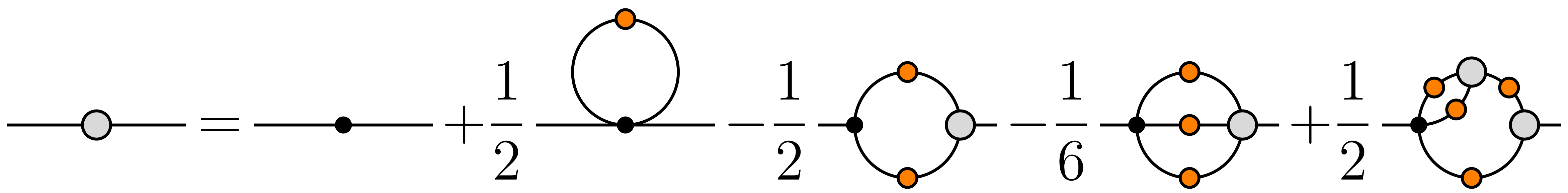
DSE

$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi} + \phi]}{\delta \phi(x)} \right\rangle$$

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi} \, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2} \int_p \hat{\varphi}(p) R_k(p^2) \hat{\varphi}(-p) + \int_x \hat{\varphi} \frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

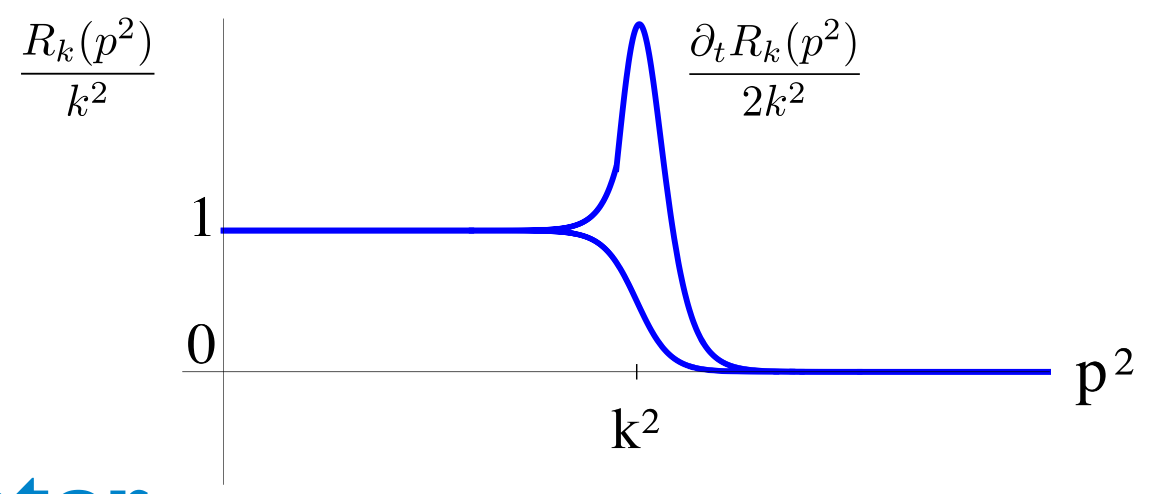


$\Gamma_k^{(2)}[\phi]$ $\quad S^{(2)}[\phi]$

Functional Renormalisation Group

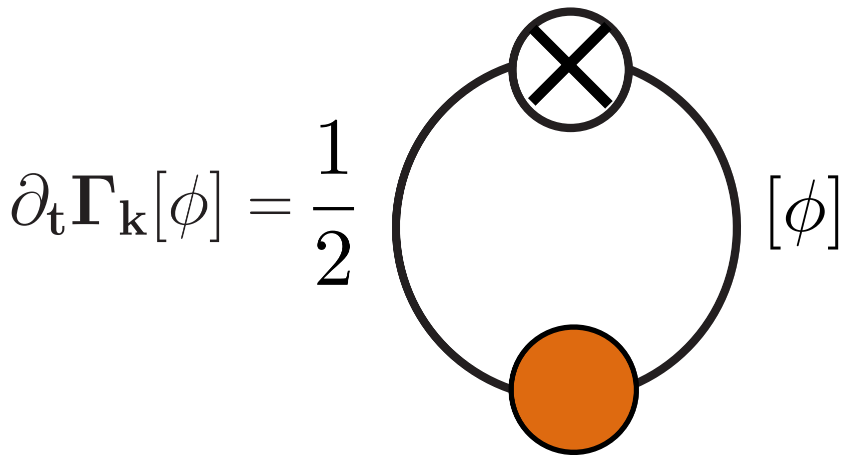
Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

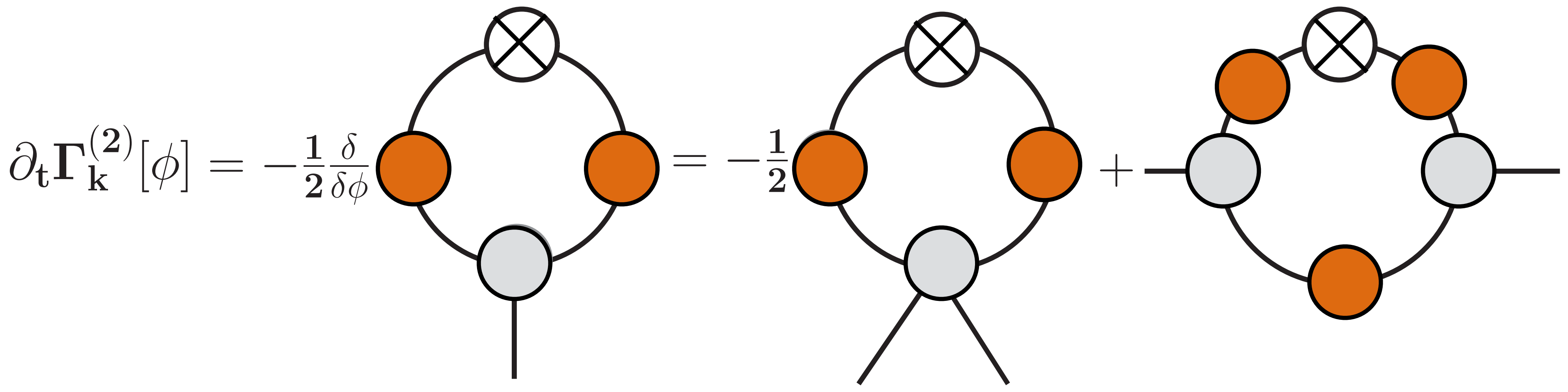


regulator

Diagrammatics



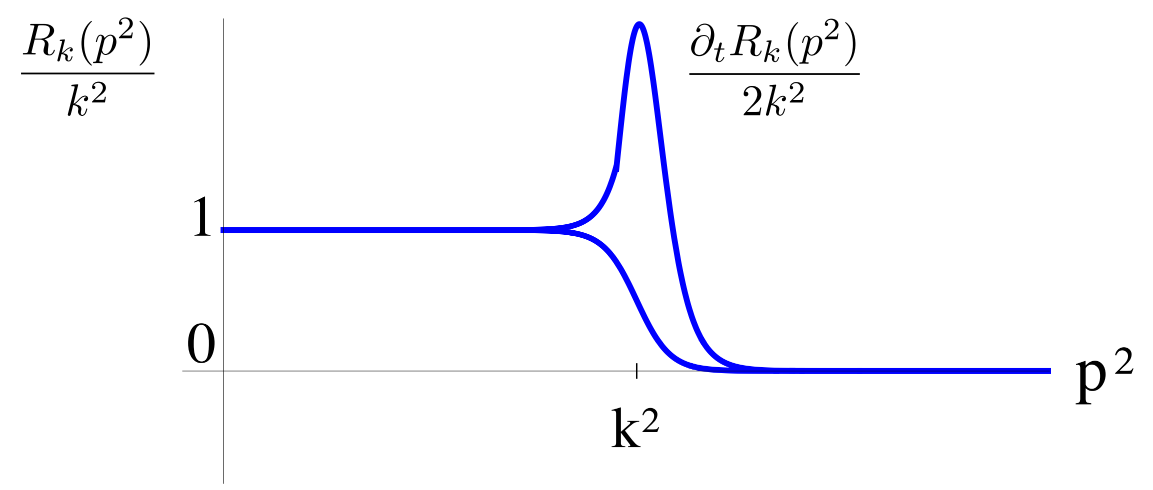
(Inverse) propagator



Functional Renormalisation Group

Flow

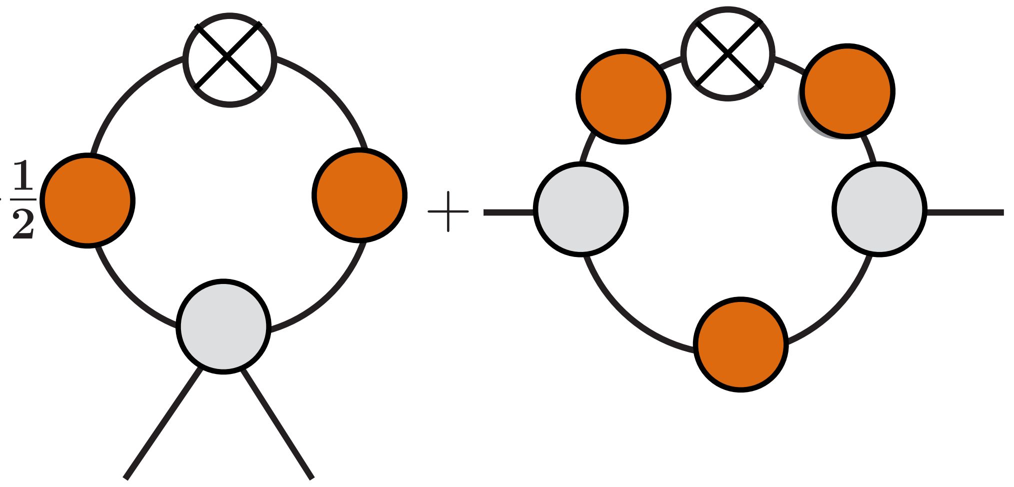
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



(Inverse) propagator

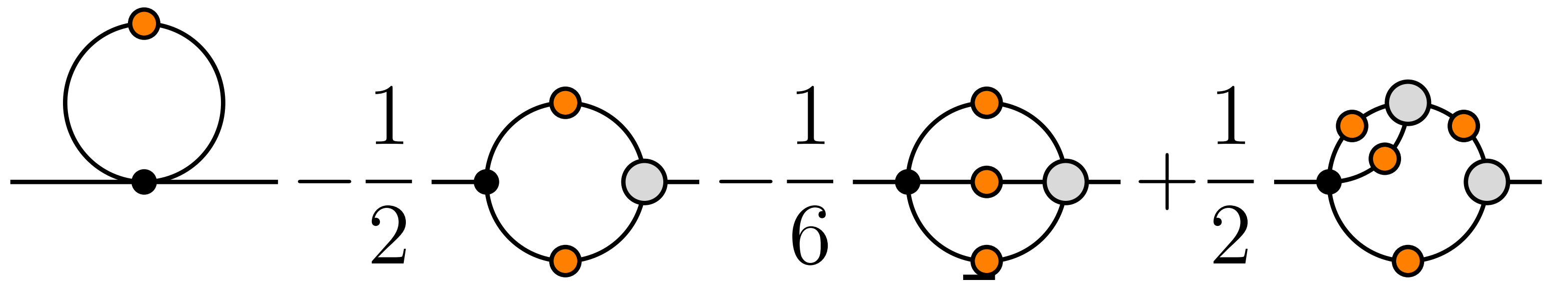
fRG

$$\partial_t \Gamma_k^{(2)}[\phi] = -\frac{1}{2}$$



DSE

$$\Gamma_k^{(2)}[\phi] - S^{(2)}[\phi] = \frac{1}{2} \quad - \frac{1}{2} \quad - \frac{1}{6} \quad + \frac{1}{2}$$



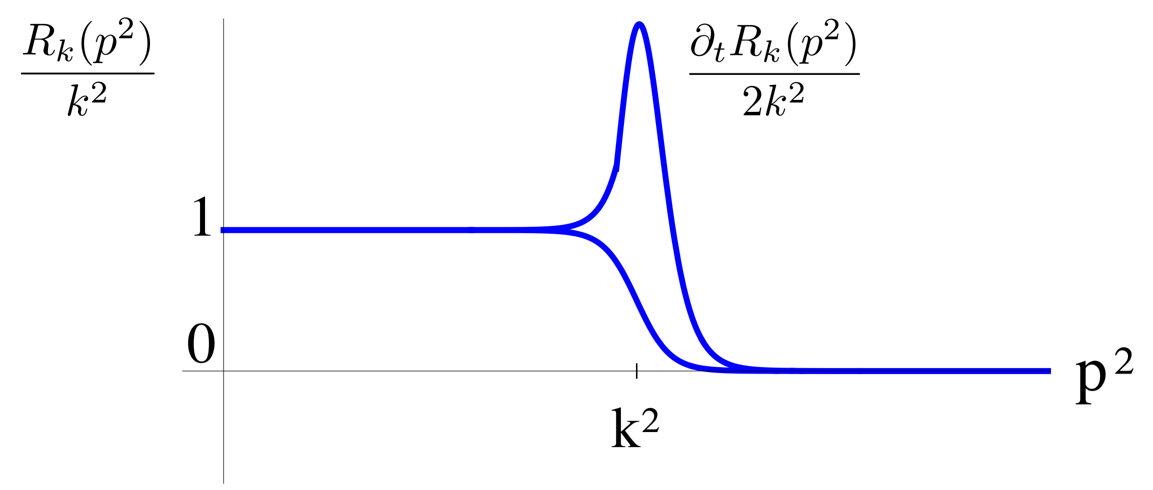
$$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$$

$$\Gamma^{(n)} = \mathrm{DSE}_n[S^{(m)}, \Gamma^{(m)}; m = 2, ..., n+2]$$

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

	fRG	DSE	2PI	3PI	4PI
• 1-loop exact	✓	–			
• closed	✓	✓			
• RG-scaling	✓	–	–	–	✓
• energy/particle-number conservation	–	–	✓	✓	✓

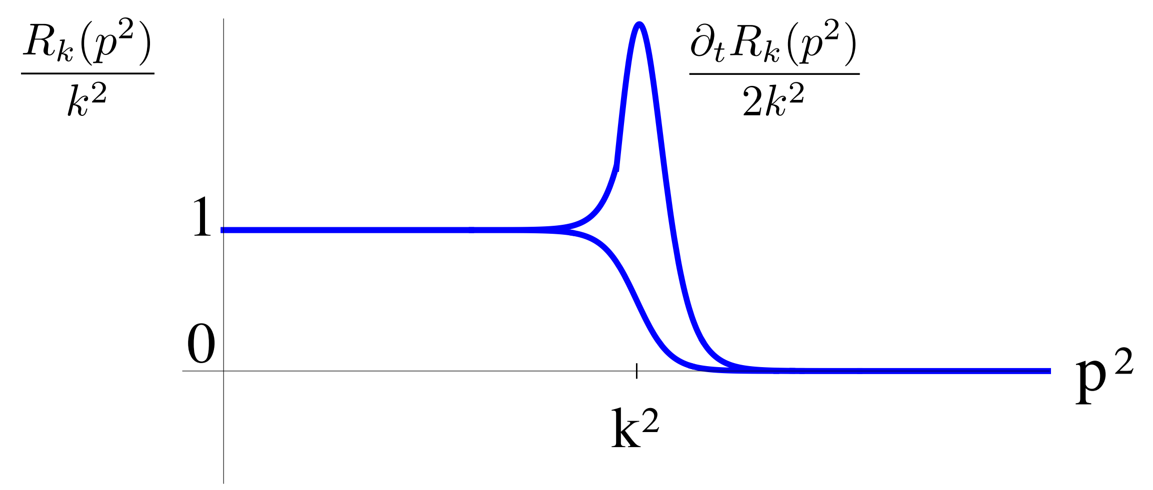
✓ automatic

– only in specific approximation schemes

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

	FunApproaches
• 1-loop exact	✓
• closed	✓
• RG-scaling	✓
• energy/particle-number conservation	✓

✓ automatic

– only in specific approximation schemes

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

- Expansion in powers of momenta
- Controlled in the presence of a mass gap m_{gap}
- Expansion parameter $\frac{p^2}{\max(k^2, m_{\text{gap}}^2)}$

Vertex expansion

- Expansion in number n of external fields
- Controlled in perturbation theory/presence of symmetries
- Expansion parameter n

Mixtures, exact resummation schemes, ...

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

$$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$$

Derivative expansion

- Expansion in powers of momenta
- Controlled in the presence of a mass gap m_{gap}
- Expansion parameter $\dfrac{p^2}{\max(k^2, m_{\mathrm{gap}}^2)}$

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi\, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi \, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \big(p^2 + V_k''(\phi)\big) \, (2\pi)^d \delta(p-q)$$

$$\partial_t V_k(\phi) = \frac{1}{d} \frac{\Omega_d}{(2\pi)^d} \frac{k^{2+d}}{k^2 + V_k''(\phi)}$$

$$R_{k,\mathrm{opt}}(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$$

$$\partial_t R_{k,\mathrm{opt}}(p^2) = 2k^2 \theta(k^2 - p^2)$$

$$\Gamma^{(2)}[\phi](p) + R_{k,\mathrm{opt}}(p^2) = \big[k^2 + V''(\phi)\big] \, \theta(k^2 - p^2) + (p^2 + V''(\phi))\theta(p^2 - k^2)$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

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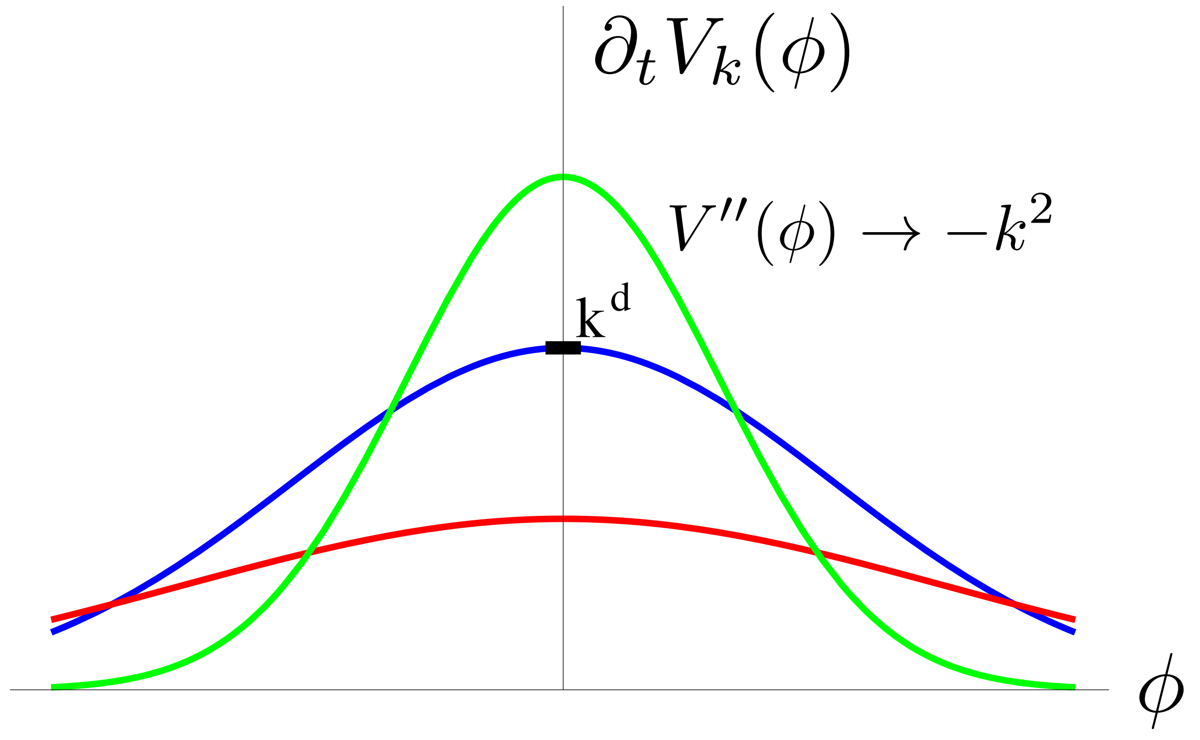
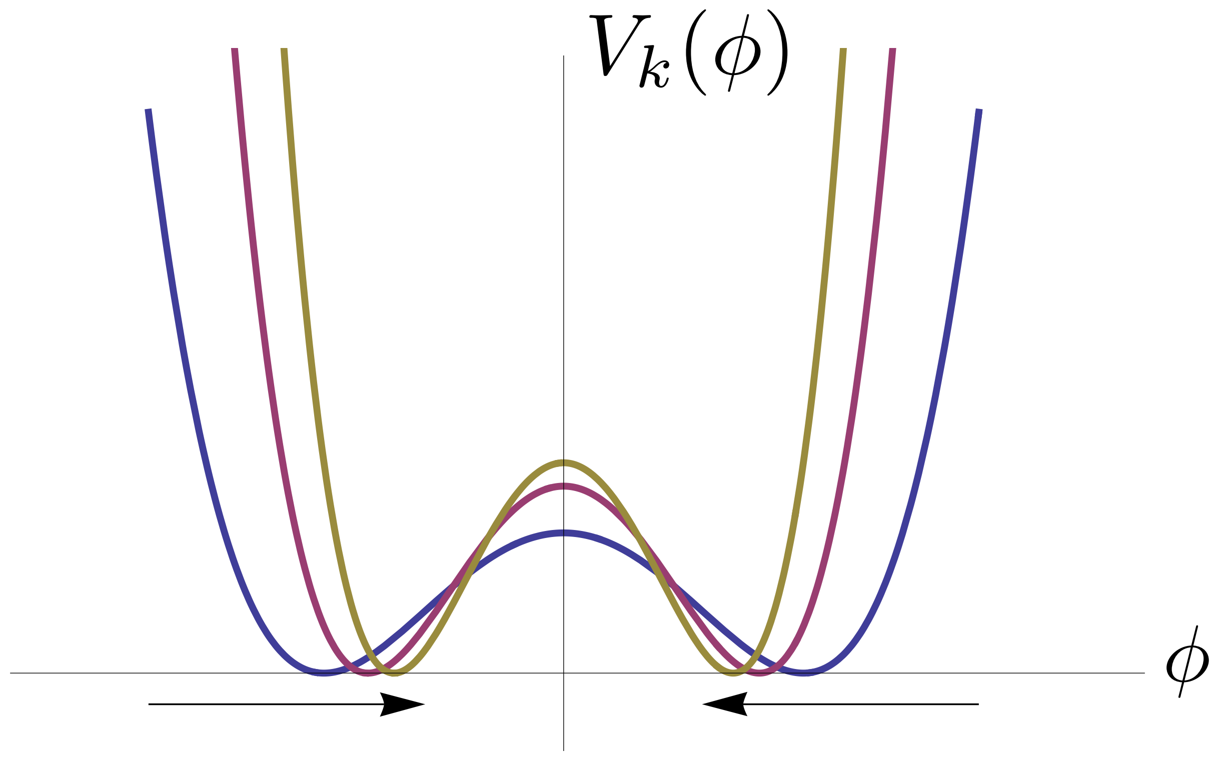
Spontaneous symmetry breaking

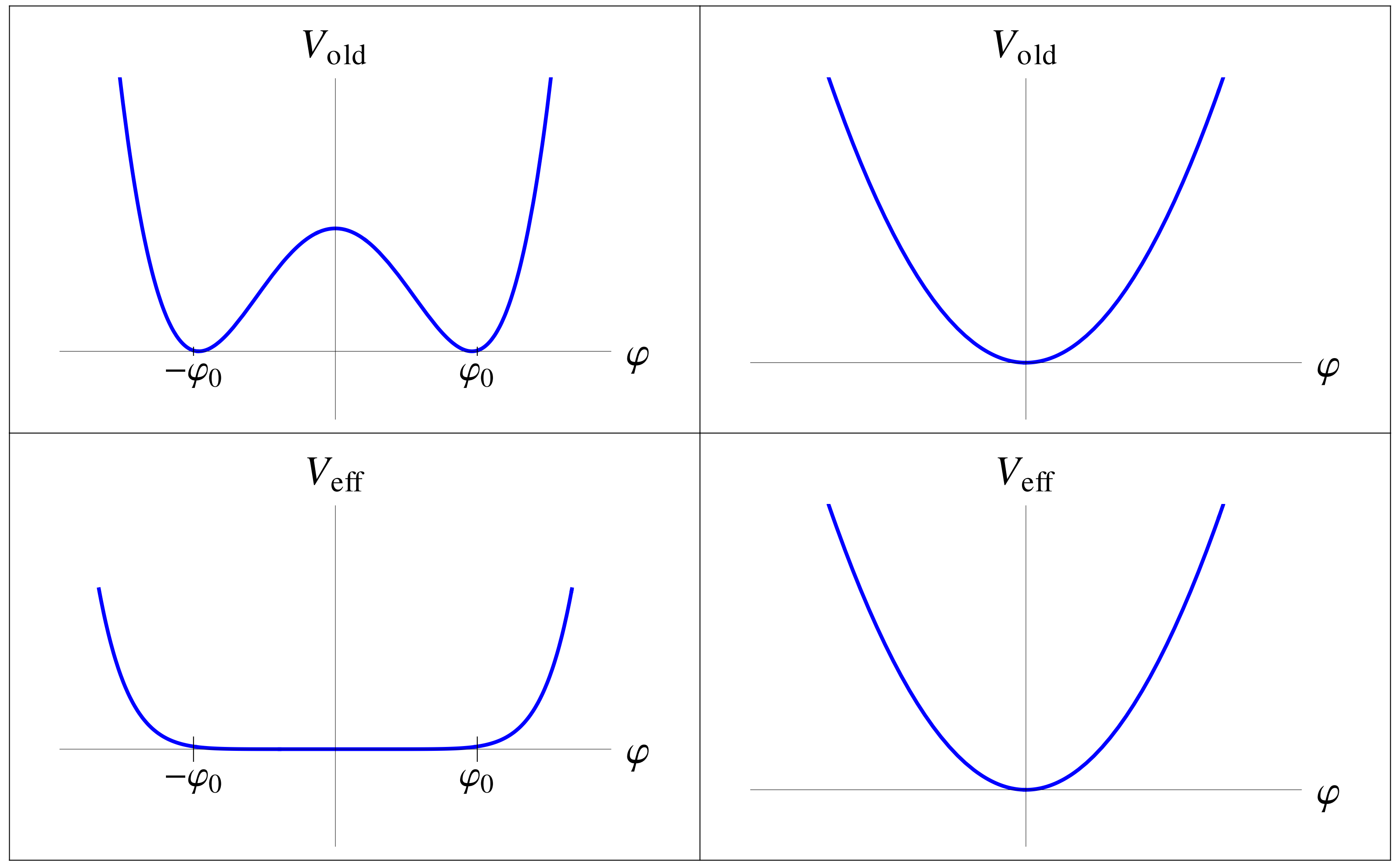
Approximation schemes & phase structure

$$\partial_t V_k(\phi) = \frac{1}{d} \frac{\Omega_d}{(2\pi)^d} \frac{k^{2+d}}{k^2 + V_k''(\phi)}$$



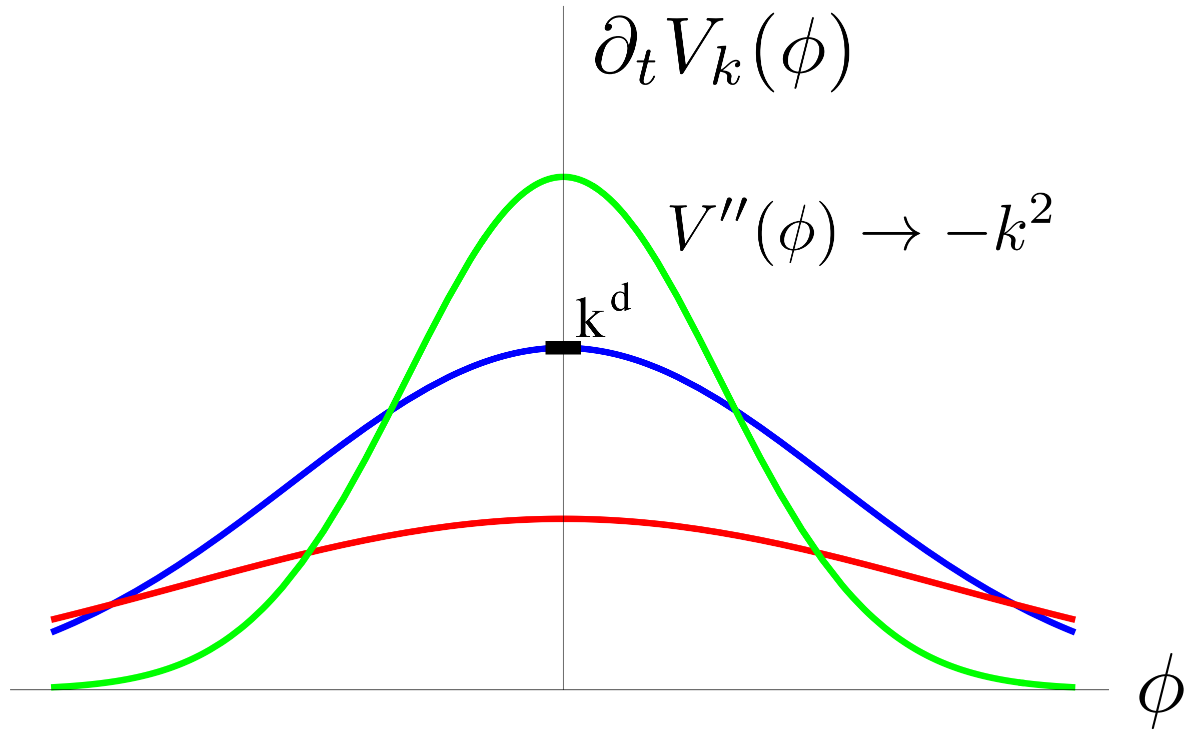


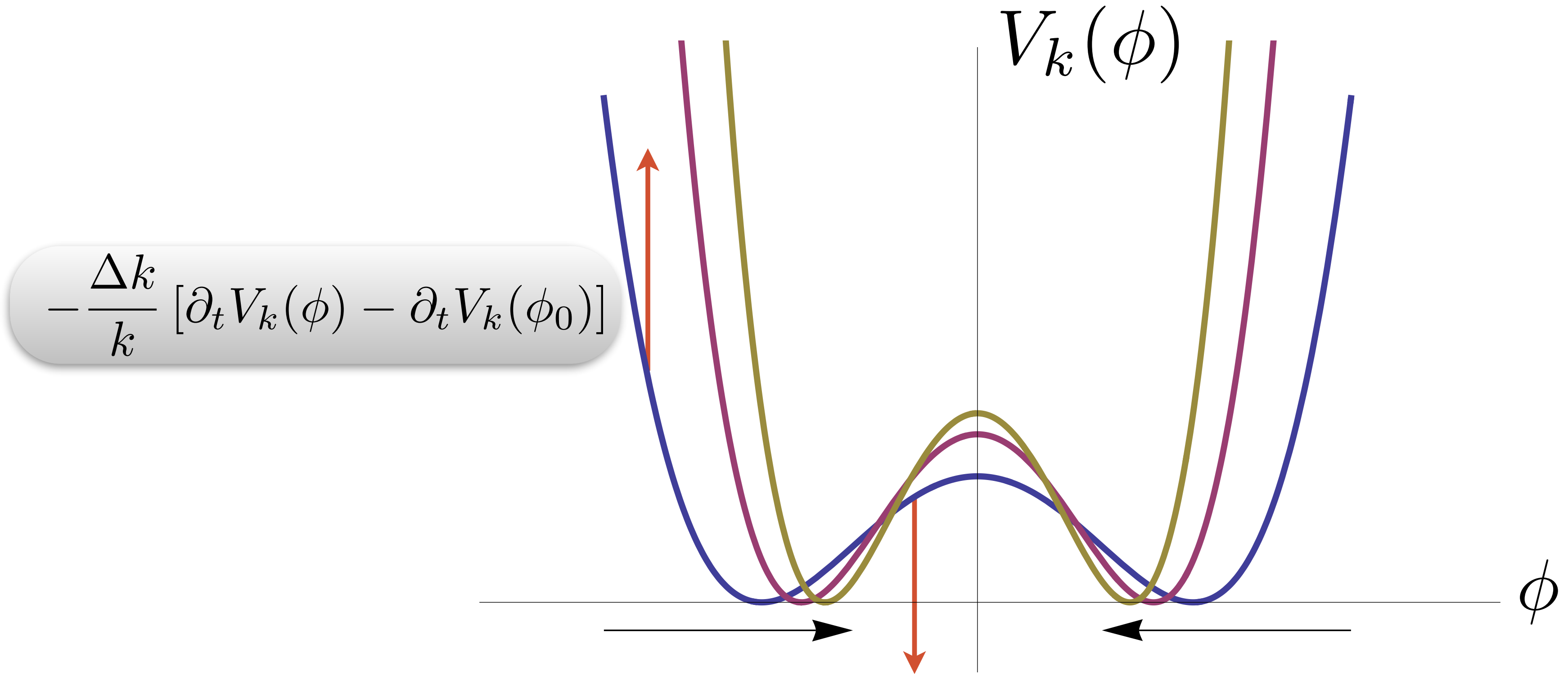
- bosonic flow is symmetry-restoring
- flow guarantees convexity of effective action



Approximation schemes & phase structure

$$\partial_t V_k(\phi) = \frac{1}{d}\frac{\Omega_d}{(2\pi)^d}\frac{k^{2+d}}{k^2 + V_k''(\phi)}$$





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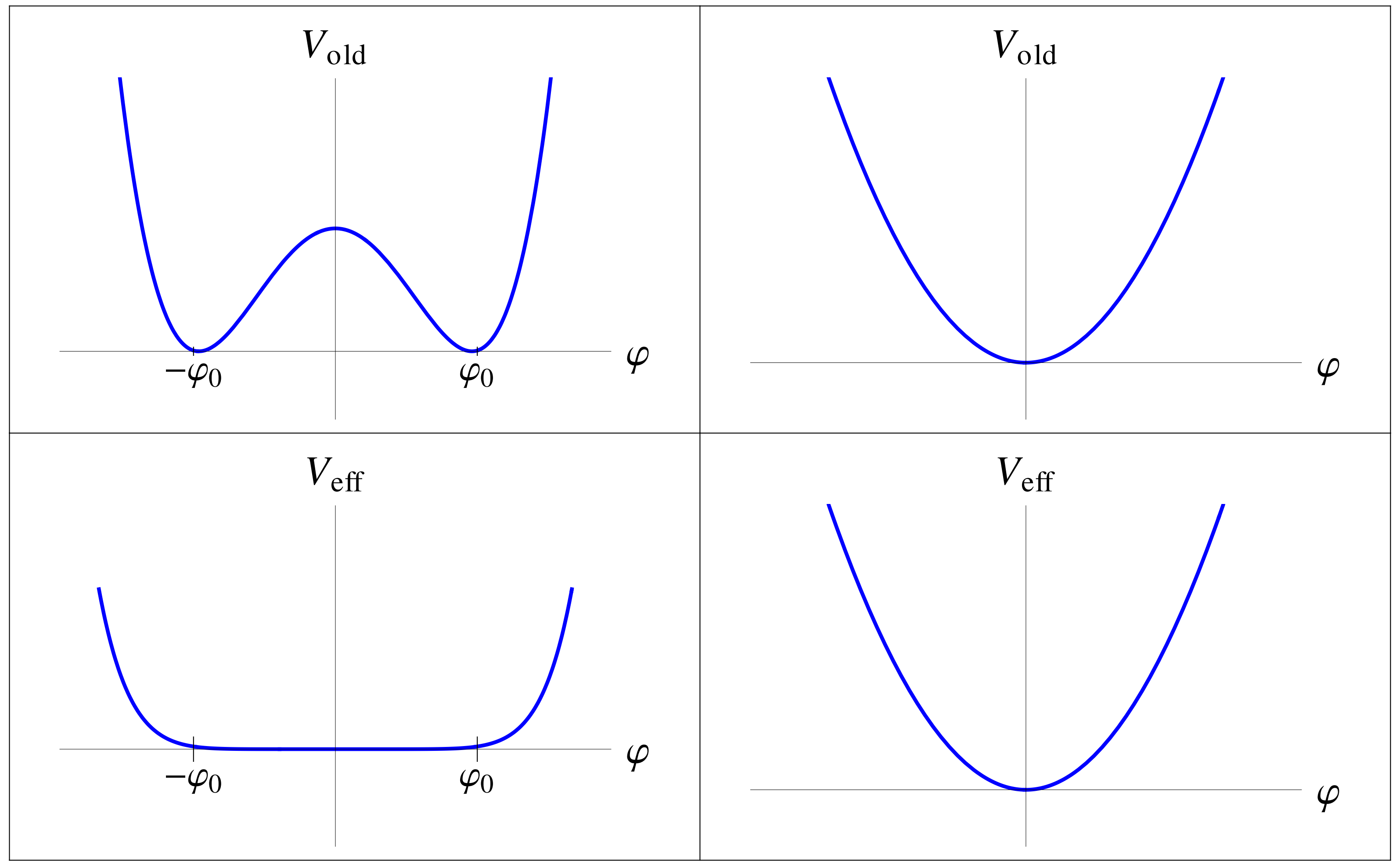
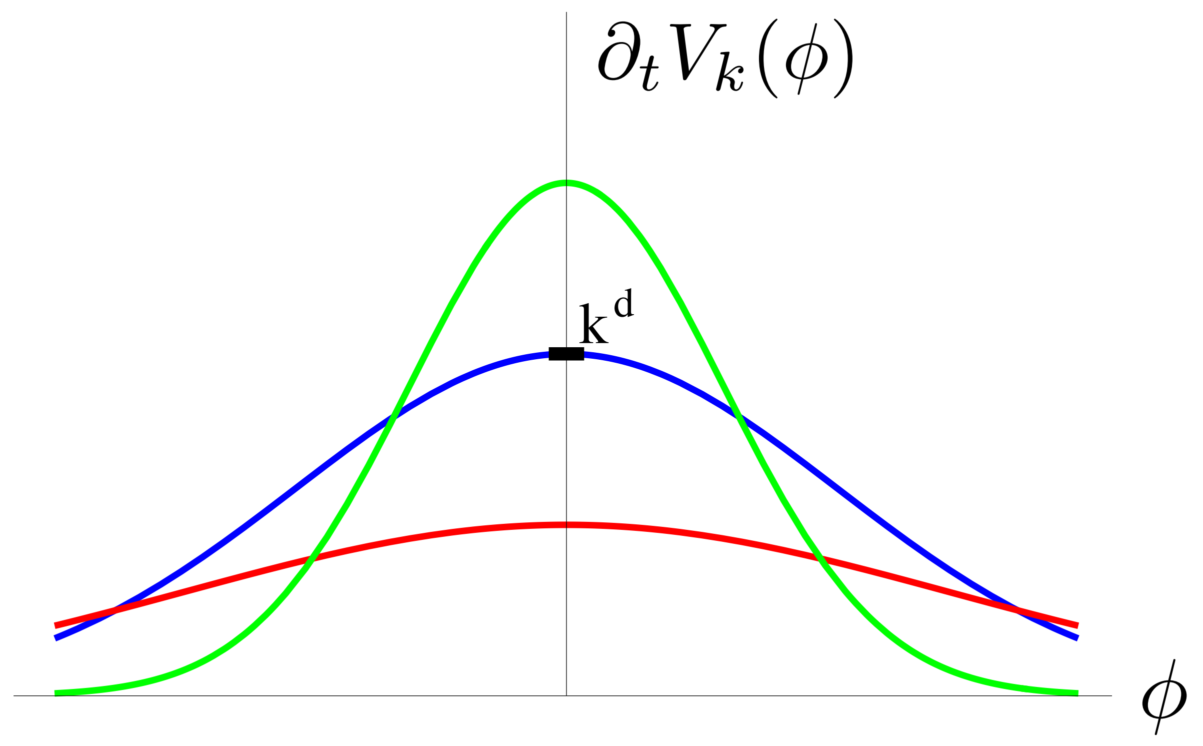


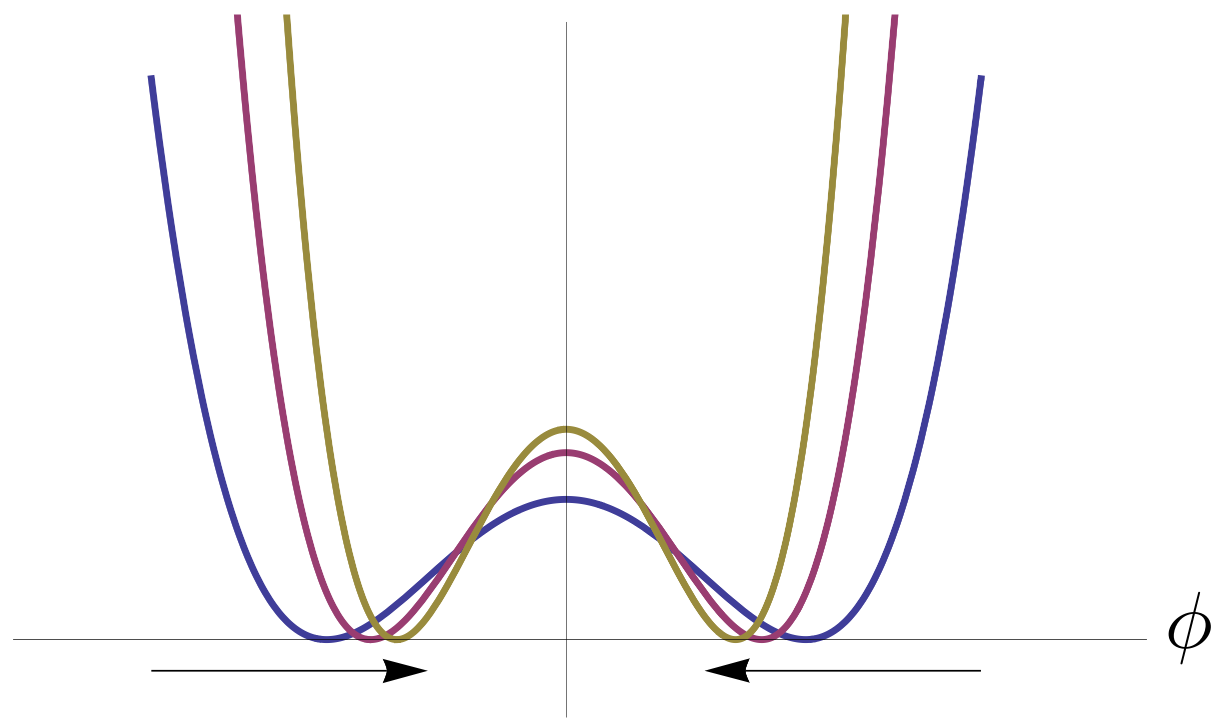
Approximation schemes & phase structure

$$\partial_t V_k(\phi) = \frac{k^d}{d} \frac{\Omega_d}{(2\pi)^d} \frac{1 - \frac{\eta_\phi}{d+2}}{1 + \frac{V_k''(\phi)}{k^2}}$$

Anomalous dimension

$$\eta_\phi = -\frac{\partial_t Z_\phi}{Z_\phi}$$





- **bosonic flow is symmetry-restoring**
- **flow guarantees convexity of effective action**

Litim, JMP, Vergara, hep-th/0602140

Example: 3d critical exponents with fRG

Simple approximation: LPA'

$$\Gamma_k[\phi] = \frac{1}{2}\int_p Z_\phi\, \phi(p)\, p^2\, \phi(-p) + \int_x V_k(\phi)$$

Taylor expansion $V_k(\phi) = \sum_{n=1}^{N_{\max}} \frac{\lambda_n}{n!}(\phi^2 - \phi_{0,k}^2)^n$

Ising universality

$N = 1: \ \nu_{\text{Ising}} = 0.630...$

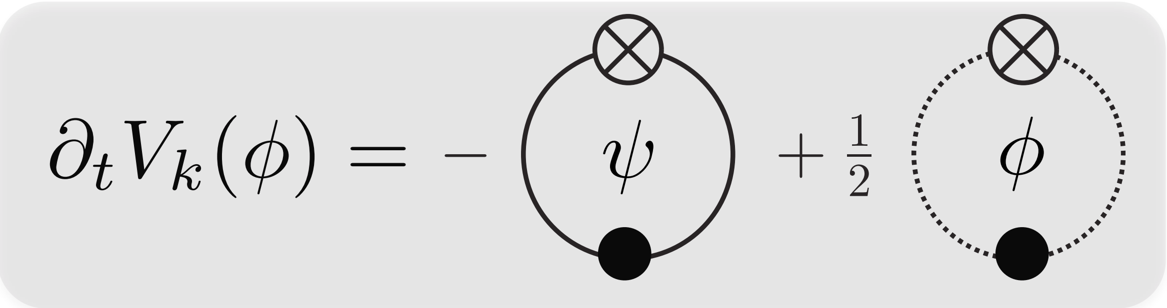
fRG: LPA'

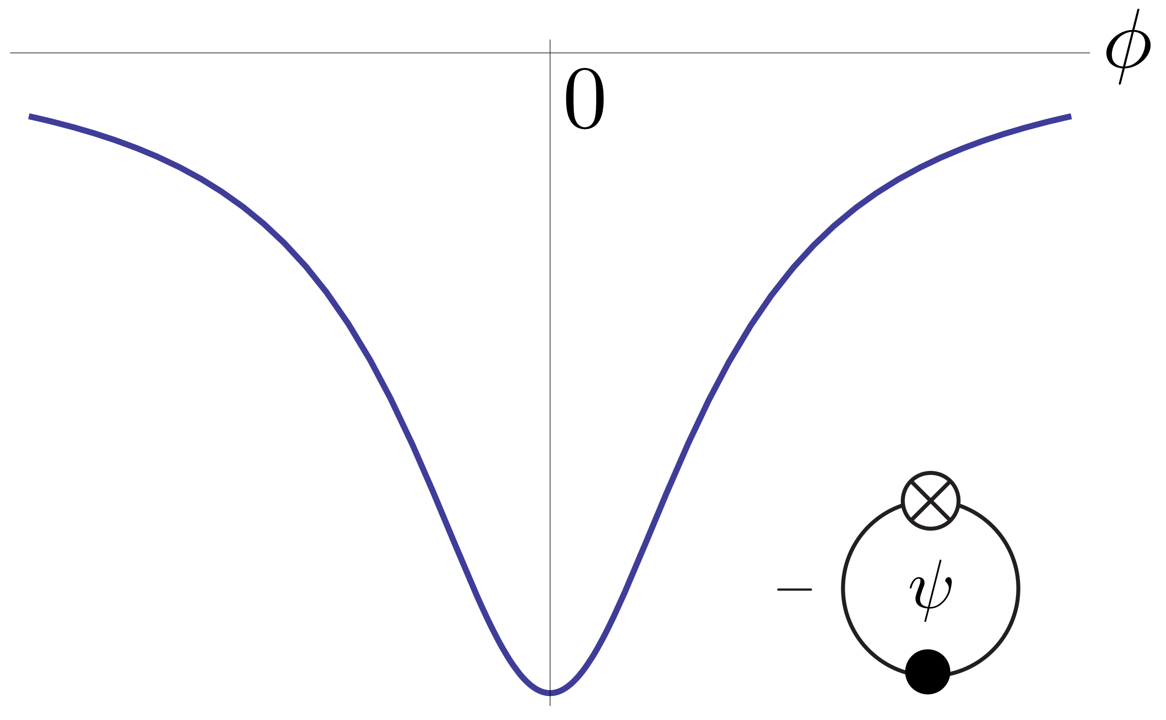
$N = 1: \ \nu_{\text{Ising}} = 0.637...$

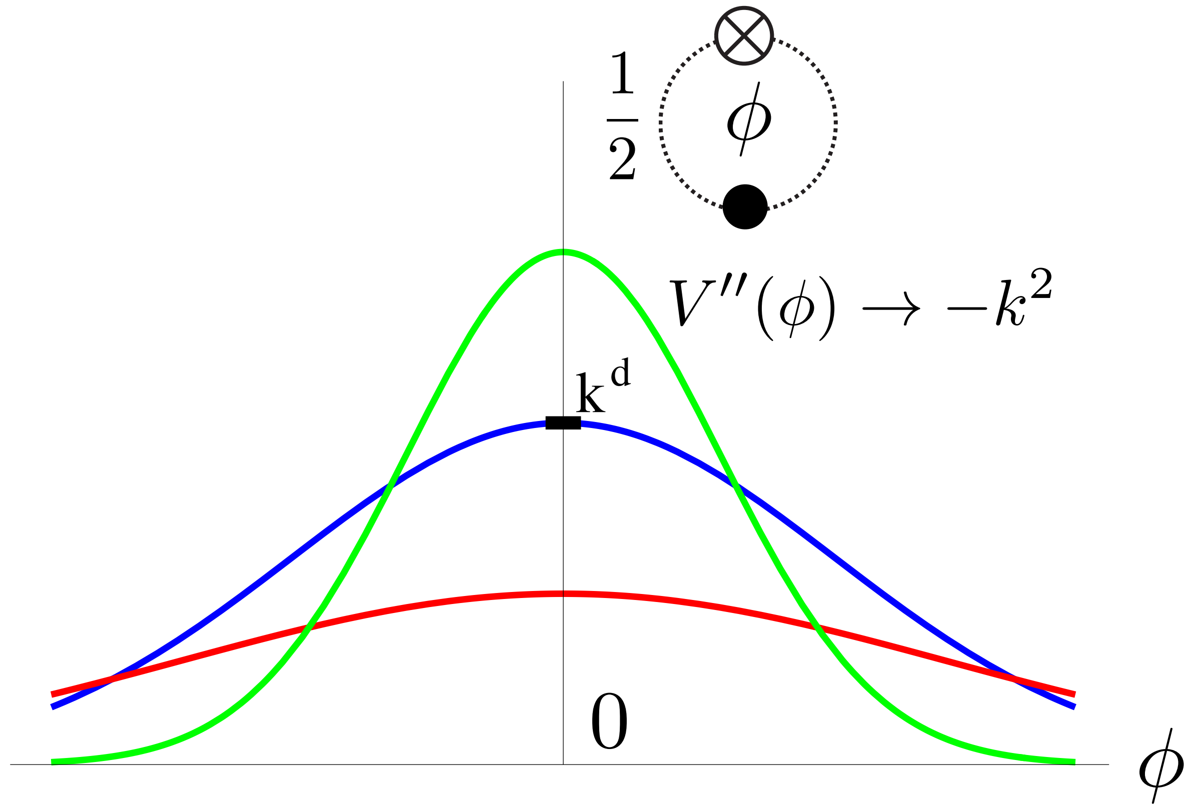
A simple program to compute critical exponents in O(N)-models with the Wetterich equation

Michael Scherer

Spontaneous symmetry breaking

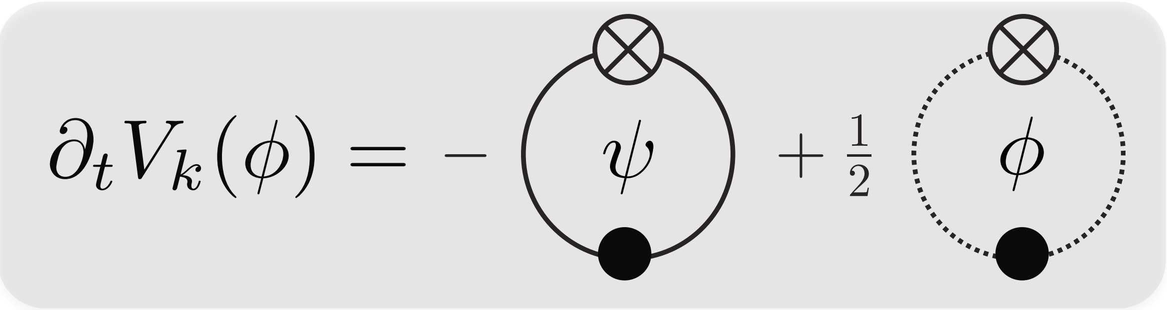


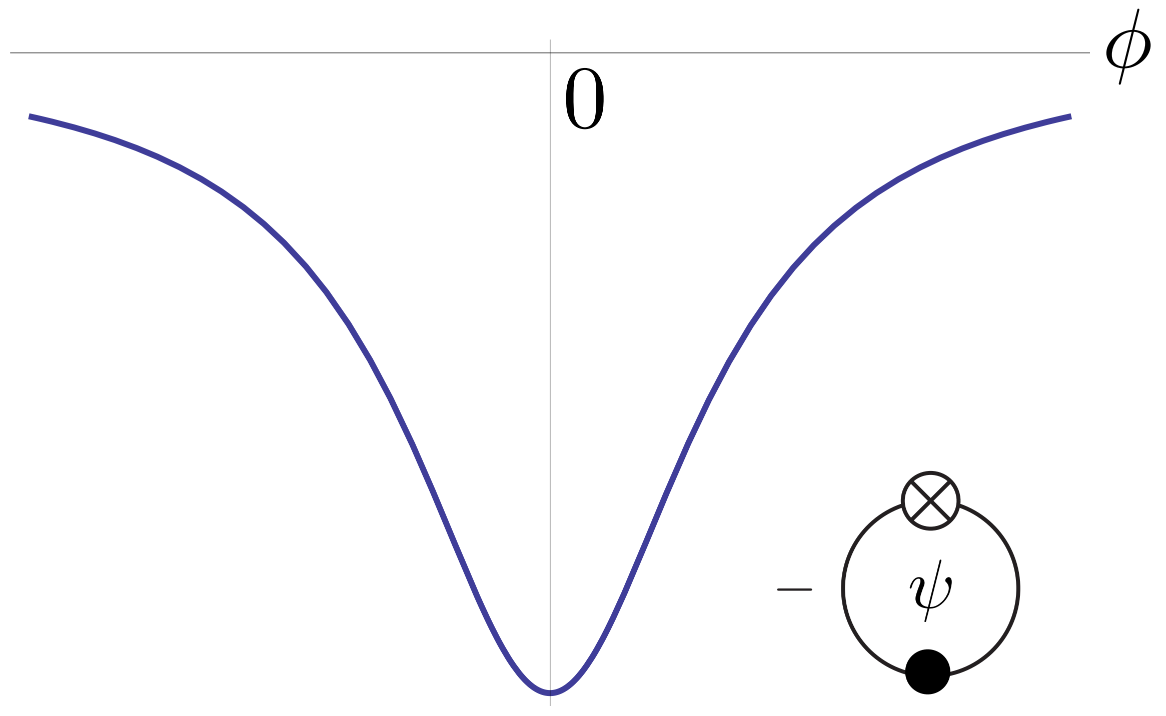




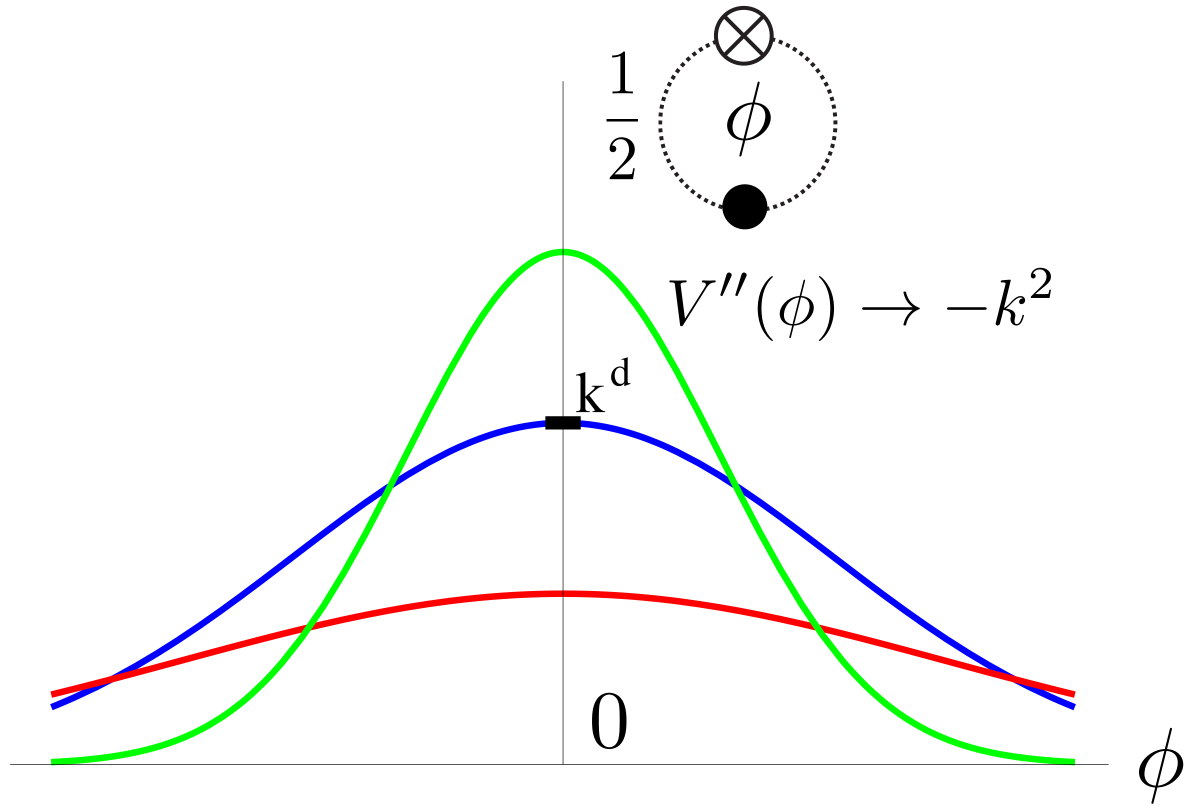
- bosonic flow is symmetry-restoring
- fermionic flow is symmetry-breaking
- competing dynamics decides about fate of symmetries
- flow guarantees convexity

Spontaneous symmetry breaking





+



'governs general phase structures'

- **bosonic flow is symmetry-restoring**
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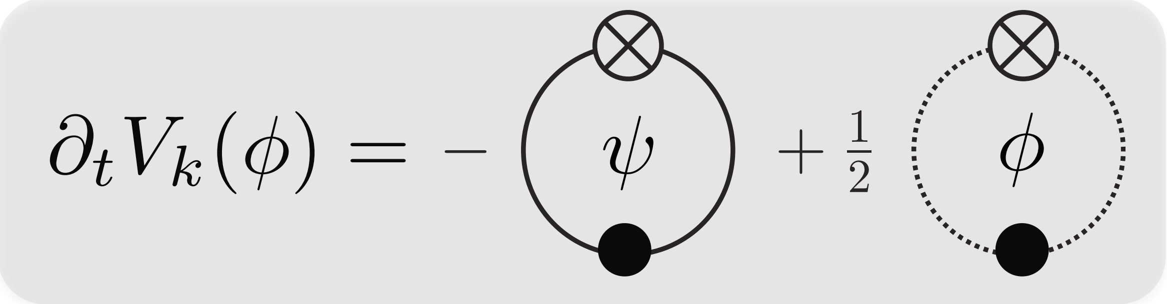
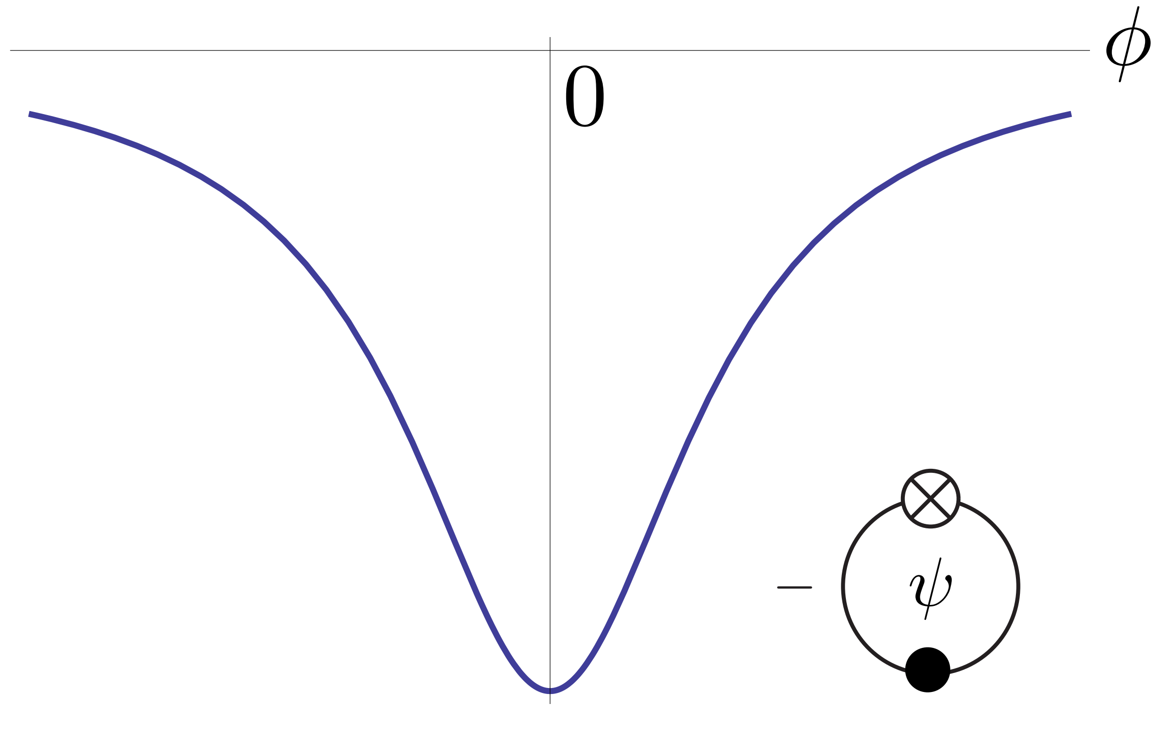
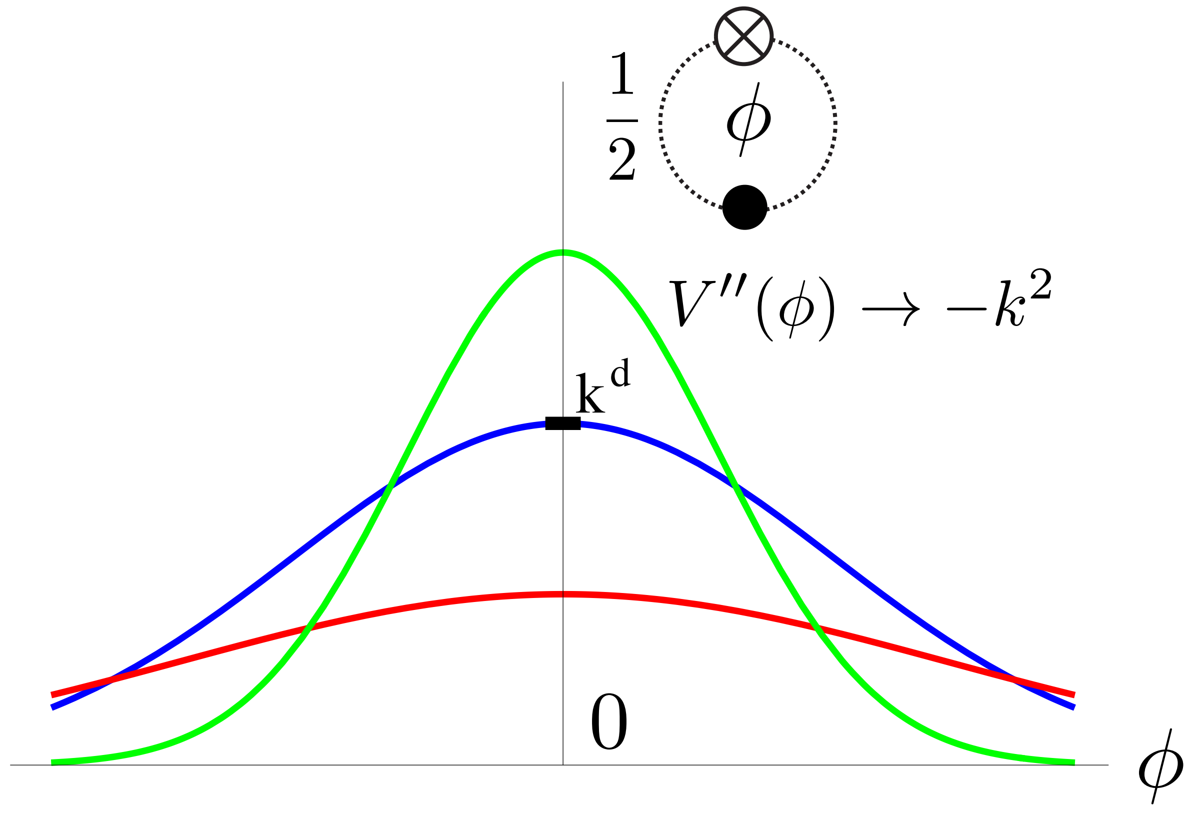
Spontaneous symmetry breaking







'governs general phase structures'

- bosonic flow is symmetry-restoring
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Direct access
to
Full order parameter potential

Direct definition/access
of
Regime with critical scaling

Spontaneous symmetry breaking

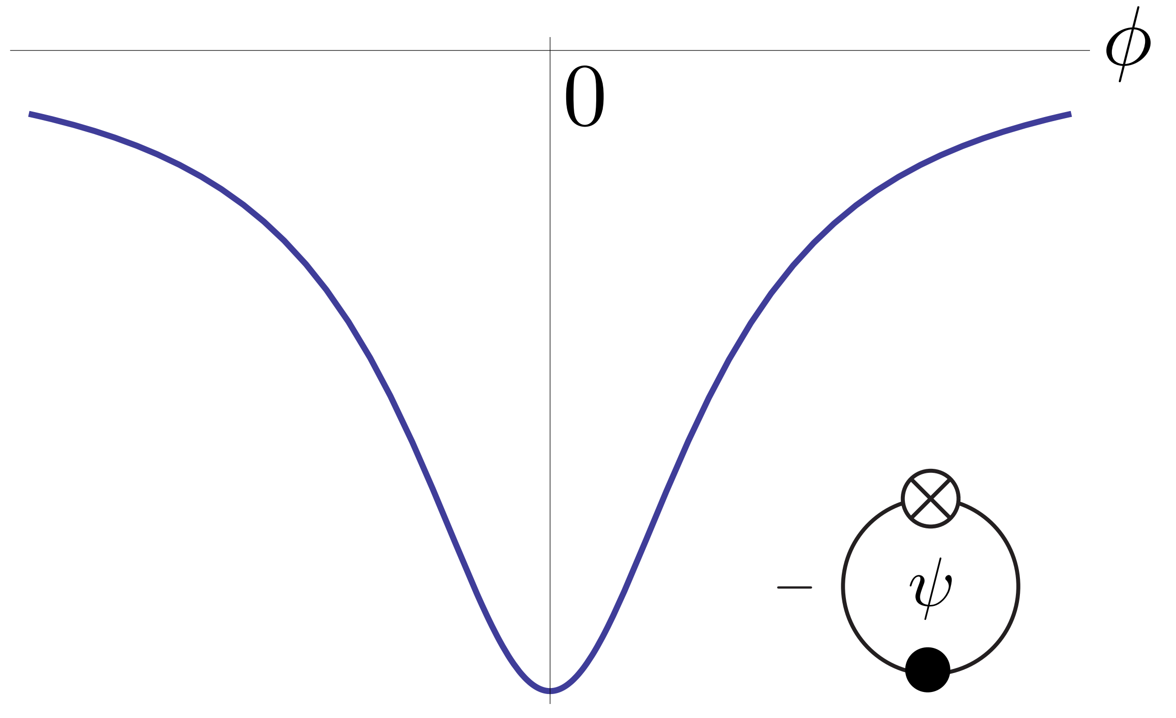
Initiating the hydro era (2019)

Grossi, Wink, SciPost Phys. Core 6 (2023)

State of the art time steppers

Ihssen, Sattler, Wink, CPC 300 (2024) 109182

$$\partial_t V_k(\phi) = - \text{[}\psi\text{ loop]} + \tfrac{1}{2}\,\text{[}\phi\text{ loop]}$$



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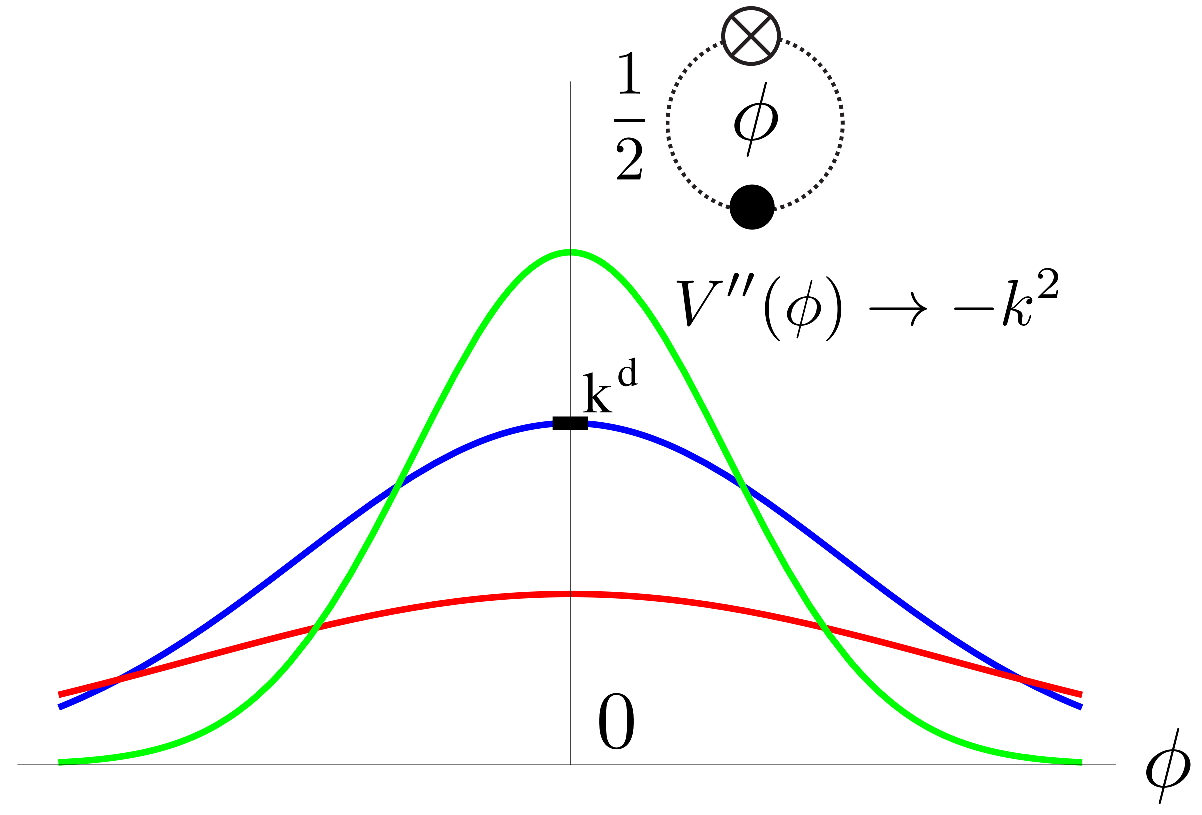

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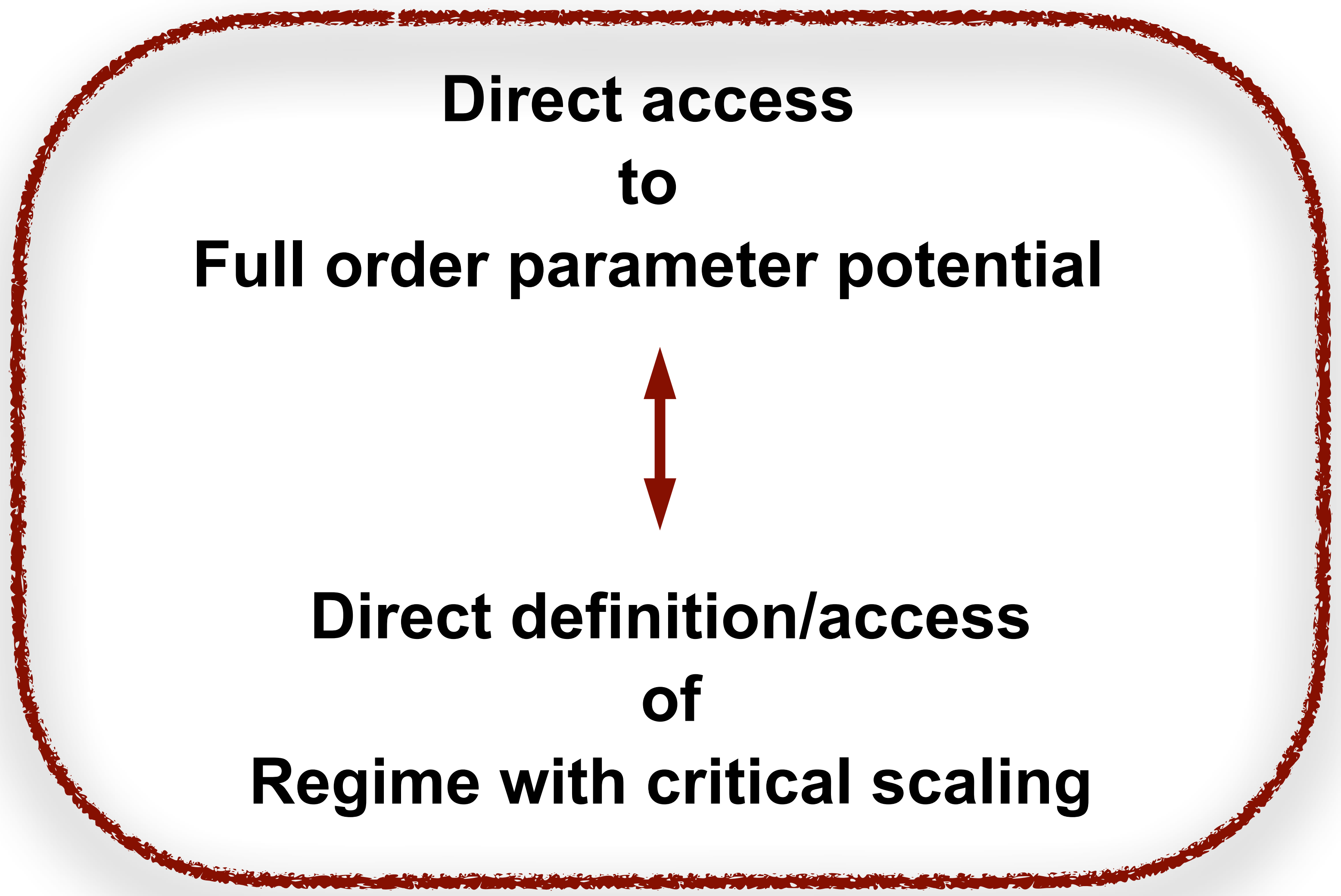
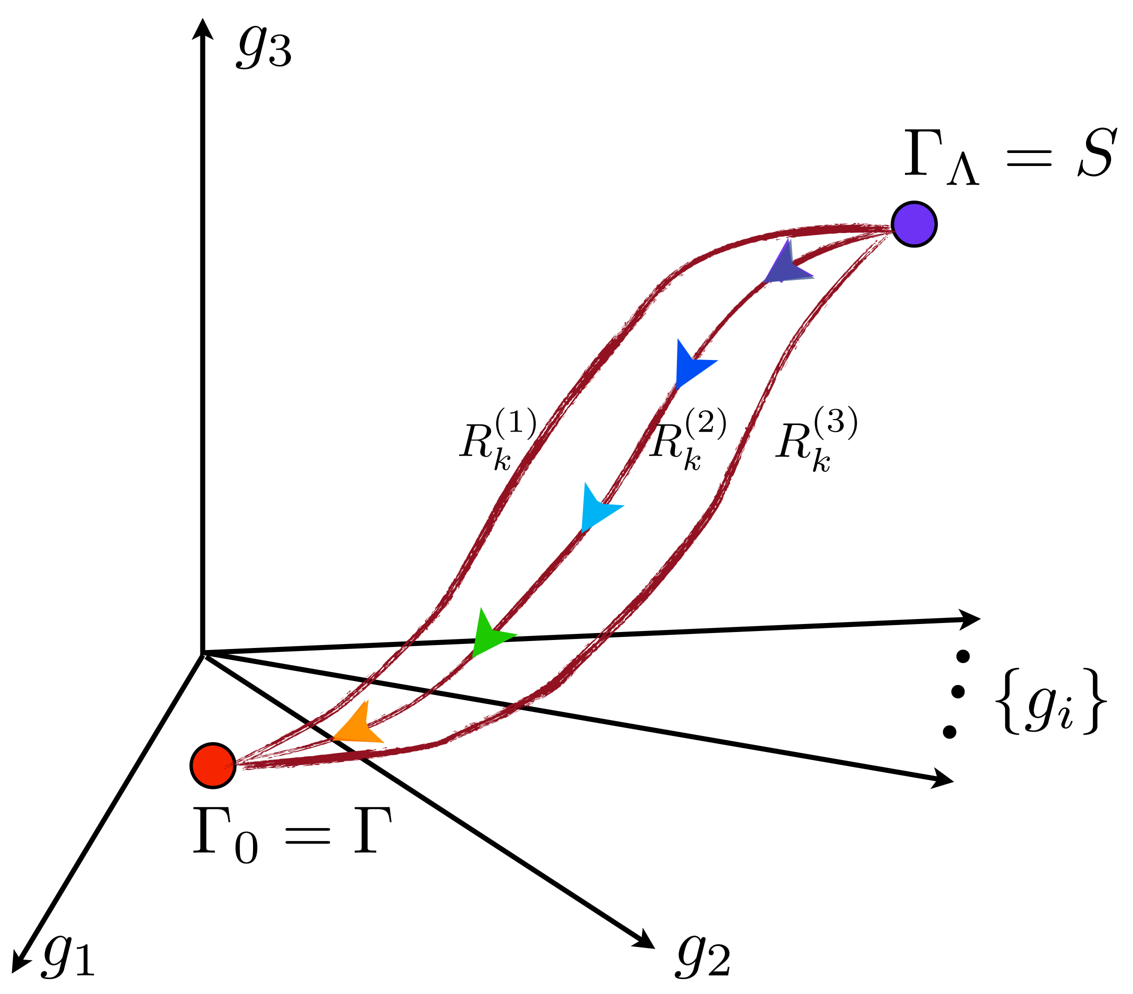


Systematic error control & optimisation

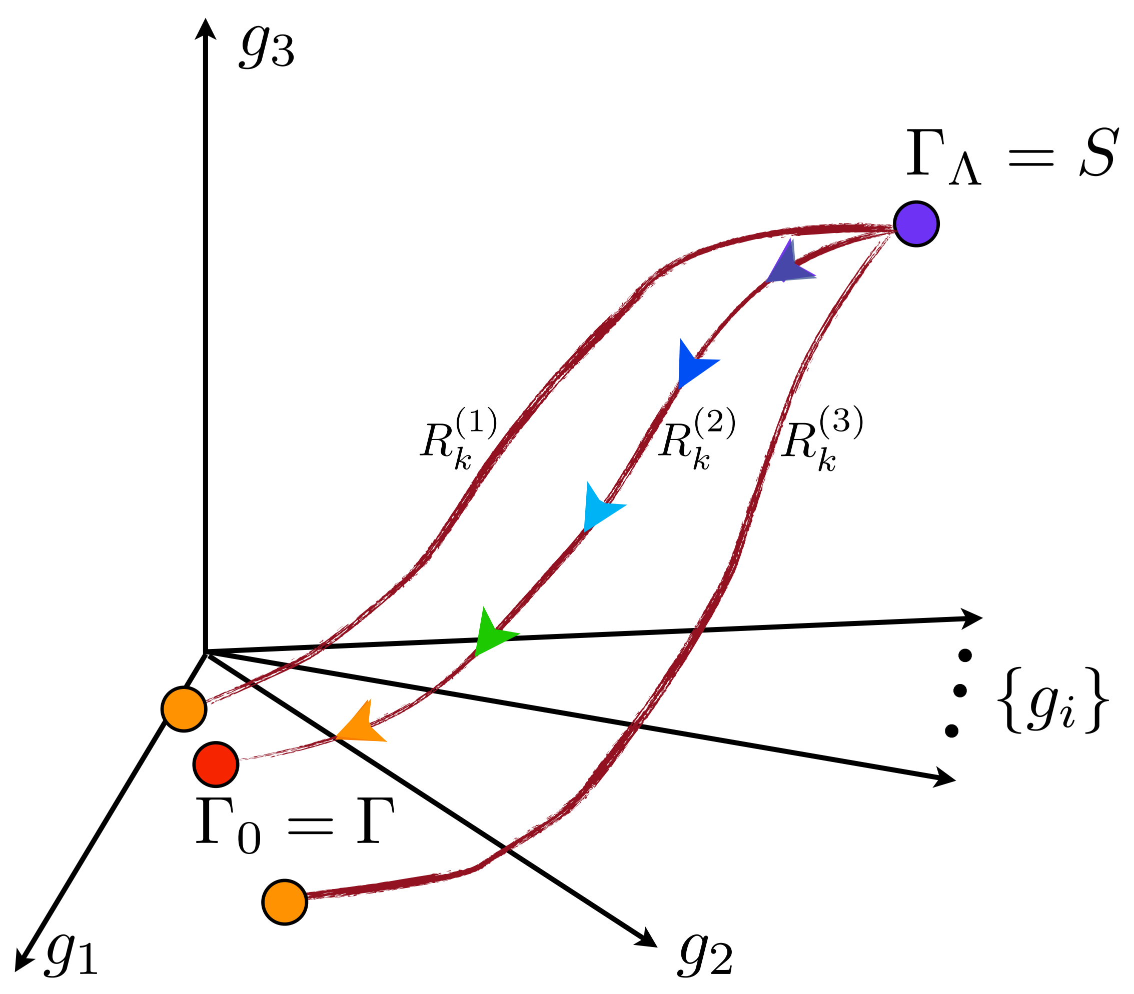
Systematic error control & optimisation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Theory space



full flow



approximated flow

Optimisation: find $R_k^{(2)}$!

Systematic error control & optimisation

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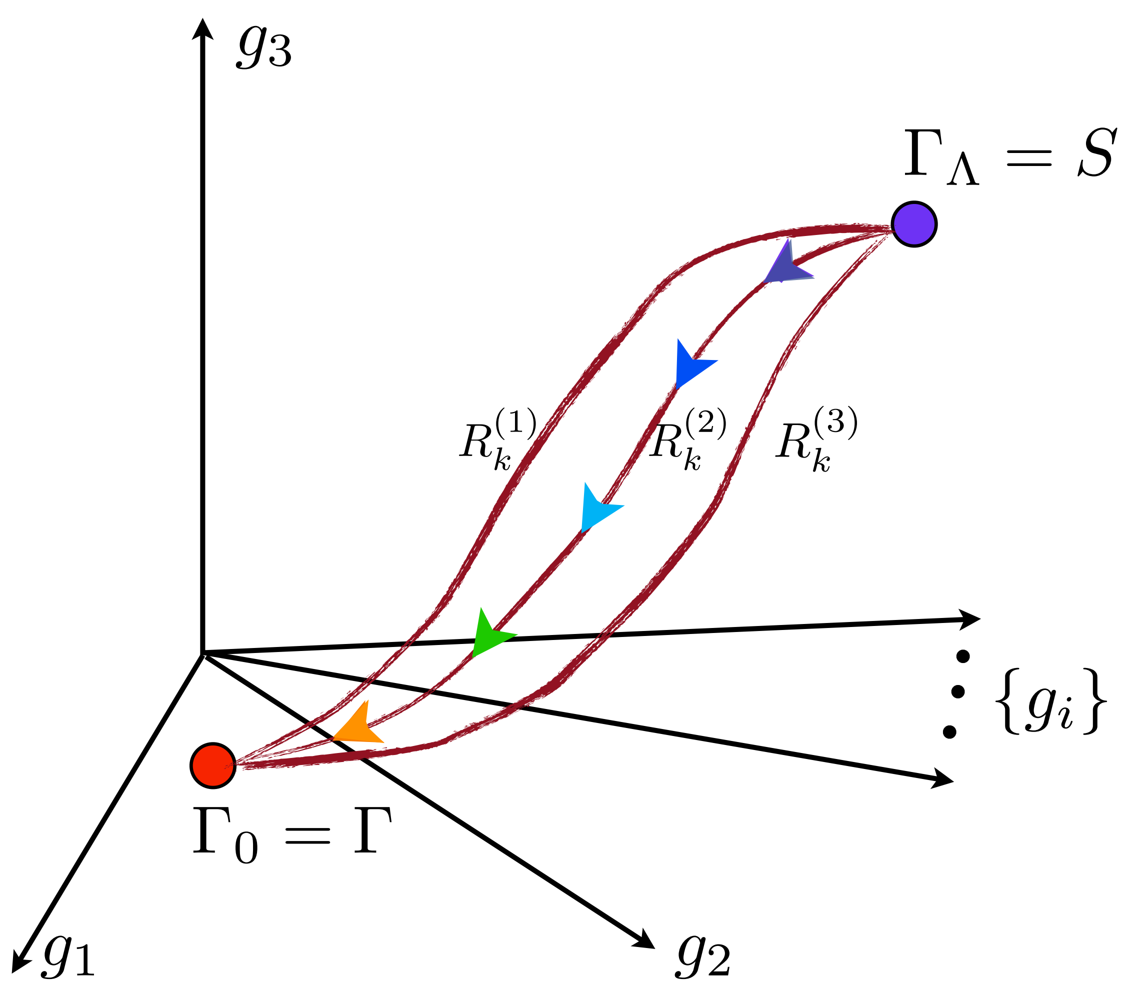

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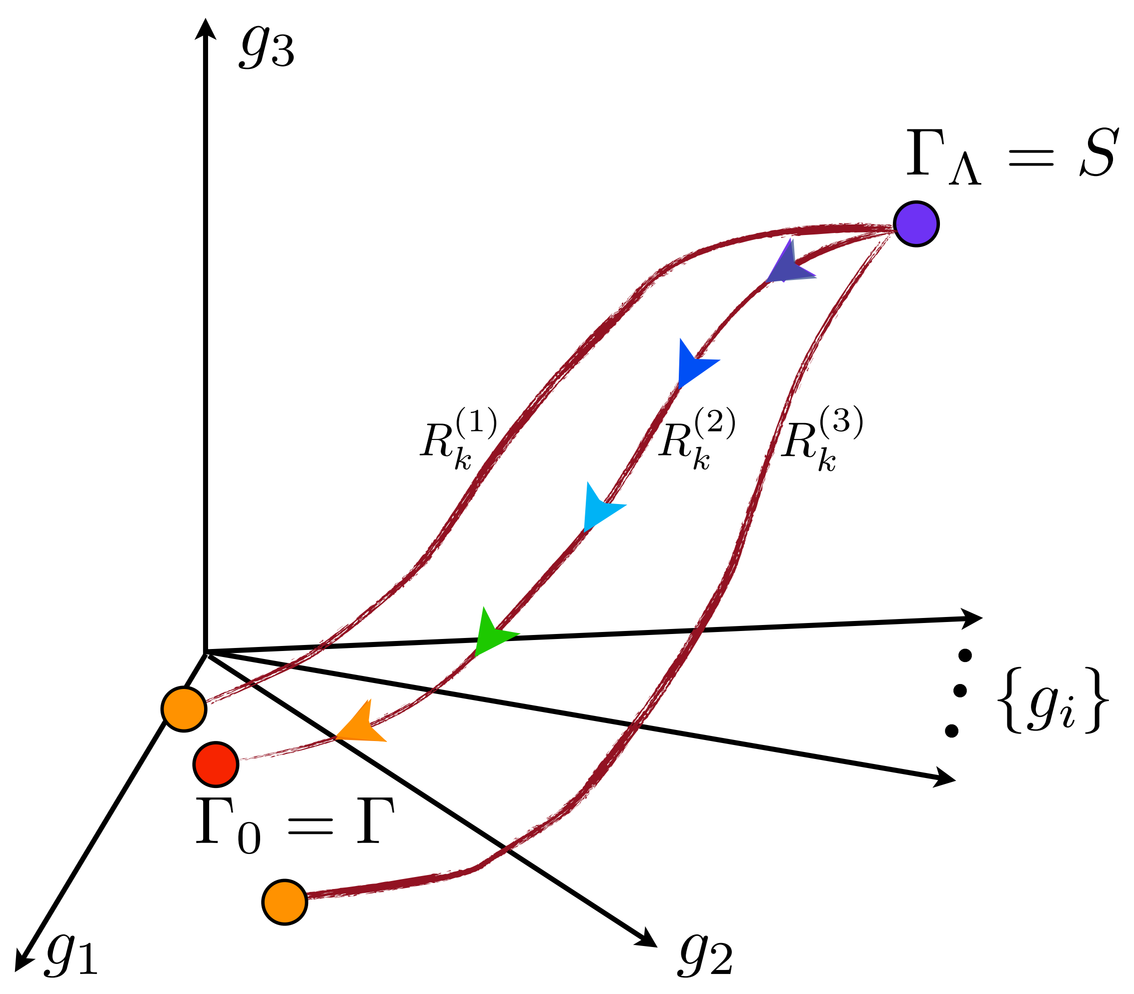

approximated flow

Optimisation: find $R_k^{(2)}$!

Principle of minimal sensitivity

eg. Liao, Polonyi, Strickland, NPB 567 (2000) 493-514
Canet, Delamotte, Mouhanna, Vidal, PRD 67 (2003) 065004

Systematic error control & optimisation

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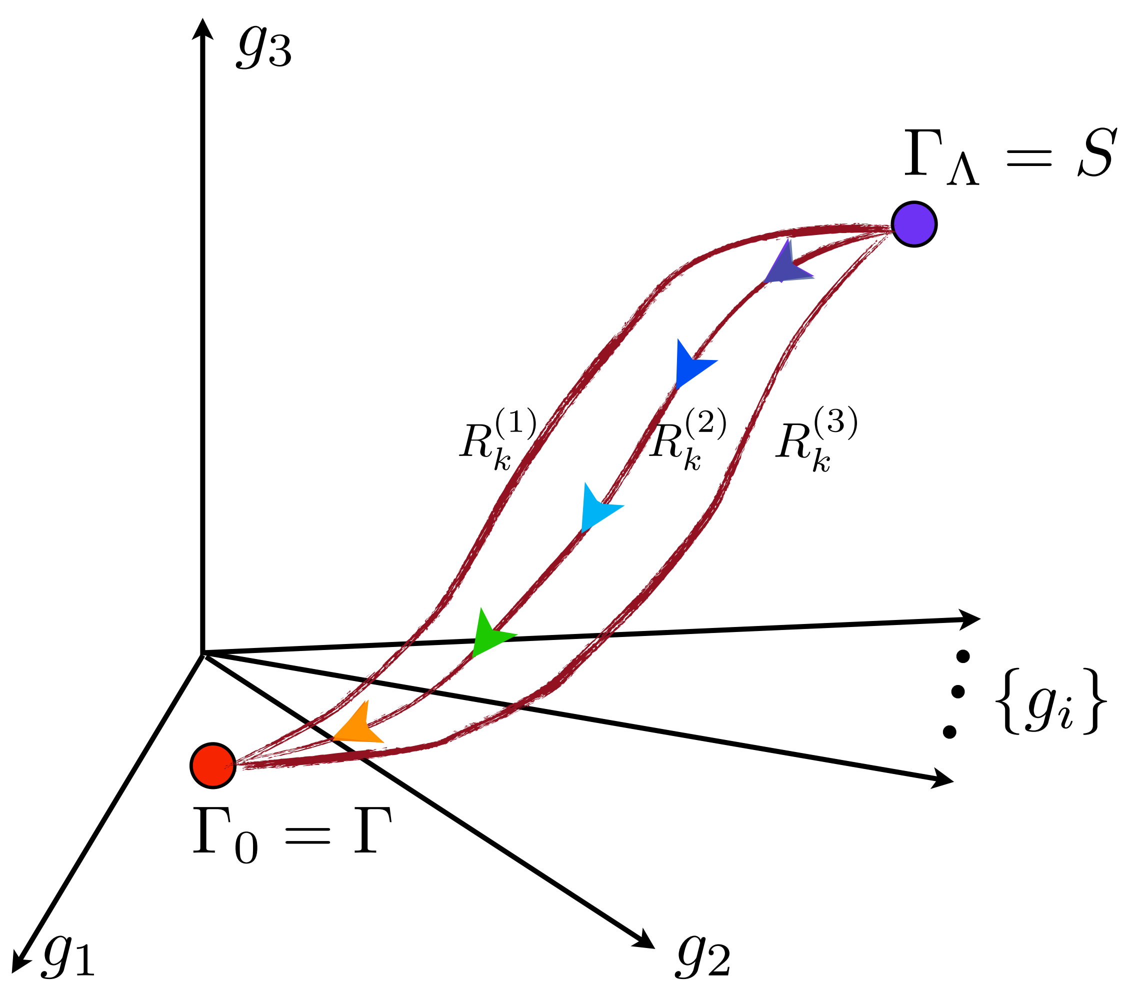
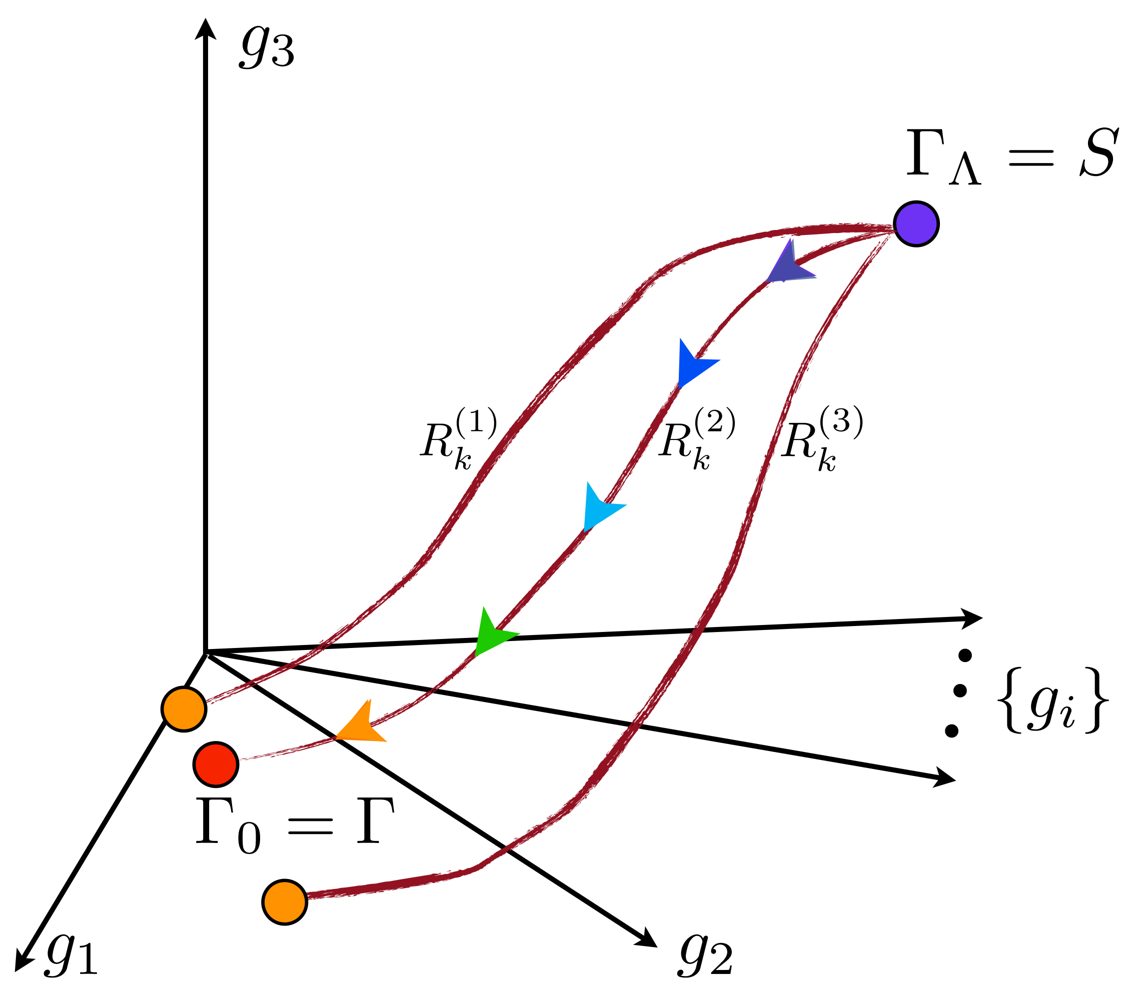


full flow



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eg. Liao, Polonyi, Strickland, NPB 567 (2000) 493-514
Canet, Delamotte, Mouhanna, Vidal, PRD 67 (2003) 065004

Most rapid convergence at fixed points

Litim, PLB 486 (2000) 92-99

Functional optimisation: Integrability

JMP, AP 322 (2007) 2831
JMP, Scherer, Schmidt, Wetzel, AP 384 (2017) 165

Systematic error control & optimisation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

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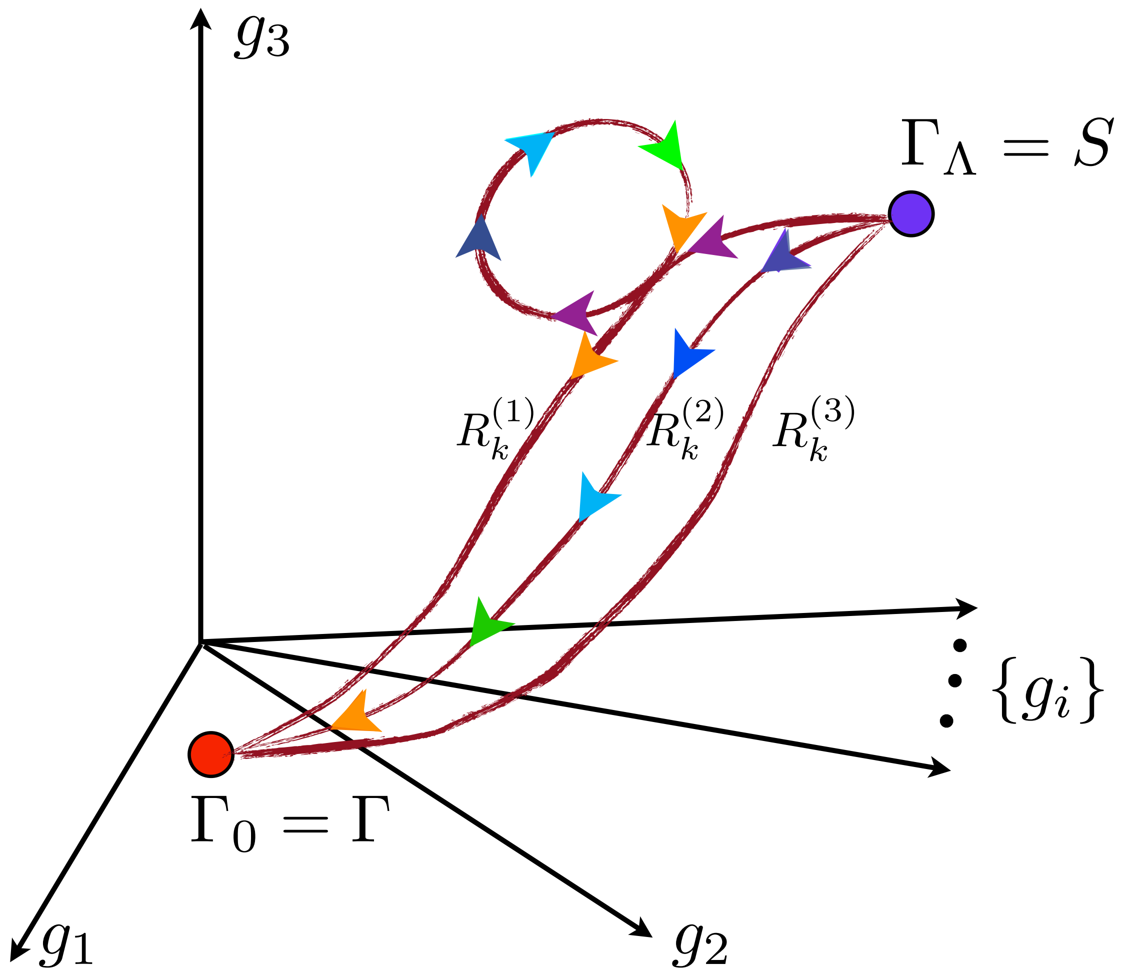
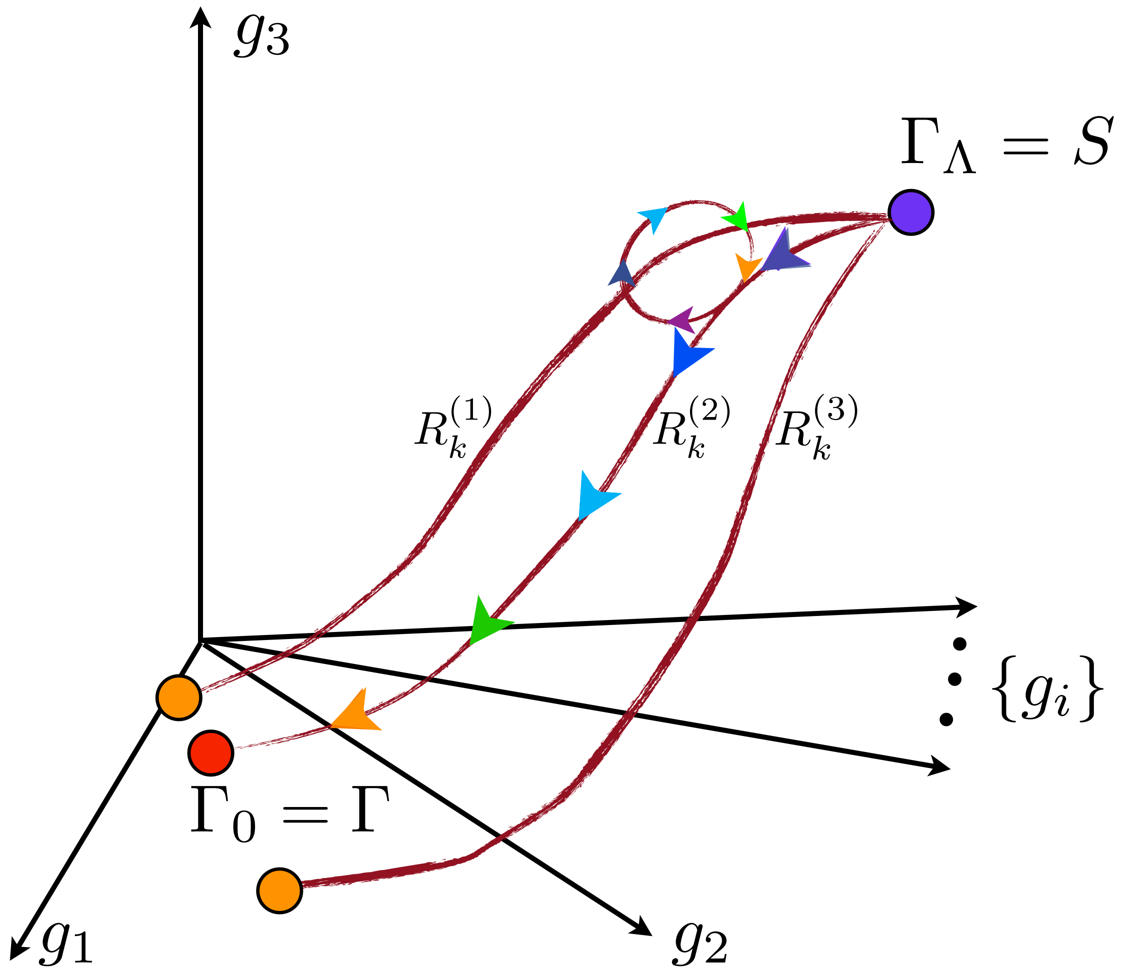
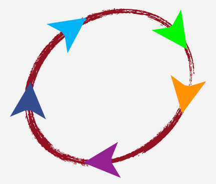


full flow



Optimised flow

Optimisation: find $R_k^{(2)}$!

$\lim_{L \to 0} \frac{1}{L}$  $\to 0$

Functional optimisation: Integrability

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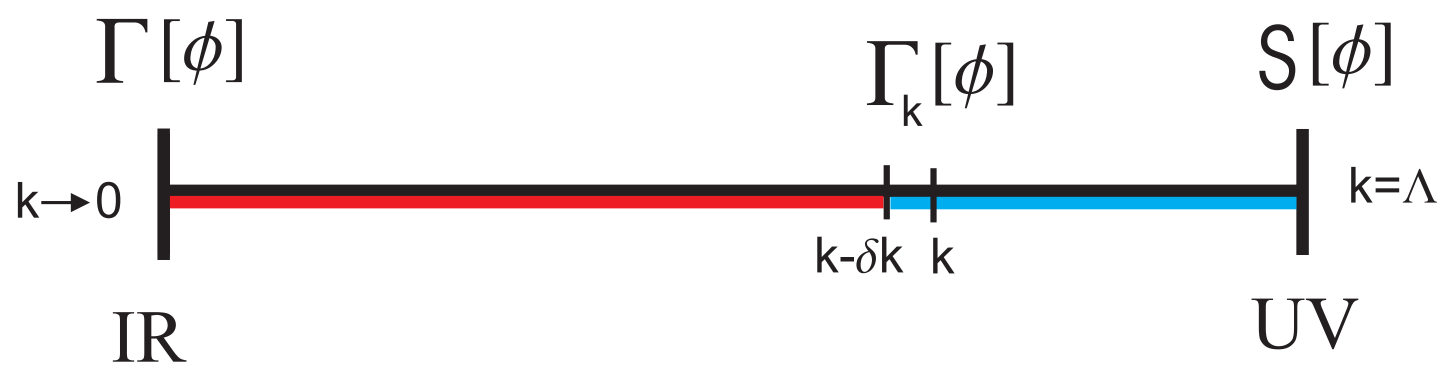
Functional flows for QCD

Flows for correlation functions
&
chiral symmetry breaking

Functional flows for QCD

Dupuis et al, Phys.Rept. (2021)
Fu, Commun.Theor.Phys. 74 (2022) 9, 097304

free energy at momentum



hadrons

quarks & gluons

ab initio

Functional flows for QCD

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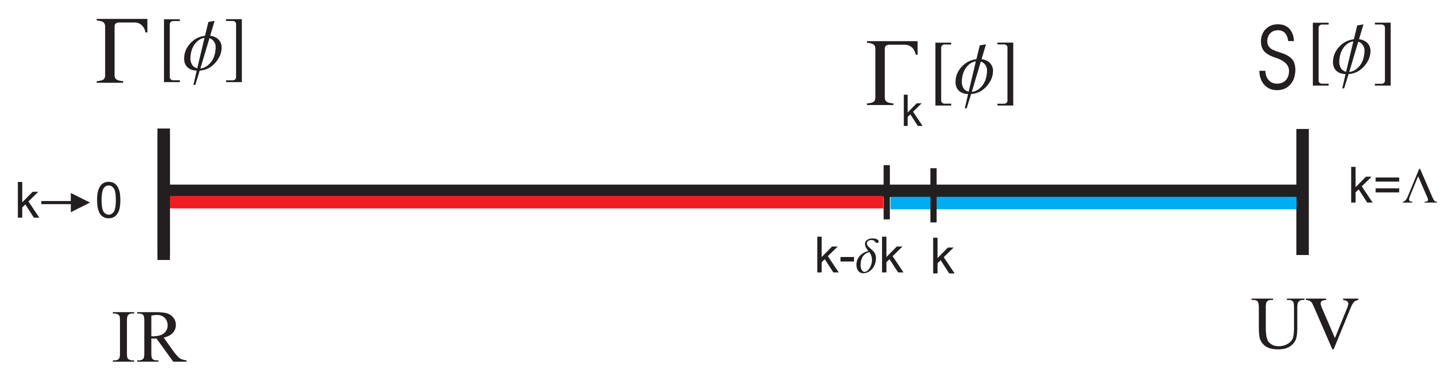


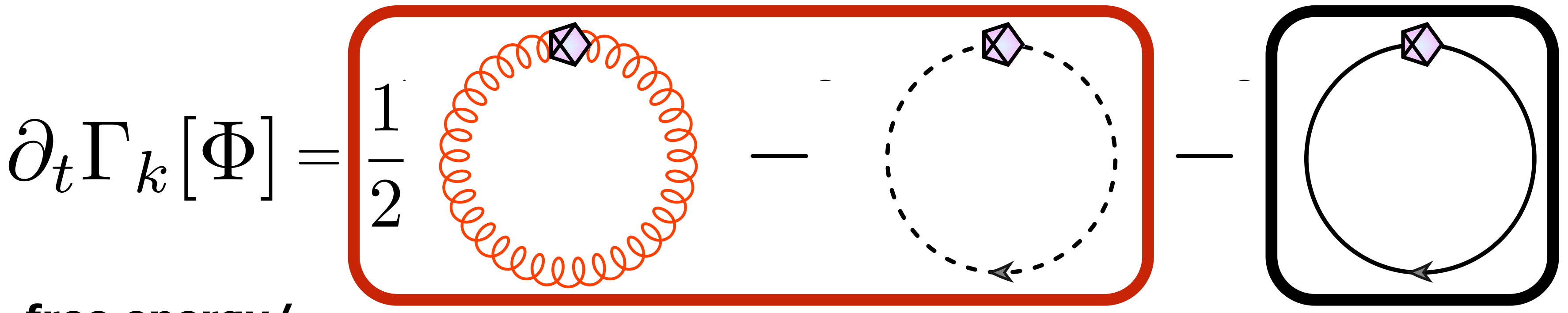
hadrons   quarks & gluons

ab initio

RG-scale k: $t = \ln k$

glue quantum fluctuations

functional RG:



free energy/ grand potential

quark quantum fluctuations

$$\Phi = (A_\mu, c, \bar{c}, q, \bar{q})$$

Functional flows for QCD

Dupuis et al, Phys.Rept. (2021)
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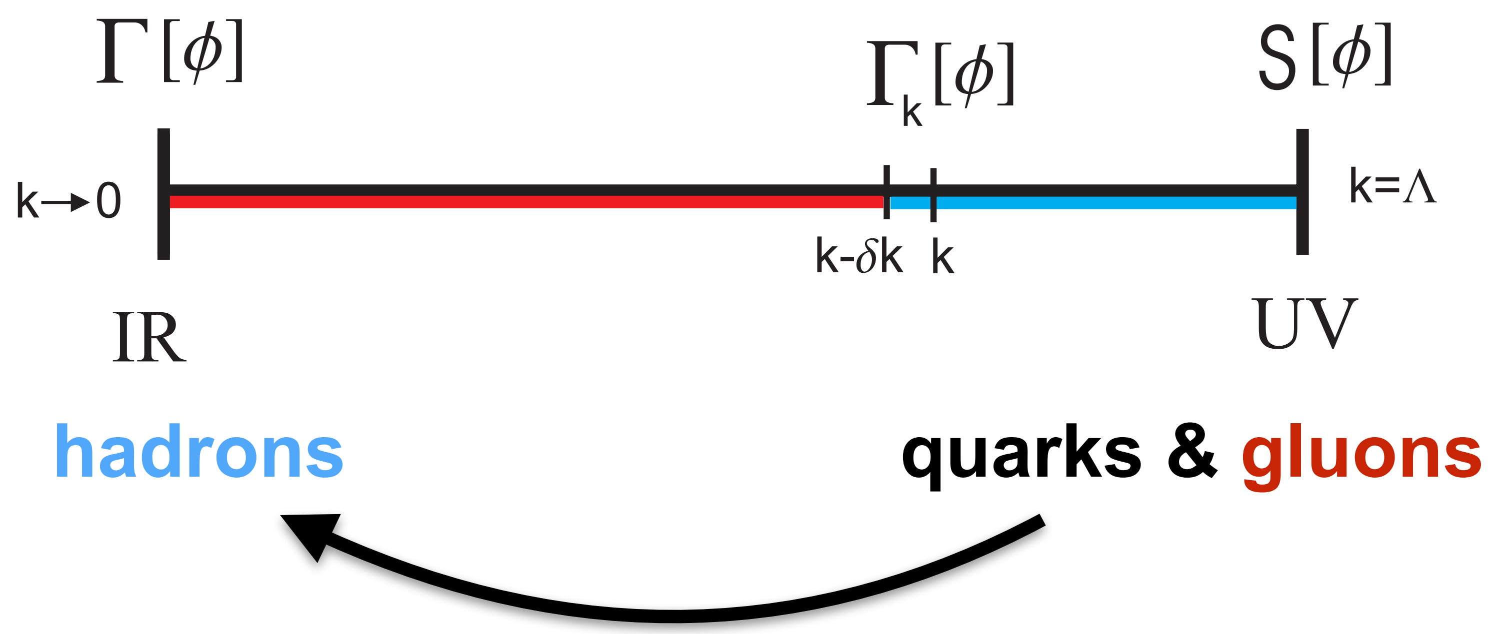
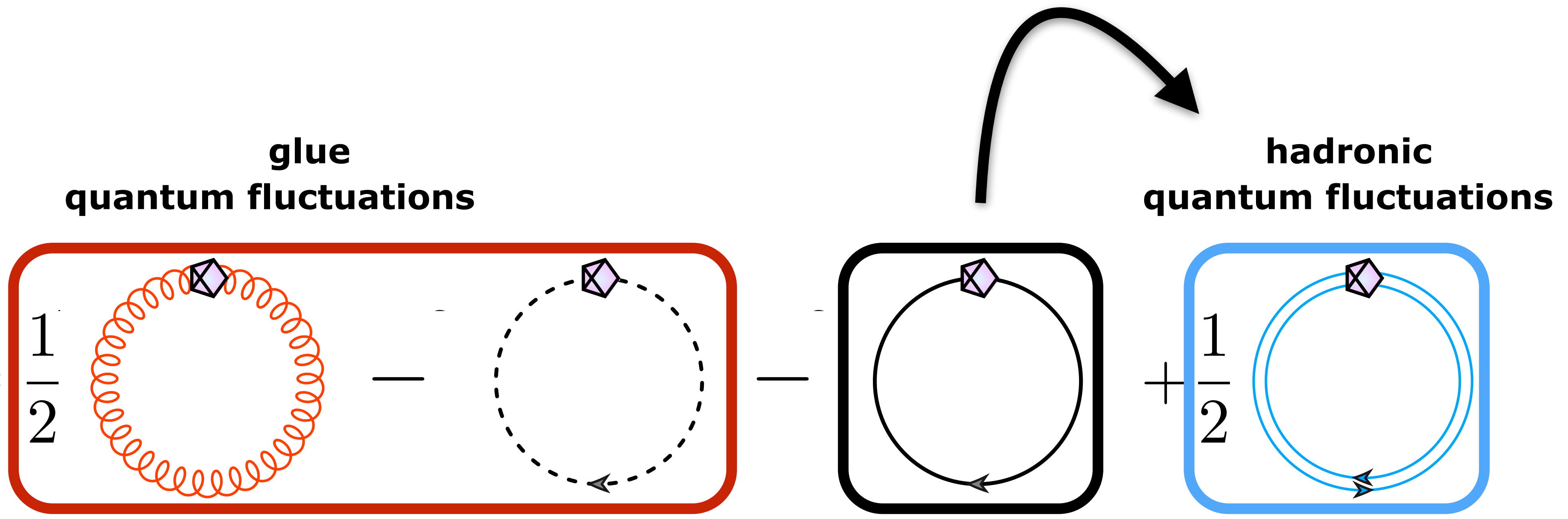


ab initio

fRG approach with emergent composites/dynamical hadronisation

functional RG: $\left(\partial_t + \int_x \dot{\Phi}\,\frac{\delta}{\delta\Phi}\right)\Gamma_k[\Phi] =$



free energy/
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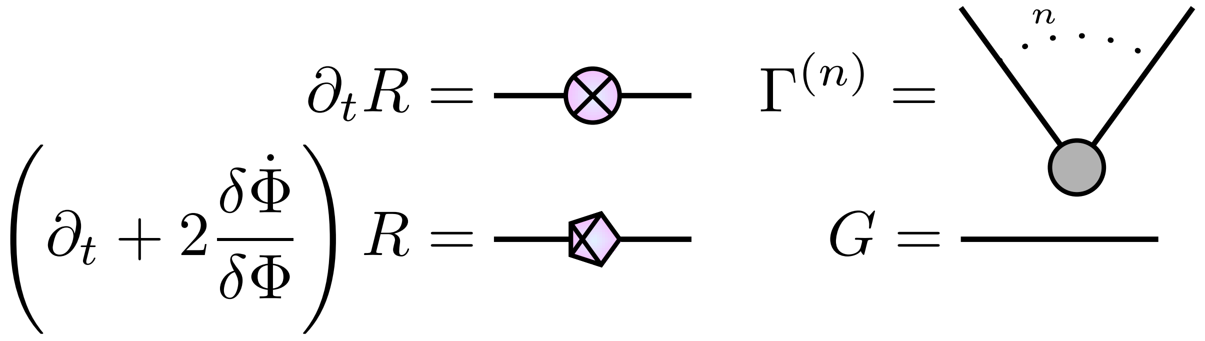
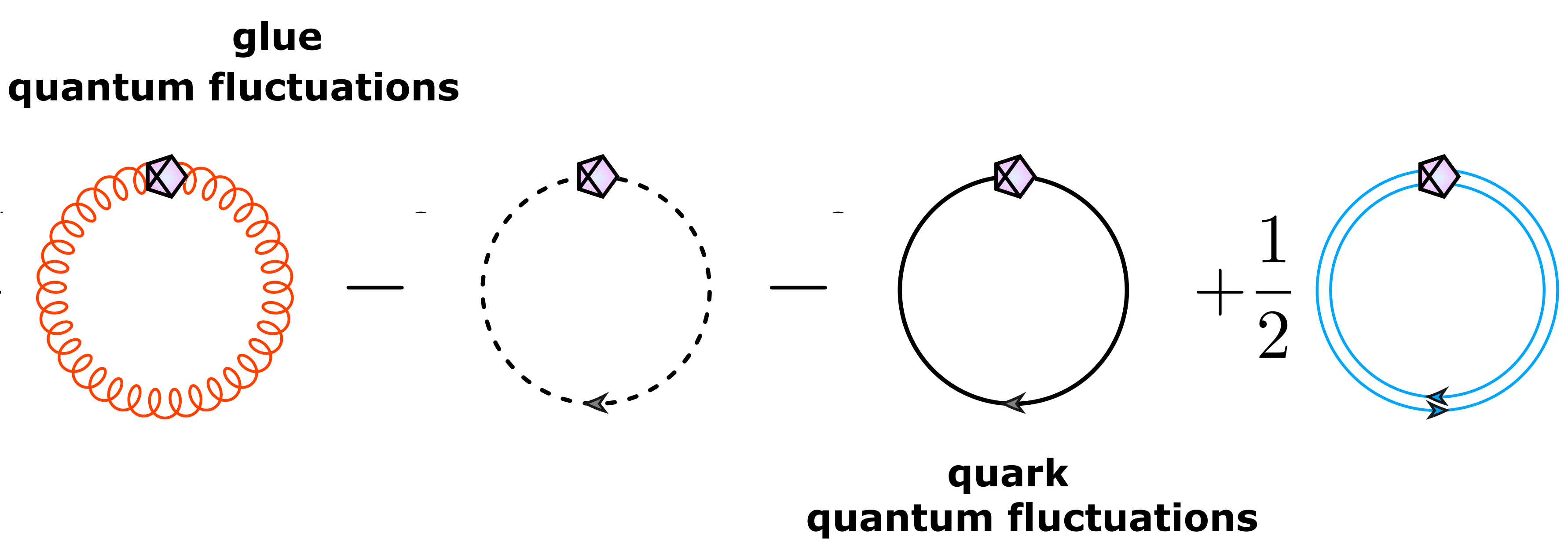


$$\Phi = (A_\mu, c, \bar{c}, q, \bar{q}, \phi)$$

ϕ **hadronic composites**

Flows for correlation functions

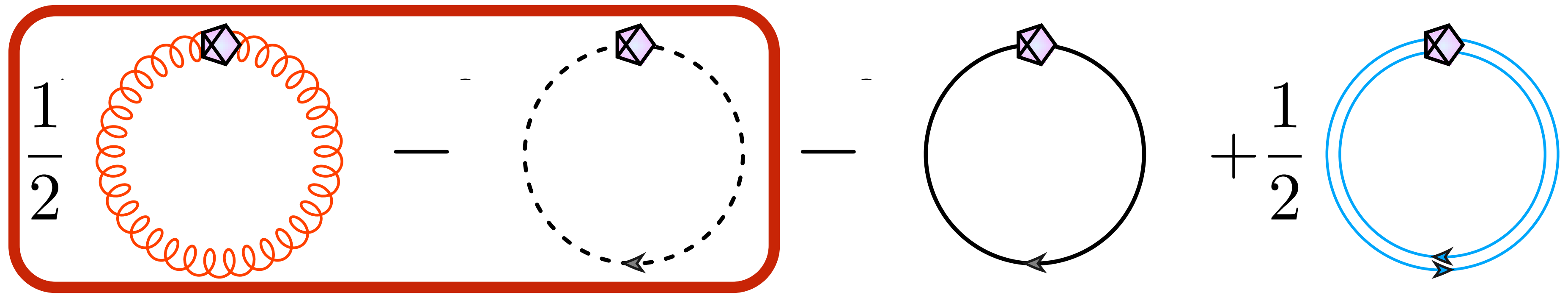
functional RG: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}$ 

free energy/
grand potential

Correlation functions

Flows for correlation functions

glue
quantum fluctuations

functional RG: $\partial_t \Gamma_k[\Phi] =$ 

free energy/
grand potential

quark
quantum fluctuations

gluon propagator

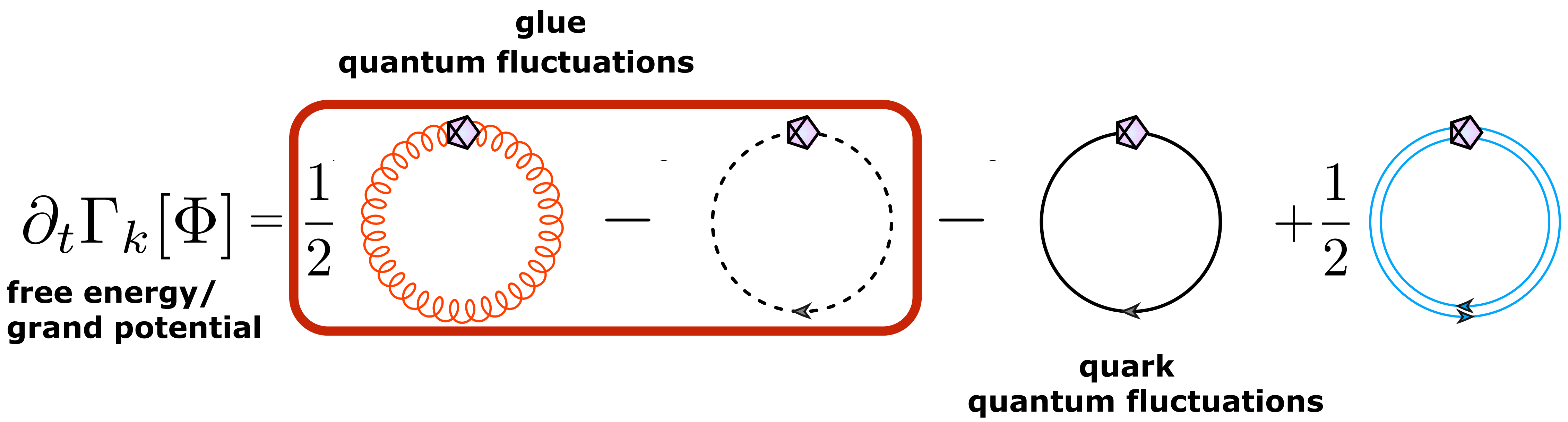
$$\langle A_\mu A_\nu \rangle (p)$$

Pure glue

Correlation functions

Flows for correlation functions

functional RG:



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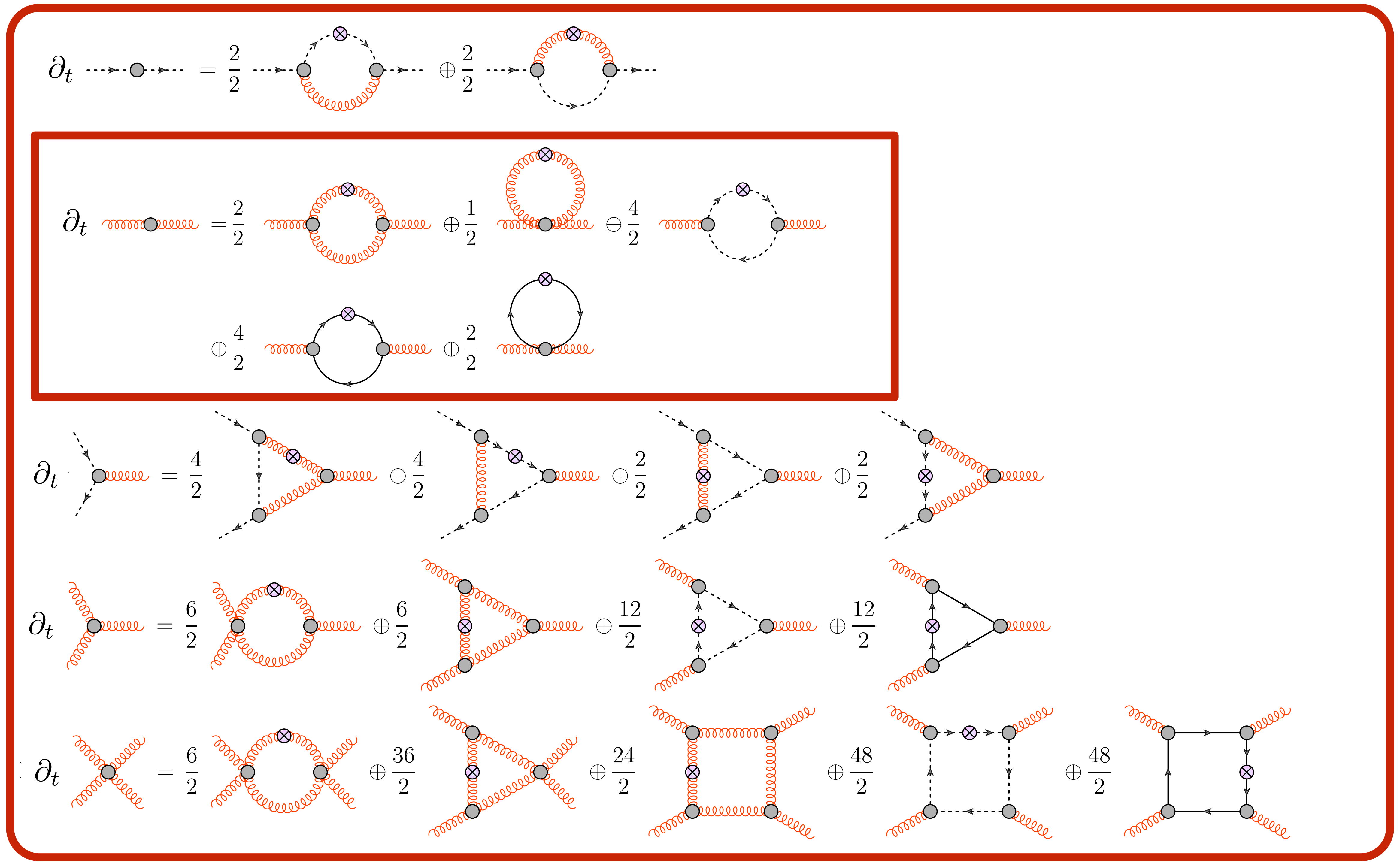


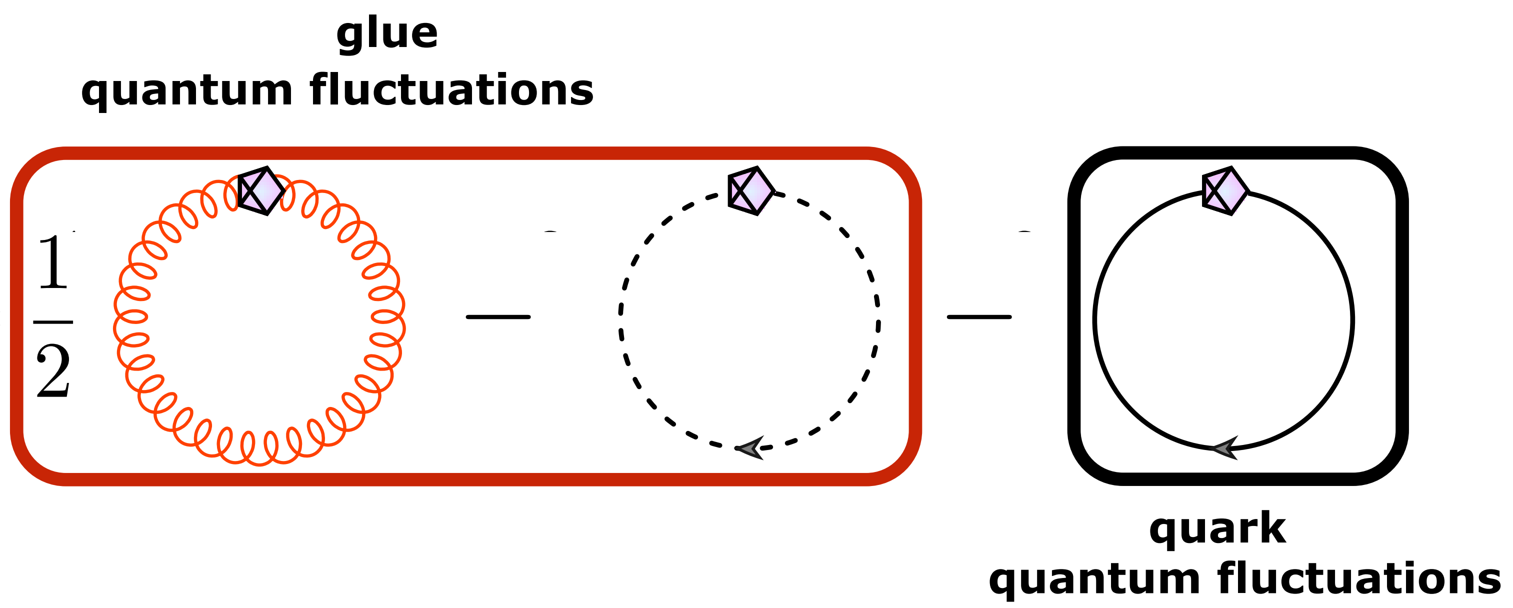
... 1-loop exact

Flows for correlation functions

no hadronic composites

$\phi = 0$

functional RG: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}$ (glue loop) $-$ (ghost loop) $-$ (quark loop)



free energy/
grand potential

Correlation functions

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quark propagator

$\langle q \bar{q} \rangle (p)$

Flows for correlation functions

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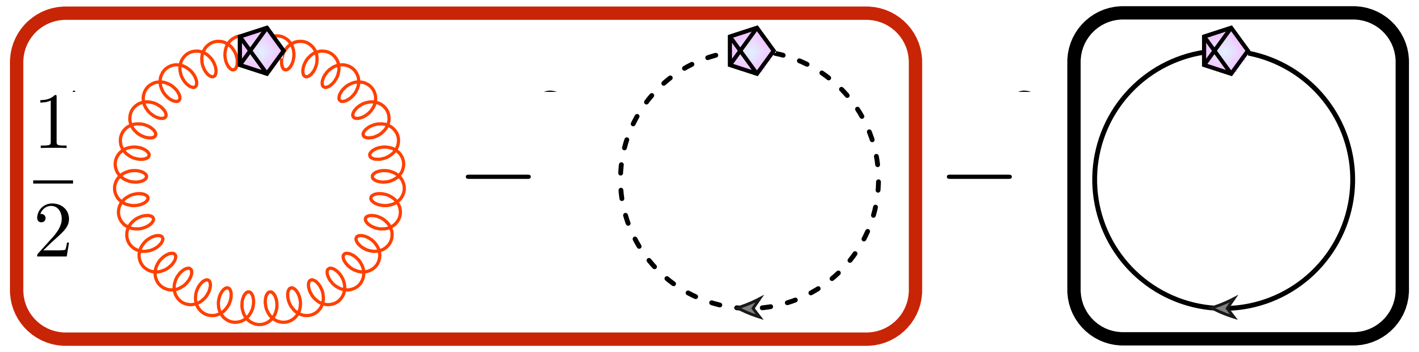
functional RG: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}$ 

free energy/
grand potential

quark
quantum fluctuations

Correlation functions

gluon propagator

$\langle A_\mu A_\nu \rangle (p)$

quark propagator

$\langle q\bar{q} \rangle (p)$

quark-gluon vertex

$\langle q\bar{q} A_\mu \rangle (p_1, p_2)$

Eight tranverse tensor structures

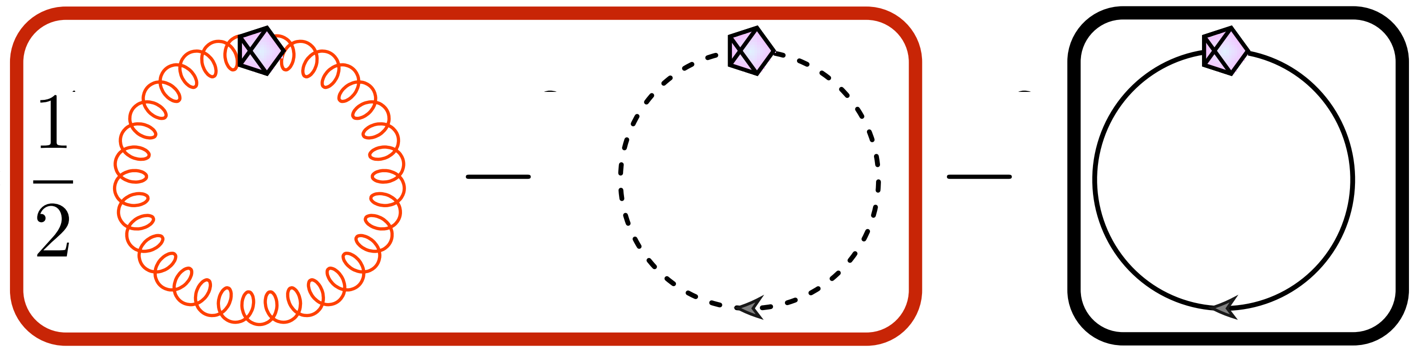
Flows for correlation functions

no hadronic composites

$\phi = 0$

glue
quantum fluctuations

functional RG: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}$



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Eight tranverse tensor structures

quark—anti-quark scattering

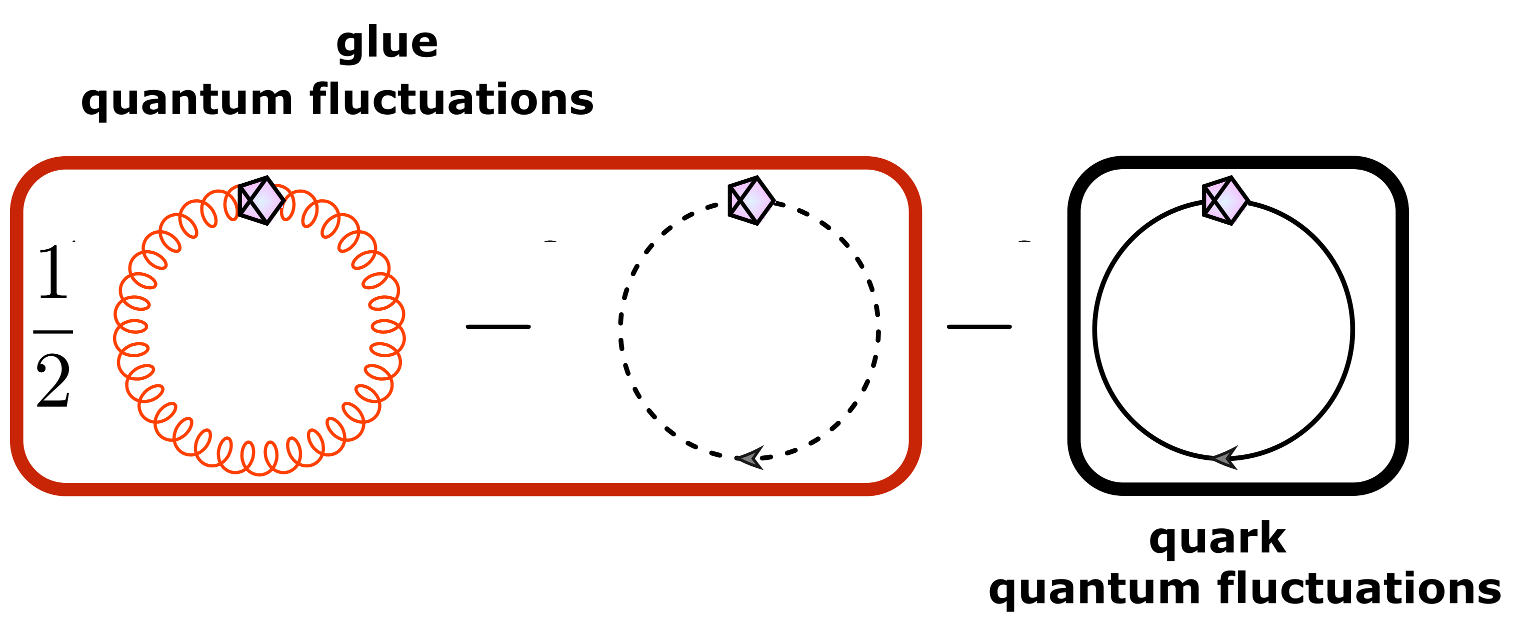
$\langle q\bar{q}q\bar{q} \rangle (p_1, p_2, p_3)$

Flows for correlation functions

no hadronic composites

$\phi = 0$

functional RG: $\partial_t \Gamma_k[\Phi] =$



free energy/
grand potential

Correlation functions

gluon propagator

$\langle A_\mu A_\nu \rangle(p)$

quark propagator

$\langle q\bar{q} \rangle(p)$

quark-gluon vertex

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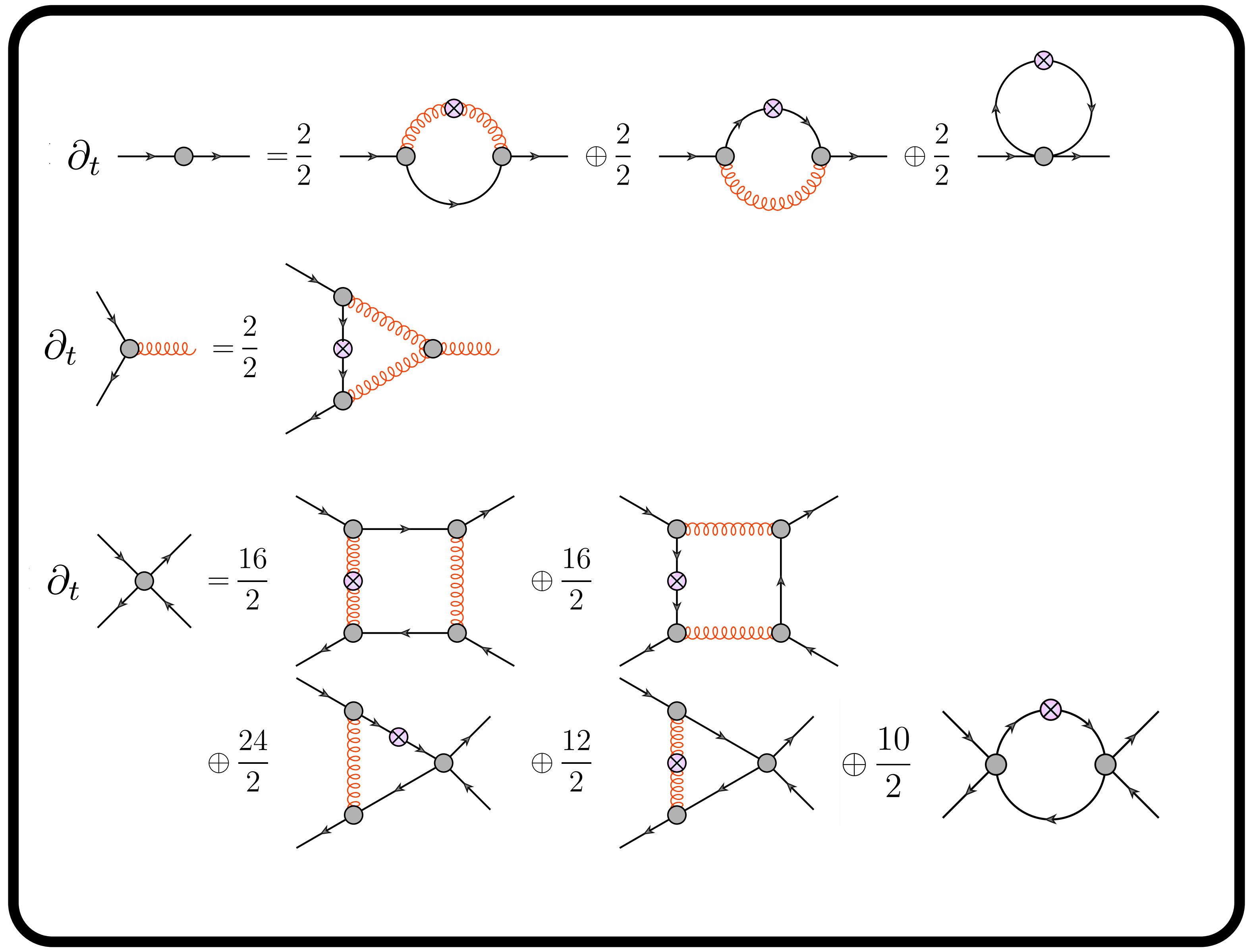
Eight tranverse tensor structures

quark—anti-quark scattering

$\langle q\bar{q} q\bar{q} \rangle(p_1, p_2, p_3)$



... 1-loop exact

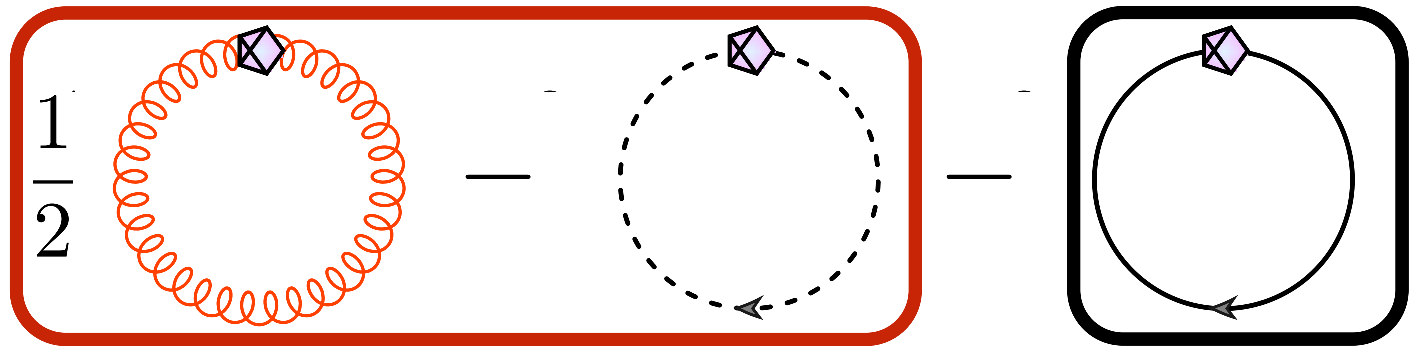
Chiral symmetry breaking & mesons

no hadronic composites

$\phi = 0$

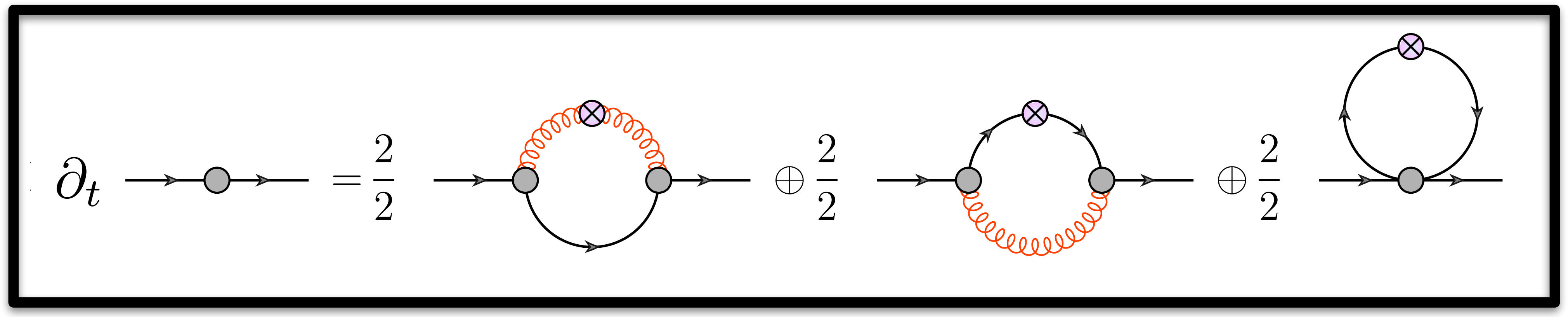
glue
quantum fluctuations

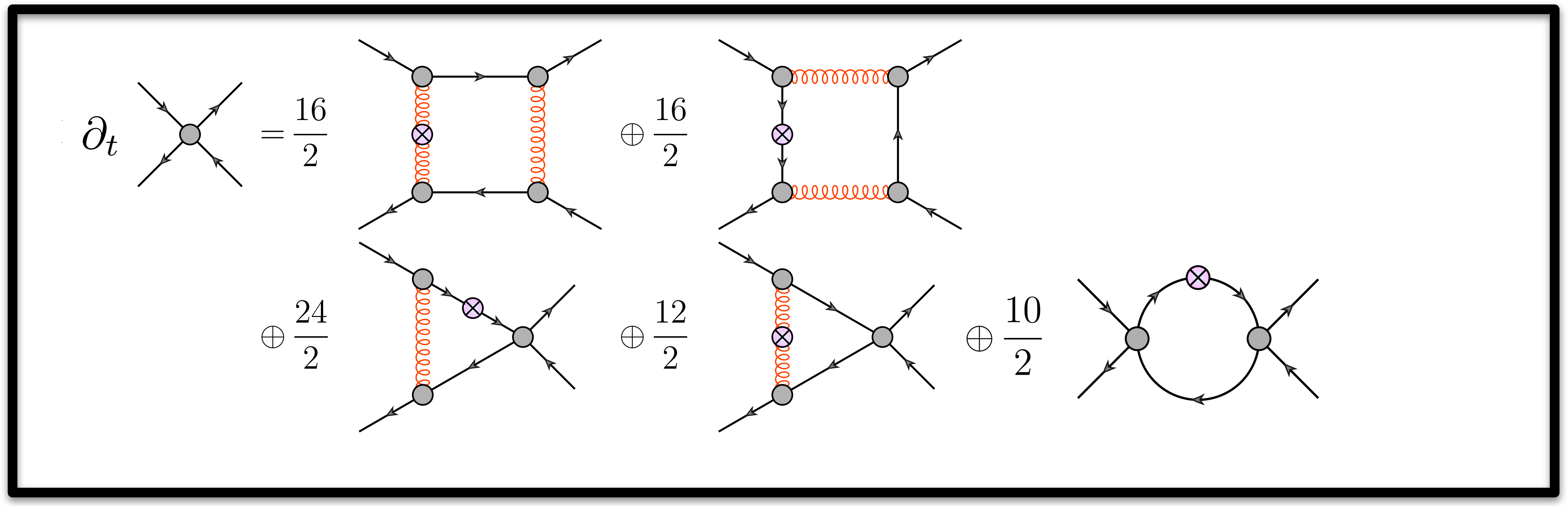
functional RG: $\partial_t \Gamma_k[\Phi] = \frac{1}{2}$ (…) $-$ (…) $-$ (…)



free energy/
grand potential

quark
quantum fluctuations





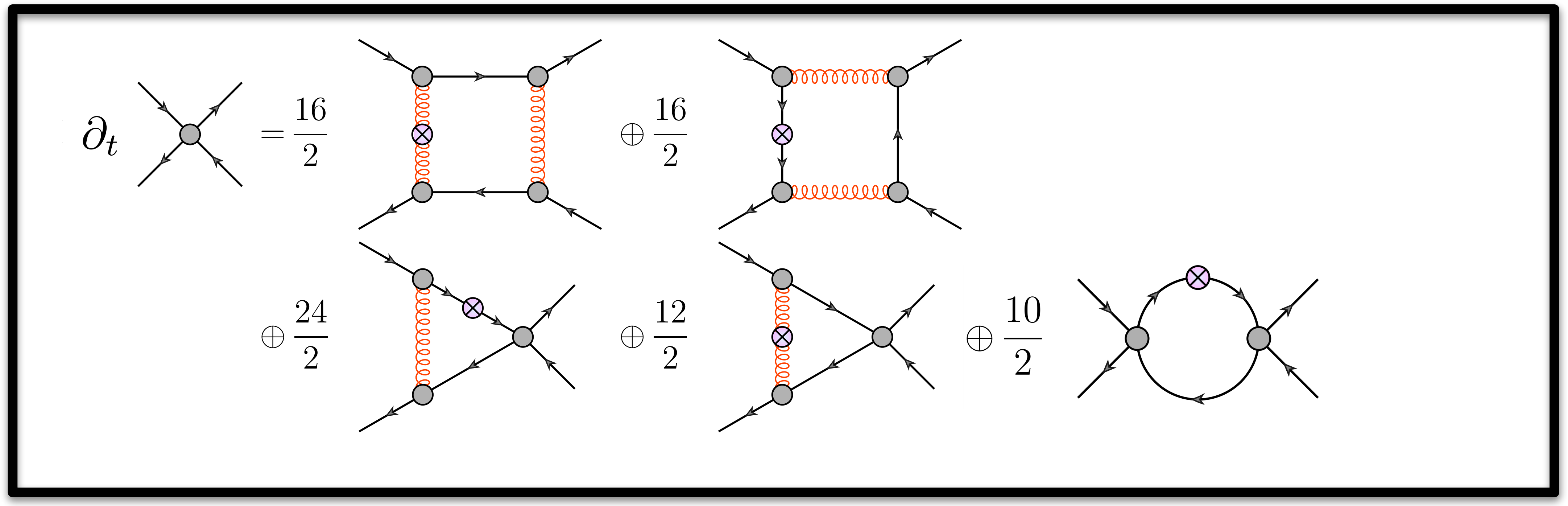
2 tensor structures

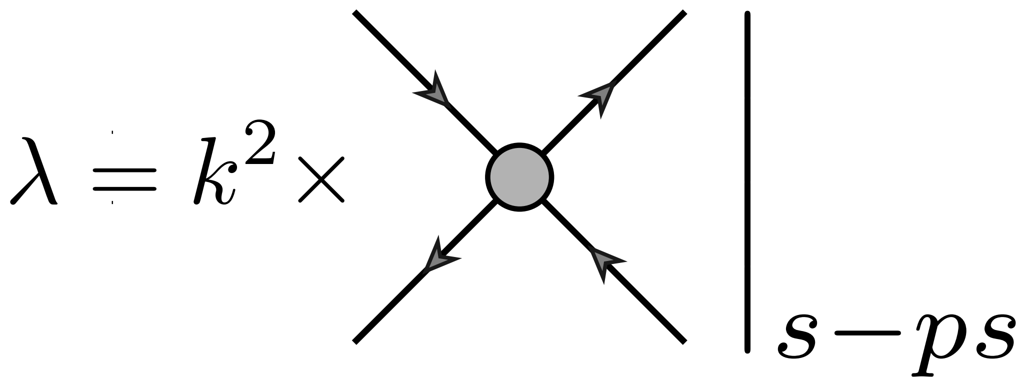
Two flavours: 10 momentum-independent tensor structures

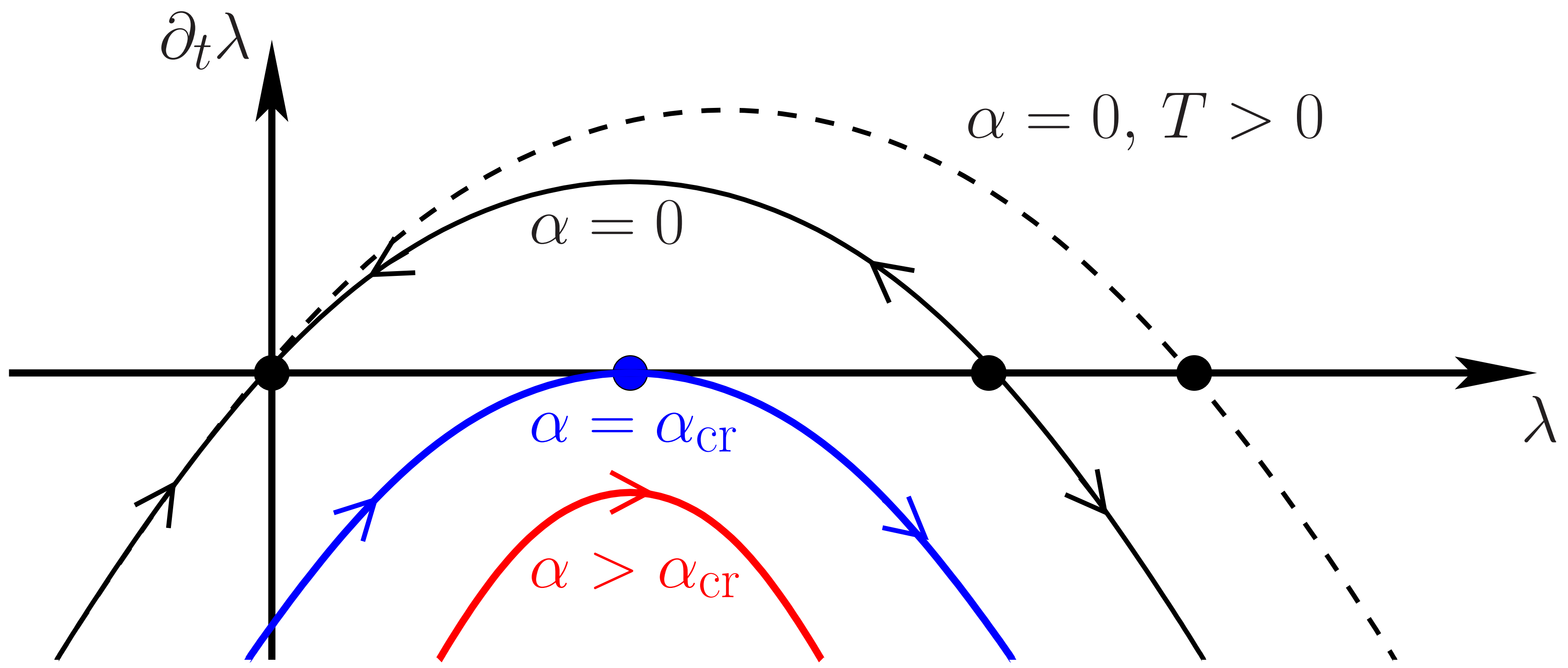
$$\Gamma_{\rm mat} = \int_p \bar q(-p)\left[Z_q(p) i \not p + M_q(p)\right] q(p) + \sum_{i=1}^{10} \int_{\mathbf{p}} \lambda^{(i)}_{\bar q^2 q^2}(\mathbf{p}) \left(\bar q^2 \mathcal{T}^{(i)}_{\bar q^2 q^2} q^2\right)(\mathbf{p}) + \cdots$$

Chiral symmetry breaking & mesons

Chiral symmetry breaking in a nutshell







Chiral symmetry breaking & mesons

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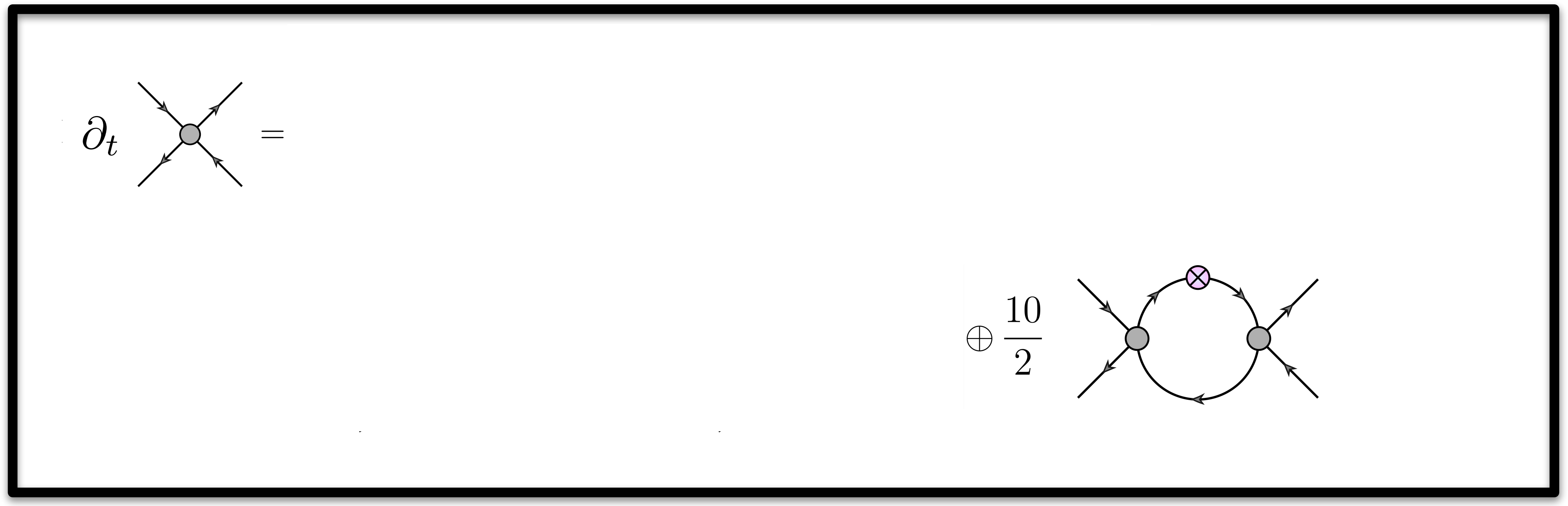
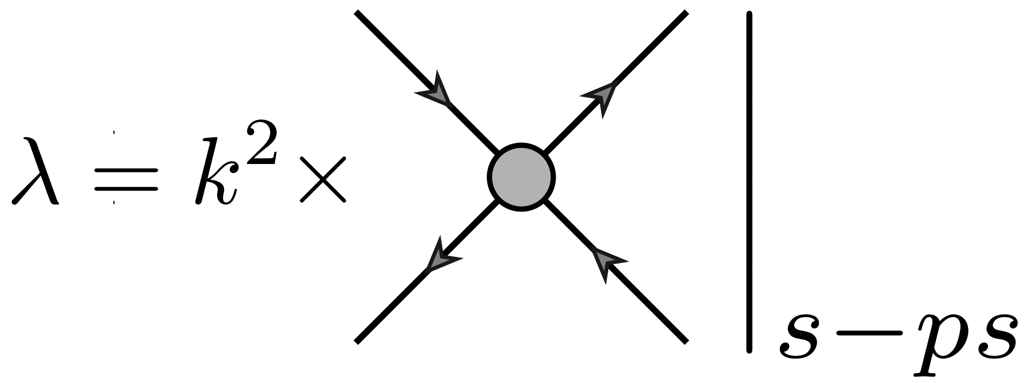
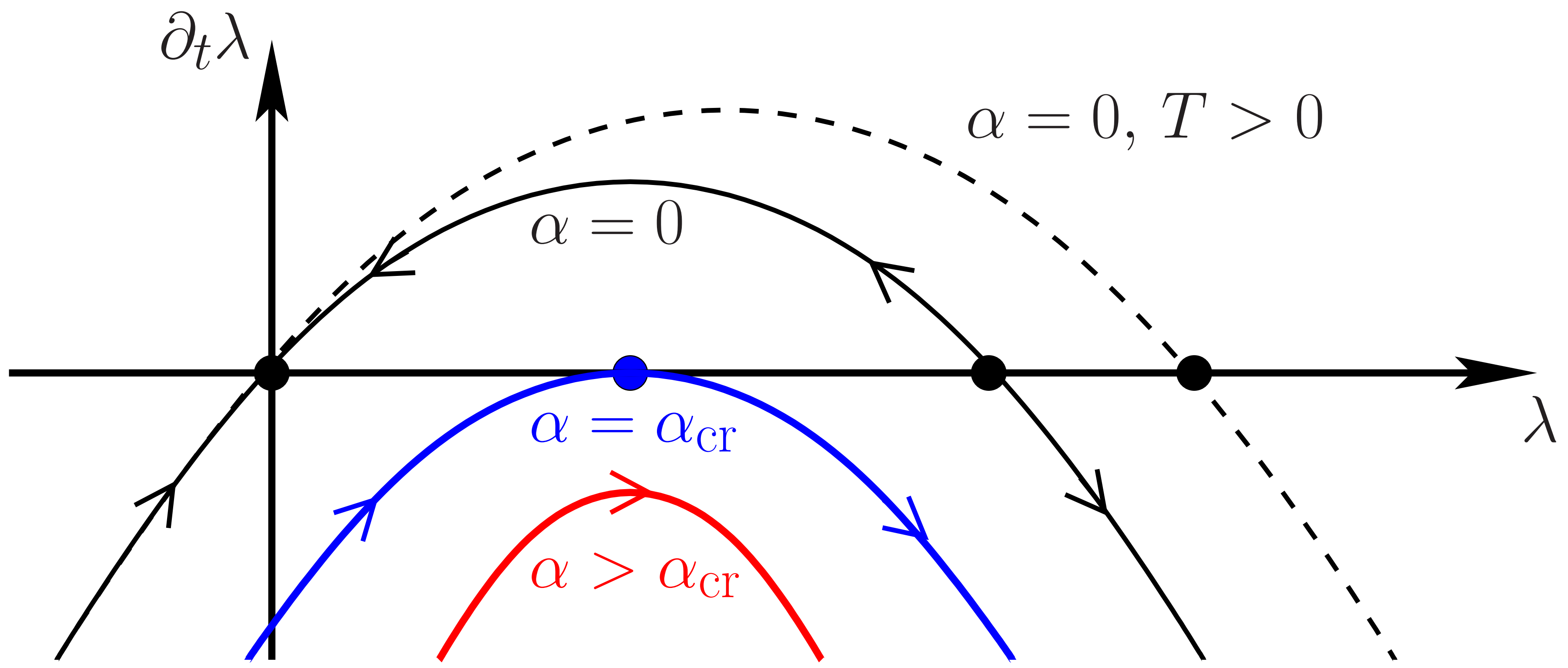


Beta-function of dimensionless scalar-pseudoscalar coupling

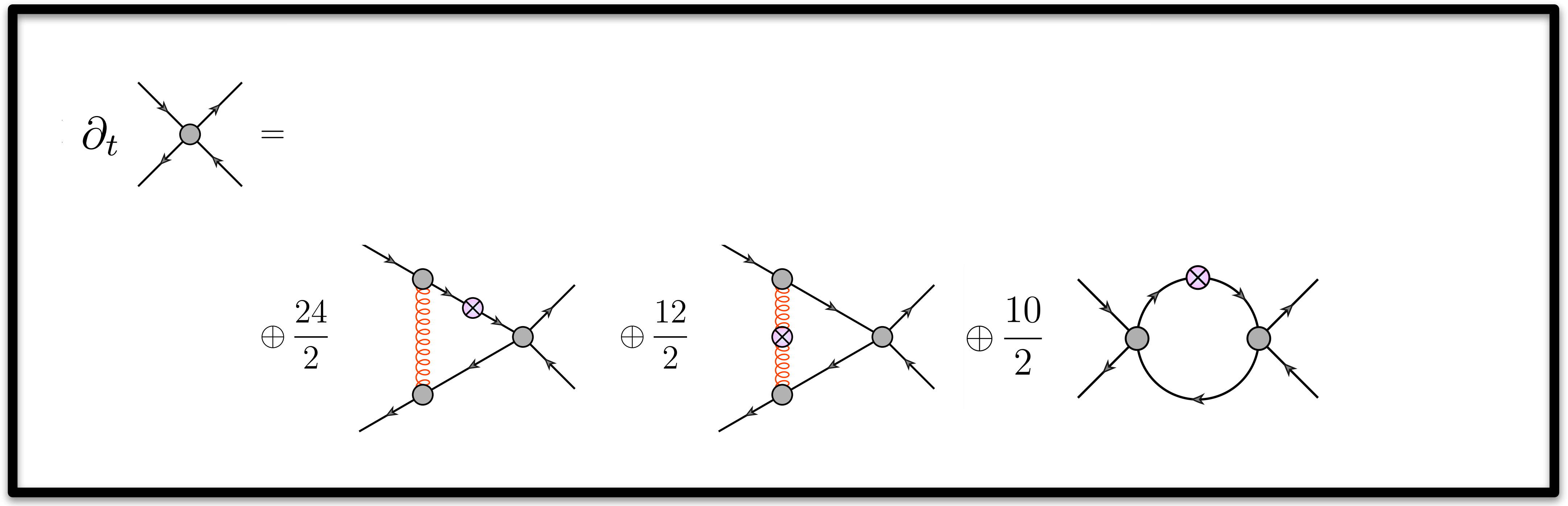
$$\partial_t \lambda = 2\lambda - A(k, M_q)\, \lambda^2$$





Chiral symmetry breaking & mesons

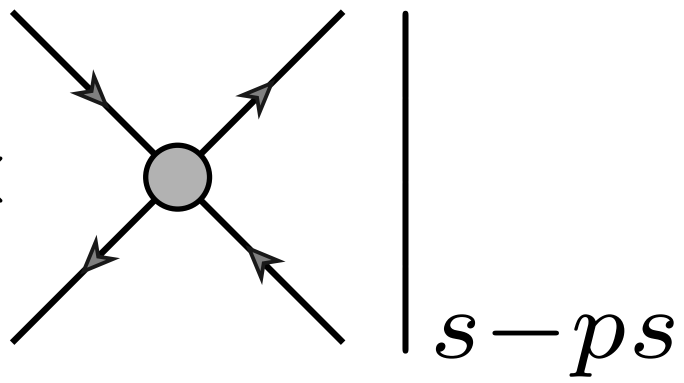
Chiral symmetry breaking in a nutshell

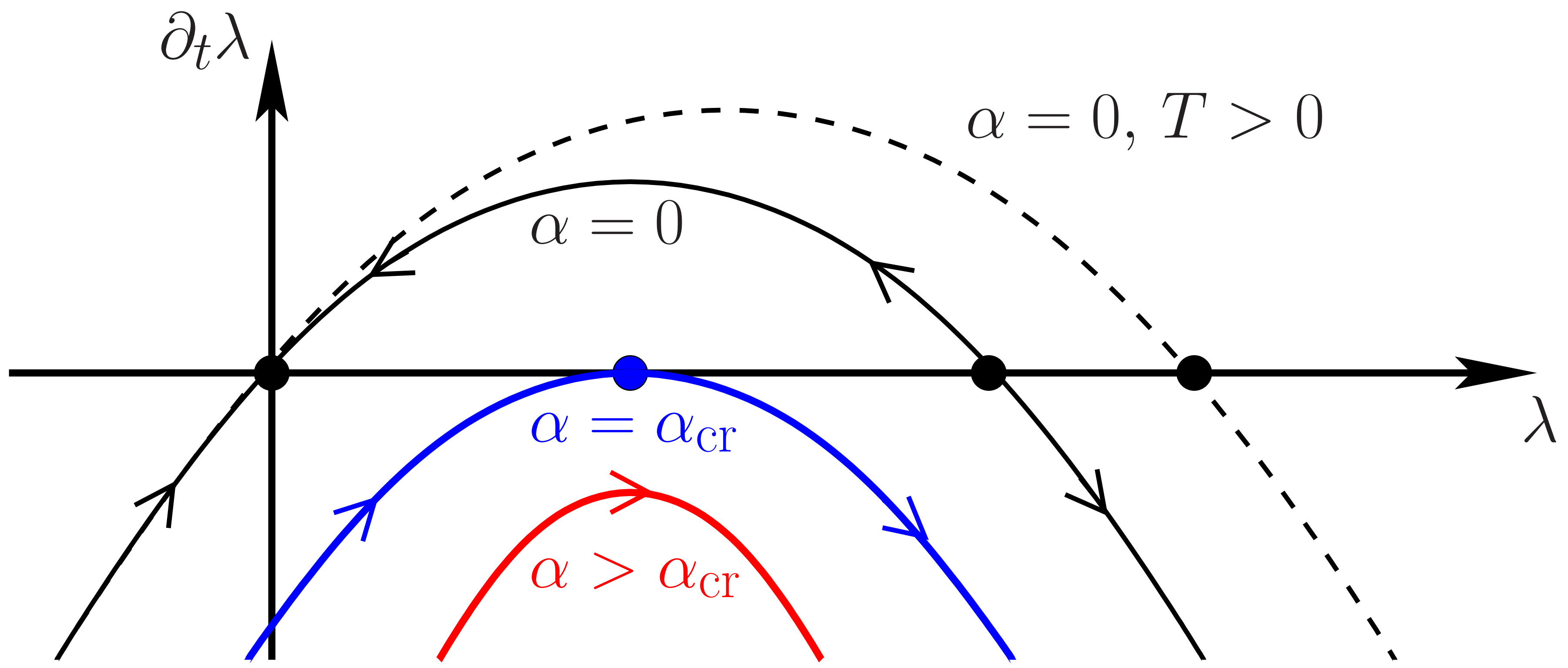


Beta-function of dimensionless scalar-pseudoscalar coupling

$$\partial_t \lambda = 2\lambda - A(k, M_q)\, \lambda^2 - B(k, M_q, M_{\text{gap}})\lambda\alpha_s$$

$\lambda = k^2 \times$





Chiral symmetry breaking & mesons

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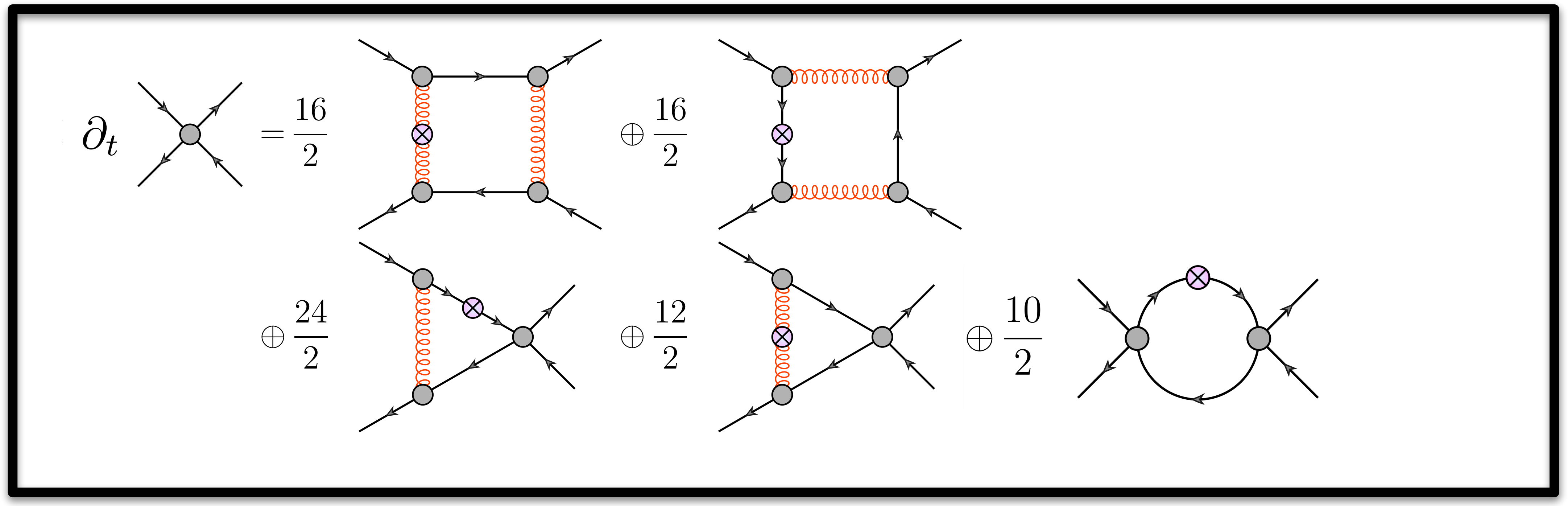
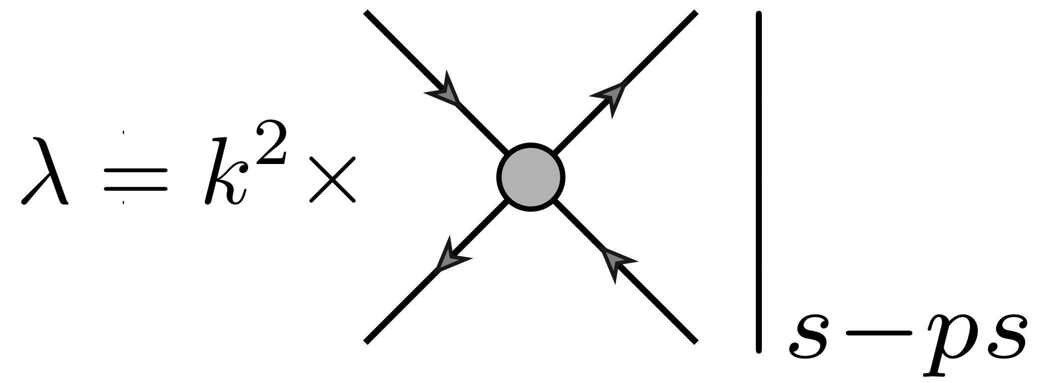
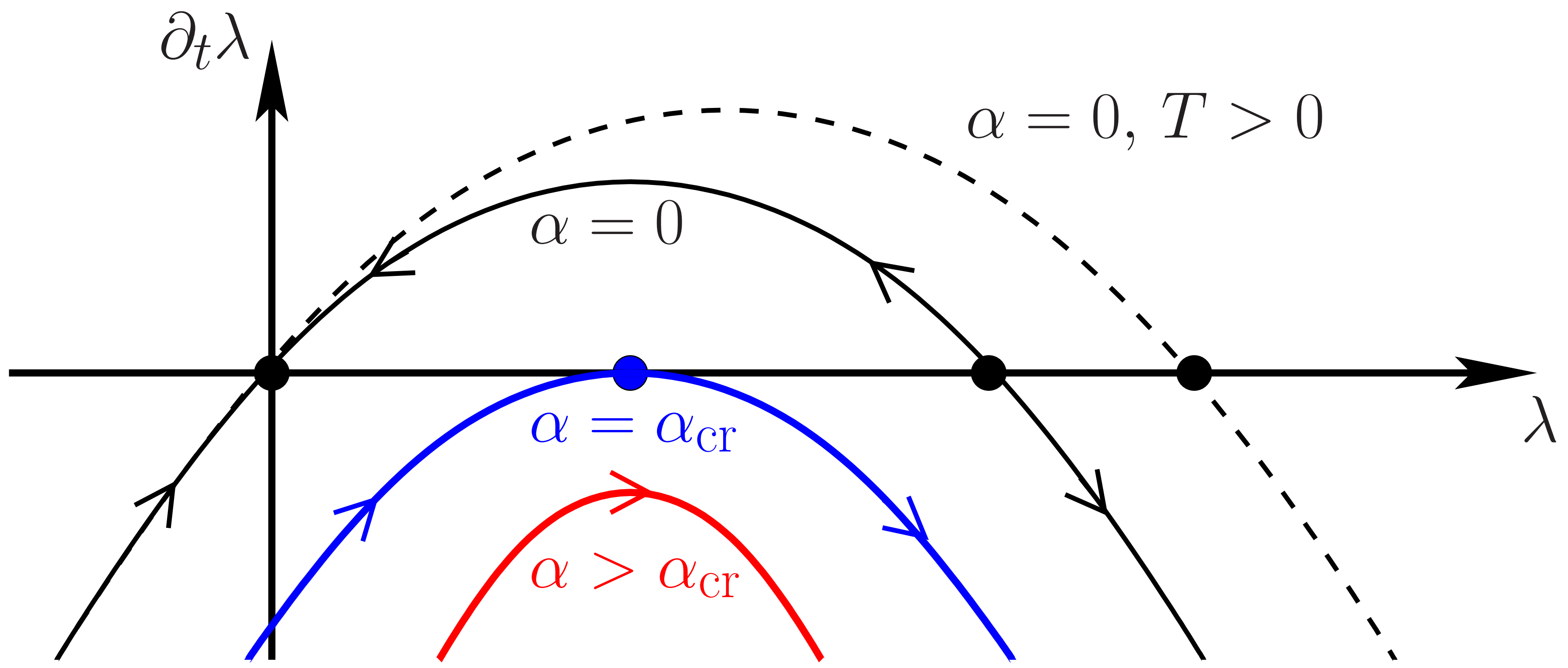


Beta-function of dimensionless scalar-pseudoscalar coupling

$$\partial_t \lambda = \Big[2 - B(k, M_q, M_{\rm gap})\alpha_s\Big]\lambda - A(k, M_q)\,\lambda^2 - C(k, M_q, M_{\rm gap})\alpha_s^2$$





Chiral symmetry breaking & mesons

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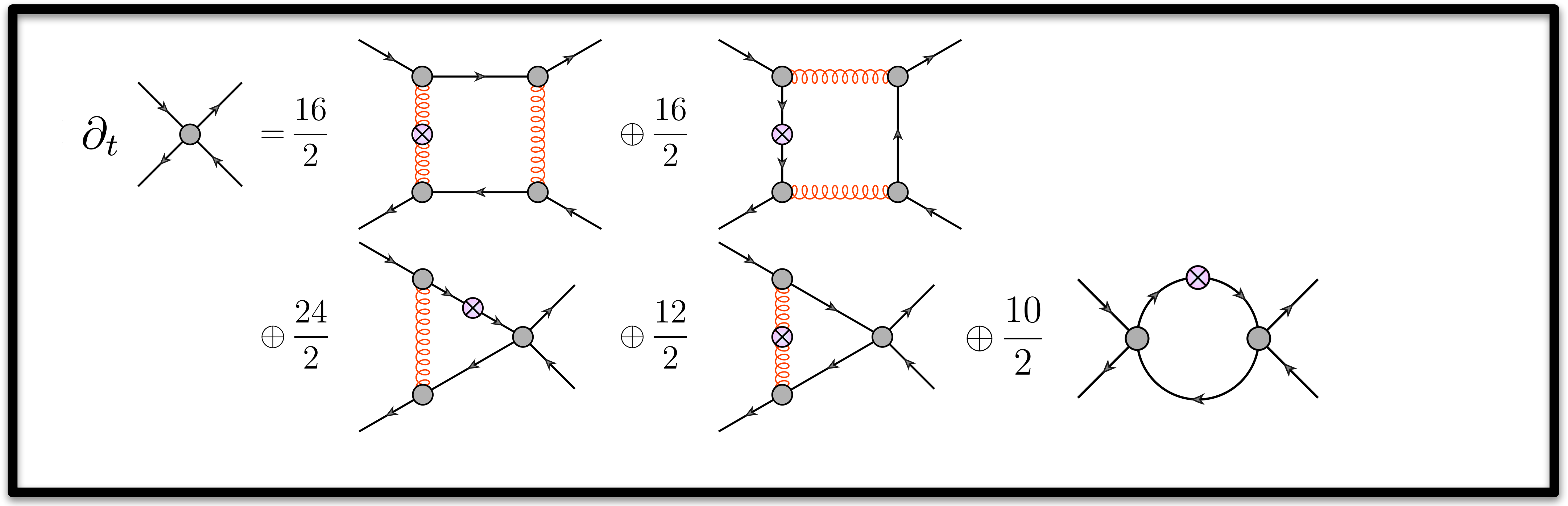

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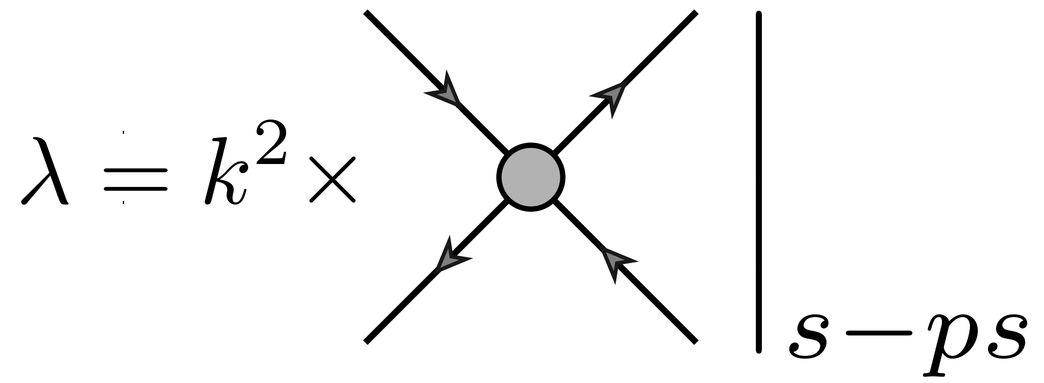
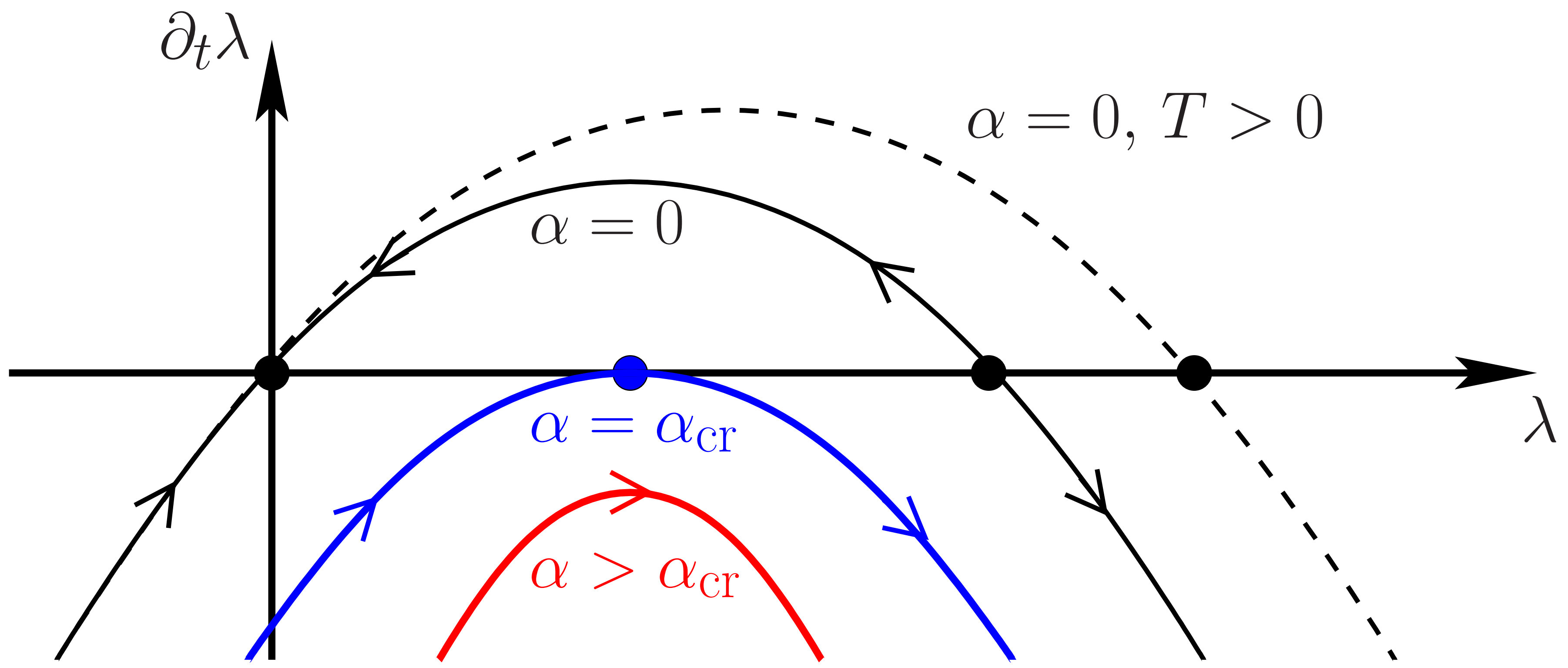




chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

fRG: Aoki, Braun, Jäckel, Gies, JMP, Terao, Wetterich, ...

Getting dynamical: emergent hadrons & diquarks

Gies, Wetterich, PRD 65 (2002) 065001
PRD 69 (2004) 025001

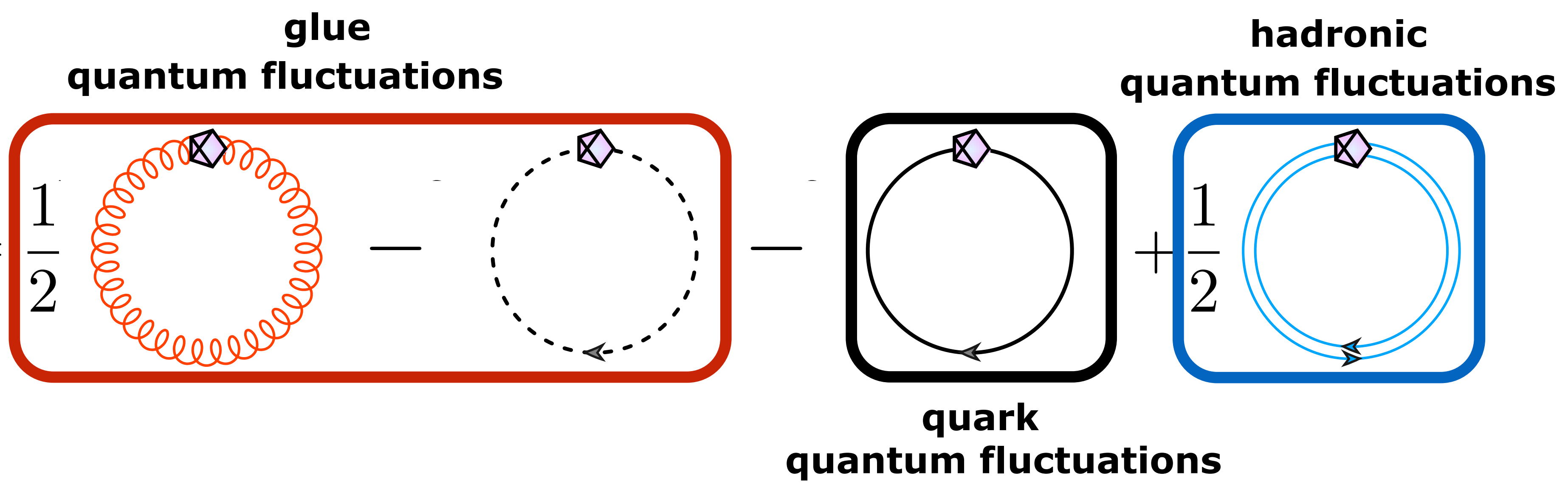
JMP, AP 322 (2007) 2831-2915
Floerchinger, Wetterich, PLB 680 (2009) 371

Fu, JMP, Rennecke, PRD 101, (2020) 054032
Fukushima, JMP, Strodthoff, 2103.01129

Dynamical hadronisation: mesons & diquarks

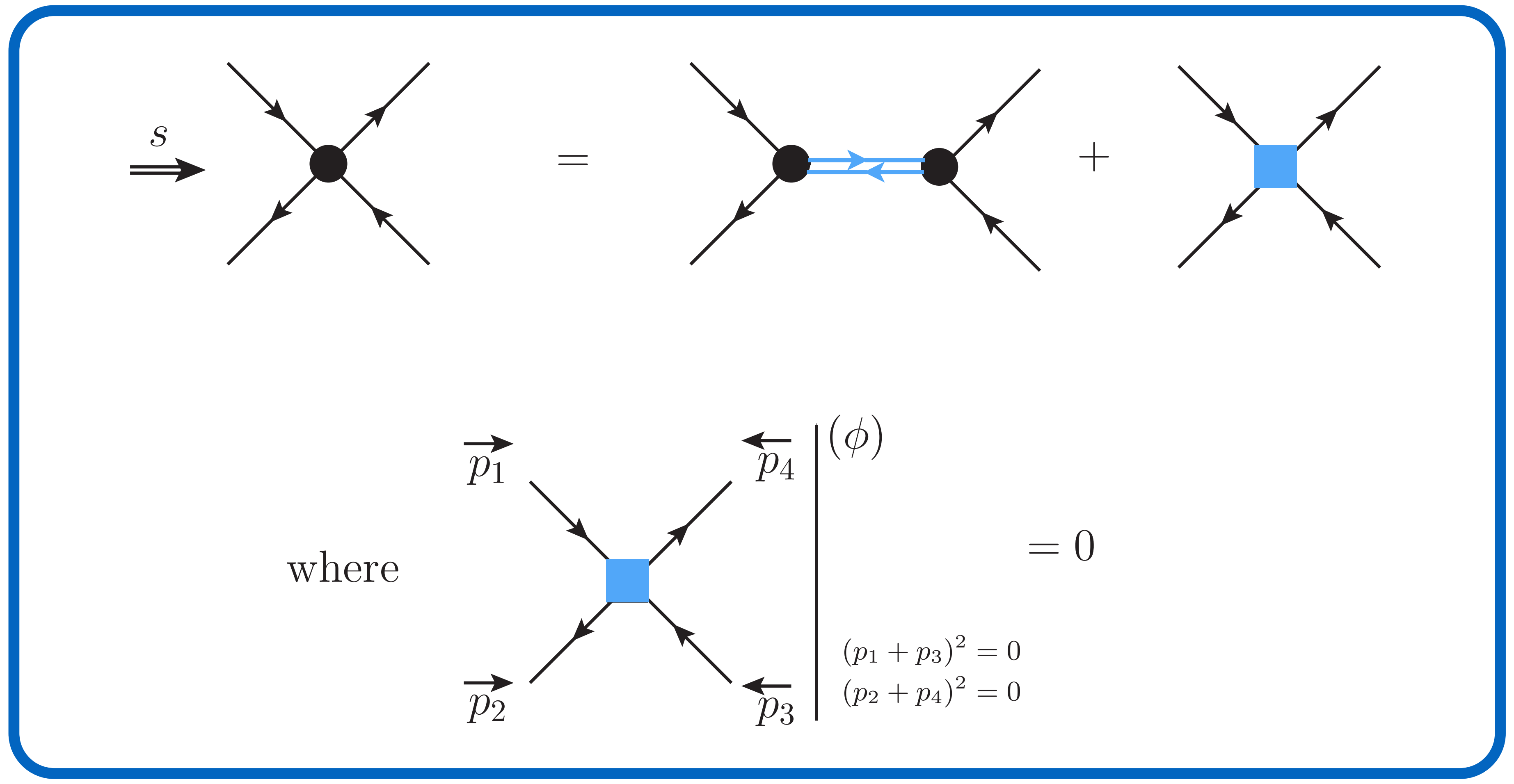
functional RG:

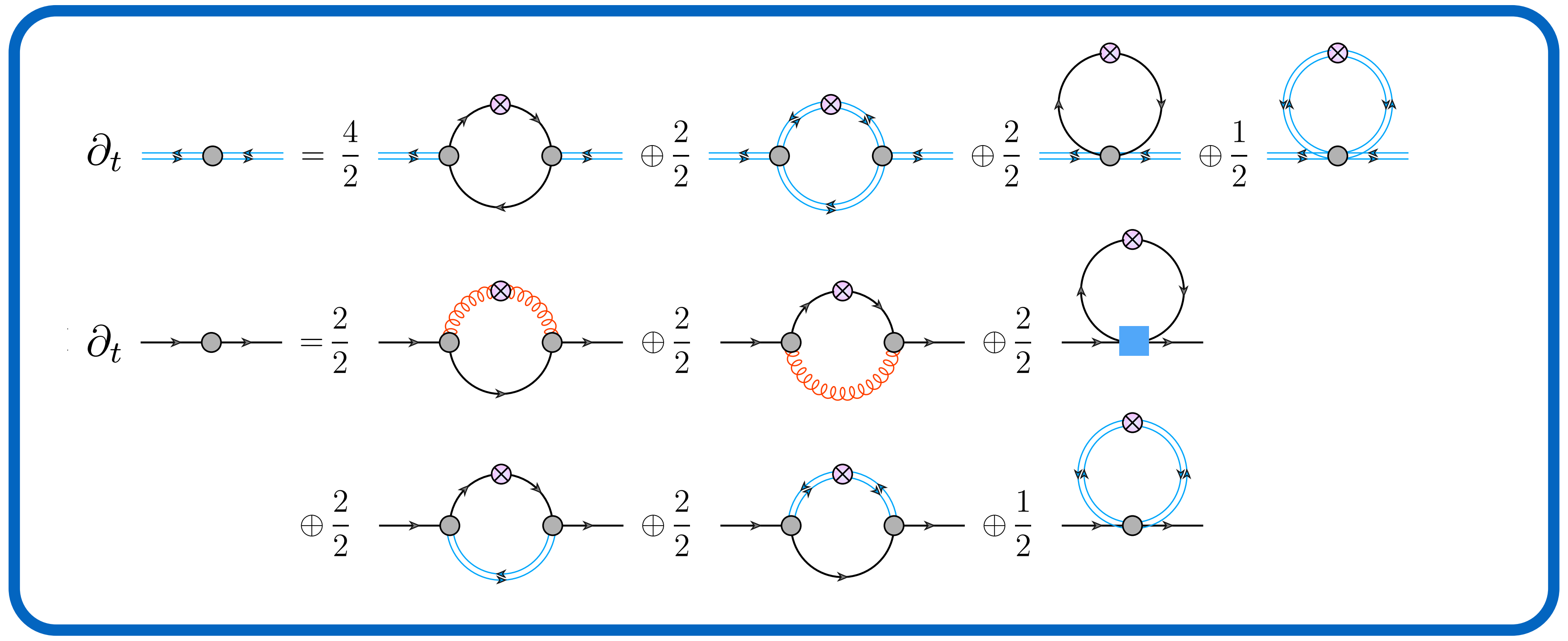
$$\left(\partial_t + \int_x \dot{\Phi}\frac{\delta}{\delta\Phi}\right)\Gamma_k[\Phi] =$$



‘DynHad for mesons & diquarks is BSE-DSE for QCD in a ‘unified’ effective action approach’

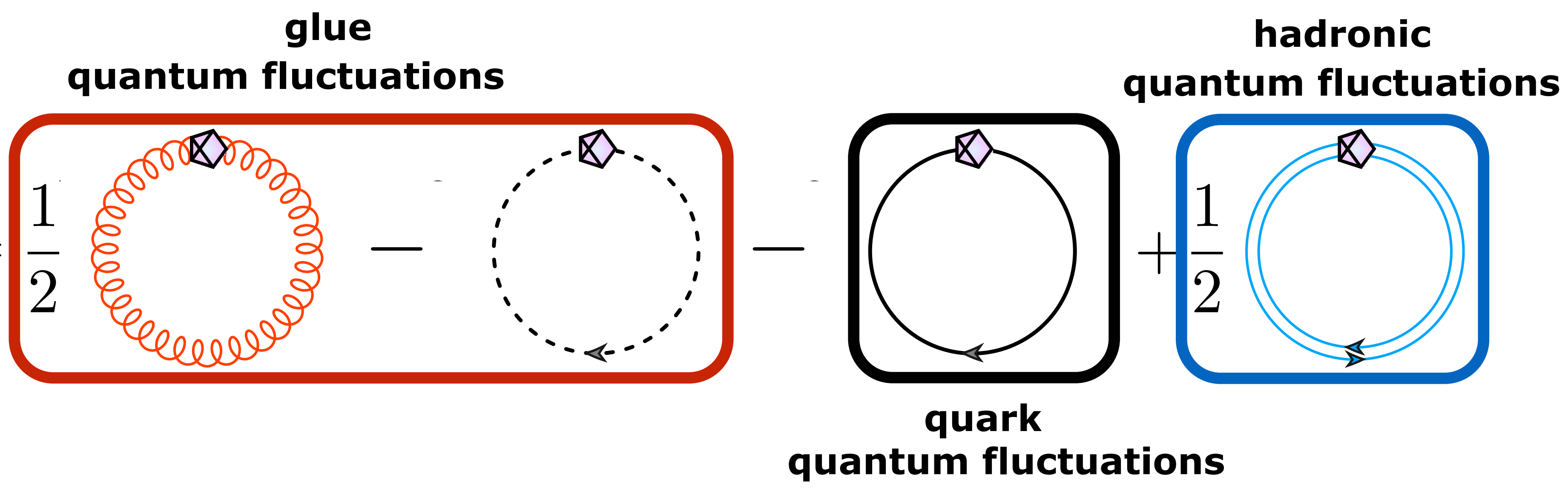
Dynamical hadronisation





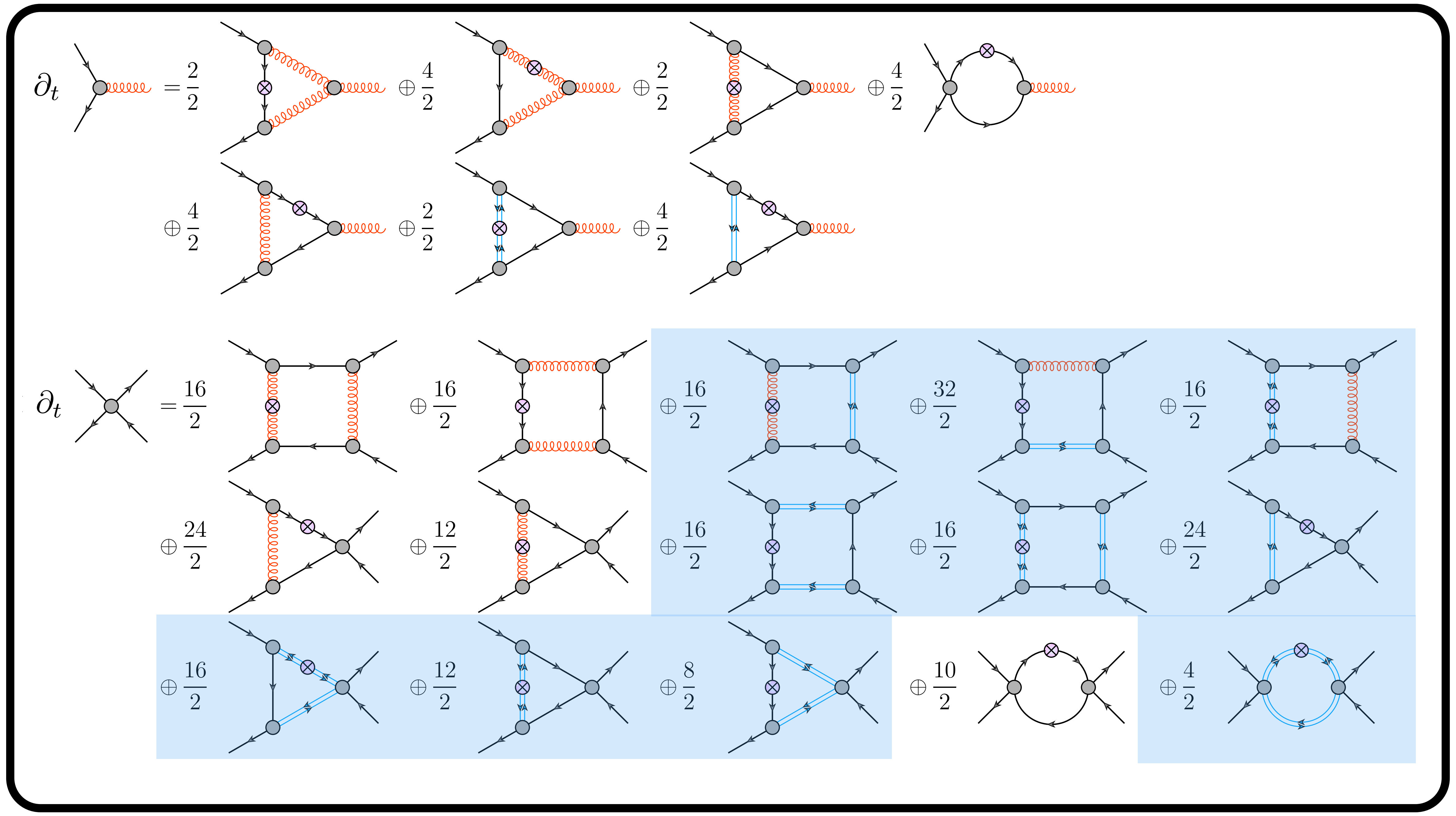
Functional flows for QCD

functional RG: $\left(\partial_t + \int_x \dot{\Phi}\,\frac{\delta}{\delta\Phi}\right)\Gamma_k[\Phi] =$

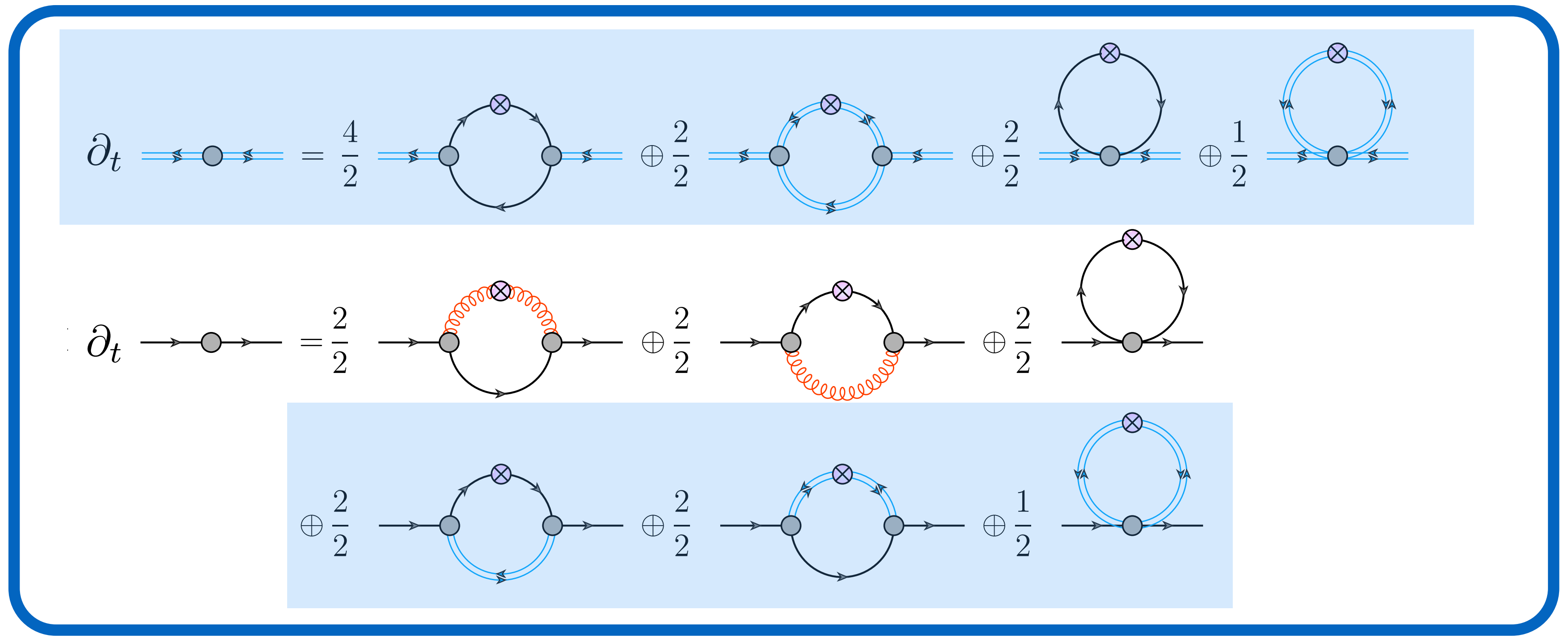


Correlation functions

gluon propagator	quark propagator	quark-gluon vertex	quark—anti-quark scattering
$\langle A_\mu A_\nu\rangle(p)$	$\langle q\bar{q}\rangle(p)$	$\langle q\bar{q}A_\mu\rangle(p_1,p_2)$ Eight tranverse tensor structures	$\langle q\bar{q}q\bar{q}\rangle(p_1,p_2,p_3)$



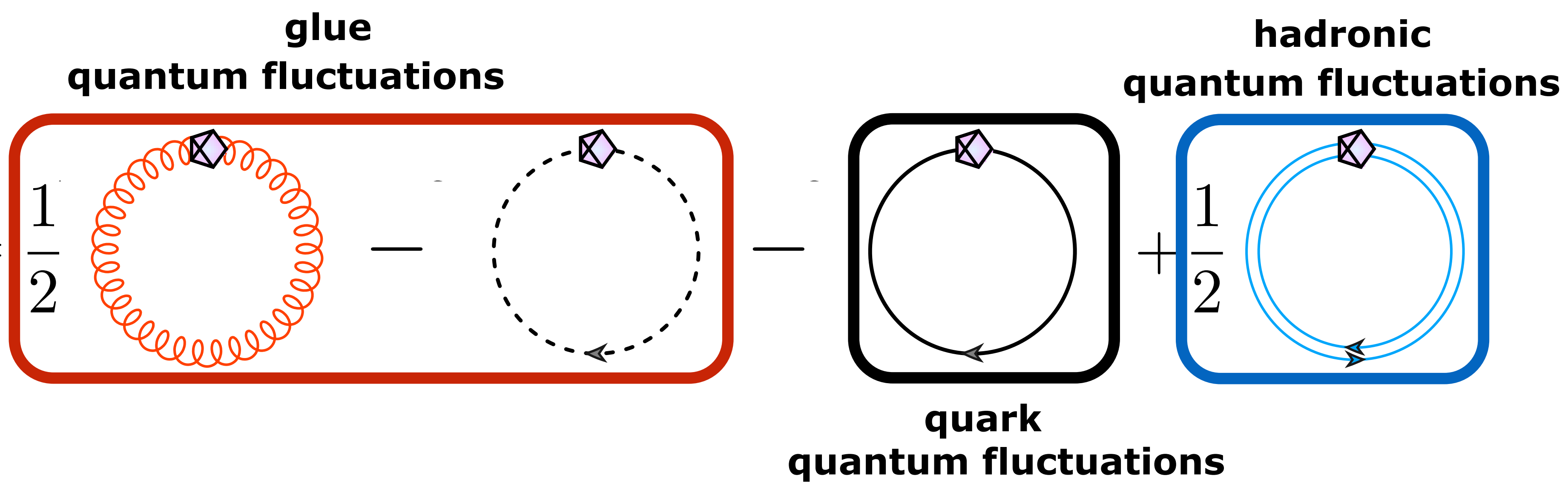
Dynamical hadronisation



Dynamical hadronisation: mesons & diquarks

Implementation:

functional RG: $$\left(\partial_t + \int_x \dot{\Phi}\frac{\delta}{\delta\Phi}\right)\Gamma_k[\Phi] =$$



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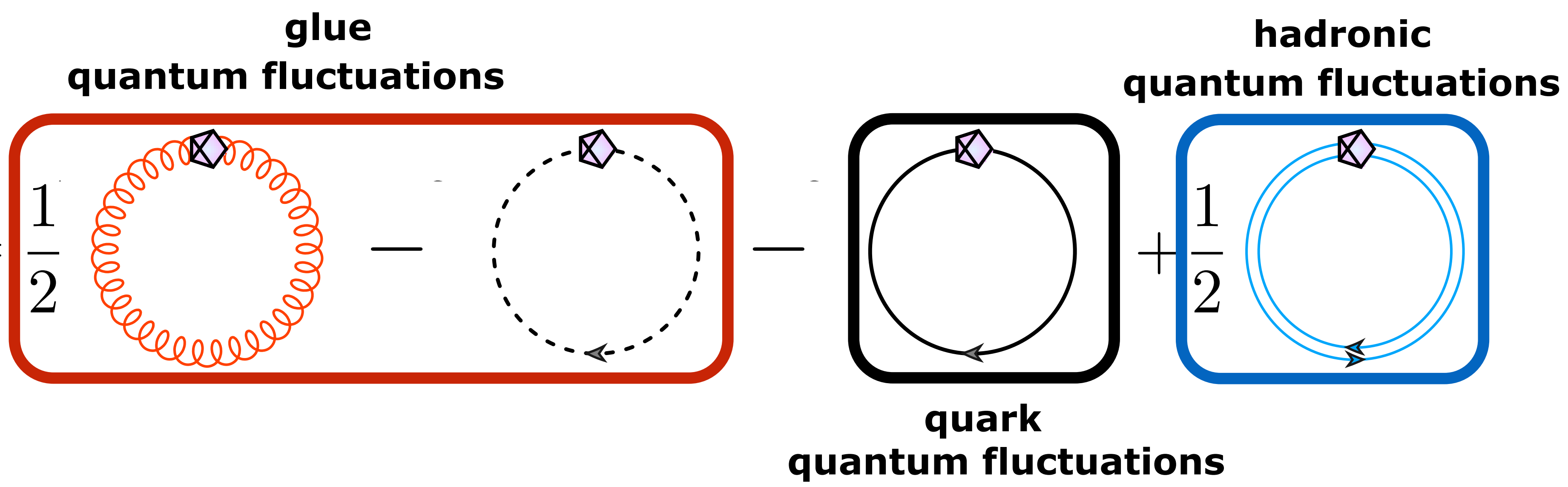


Consider path integral in the presence of sources for composite operators

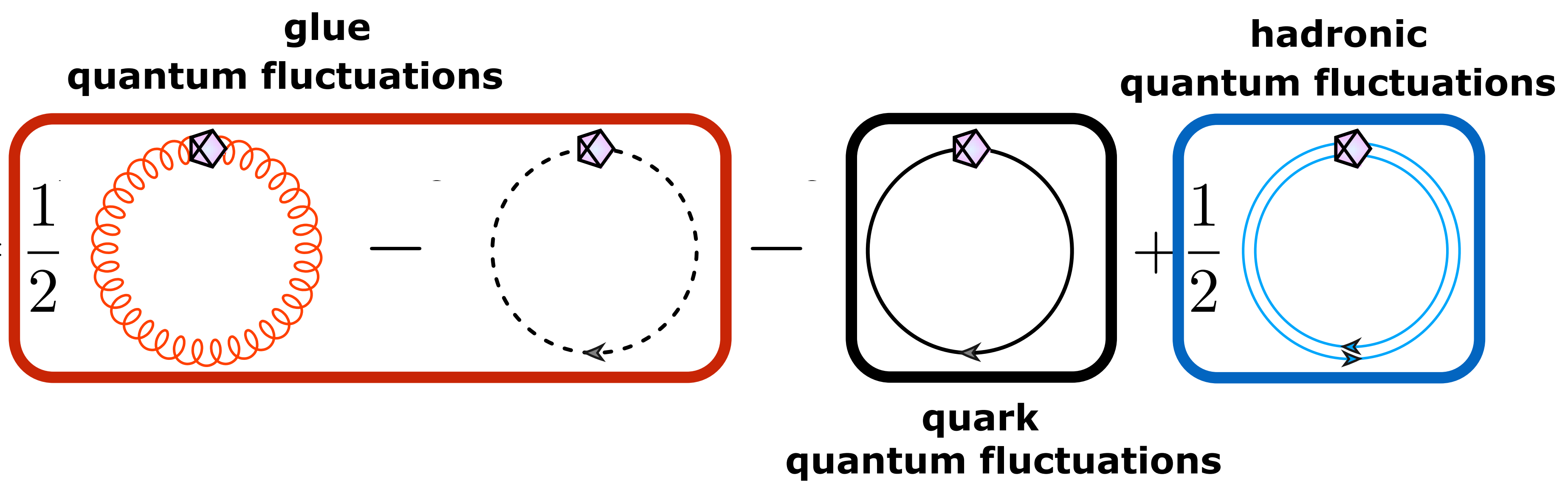
JMP, AP 322 (2007) 2831-2915

$$Z[J_q, J_{\bar{q}}, J_{\mathcal{O}}] = \int dq d\bar{q}\, e^{-S[q,\bar{q}] + \int J_q q - \bar{q} J_{\bar{q}} + \int J_{\mathcal{O}}\, \mathcal{O}[q,\bar{q}]}$$

Dynamical hadronisation: mesons & diquarks

Implementation:

functional RG: $\left(\partial_t + \int_x \dot{\Phi} \frac{\delta}{\delta \Phi}\right) \Gamma_k[\Phi] =$



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Choose scale-dependent $\mathcal{O}_k[q,\bar{q}]$ ‘to optimise dynamics’!

$$\partial_t \Gamma_k[A_\mu, q, \bar{q}] \longrightarrow \partial_t \Gamma_k[\Phi] + \partial_t \mathcal{O}_k^{(i)}[\Phi] \frac{\delta \Gamma_k}{\delta \Phi_i}$$

$$\Phi = (A_\mu, q, \bar{q}, \langle \mathcal{O}^{(1)} \rangle, ...)$$

Dynamical hadronisation: mesons & diquarks

Implementation:

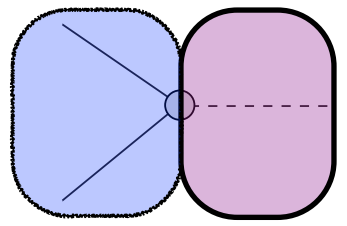
2001 - : Braun, Flörchinger, Fu Gies, JMP, Rennecke, Wetterich, …

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

$$\lambda_\psi=\frac{h^2}{m_\phi^2}$$

$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$



Hubbard-Stratonovich

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Common choices

$$T^i=(1\,,\,\gamma_5\vec\sigma)$$

Scalar-pseudoscalar channel

Dynamical hadronisation: mesons & diquarks

Implementation:

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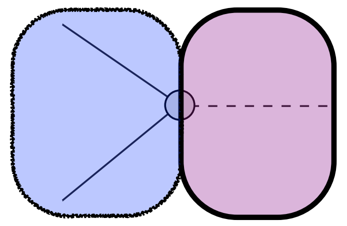

$$\Phi=(\sigma,\vec\pi)$$

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Common choices

$$T^i=(1\,,\,\gamma_5\vec\sigma)$$

Scalar-pseudoscalar channel

$$T^i=\gamma_0$$

Density channel
(part of vector multiplet)

Dynamical hadronisation: mesons & diquarks

Implementation:

2001 - : Braun, Flörchinger, Fu Gies, JMP, Rennecke, Wetterich, …

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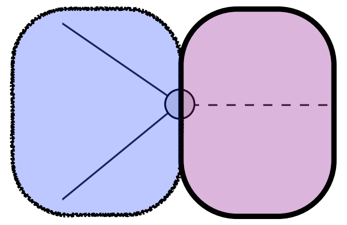

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JMP, AP 322 (2007) 2831-2915

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Common choices

$$T^i=(1\,,\,\gamma_5\vec\sigma)$$

Scalar-pseudoscalar channel

$$T^i=(\gamma_0\,,\,\vec\gamma)$$

Density channel
(part of vector multiplet)

Dynamical hadronisation: mesons & diquarks

Implementation:

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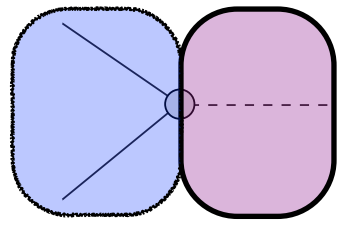
Hubbard-Stratonovich



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Scalar-pseudoscalar channel

$$T^i=(\gamma_0\,,\,\vec\gamma)$$

Density channel
(part of vector multiplet)

$$\psi\tilde T^i\psi$$

Diquark channels

Dynamical hadronisation: mesons & diquarks

Implementation:

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

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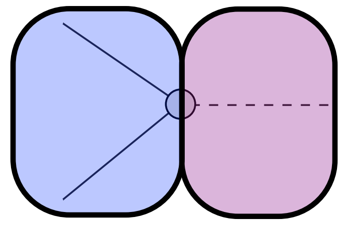

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Density channel

(part of vector multiplet)

$$\psi\tilde T^i\psi$$

Diquark channels

Complete basis

$$\mathrm{N_f}=2:10$$

$$\mathrm{N_f}=3:26$$

Momentum-independent tensor structures

Dynamical hadronisation: mesons & diquarks

Implementation:

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

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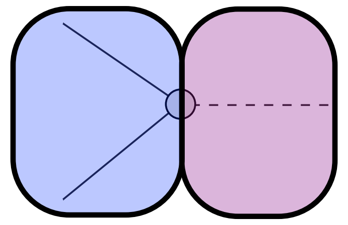

Hubbard-Stratonovich

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Diquark channels

Complete basis

$$\mathrm{N_f}=2:\mathbf{10}$$

$$\mathrm{N_f}=3:\mathbf{26}$$

Momentum-independent tensor structures

All tensor structures for $\mathrm{N_f}=2:$ **256**

Dynamical hadronisation: mesons & diquarks

Implementation:

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

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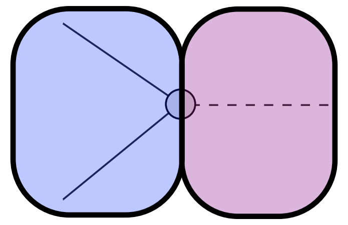


$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$

General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\Big|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,\ldots,n,\bar n,\ldots)$$

mesons **baryons**

How to fix ϕ_k & $\dot\phi_k$?

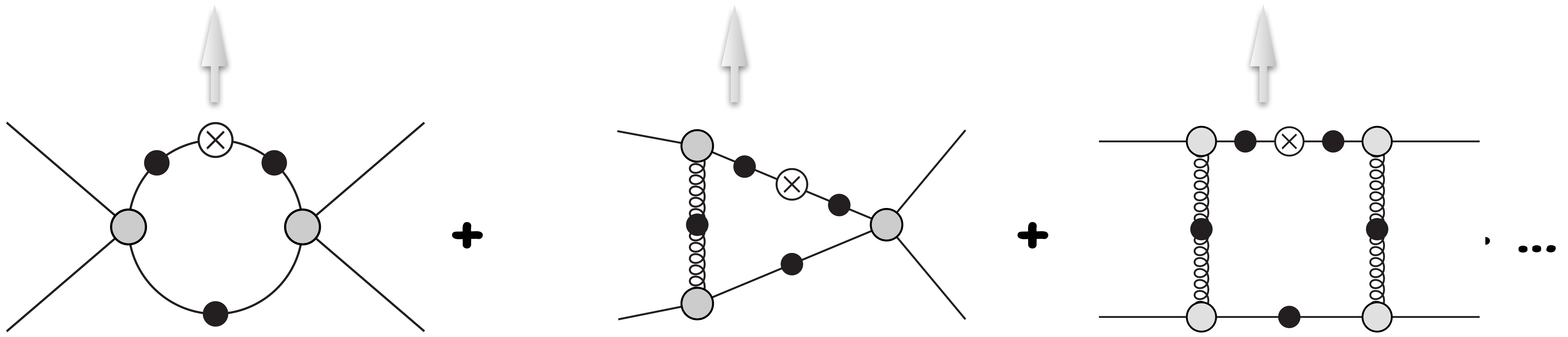
$$\dot\Phi_k\simeq\dot A_k\bar\psi\tau\psi+\dot B_k\Phi_k+\dot C_k$$

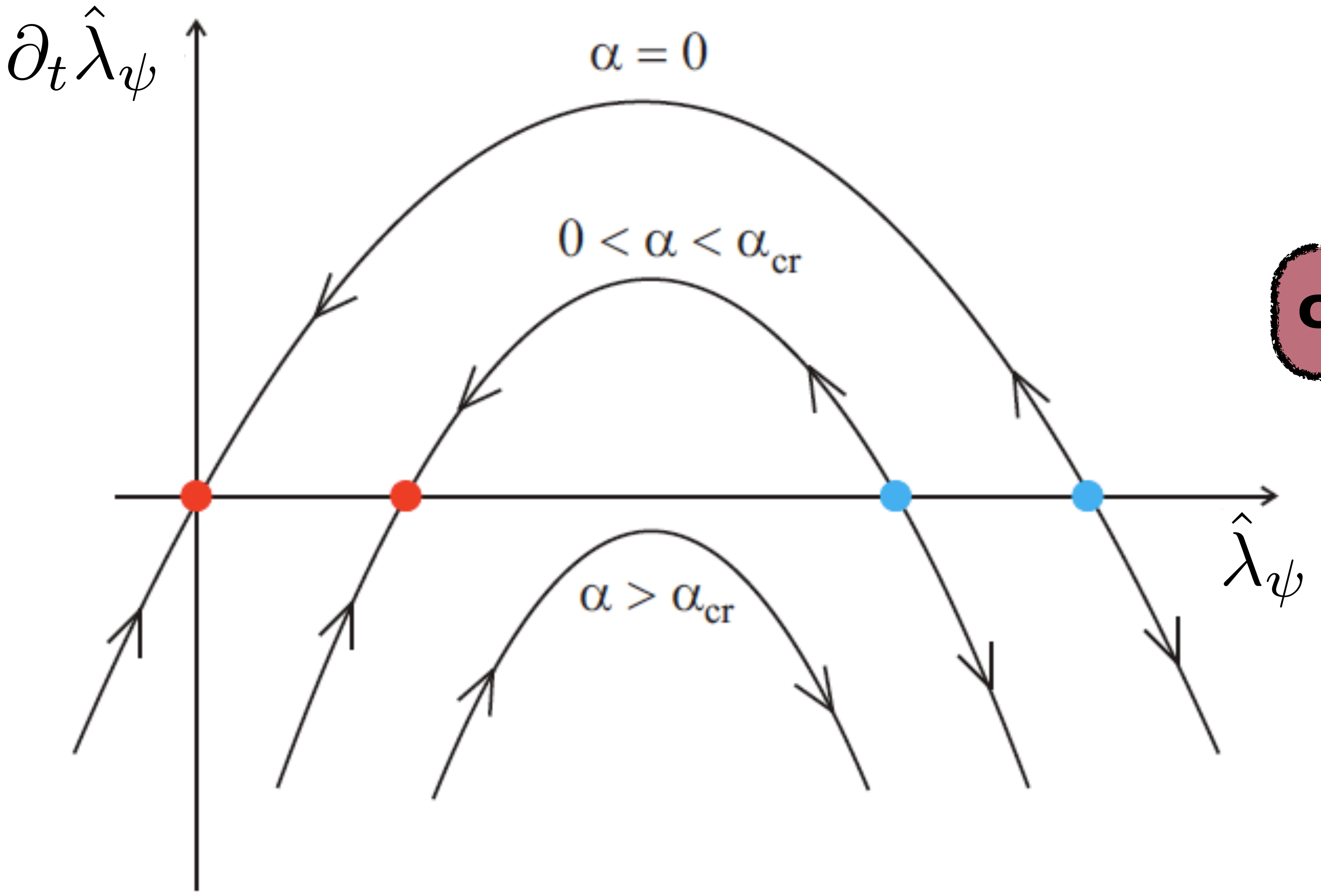
Dynamical hadronisation: mesons & diquarks

Implementation:

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ **with infrared scale** k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + A\left(\frac{T}{k}\right)\hat{\lambda}_\psi^2 + B\left(\frac{T}{k}\right)\hat{\lambda}_\psi\,\alpha_s + C\left(\frac{T}{k}\right)\alpha_s^2 + \ldots$$





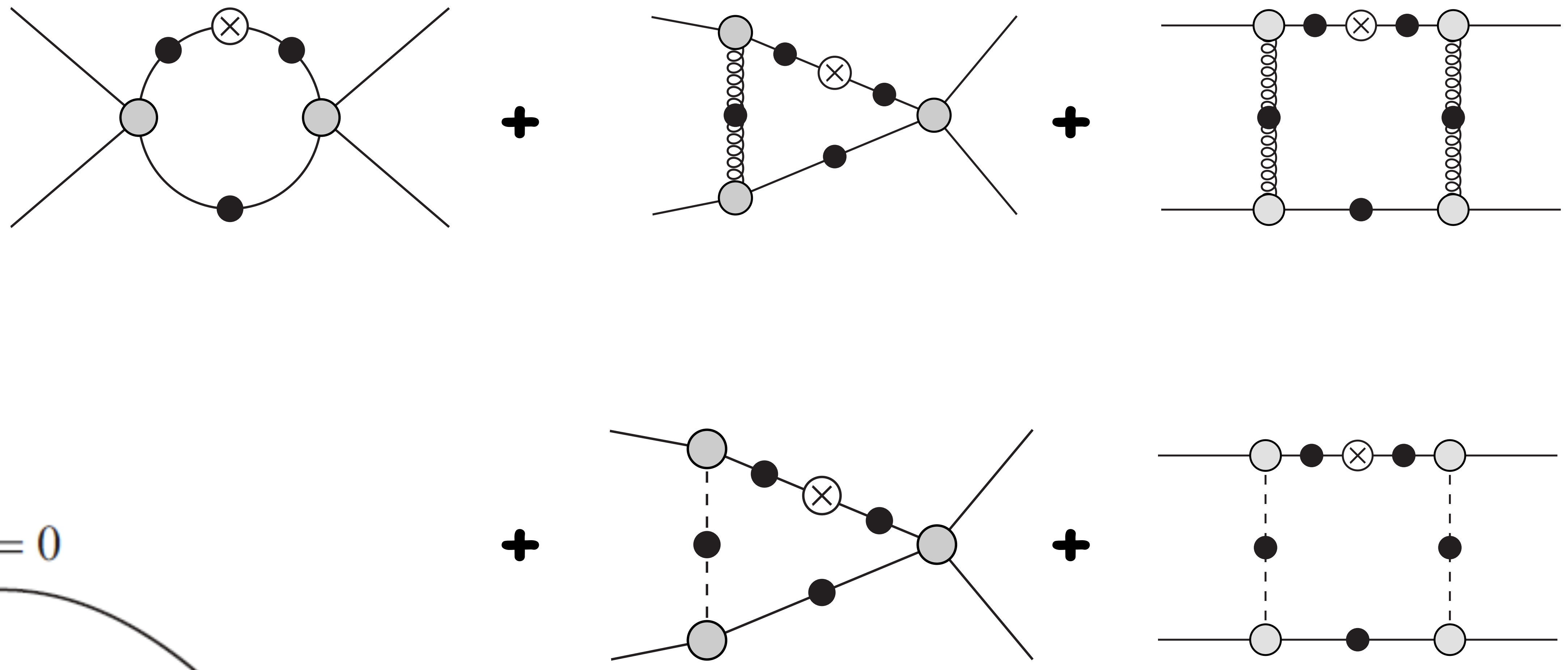
chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

Dynamical hadronisation: mesons & diquarks

Implementation:

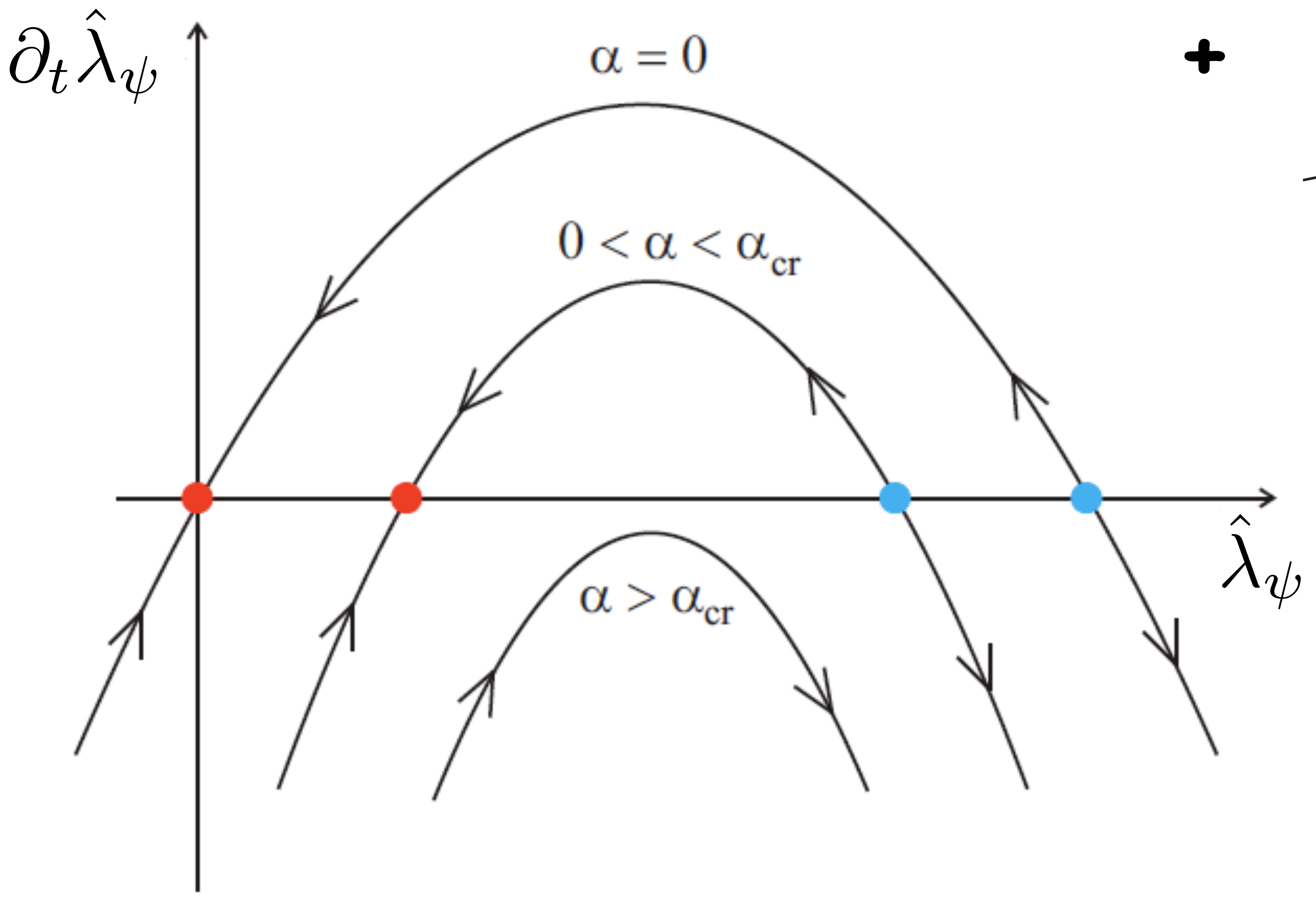
Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ **with infrared scale** k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi +$$



$+ \ \partial_t \Phi$ - terms

+ ...

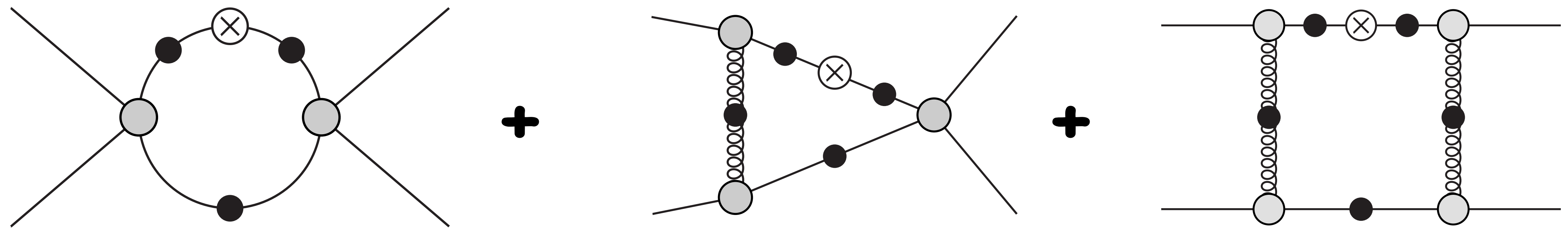


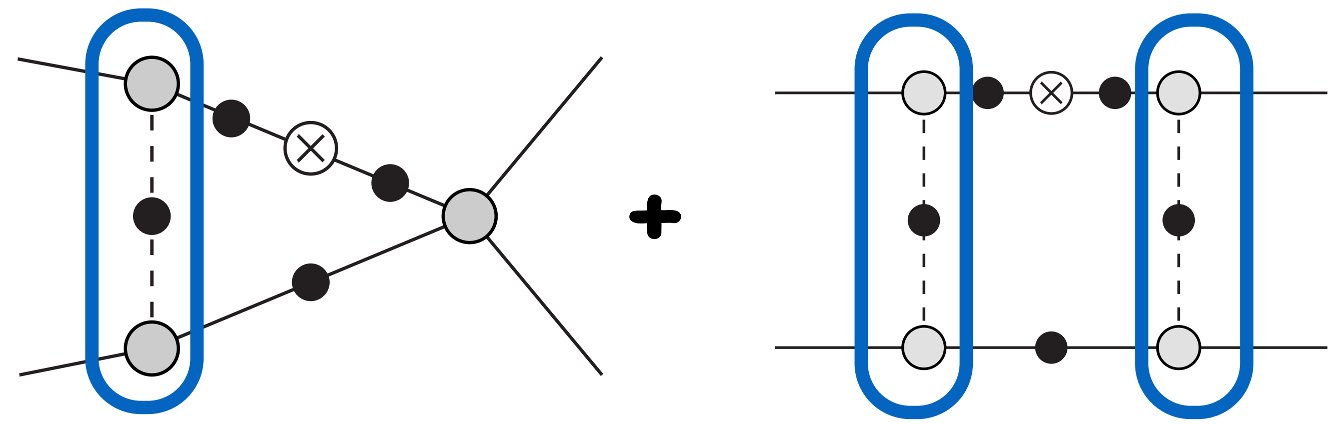
Dynamical hadronisation: mesons & diquarks

Implementation:

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ **with infrared scale** k

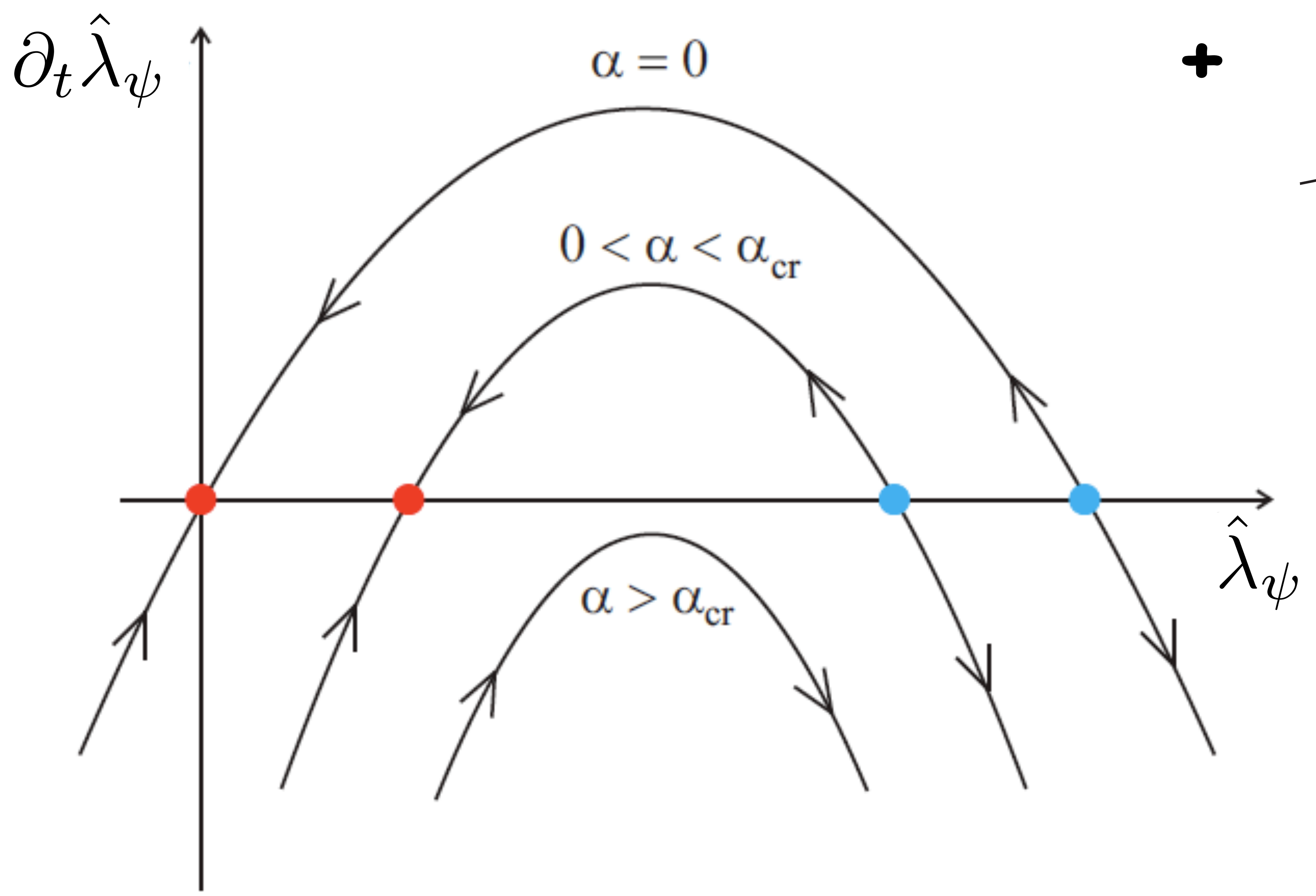
$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi +$$



\+ 

\+ $\partial_t \Phi$ - terms

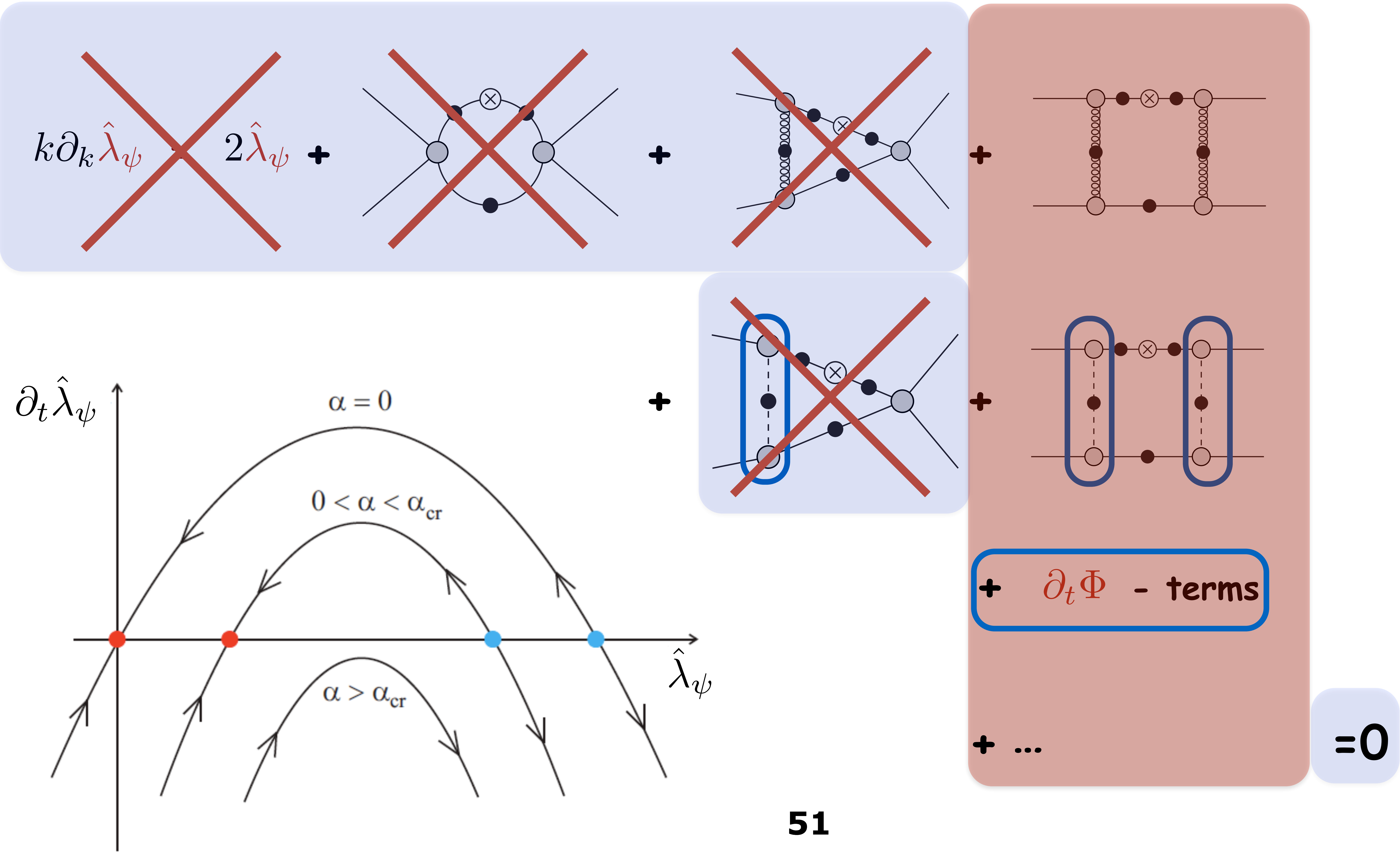
\+ ...



Dynamical hadronisation: mesons & diquarks

Implementation:

Full bosonisation $\hat{\lambda}_\psi = 0$



Dynamical hadronisation: mesons & diquarks

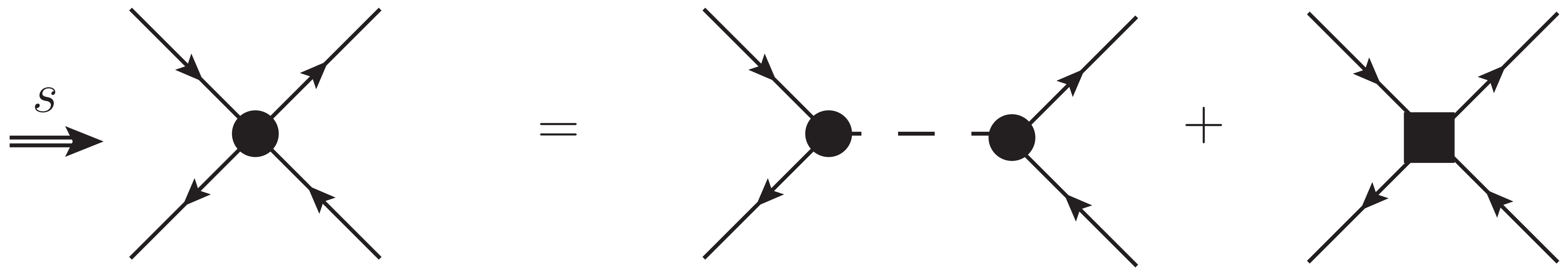
! Reminder !

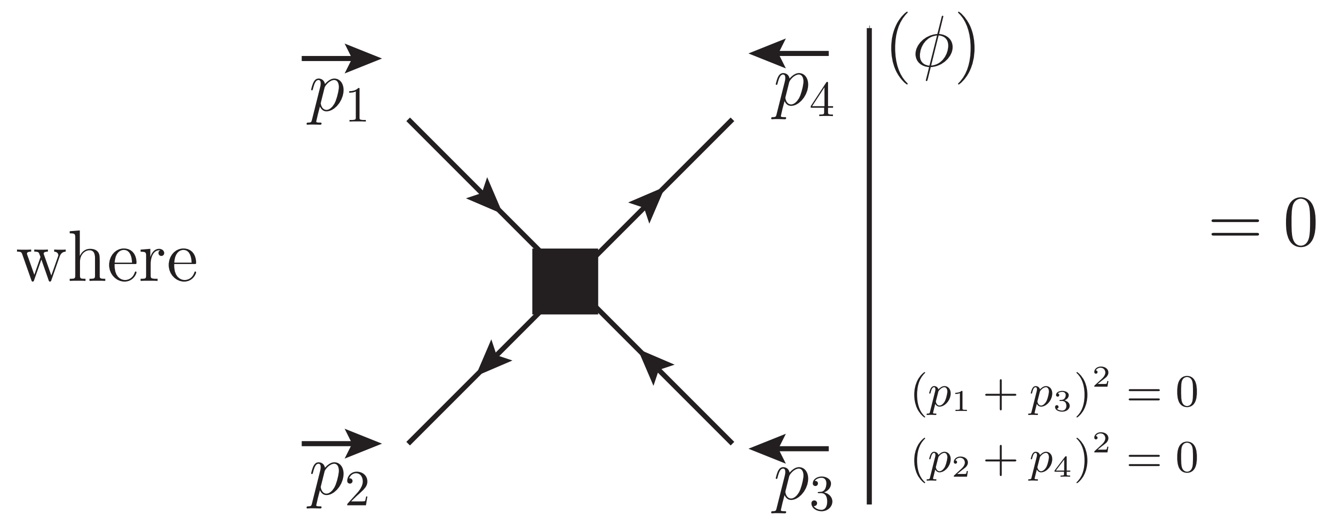
Full bosonisation $\hat{\lambda}_\psi = 0$ **Really?**

Dynamical hadronisation: mesons & diquarks

! Reminder !

Full bosonisation $\hat{\lambda}_\psi = 0$ **Really?**





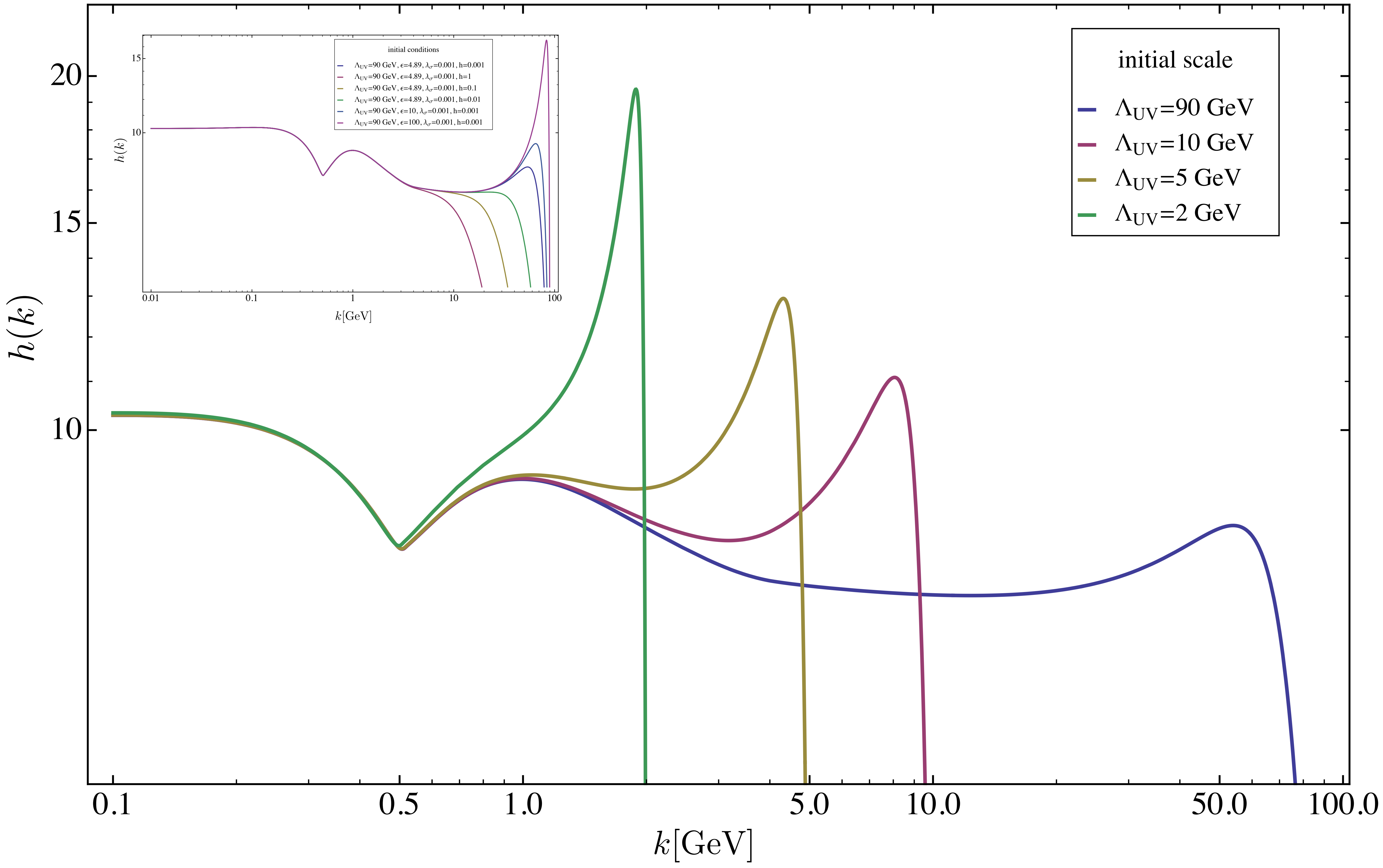
(i) Complete dynamical hadronisation of one tensor channel removes one momentum channel!

(ii) Residual four-quark vertex left!

Dynamical hadronisation at work

Dynamical hadronisation

Stability & decoupling

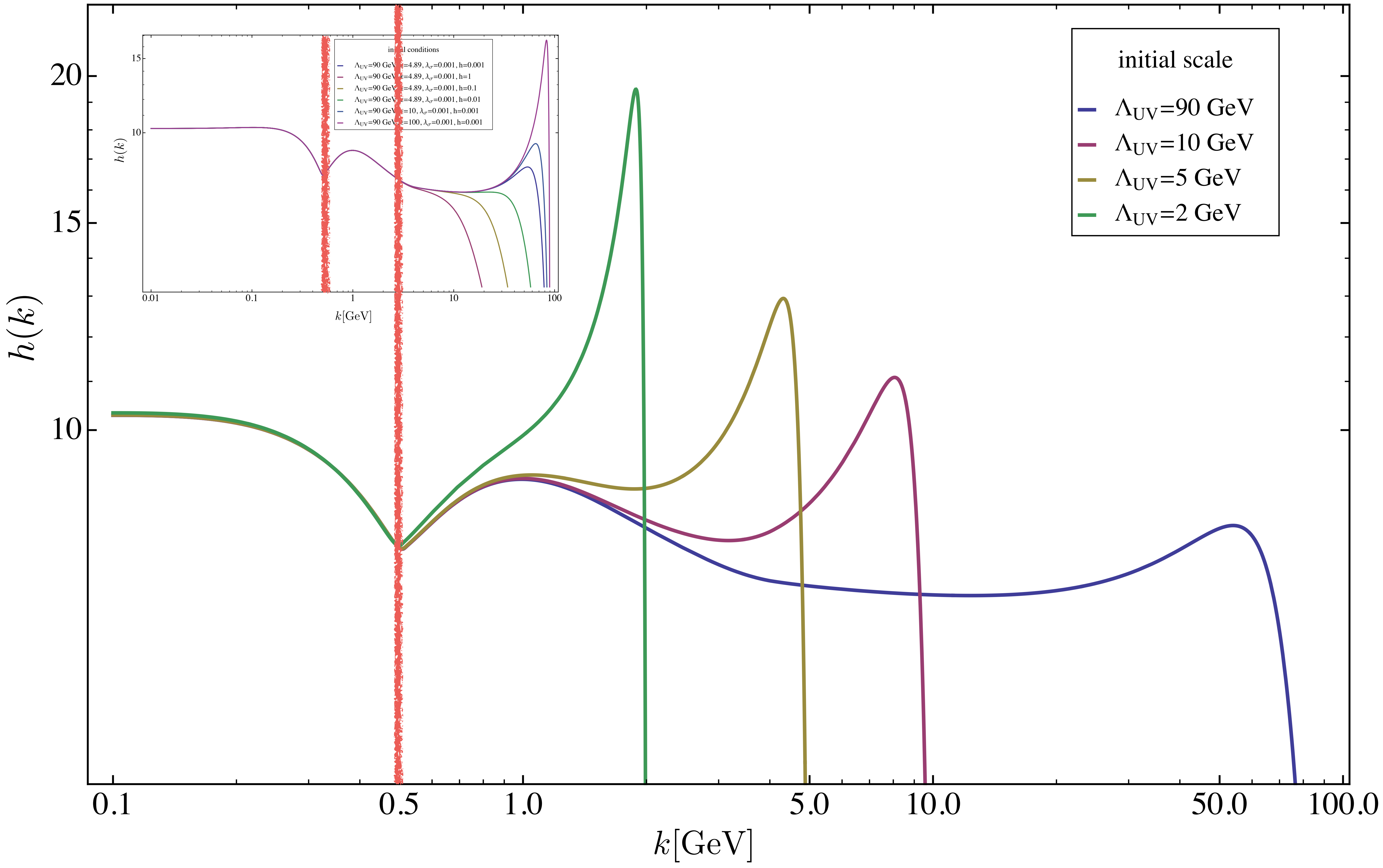


Braun, Fister, JMP, Rennecke, PRD 94 (2016) 034016

Dynamical hadronisation

Cutoff scale of dynamical chiral symmetry breaking

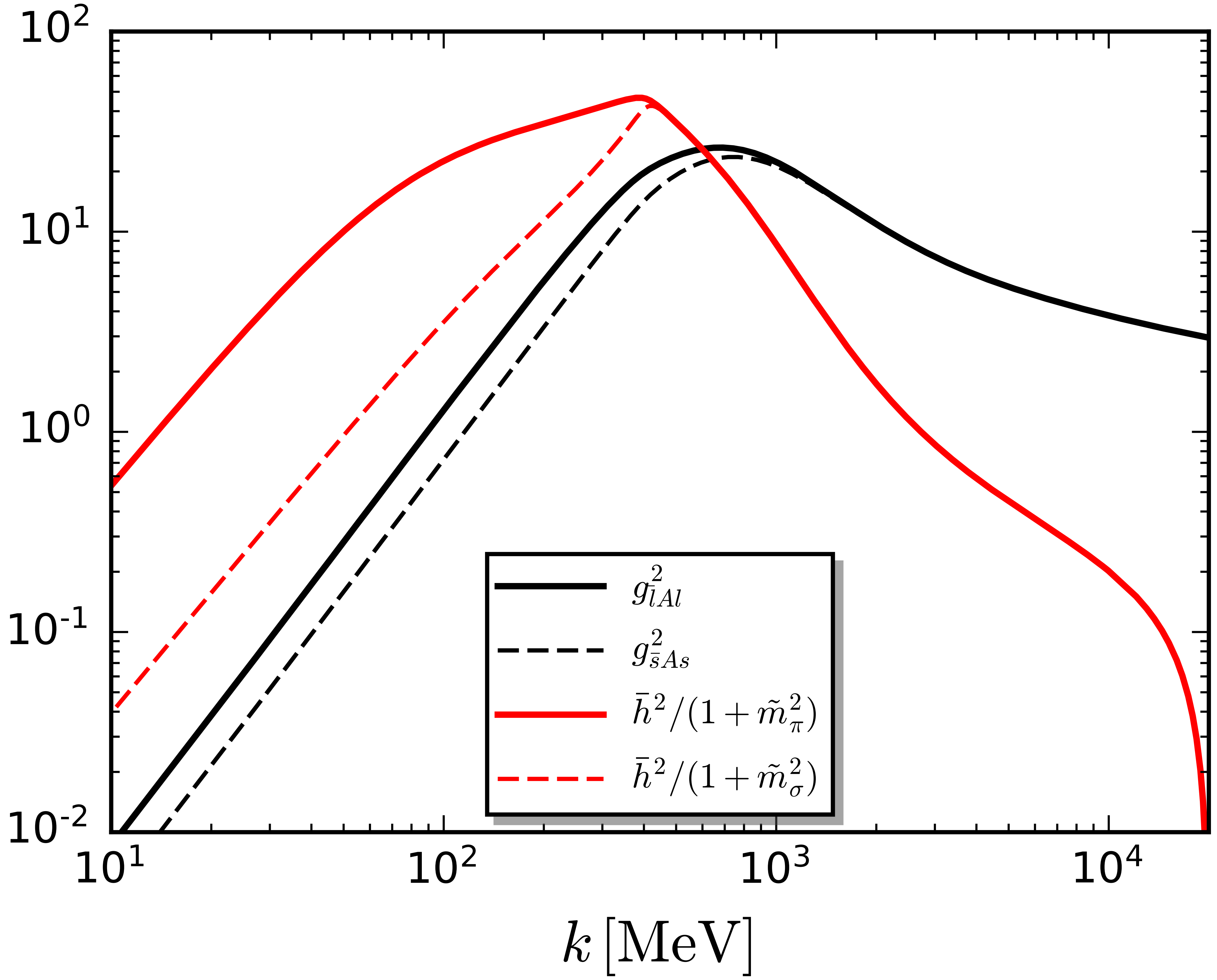
Stability & decoupling



Braun, Fister, JMP, Rennecke, PRD 94 (2016) 034016

Dynamical hadronisation

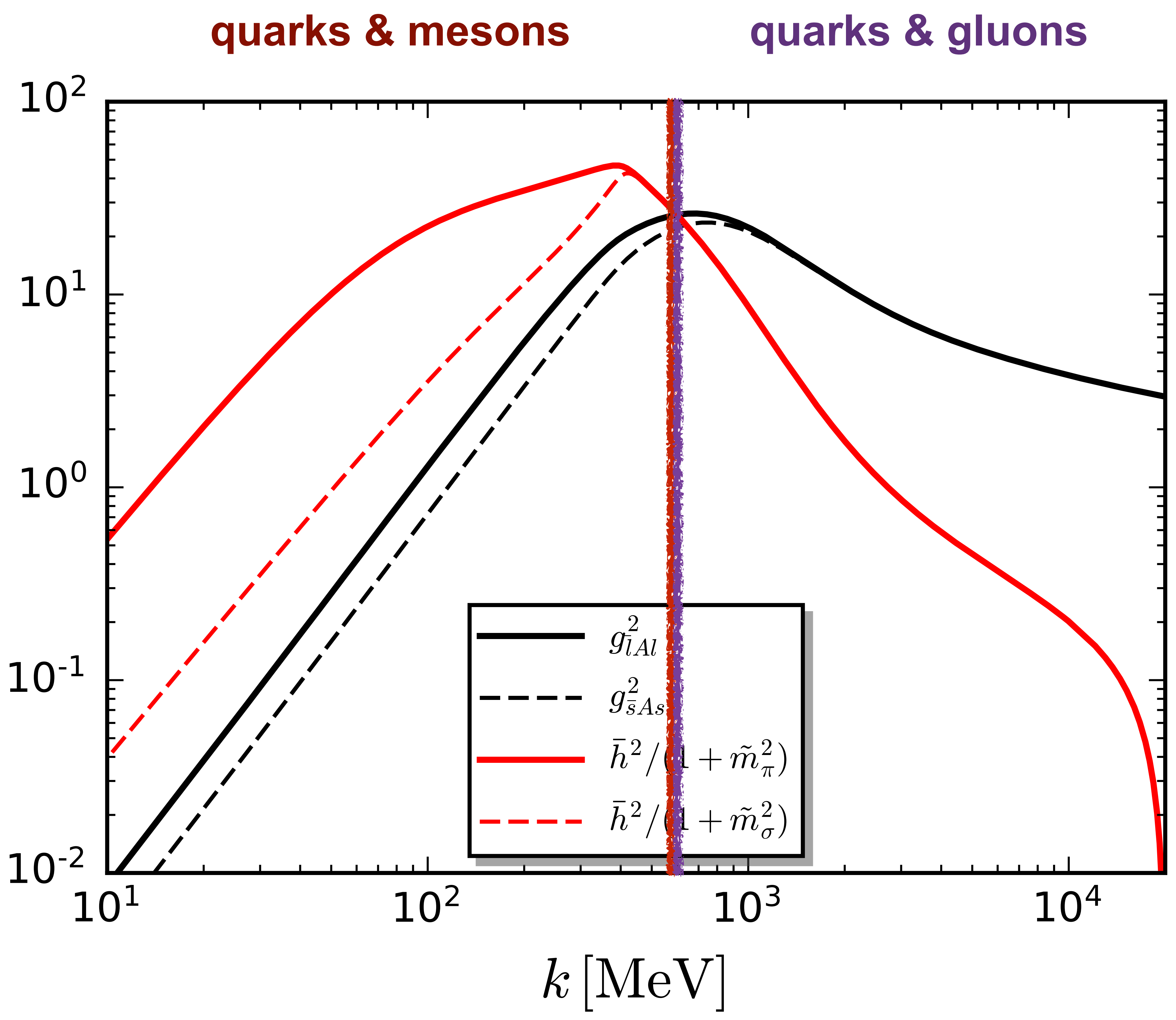
Stability & decoupling



Fu, JMP, Rennecke, PRD 101, (2020) 054032

Dynamical hadronisation

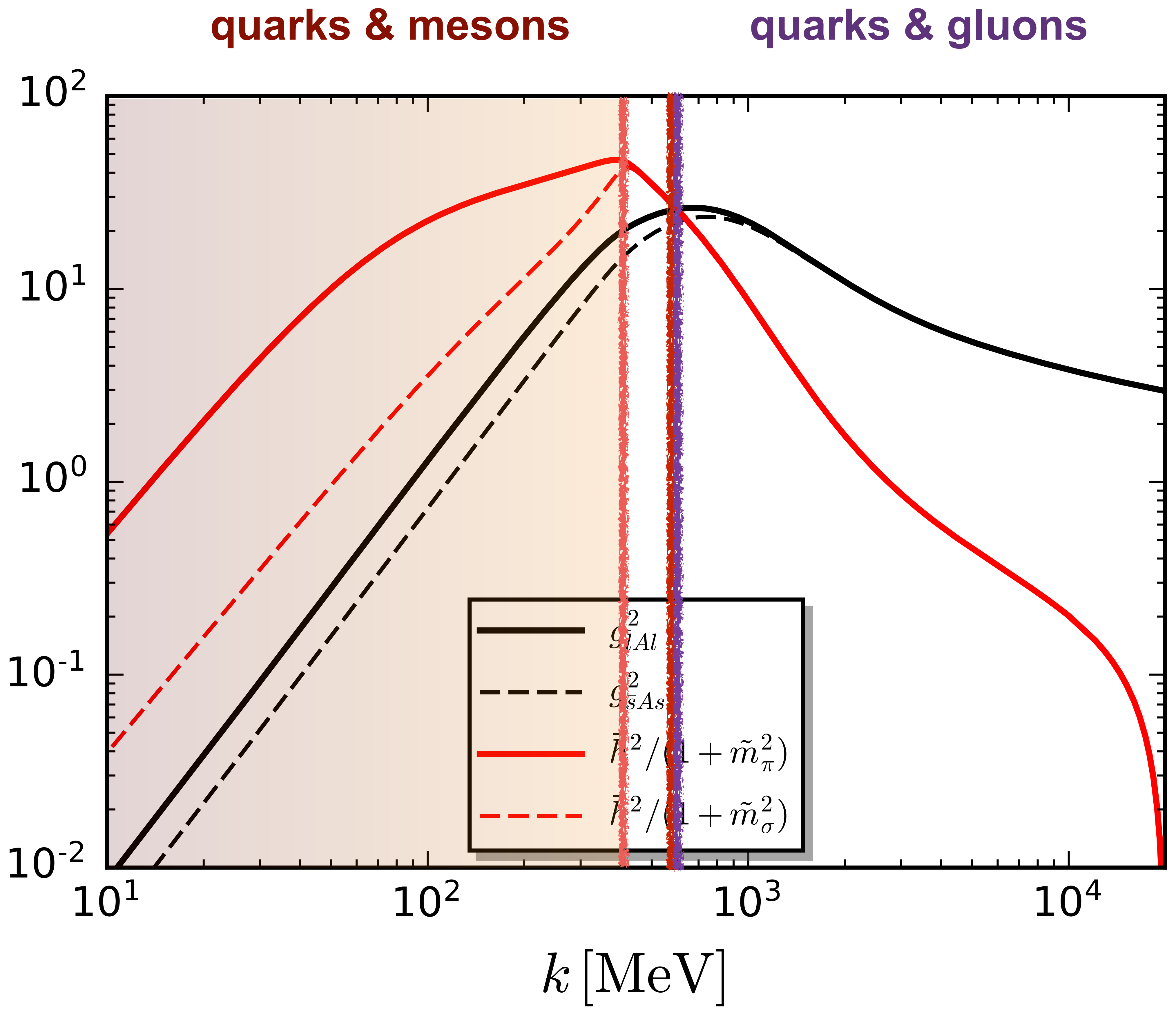
Stability & decoupling



Fu, JMP, Rennecke, PRD 101, (2020) 054032

Dynamical hadronisation

Stability & decoupling



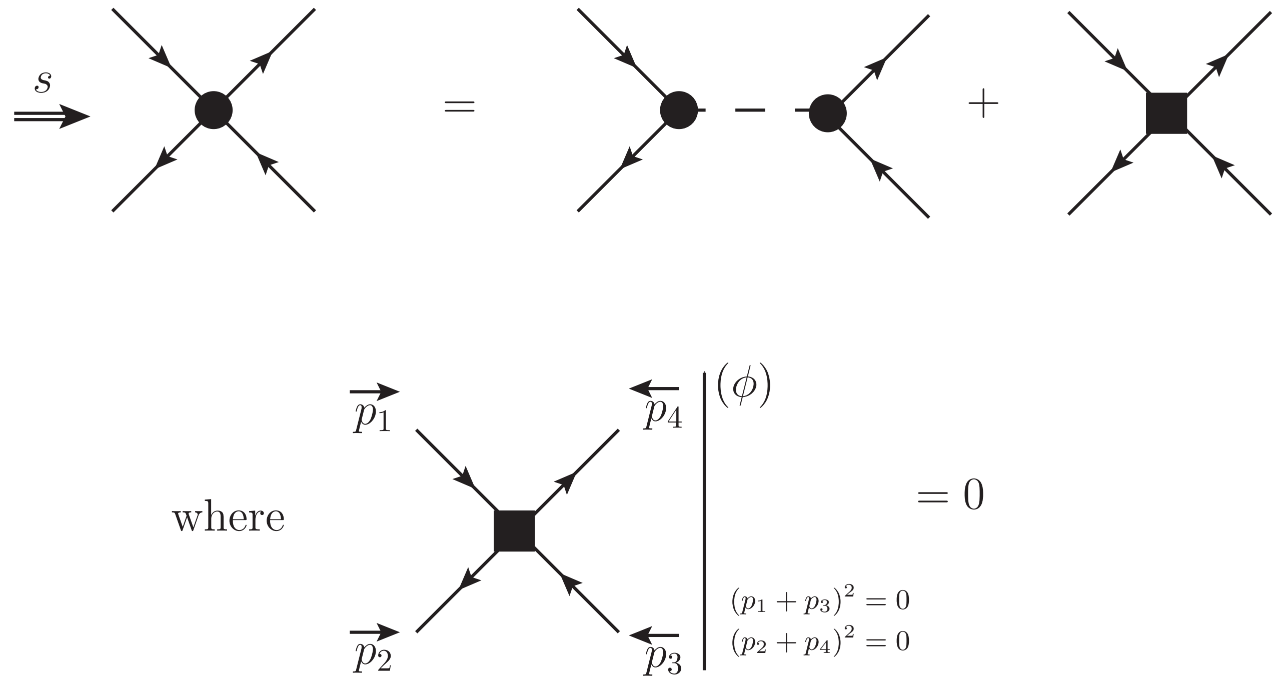
Pions: Chiral perturbation theory

Fu, JMP, Rennecke, PRD 101, (2020) 054032

Dynamical hadronisation at work

Mesons & diquarks:

(i)

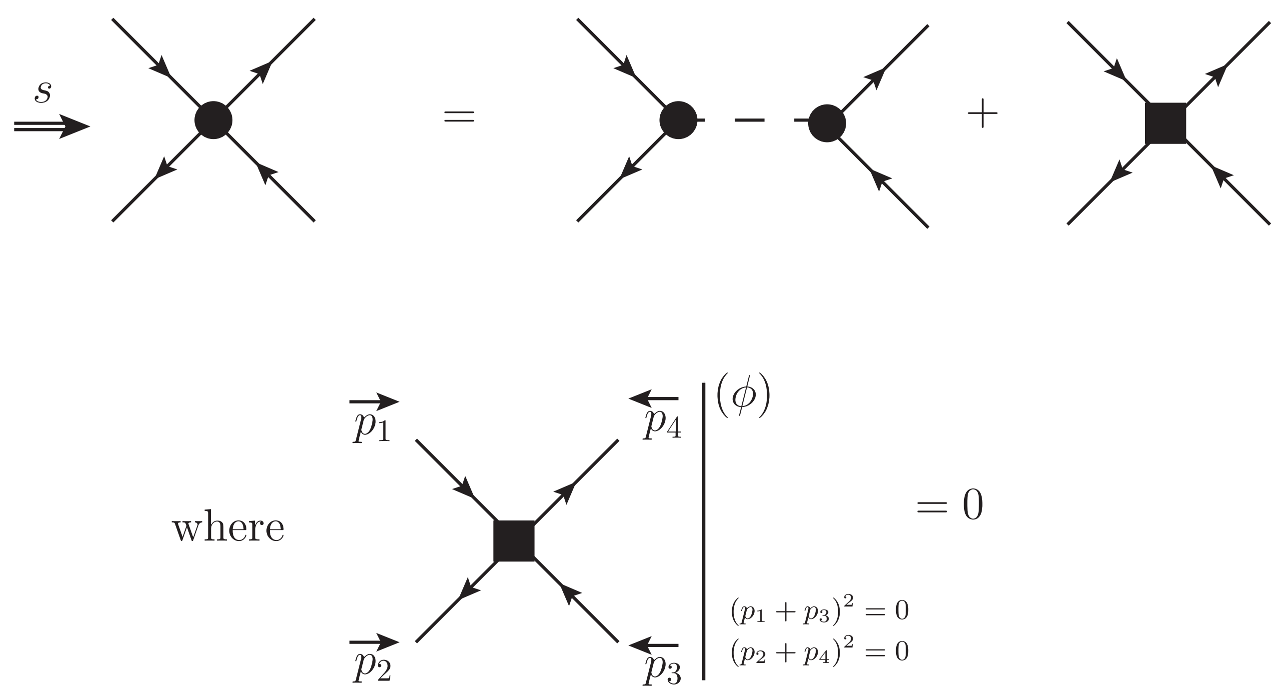


Fukushima, JMP, Strodthoff, 2103.01129

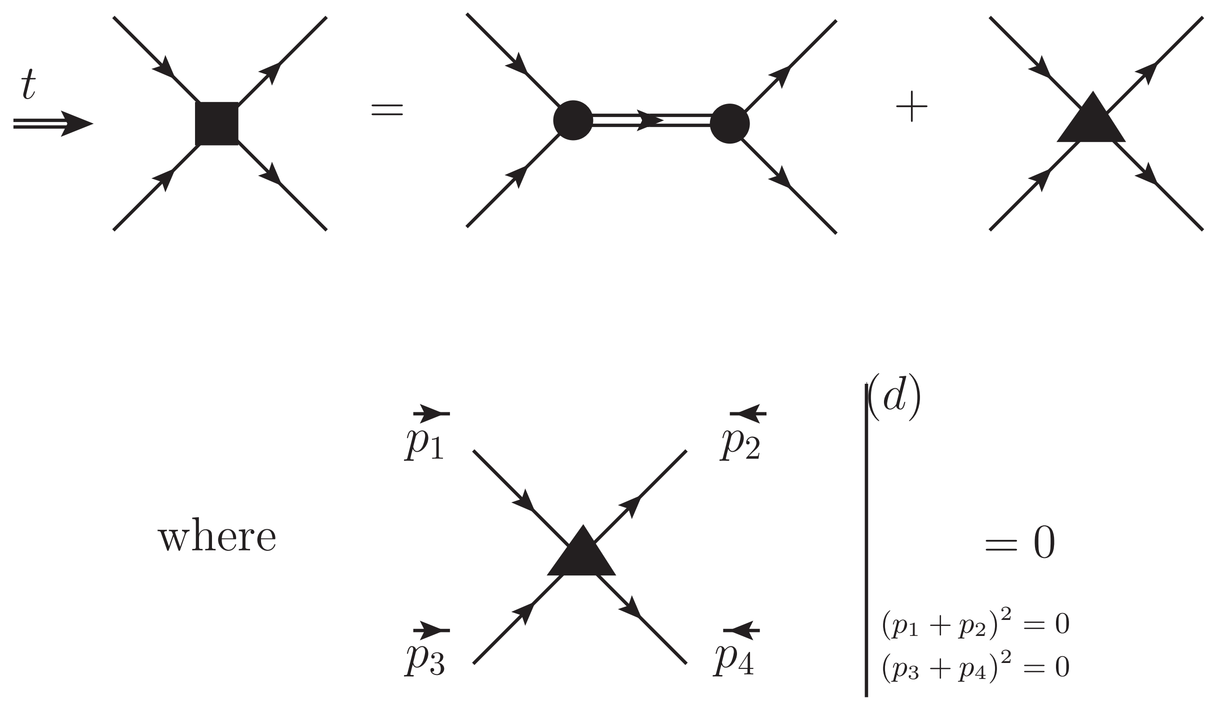
Dynamical hadronisation at work

Mesons & diquarks:

(i)



(ii)

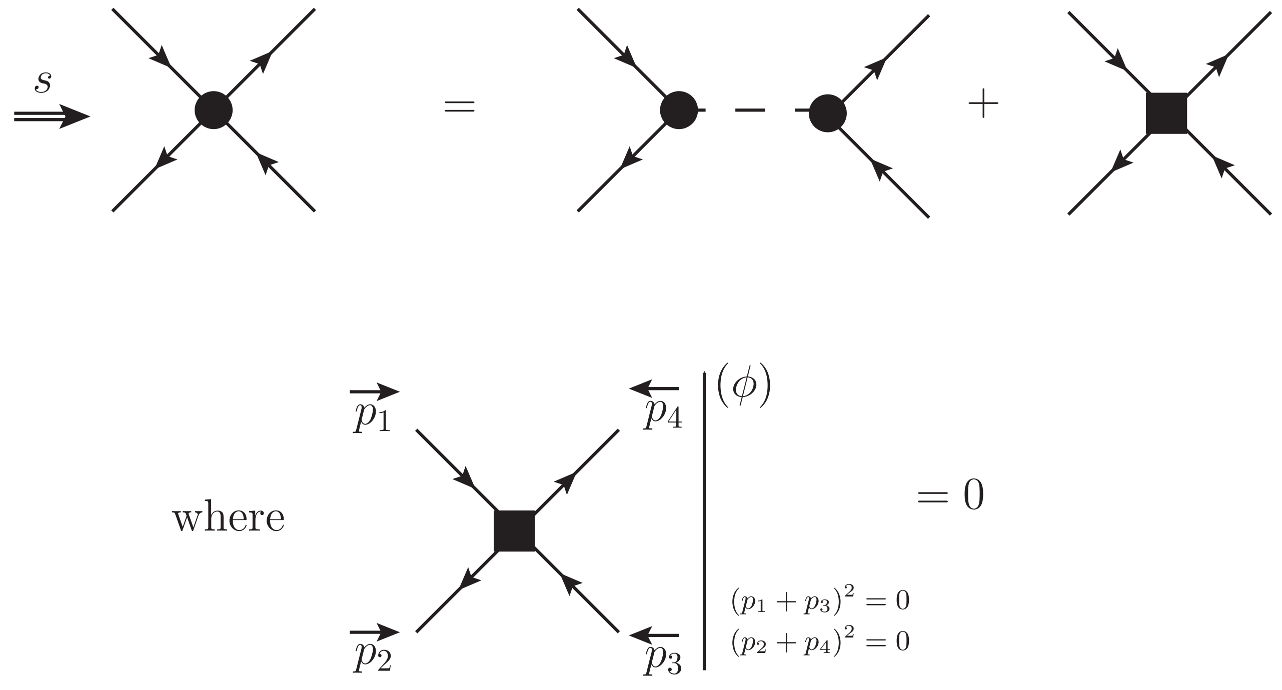


Fukushima, JMP, Strodthoff, 2103.01129

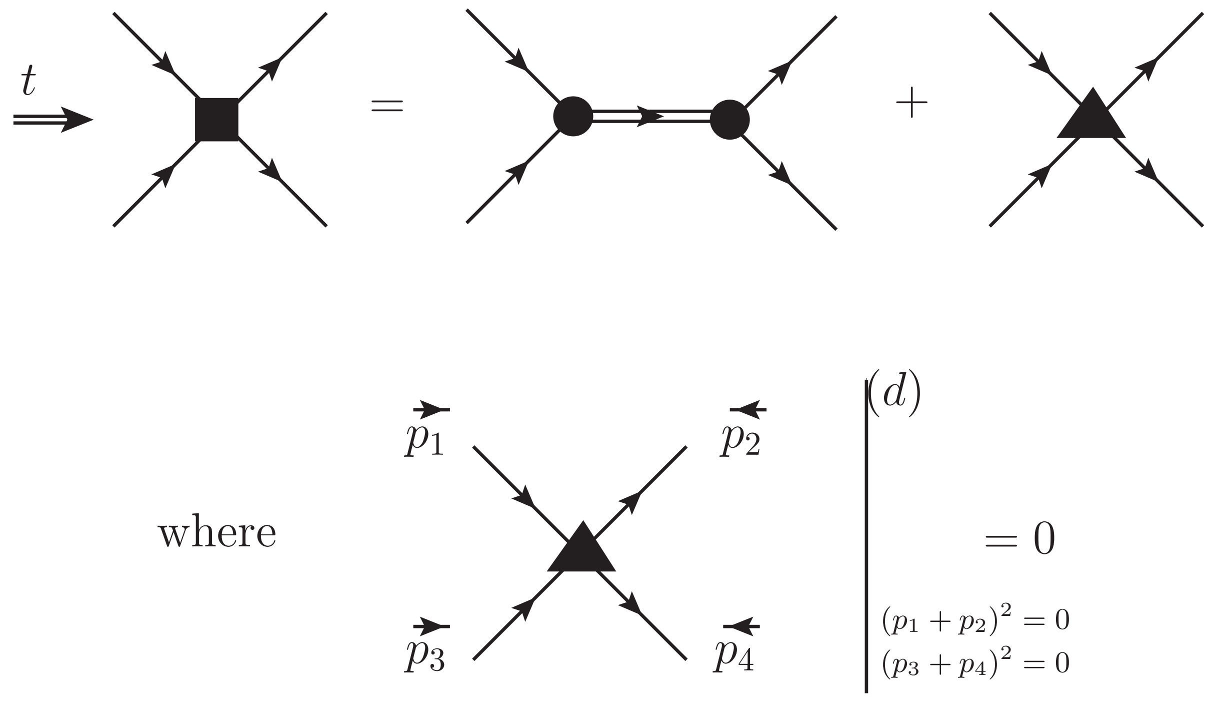
Dynamical hadronisation at work

Mesons & diquarks:

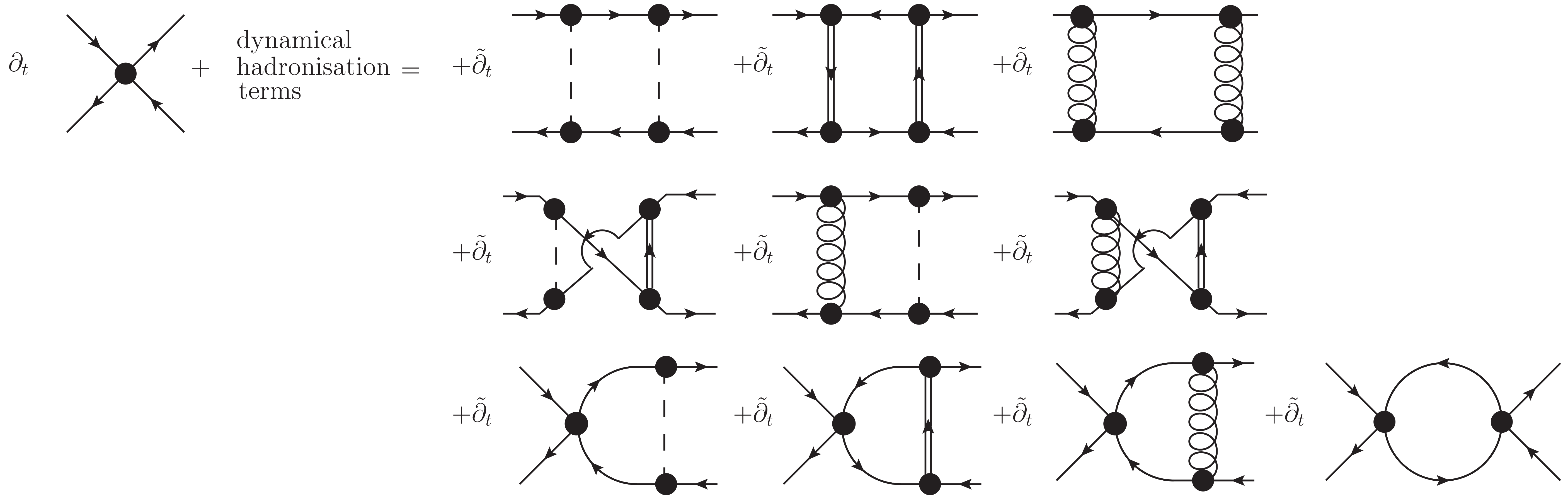
(i)



(ii)



Schematical flow

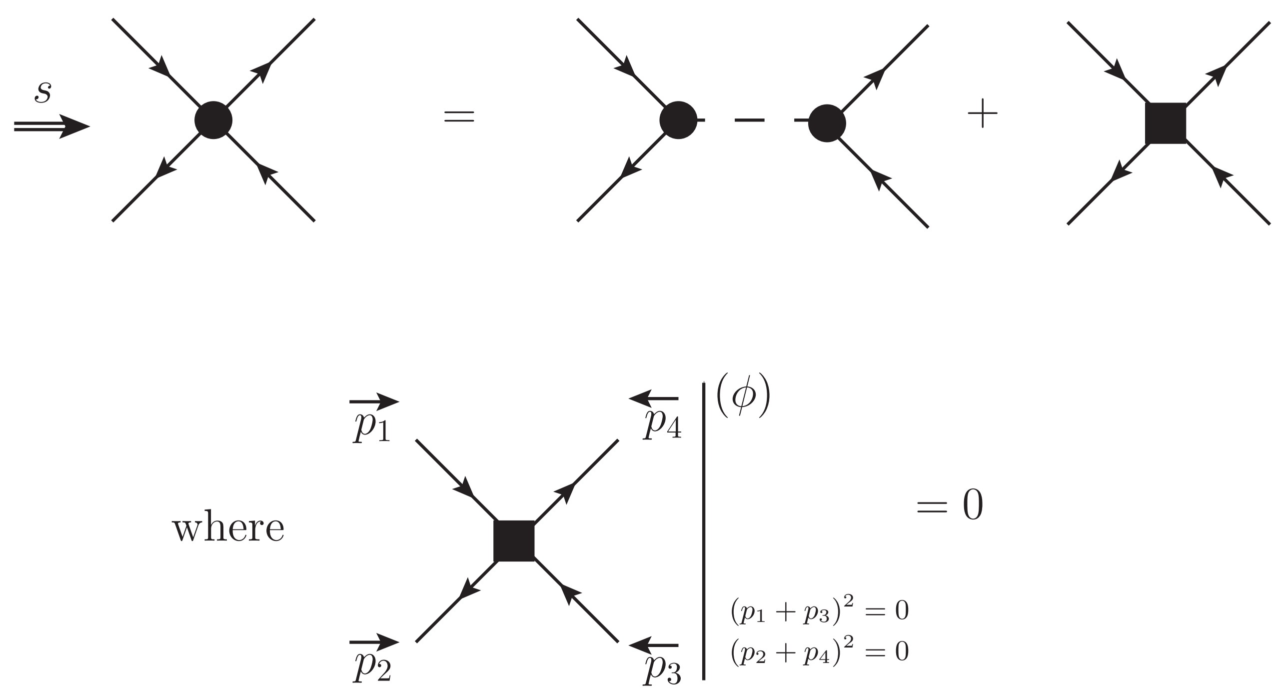


Fukushima, JMP, Strodthoff, 2103.01129

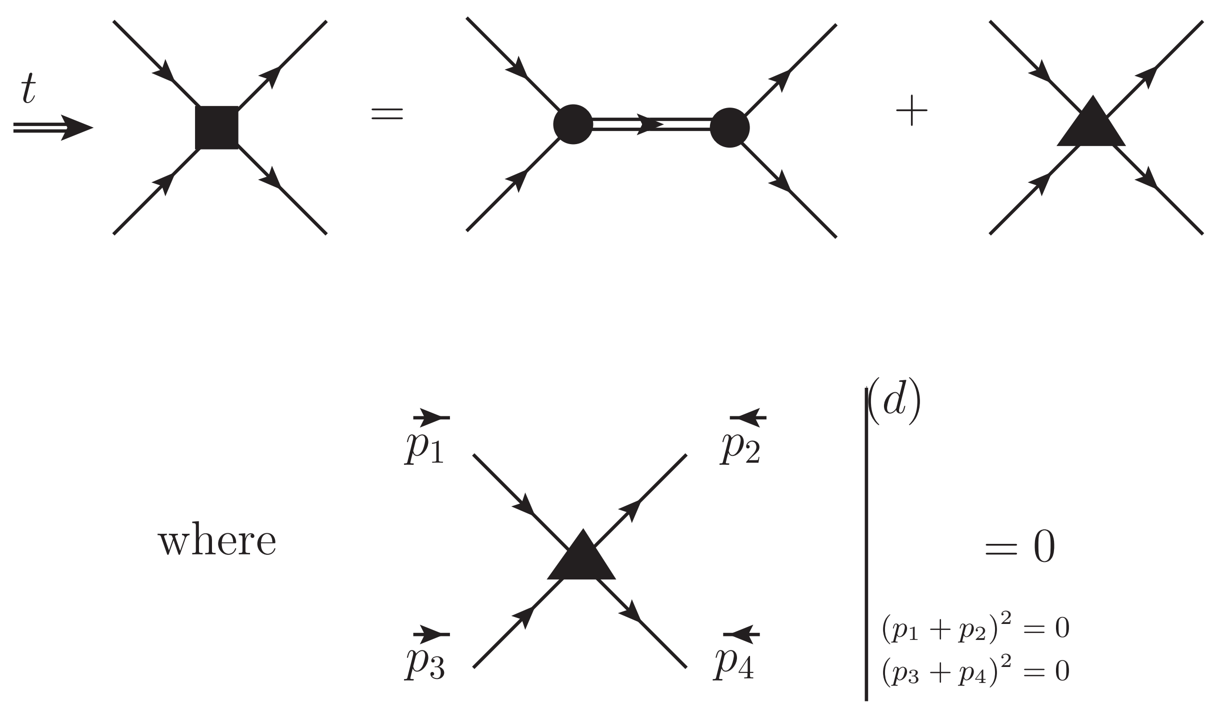
Dynamical hadronisation at work

Mesons & diquarks:

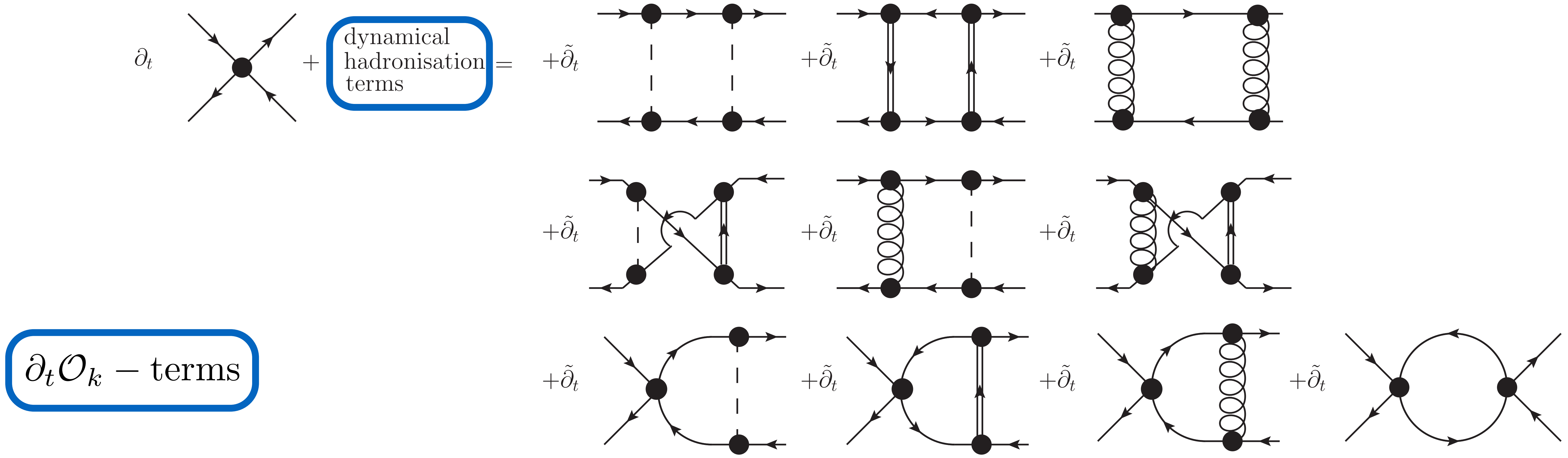
(i)



(ii)



Schematical flow

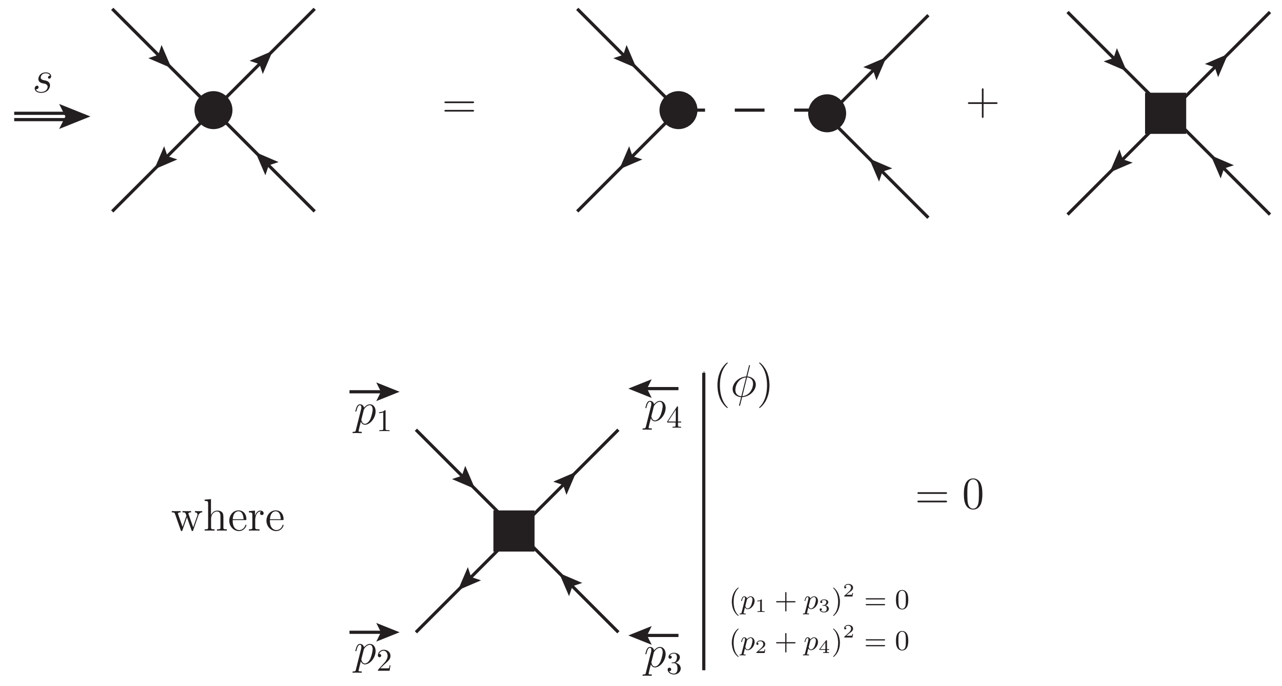


Fukushima, JMP, Strodthoff, 2103.01129

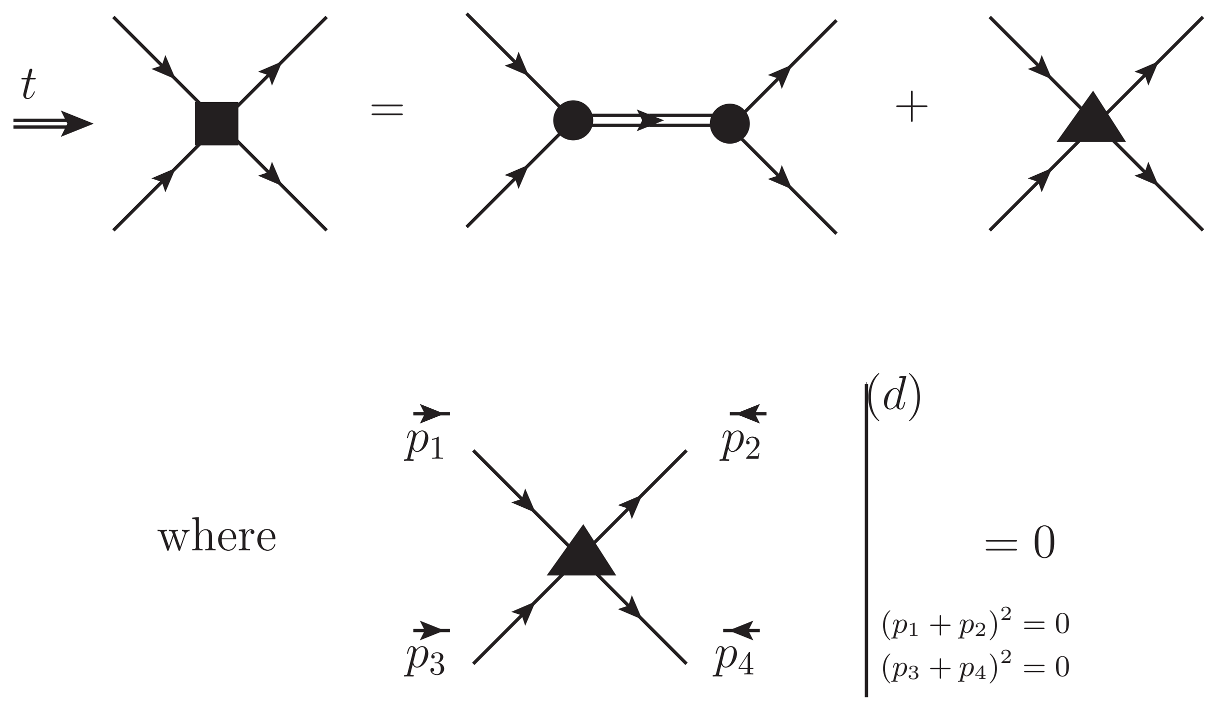
Dynamical hadronisation at work

Mesons & diquarks:

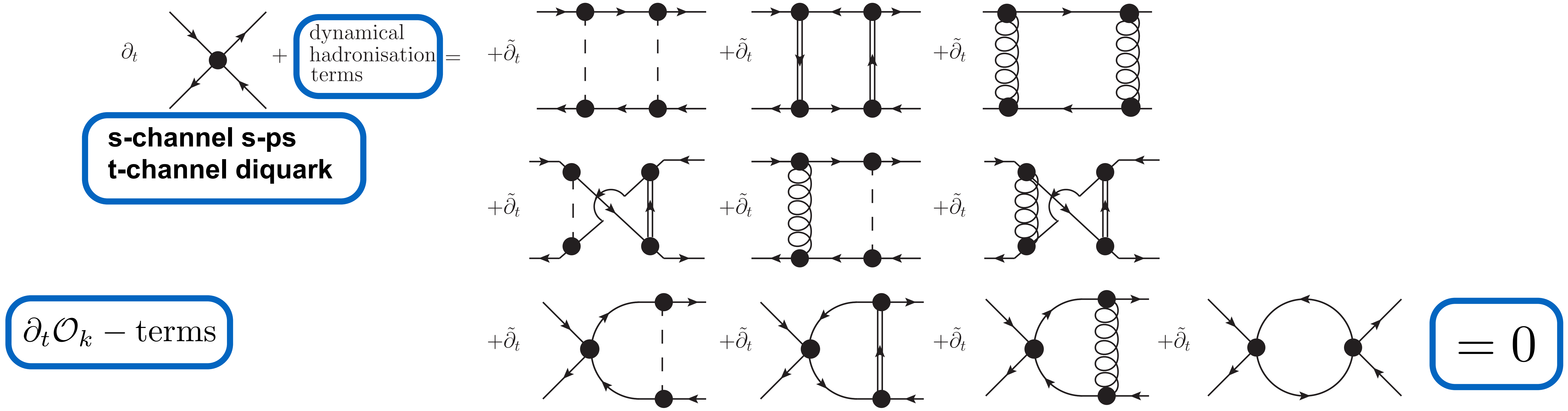
(i)



(ii)



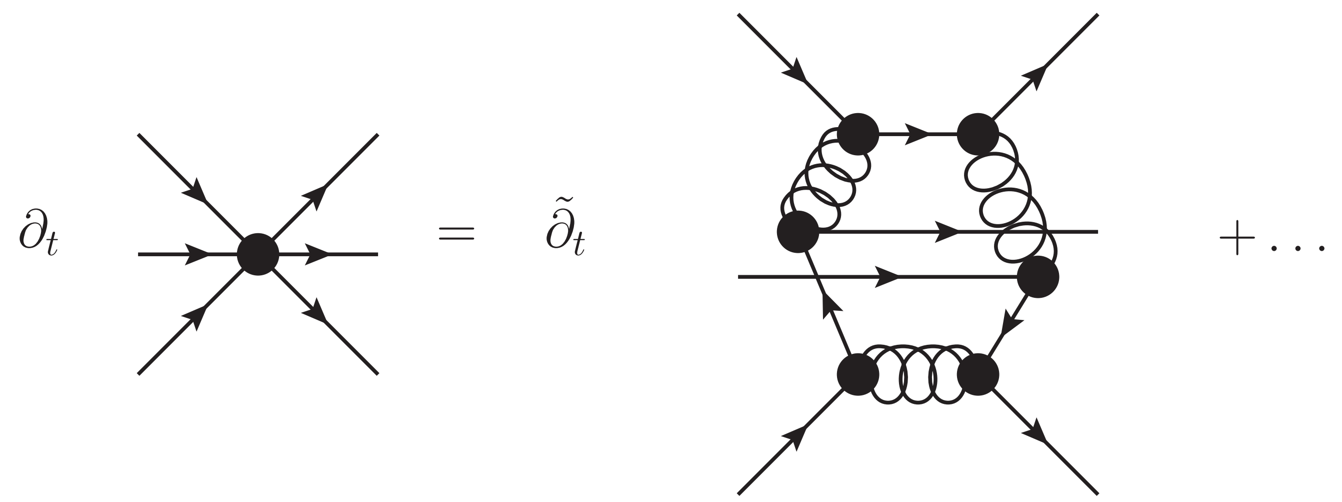
Schematical flow



Fukushima, JMP, Strodthoff, 2103.01129

Dynamical hadronisation at work

baryons:

Dominant UV-process: 

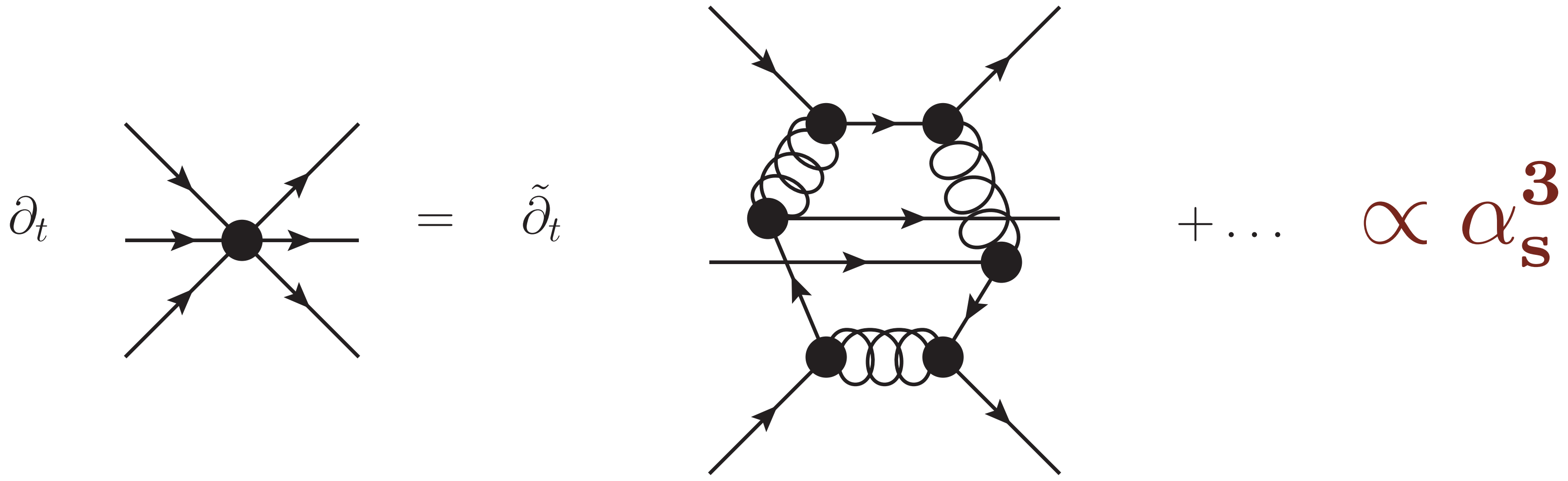
$\propto \alpha_s^3$

Fukushima, JMP, Strodthoff, 2103.01129

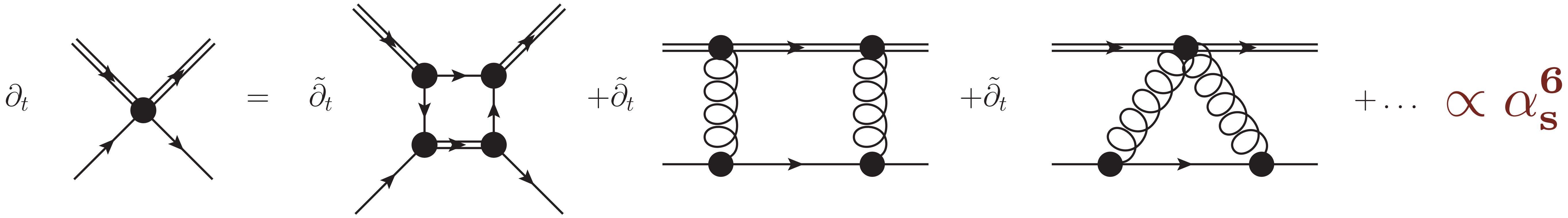
Dynamical hadronisation at work

baryons:

Dominant UV-process:



UV-subdominant:



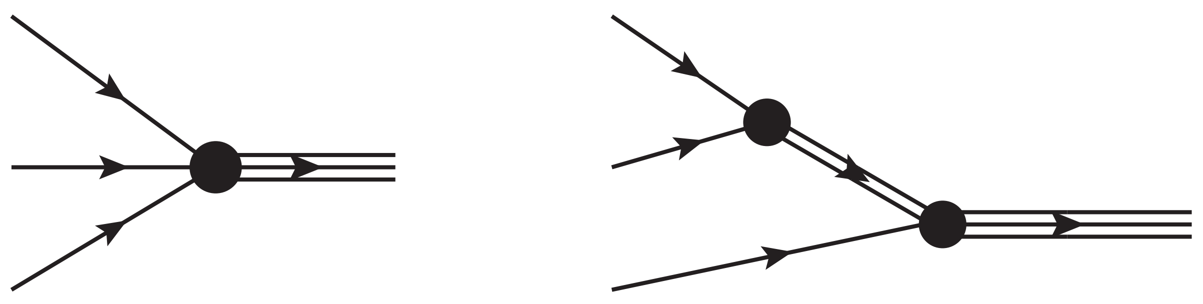
Fukushima, JMP, Strodthoff, 2103.01129

Dynamical hadronisation at work

baryons:

Baryonisation

Baryon formation processes



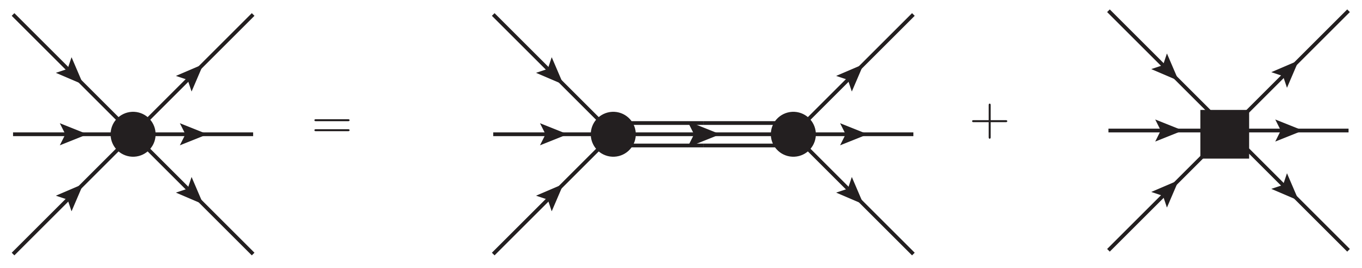
Fukushima, JMP, Strodthoff, 2103.01129

Dynamical hadronisation at work

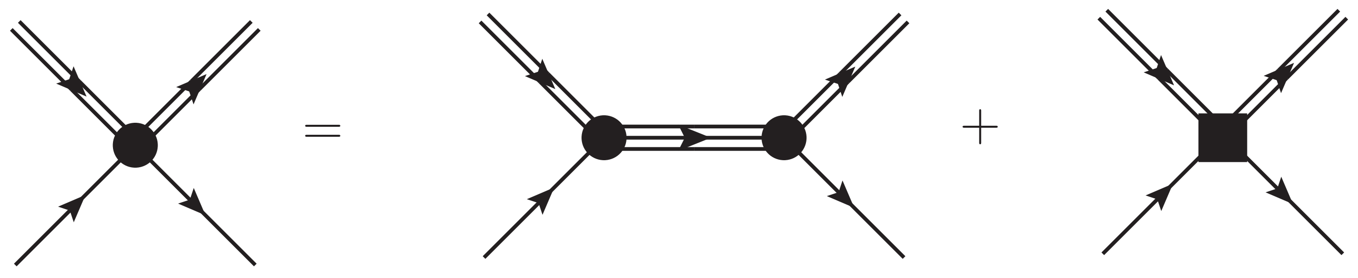
baryons:

Baryonisation

three-quark scattering



quark-diquark scattering



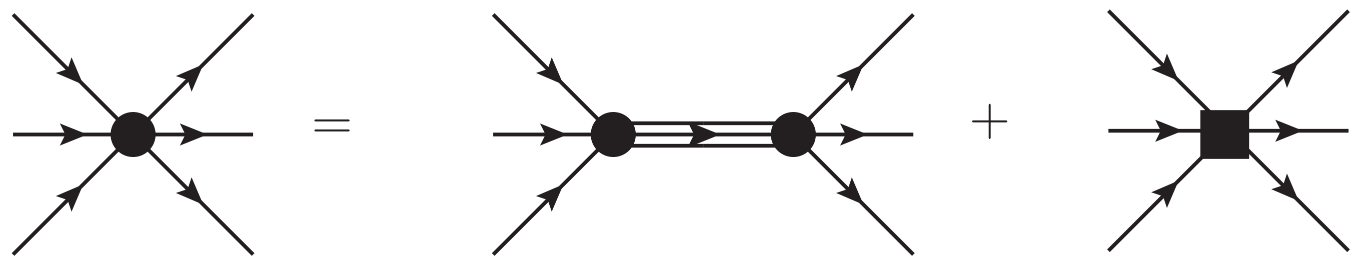
Fukushima, JMP, Strodthoff, 2103.01129

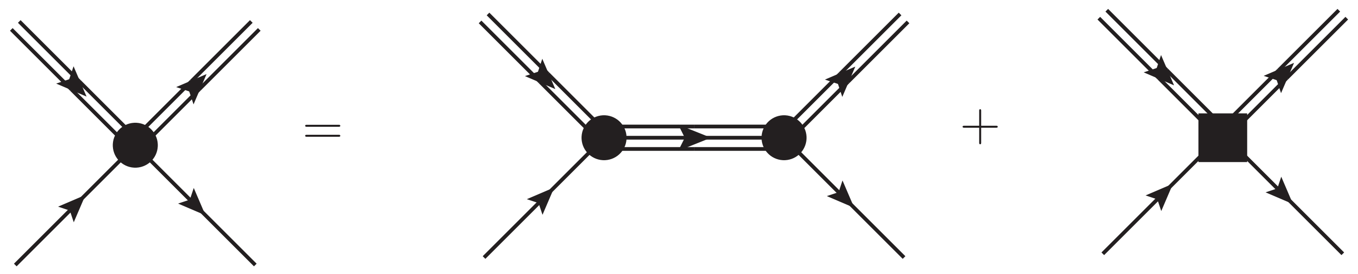
Dynamical hadronisation at work

baryons:

Baryonisation

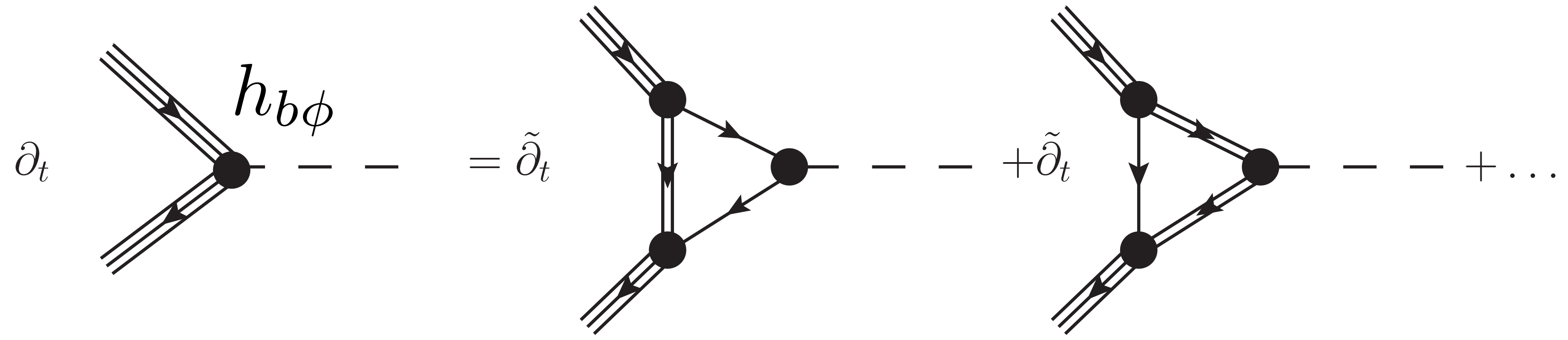
three-quark scattering **quark-diquark scattering**



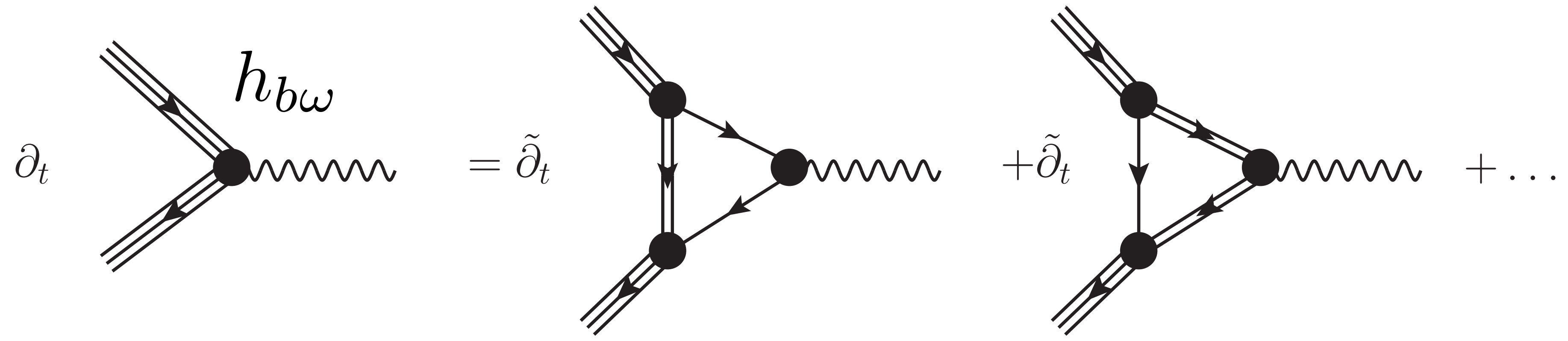


Yukawa-flows with baryonisation

nucleon-nucleon — $(\vec{\pi}, \sigma)$ **scattering:**



nucleon-nucleon — ω_μ **scattering:**

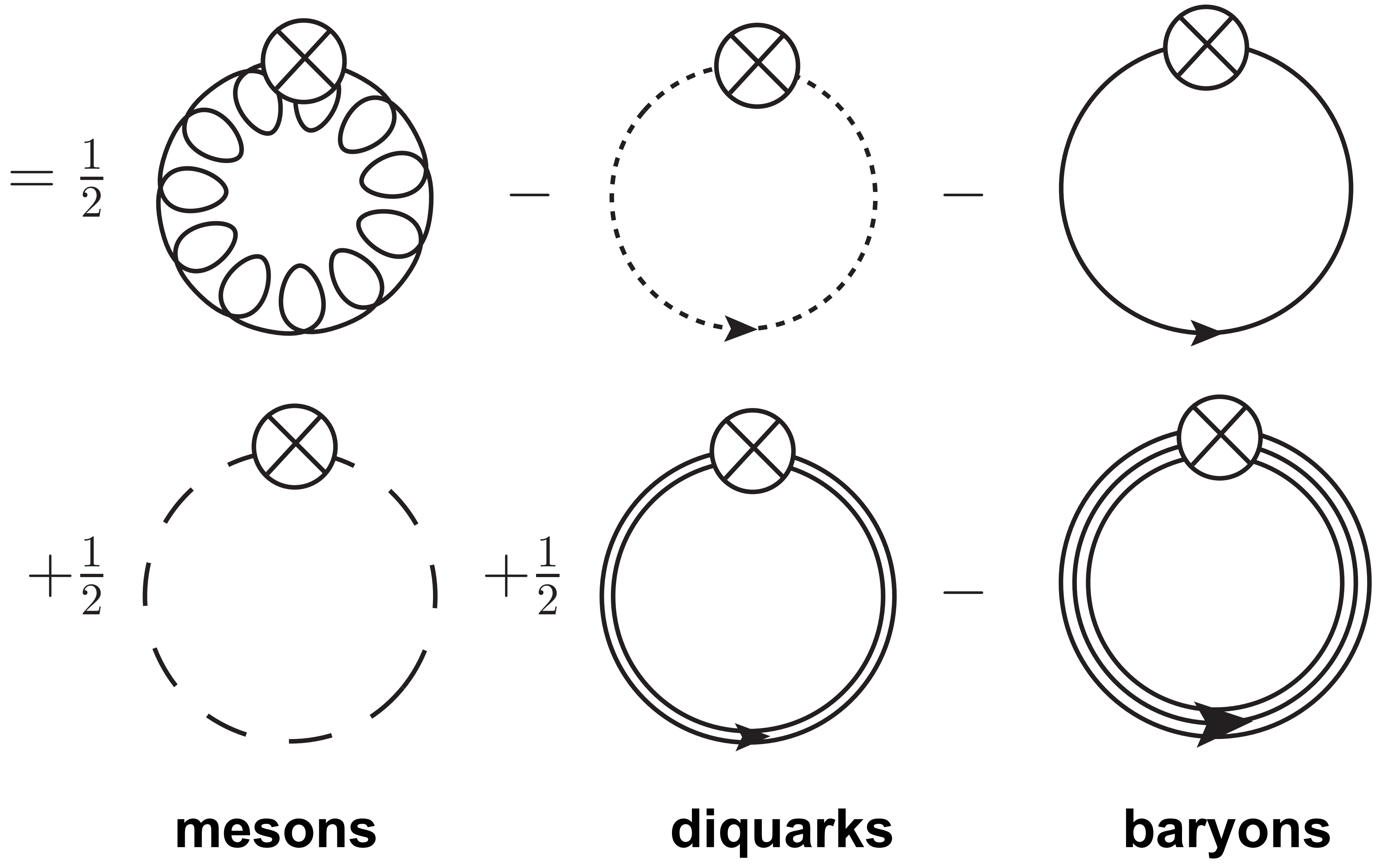


Fukushima, JMP, Strodthoff, 2103.01129

Dynamical hadronisation at work

‘DynHad for mesons, diquarks & baryons is Faddeev-BSE-DSE for QCD in a ‘unified’ effective action approach’

$$\left(\partial_t + \partial_t \Phi_{i,k}[\Phi]\,\frac{\delta}{\delta\Phi_i}\right)\left(\Gamma_k[\Phi] + c_{\Phi_i}\,\Phi_i\right)$$



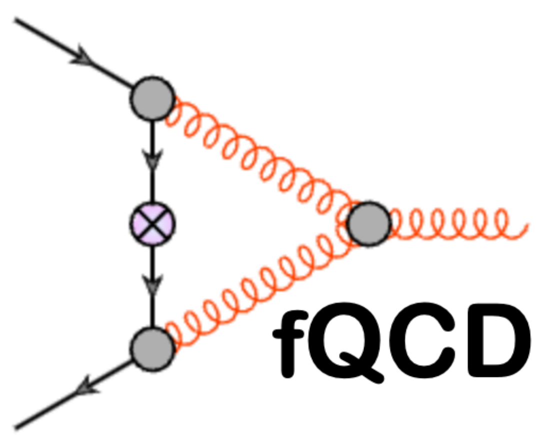
Fukushima, JMP, Strodthoff, 2103.01129

(II) Functional QCD and the QCD phase structure

- **QCD at finite temperature and density**
  - Benchmarks in the vacuum
  - Correlation functions at finite temperature
  - Polyakov loop from functional approaches

- **QCD phase structure**
  - Locating the QCD phase boundary and the critical end point
  - The unreasonable effectiveness of low energy effective theories and how to use them
  - Fluctuations of conserved charges: Ripples of the critical end point

fQCD collaboration

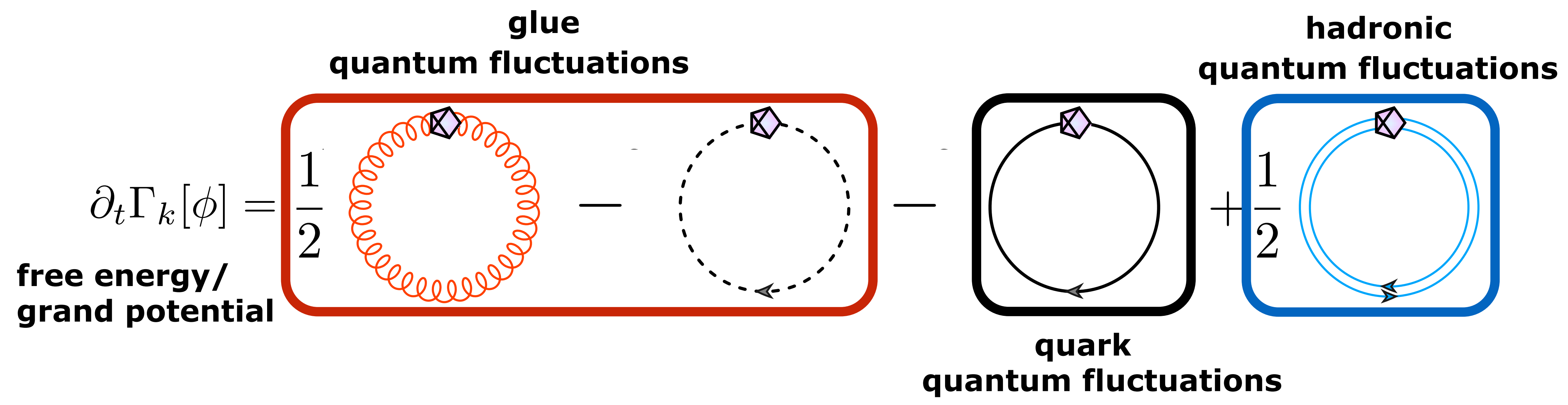


Dalian, Beijing, Darmstadt, Heidelberg, Gießen

Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

Functional flows for QCD

functional RG:



Correlation functions

gluon propagator

$$\langle A_\mu A_\nu \rangle (p)$$

quark propagator

$$\langle q\bar{q} \rangle (p)$$

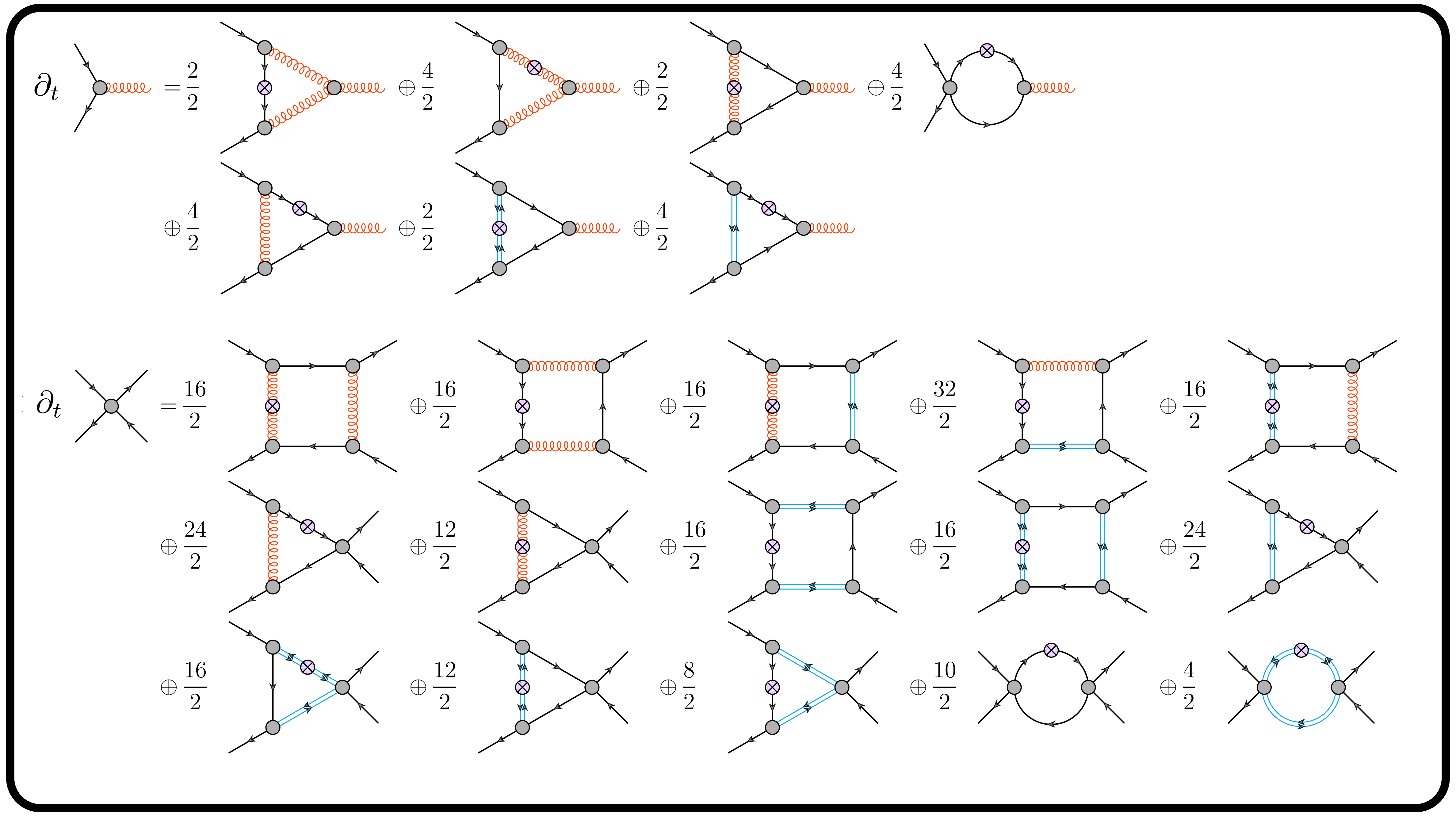
quark-gluon vertex

$$\langle q\bar{q} A_\mu \rangle (p_1, p_2)$$

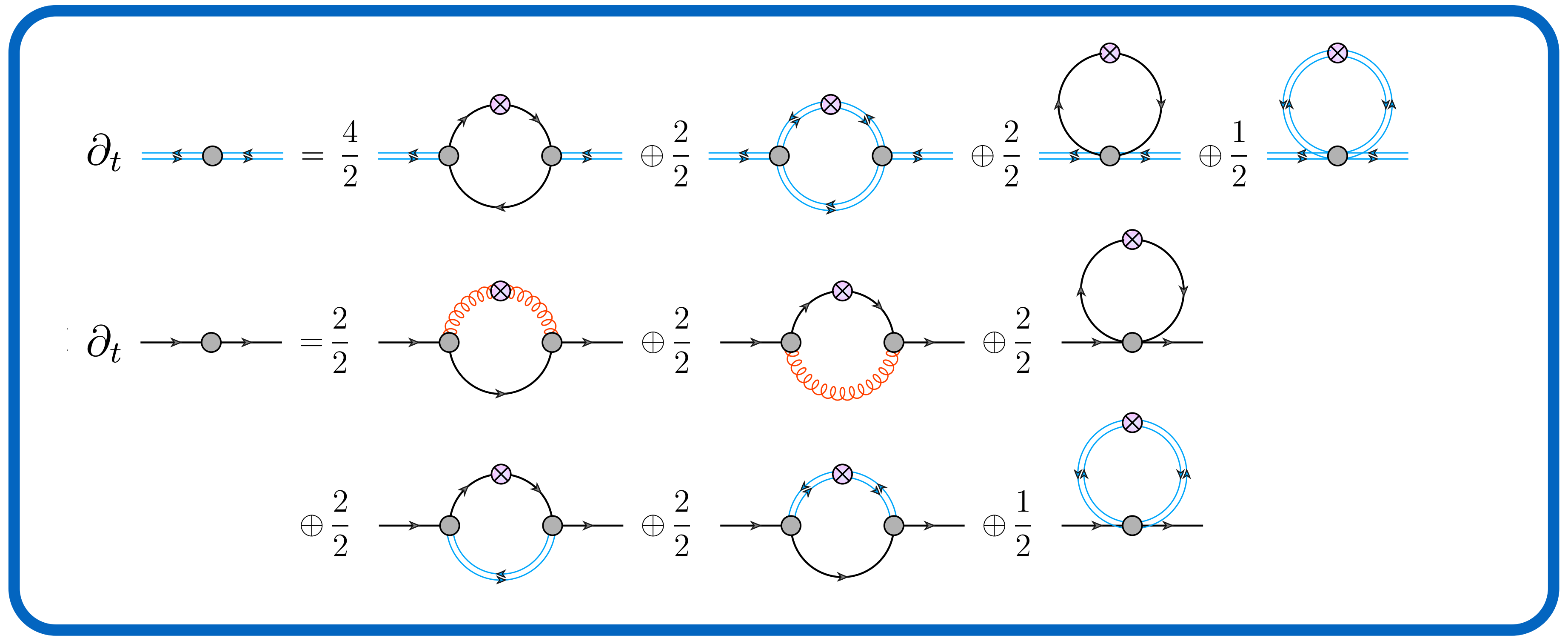
Eight tranverse tensor structures

quark—anti-quark scattering

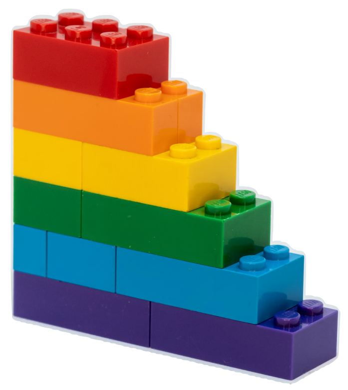
$$\langle q\bar{q}q\bar{q} \rangle (p_1, p_2, p_3)$$

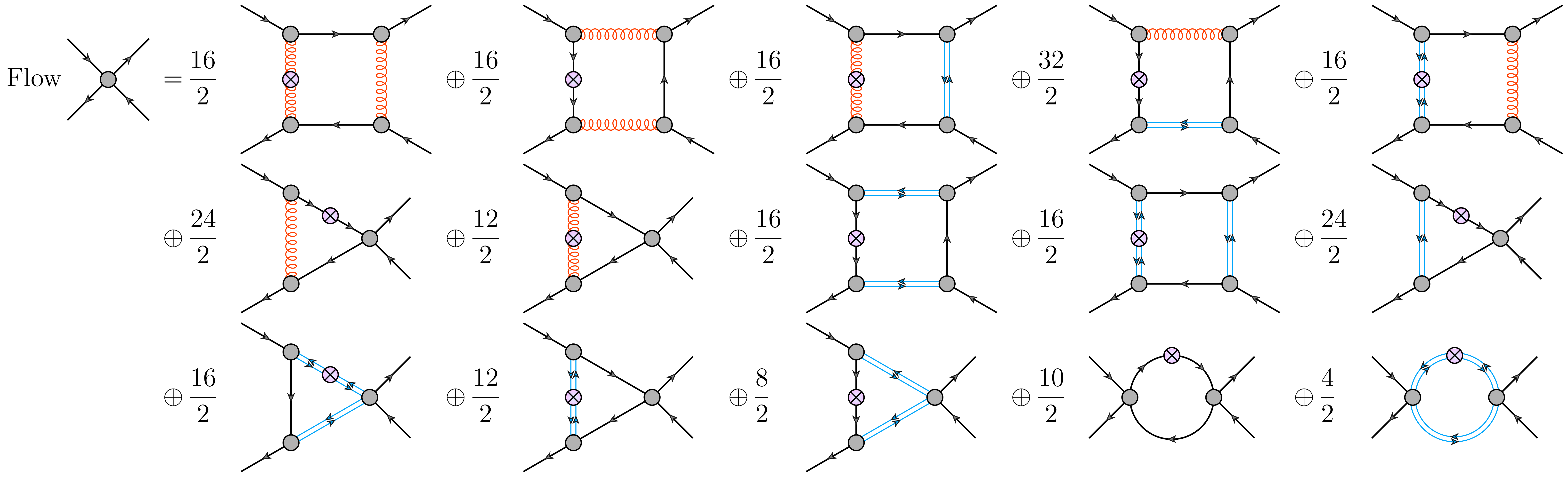


Dynamical hadronisation

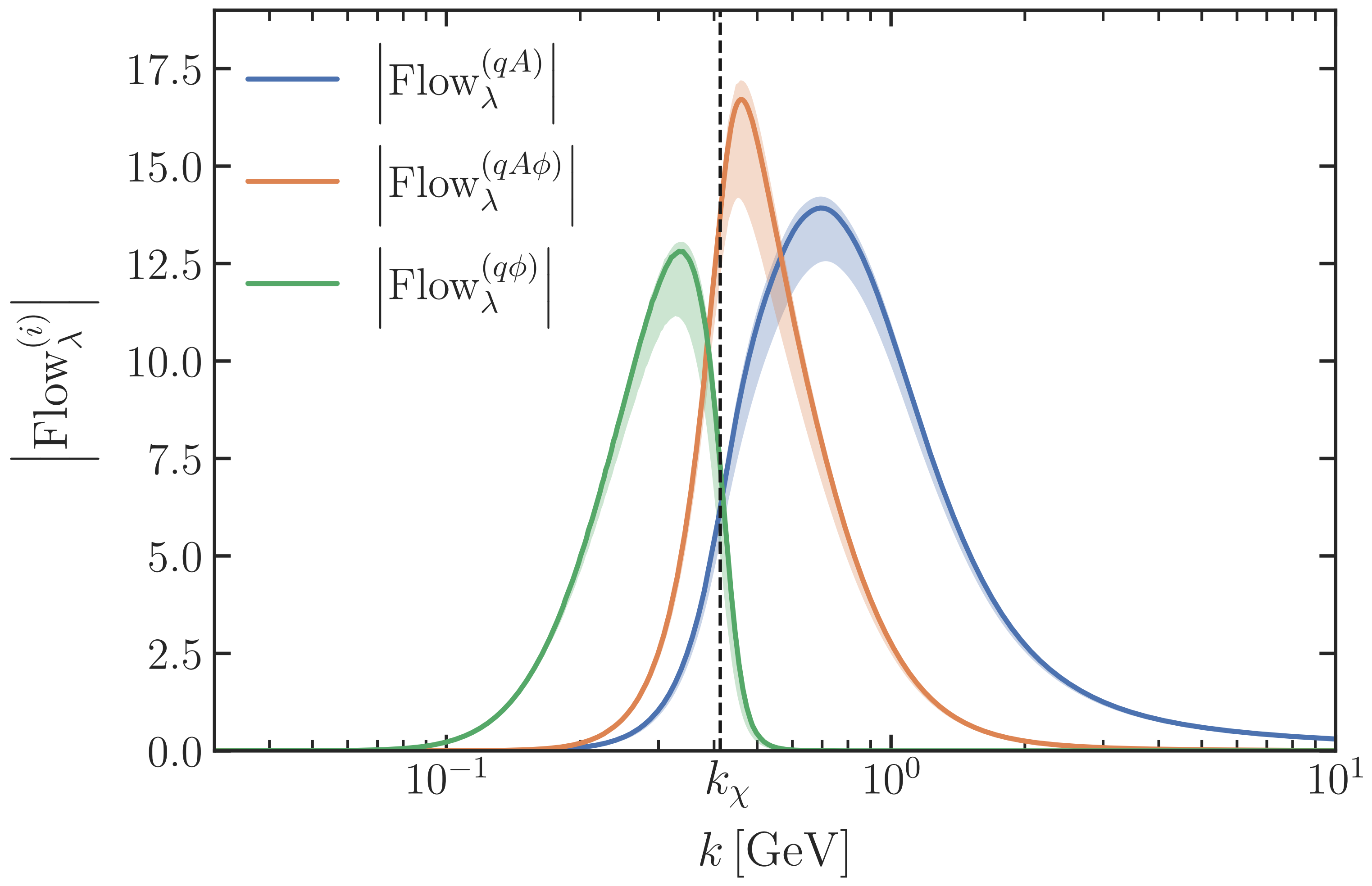


How to: systematic error estimates & the LEGO® principle





Example: 4-quark scattering vertex



Ihssen, JMP, Sattler, Wink, in preparation

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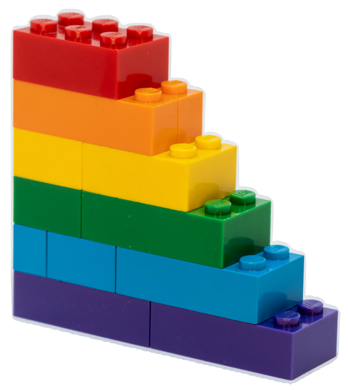
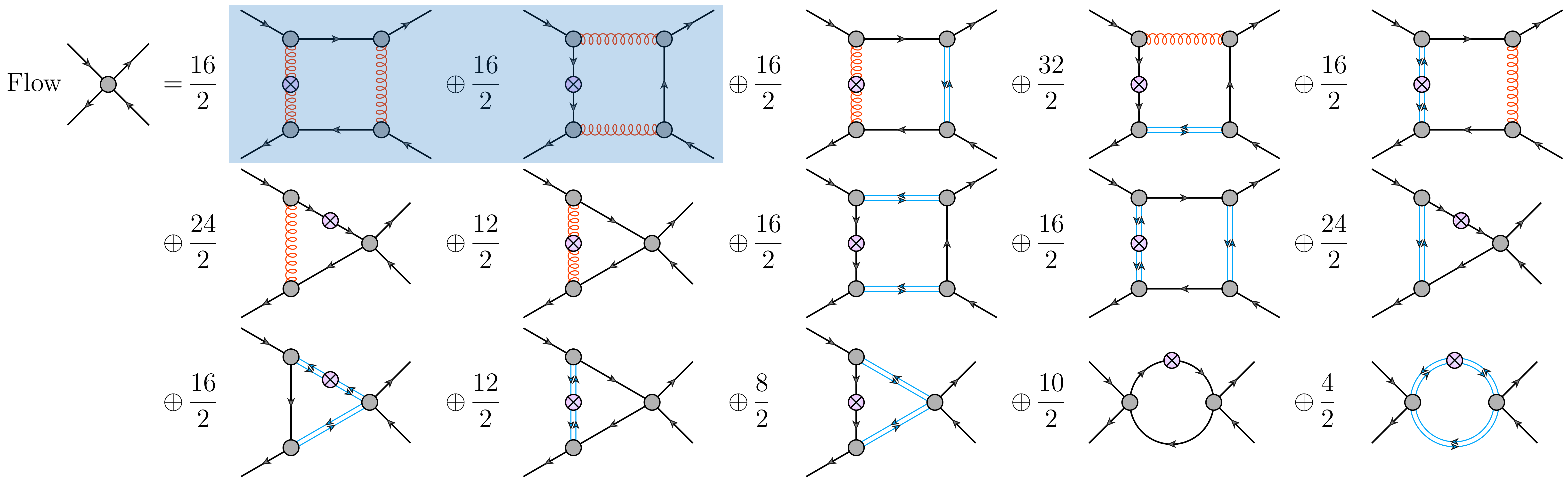
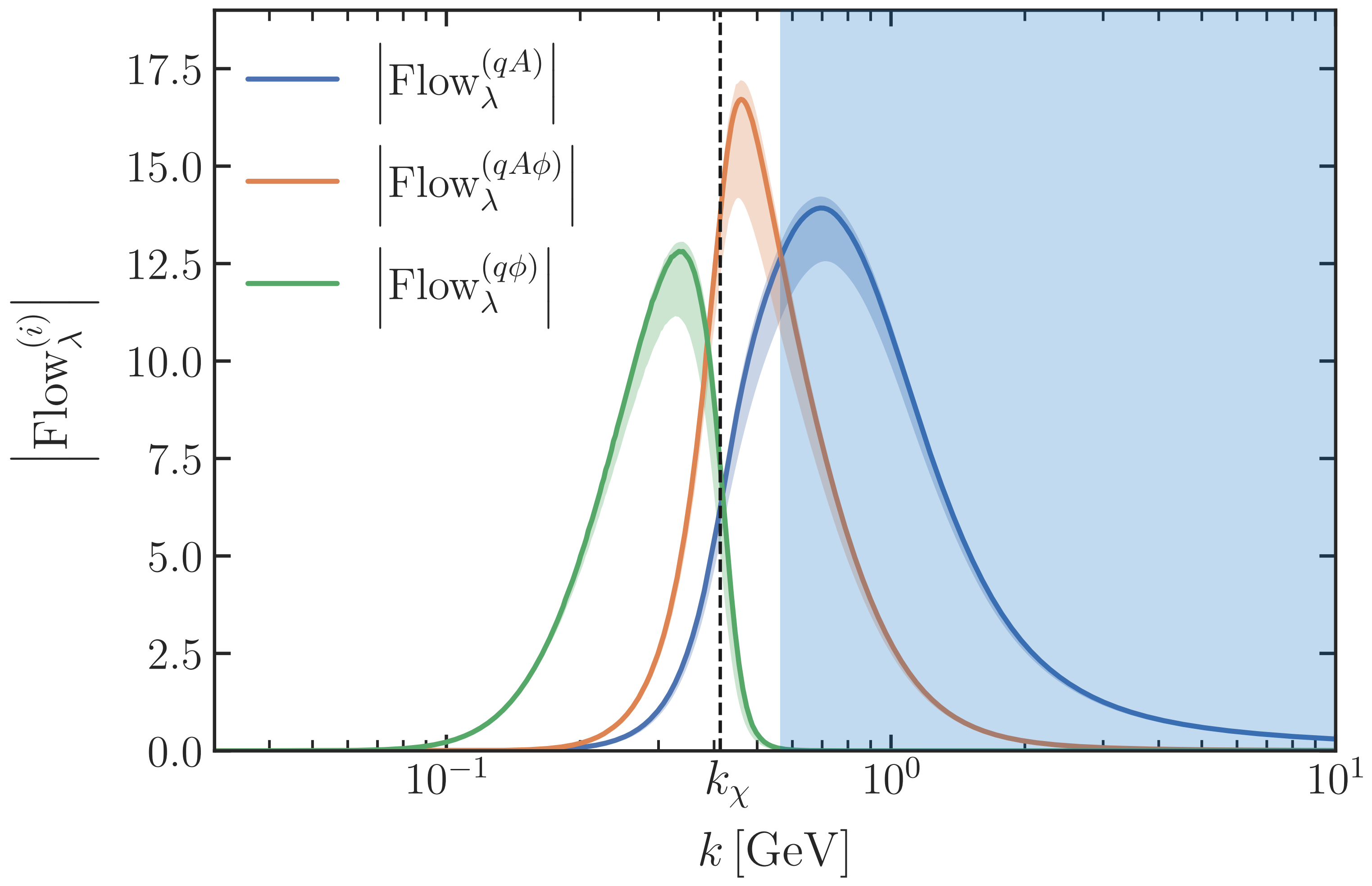


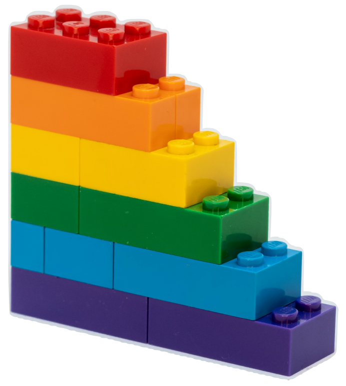


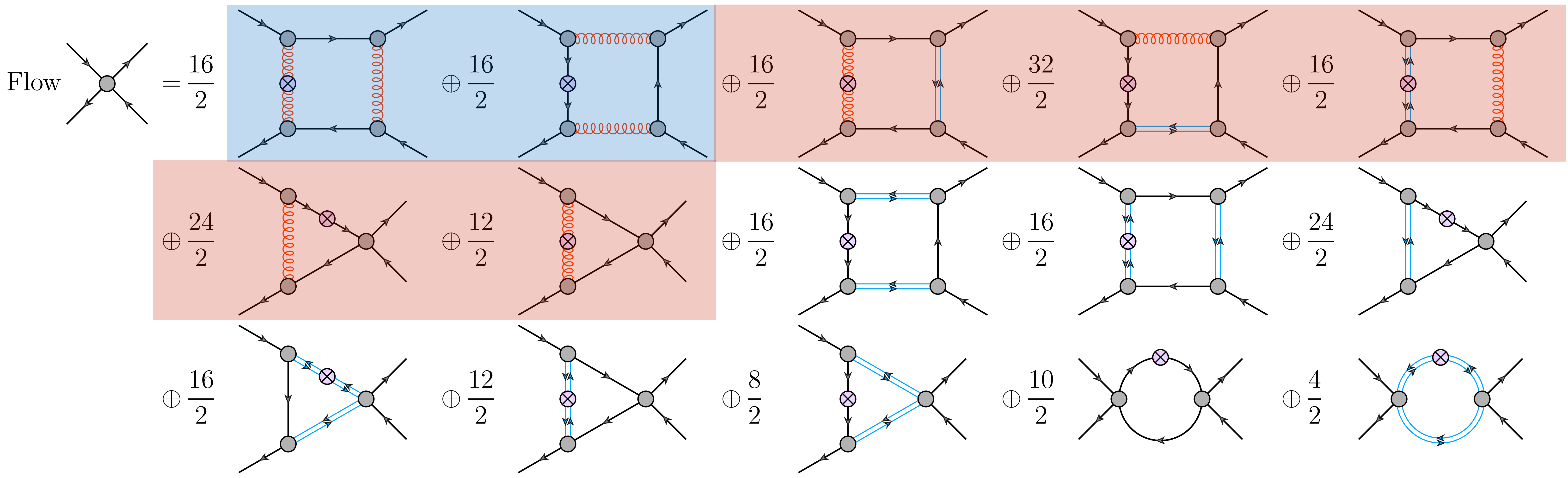
Example: 4-quark scattering vertex



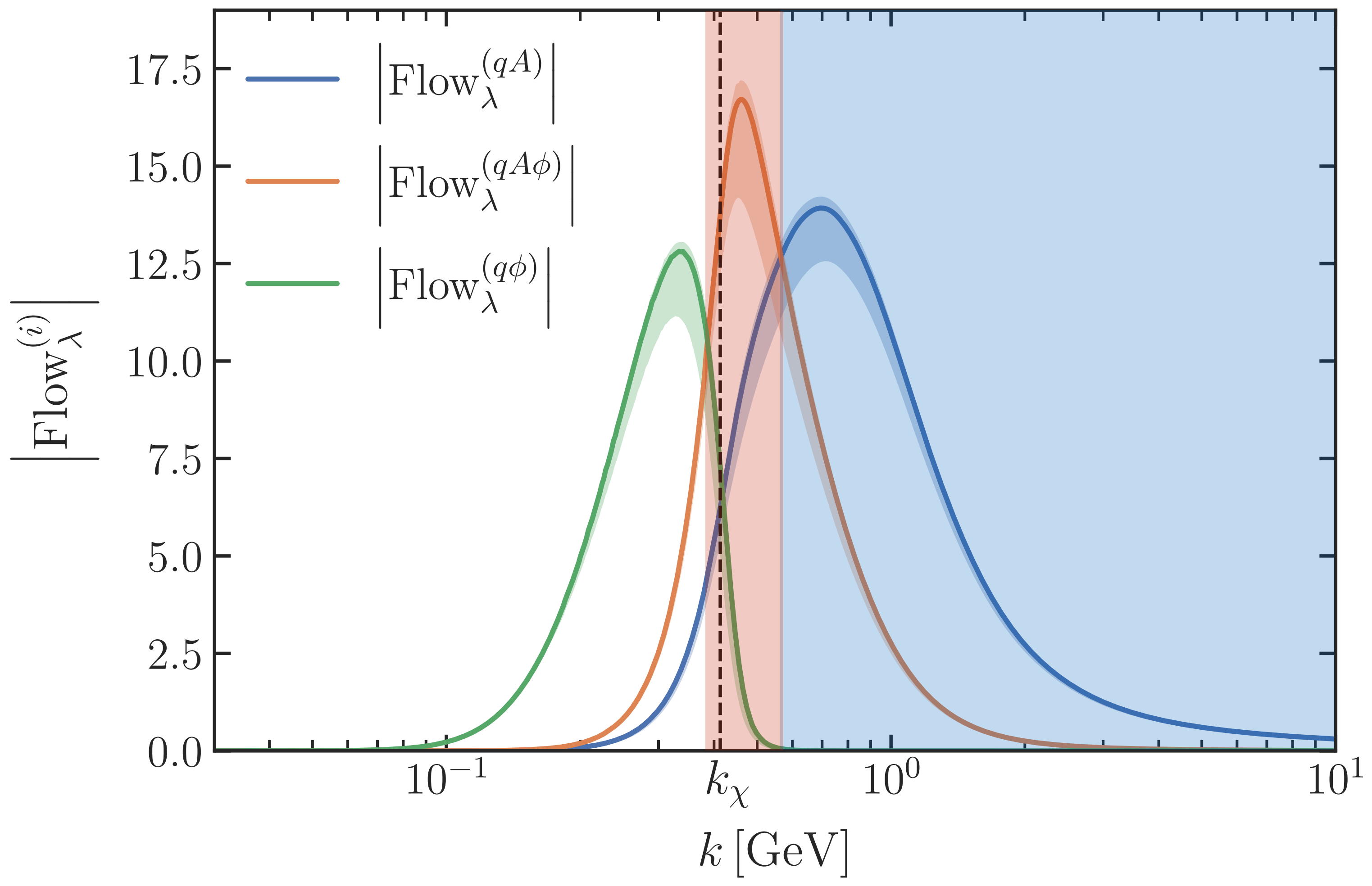
Ihssen, JMP, Sattler, Wink, in preparation

How to: systematic error estimates & the LEGO® principle



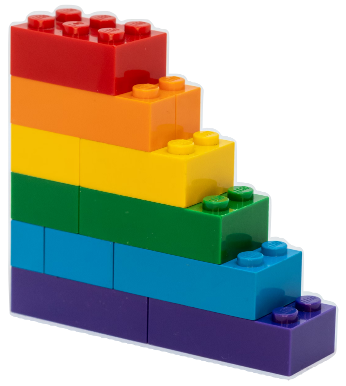


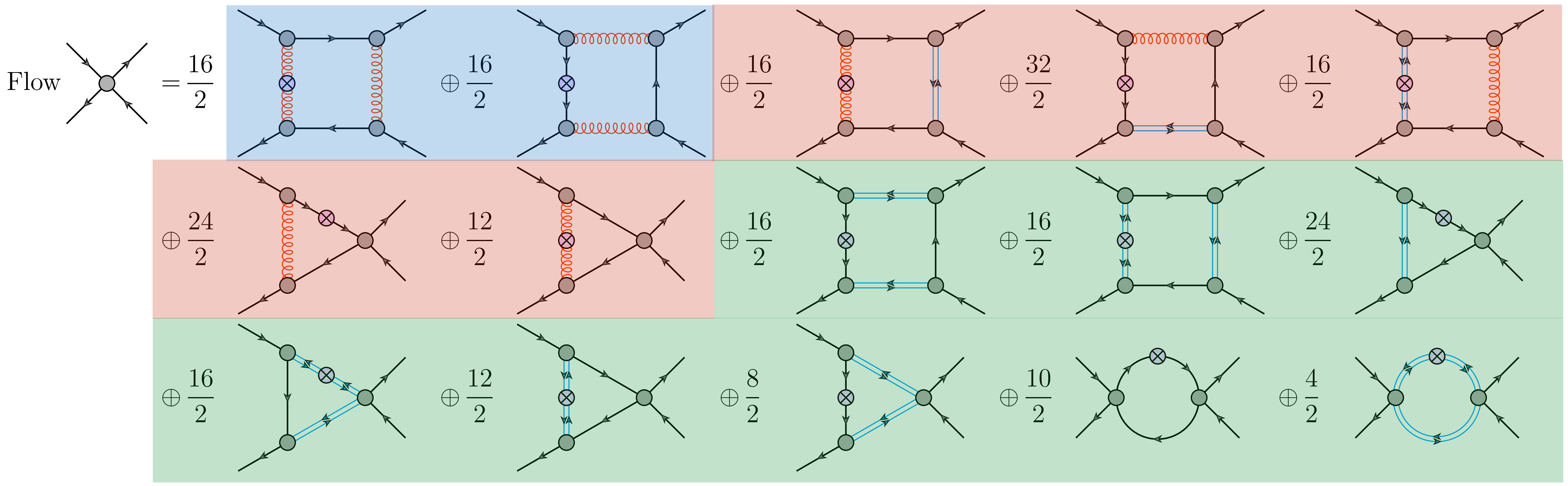
Example: 4-quark scattering vertex



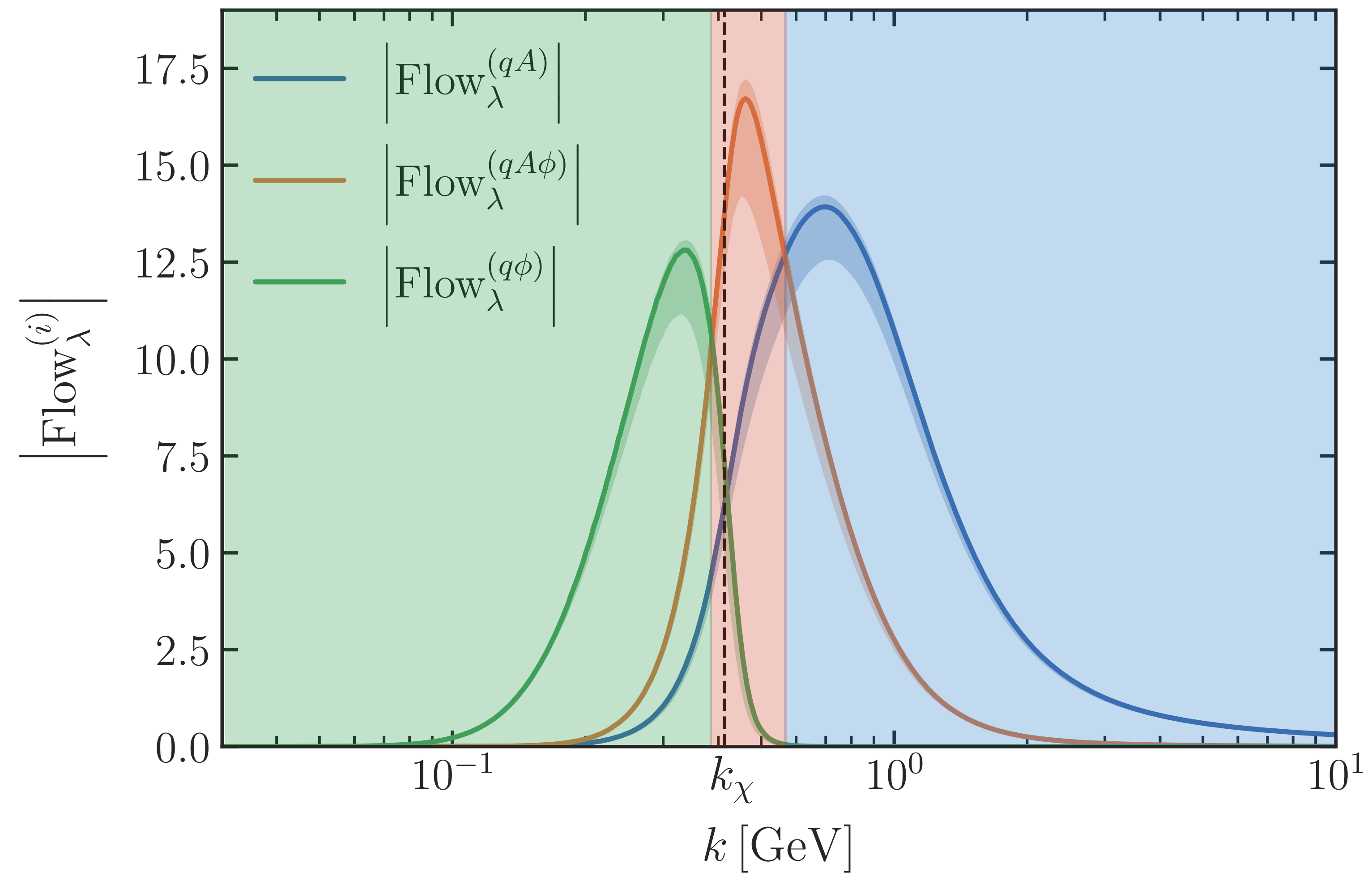
Ihssen, JMP, Sattler, Wink, in preparation

How to: systematic error estimates & the LEGO® principle



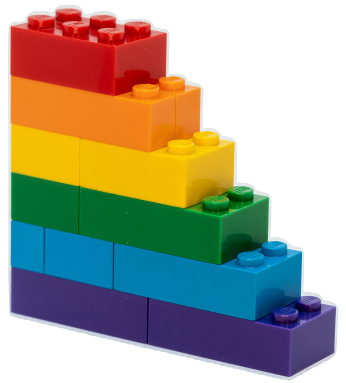


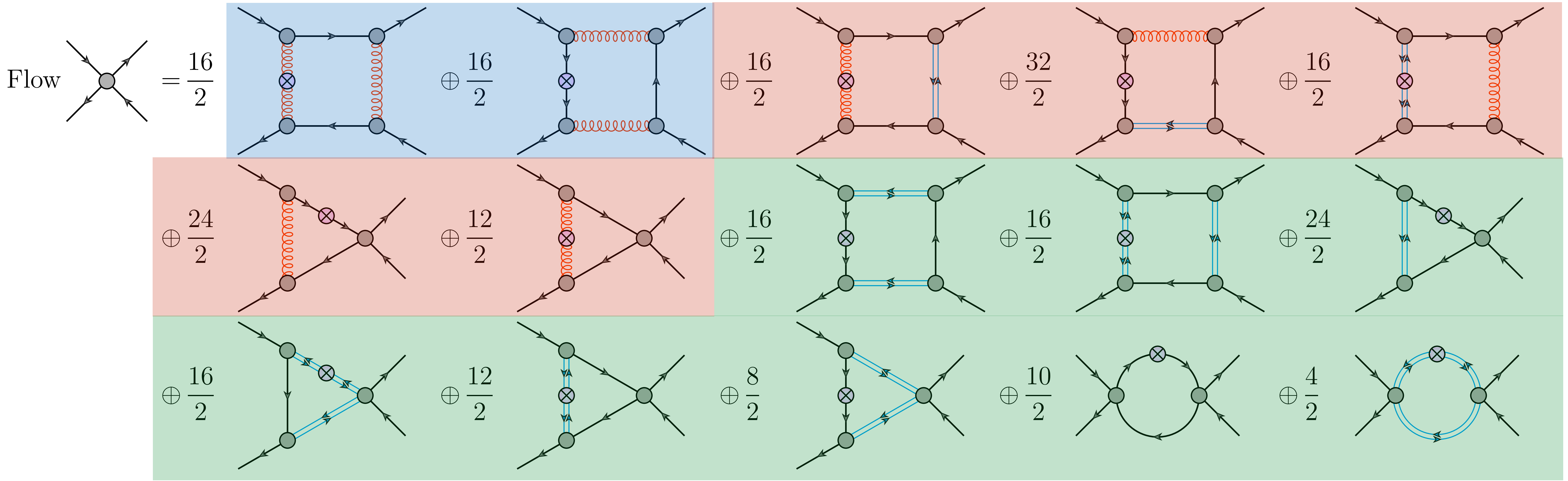
Example: 4-quark scattering vertex



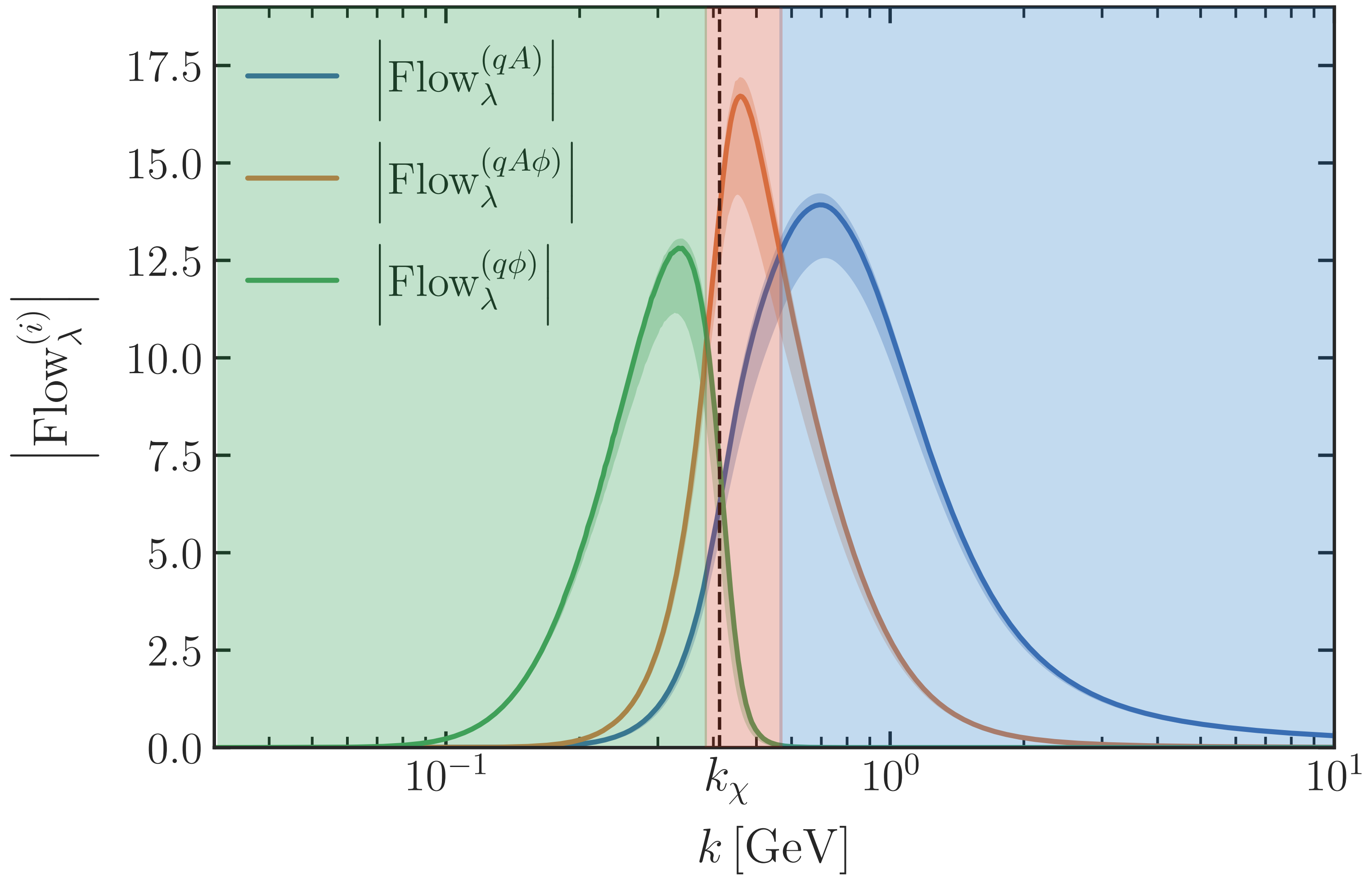
Ihssen, JMP, Sattler, Wink, in preparation

How to: systematic error estimates & the LEGO® principle





The unreasonable effectiveness of low energy effective theories



Access and combined use of
error estimates
from functional QCD & LEFTs

Ihssen, JMP, Sattler, Wink, in preparation

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
Denz, Held, Pawlowski, Rodigast; unpub.

QMeS, DoFun

Mathematica packages for the derivation of functional equations

QMeS: **JMP, Schneider, Wink, CPC287 (2023) 108711**

DoFun: **Braun, Huber, CPC 183 (2012) 1290**
Huber, Cyrol, JMP, CPC 183 248 (2020) 107058

Action

QMeS, DoFun / ***VertEXpand***

QMeS / ***DoFun***

Vertices/
Feynman Rules

Symbolic
Flow Equations

FormTracer High-performance, general purpose, easy-to-use Mathematica tracing tool using FORM
Cyrol, Mitter, Pawlowski, Strodthoff;
Cyrol, Mitter, Strodthoff; CPC 219 (2017) 346

Algebraic
Flow Equations

CreateKernels Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
fQCD

Compilable Kernels

frgsolver Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
fQCD

Numerical solution

GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung

Alexander von Humboldt Stiftung/Foundation

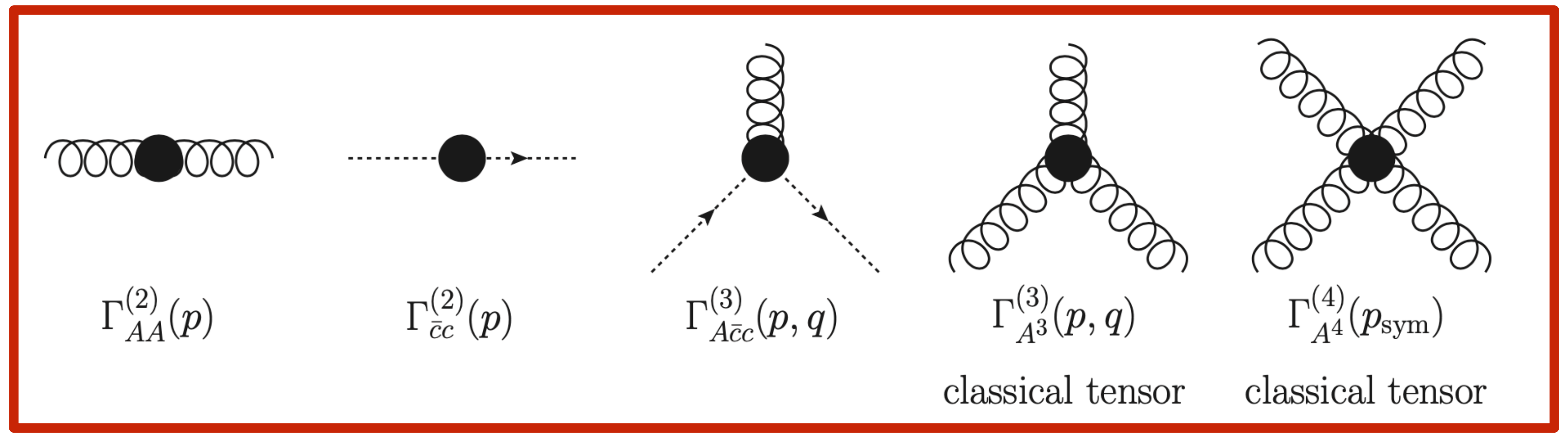
FWF Der Wissenschaftsfonds.

erc European Research Council
Established by the European Commission

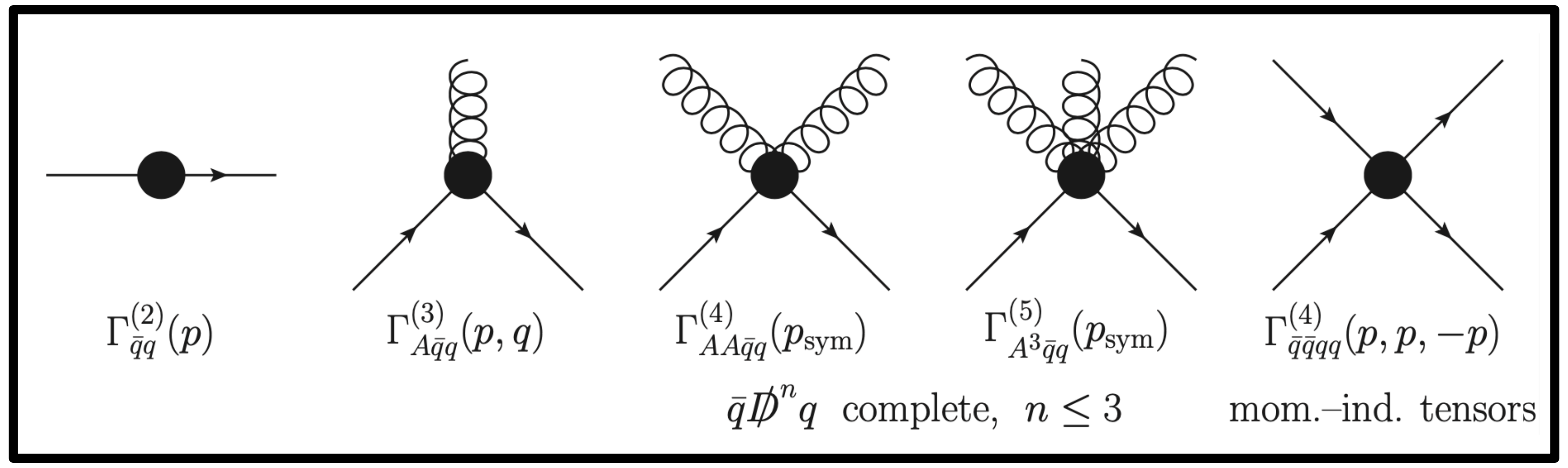
QCD at finite temperature and density

Benchmarks in the vacuum

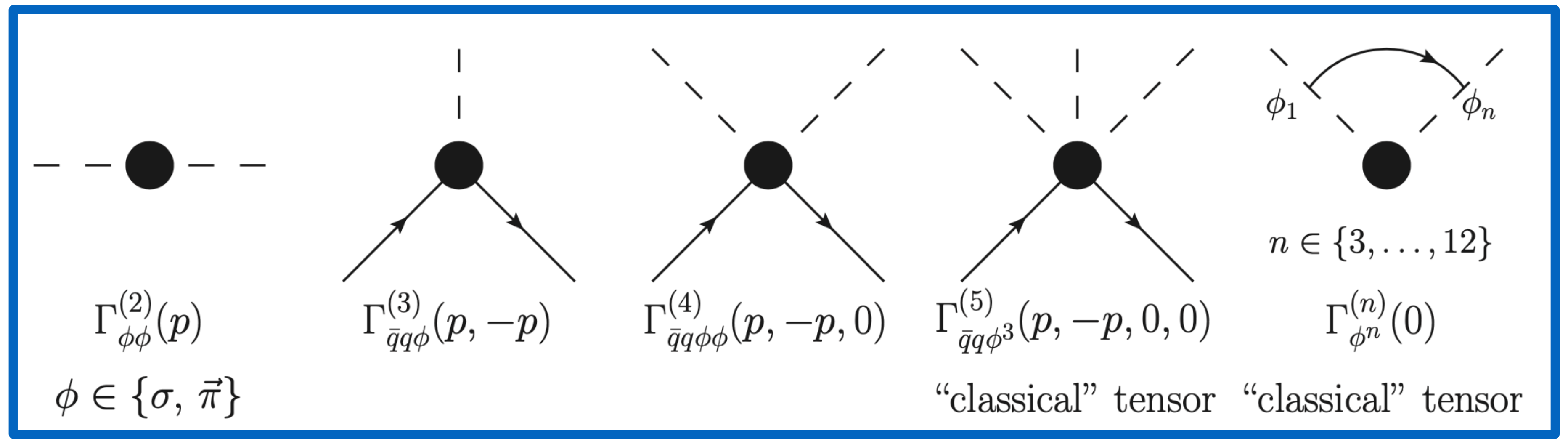
Current set of correlation functions



glue sector



quark-glue sector



quark-meson sector

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006,
PRD 97 (2018) 054015

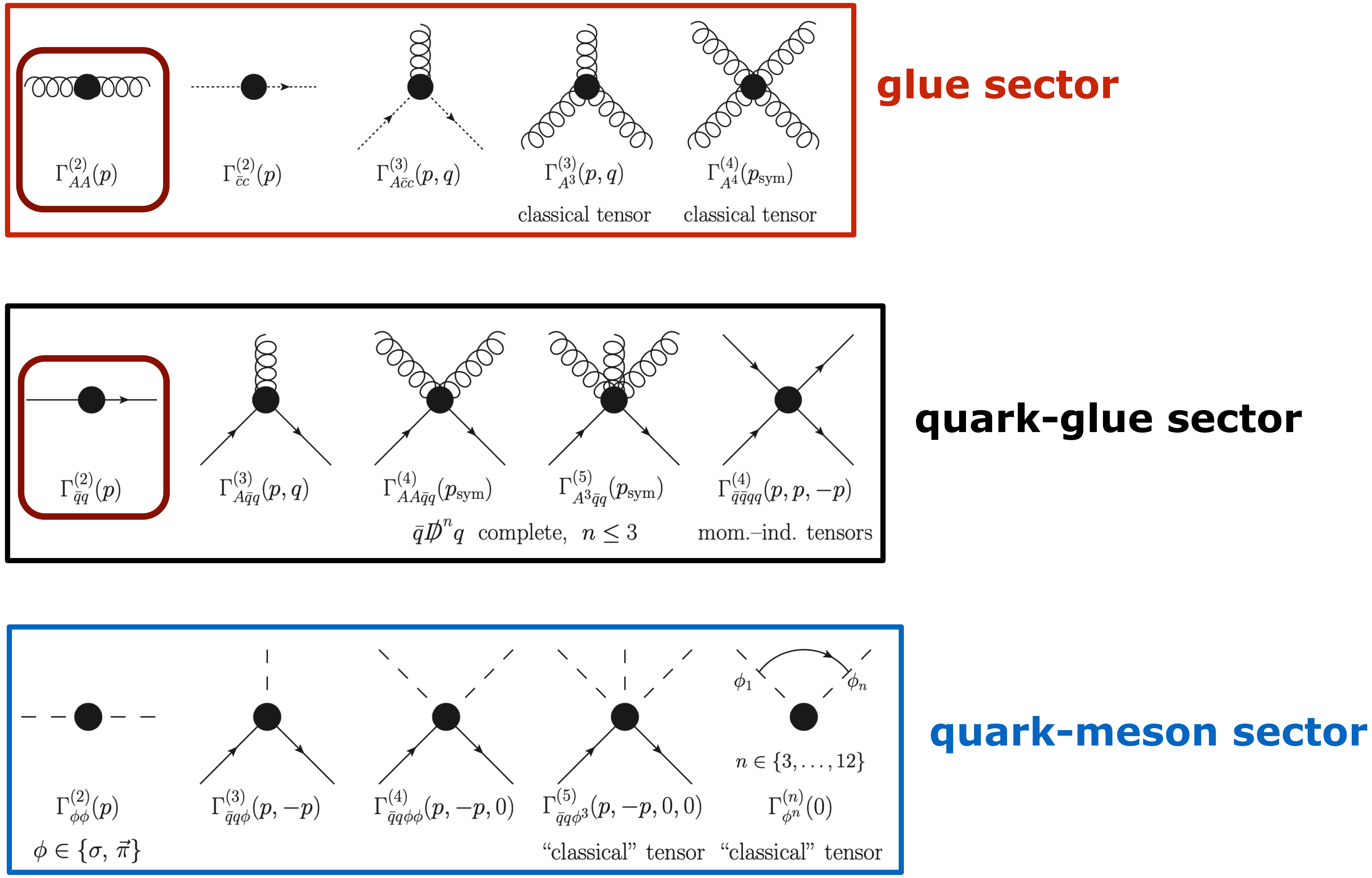
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Extension, work in progress:

Fu, Huang, Ihssen, JMP, Rennecke, Sattler, Tan

Current set of correlation functions



Aiming at apparent convergence

Extension, work in progress:

Fu, Huang, Ihssen, JMP, Rennecke, Sattler, Tan

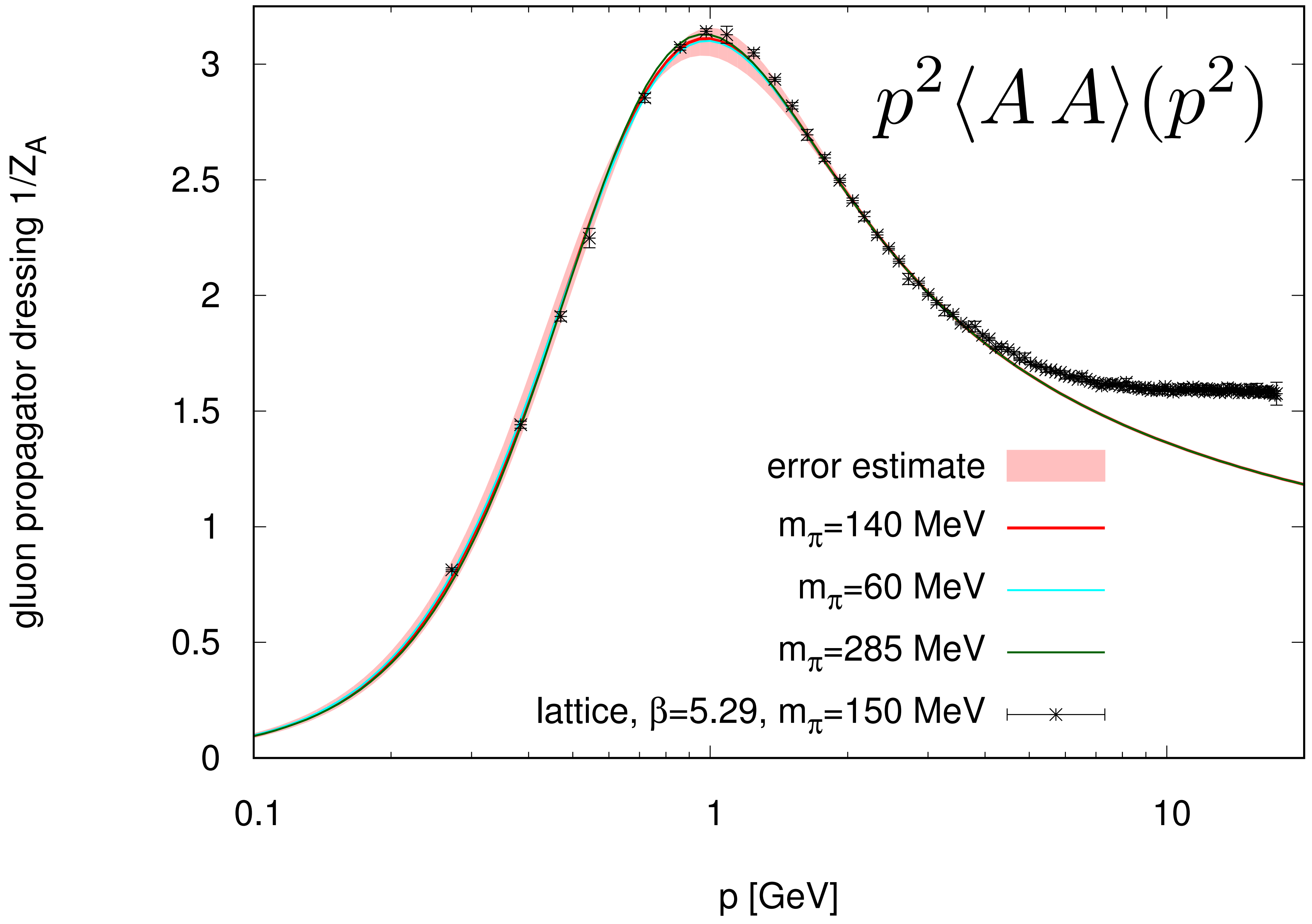
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006,
PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

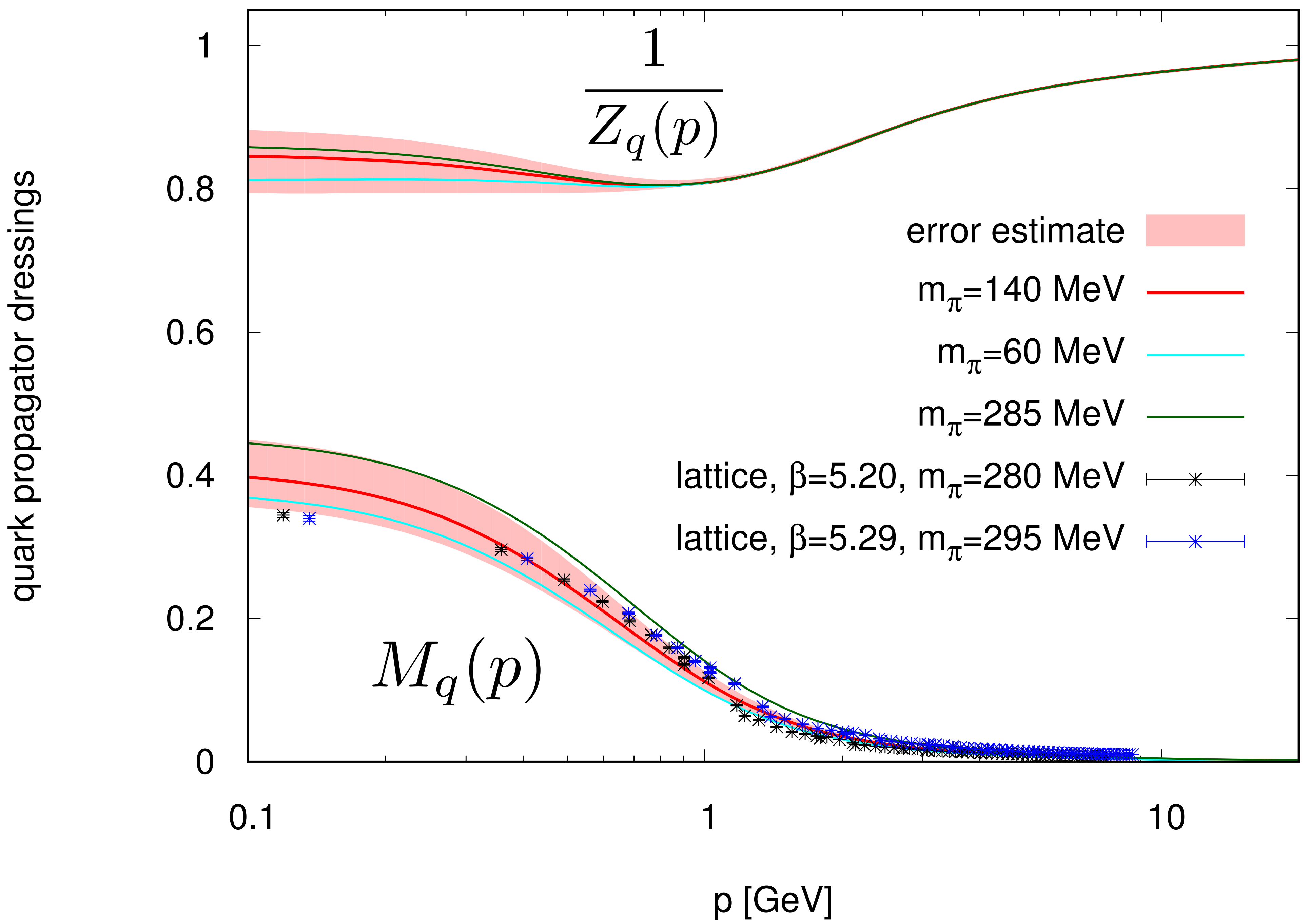
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Euclidean propagators

Two-flavour QCD



$$\frac{1}{Z_q(p)}\, \frac{1}{i\,\not{p} + M_q(p)}$$



lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363

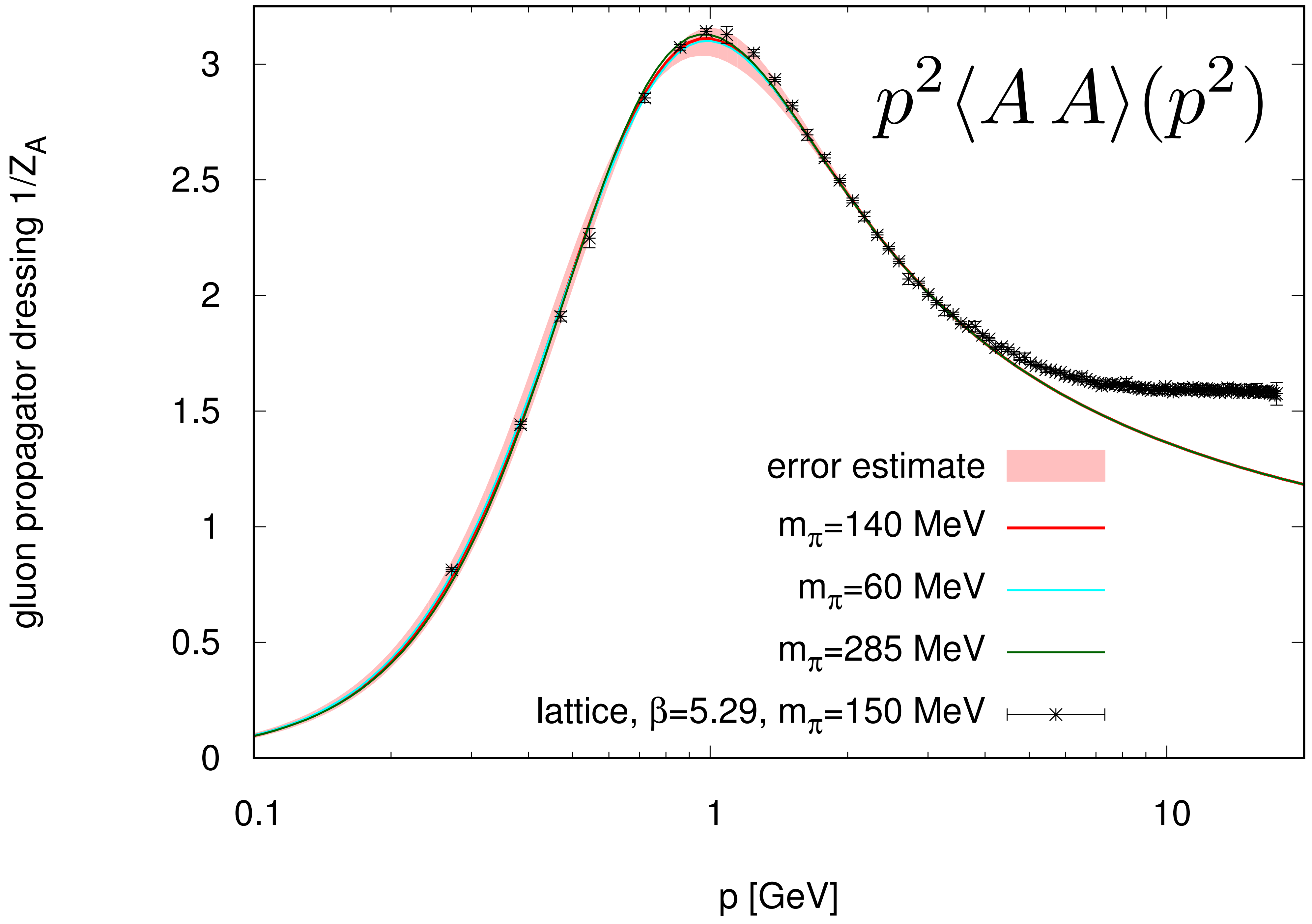
Sternbeck et al, PoS LATTICE2016 (2017)

A. Athenodorou et al, PLB 761 (2016) 444

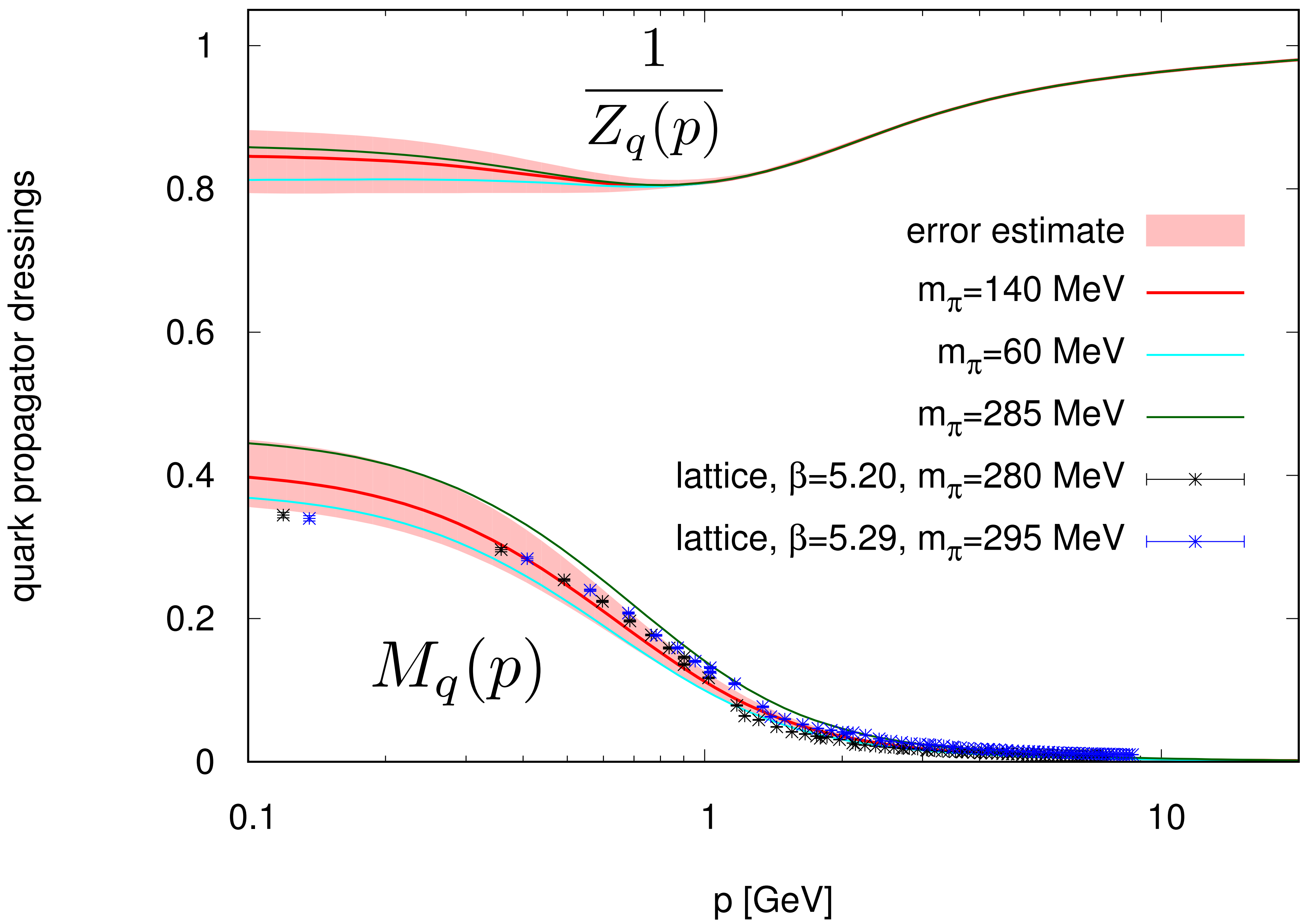
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Euclidean propagators

Two-flavour QCD



$$\frac{1}{Z_q(p)}\,\frac{1}{i\not{p}+M_q(p)}$$

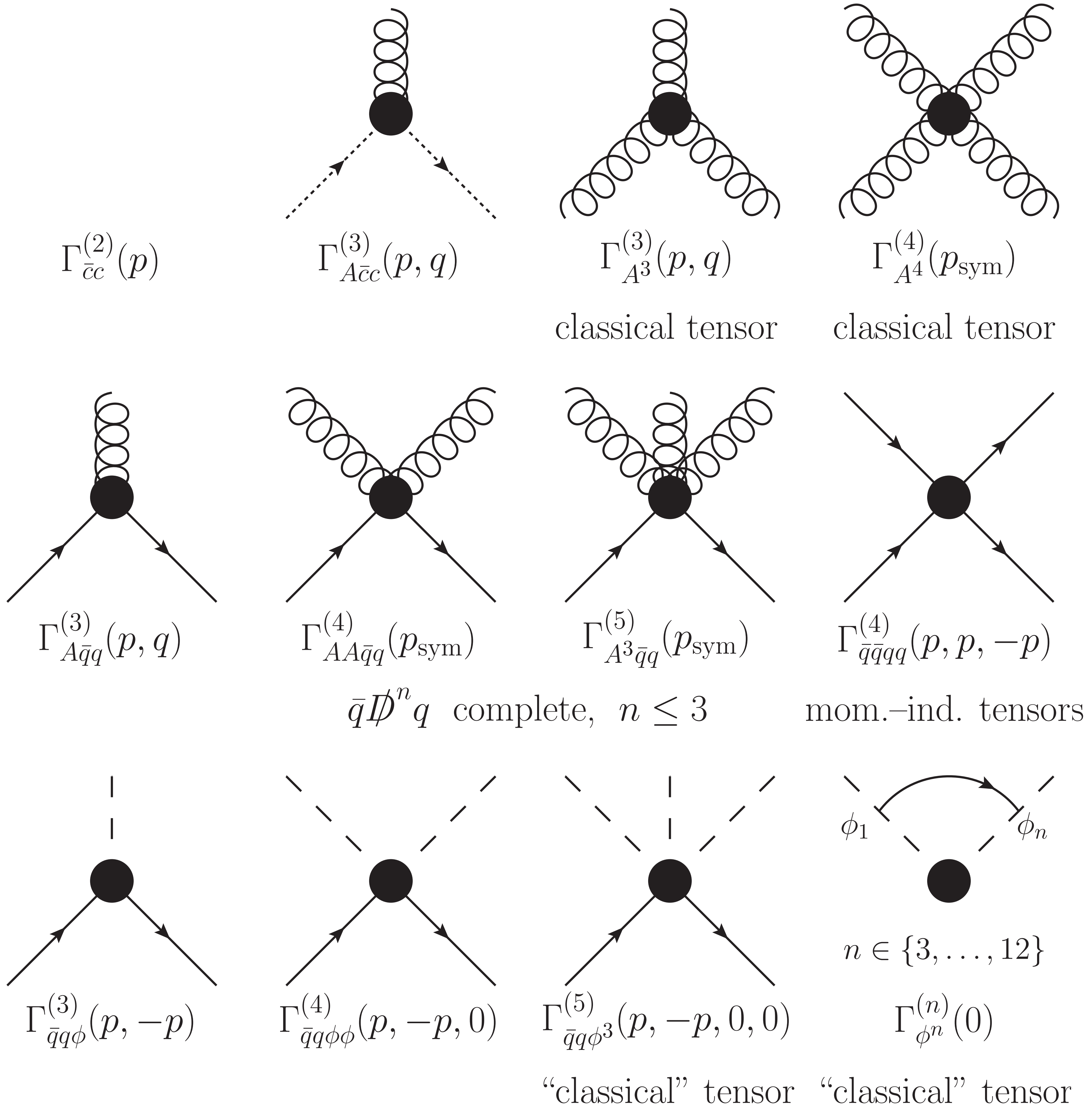


lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363
Sternbeck et al, PoS LATTICE2016 (2017)
A. Athenodorou et al, PLB 761 (2016) 444

simple correlations

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

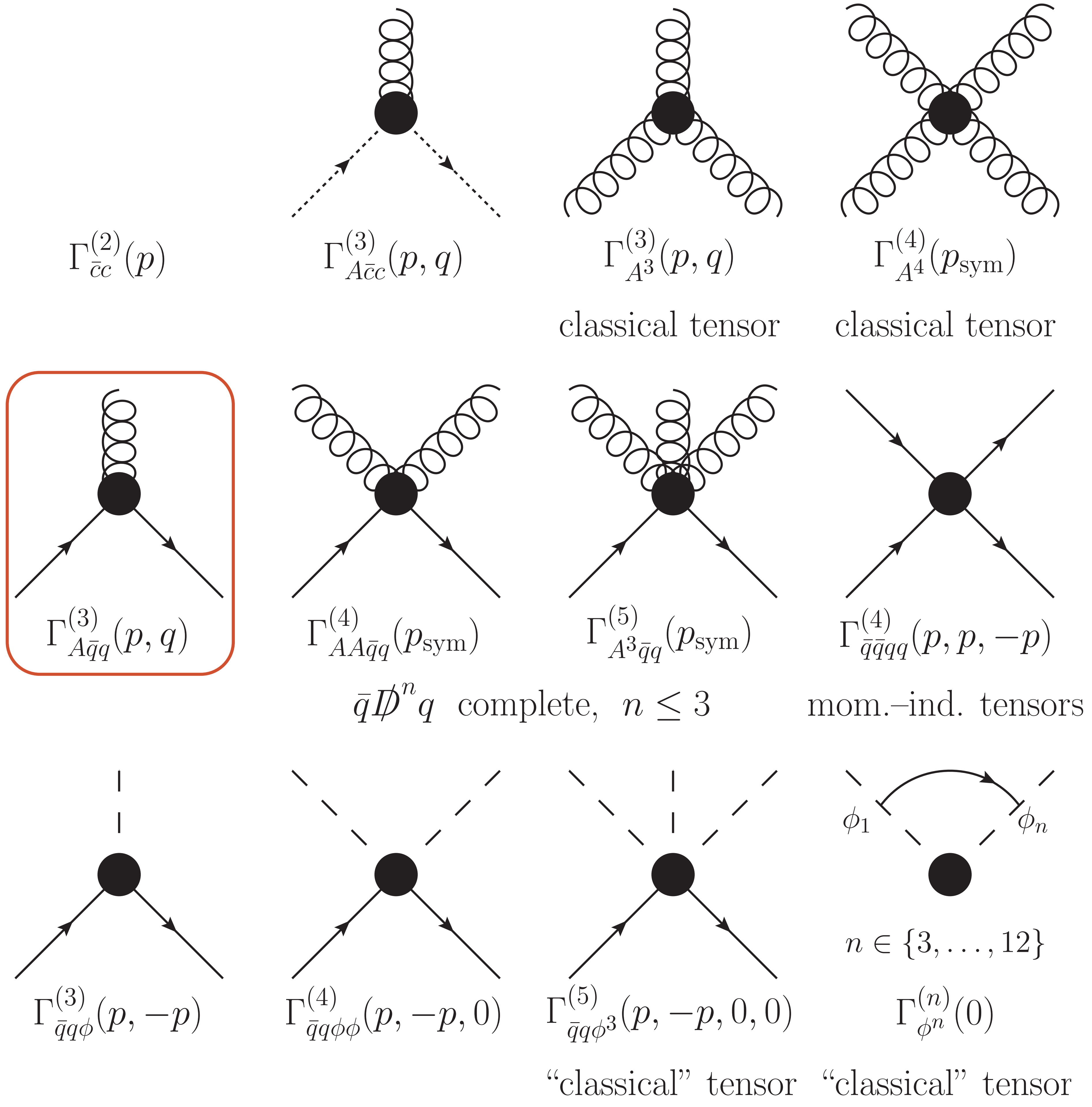
Vertices



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Vertices



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q:$ $\quad \left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$

$\bar{q}\not{D}^2 q:$ $\quad \left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, 1_{4\times 4}$

$$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p} - \not{q})\gamma_\mu$$

$$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p} + \not{q})\gamma_\mu$$

$\bar{q}\not{D}^3 q:$ $\quad \left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p} + \not{q})(p-q)_\mu$

$$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p} - \not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p}, \not{q}]\gamma_\mu$$

Aiming at apparent convergence

quenched: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

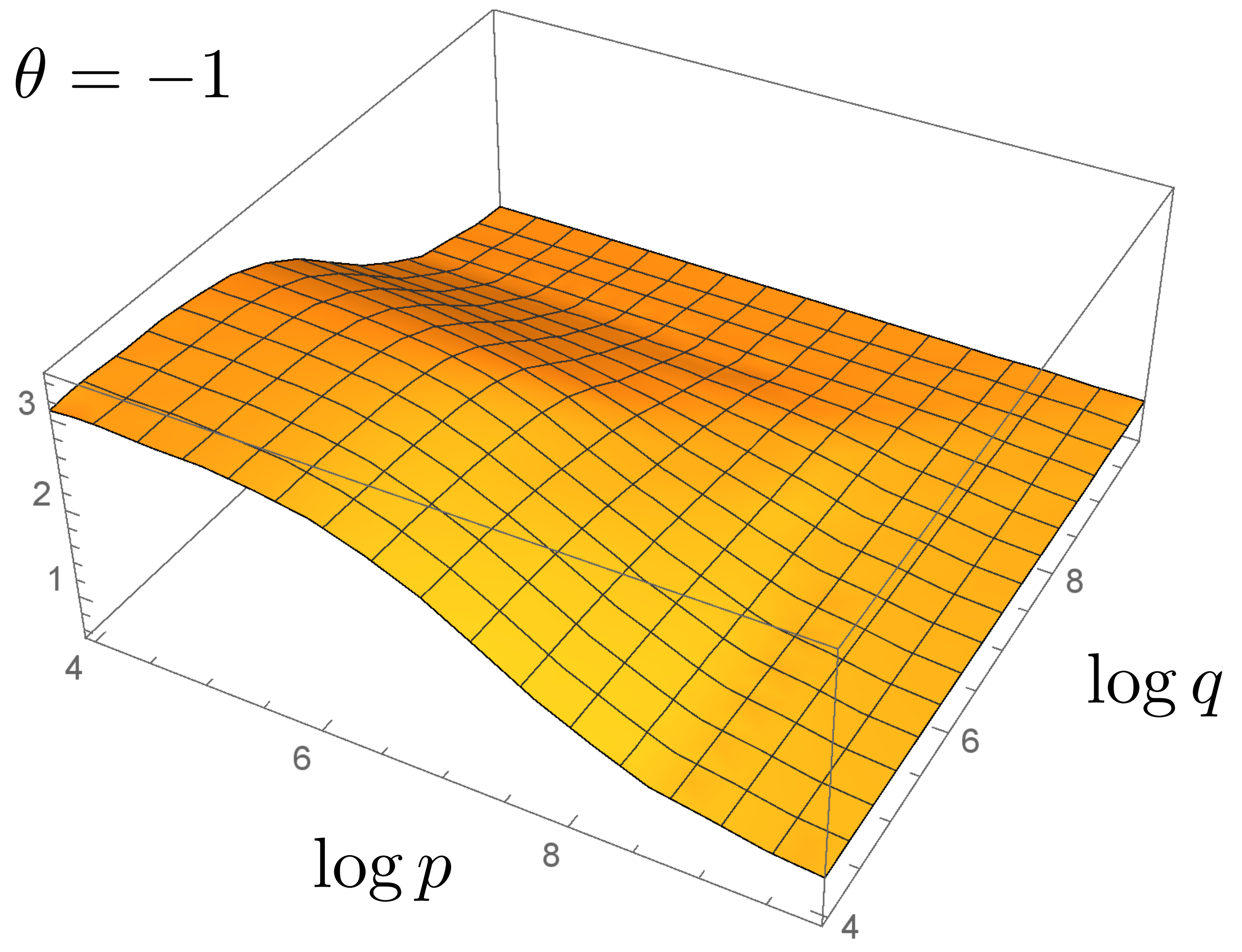
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

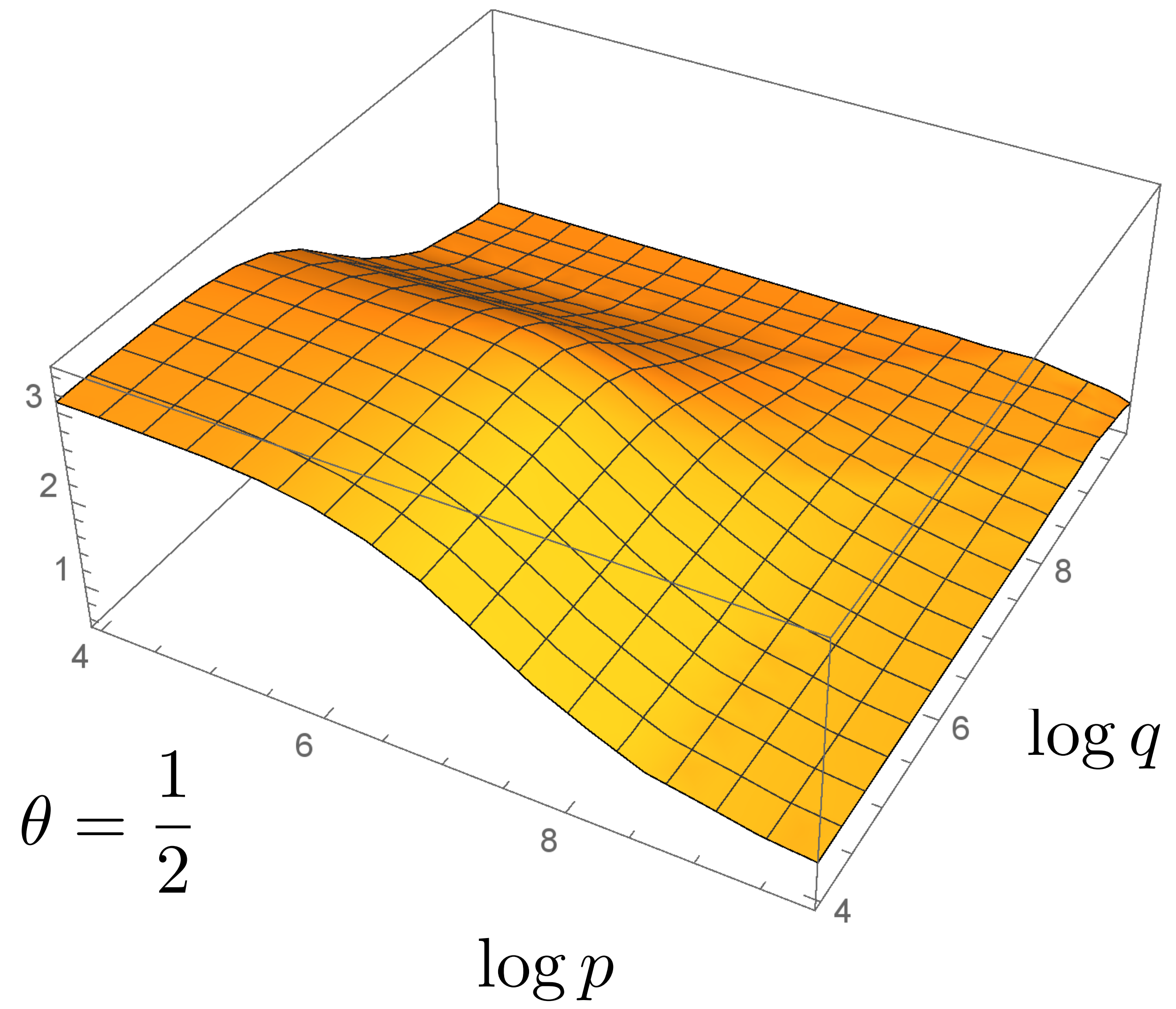
Quark-gluon vertex

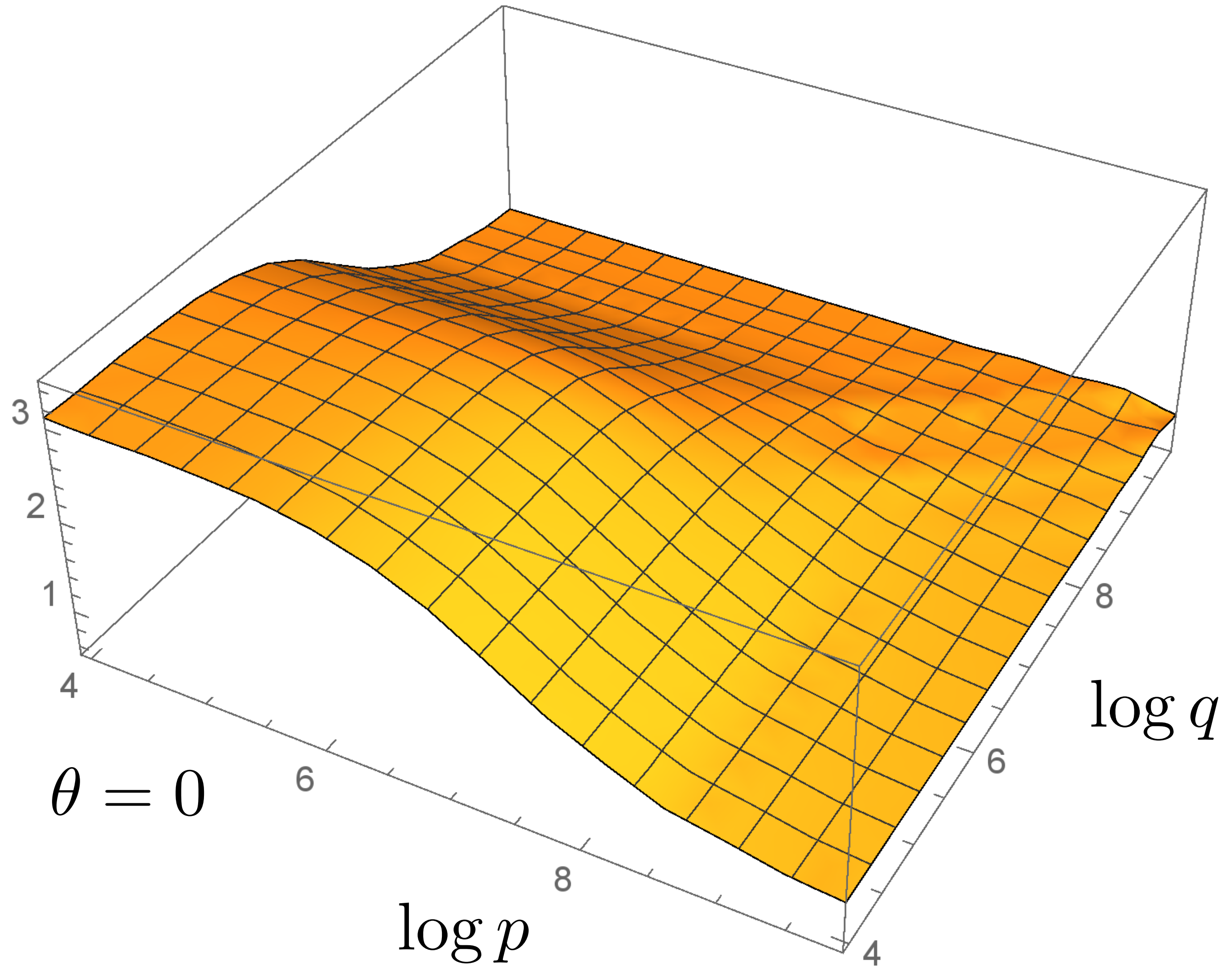
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

$\lambda_1(p, q)$







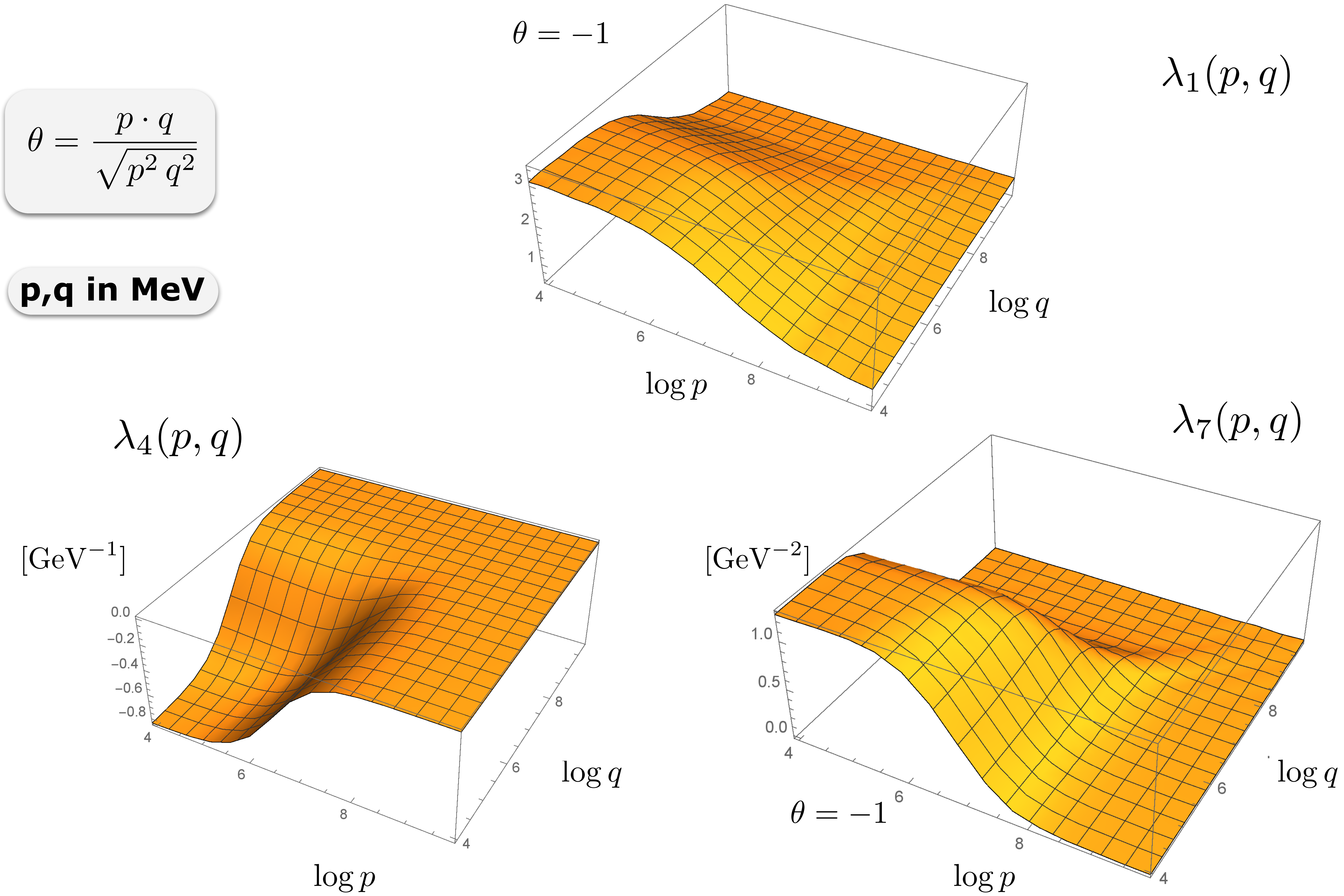
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

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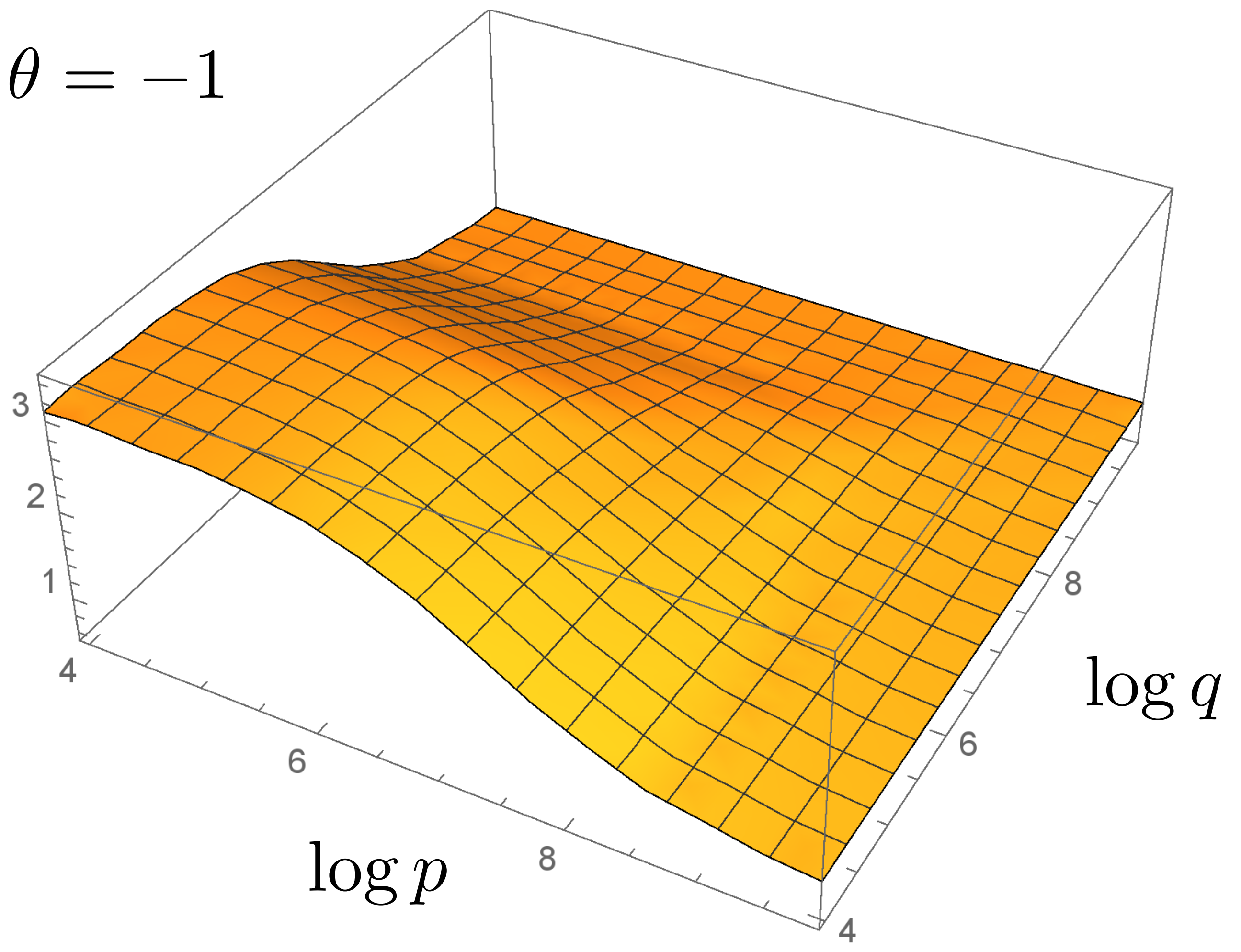


$\lambda_1(p, q)$

up-to-date 1st principles works:

FunMethods: **Williams, EPJ A51 (2015) 57**
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026

Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, arXiv:1610.06158

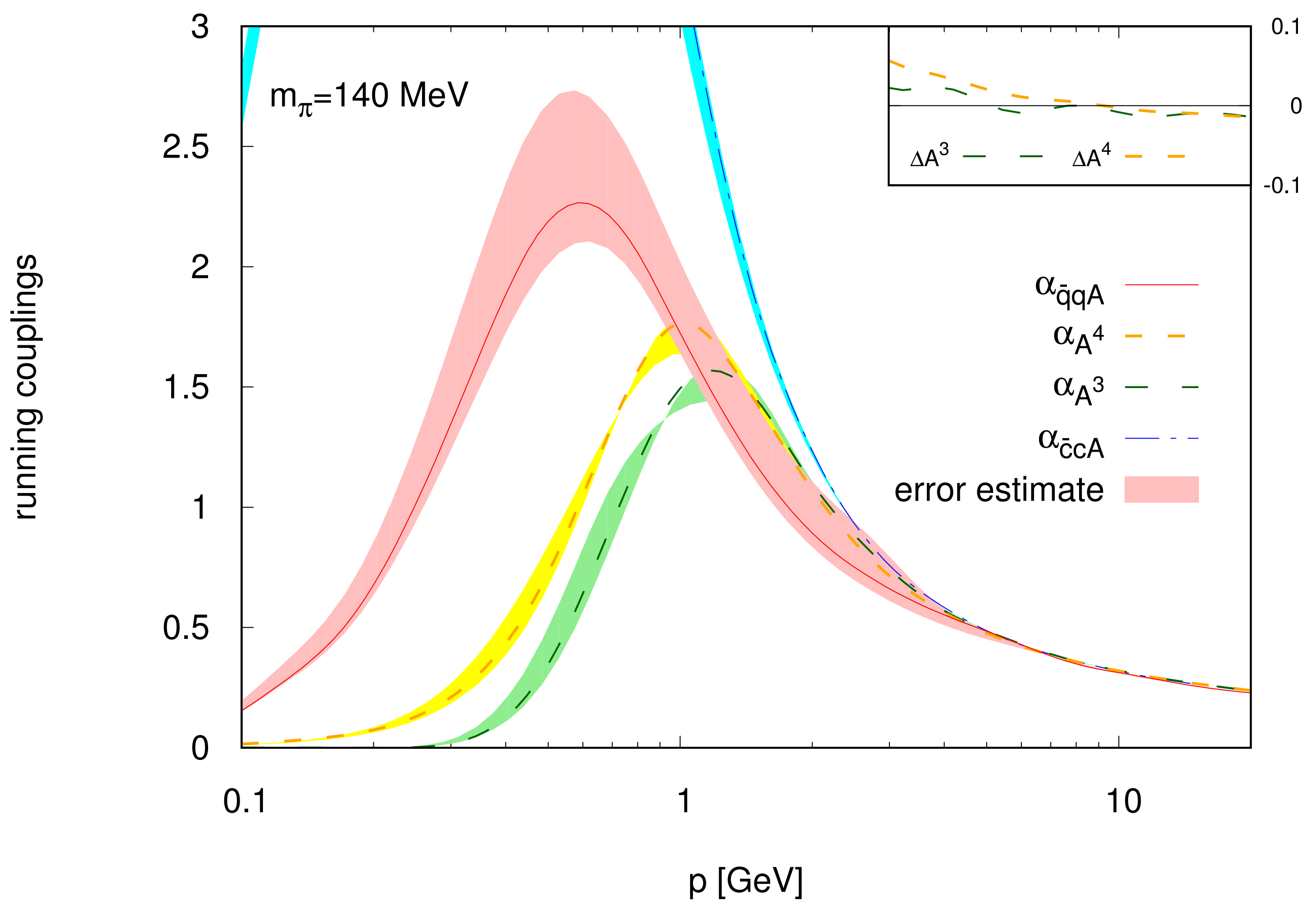
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**

Beware of BRST



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

fRG/DSE-assisted DSE/fRG

Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

More generally: X-assisted Y

X=fRG, DSE, nPI,
lattice, exp. data

Y=fRG, DSE, nPI

Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

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Example: use

(a) 2+1 fRG-assisted gluon

(b) optional: two-flavour fRG quark-gluon vertex

in 2+1 flavour DSE quark gap eq.

Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

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lattice, exp. data
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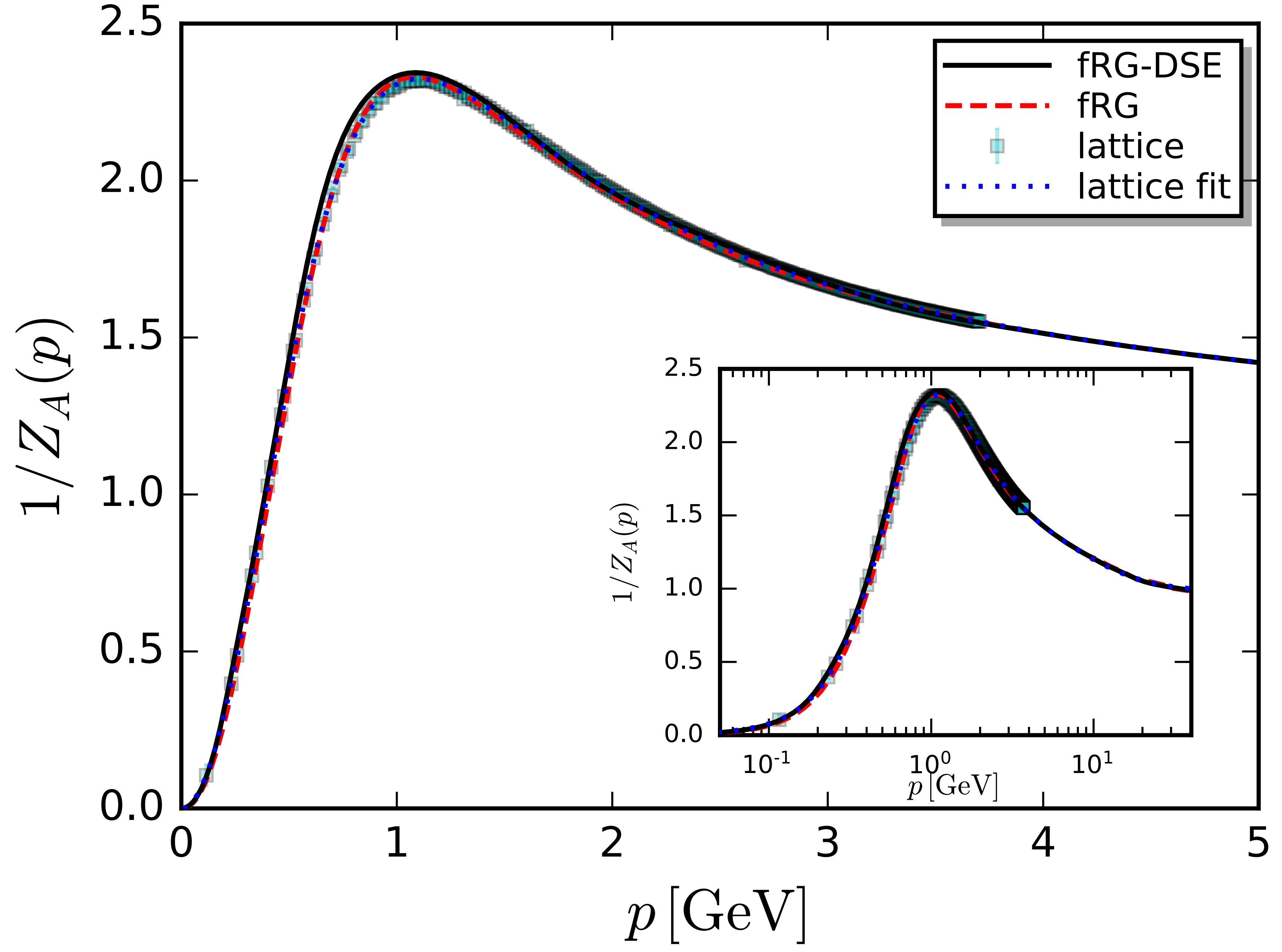
Input: 2+1 flavour gluon propagator



fRG: Fu, JMP, Rennecke, PRD 97 (2018) 054006

fRG-DSE: Gao, JMP, PLB 820 (2021) 136584
PRD 102 (2020) 034027

lattice: Zafeiropoulos et al, PRL 122 (2019) 16, 162002
Cui et al, CPC 44 (2020) 8, 083102

FunResults based on:

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

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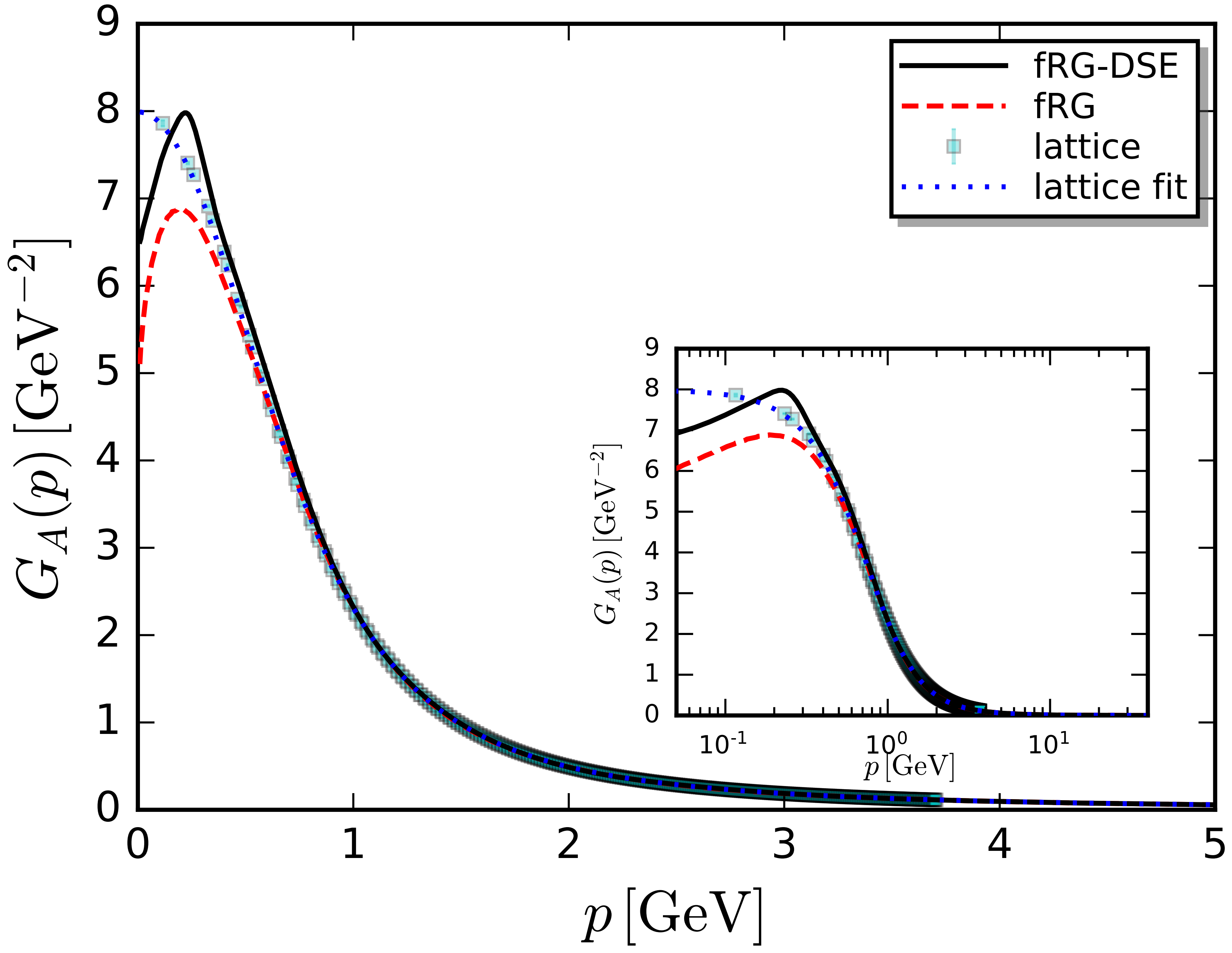

fRG: Fu, JMP, Rennecke, PRD 97 (2018) 054006

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PRD 102 (2020) 034027

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Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

More generally: X-assisted Y

X=fRG, DSE, nPI, lattice, exp. data

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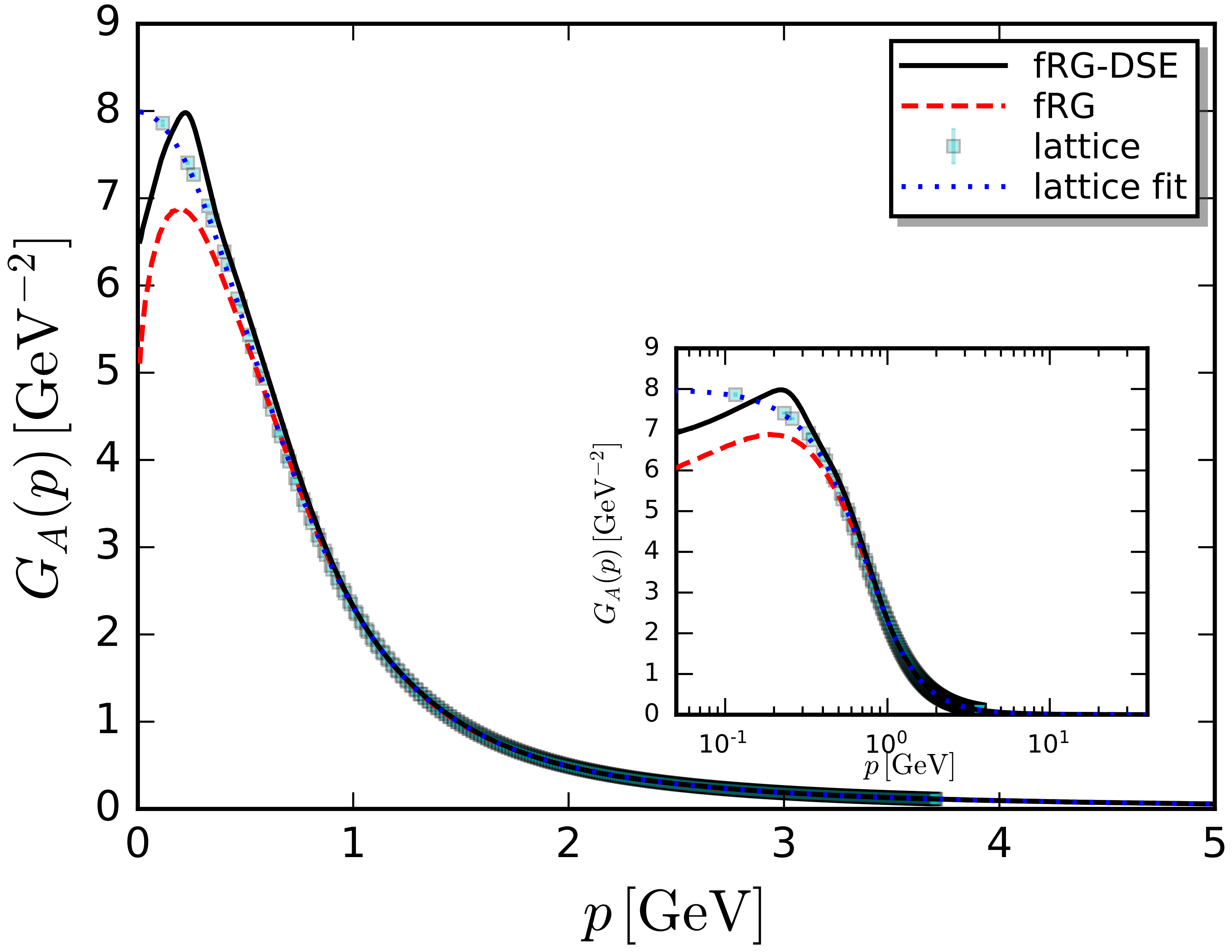


fRG: Fu, JMP, Rennecke, PRD 97 (2018) 054006

fRG-DSE: Gao, JMP, PLB 820 (2021) 136584
PRD 102 (2020) 034027

lattice: Zafeiropoulos et al, PRL 122 (2019) 16, 162002
Cui et al, CPC 44 (2020) 8, 083102

FunResults based on:

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Further example, e.g. lattice-assisted 2+1 flavour DSE

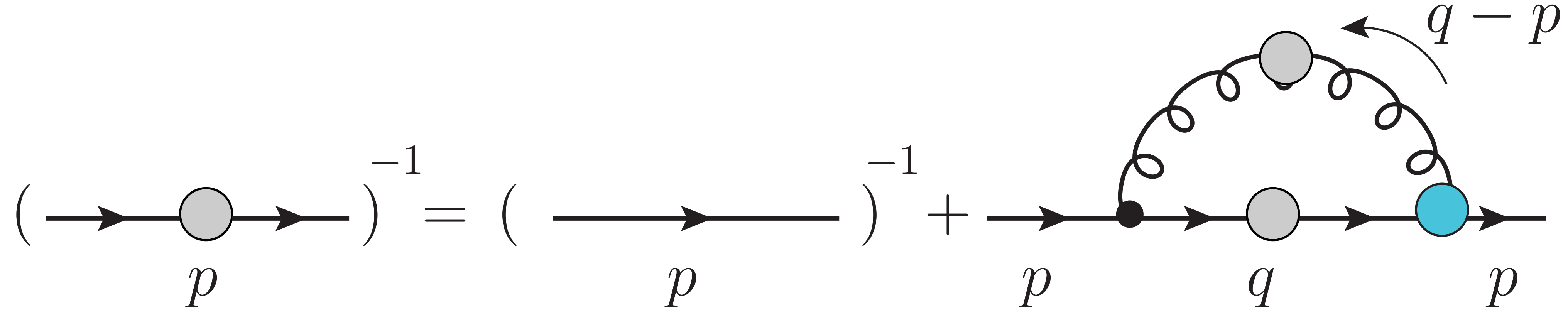
Aguilar et al, EPC 80 (2020) 2, 154

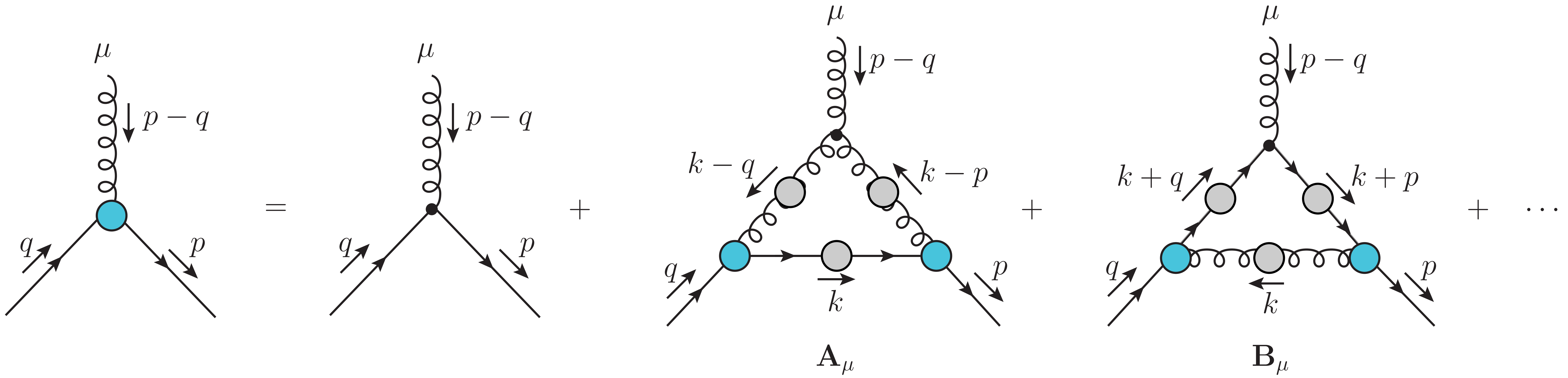
Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

Example: use (a) 2+1 fRG-assisted gluon (b) optional: two-flavour fRG quark-gluon vertex in 2+1 flavour DSE quark gap eq.

Full quark gluon





Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

Example: use

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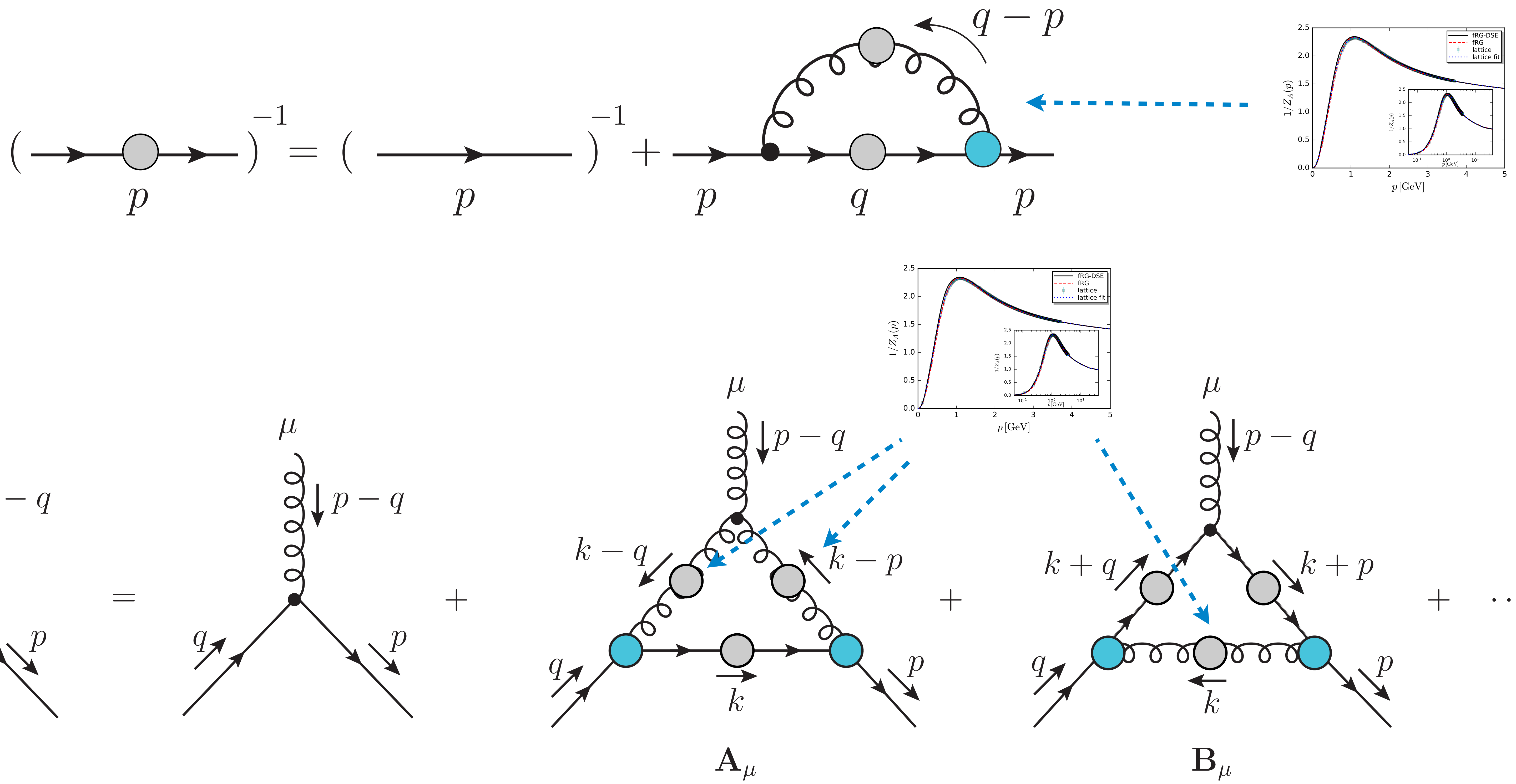
in 2+1 flavour DSE quark gap eq.

Full quark gluon

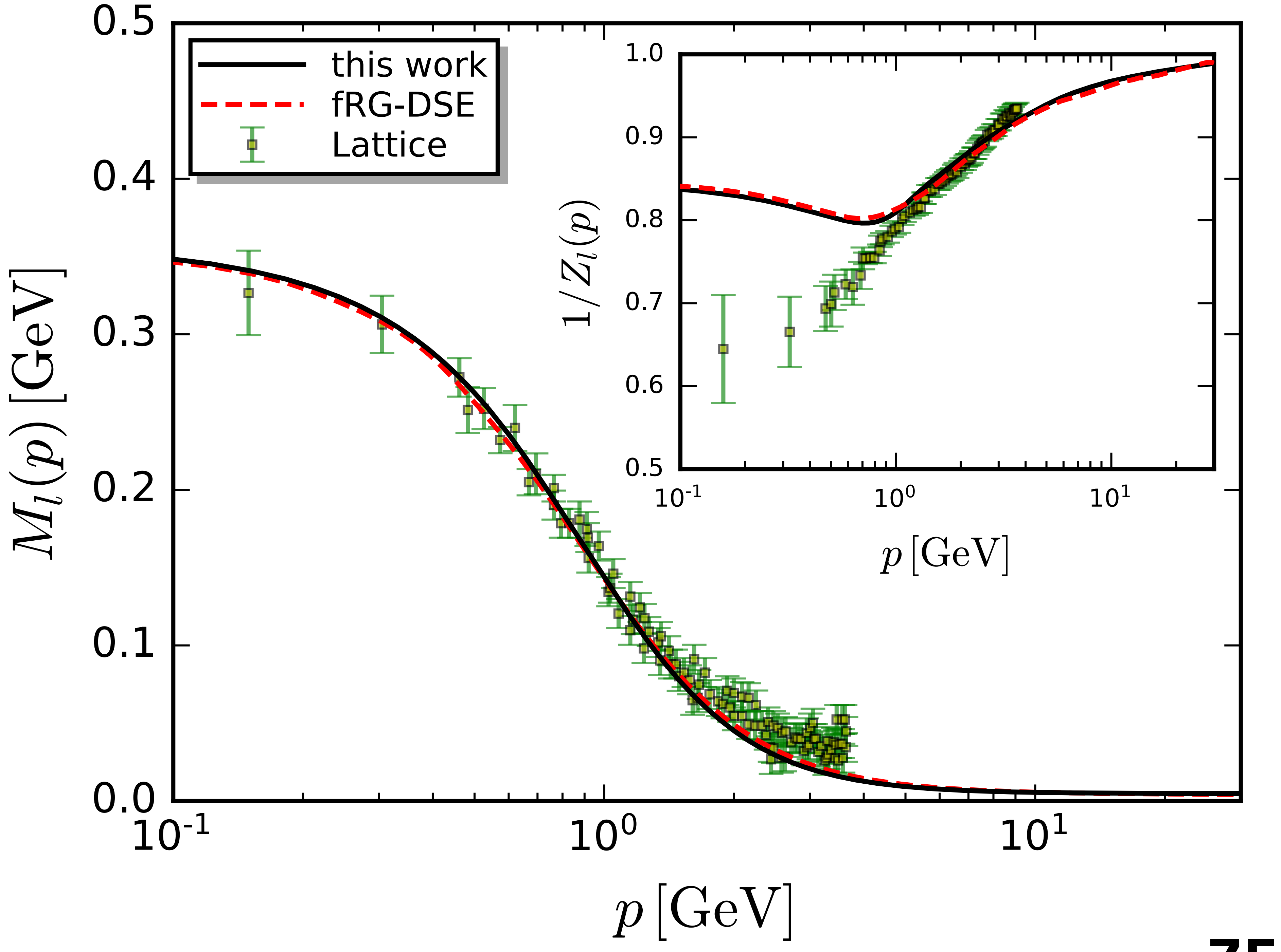


Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

Results

2+1 quark propagator



Chiral condensate

DSE: $\Delta_{l,\chi}(2\,\mathrm{GeV}) = (269.3(7)\,\mathrm{MeV})^3$

Lattice: $\Delta_{l,\chi}(2\,\mathrm{GeV}) = (272(5)\,\mathrm{MeV})^3$

FLAG: Aloki et al, EPJC 80 (2020) 2, 113

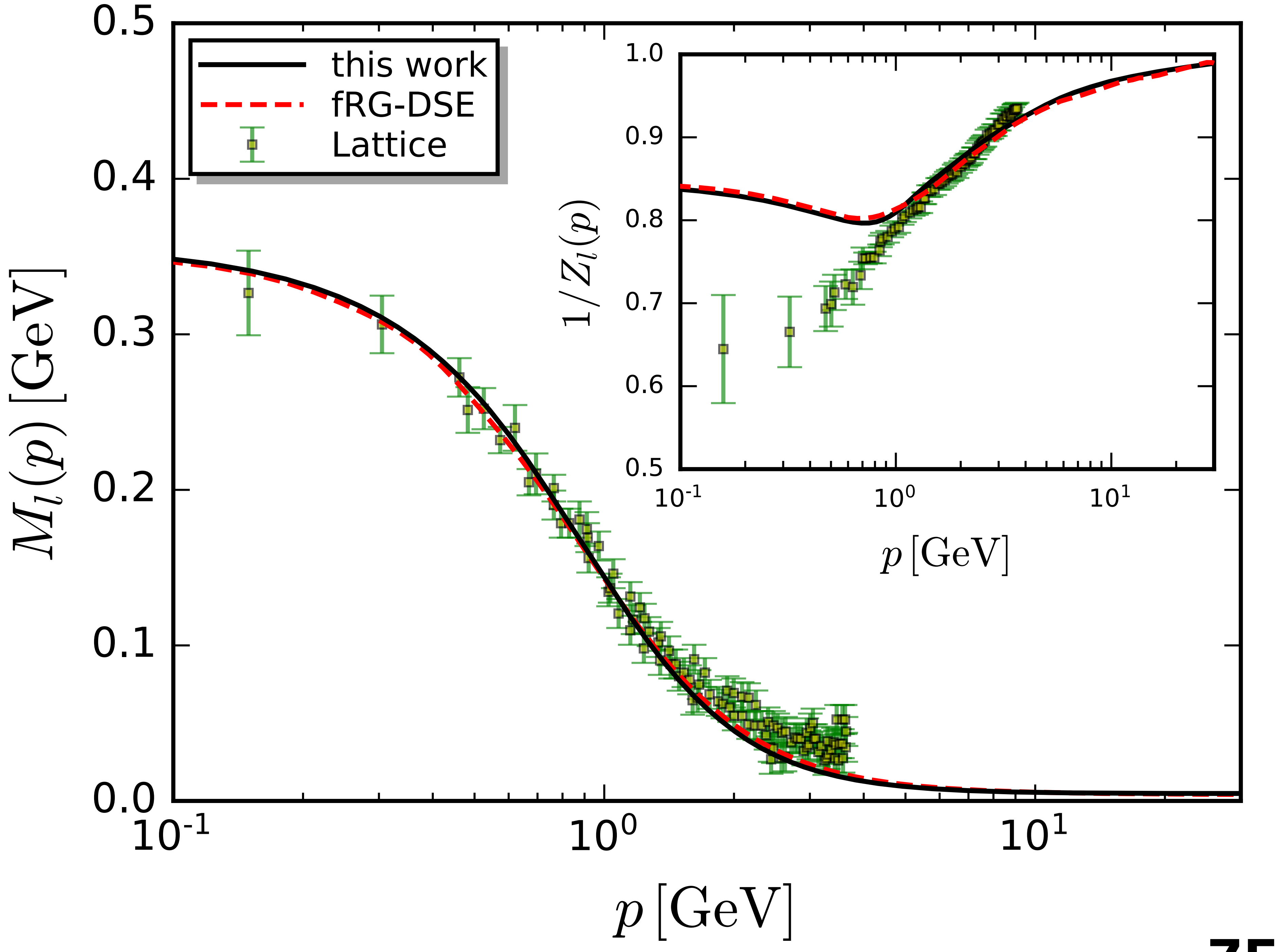
Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

fRG/DSE-assisted DSE/fRG

Example: use **(a) 2+1 fRG-assisted gluon** **(b) optional: two-flavour fRG quark-gluon vertex** **in 2+1 flavour DSE quark gap eq.**

Results

2+1 quark propagator



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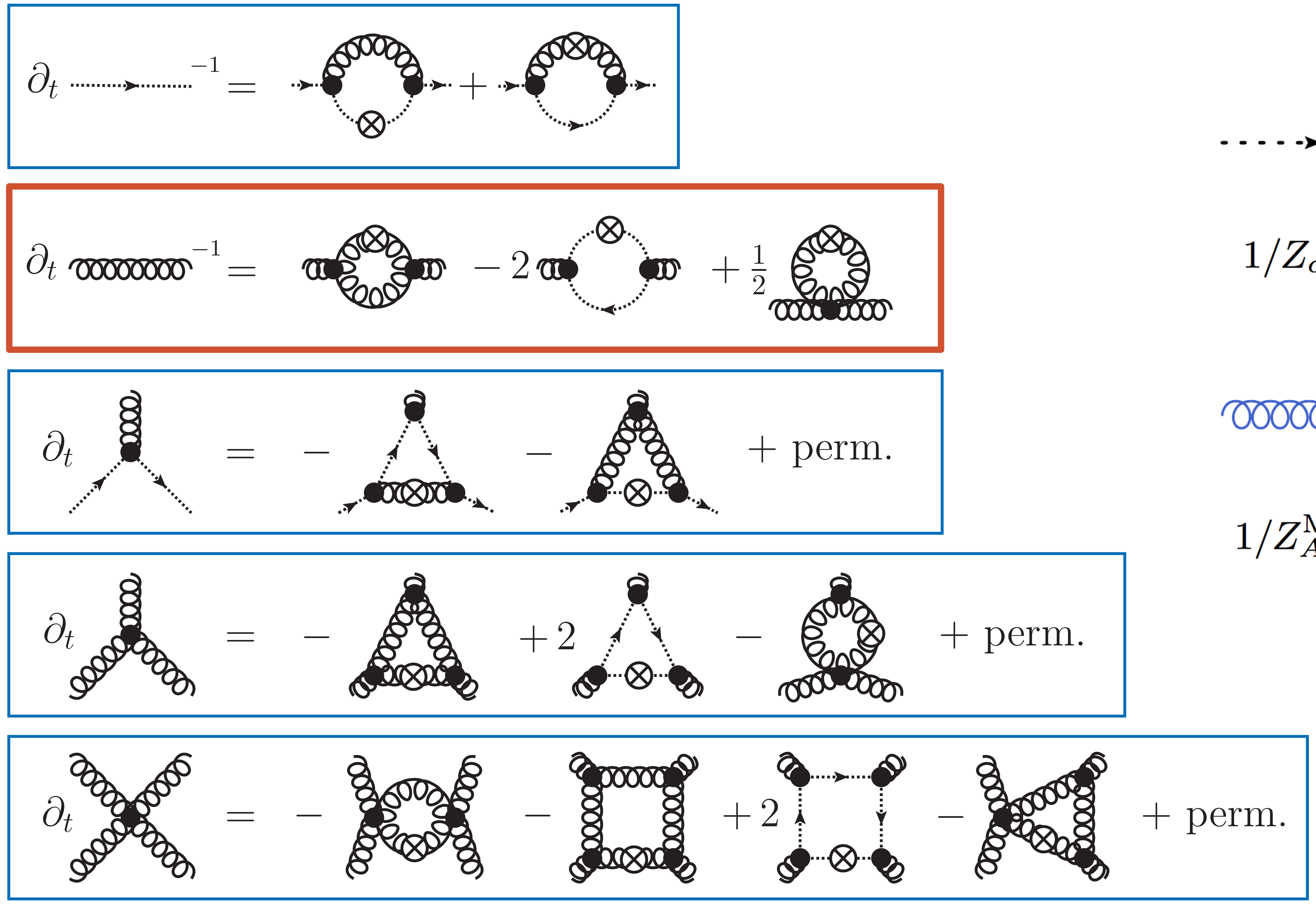
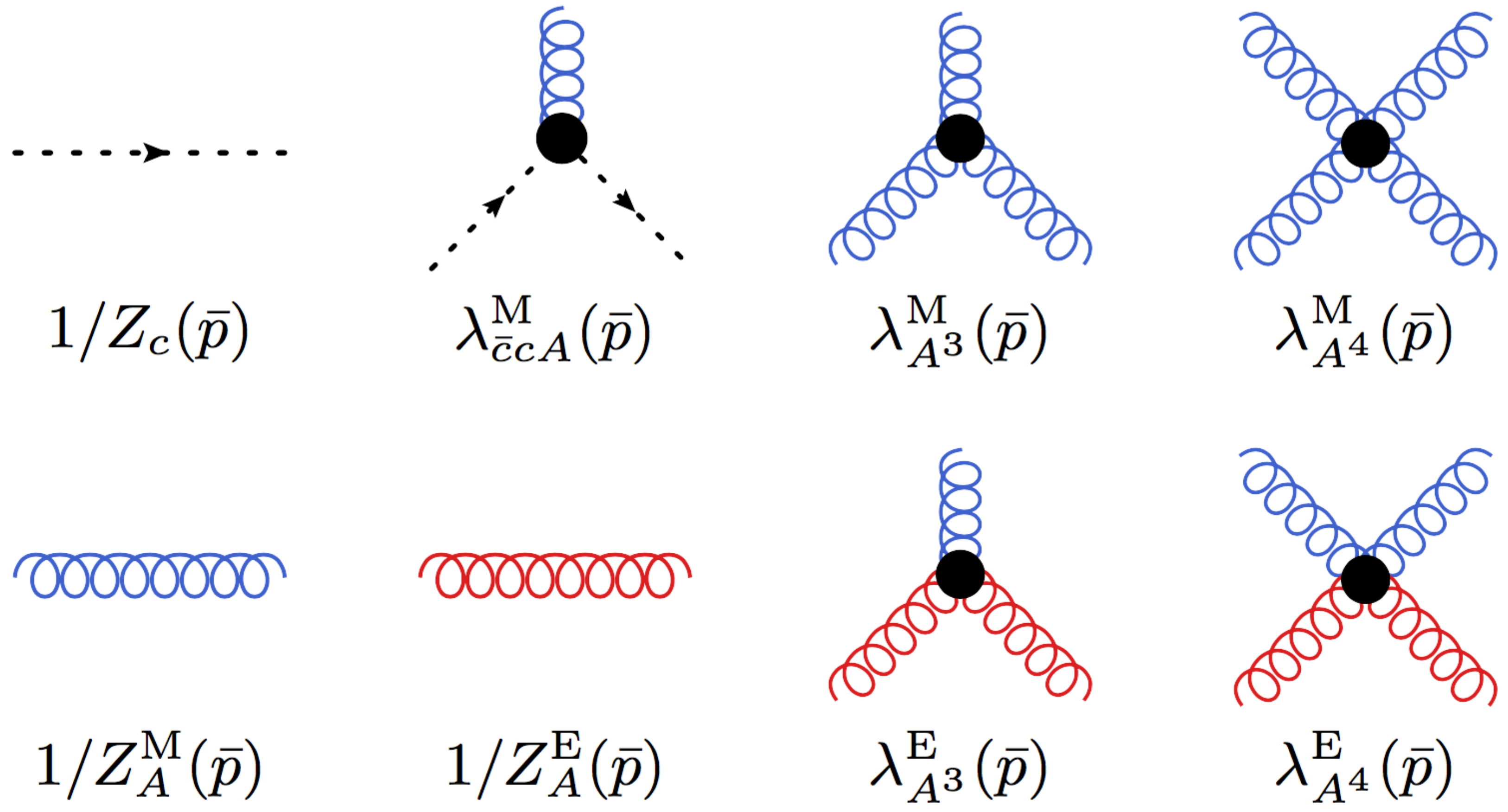
FLAG: Aloki et al, EPJC 80 (2020) 2, 113

Gao, Papavassiliou, JMP, PRD 103 (2021) 094013

Correlation functions at finite temperature

YM-theory: gluonic correlation functions

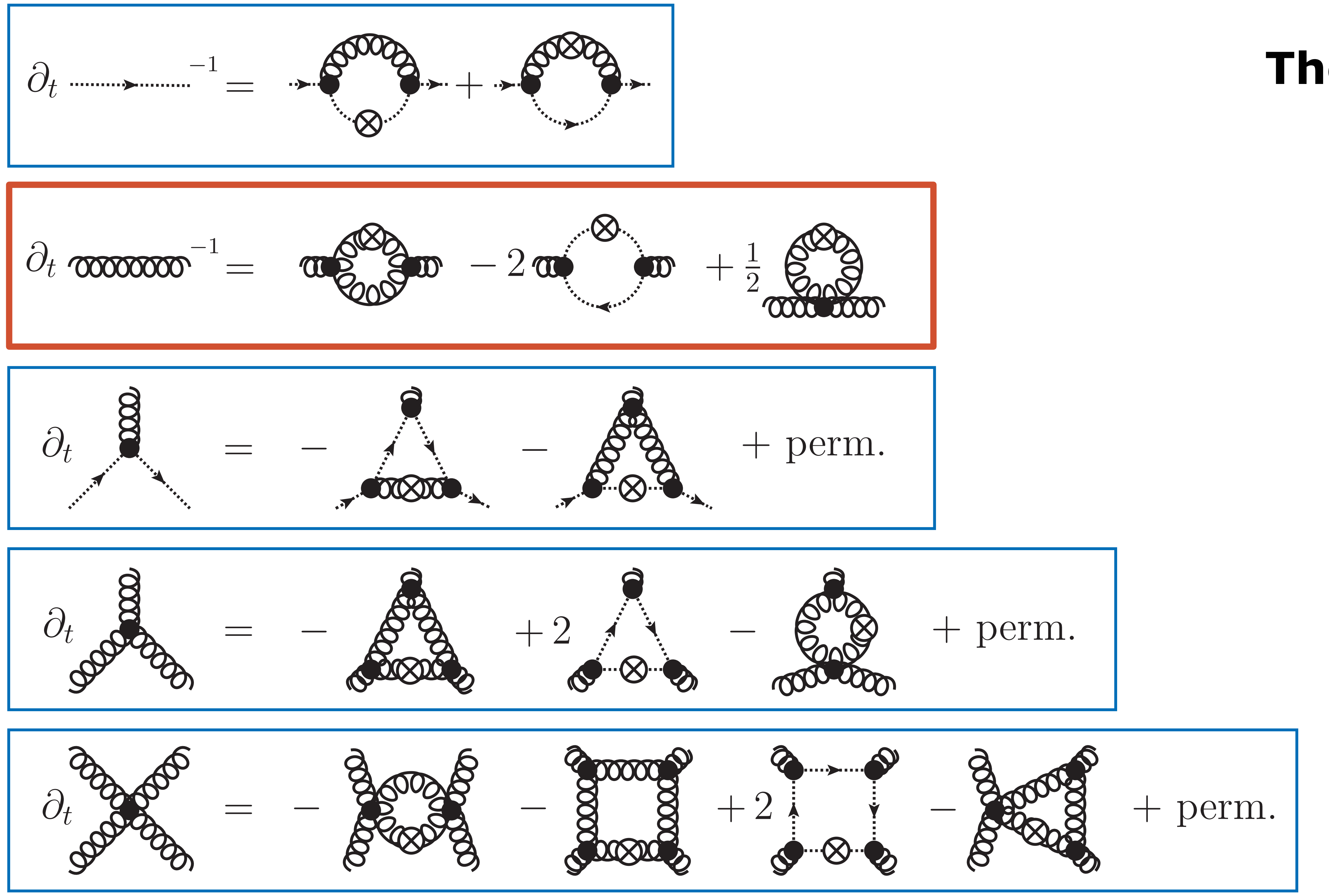




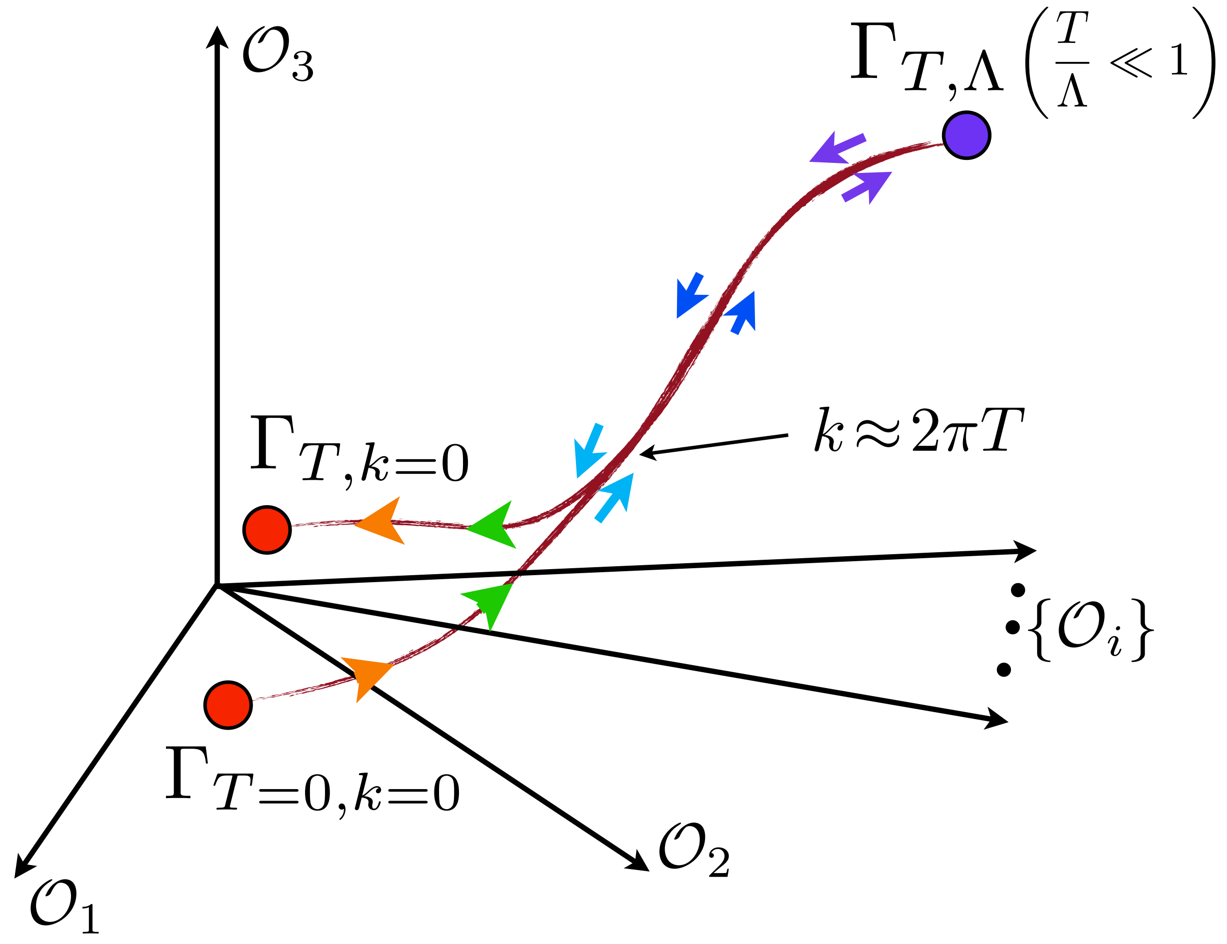
Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

YM-theory: gluonic correlation functions



Thermal flows

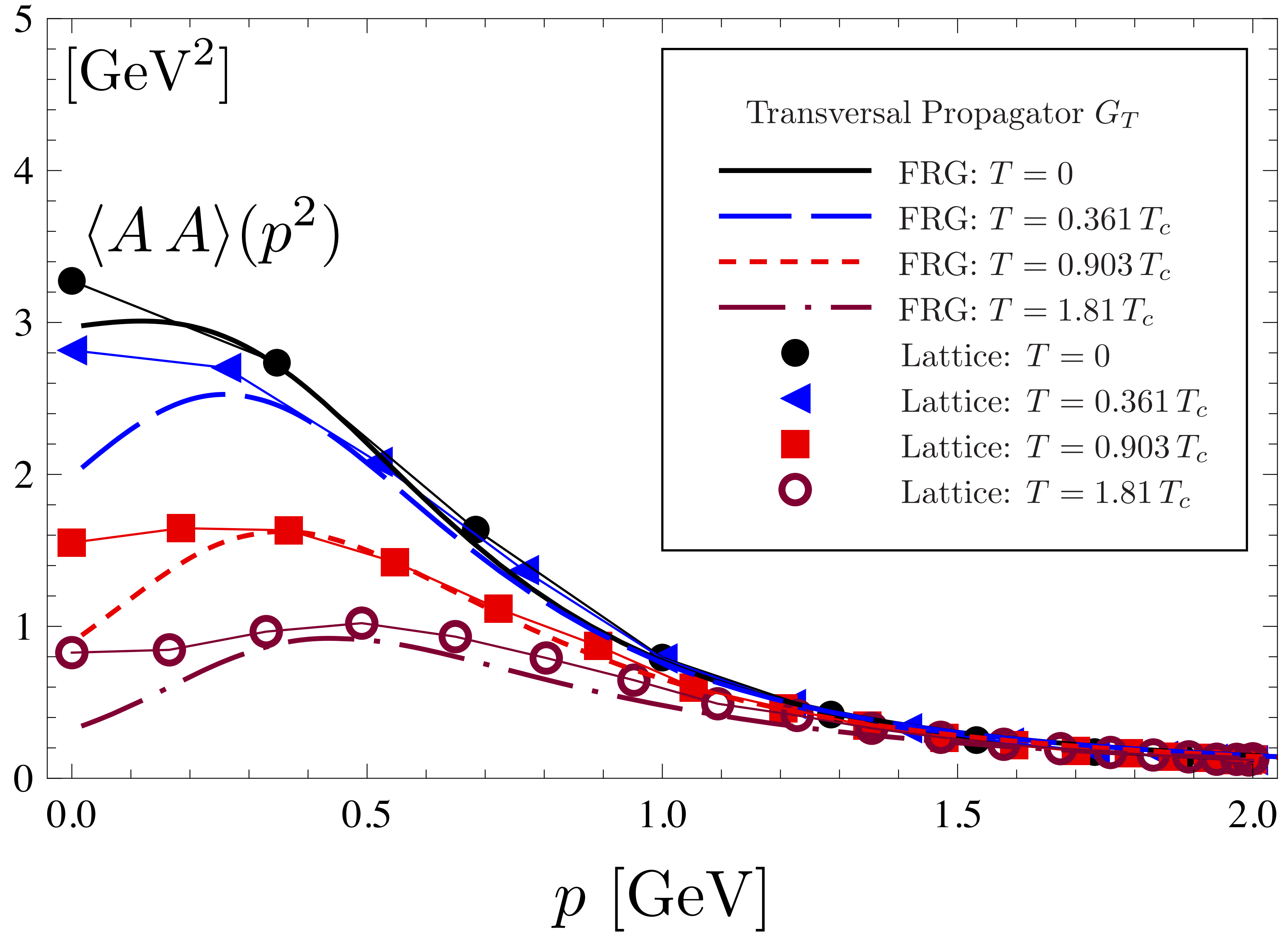


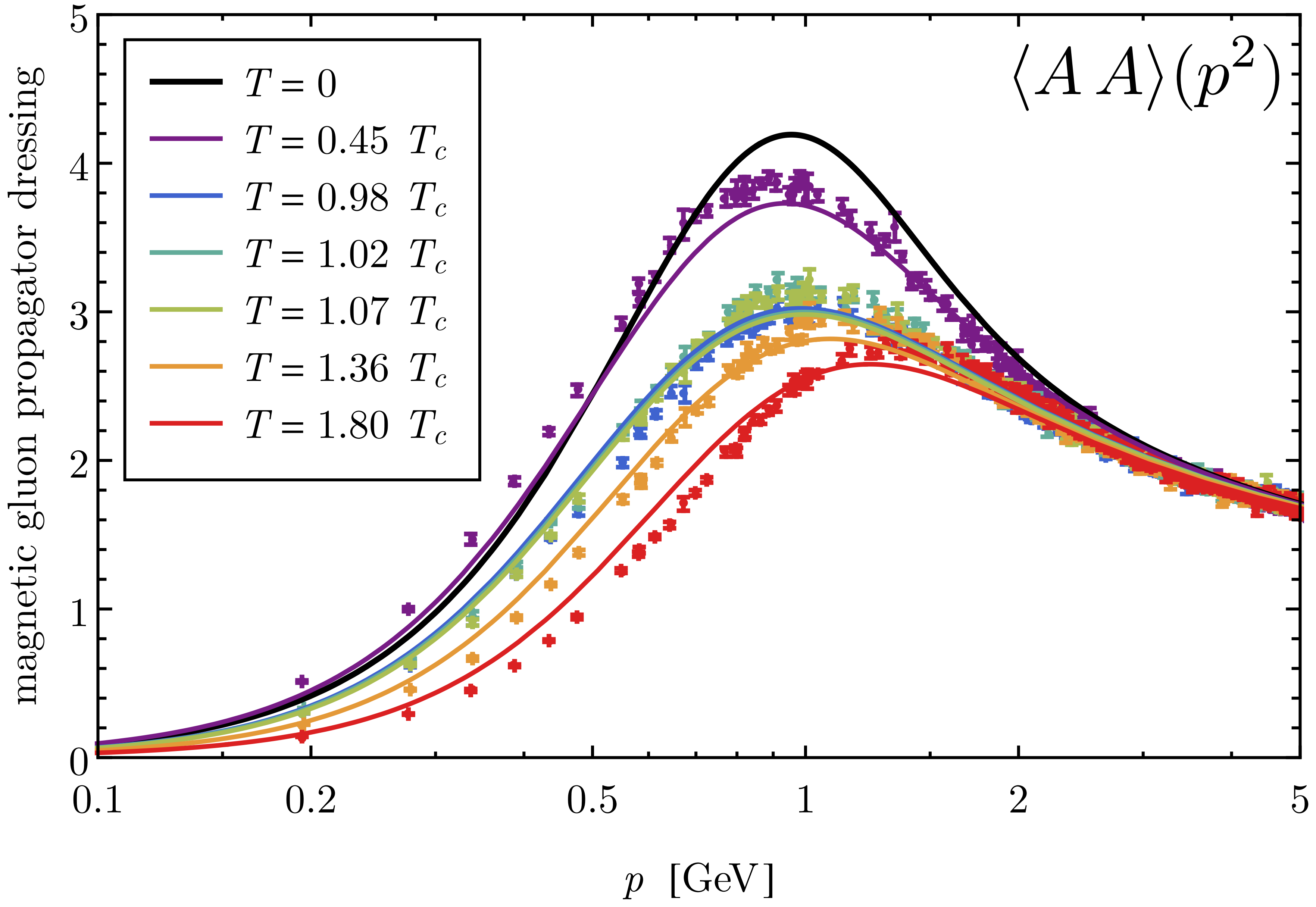
Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

Euclidean gluon propagator at finite T

chromo-magnetic propagator





Fister, JMP, arXiv:1112.5440

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

Lattice: Silva, Oliviera, Bicudo, Cardoso, PRD89 (2014) 7, 074503

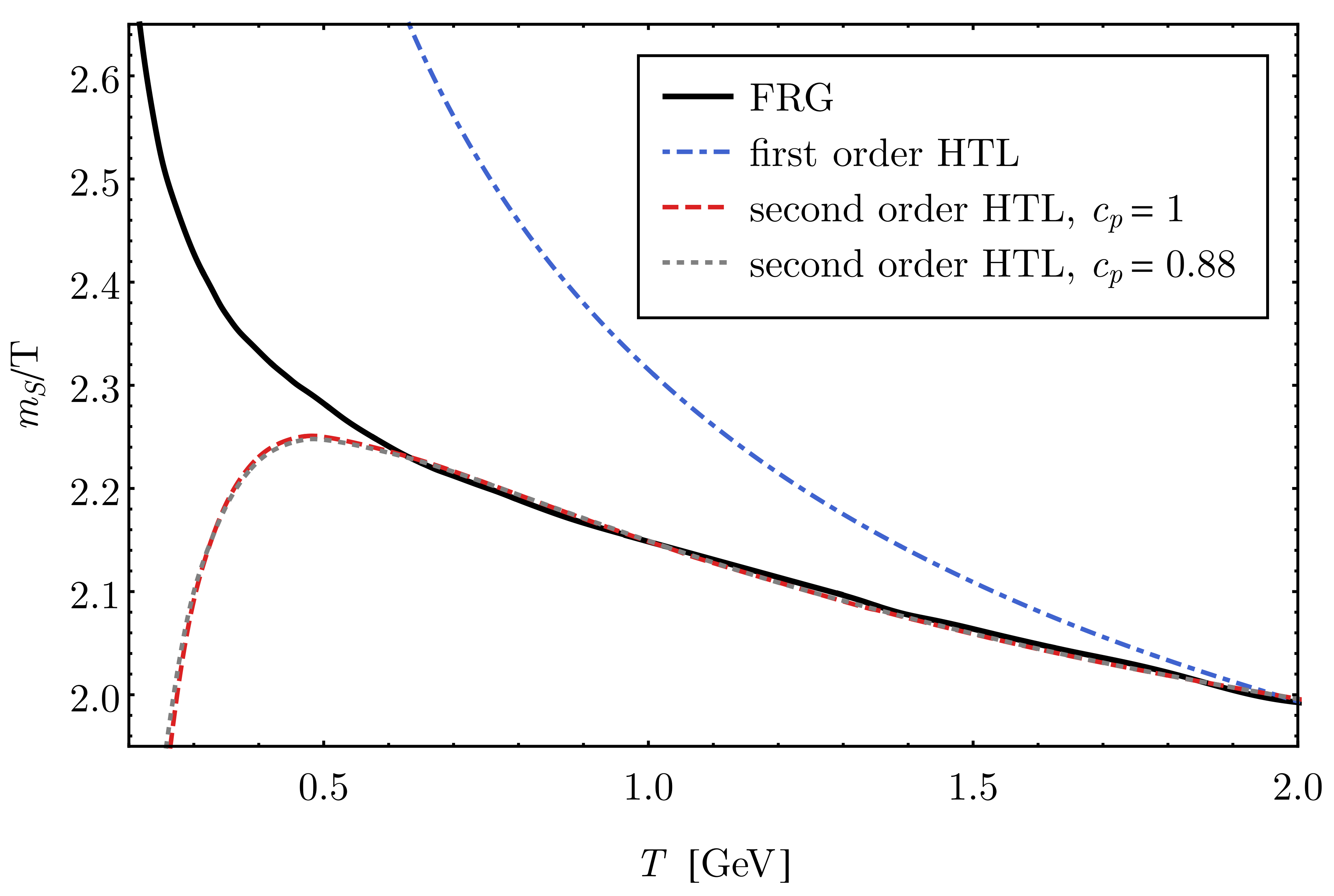
CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

Aiming at apparent convergence

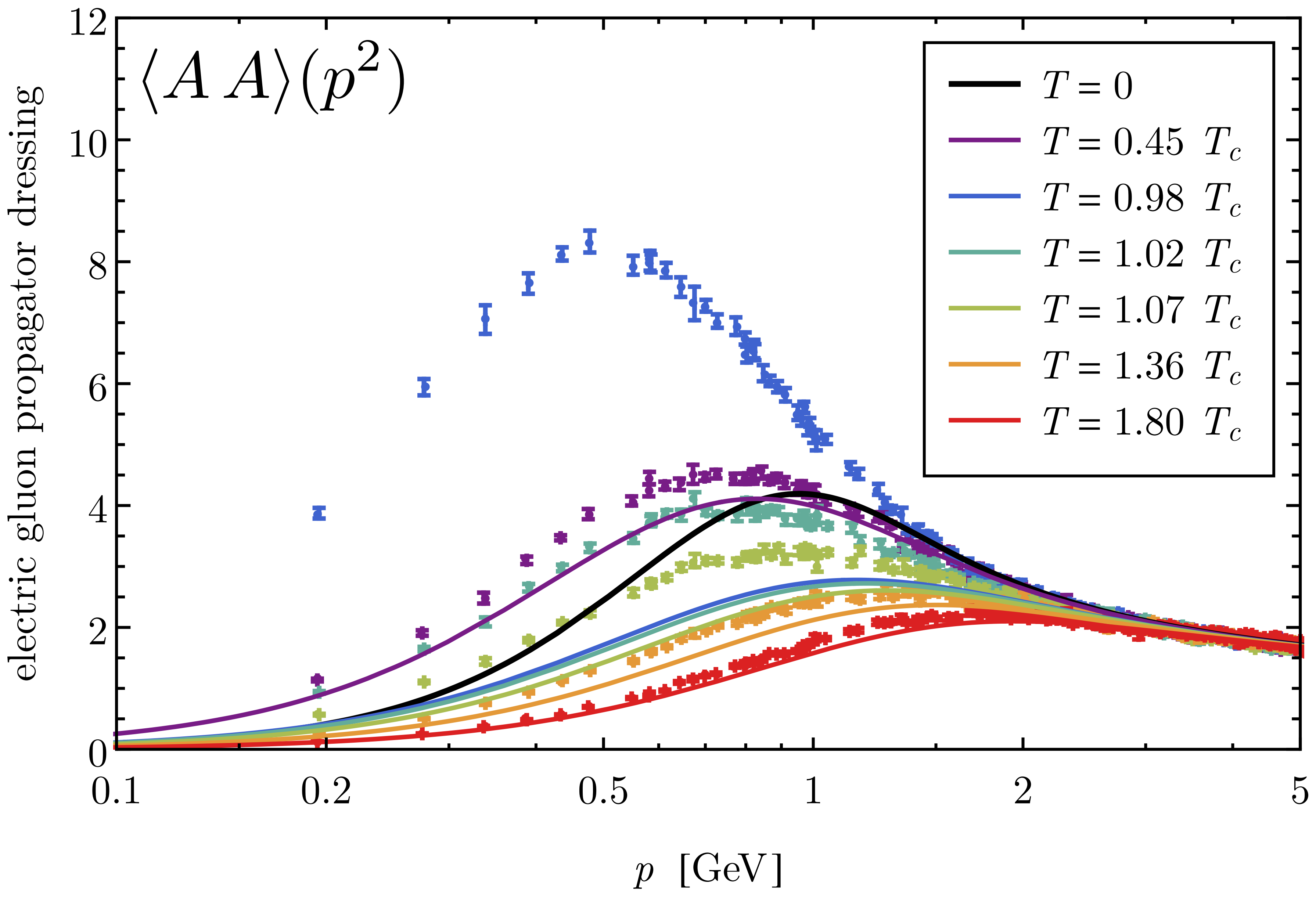
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

Euclidean gluon propagator at finite T

Debye mass (chromo-electric)



chromo-electric propagator



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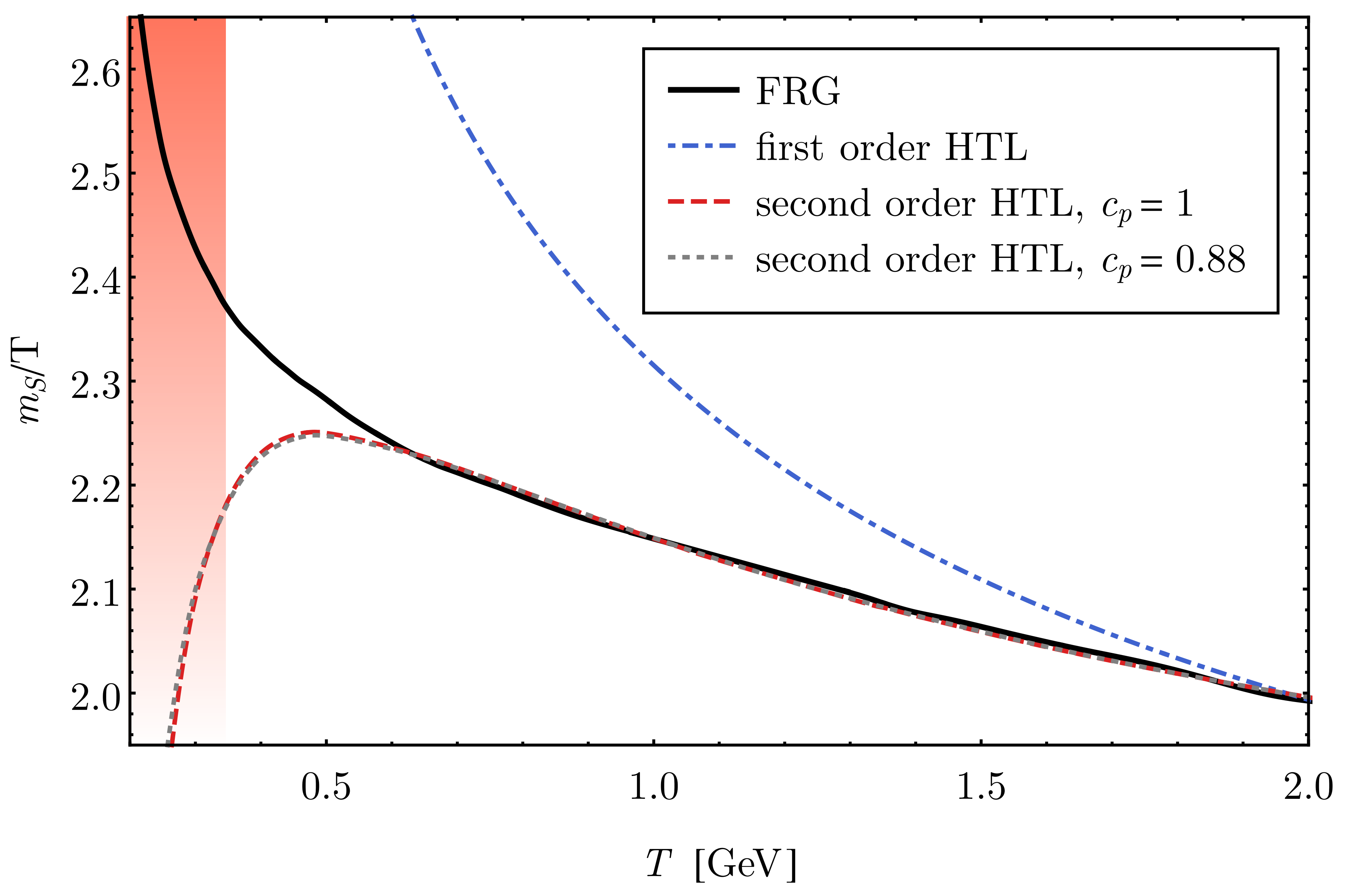
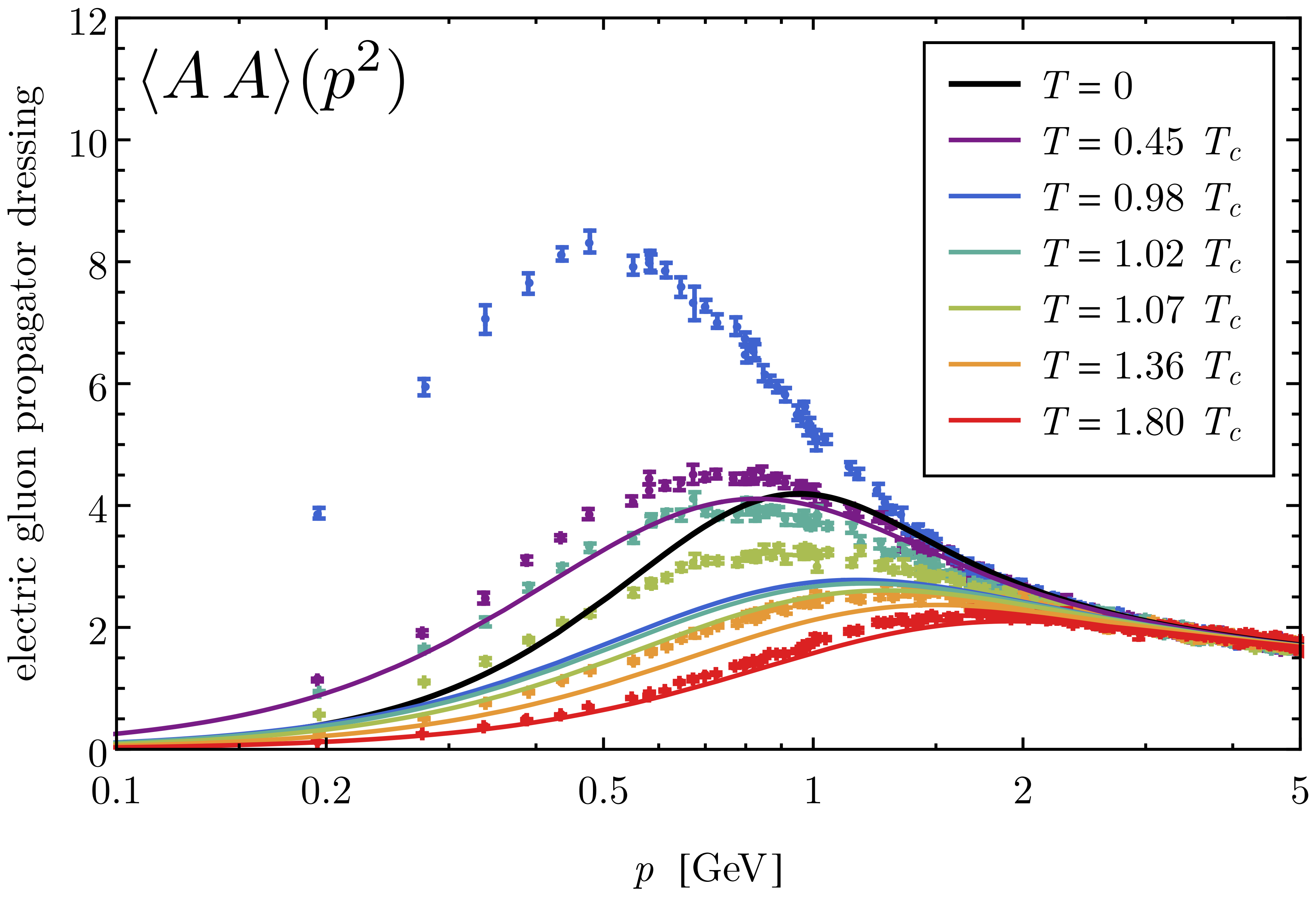


$\langle A_0 \rangle \neq 0$

chromo-electric propagator



Lattice: Silva, Oliveira, Bicudo, Cardoso, PRD89 (2014) 7, 074503

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 5, 054015

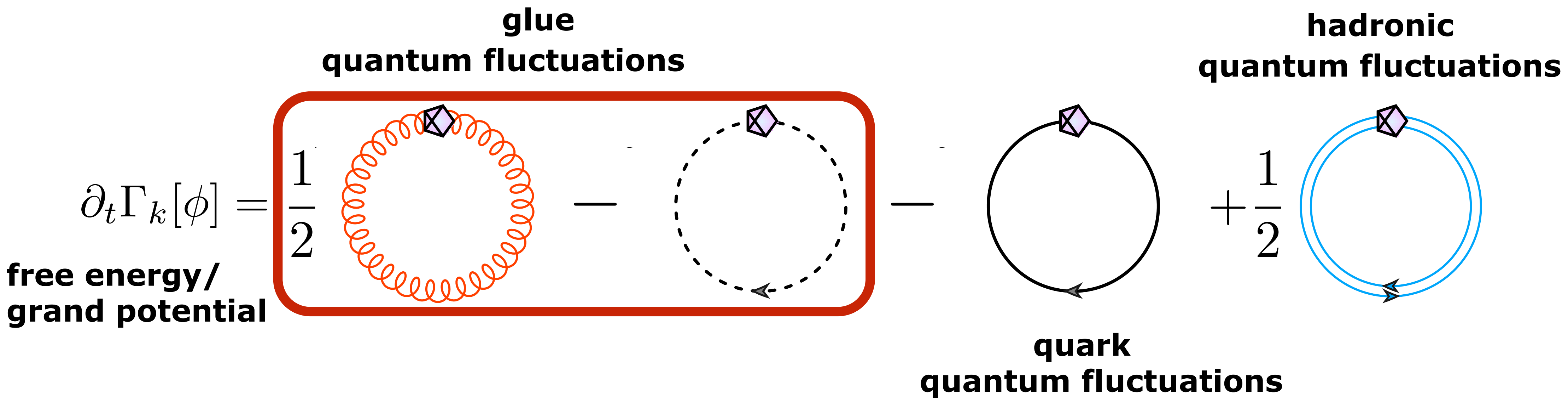
Polyakov loop from functional approaches

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}}\mathbf{tr}\,\mathcal{P}\mathbf{e}^{\mathbf{i\,g}\int_0^\beta \mathbf{A_0(x)}}$$

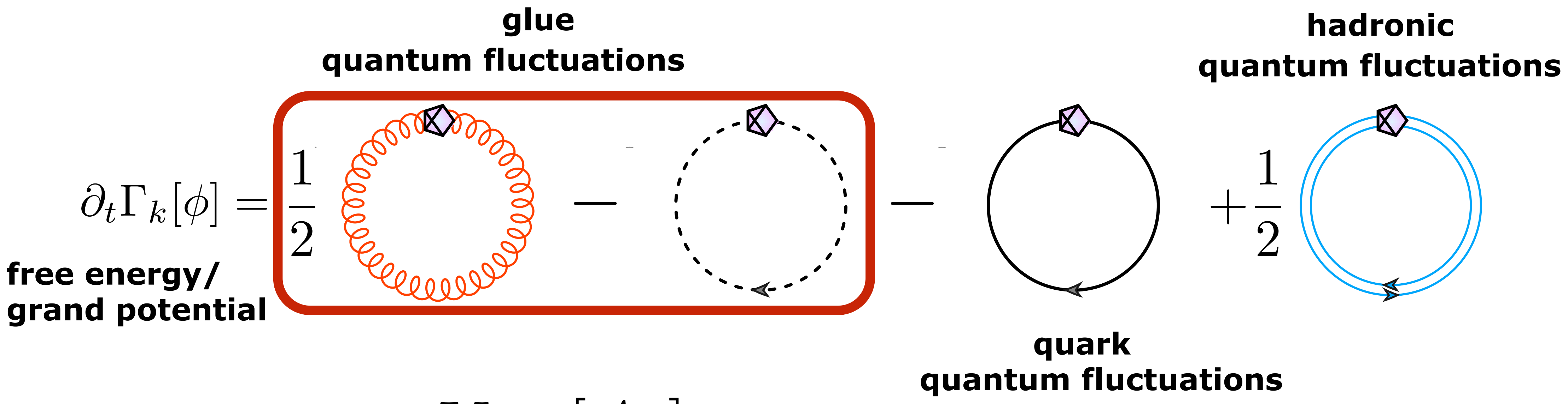


Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

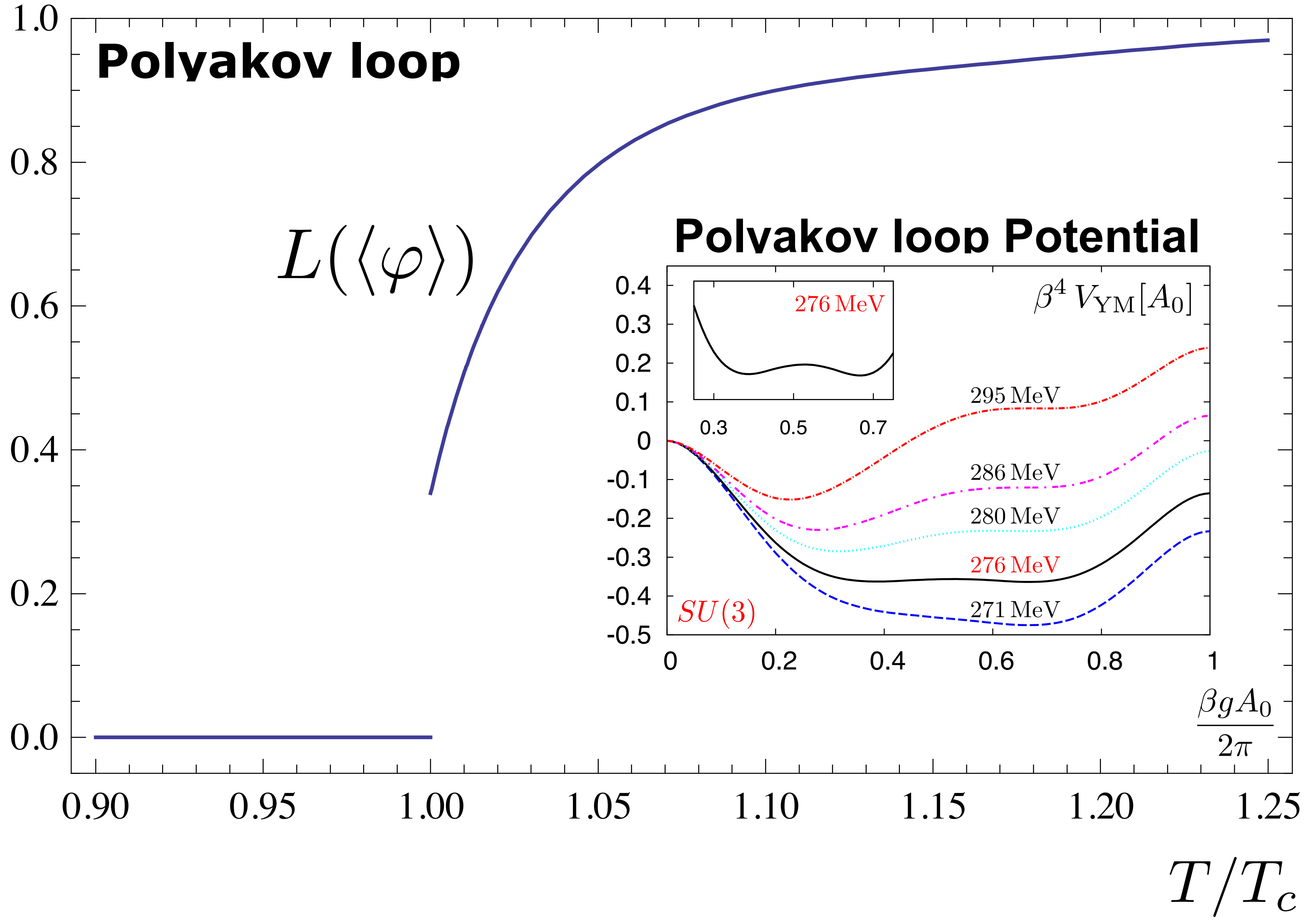
FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i g \int_0^\beta A_0(x)}$$



Polyakov loop Potential: $V_{\mathrm{YM}}[A_0]$

$$\mathcal{P} e^{i g \int_0^\beta A_0(x)} = e^{i\varphi}$$

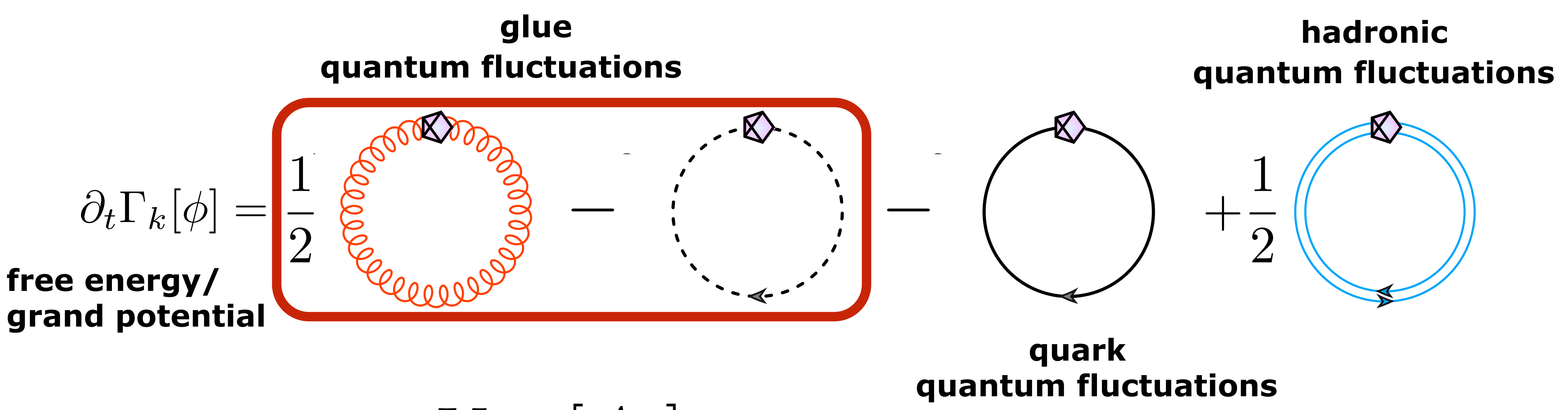


Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{tr}\, \mathcal{P} \mathbf{e}^{\mathbf{i\,g}\int_0^\beta \mathbf{A_0(x)}}$$



Polyakov loop Potential: $V_{\rm YM}[A_0]$

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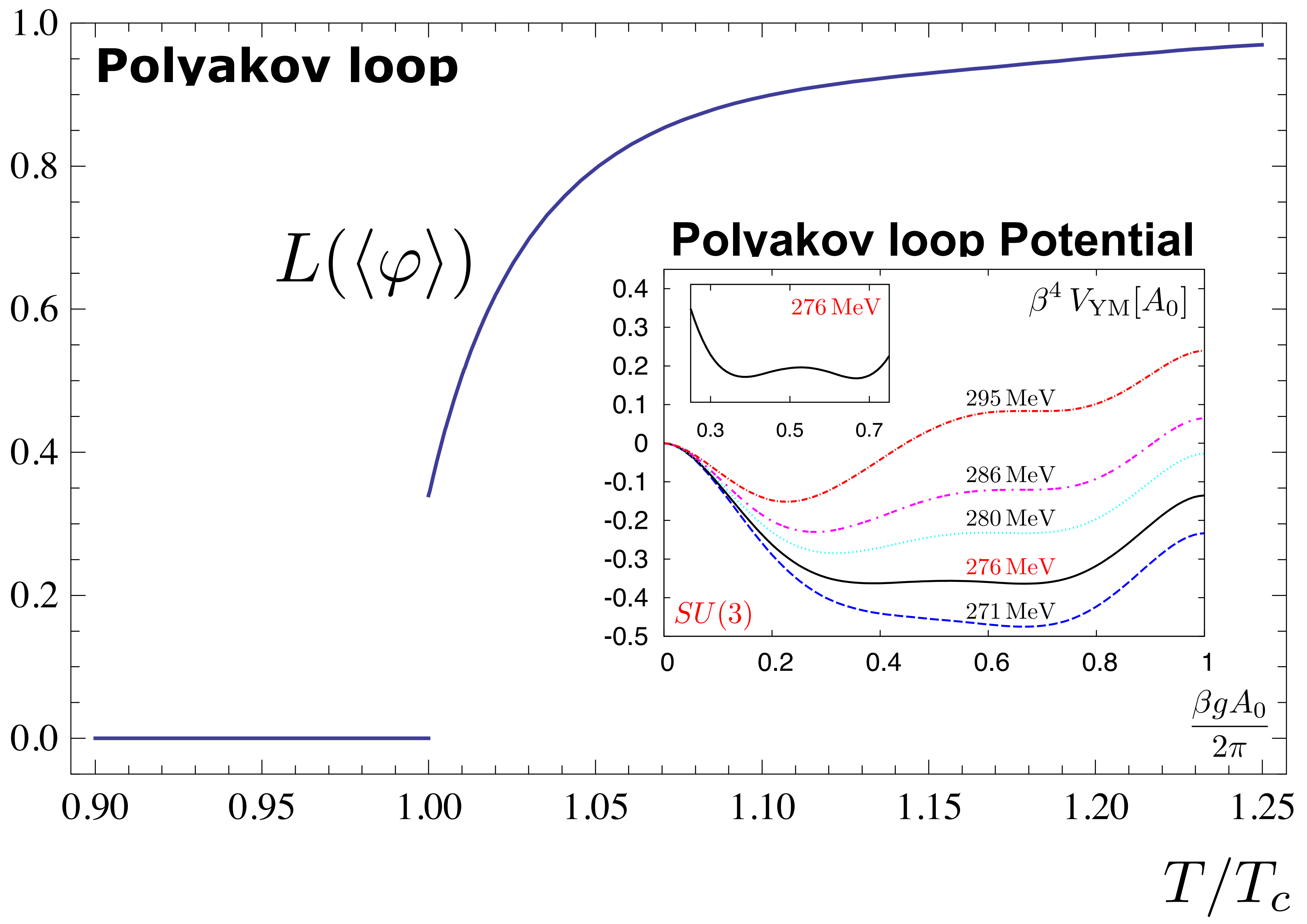


$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

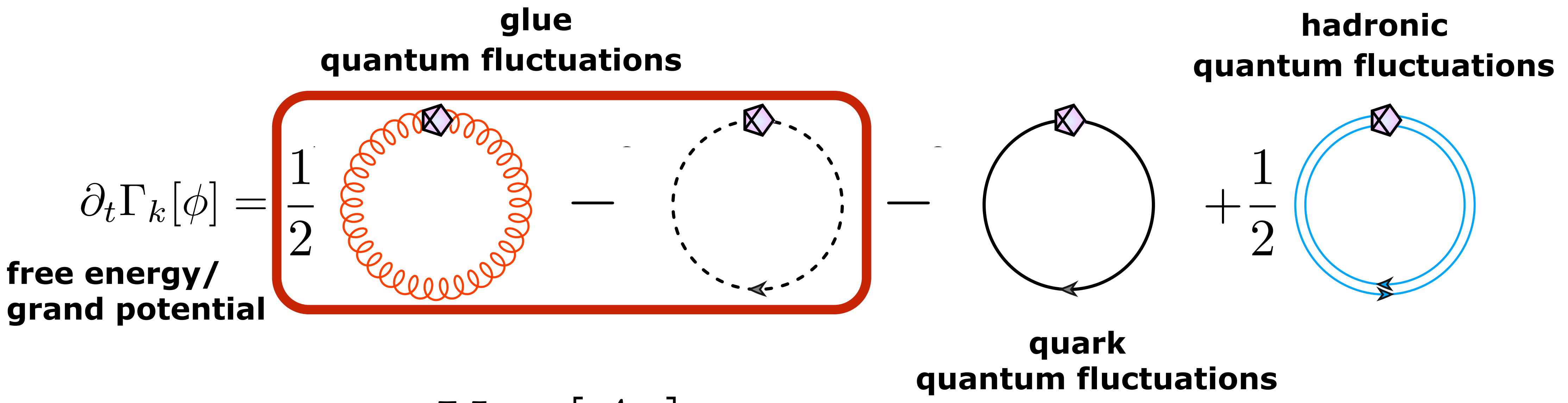
$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{tr}\, \mathcal{P} \mathbf{e}^{\mathbf{i\,g} \int_0^\beta \mathbf{A_0(x)}}$$



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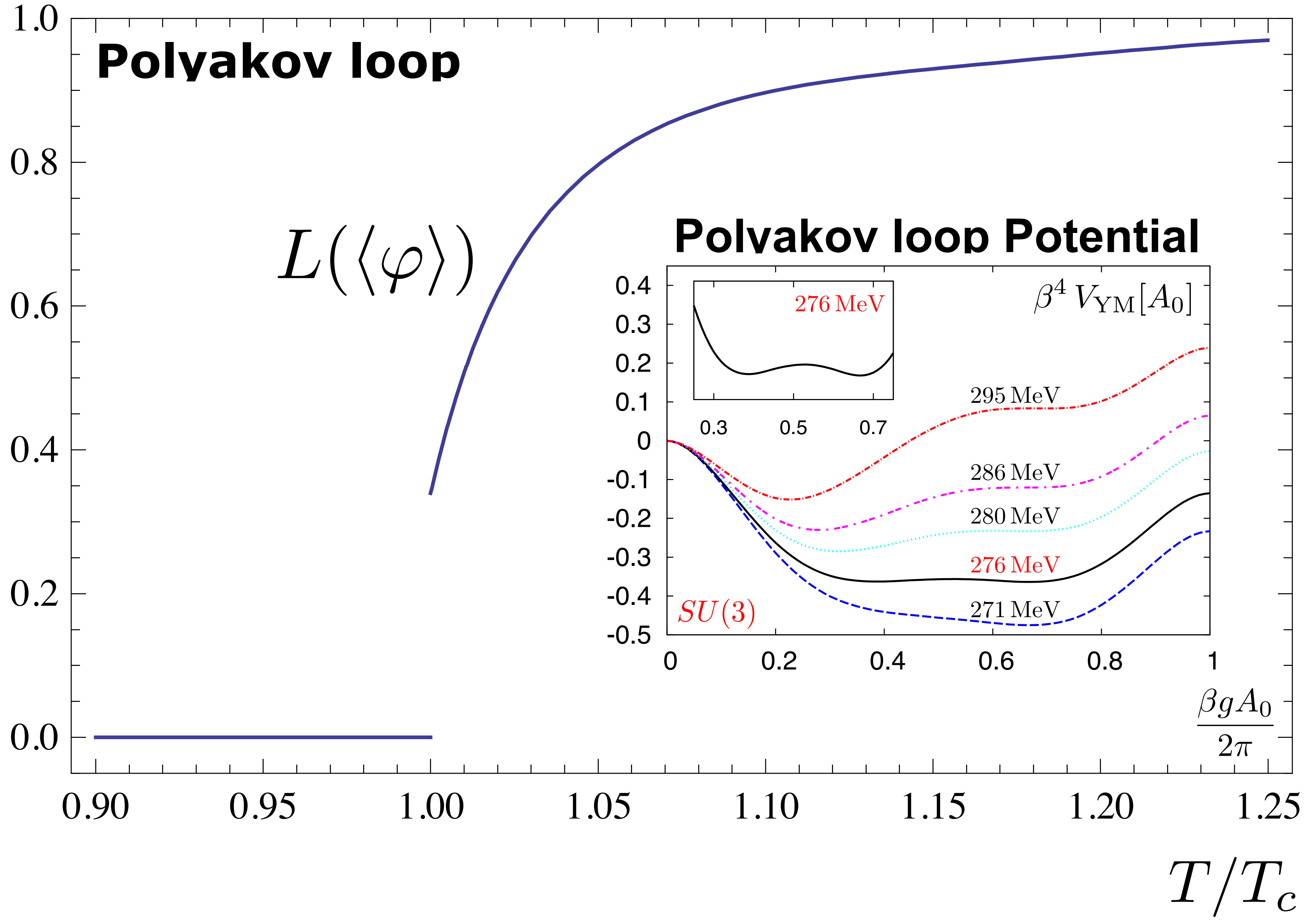
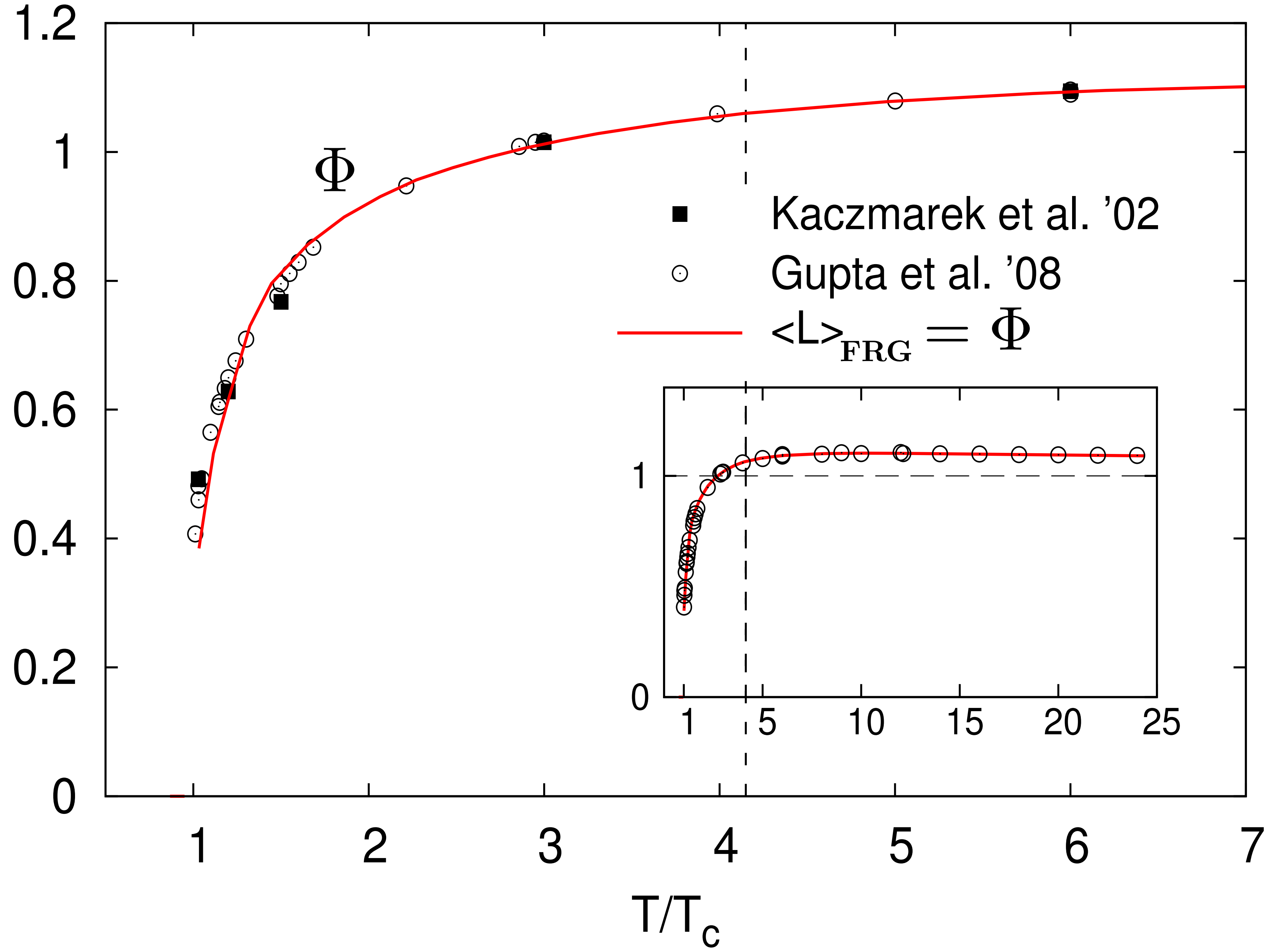
$$\mathcal{P} e^{i\,g \int_0^\beta A_0(x)} = e^{i\varphi}$$



Herbst, Luecker, JMP, arXiv:1510.03830

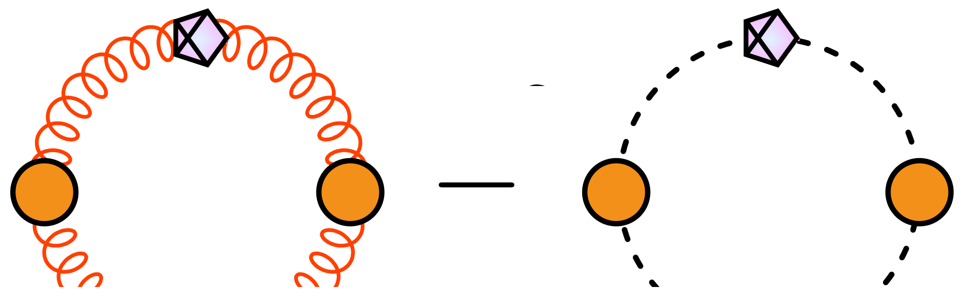


Confinement

Herbst, Luecker, JMP, arXiv:1510.03830

Flow equation for the Polyakov loop expectation value

$$\partial_t \langle L[A_0] \rangle = -\frac{1}{2}$$



$$\frac{\delta^2 \langle L[A_0] \rangle}{\delta A^2} \qquad \frac{\delta^2 \langle L[A_0] \rangle}{\delta c \delta \bar{c}}$$

quation for composite operators

JMP, AP 322 (2007) 2831

Igarashi, Itoh, Sonoda, PTP Suppl. 181 (2010) 1

Pagani, PRD 94 (2016) 045001

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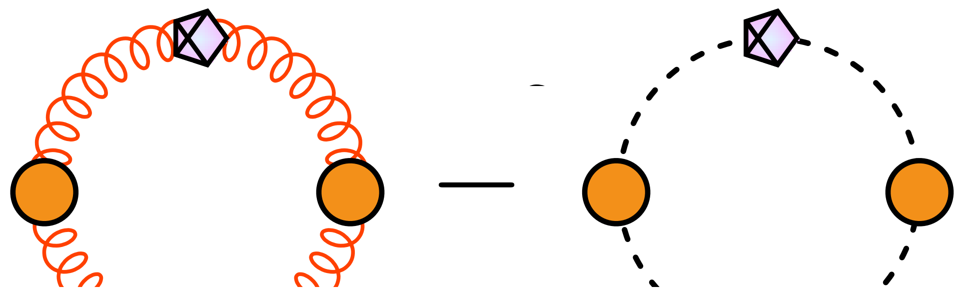

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$$\langle L[A_0] \rangle = Z_L[\bar{A}, \phi] \cdot L[A_0]$$

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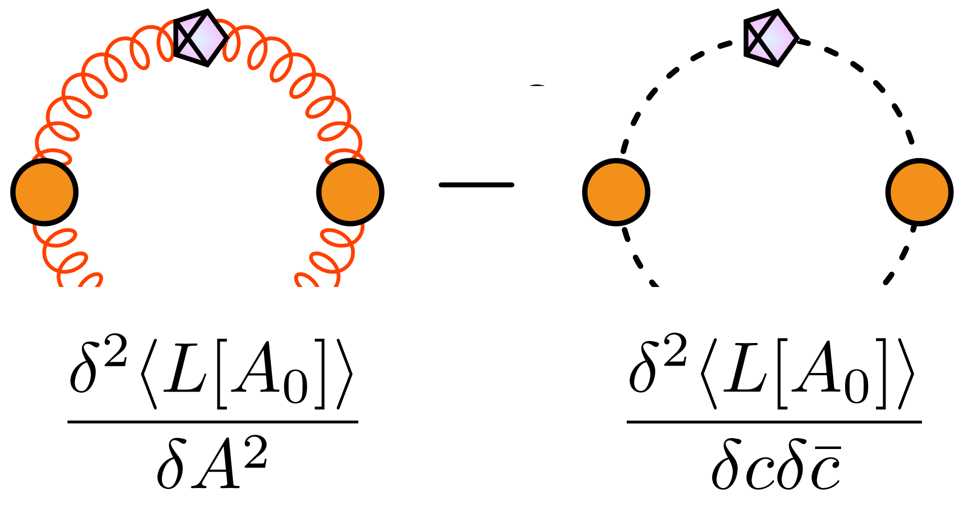

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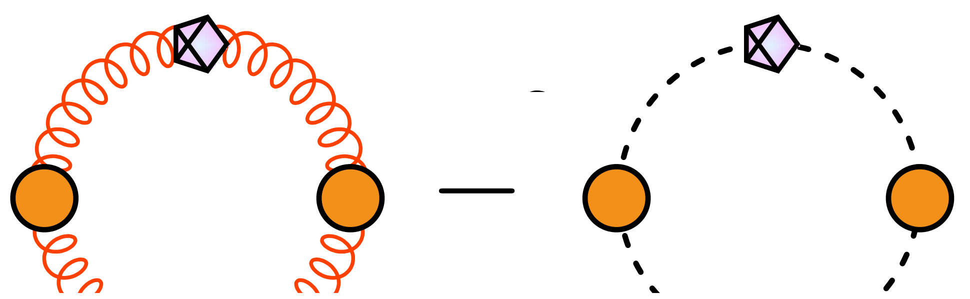

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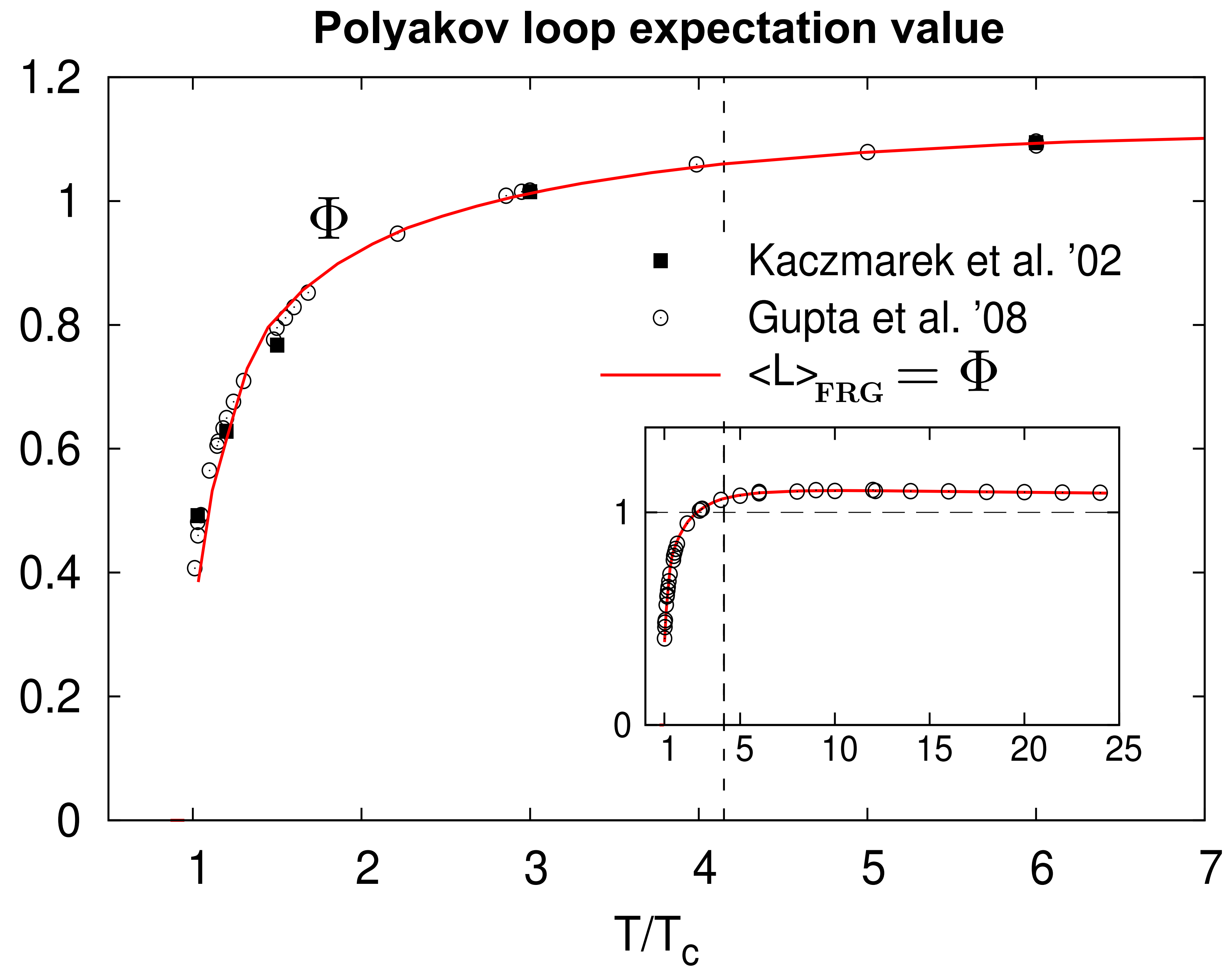


QCD phase structure

Locating the QCD phase boundary and the critical end point

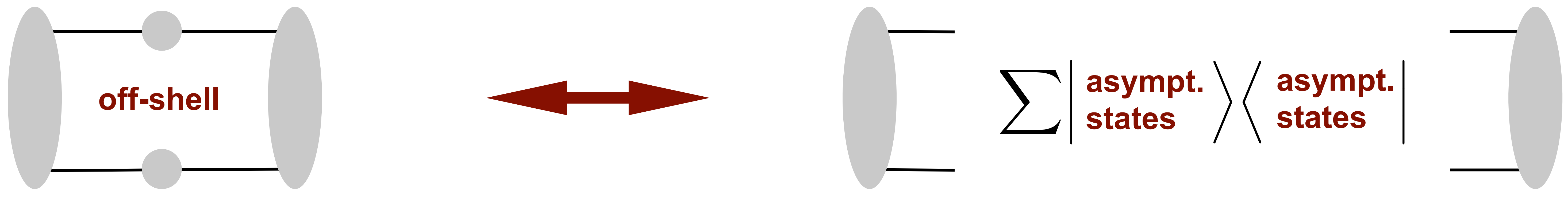
Three remarks on Functional Approaches for QCD

- **off-shell representation of thermodynamic observables**

'… and now for something completely different …'

e.g. $\mathrm{Tr}\,\langle q(x)\bar{q}(x)\rangle$

on-shell



pressure, trace anomaly, fluctuations, volume flucs., …

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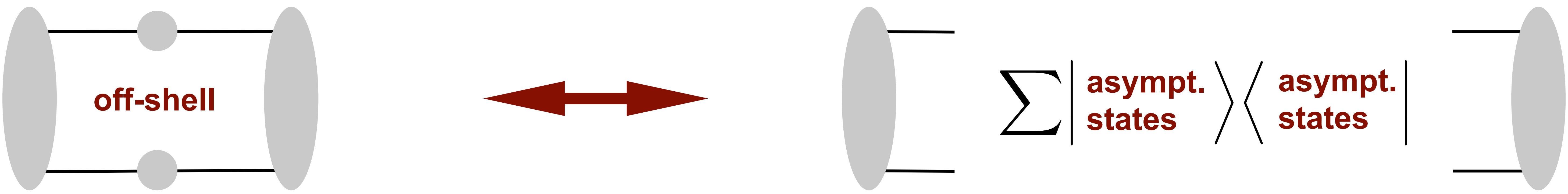

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$$\langle q(x_1)\cdots\bar{q}(x_{2n})\,A_\mu(y_1)\cdots A_\mu(y_m)\,h(z_1)\cdots h(z_l)\rangle$$

Consequences

I: simple correlations

II: Difficult access to some observables

‘No free lunch theorem’

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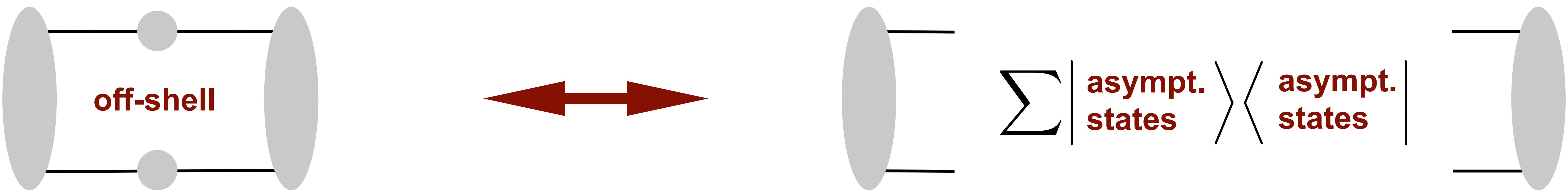

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Consequences

I: simple correlations

II: Difficult access to some observables

‘No free lunch theorem’

- **‘Your mean field is not my mean field’**

$$\left.\frac{\delta S_{\rm cl}[\phi]}{\delta\phi}\right|_{\phi=\bar\phi}=0 \qquad\qquad \left.\frac{\delta\Gamma[\phi]}{\delta\phi}\right|_{\phi=\bar\phi_{\rm quant}}=0$$

full quantum equation of motion

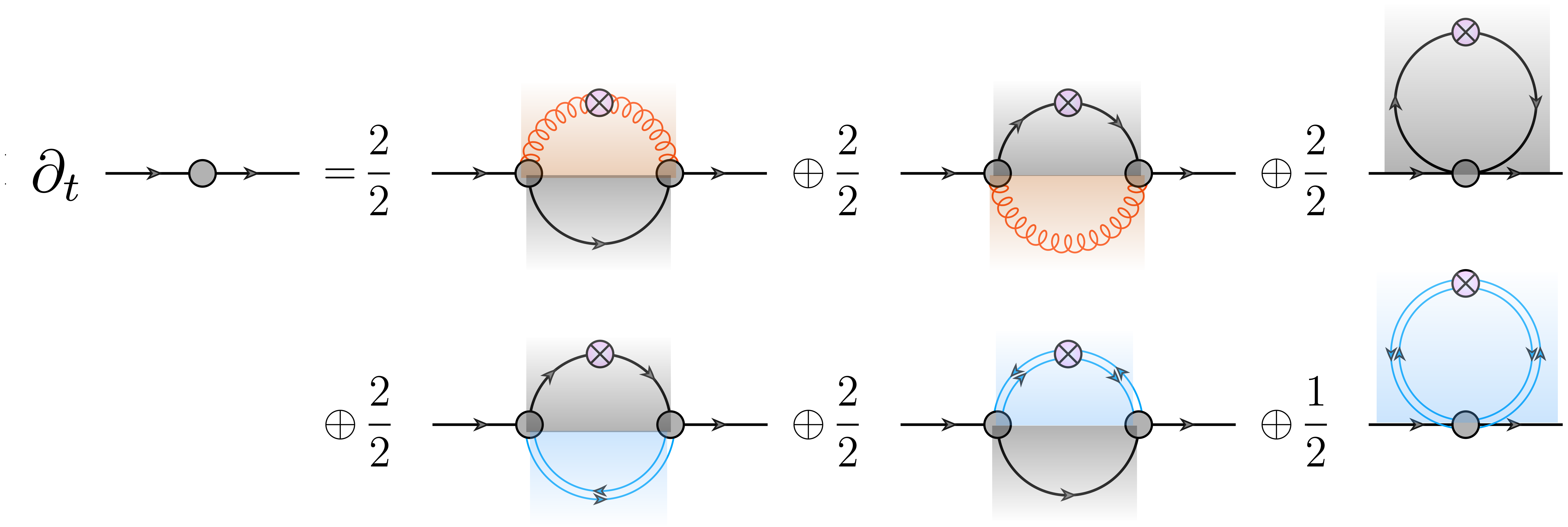
Correlation functions at finite density from functional QCD

To QCD or not to QCD….a minimal point of view

'… and now for something completely different …'

- **Self-consistent truncations to functional relations define analytic functions in μ_{B}, eg:**

$$\partial_t \Big\langle q(x)\bar{q}(y)\Big\rangle^{-1}(\mu_B) = \text{Loop}\left[\Big\langle q(x)\bar{q}(y)\Big\rangle(\mu_B),\ \Big\langle q(x)A_\mu(y)\bar{q}(z)\Big\rangle(\mu_B), \cdots ;\ \mu_B\right]$$

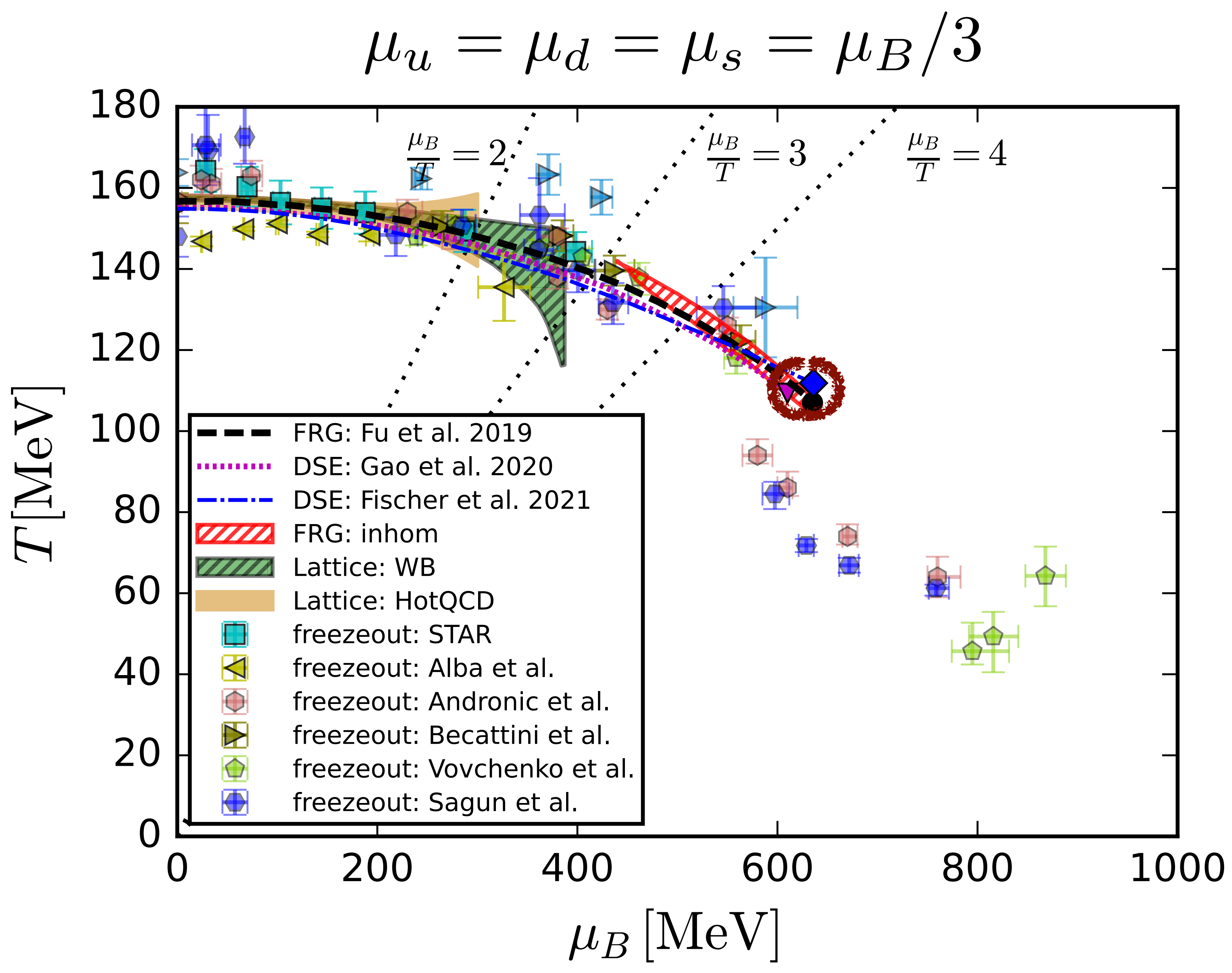


fQCD collaboration

Dalian, Beijing, Darmstadt, Heidelberg, Gießen

Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

fRG: **Fu, JMP, Rennecke, PRD 101 (2020) 054032**

DSE: **Gao, JMP, PLB 820 (2021) 136584**

Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600 - 650\,,\,105 - 115)\ \mathrm{MeV}$$

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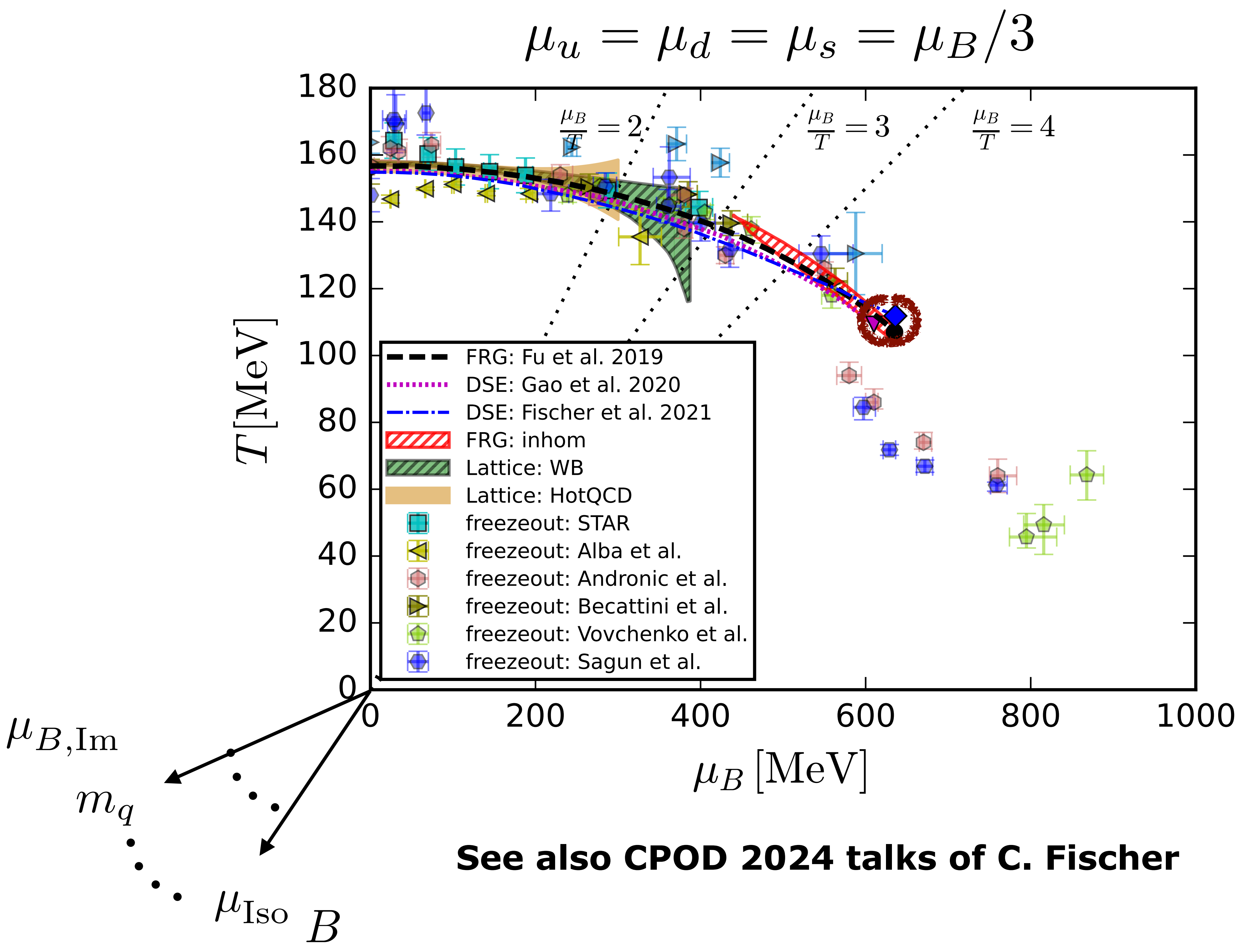

See also CPOD 2024 talks of C. Fischer

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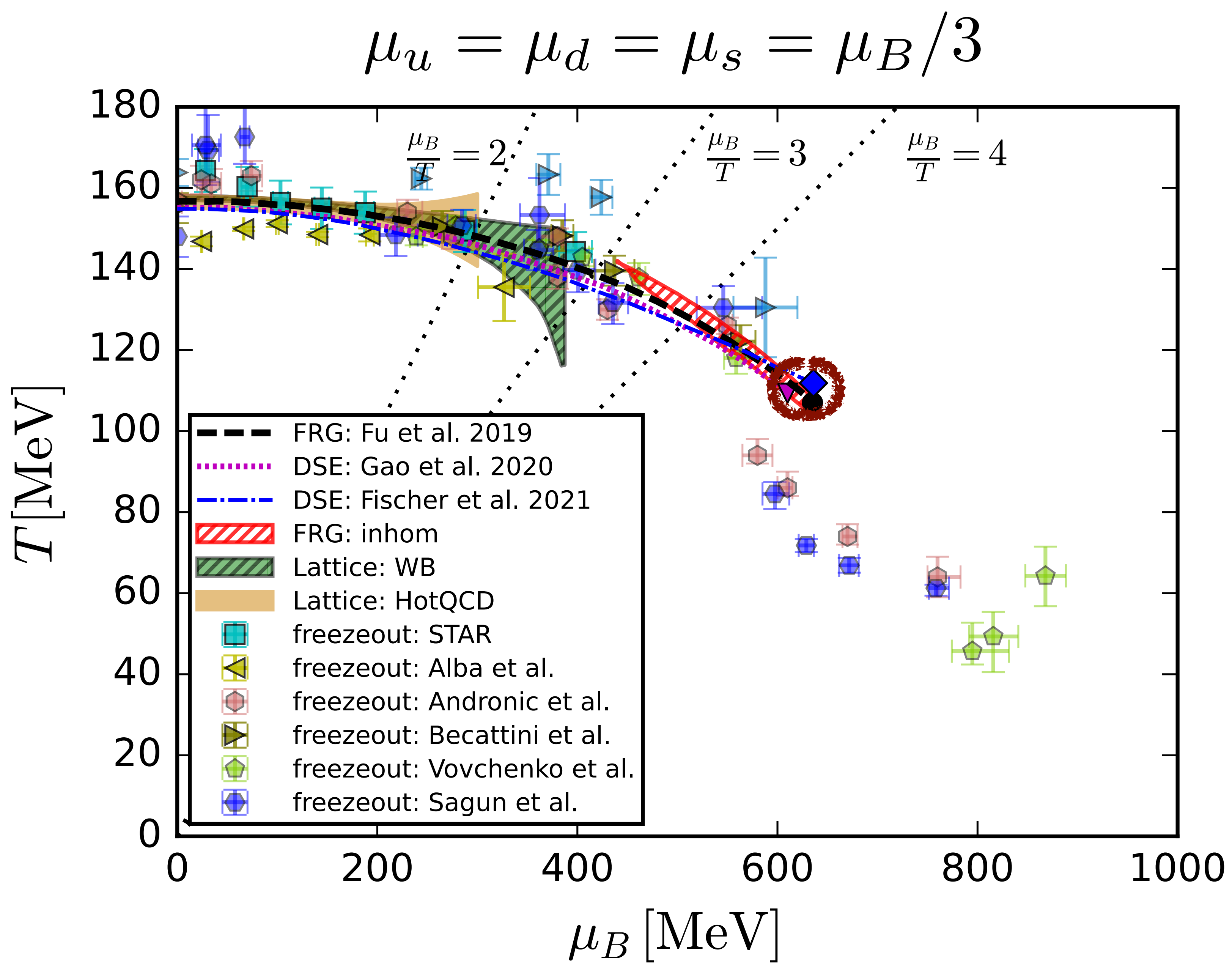
Collect all possible information/structure
for
physics understanding & extrapolations

Phase structure of QCD and the CEP



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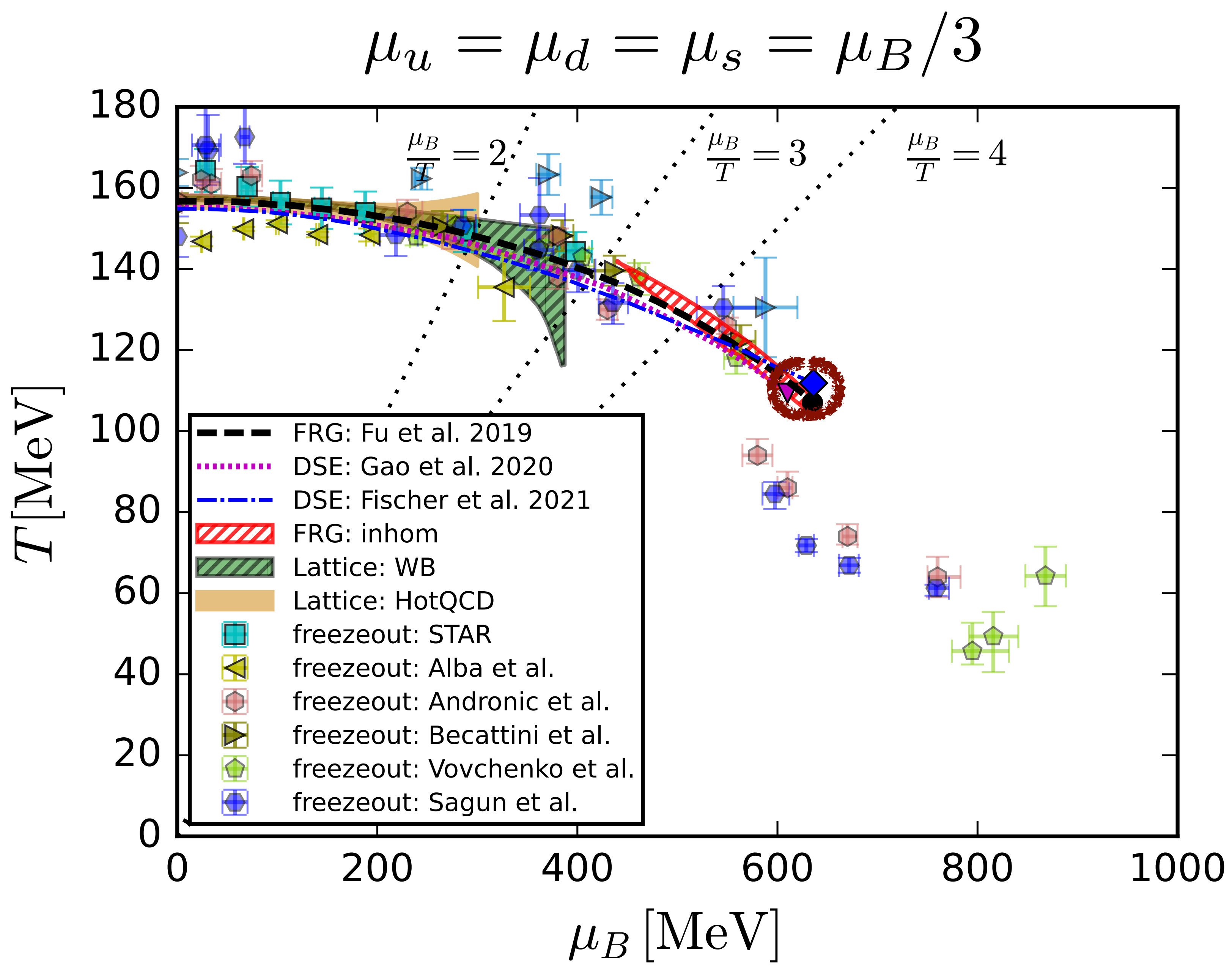

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Requires a discussion of the explicit & implicit assumptions

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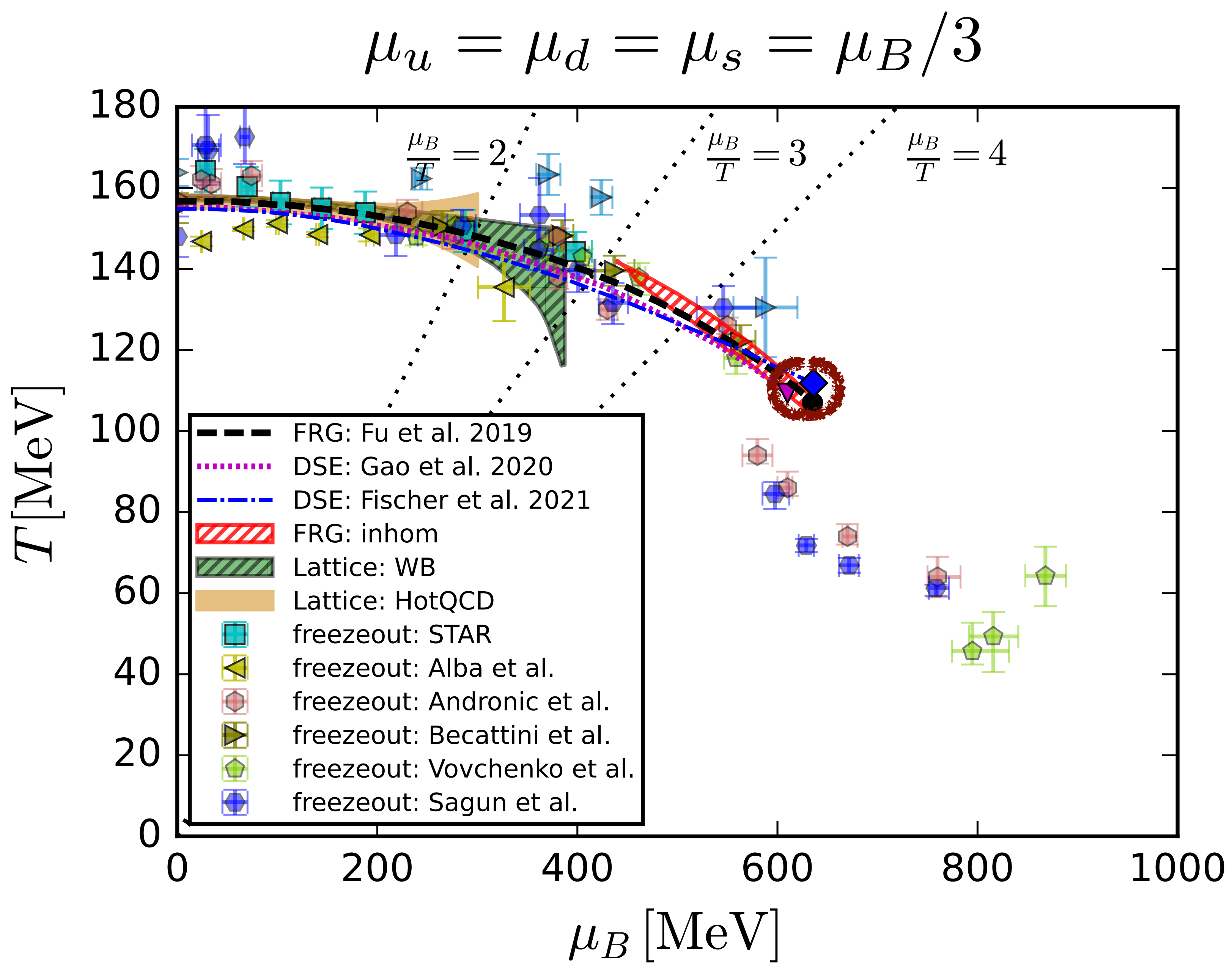

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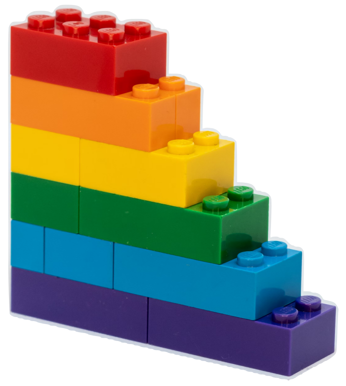
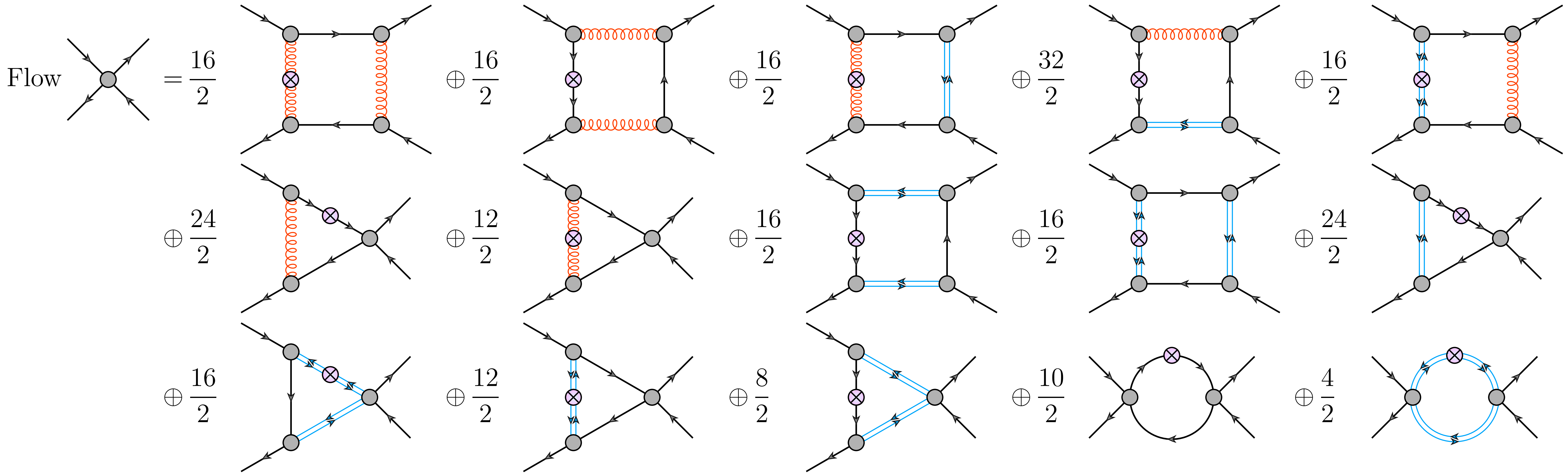
Lattice extrapolations

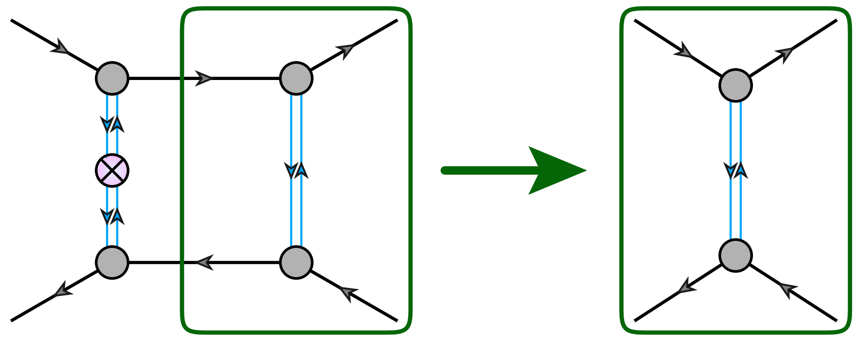
low energy effective theories:
QM, NJL, PQM, PNJL, …, Holography

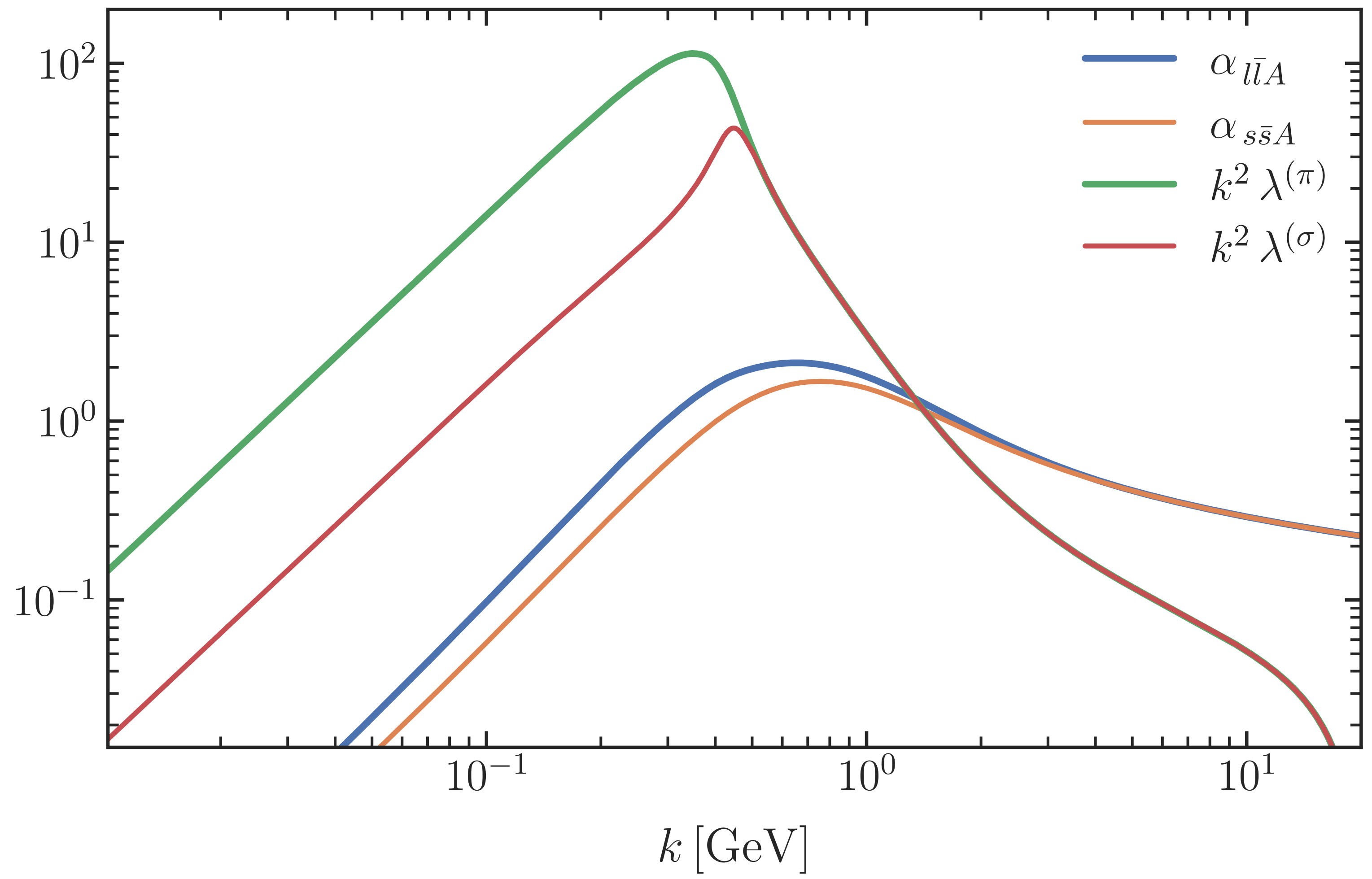
Functional QCD: systematic error estimates & the LEGO® principle

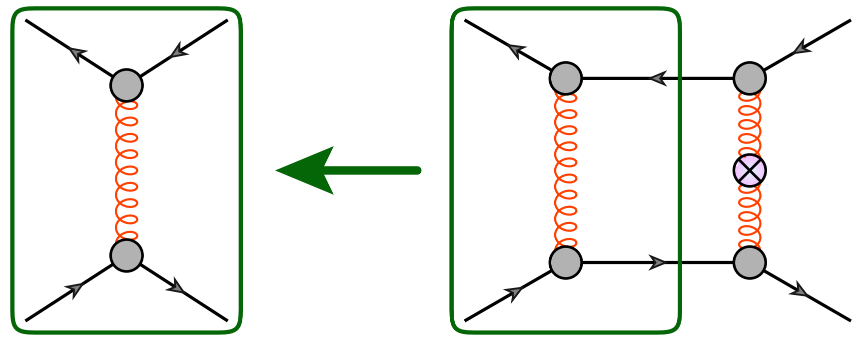




Example: 4-quark scattering vertex

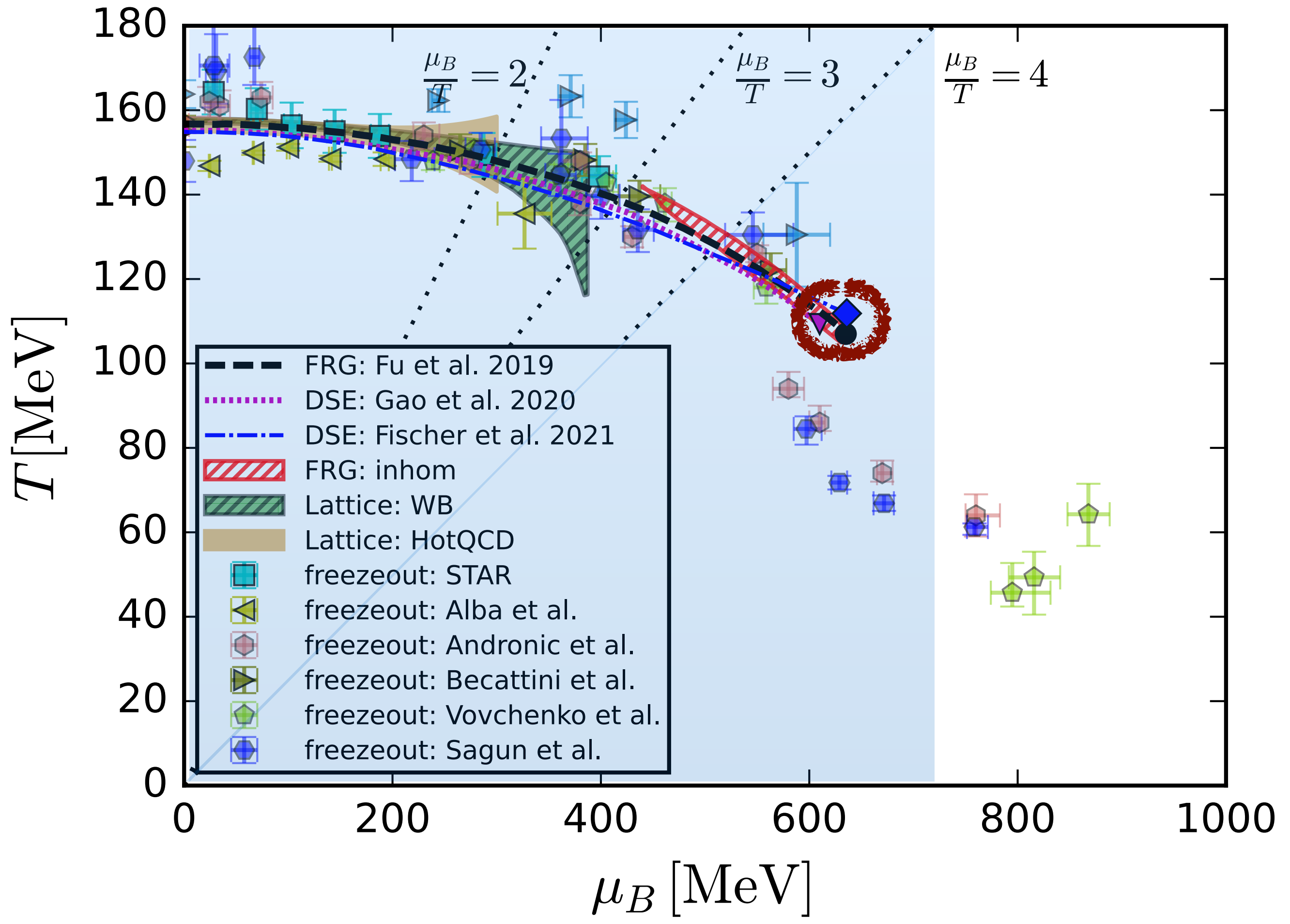




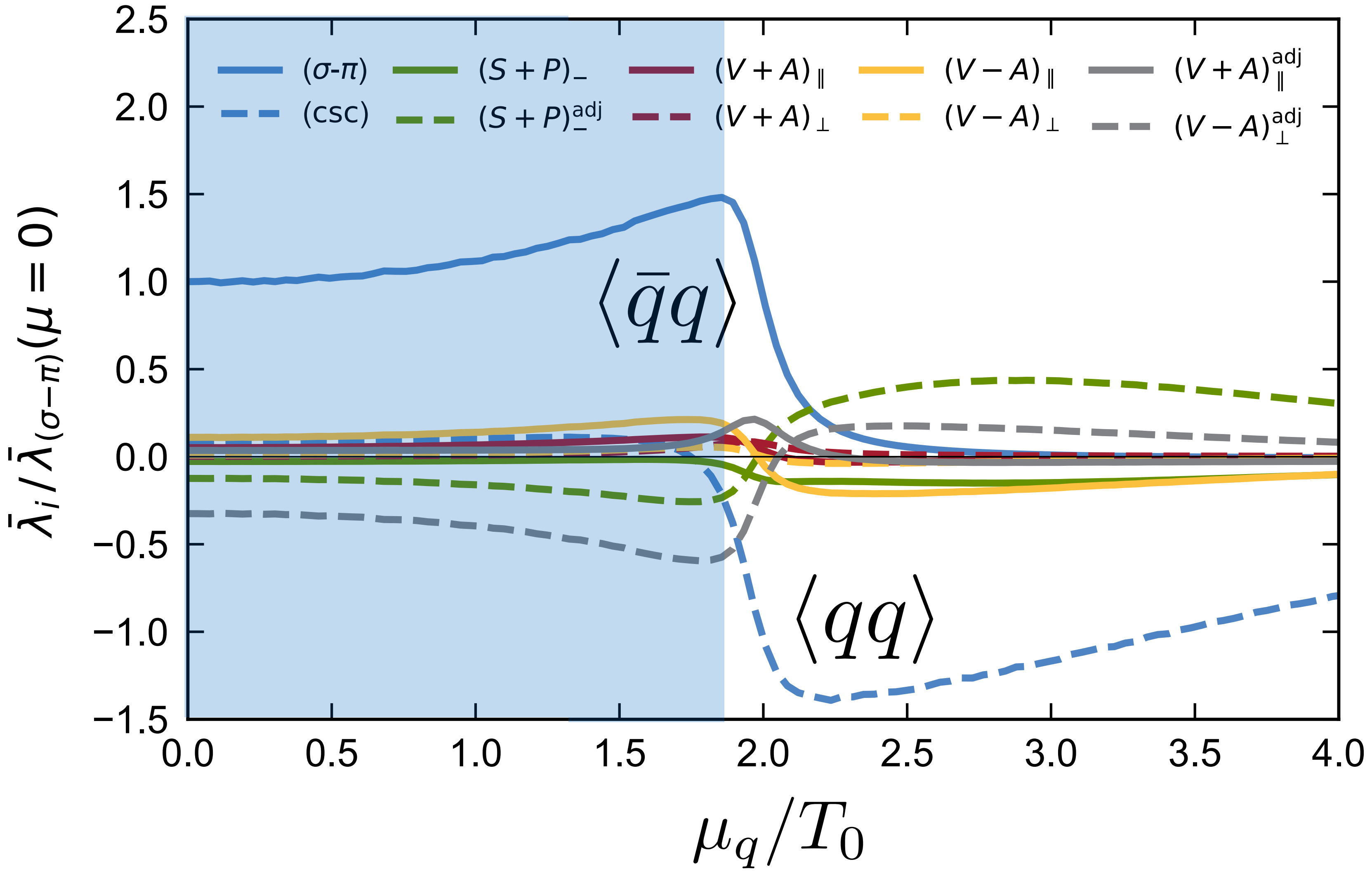


Ihssen, JMP, Sattler, Wink, in preparation

Predictions & estimates



Four-quark scattering channels

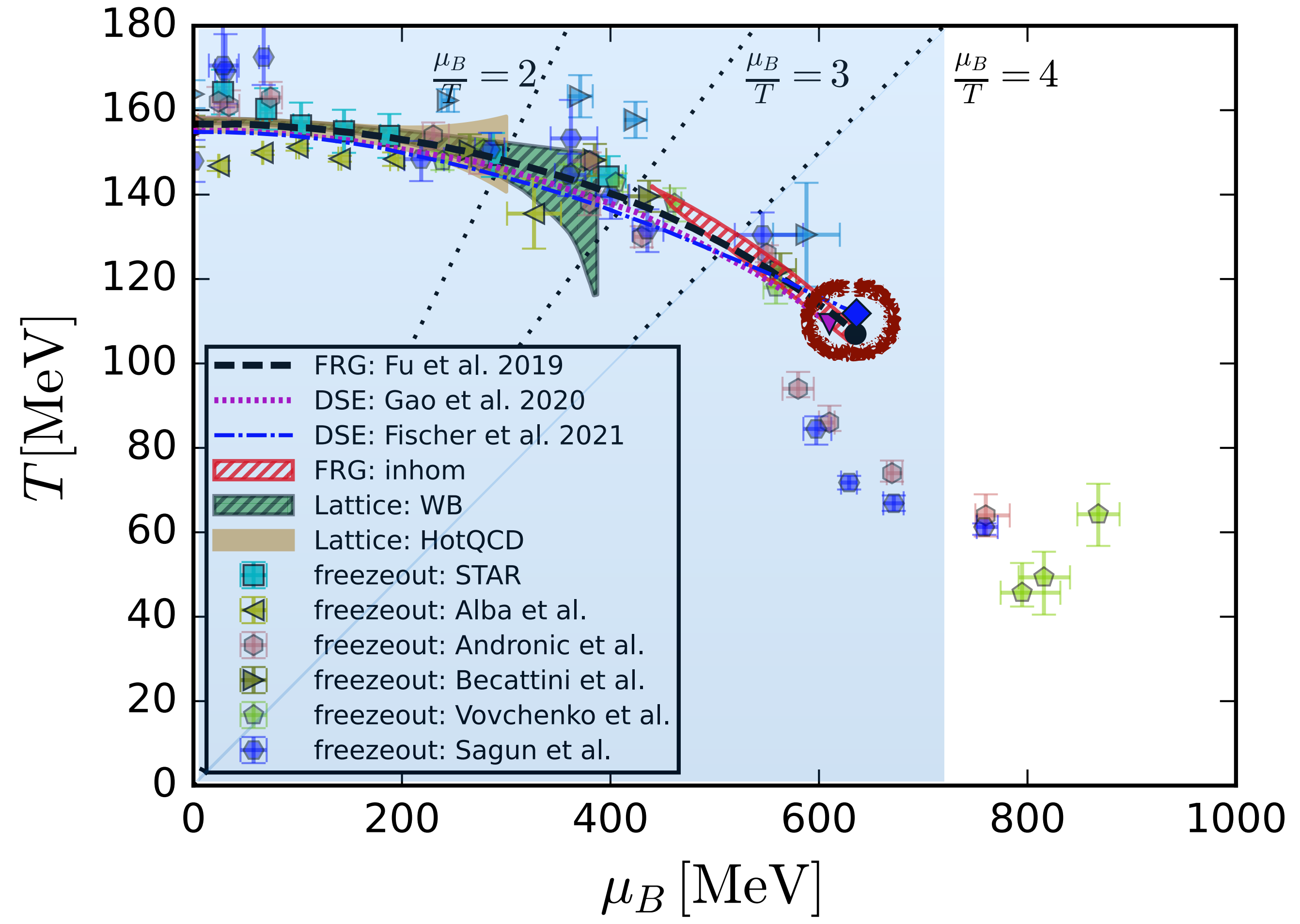


Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Dominance of scalar-pseudoscalar fluctuations

Pions & sigma mode

Predictions & estimates



Full chiral dynamics

Fu, JMP, Rennecke, PRD 101 (2020) 054032 (fRG)

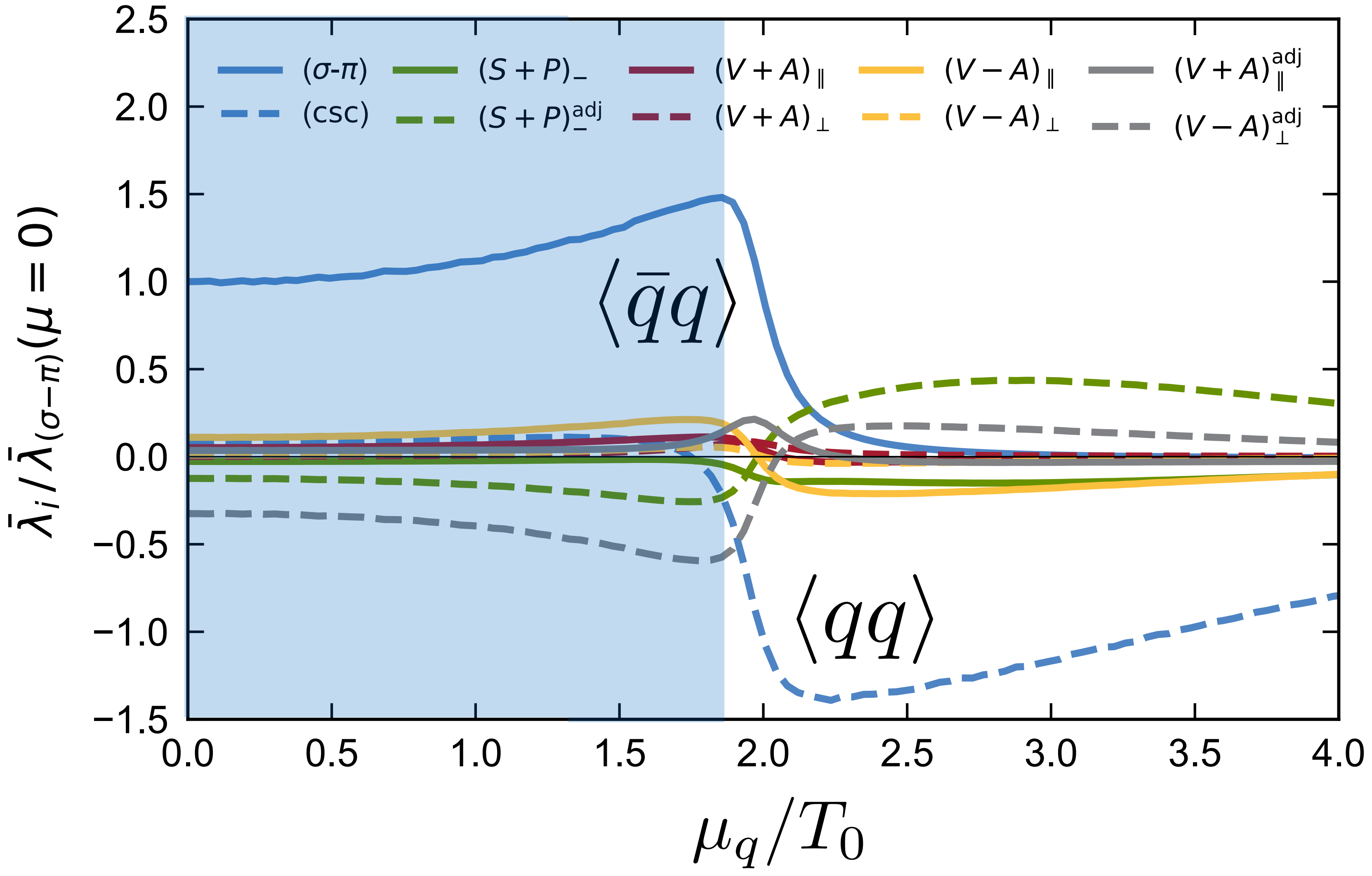
Dominant chiral dynamics

Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)

Full quark-gluon dynamics

Gao, JMP, PLB 820 (2021) 136584 (DSE)

Four-quark scattering channels



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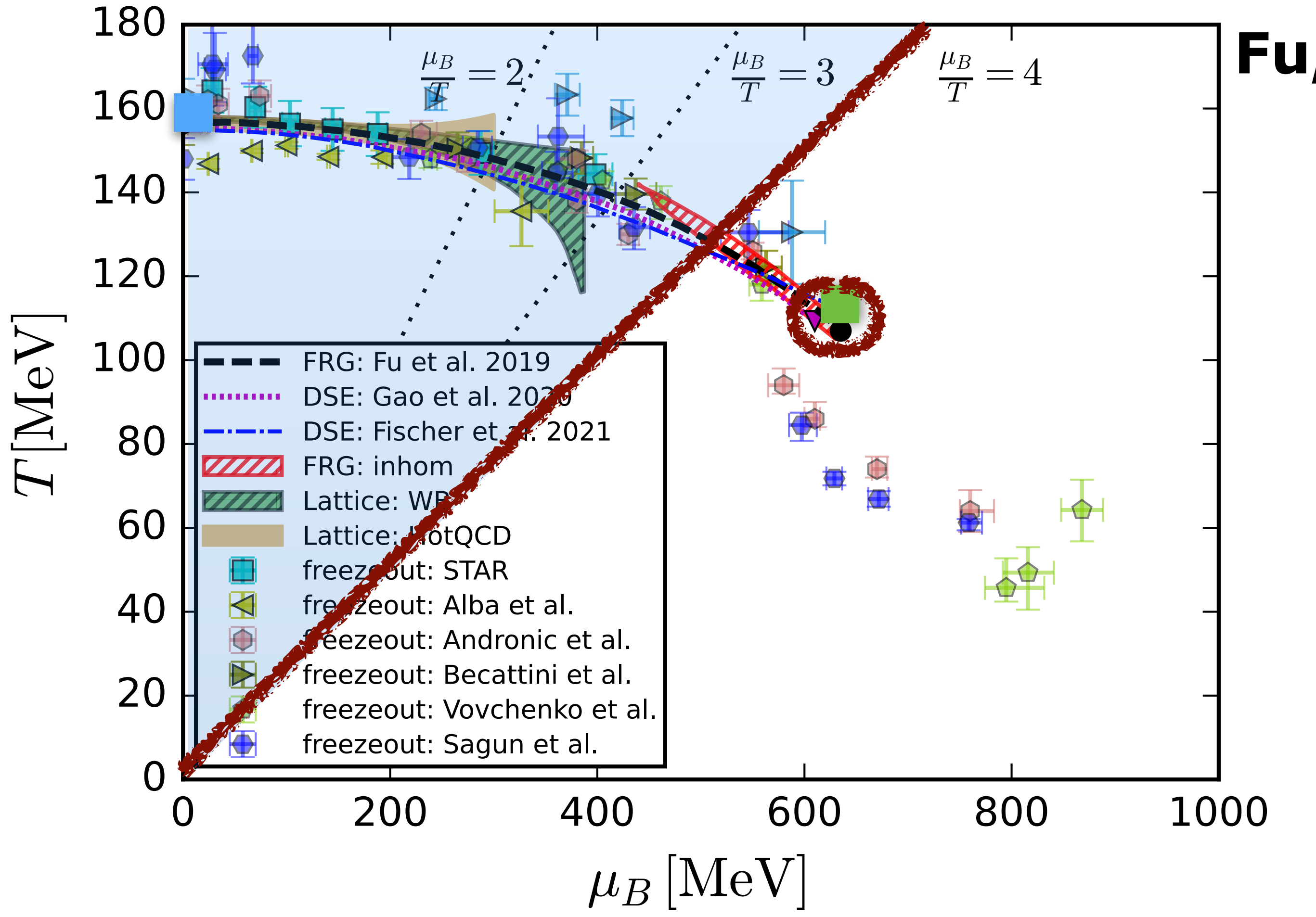


Fu, JMP, Rennecke, PRD 101 (2020) 054032

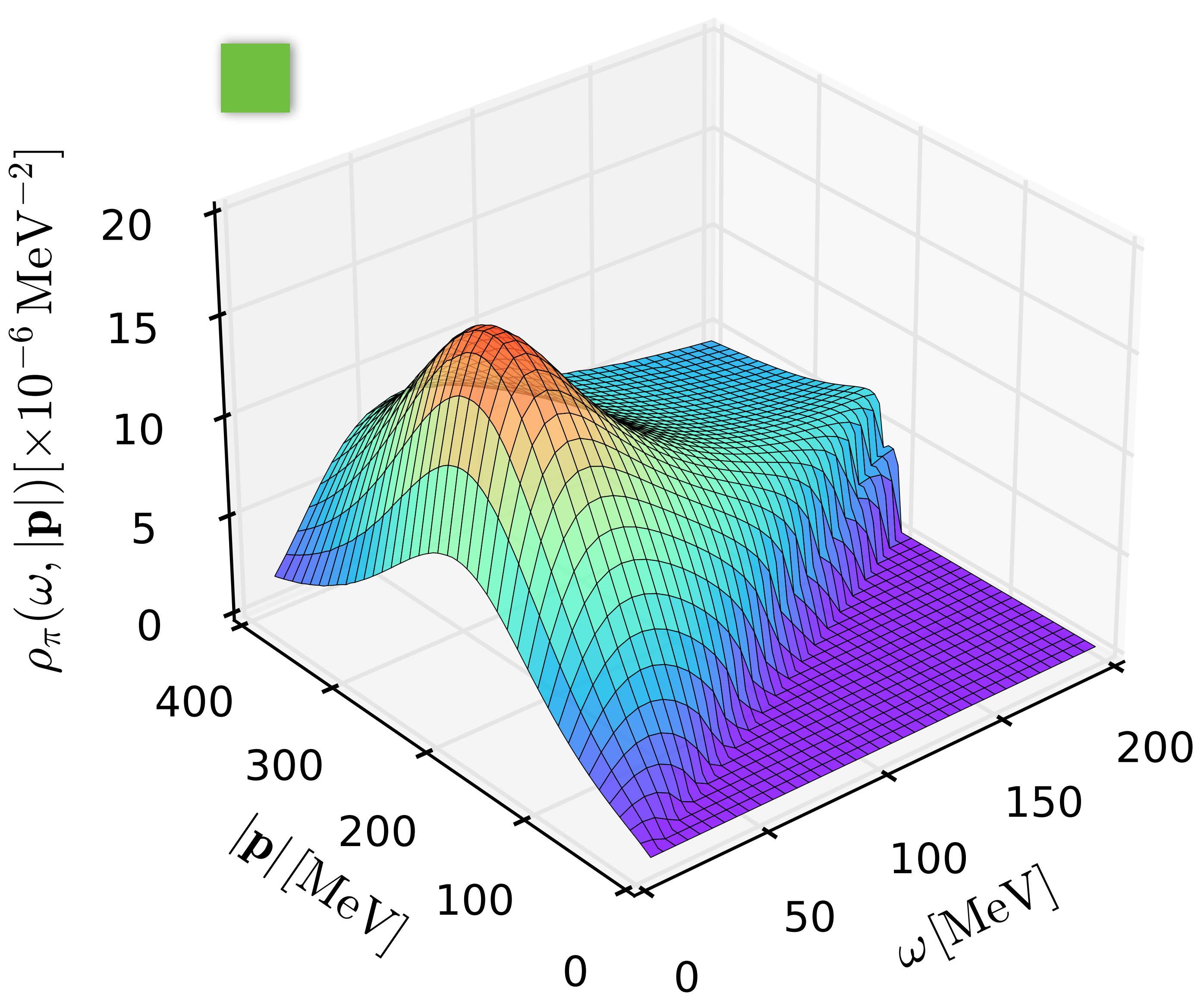
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

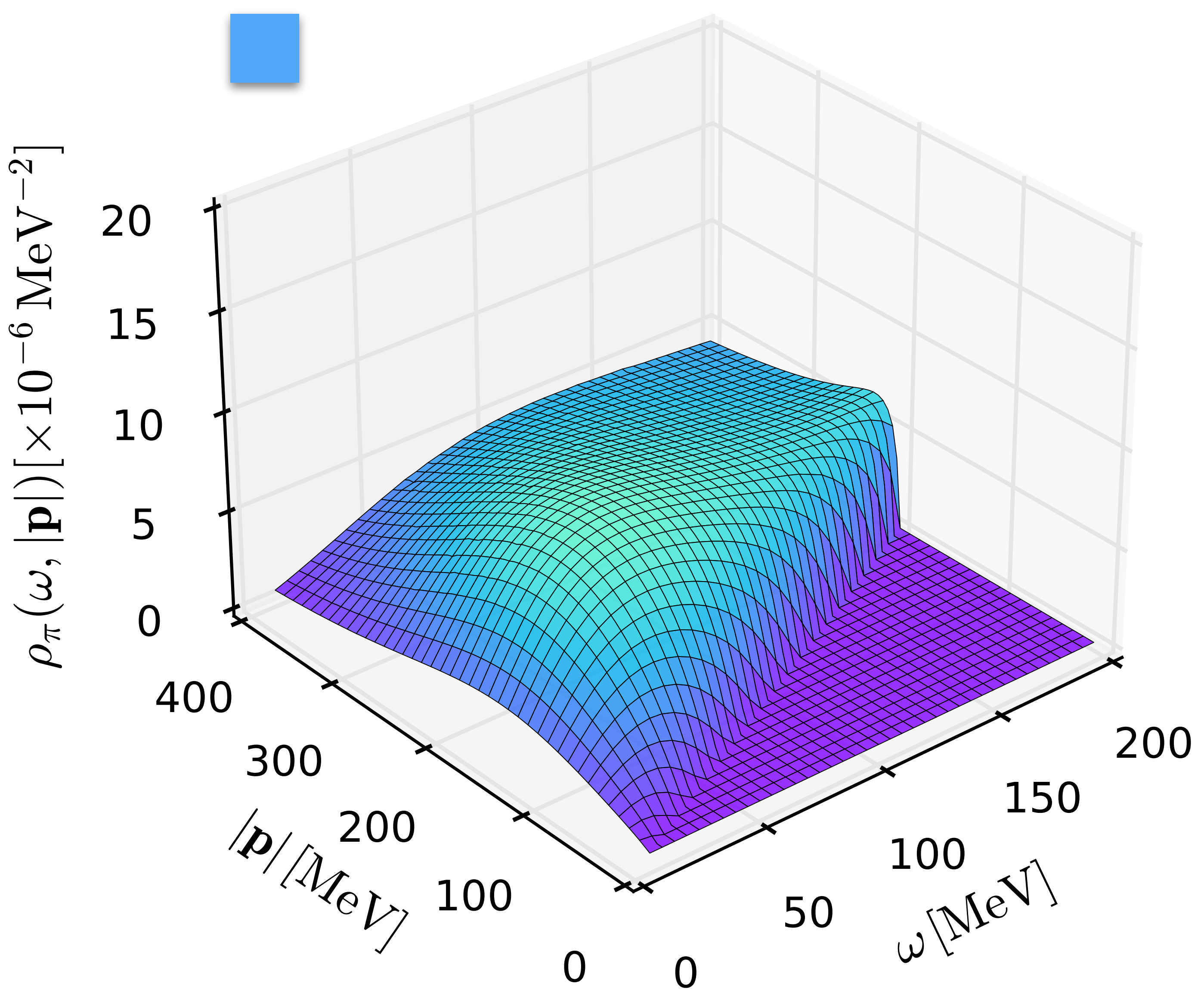
see talk of Fabian Rennecke

Regime of quantitative reliability of current best truncation

T=114 MeV & μ_B=630 MeV



T=160 MeV & μ_B =0 MeV

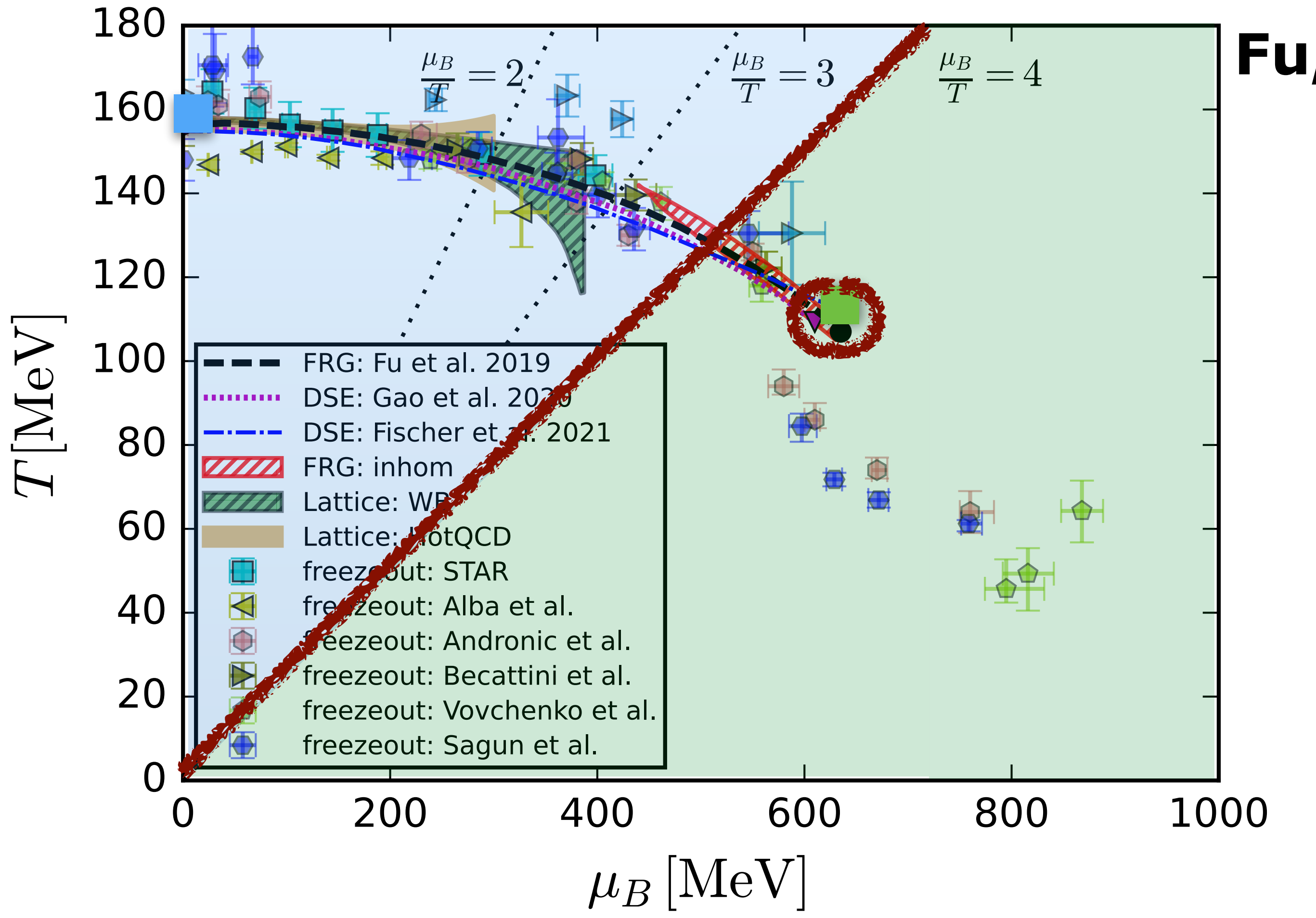


Pion spectral functions

Fu, JMP, Pisarski, Rennecke, Wen, Yin, in prep

Predictions & estimates

Prediction



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Regime of quantitative reliability of current best truncation

Estimate

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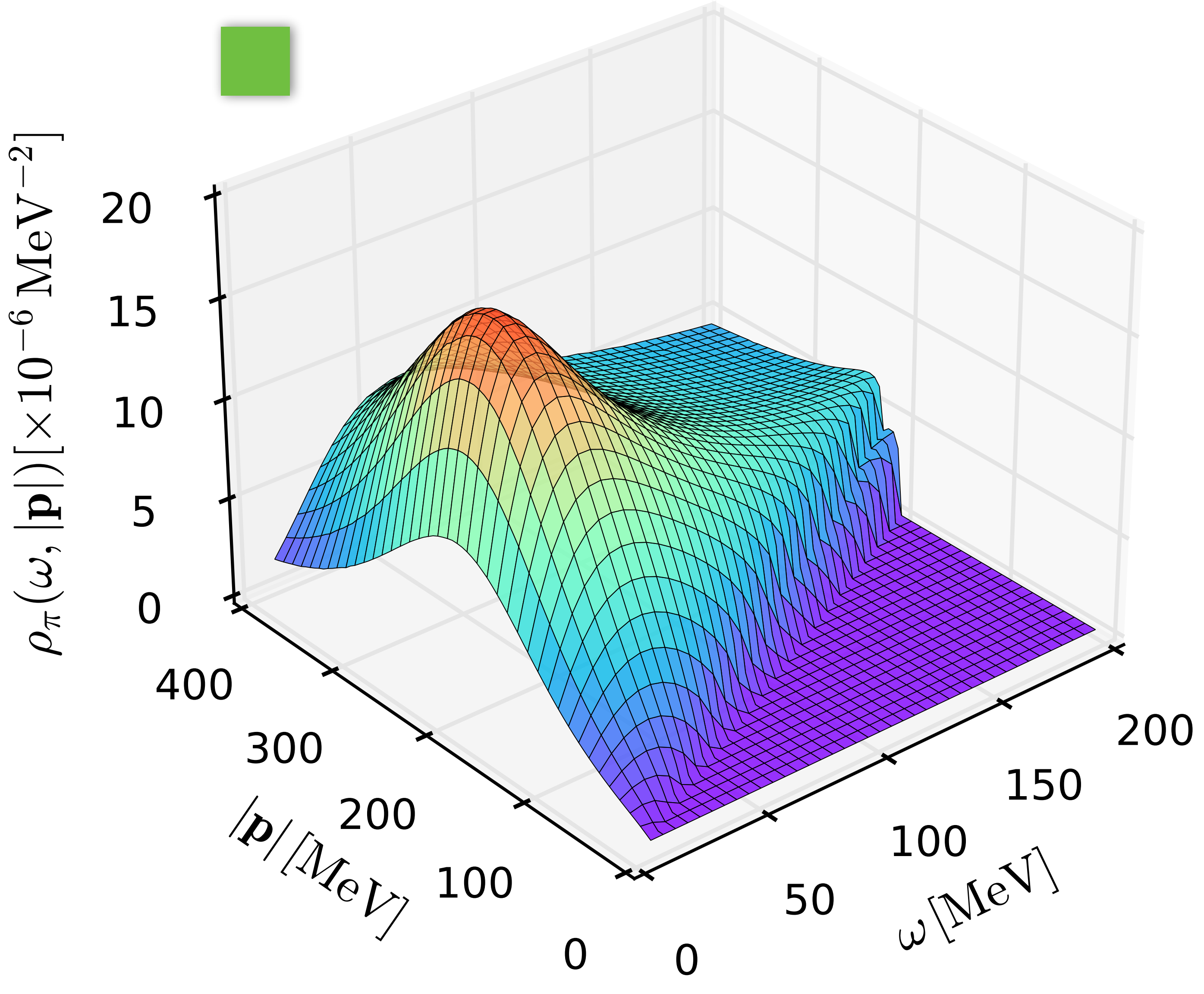

Moat regime is not captured quantitatively

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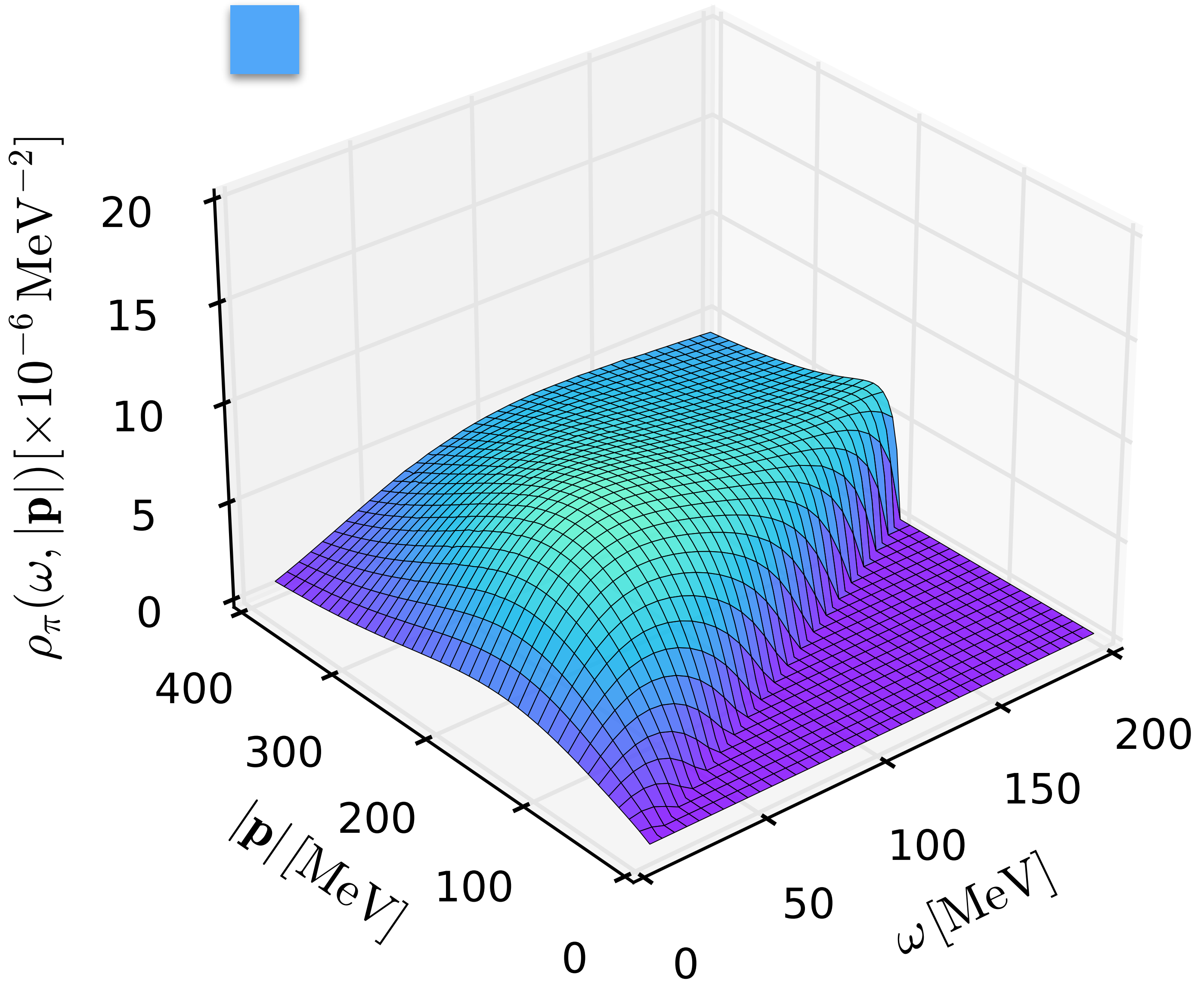
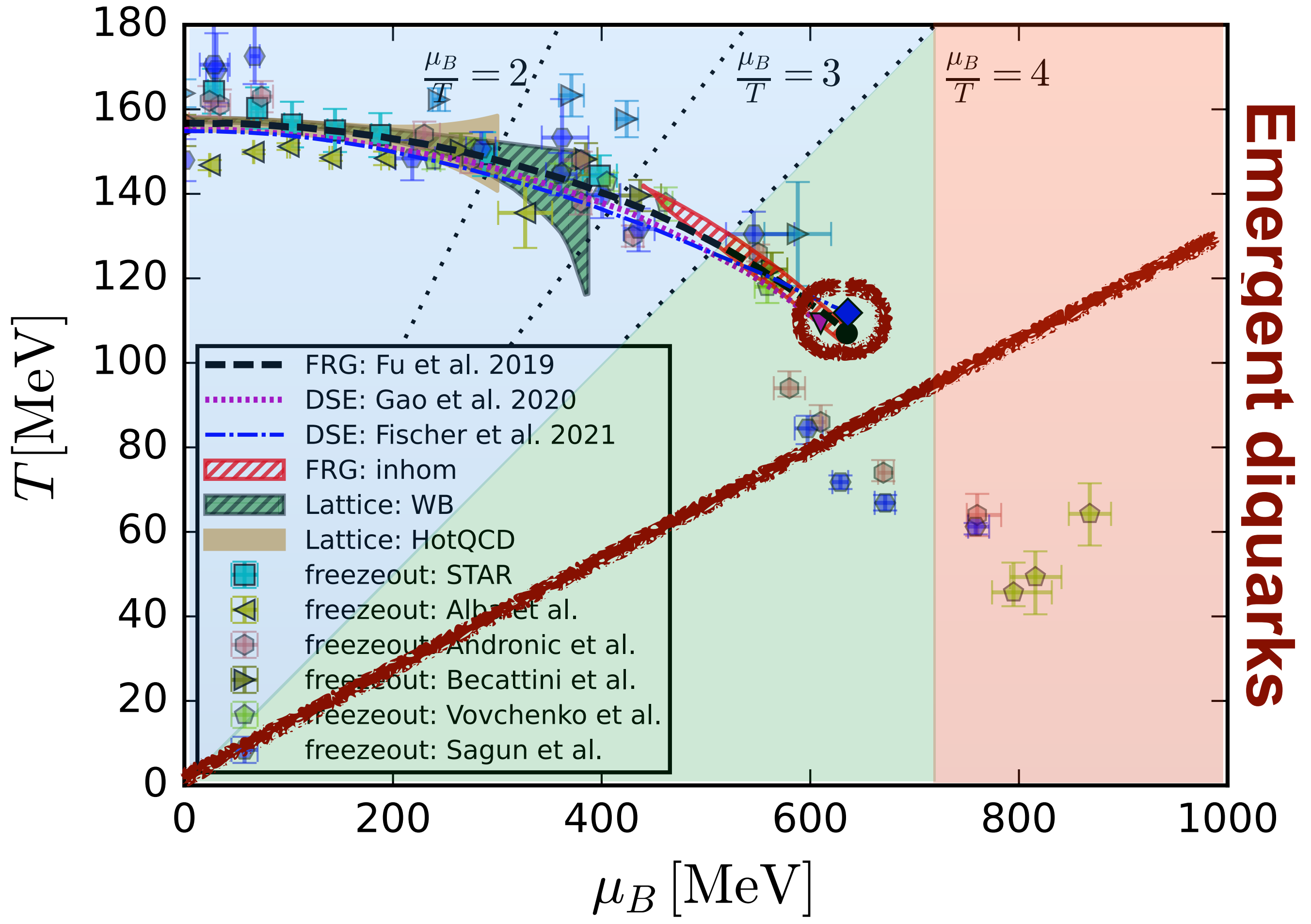


Predictions & estimates

Prediction

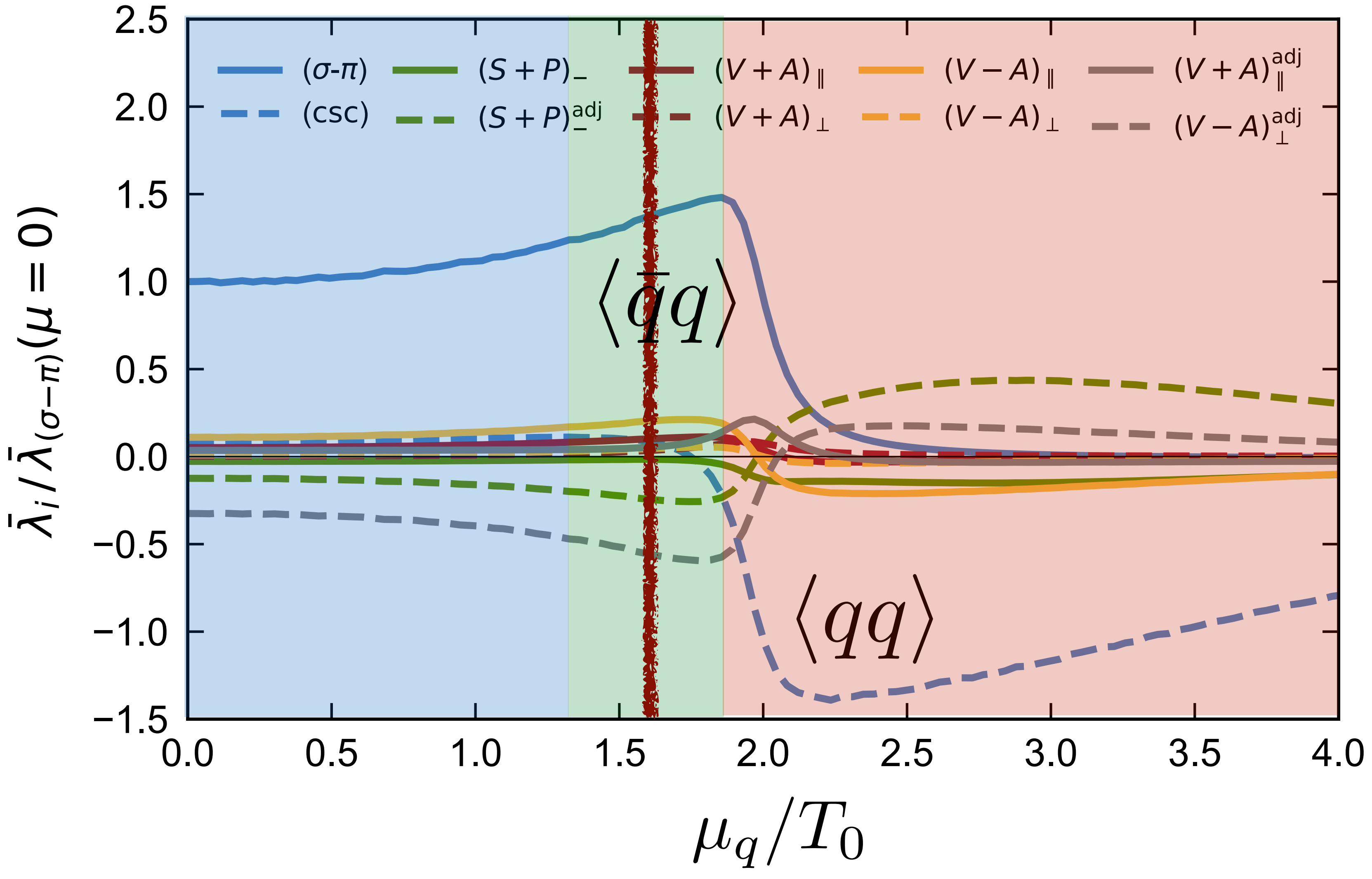


Estimate

Emergent diquarks

Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)



Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Predictions & estimates

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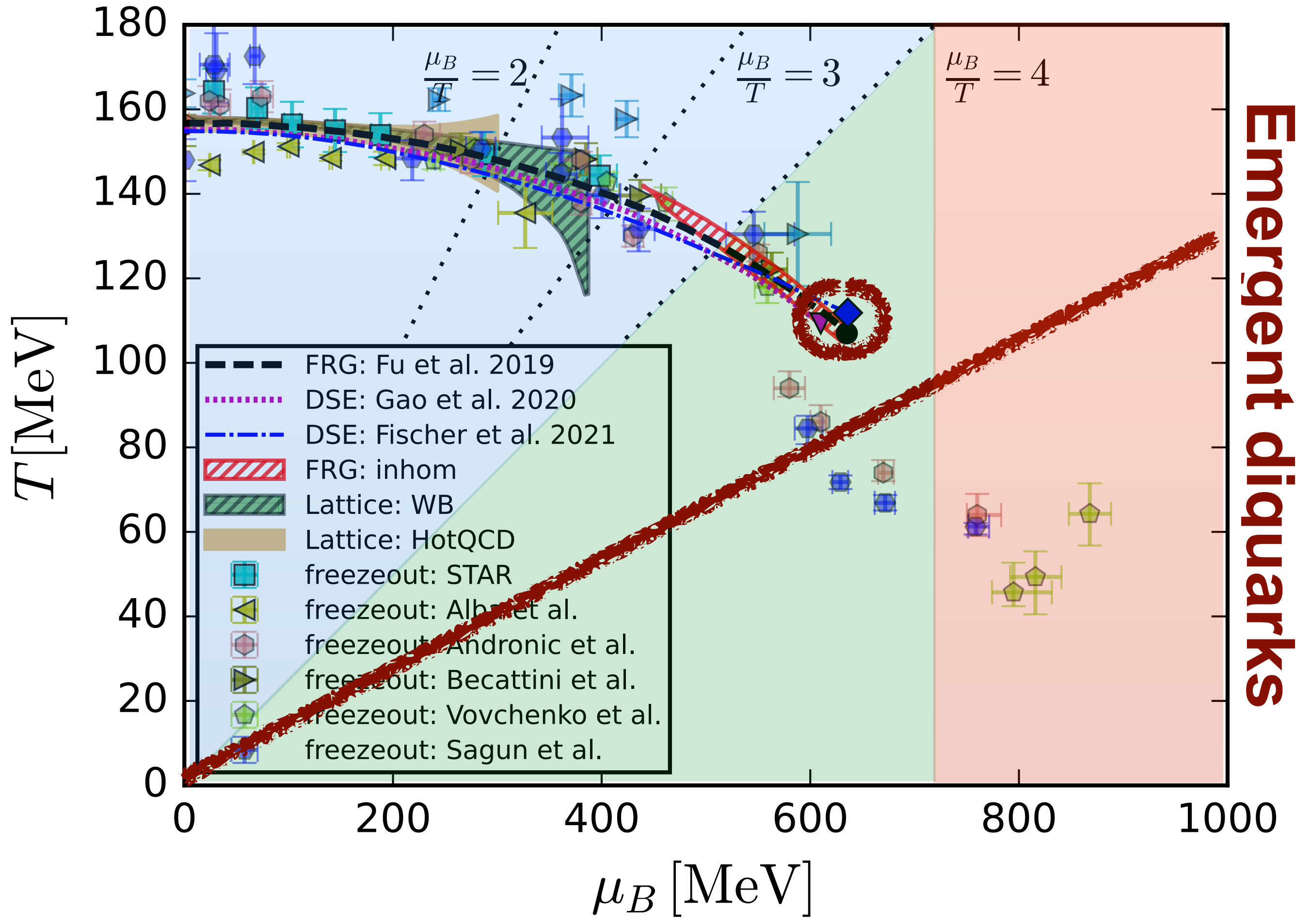

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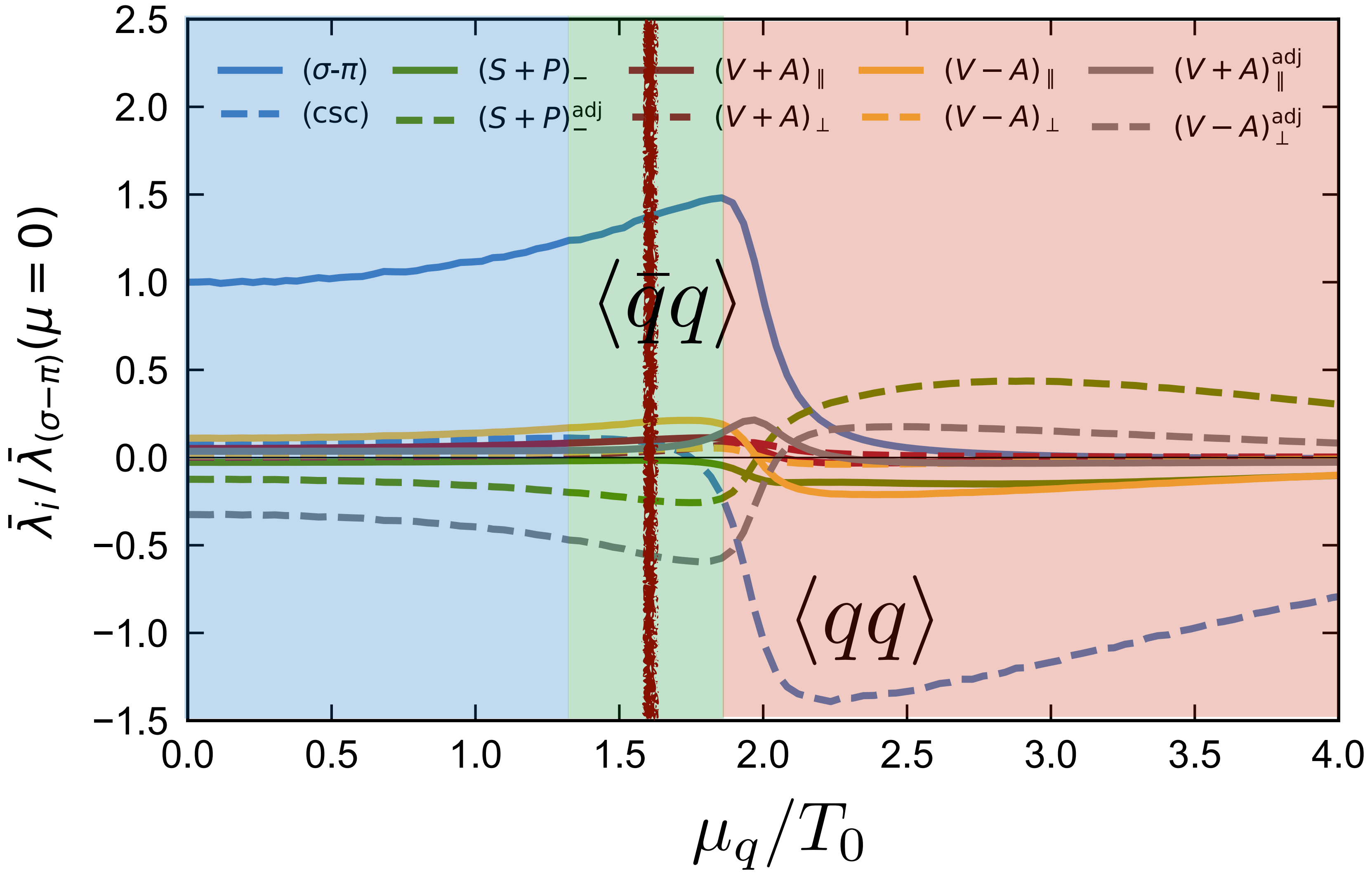


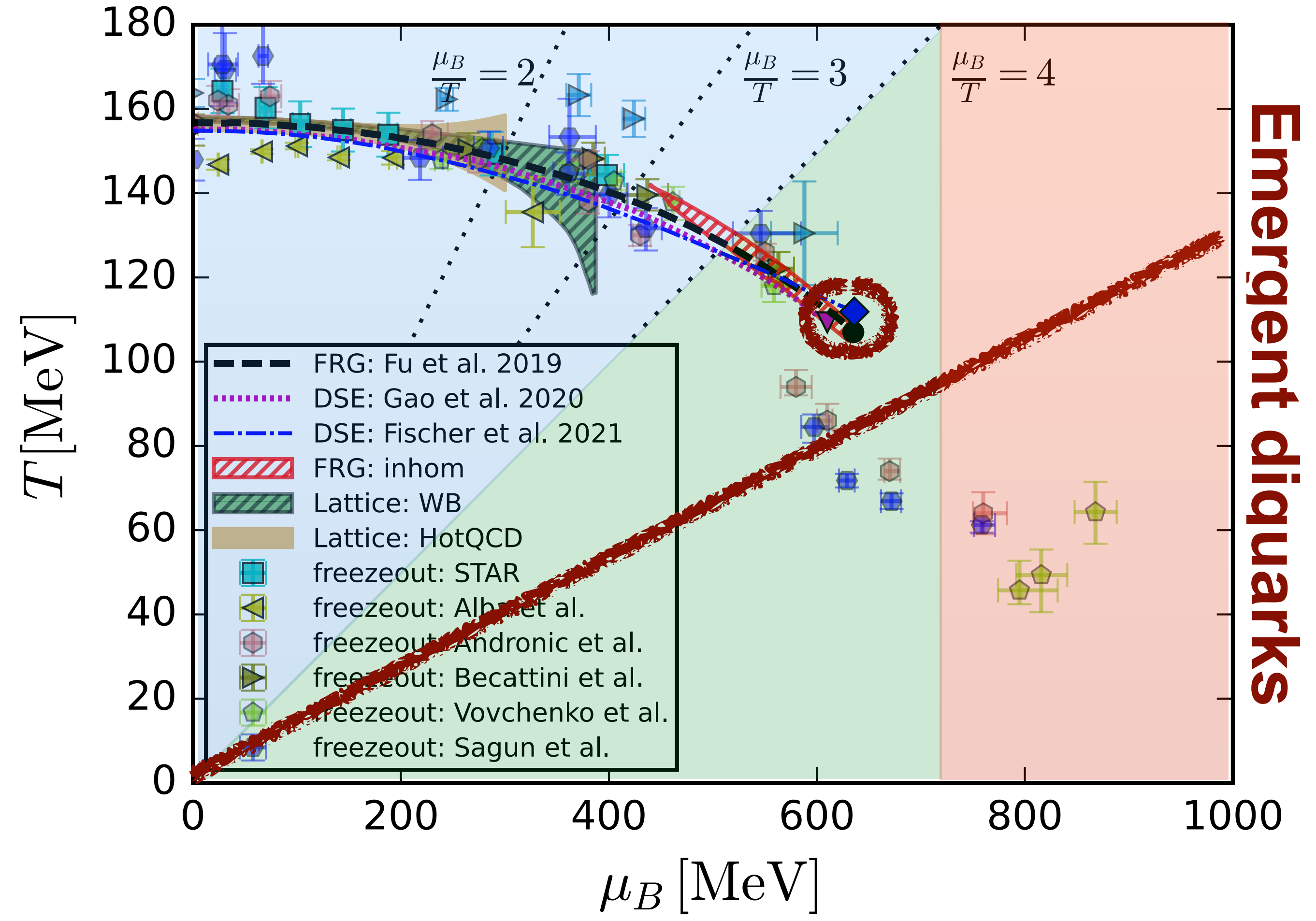
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Emergent diquarks are not captured by extrapolations

Predictions & estimates

Prediction

Emergent diquarks



Regime of quantitative reliability of current best truncation

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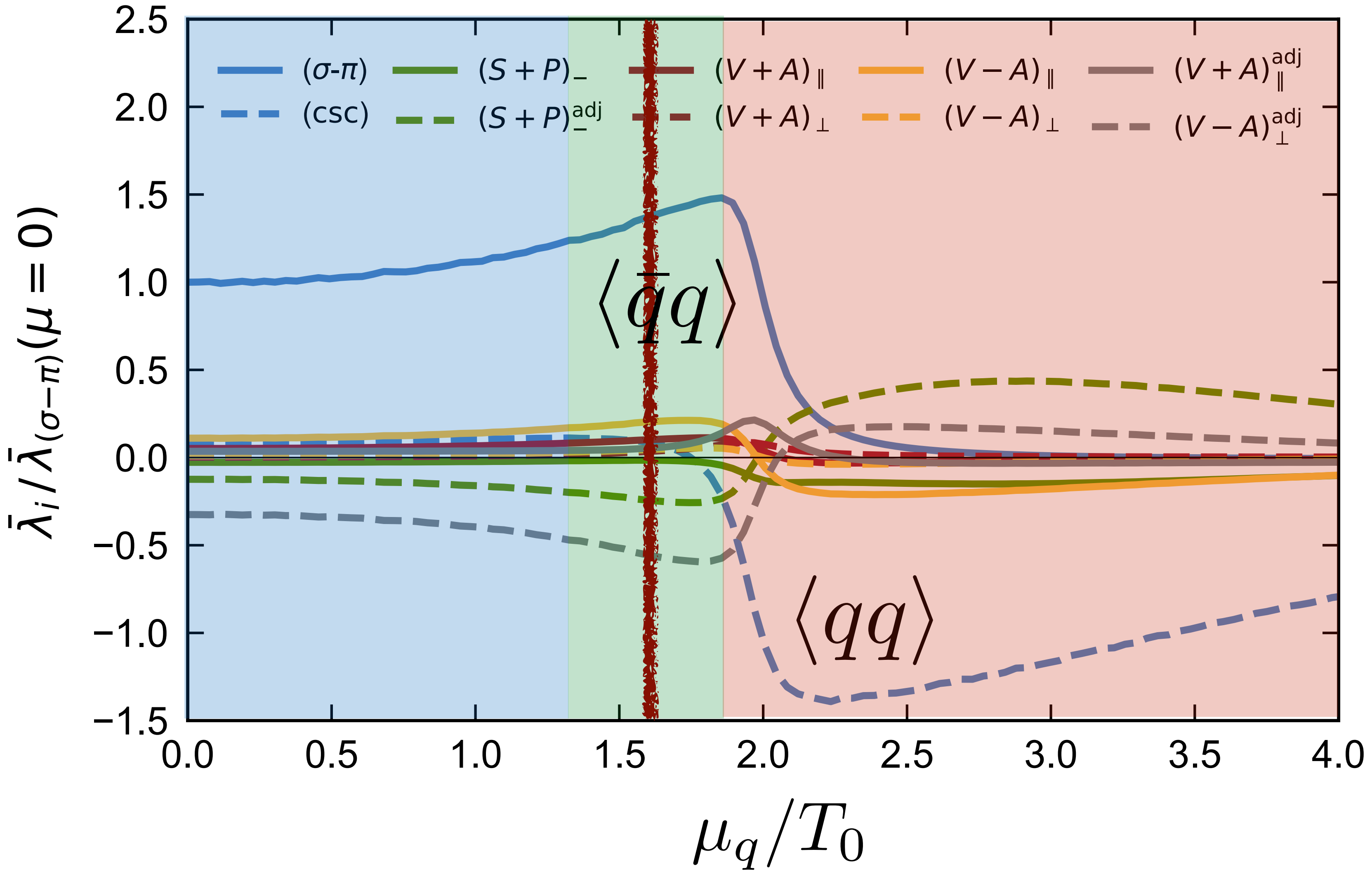
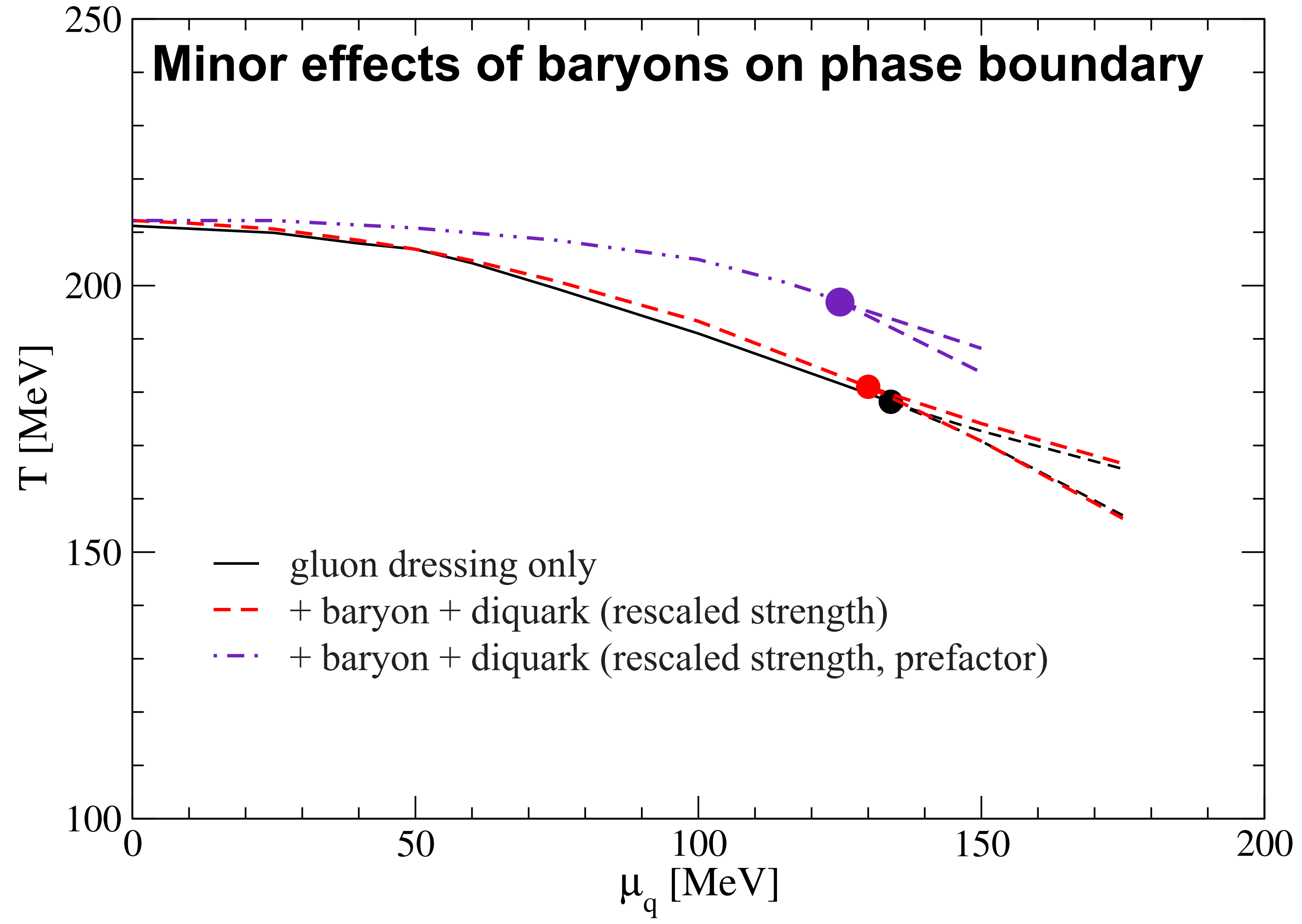


Emergent baryons (DSE)



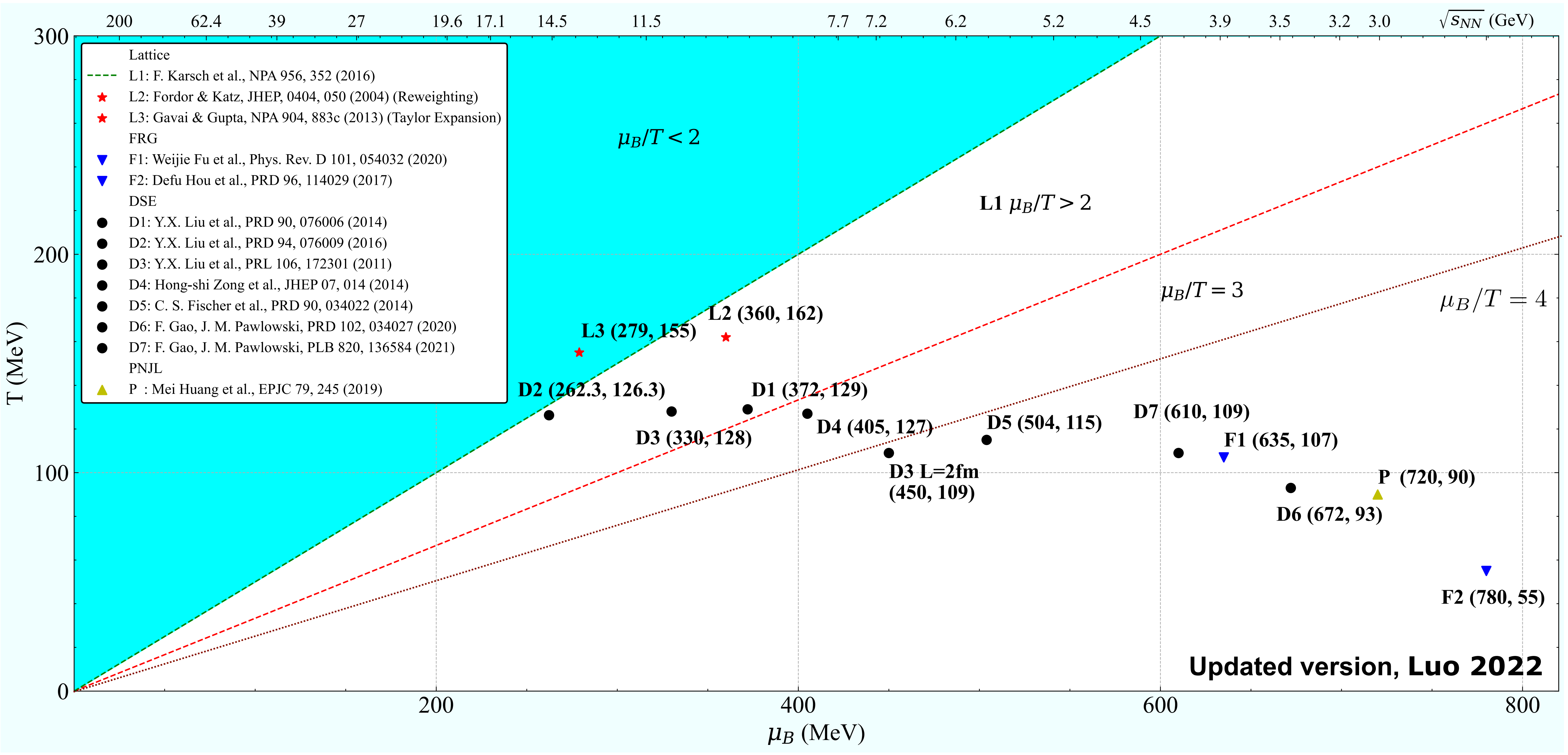
Braun, Leonhardt, Pospiech, PRD 101 (2020) 036004

Eichmann, Fischer, Welzbacher, PRD 93 (2016) 034013

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Xiaofeng Luo The 10th RHIC BES theory and experiment online seminar, Oct. 6th, 2020 9

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them

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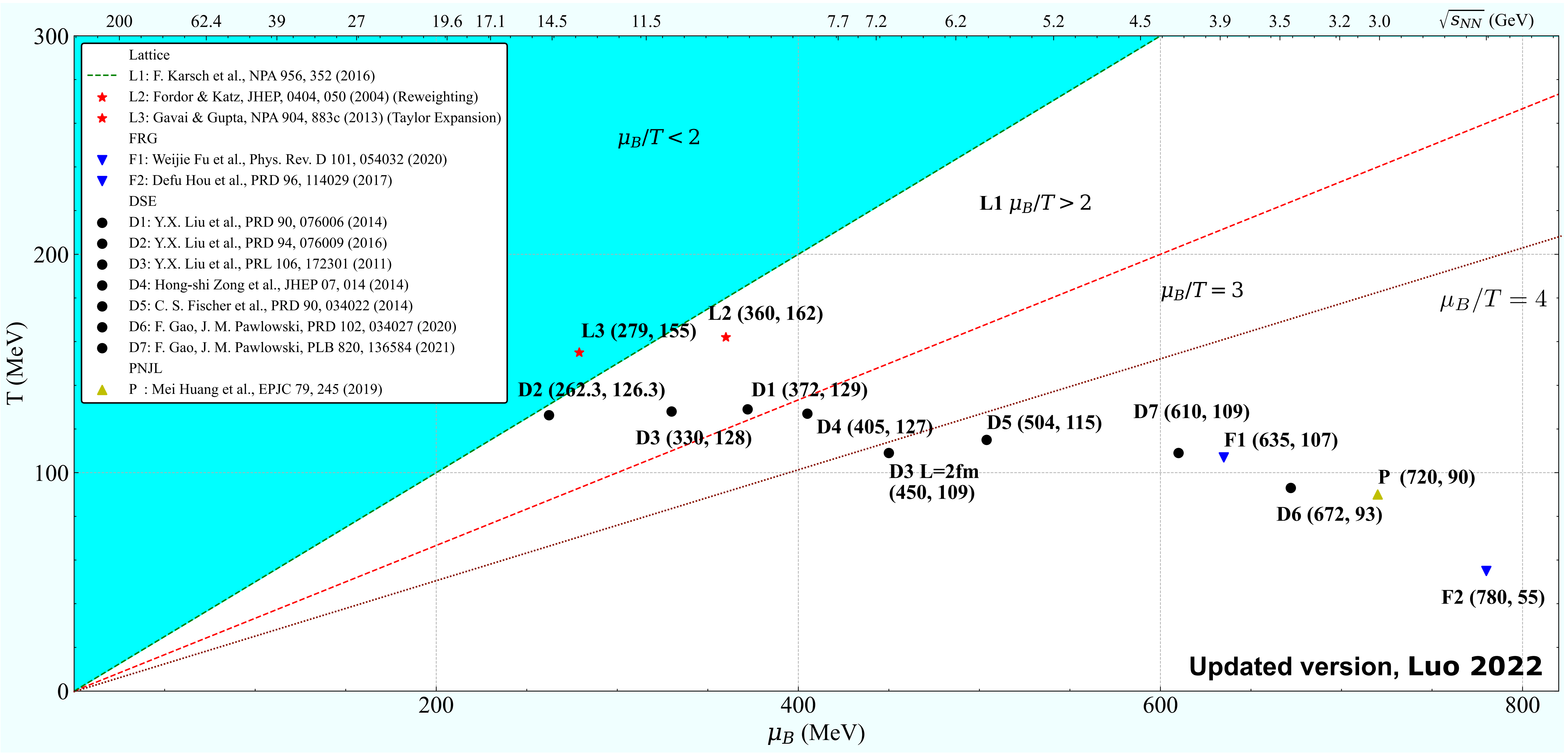

Common folklore since ~2004 ⟷ **Large** uncertainties for the estimation of CP location.

Xiaofeng Luo The 10th RHIC BES theory and experiment online seminar, Oct. 6th, 2020 9

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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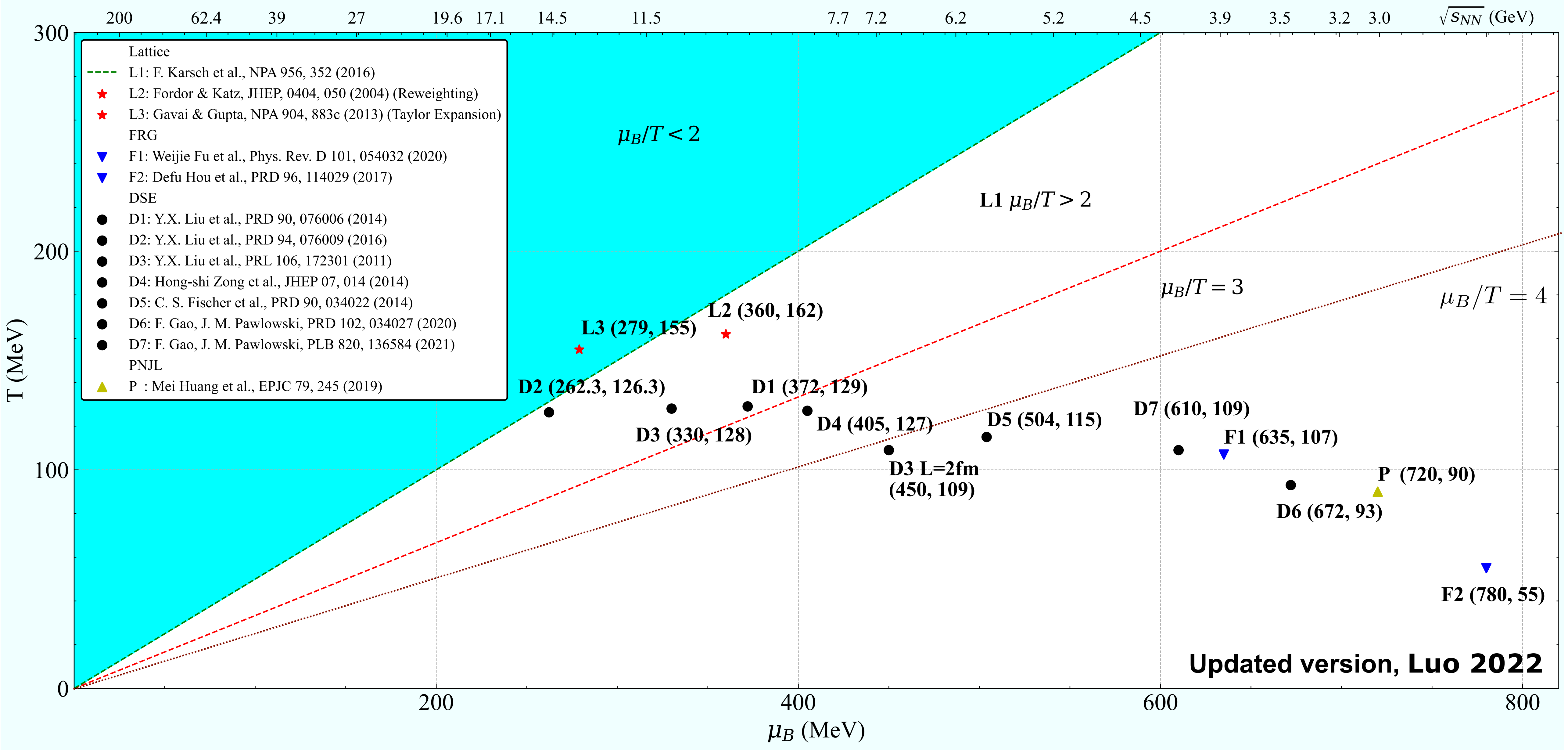

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Xiaofeng Luo The 10th RHIC BES theory and experiment online seminar, Oct. 6th, 2020 9

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

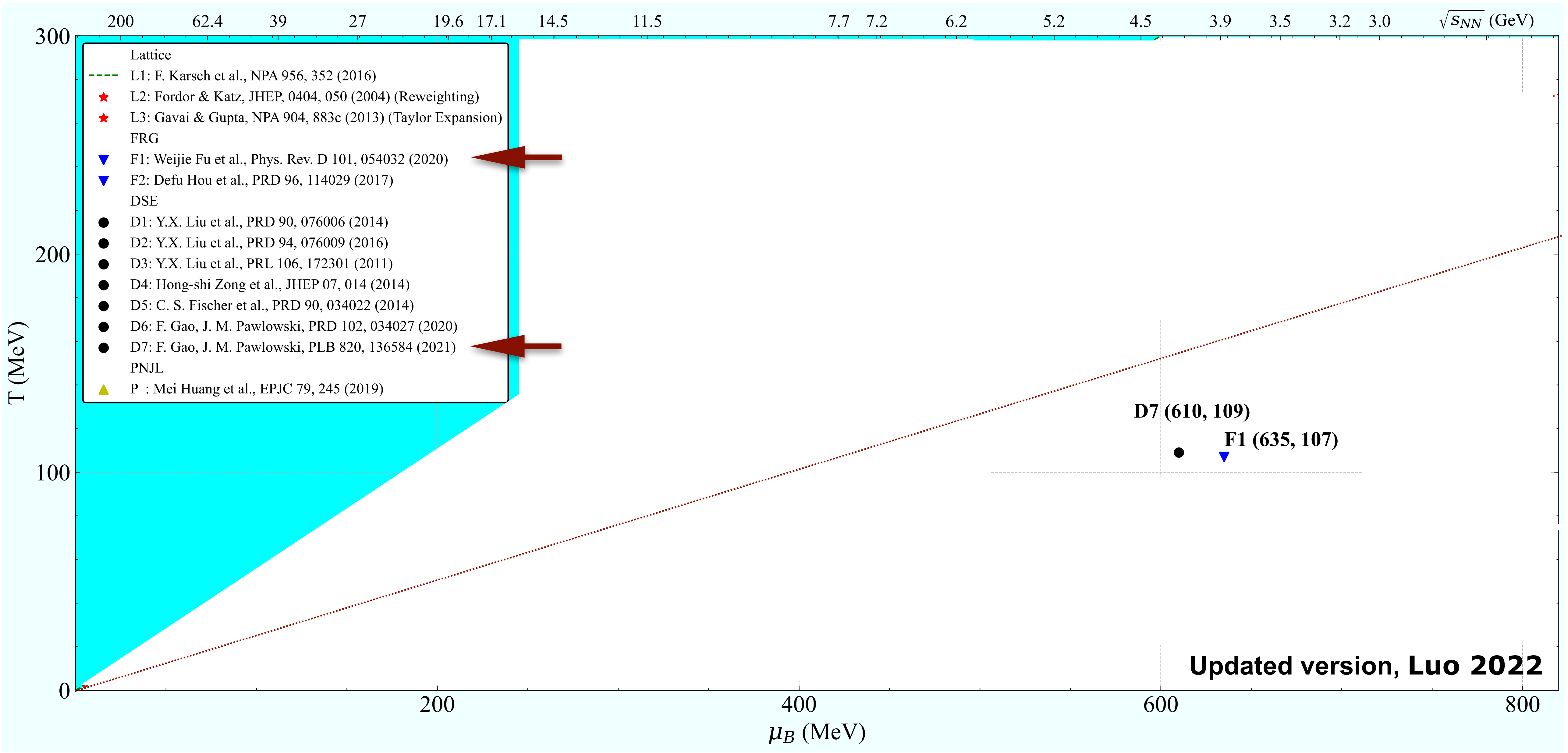
(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to judge them

Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at μ_B =0

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to judge them

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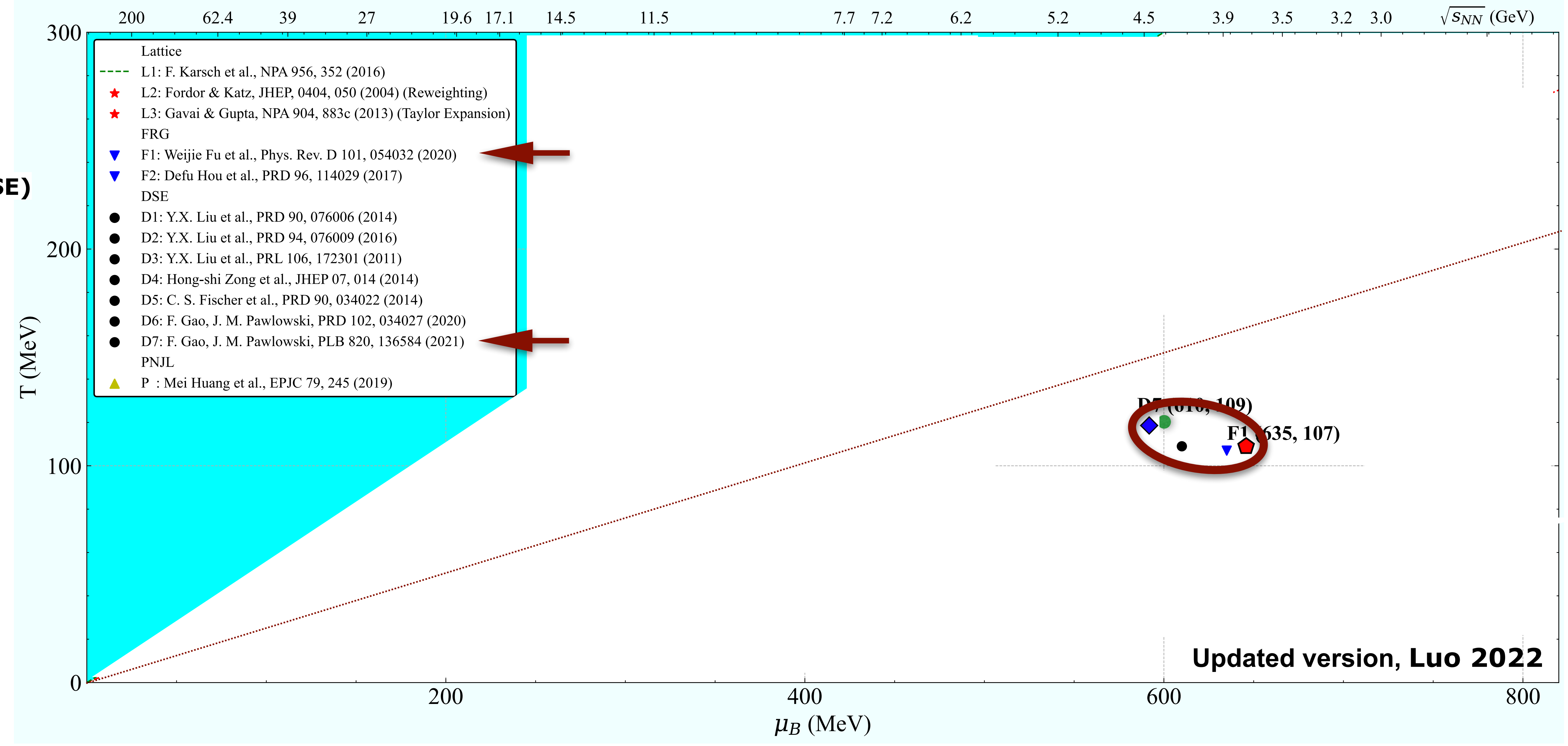
- **Gao, Lu, JMP, Schneider, in prep (DSE)**
- **Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)**
- **Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)**



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Xiaofeng Luo The 10th RHIC BES theory and experiment online seminar, Oct. 6th, 2020 9

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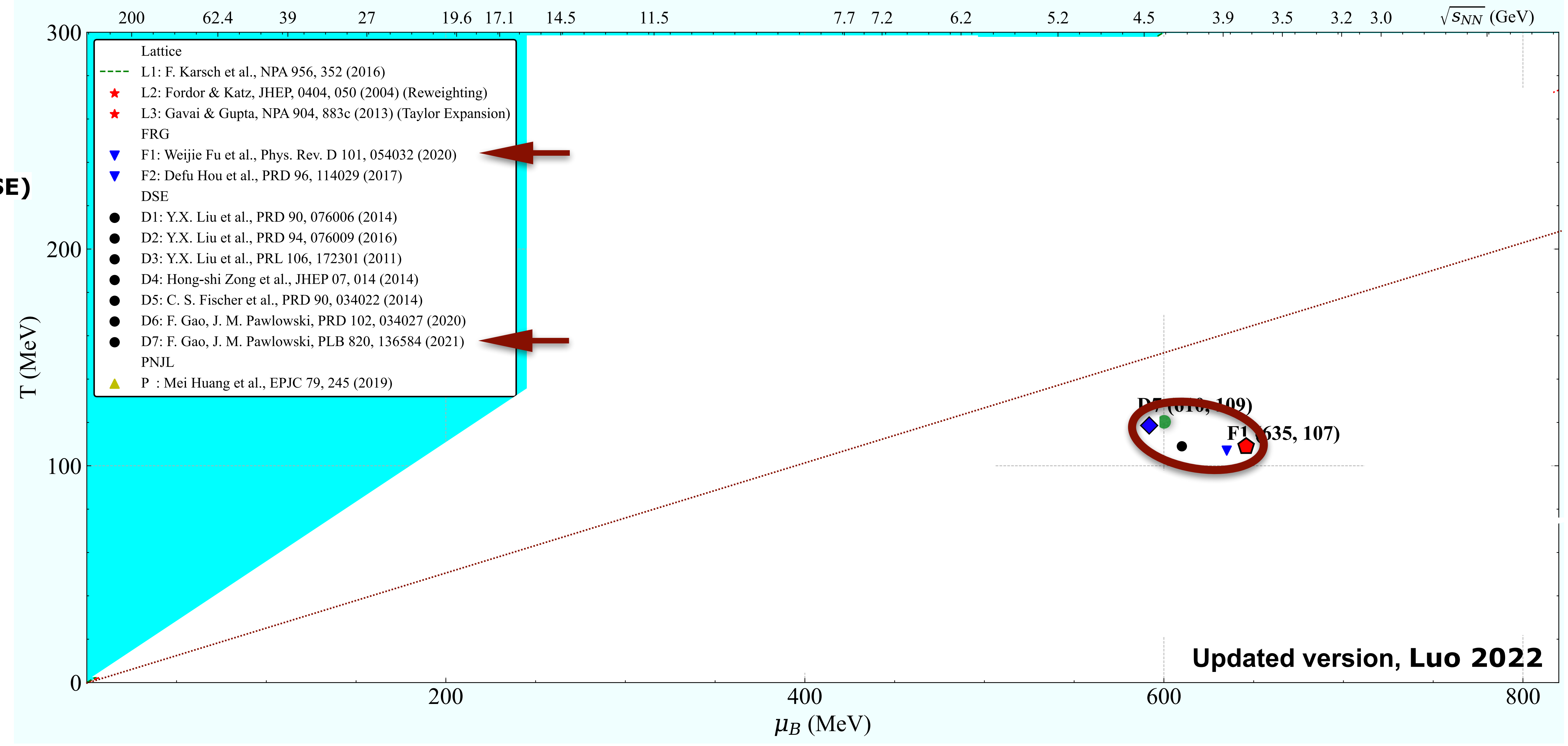

Still uncertainties for the estimation of CP location.

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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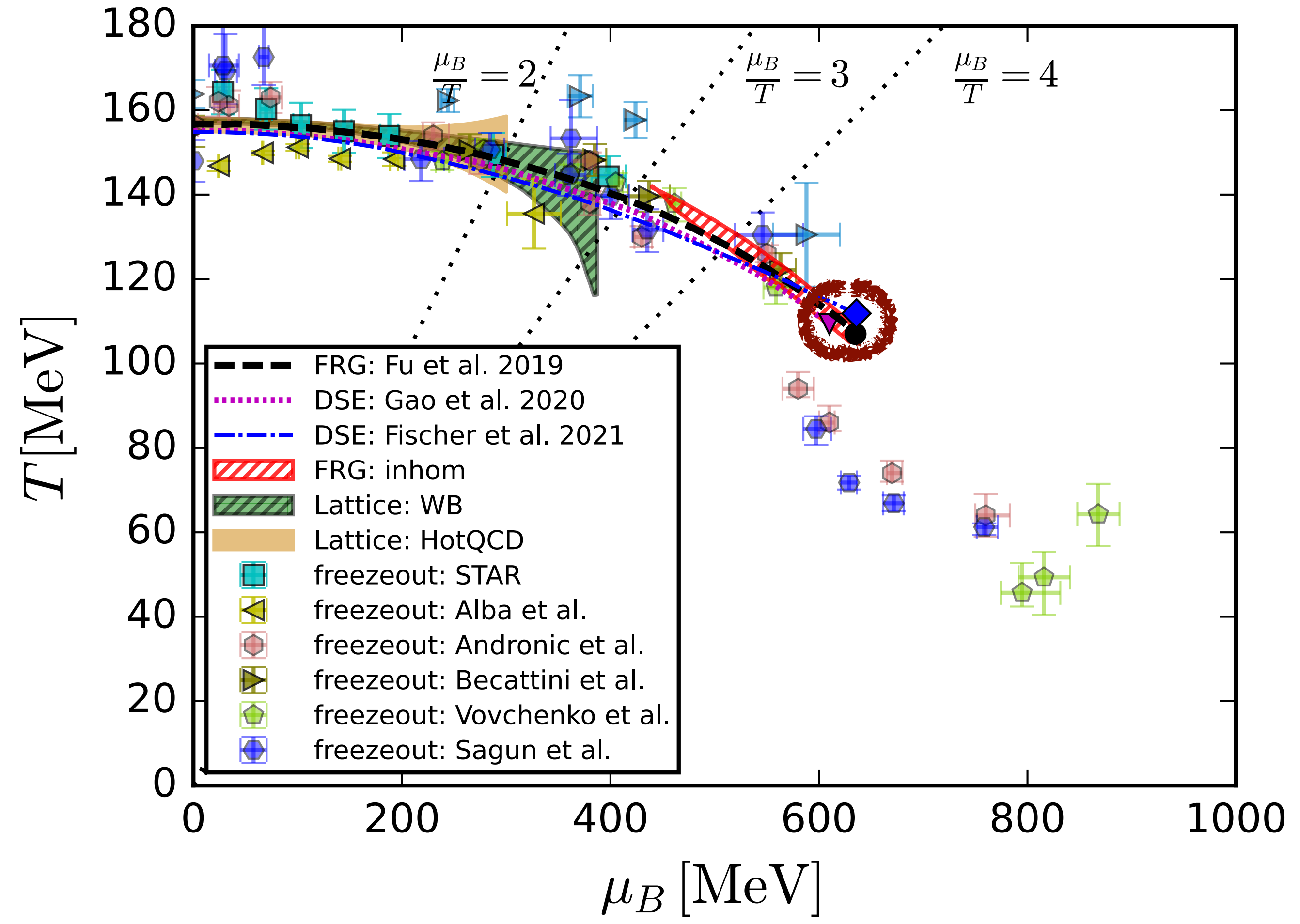
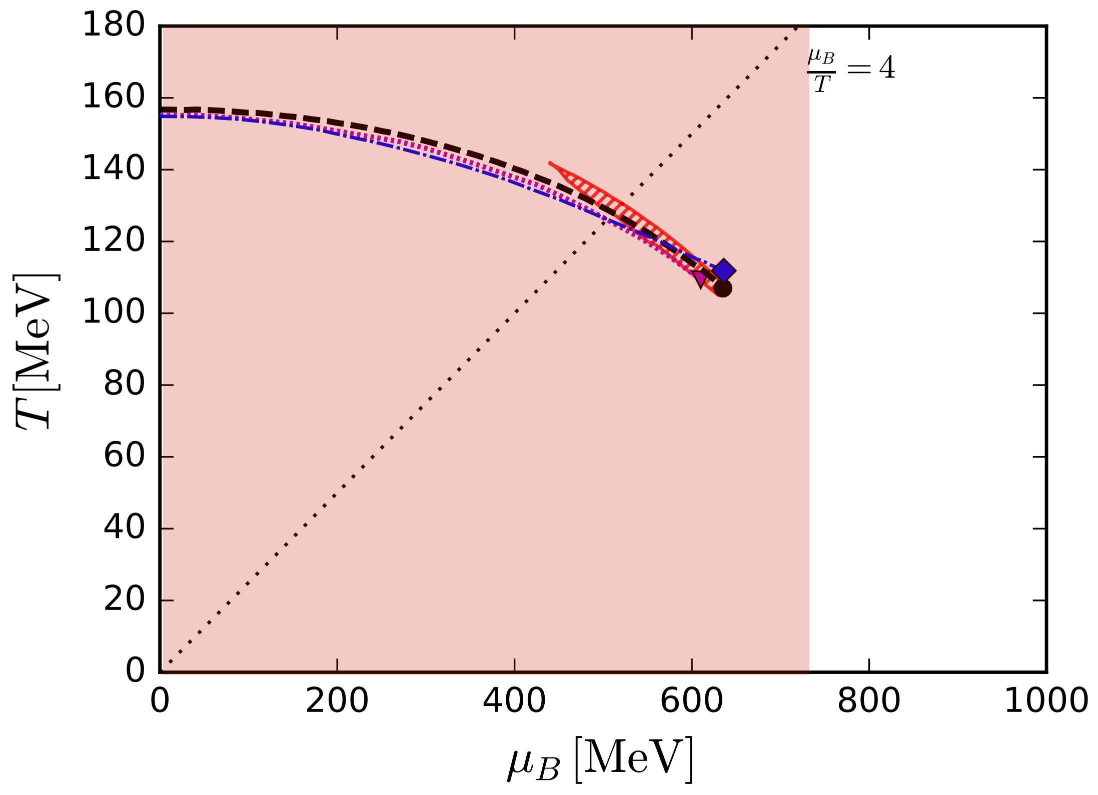
Predictions, estimates & extrapolations and how to use them



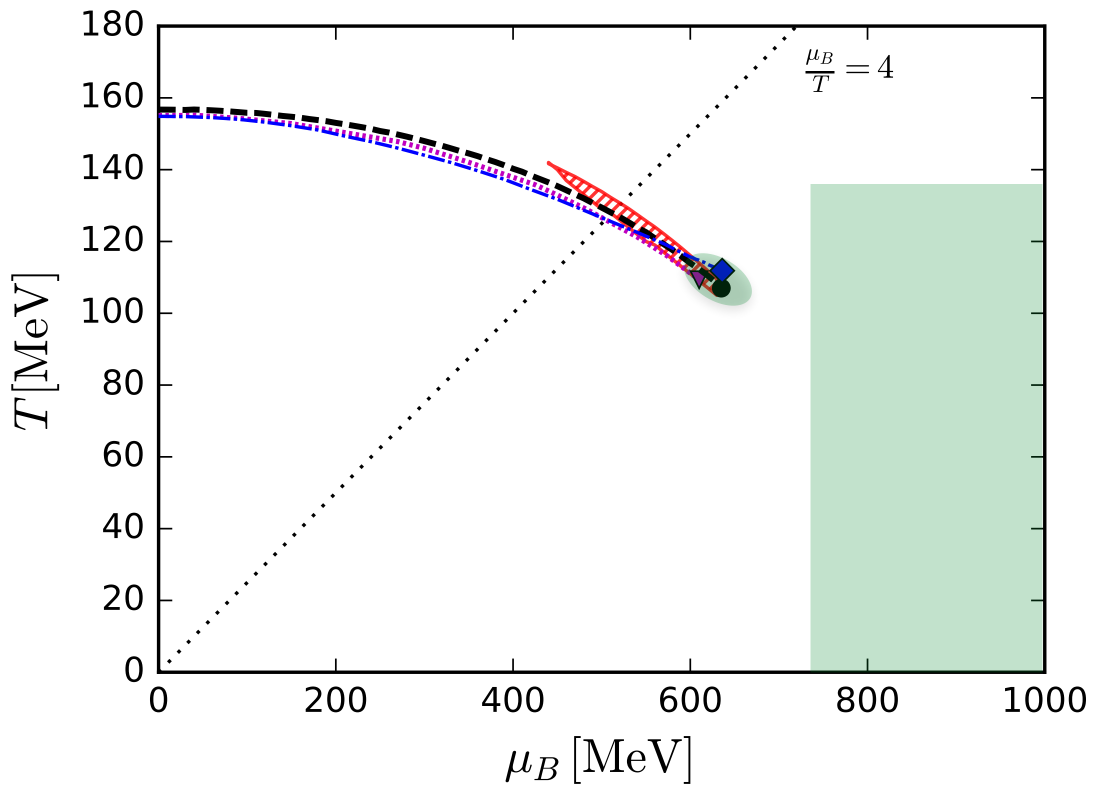
Scenario I

Scaling



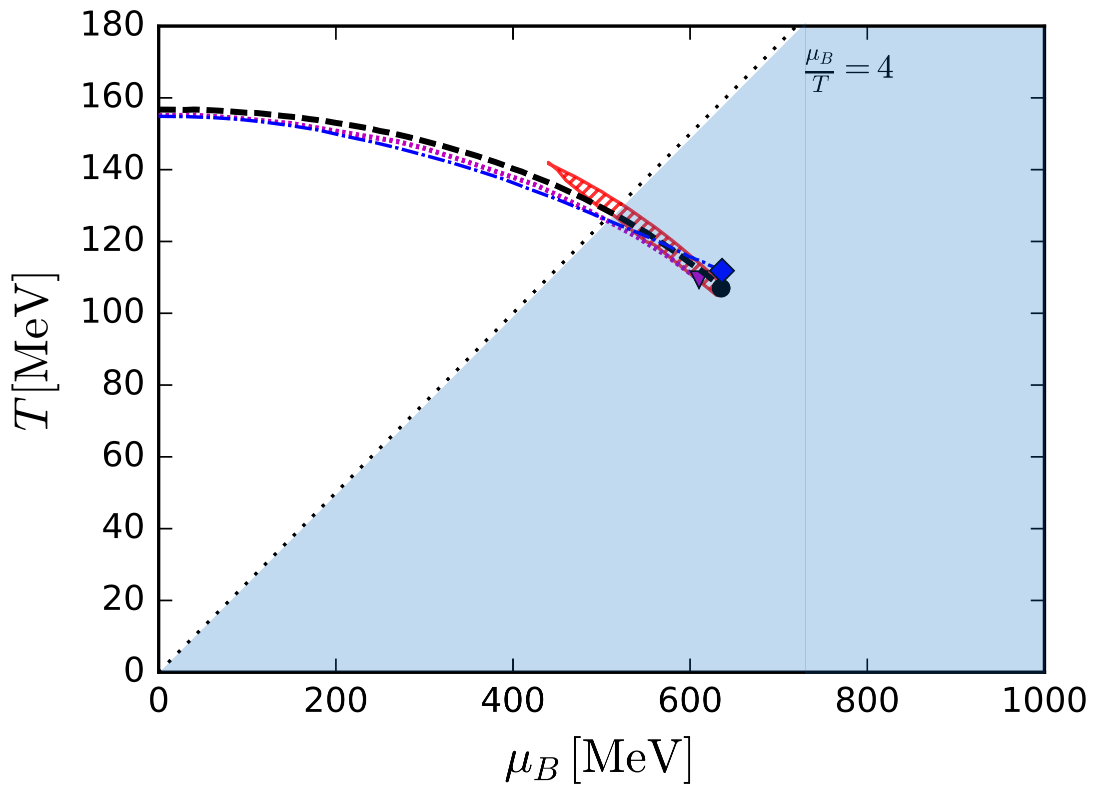
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



Extrapolations for Pheno	Scenario I	Scenario II	Scenario III
	✓	✓	✗

Predictions, estimates & extrapolations and how to use them

Out by the LEGO ® principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

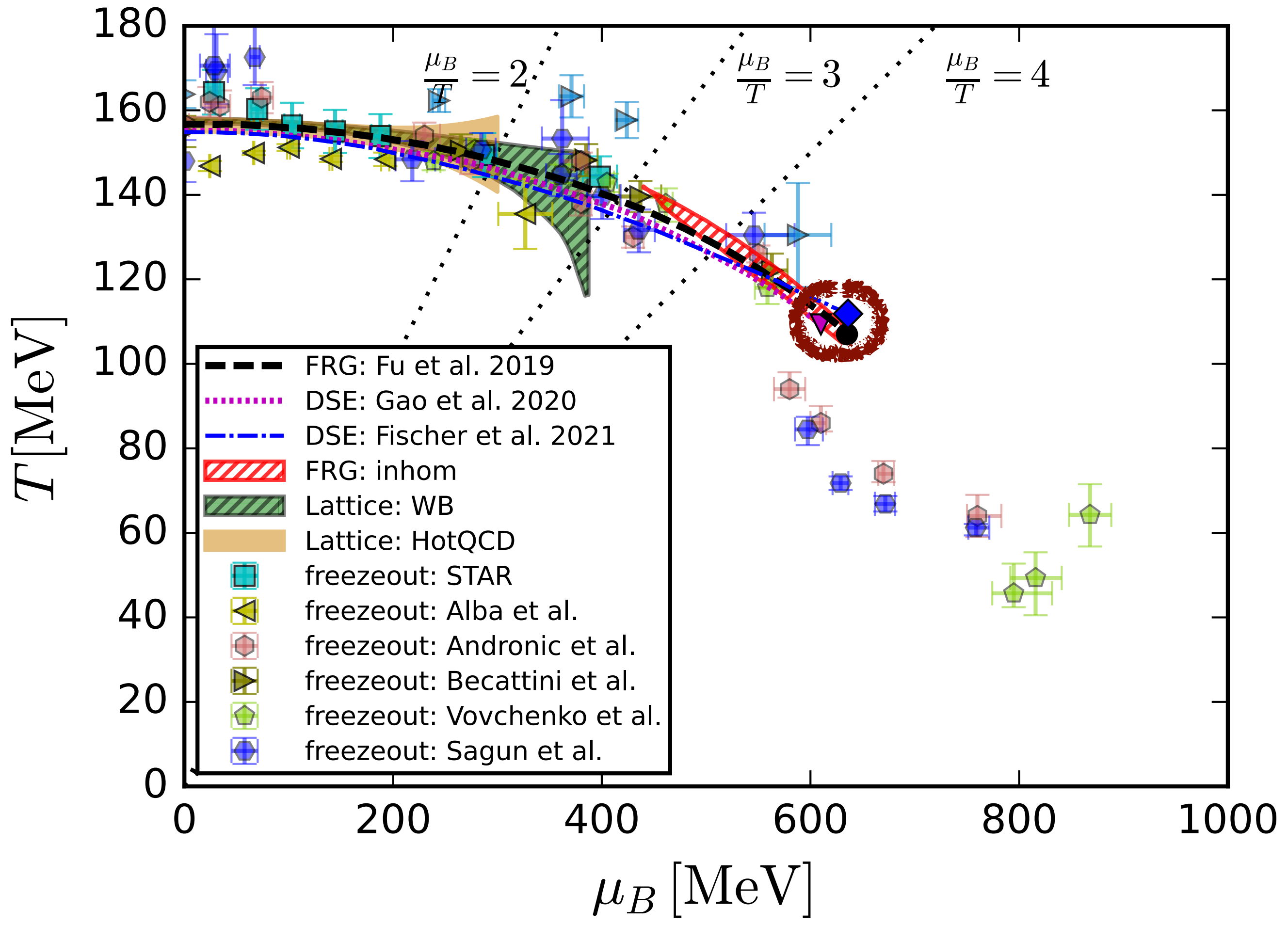
+

Size of scaling regime in LEFTs

Schaefer, Wambach, PRD 75 (2007) 085015

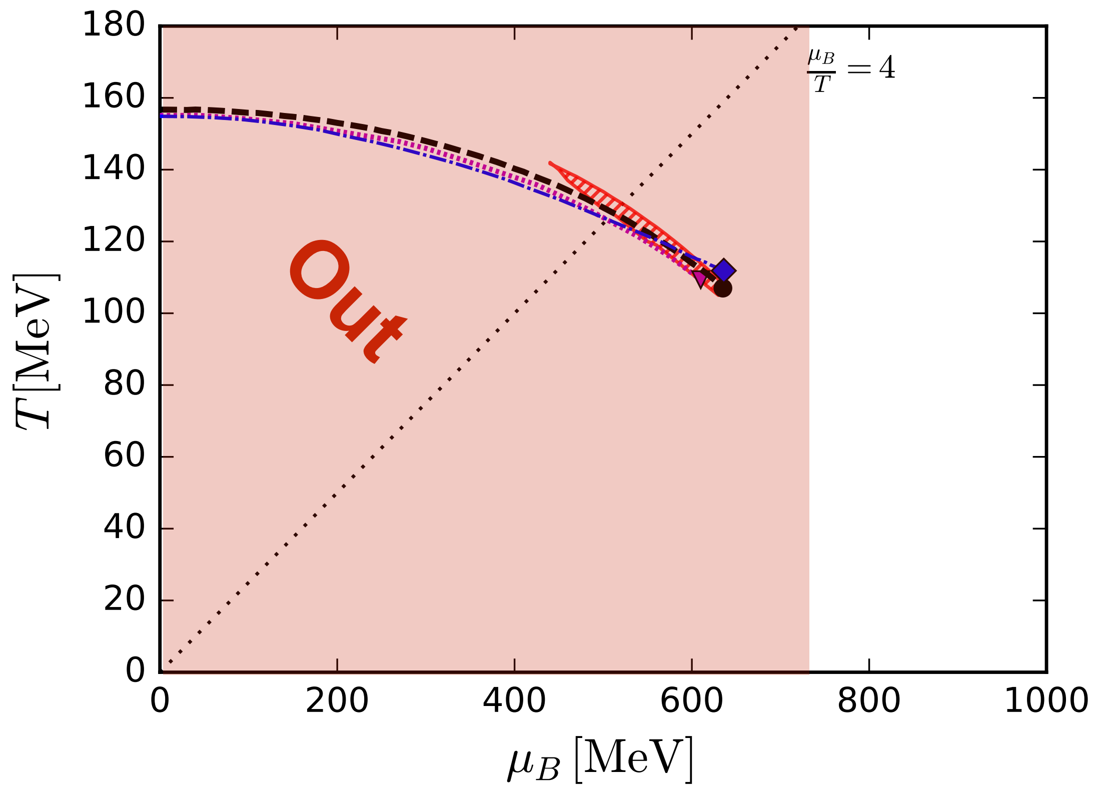
Braun, Klein, Piasecki, EPJC 71 (2011) 1576

⋮



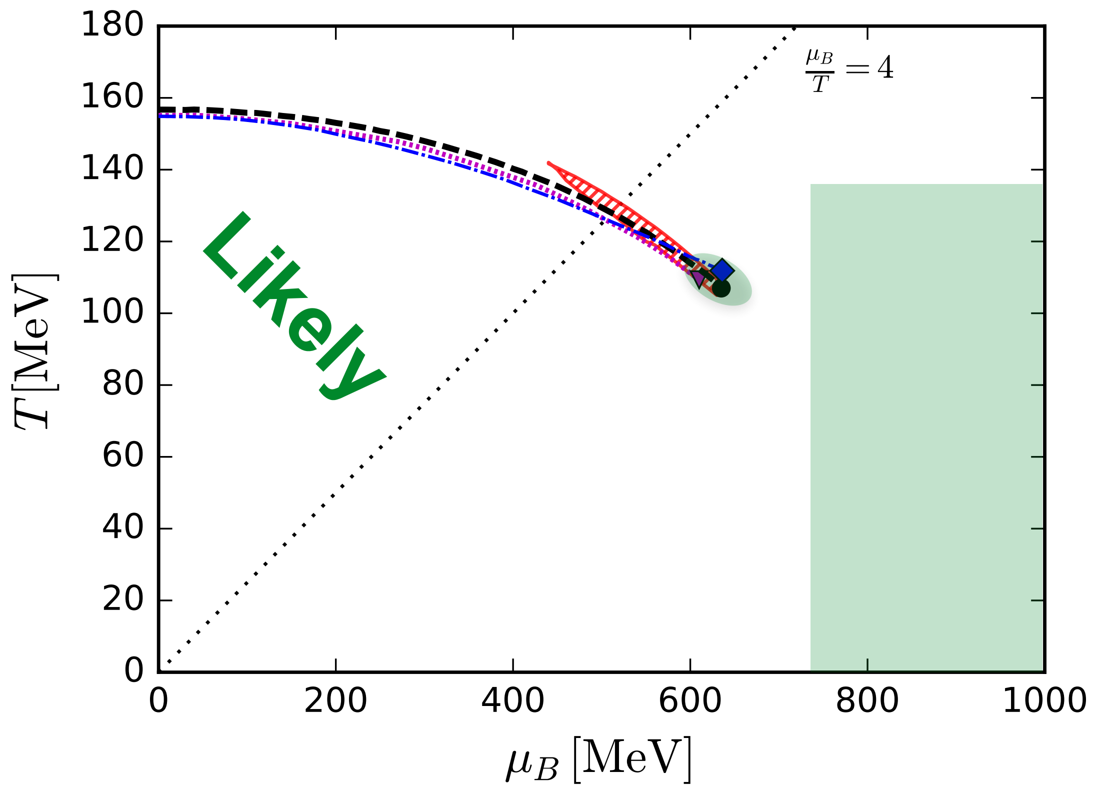
Scenario I

Scaling



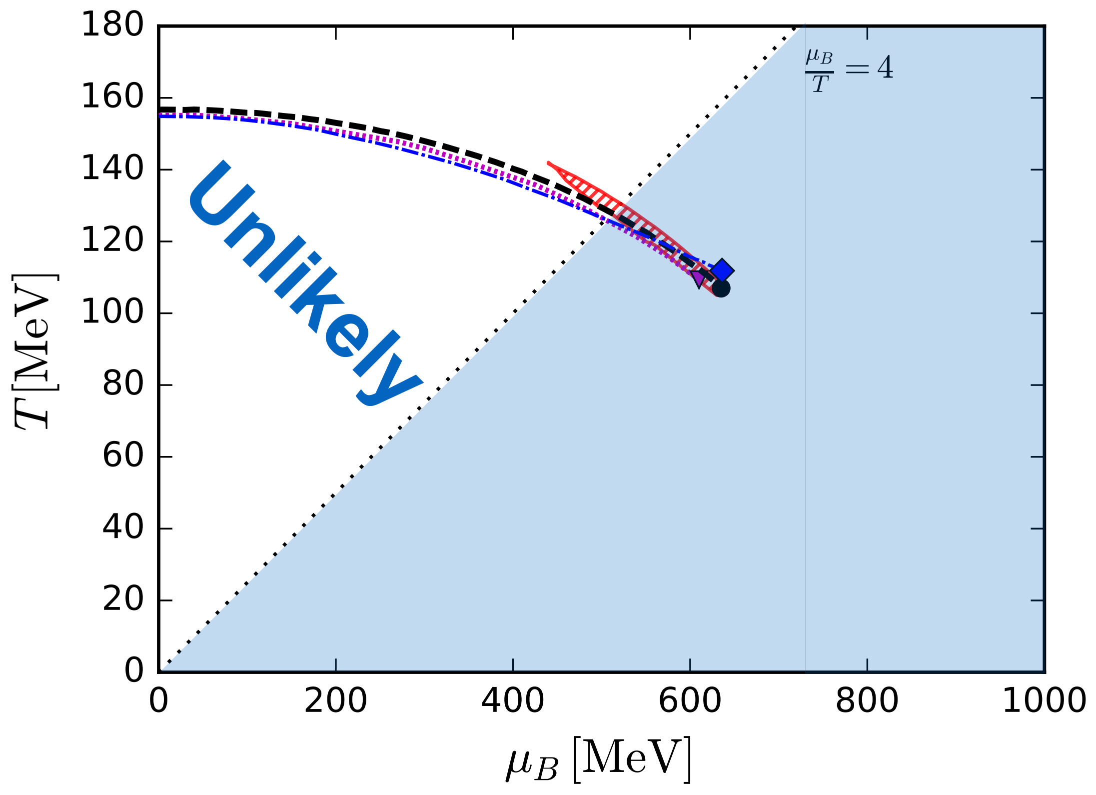
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

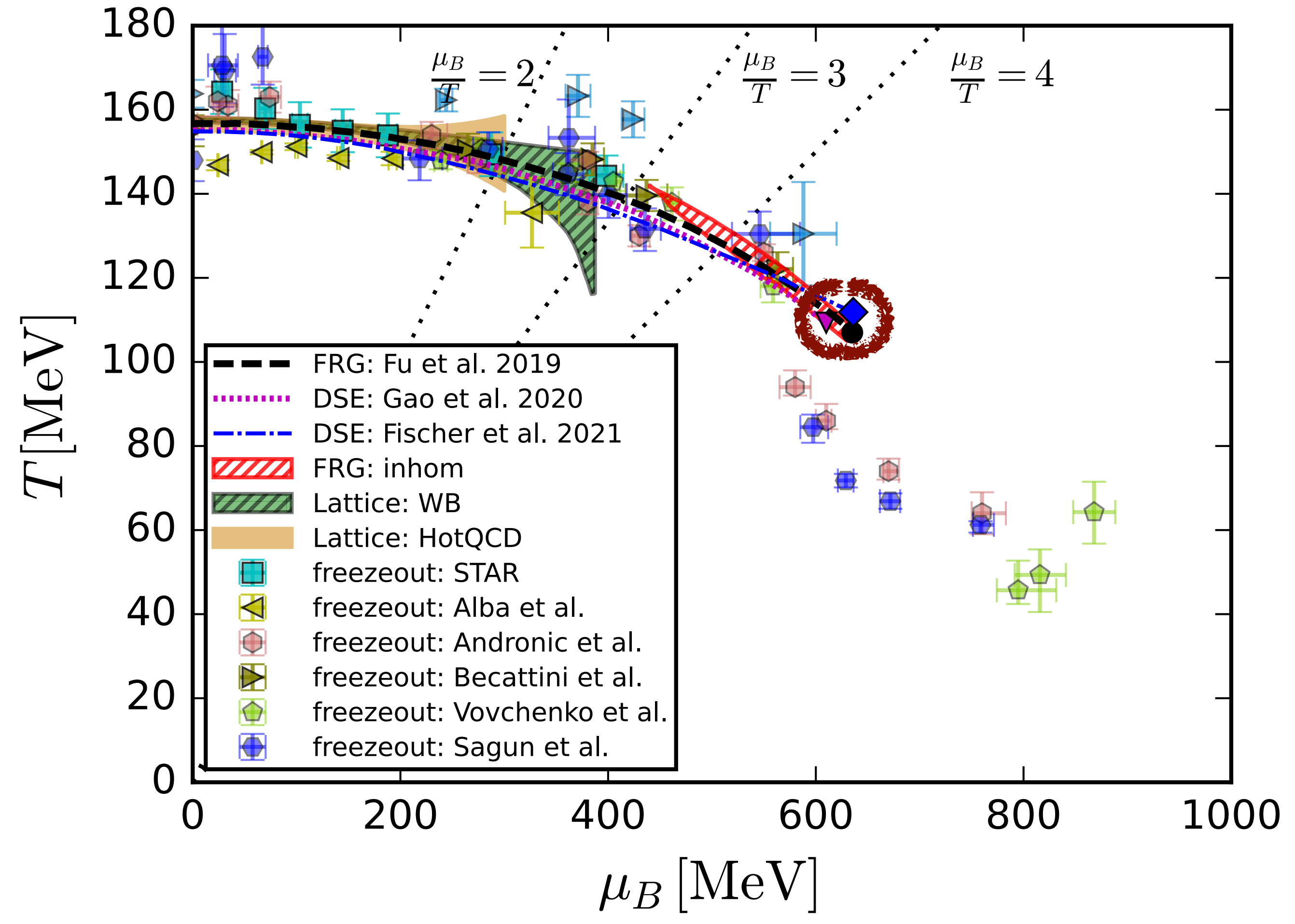
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: **Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853**

+ many results in dynamical low energy effective theories

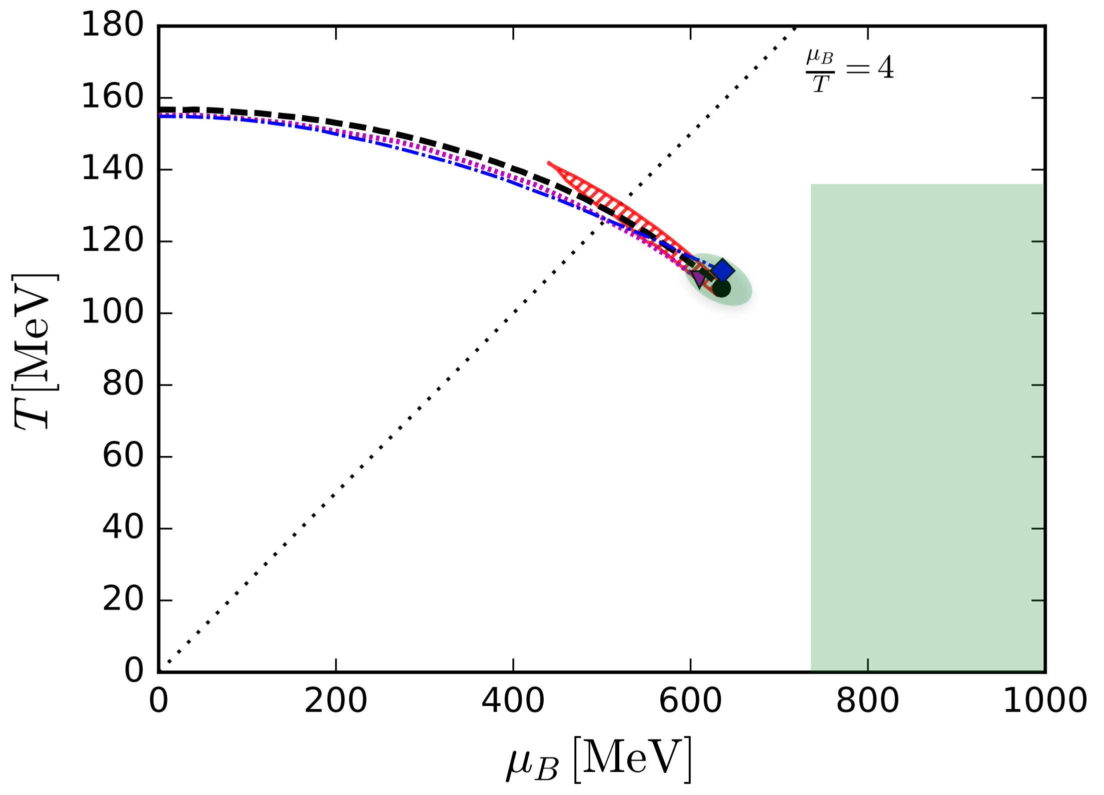
LEGO® principle

Predictions, estimates & extrapolations and how to use them



Scenario II

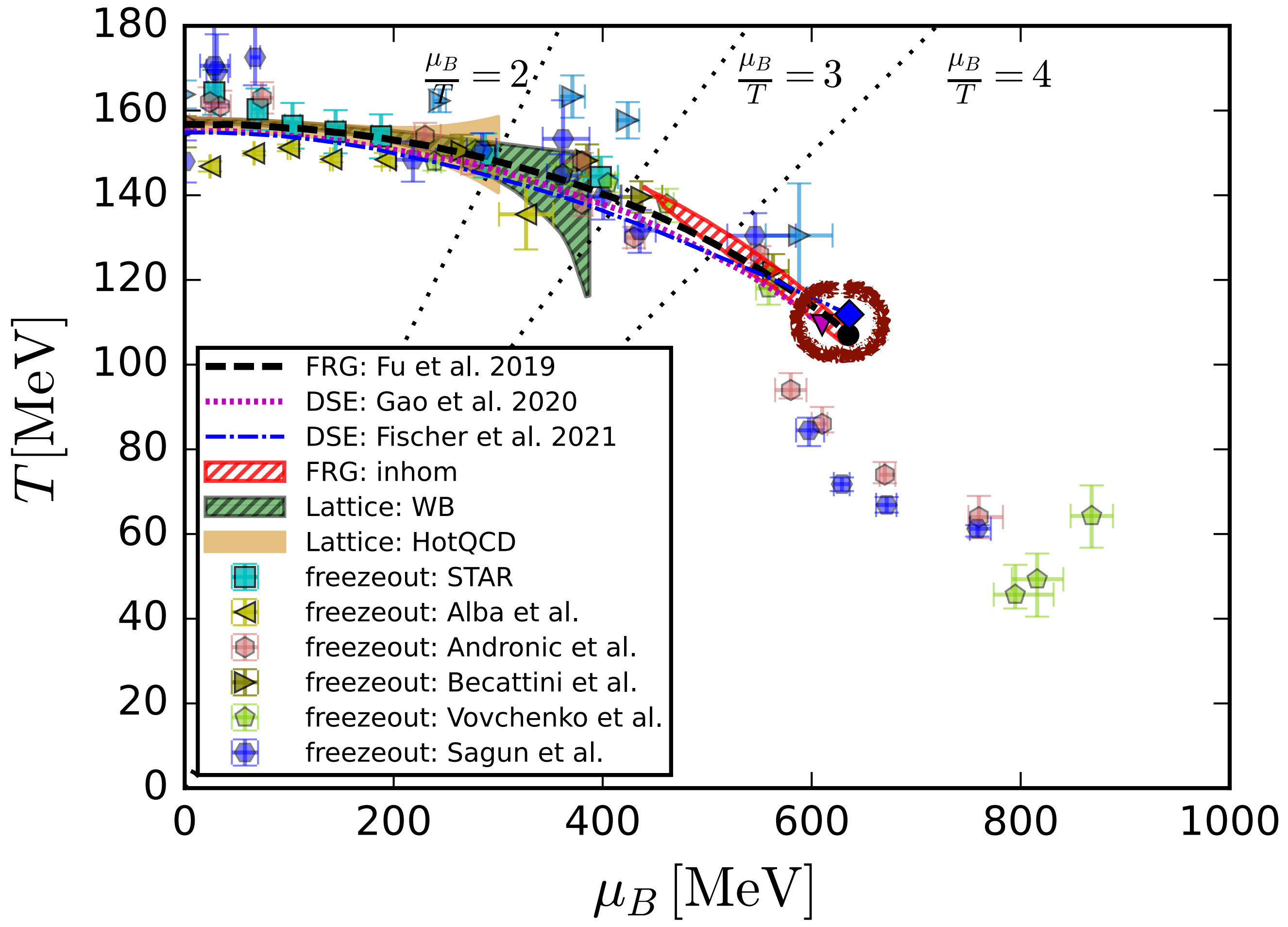
Minimal scaling and new phases



Extrapolations for Pheno

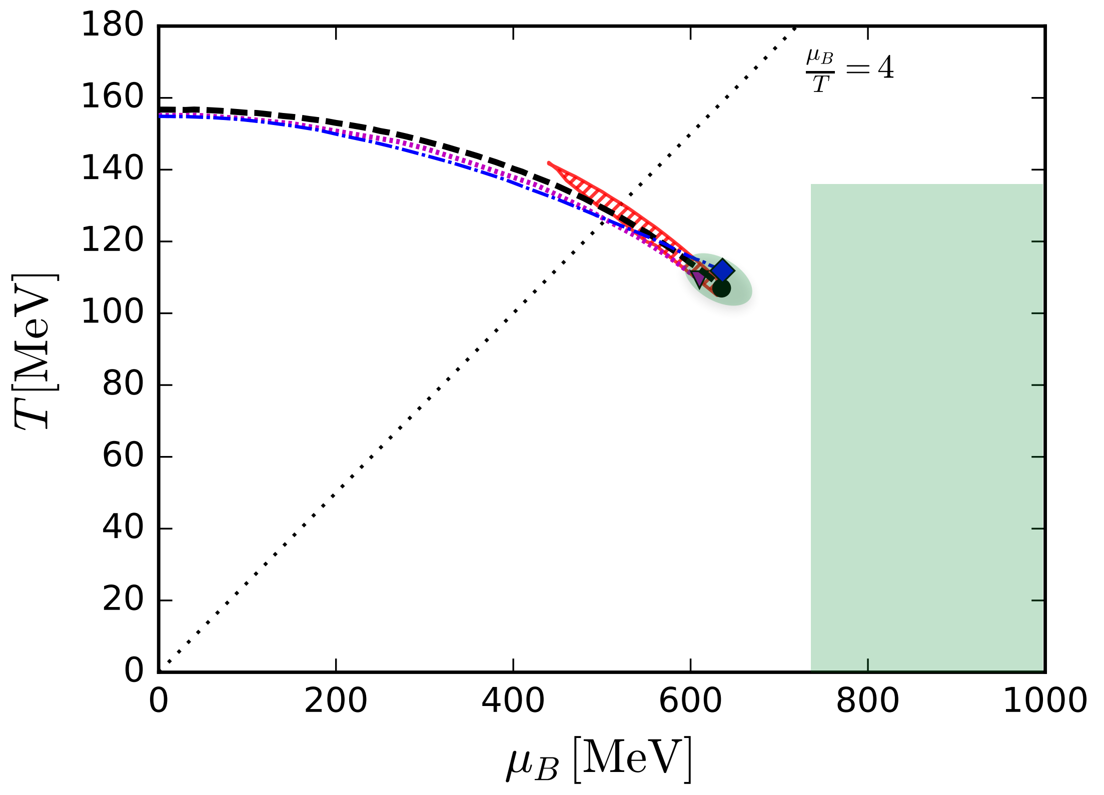
✓

Predictions, estimates & extrapolations and how to use them



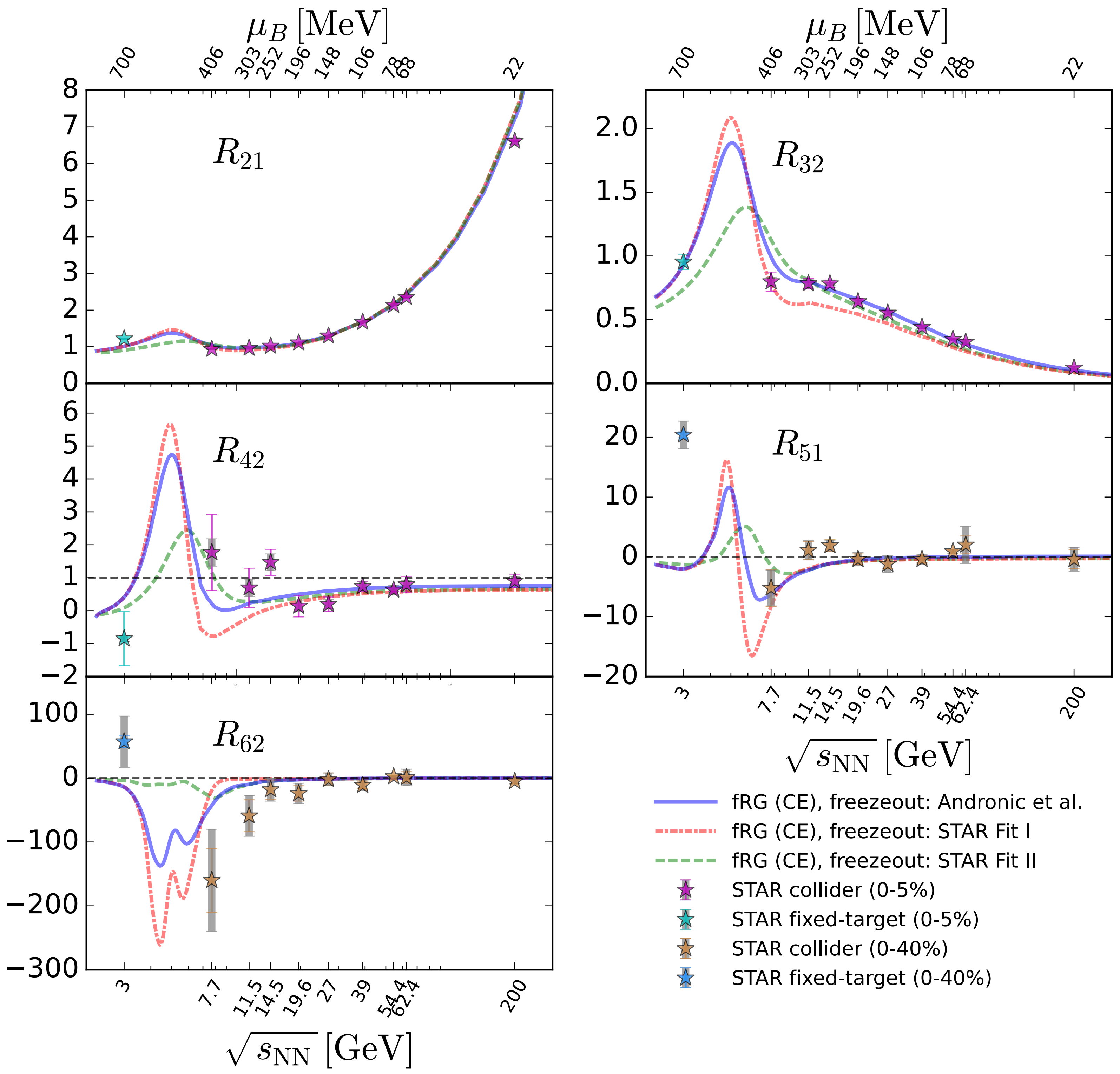
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations



see talk of Wei-jie Fu

Extrapolations for Pheno

✓

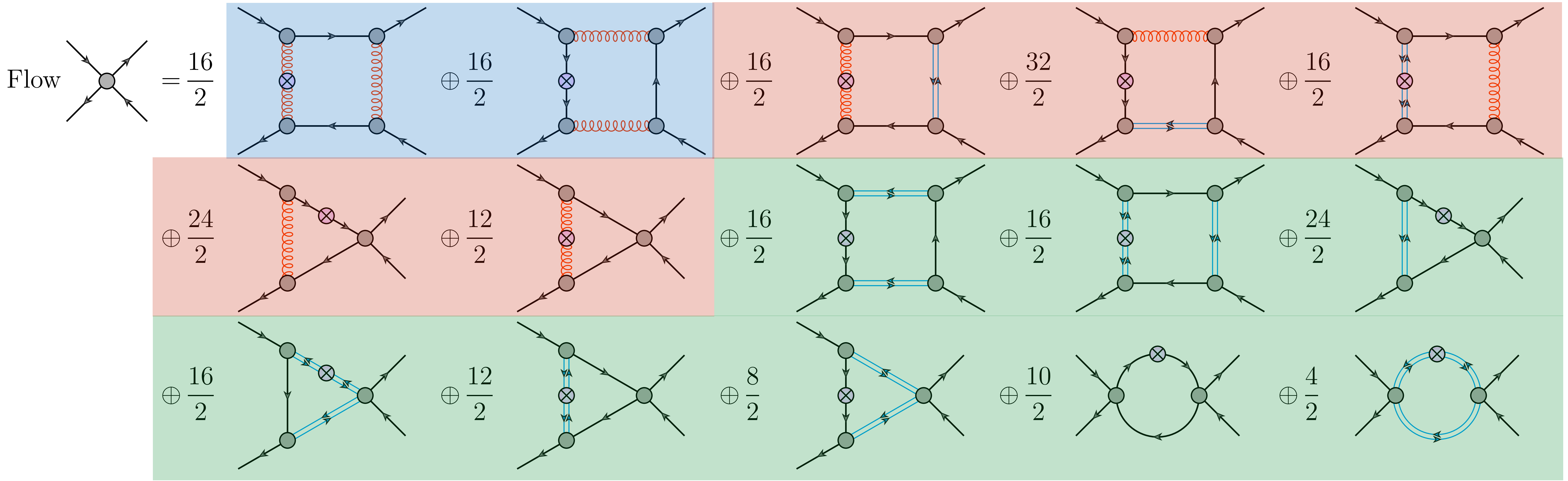
Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

The unreasonable effectiveness of low energy effective theories

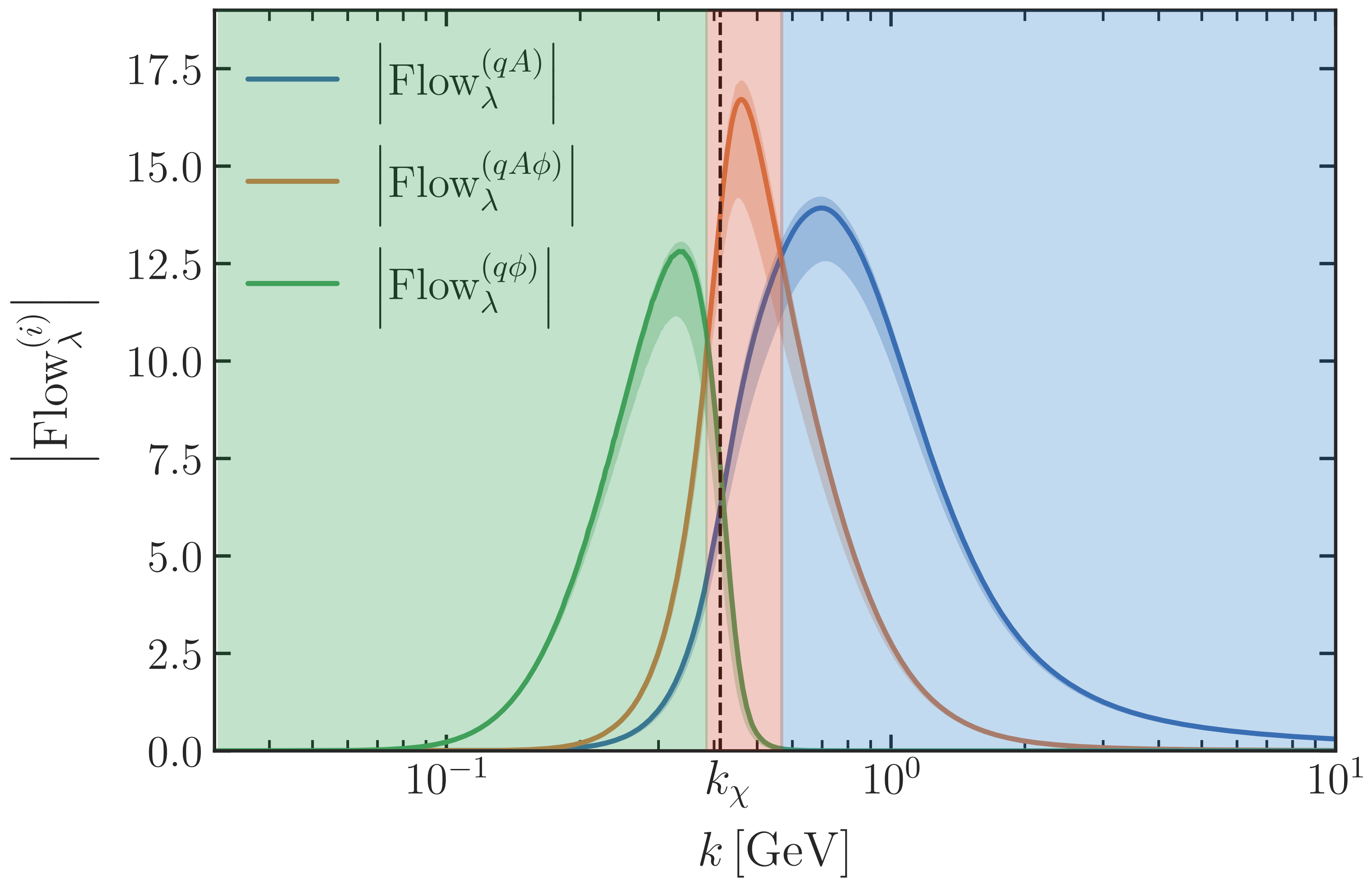
or

the LEGO® principle at work

The LEGO® principle at work

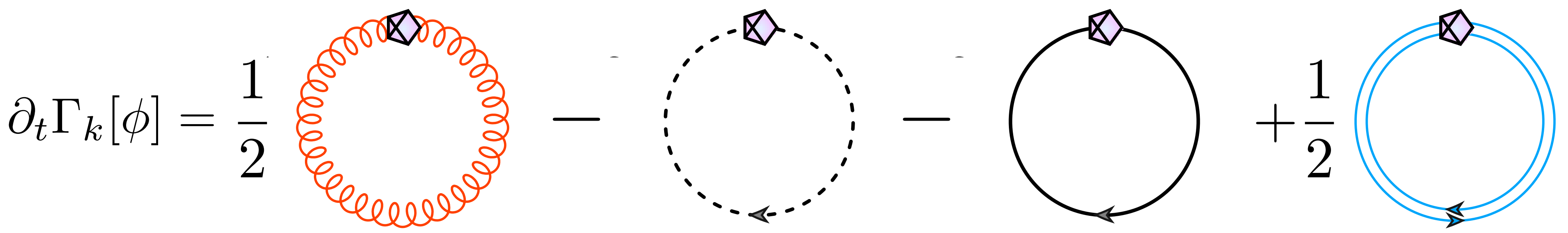


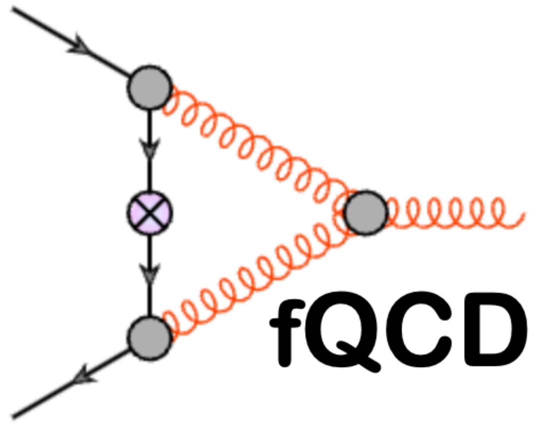
The unreasonable effectiveness of low energy effective theories



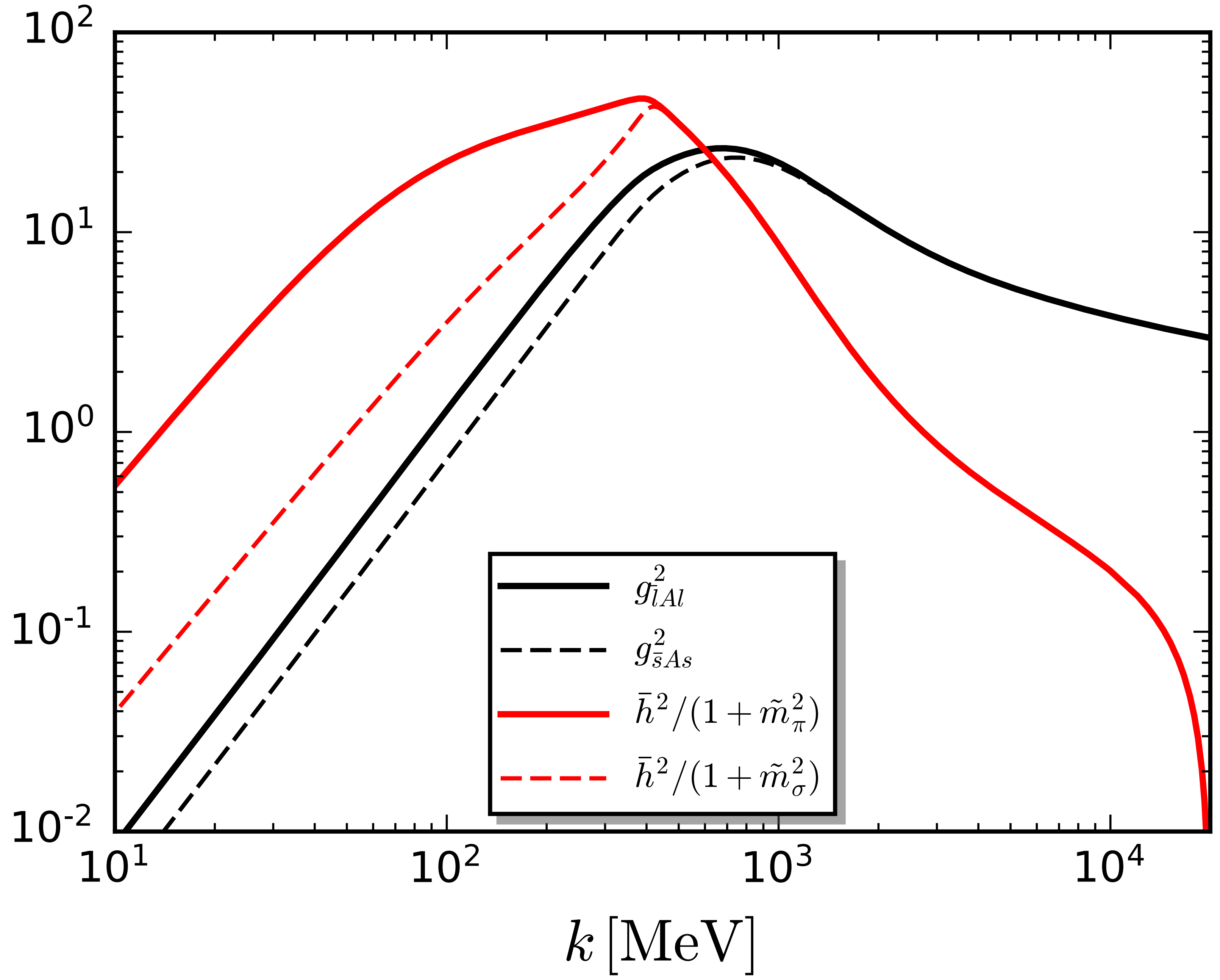
Access and combined use of
error estimates
from functional QCD & LEFTs

On the unreasonable effectiveness of low energy effective theories

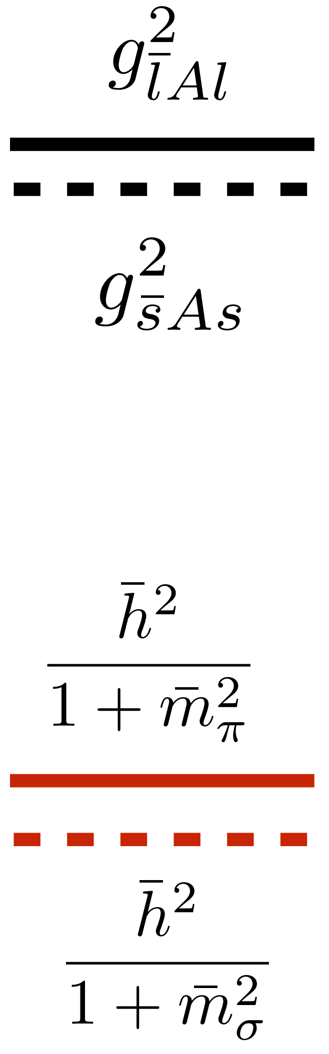




Sequential decoupling of gluon, quark, sigma, pion fluctuations



Fu, JMP, Rennecke, PRD 101, (2020) 054032

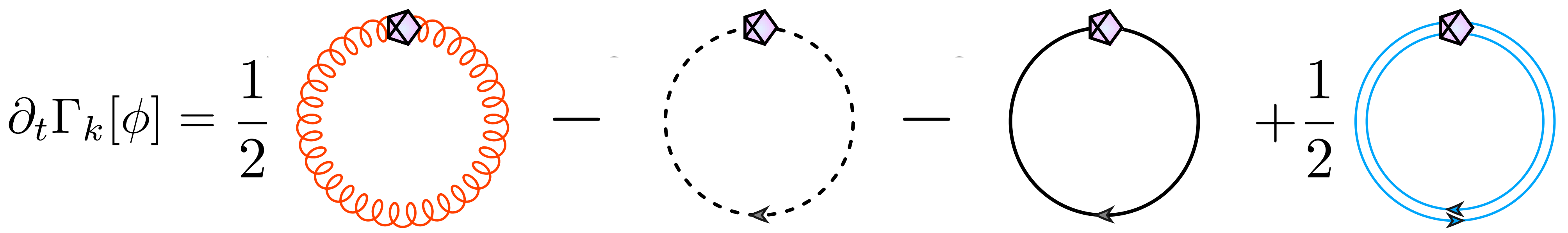


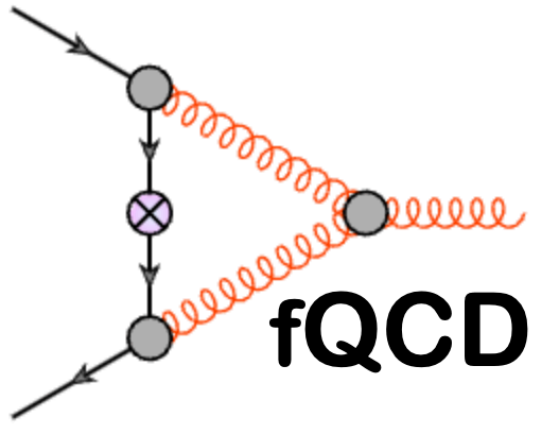
Based on:

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories





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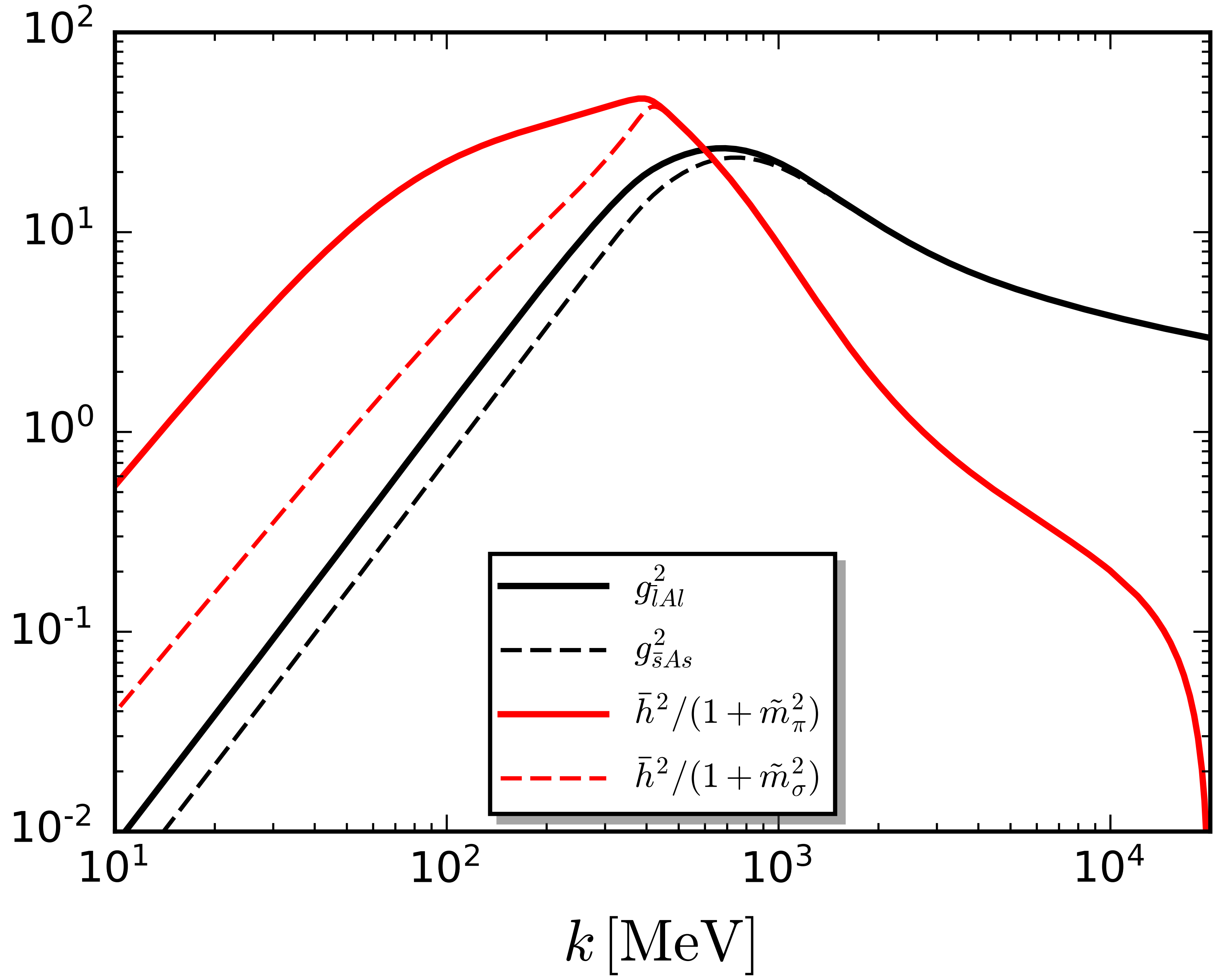

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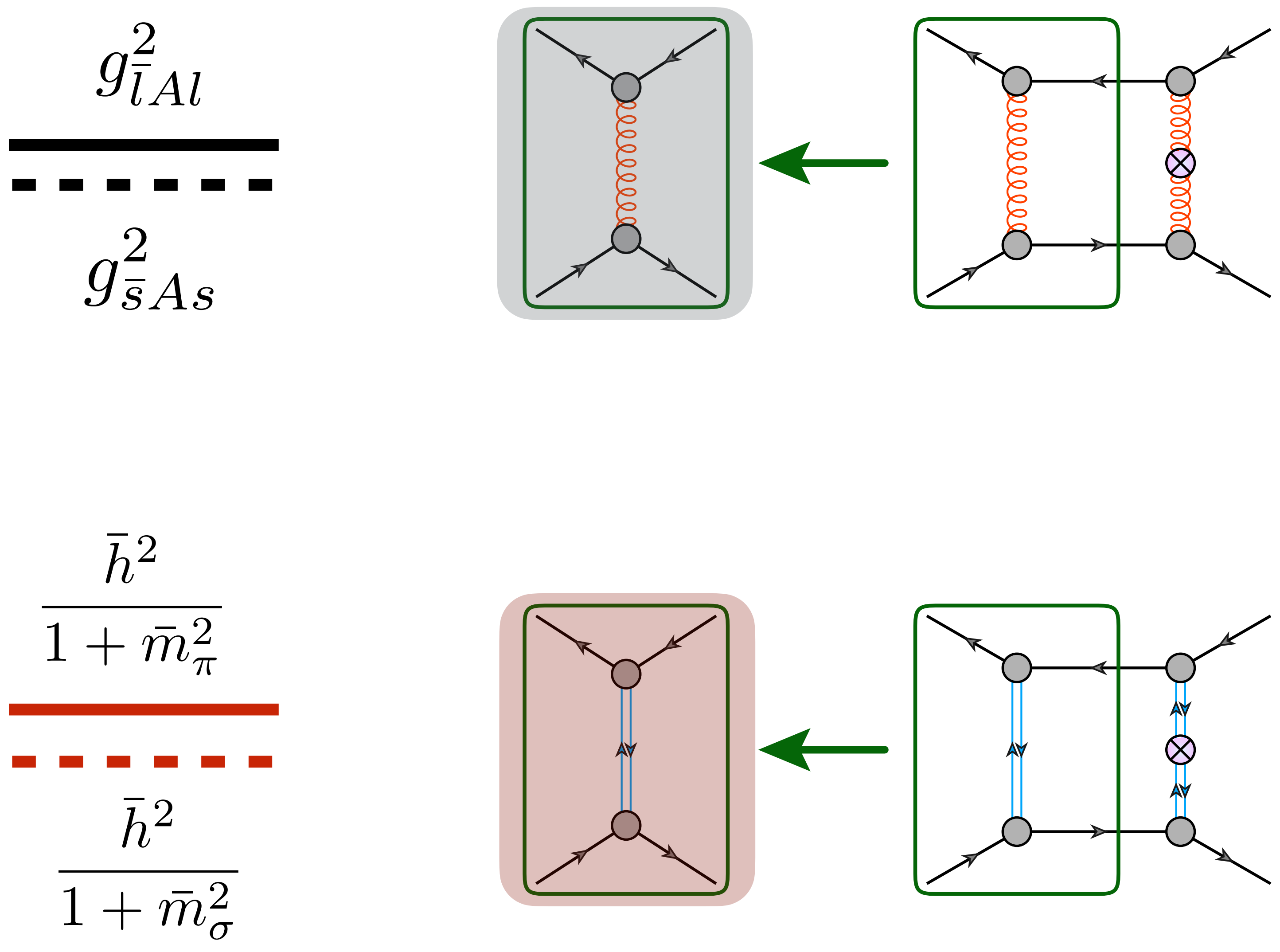

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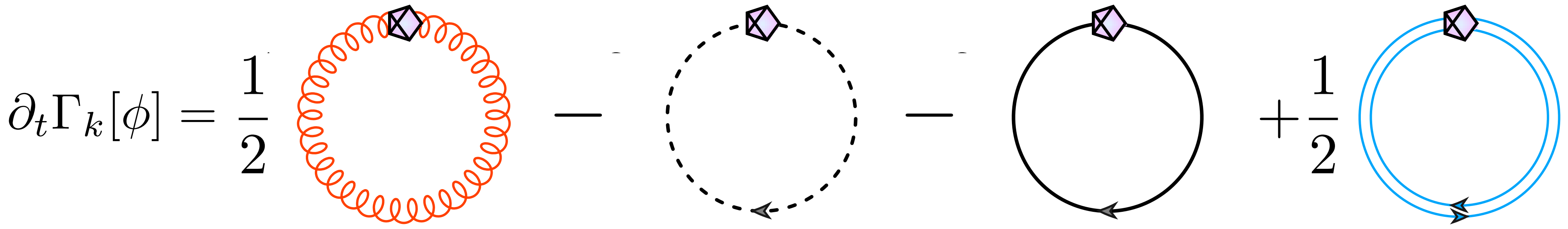

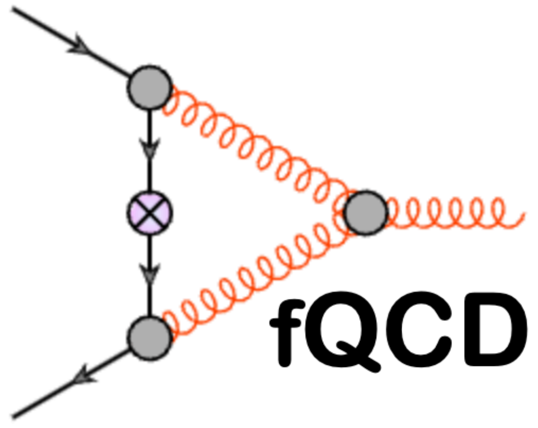

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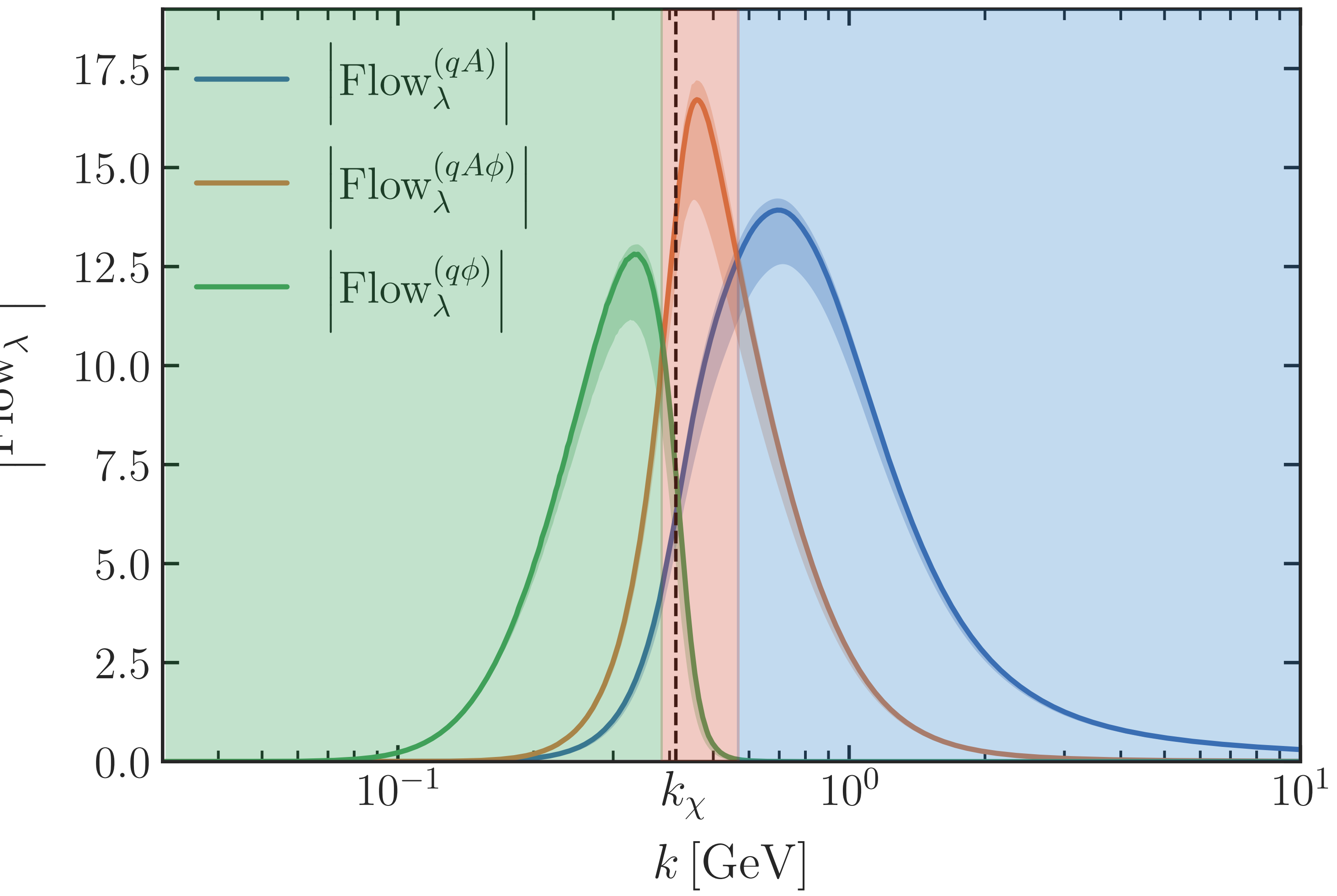

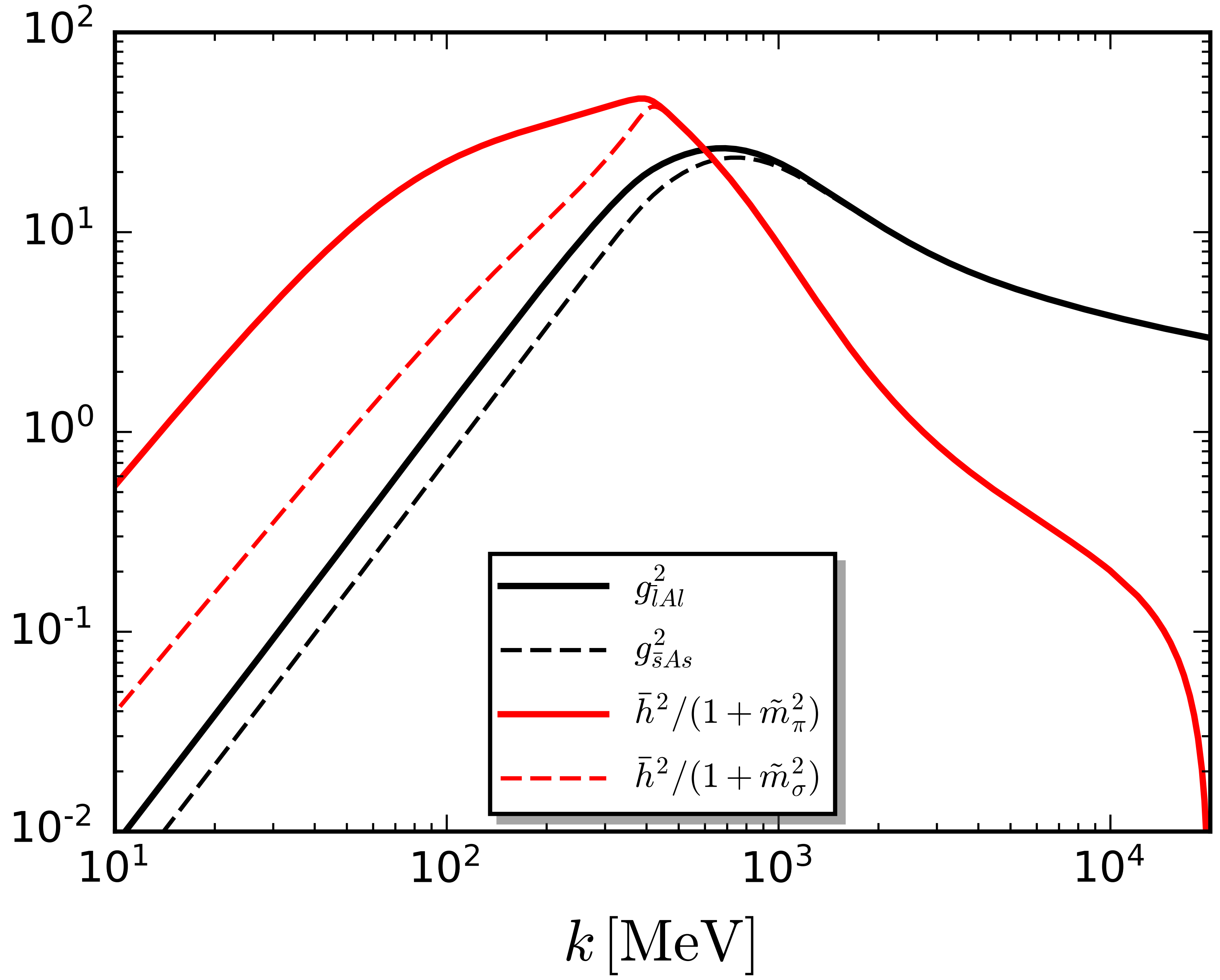

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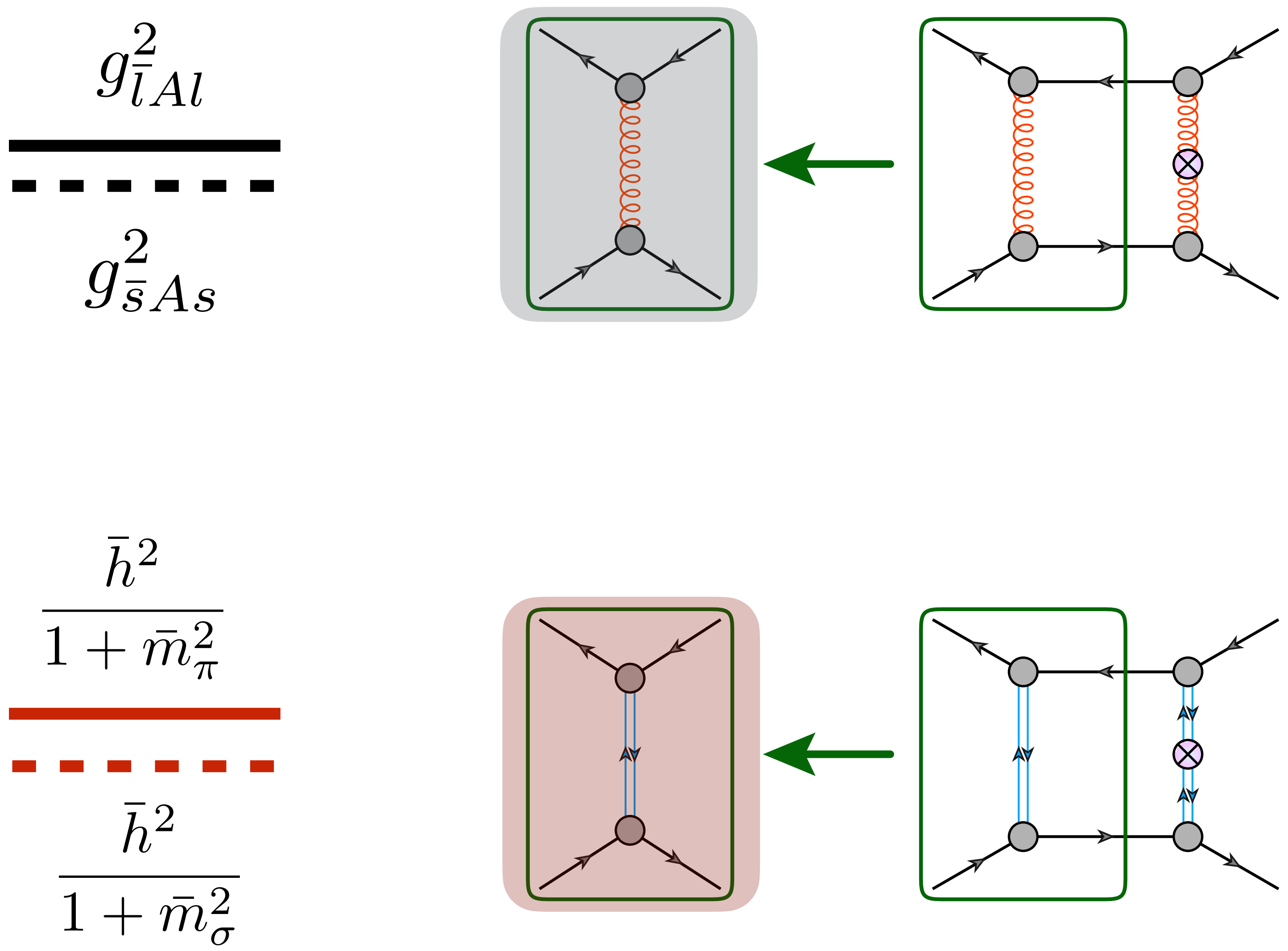

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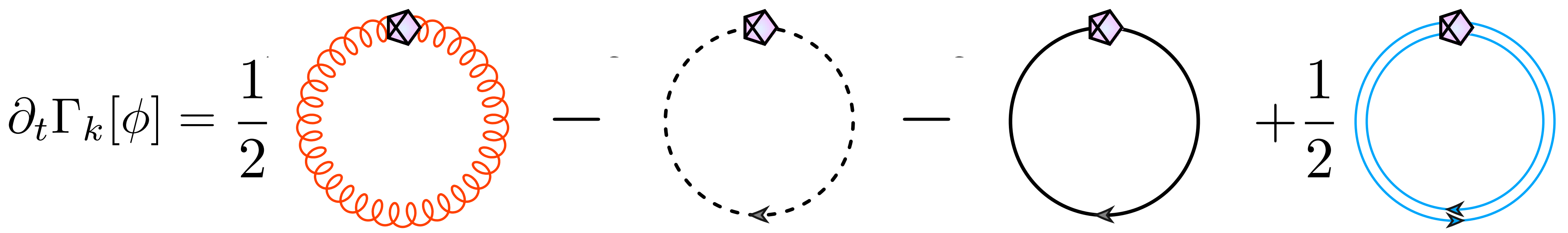

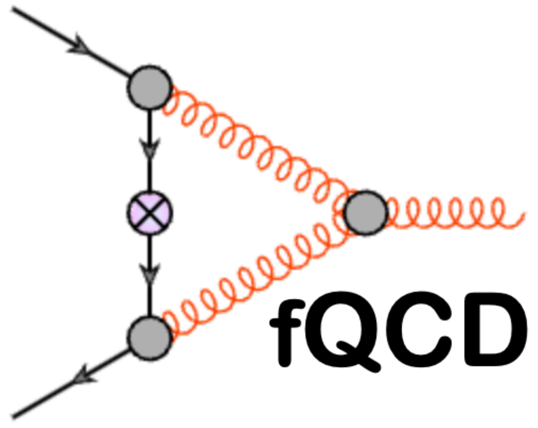

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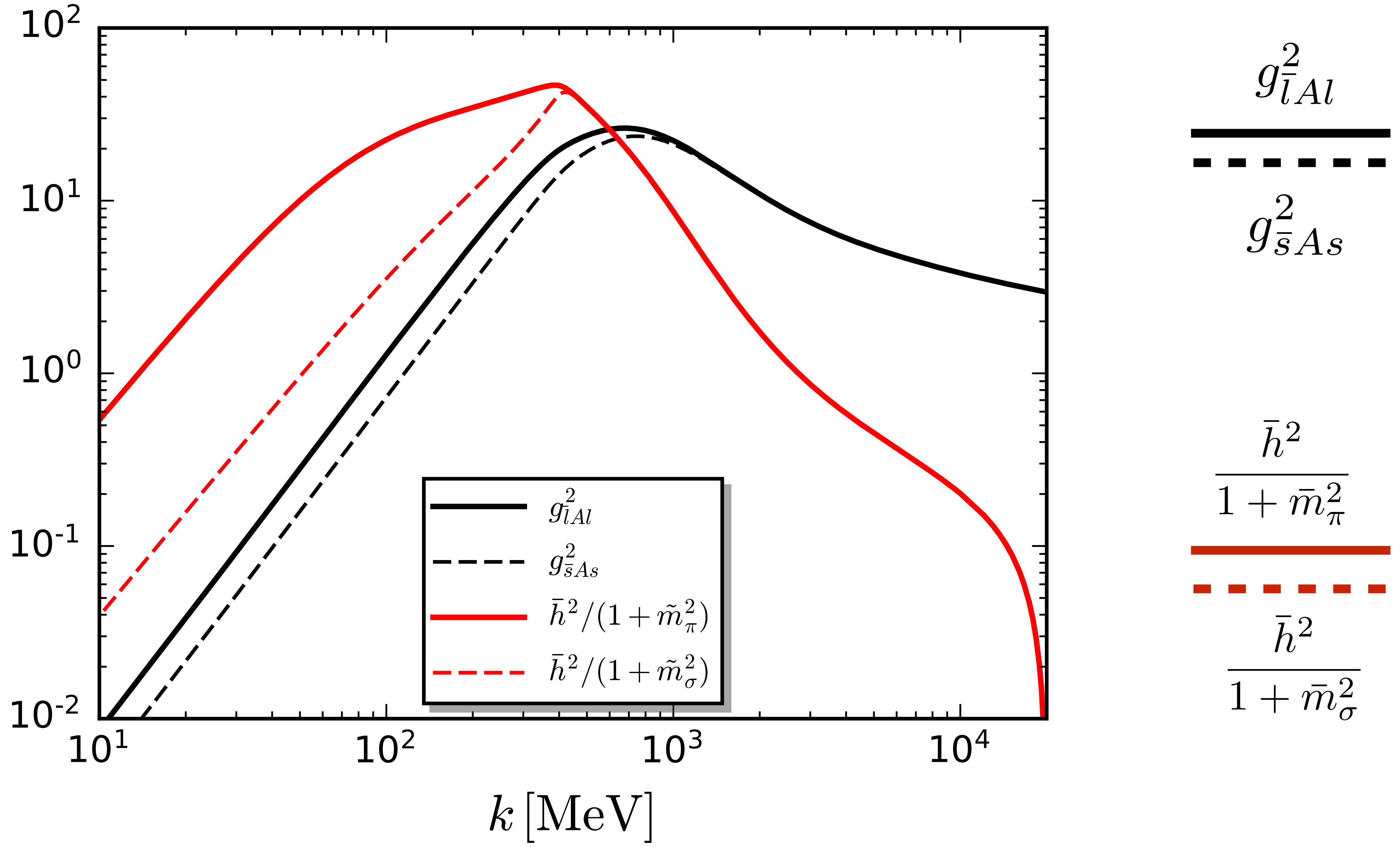

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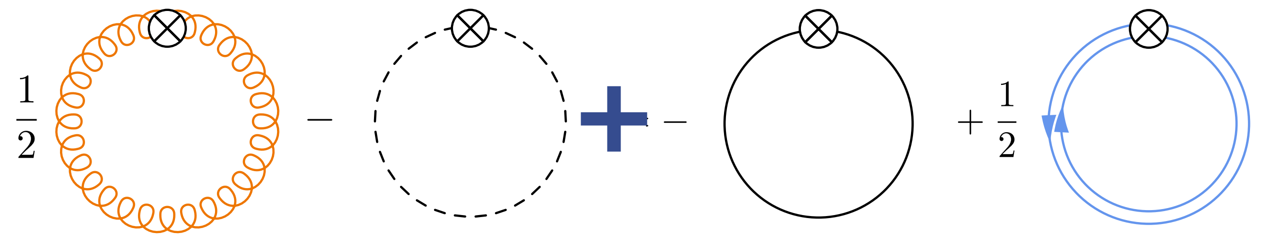
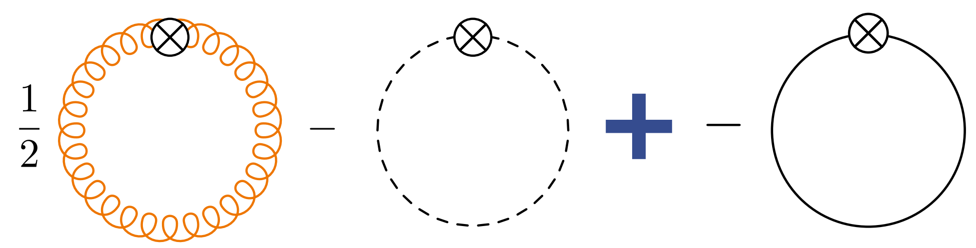
PQM-model



PNJL-model



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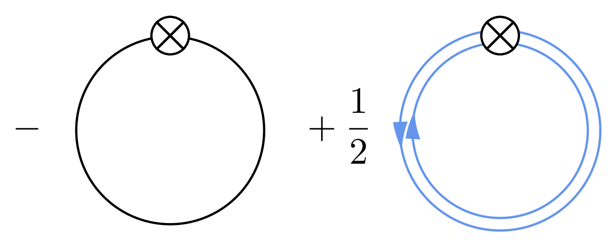
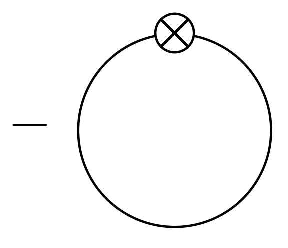


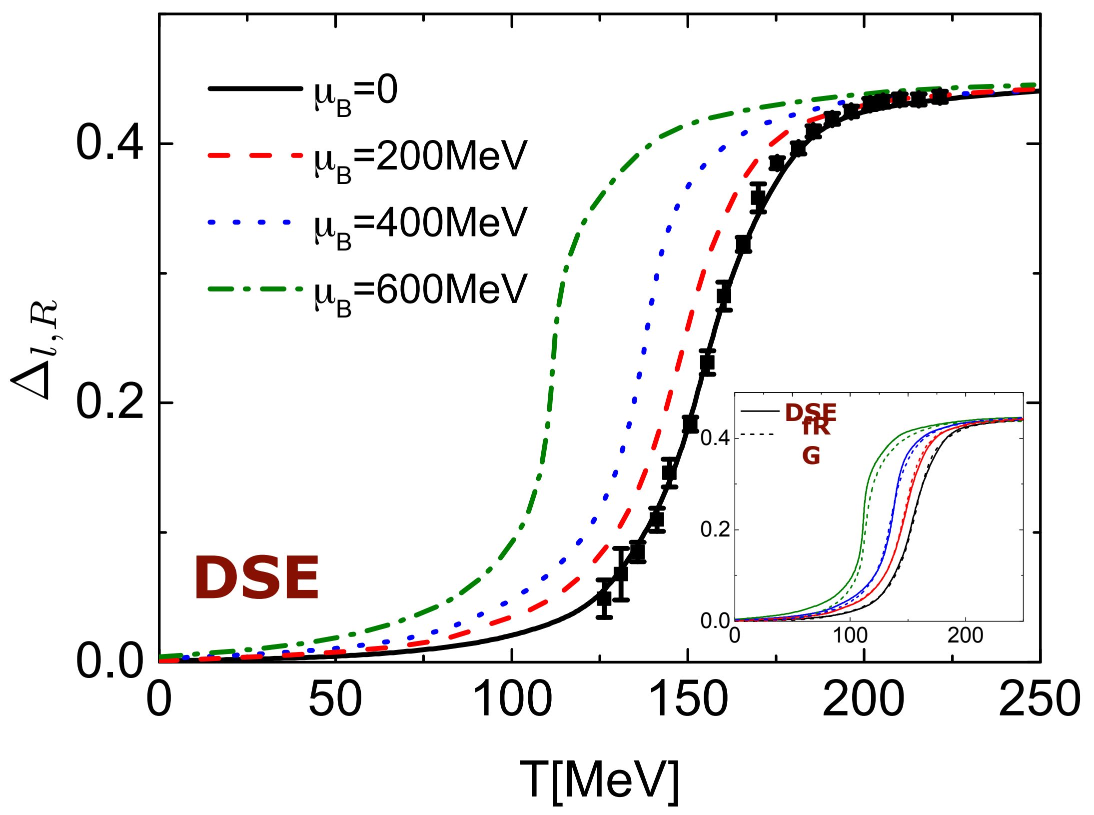
NJL-model

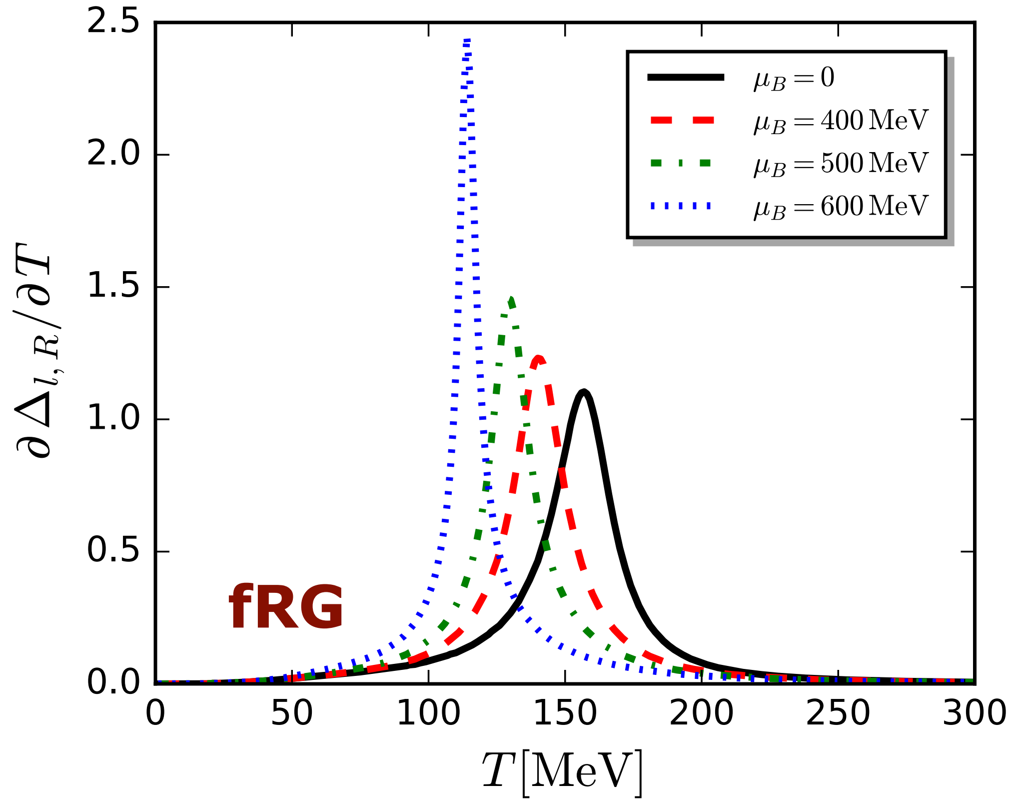


QCD-assisted low energy effective theories

Chiral condensates

renormalised condensate





$$\Delta_{l,R}(T,\mu_B) \simeq \Delta_l(T,\mu_B) - \Delta_l(0,0)$$

$$\Delta_q(T,\mu_B) = \frac{T}{\mathcal{V}}\, m_q^0 \int_x \langle \bar{q}(x) q(x)\rangle$$

lattice: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 09, 073 (2010)

DSE: quark condensates

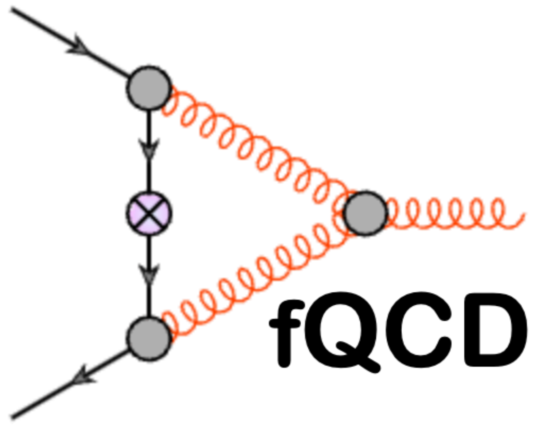
See also

Fischer, Luecker, PLB 718 (2013) 1036
Fischer, Luecker,Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011

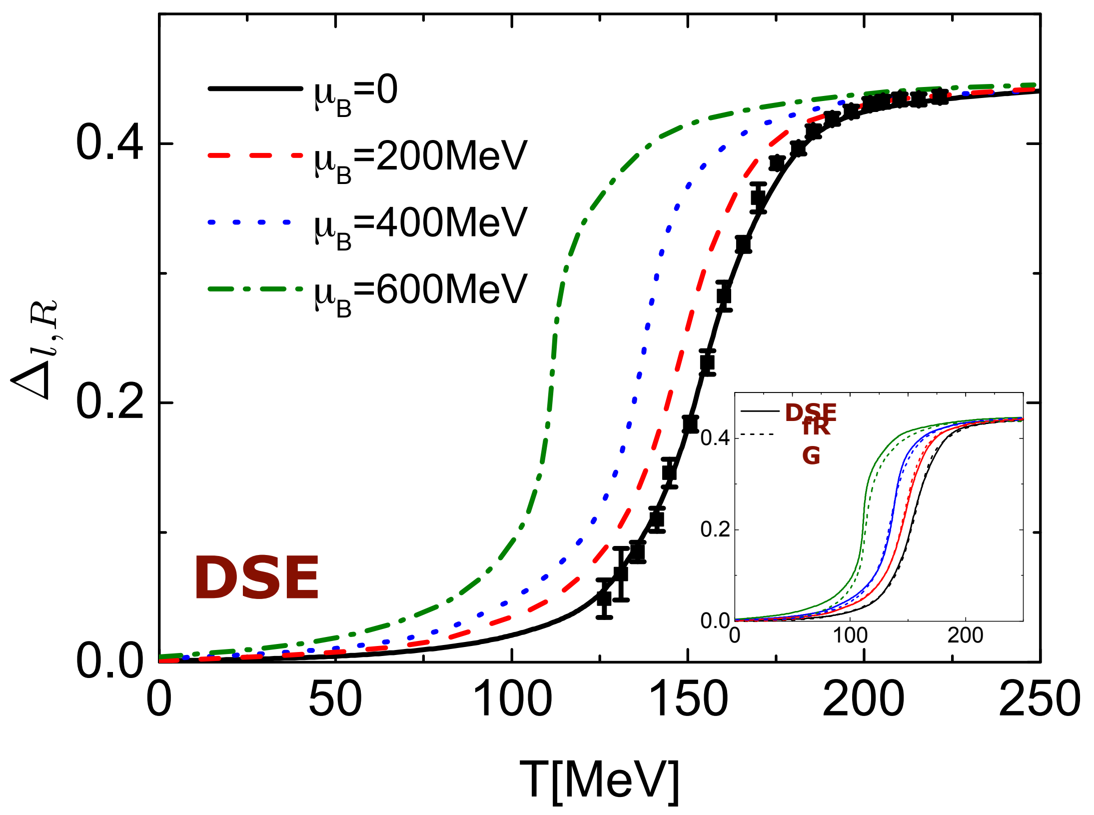
fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

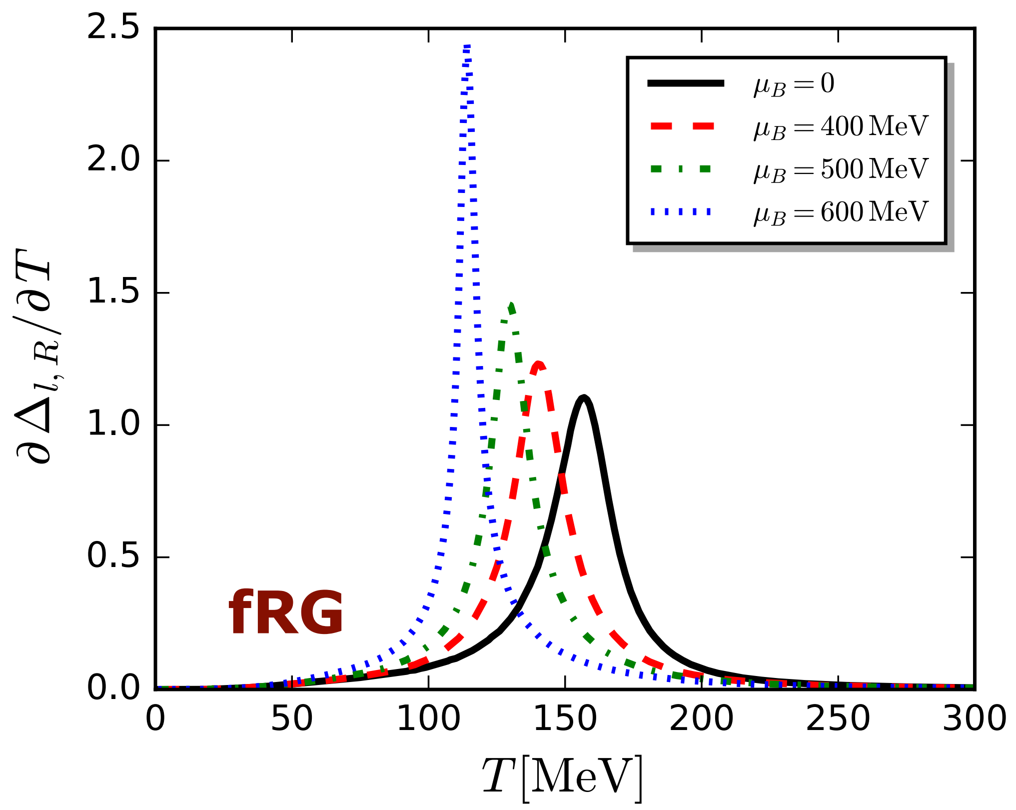
DSE: Gao, JMP, PLB 820 (2021) 136584

Chiral condensates



renormalised condensate





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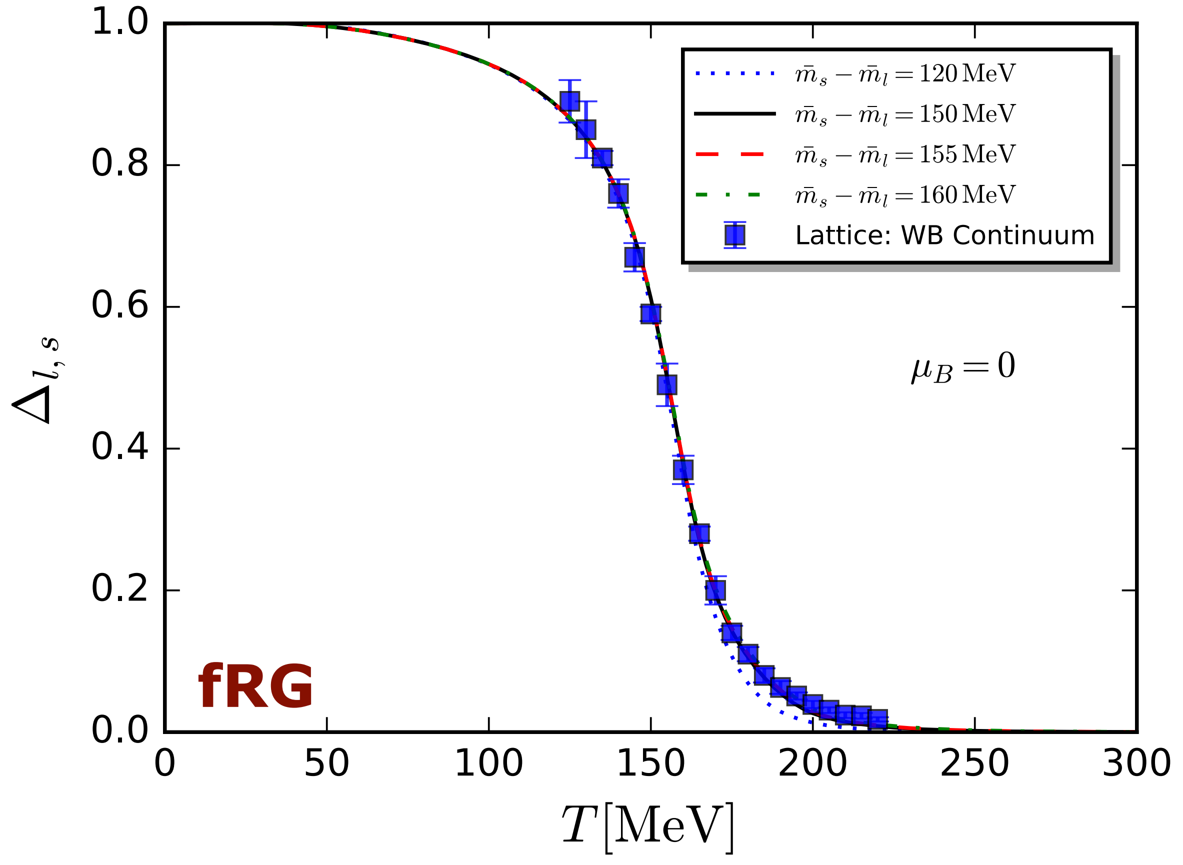
reduced condensate



$$\Delta_{l,s}(T,\mu_B) = \frac{\Delta_l(T,\mu_B) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_B)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

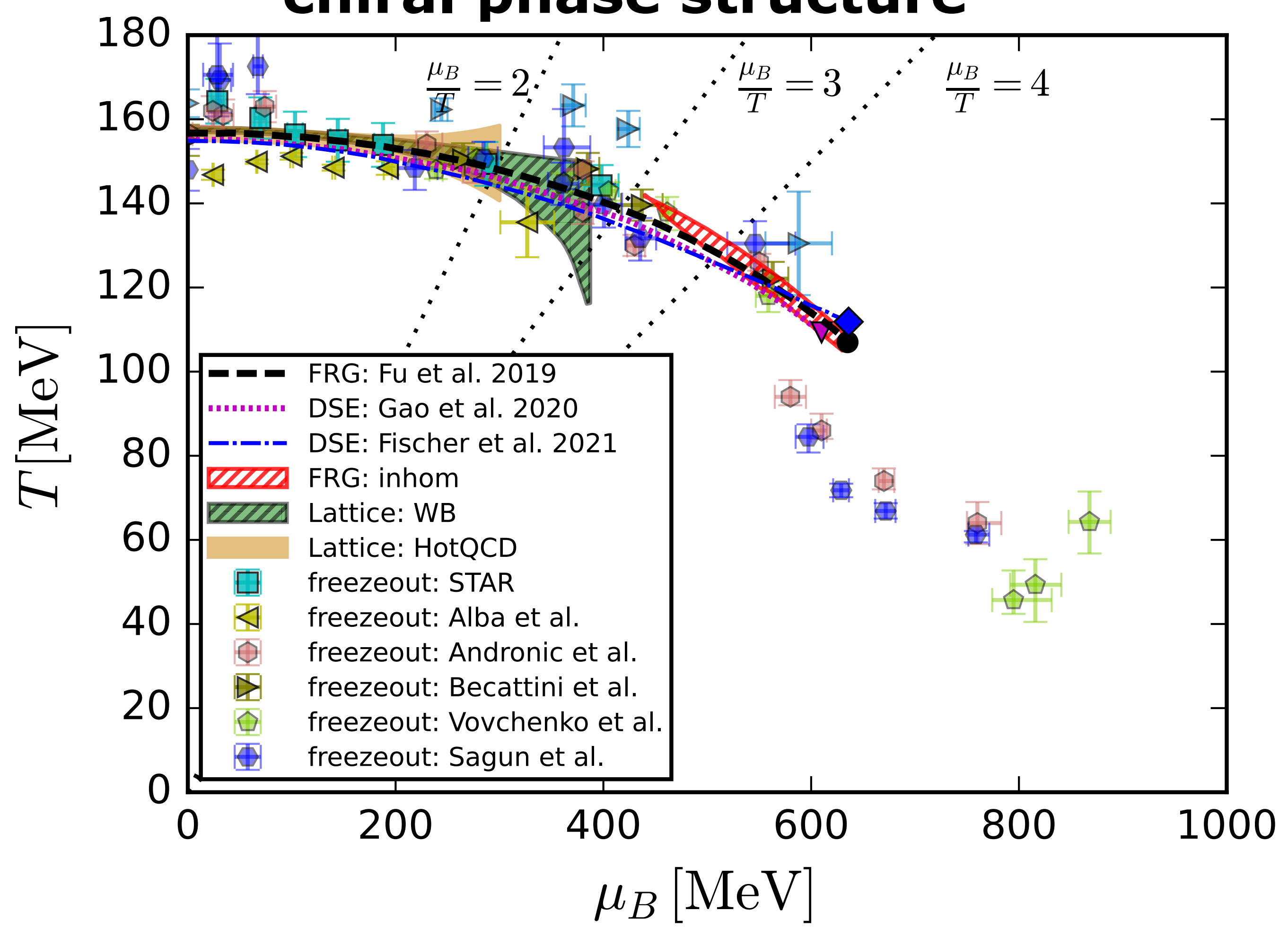
fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584

QCD-assisted low energy effective theory

Direct QCD input

chiral phase structure



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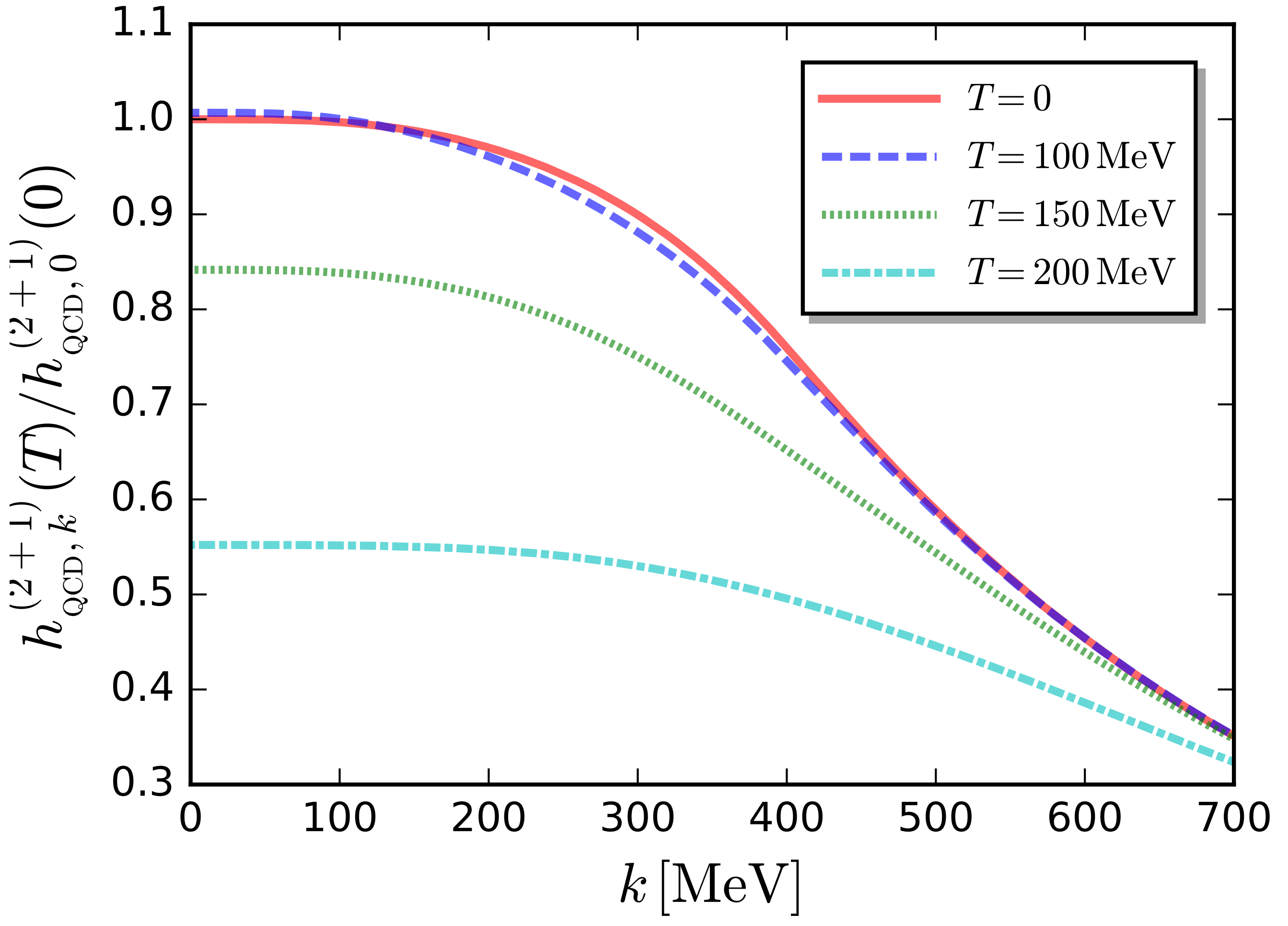
quark-antiquark—meson scattering

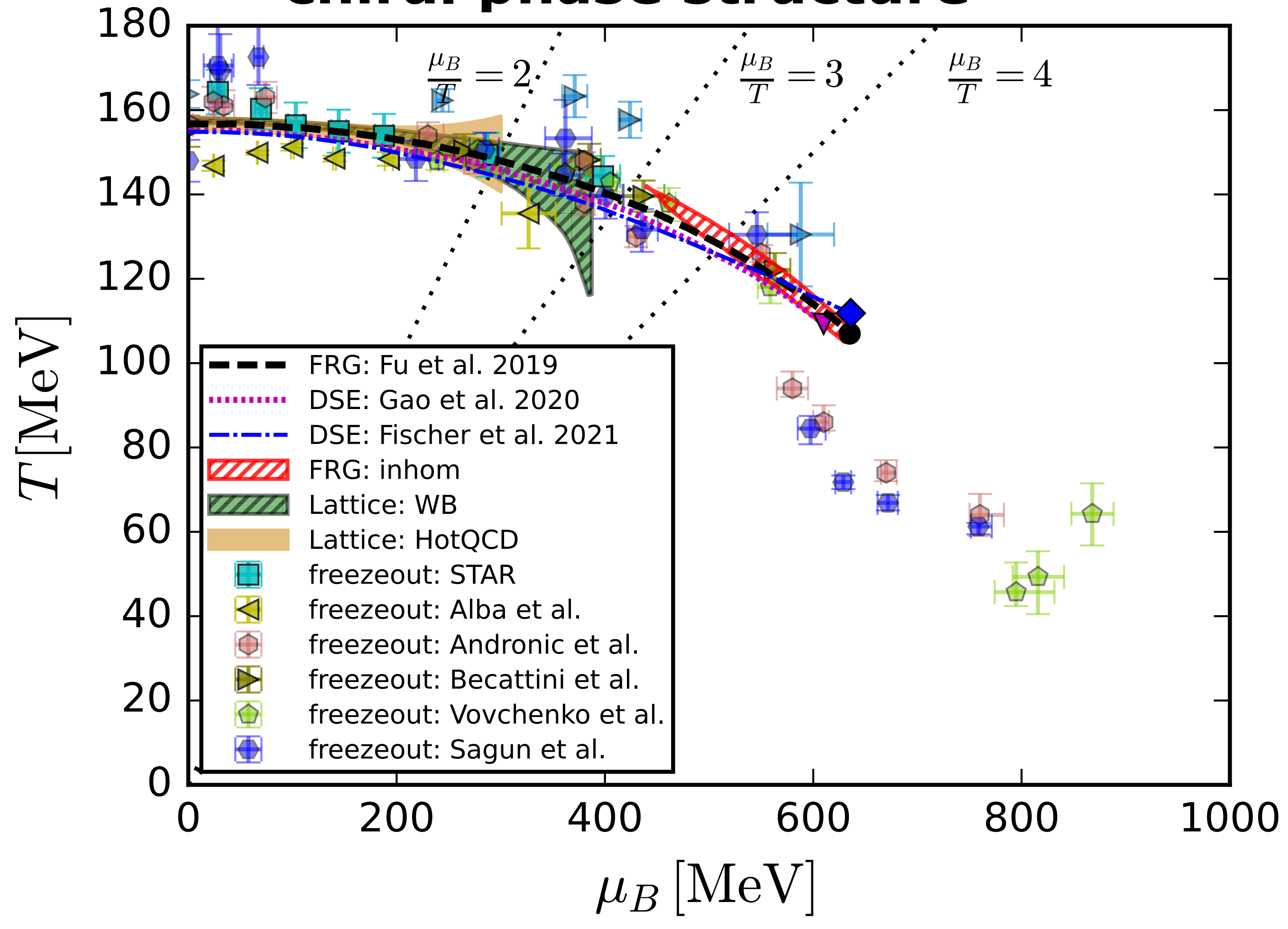


Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

QCD-assisted low energy effective theory

Direct QCD input

chiral phase structure



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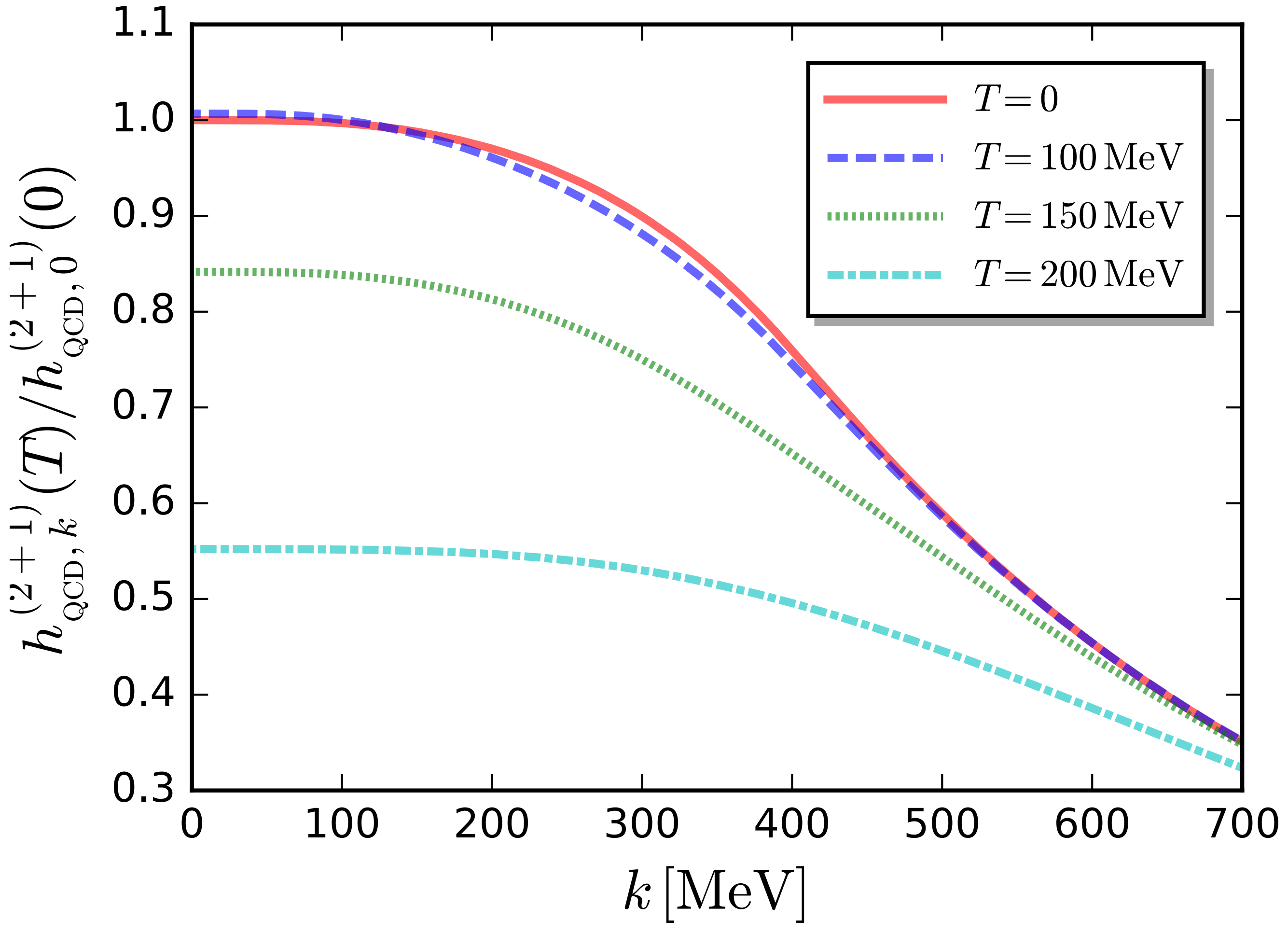


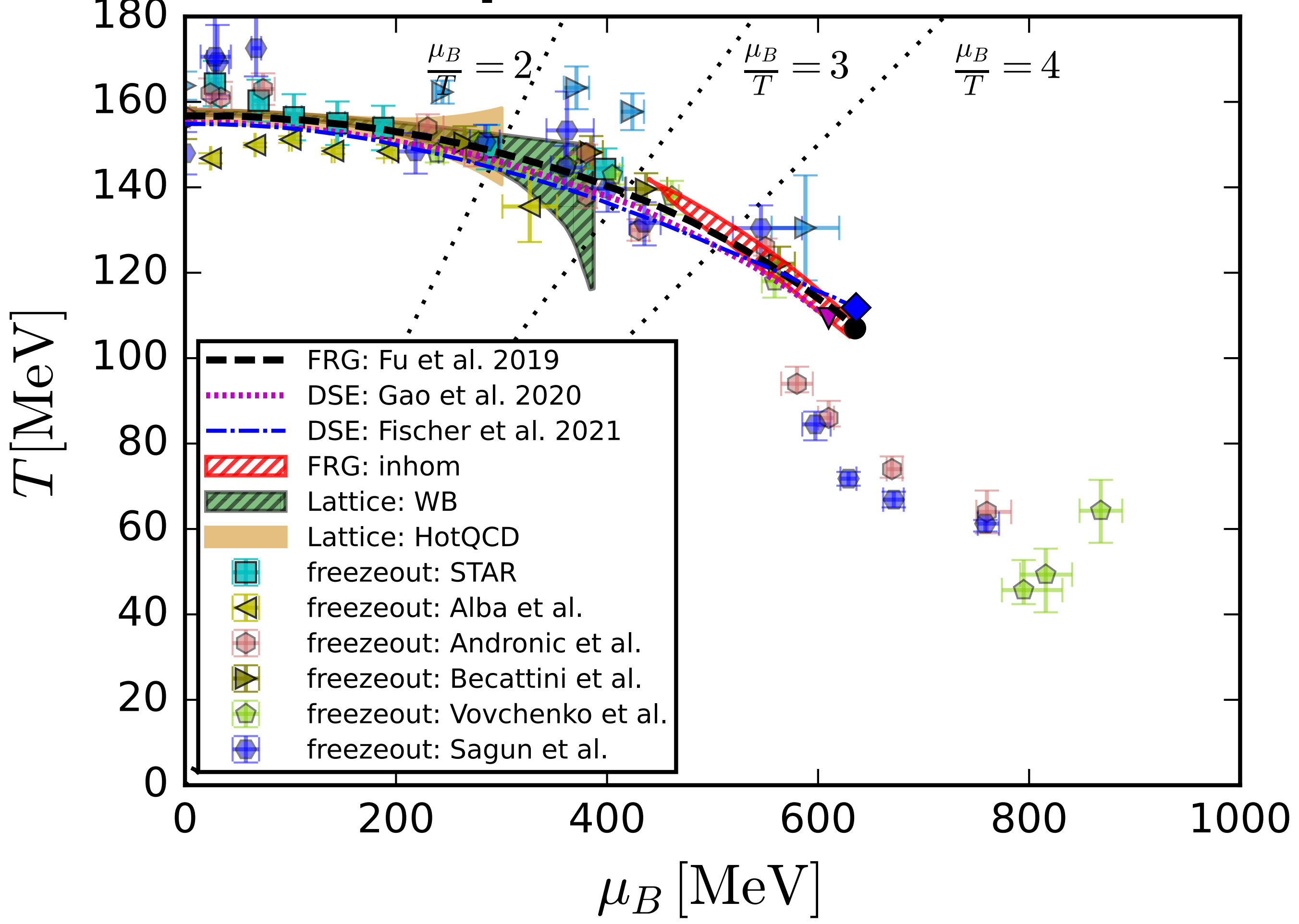
Low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

QCD-assisted low energy effective theory

Direct QCD input

chiral phase structure



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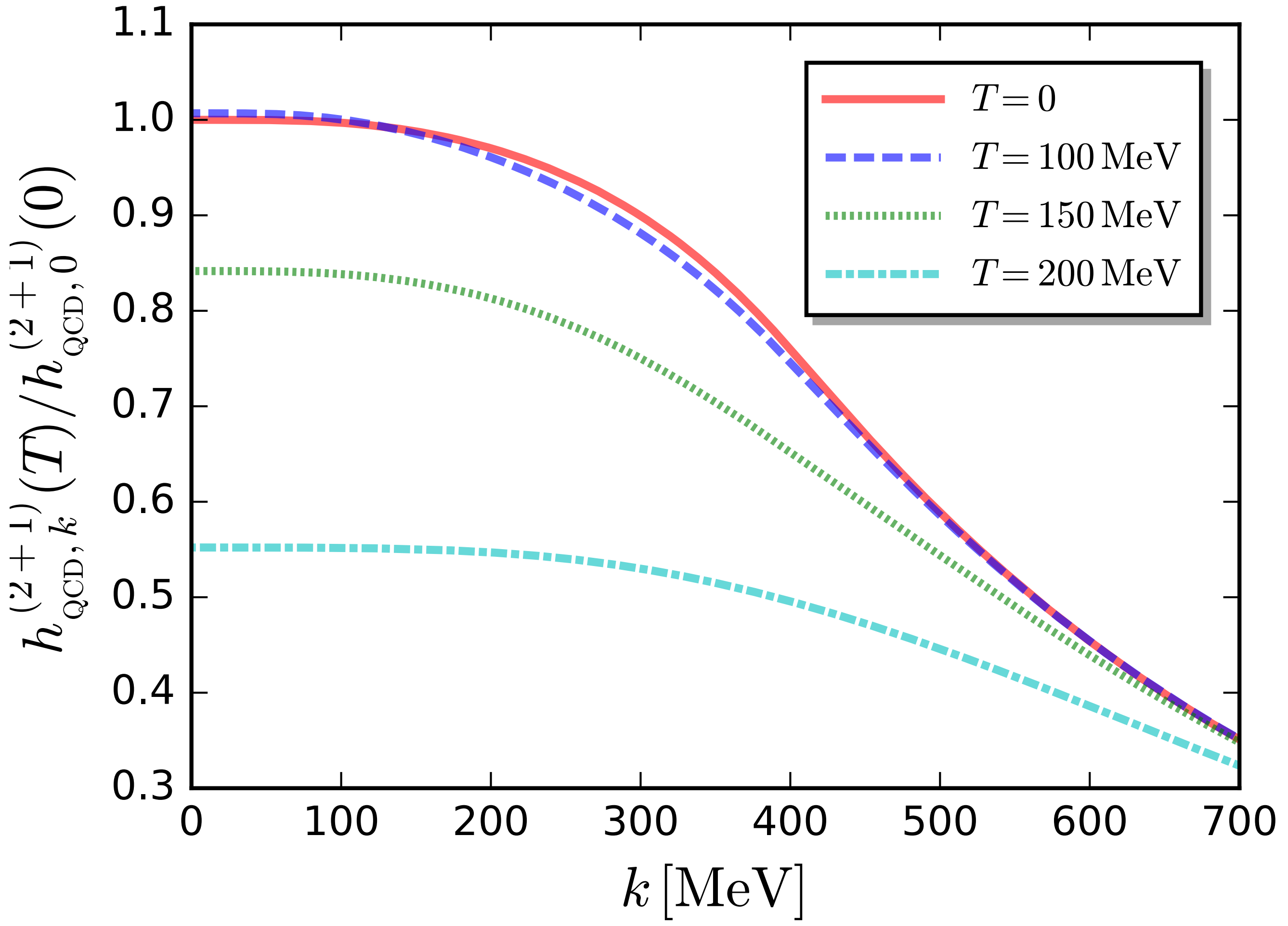
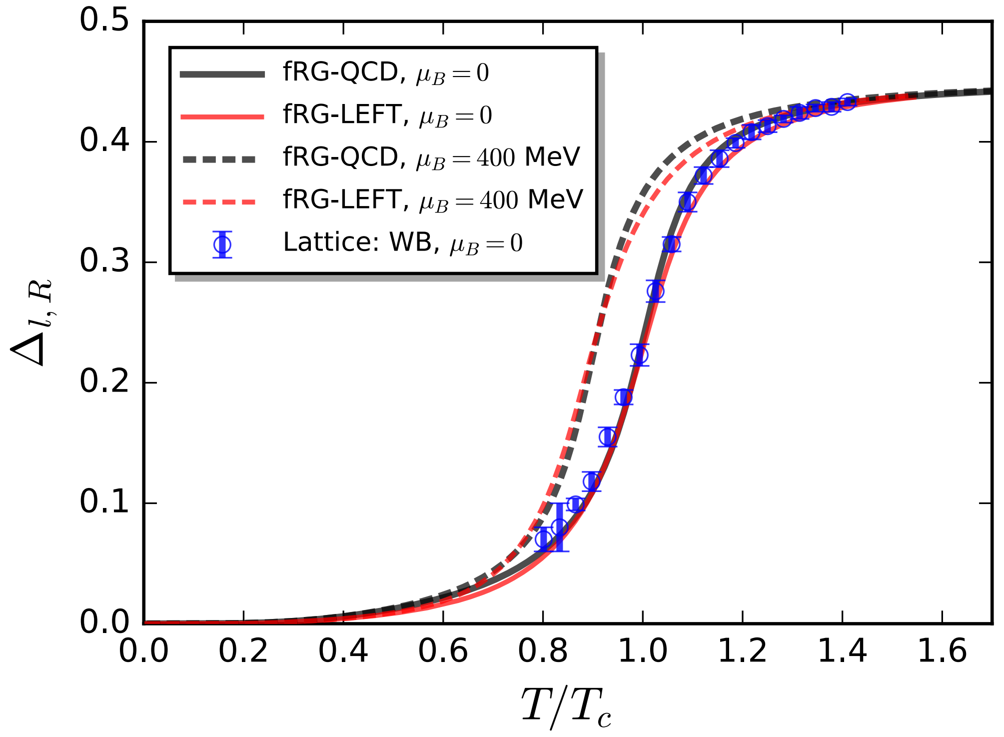
quark-antiquark—meson scattering



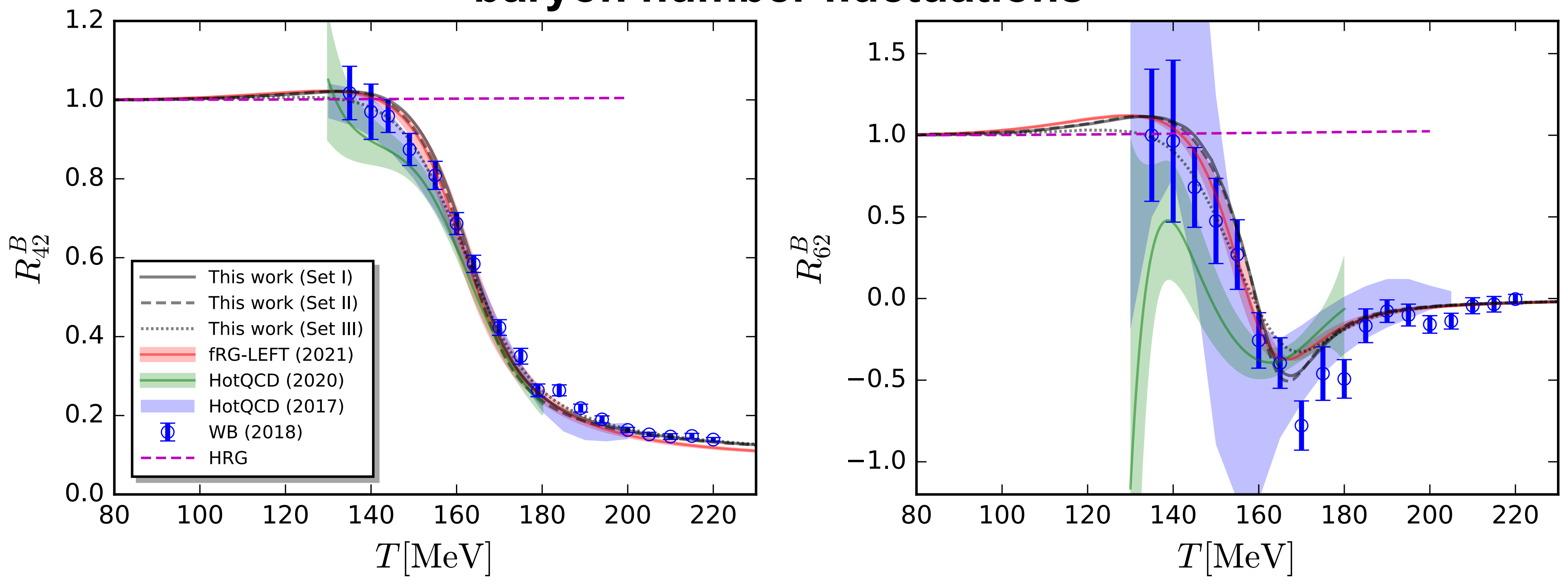
Low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

Benchmarks with lattice and fQCD at vanishing density and fQCD at finite density

renormalised chiral condensate



baryon number fluctuations



Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

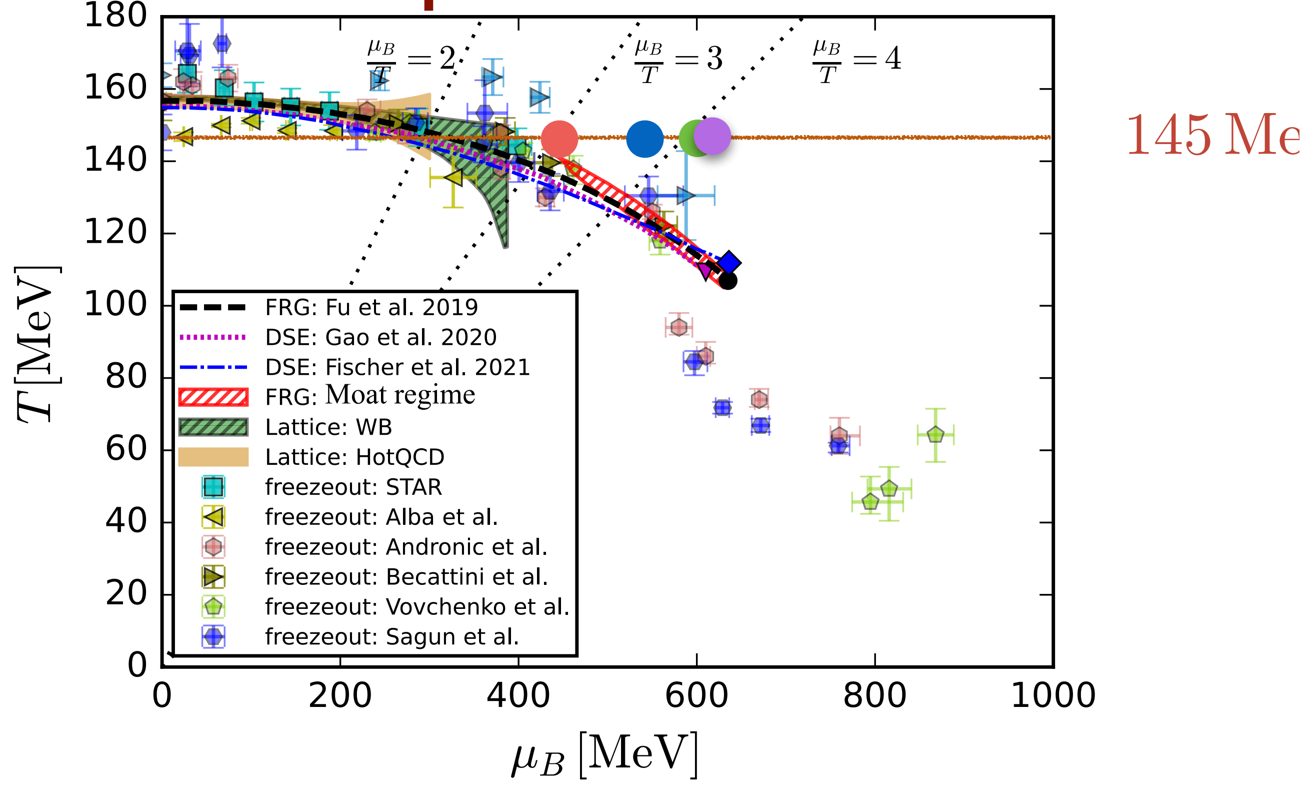
QCD-assisted heavy ion physics: compilation of functional QCD results

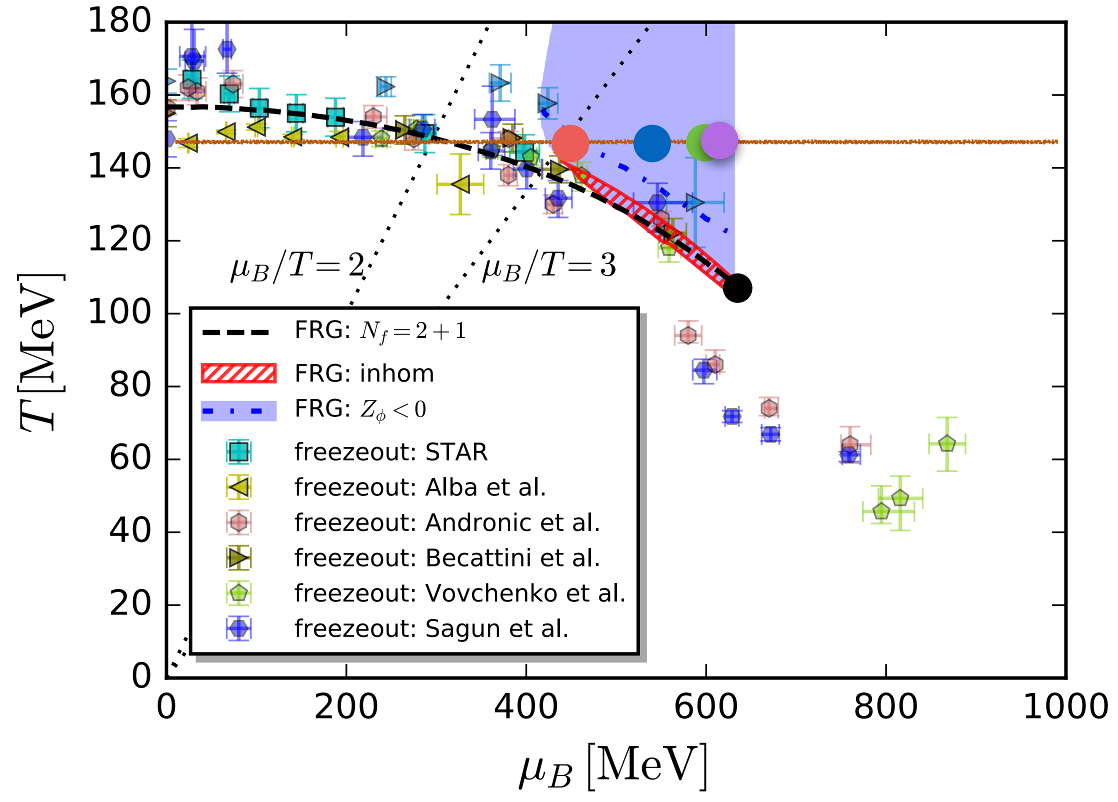
Thermodynamics & spectral properties

Sneak preview on the QCD moat

fQCD

Chiral phase structure

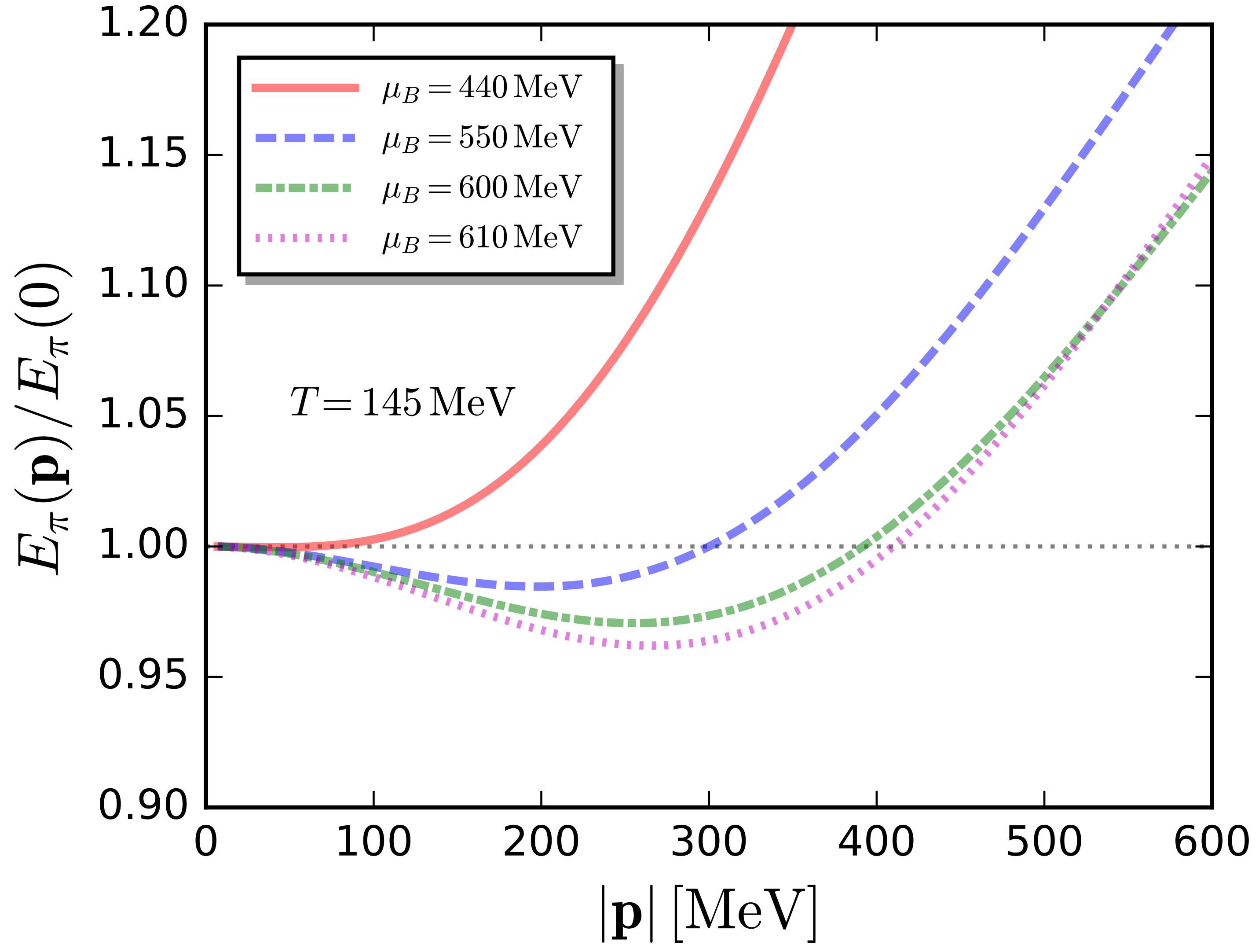




Fu, JMP, Rennecke, PRD 101 (2020) 054032

preliminary

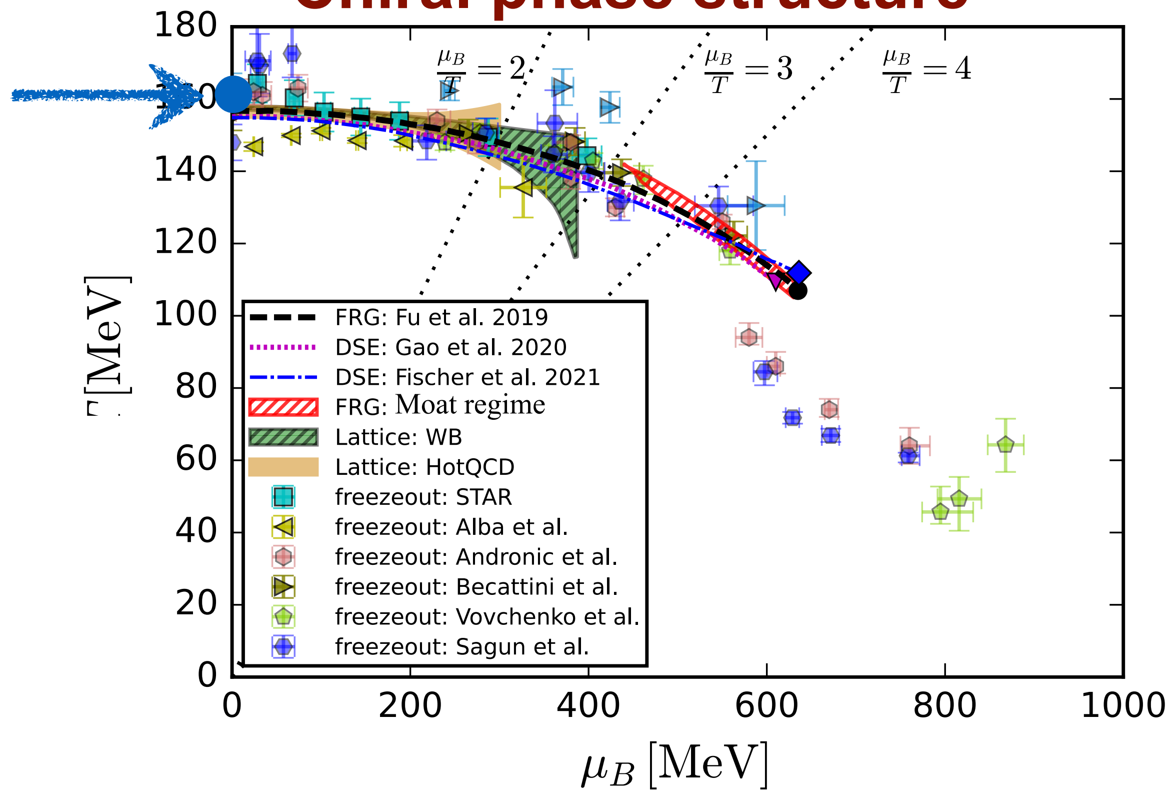
Energy



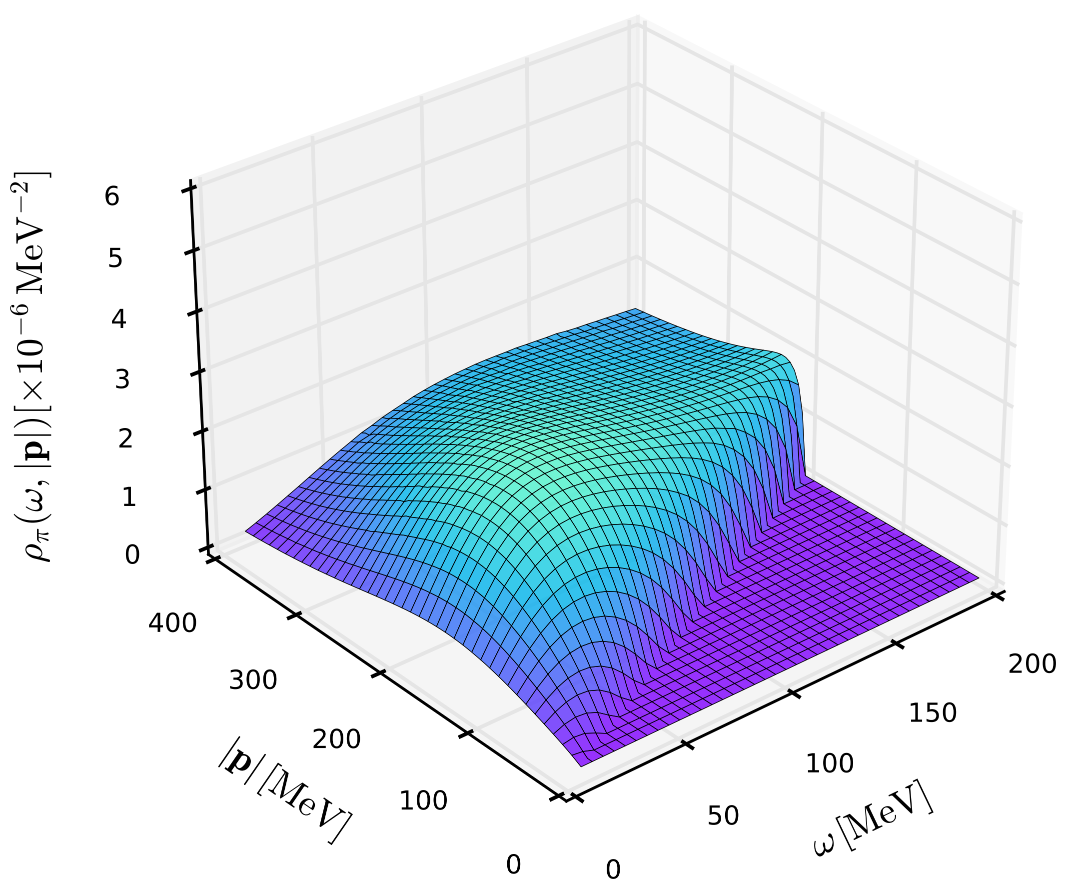
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, in preparation

Sneak preview on the QCD moat

Chiral phase structure



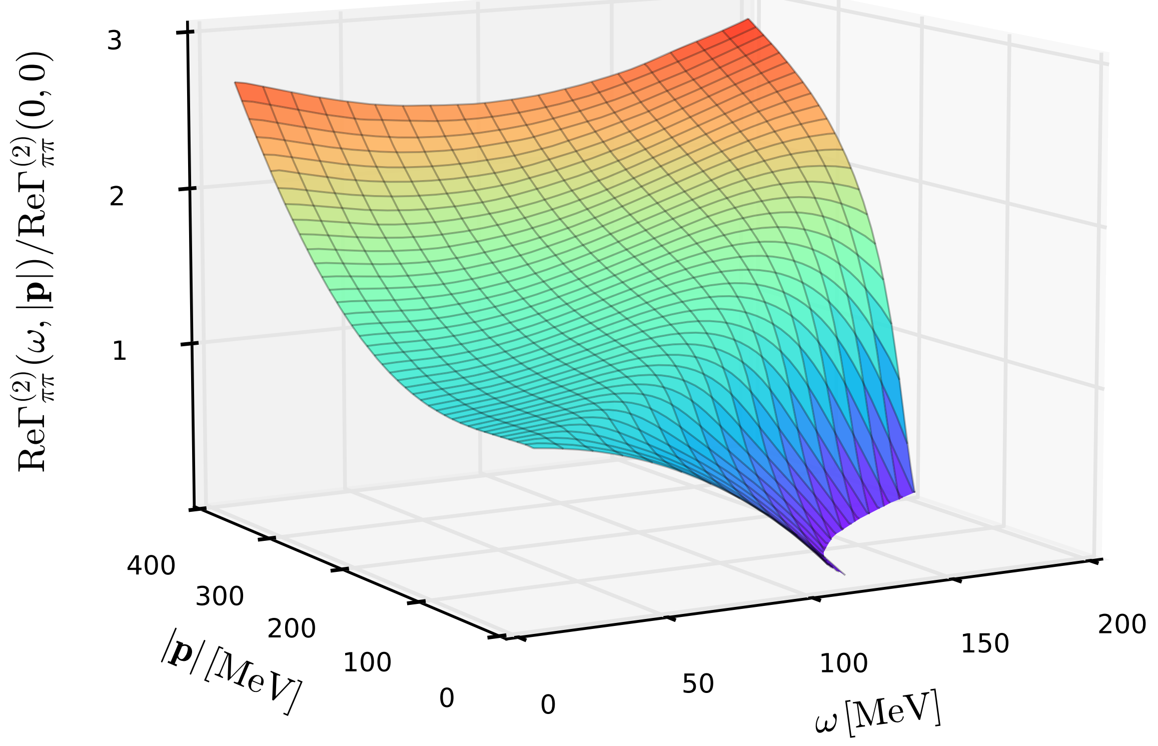
Pion spectral function



preliminary

$$(T\,,\,\mu_B) = (160\,\mathrm{MeV}, 0)$$

Pion correlation function



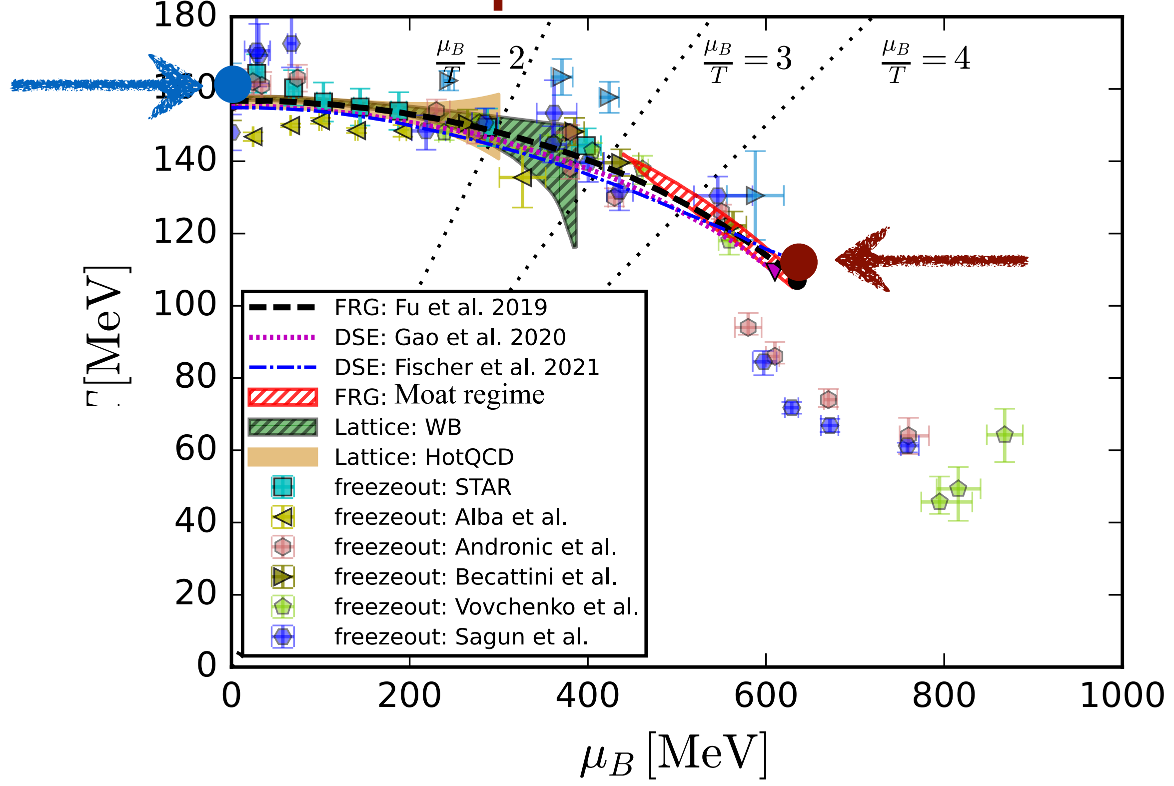
fQCD

Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, in preparation

Sneak preview on the QCD moat

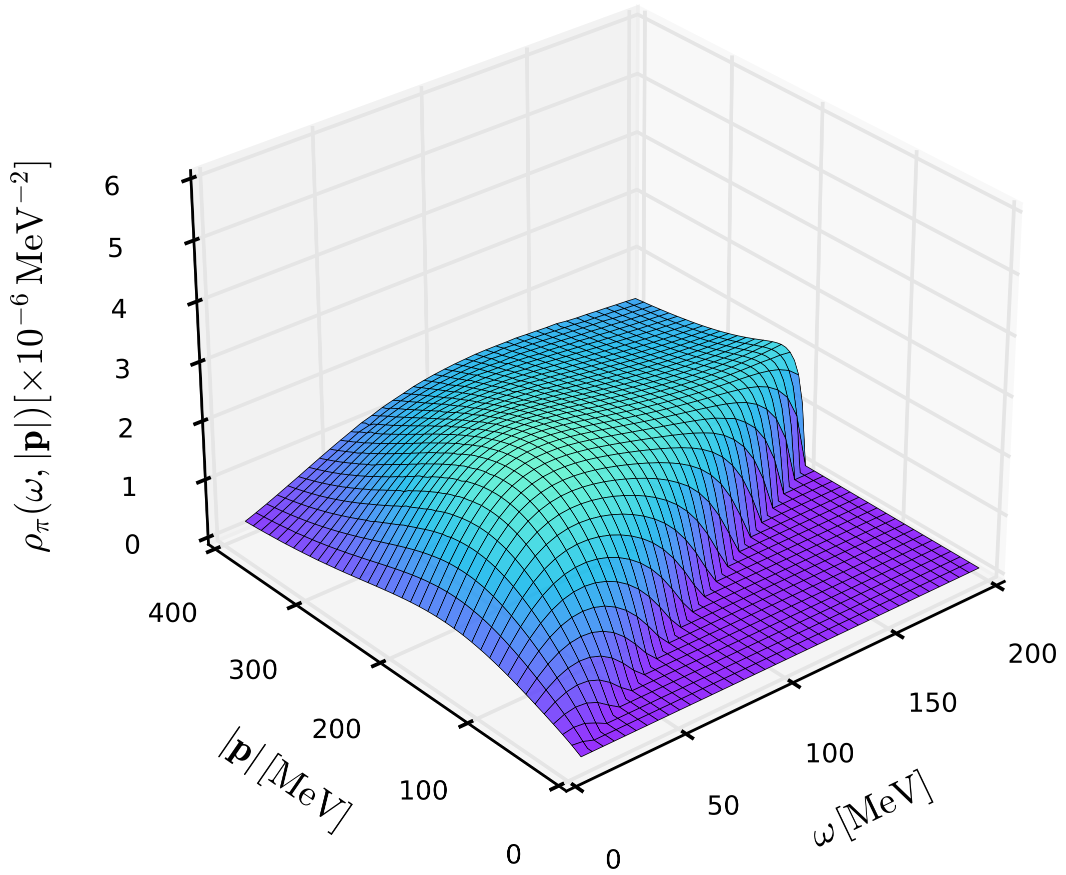
fQCD

Chiral phase structure



Pion spectral function

Pion correlation function



preliminary

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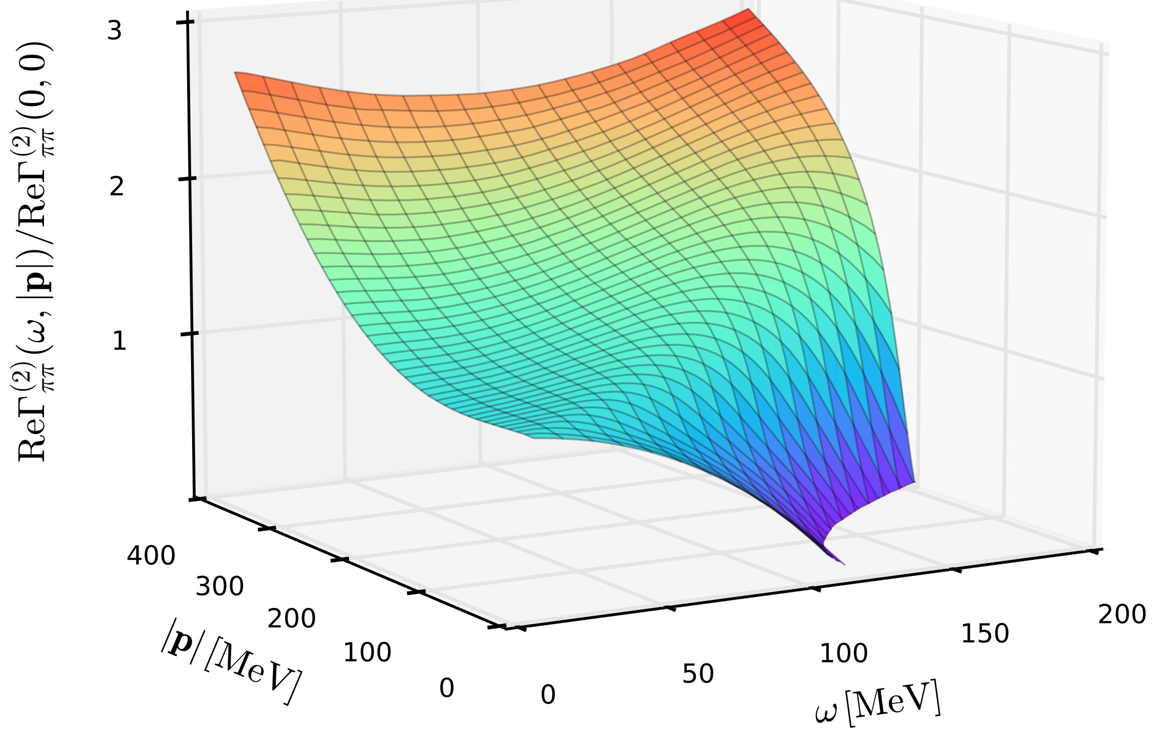
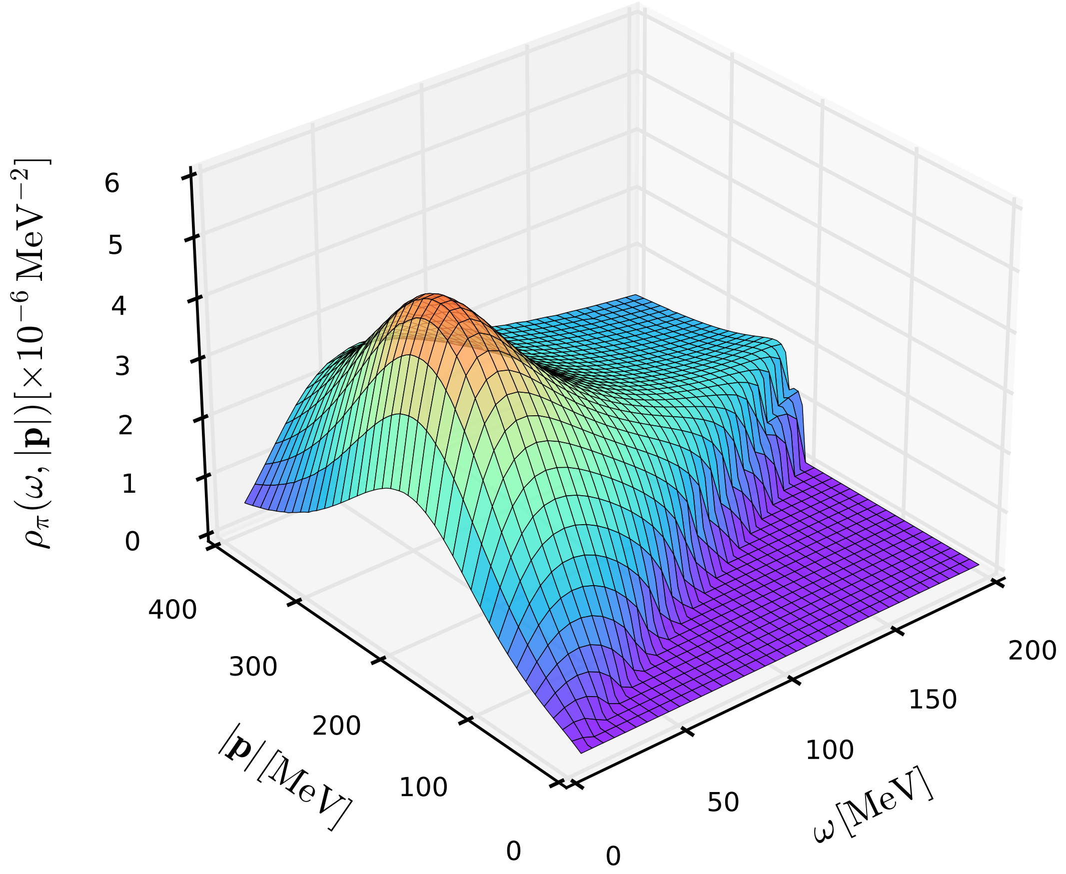
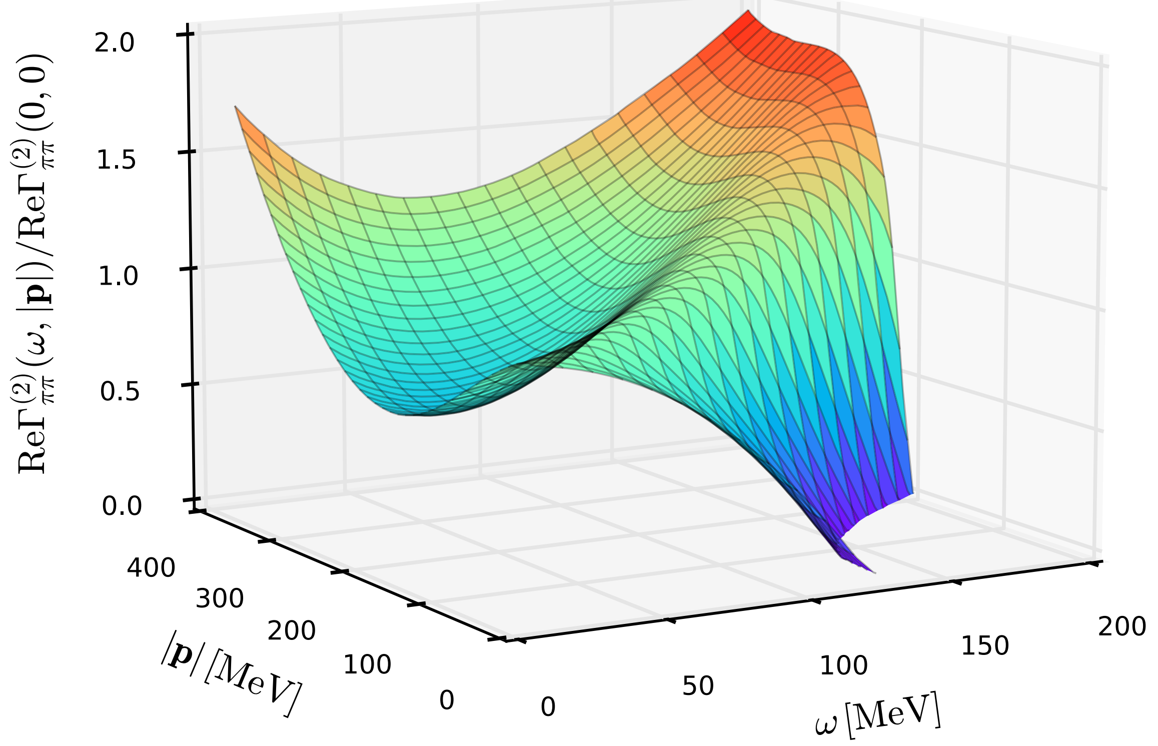




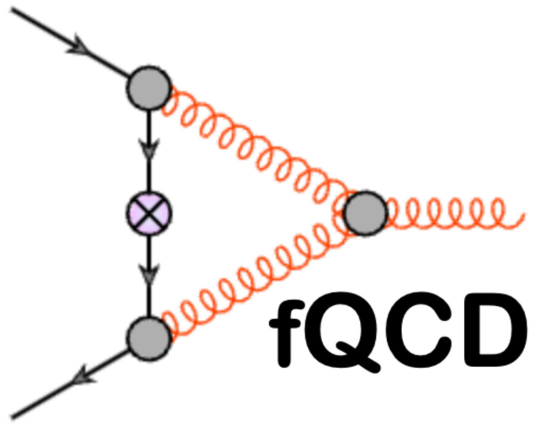
Moat

$$(T\,,\,\mu_B) = (114\,\mathrm{MeV}, 630\,\mathrm{MeV})$$

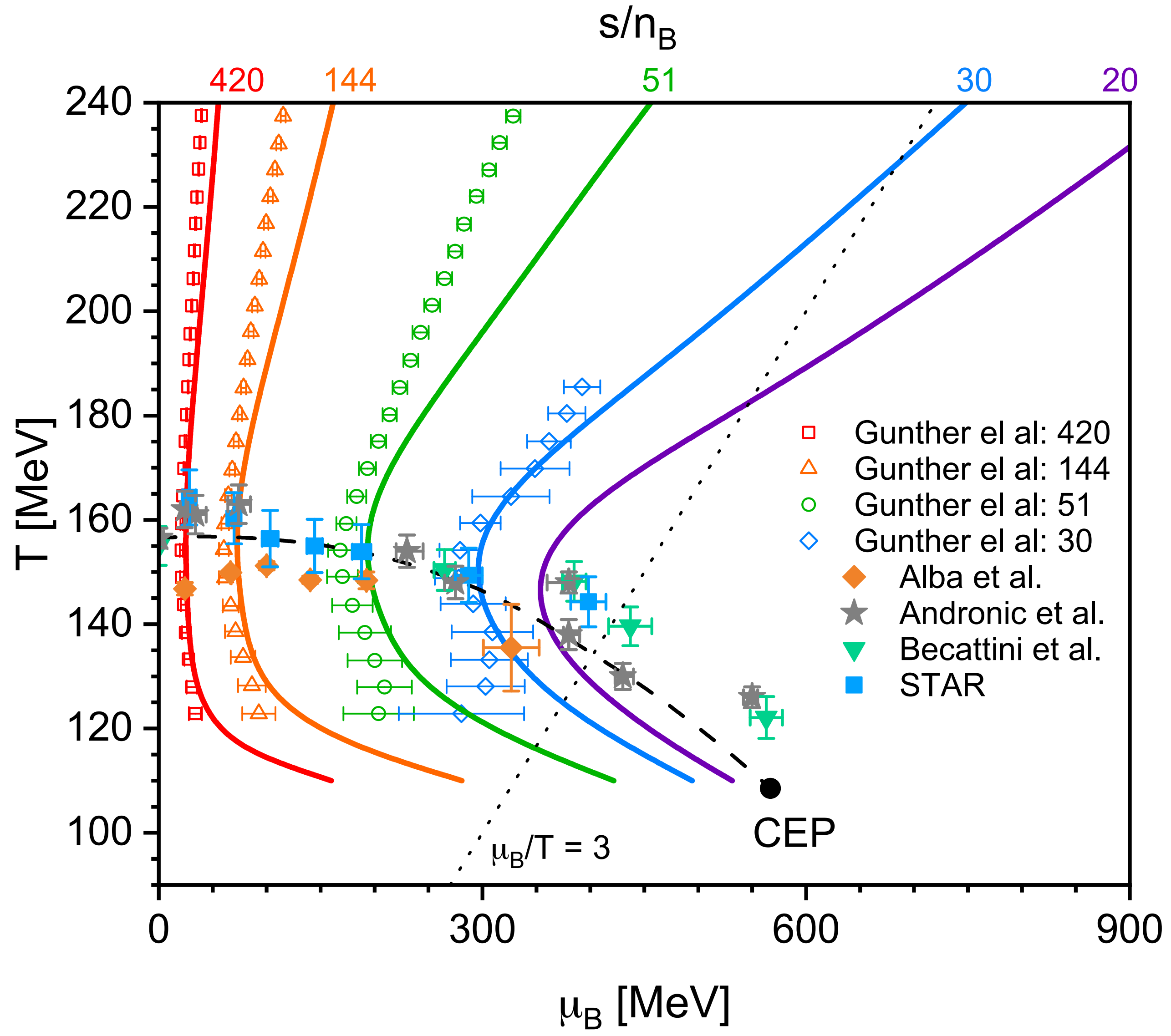


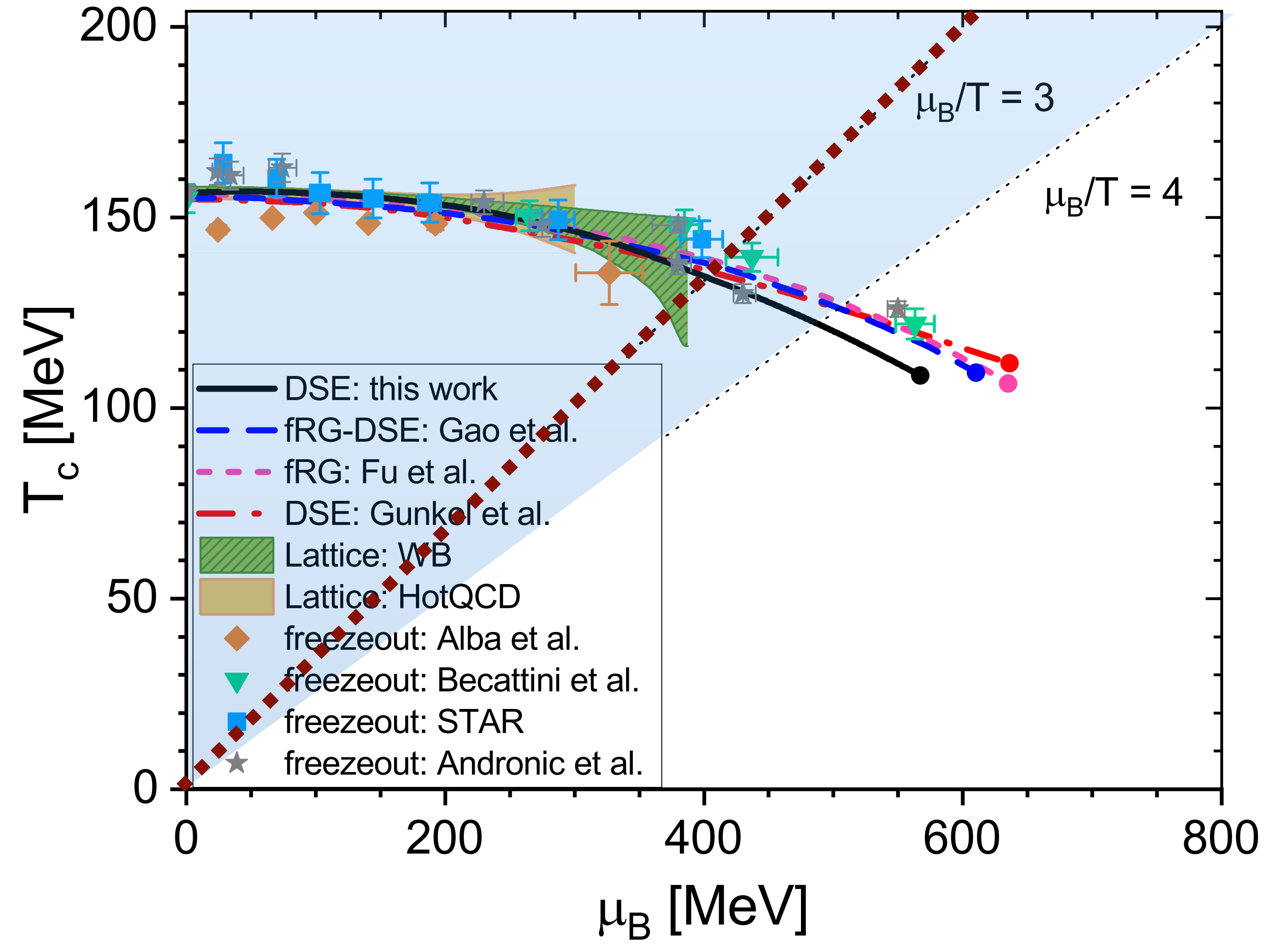
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, in preparation

EoS with the minimal DSE

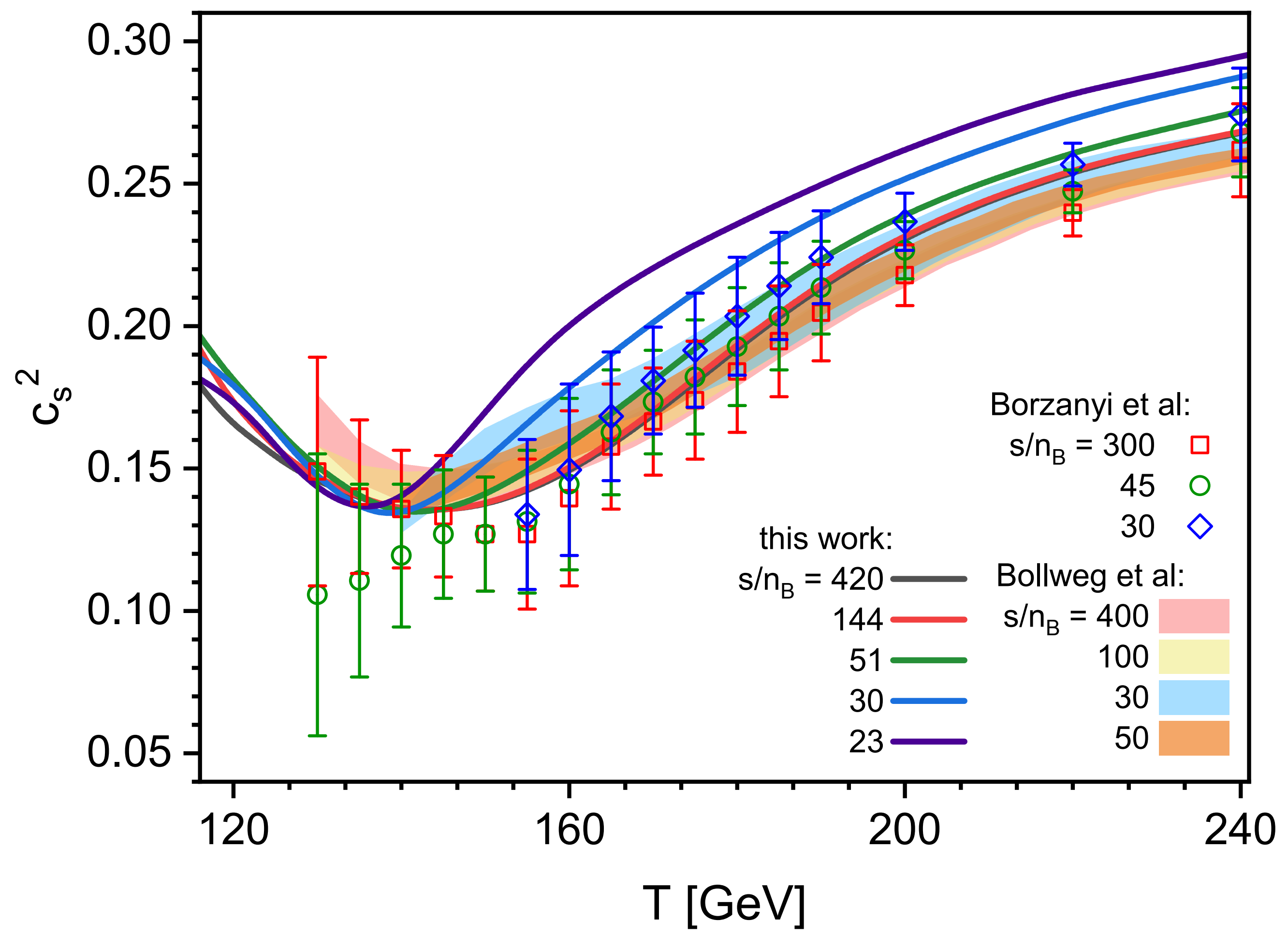


Isentropes

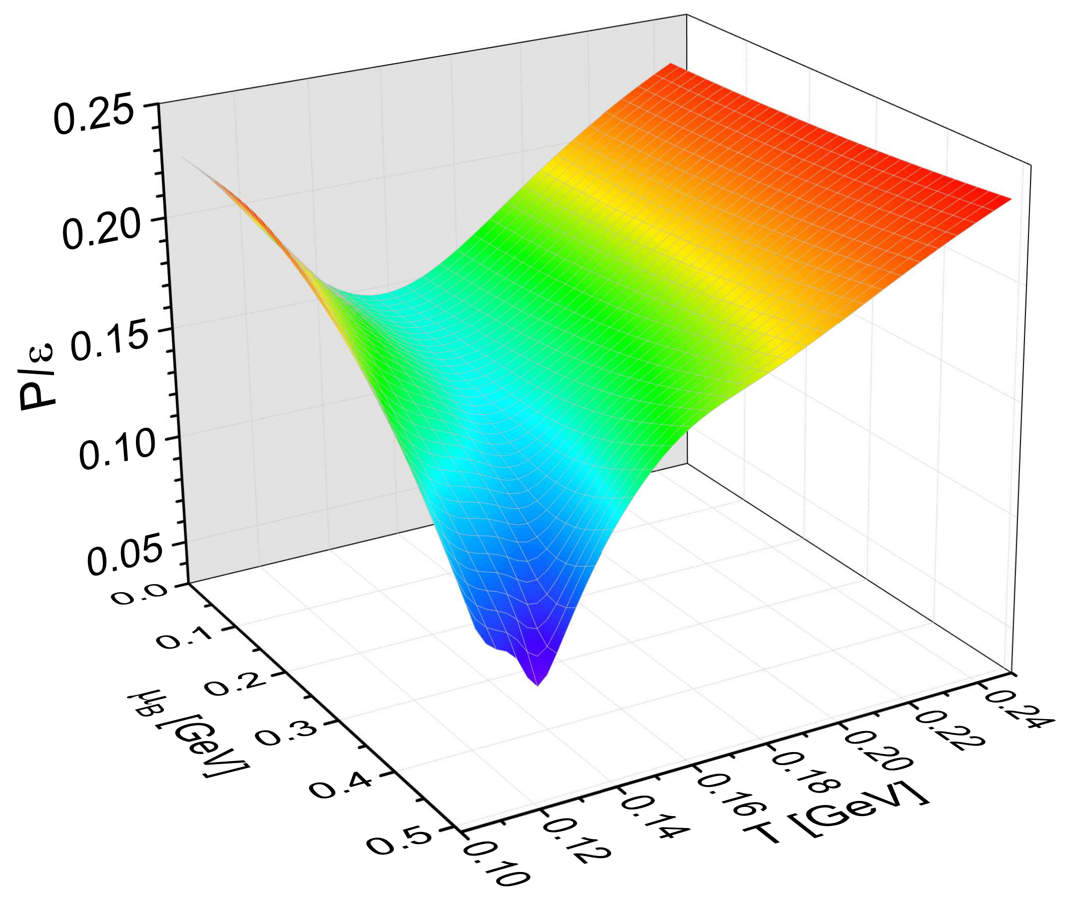




Speed of sound



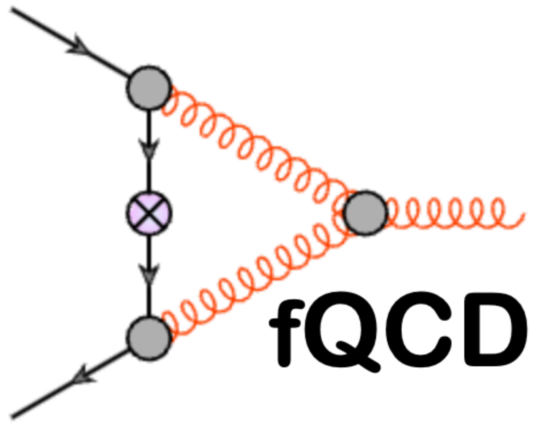
Pressure over energy



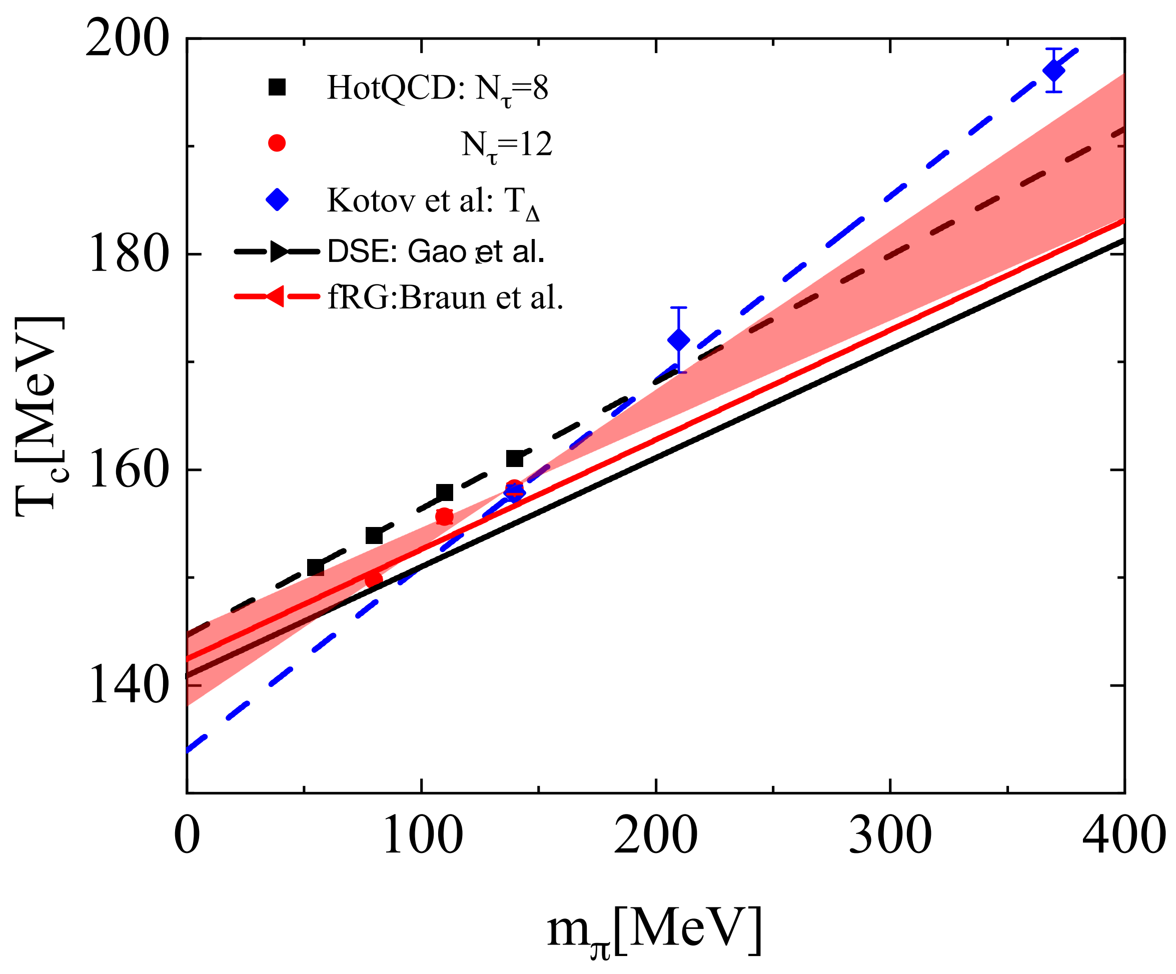
Yi Lu, Gao, Liu, JMP, 2310.18383 (accepted with PRD)

Chiral dynamics & soft modes

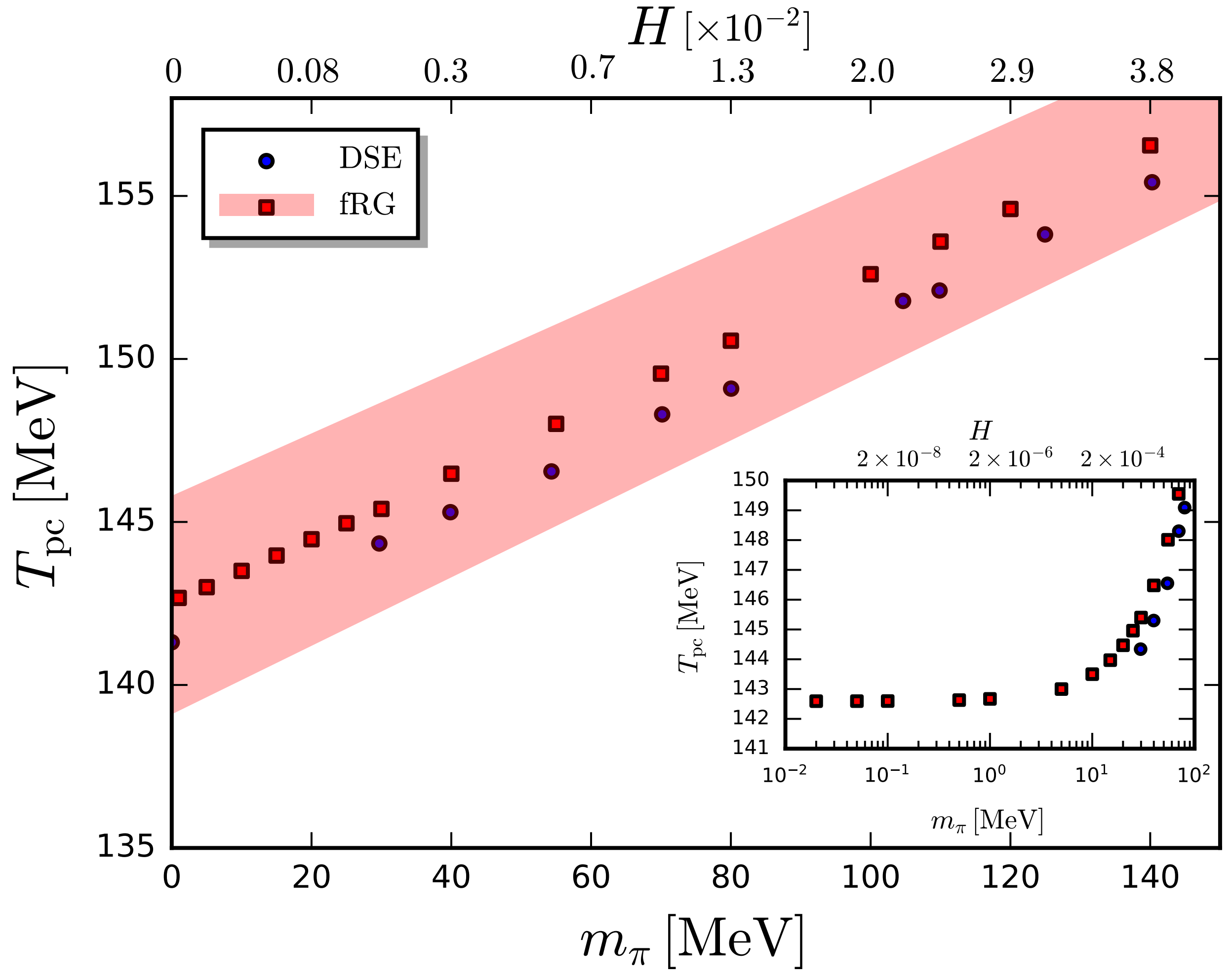
To be (critical) or not (to be)

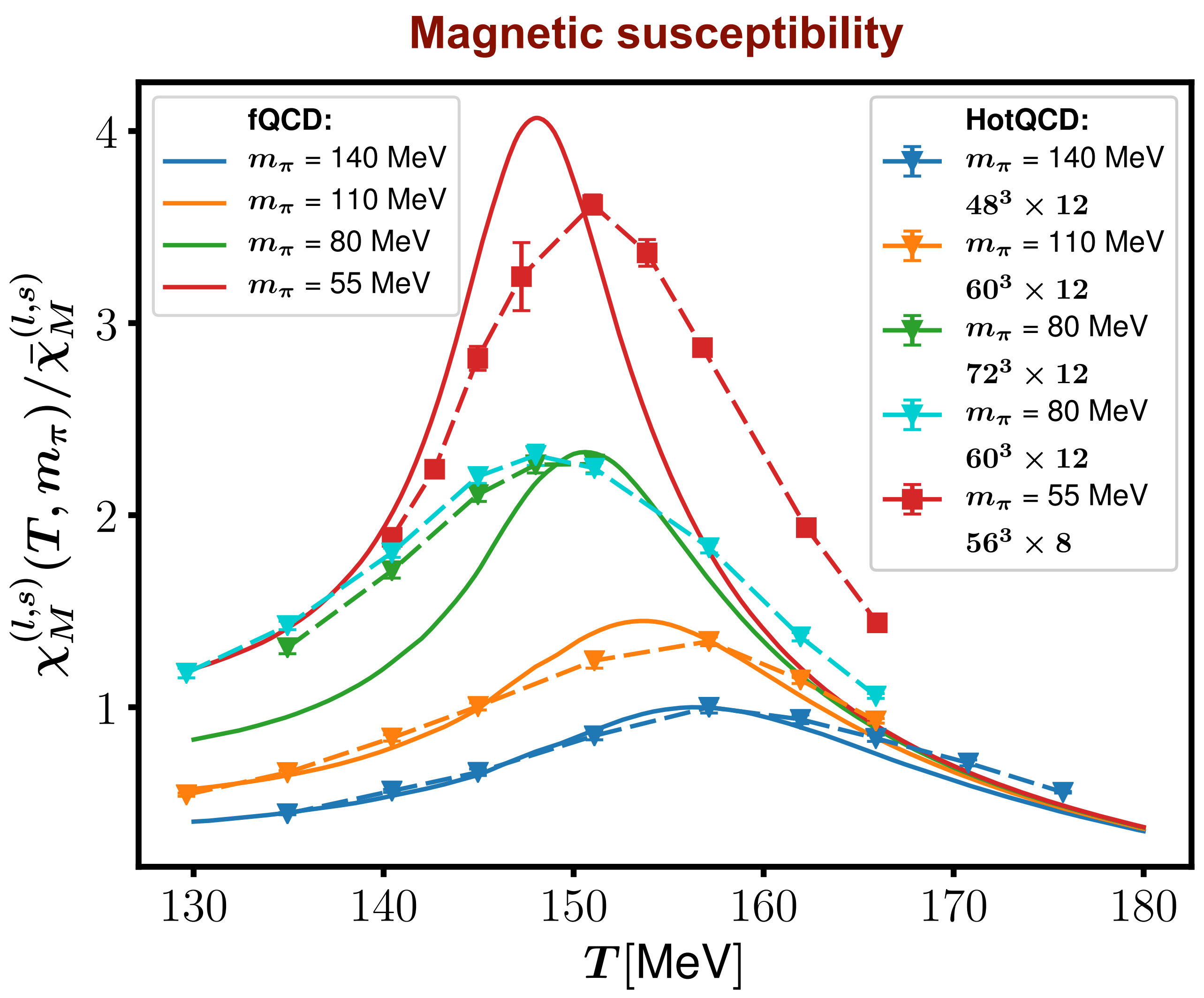


Chiral transition temperature



$$H = \frac{m_l}{m_s}$$





Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

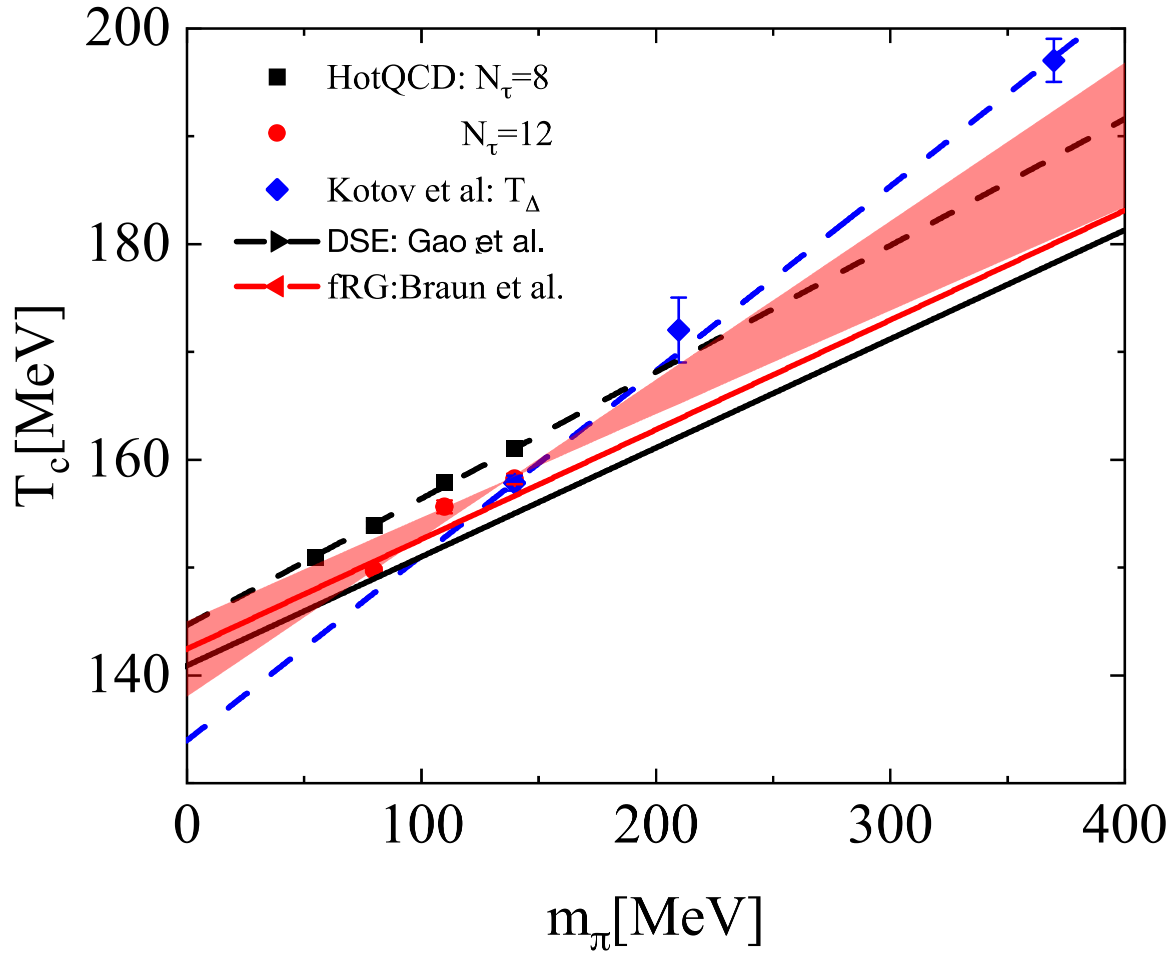
Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

To be (critical) or not (to be)

fQCD

Chiral transition temperature



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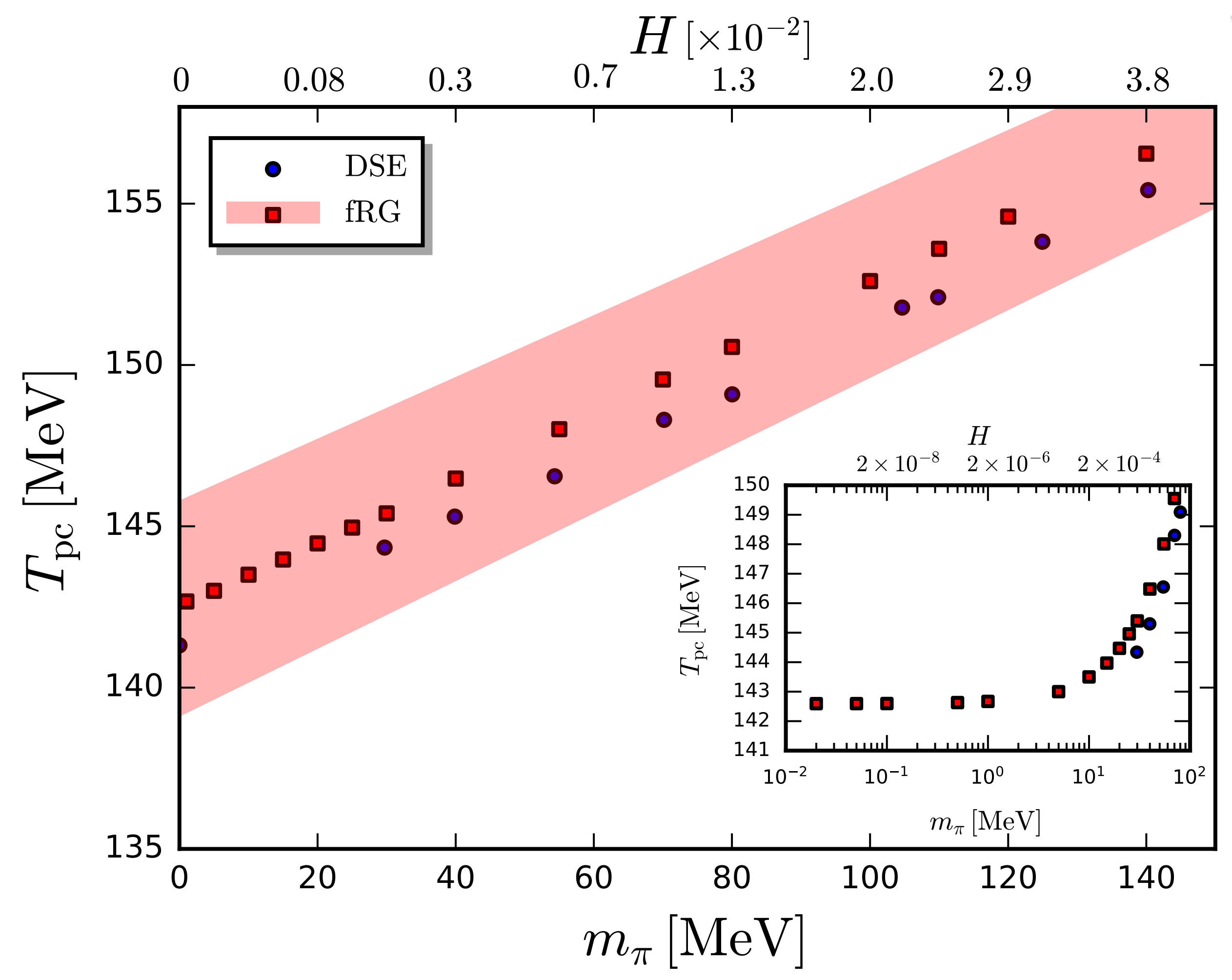
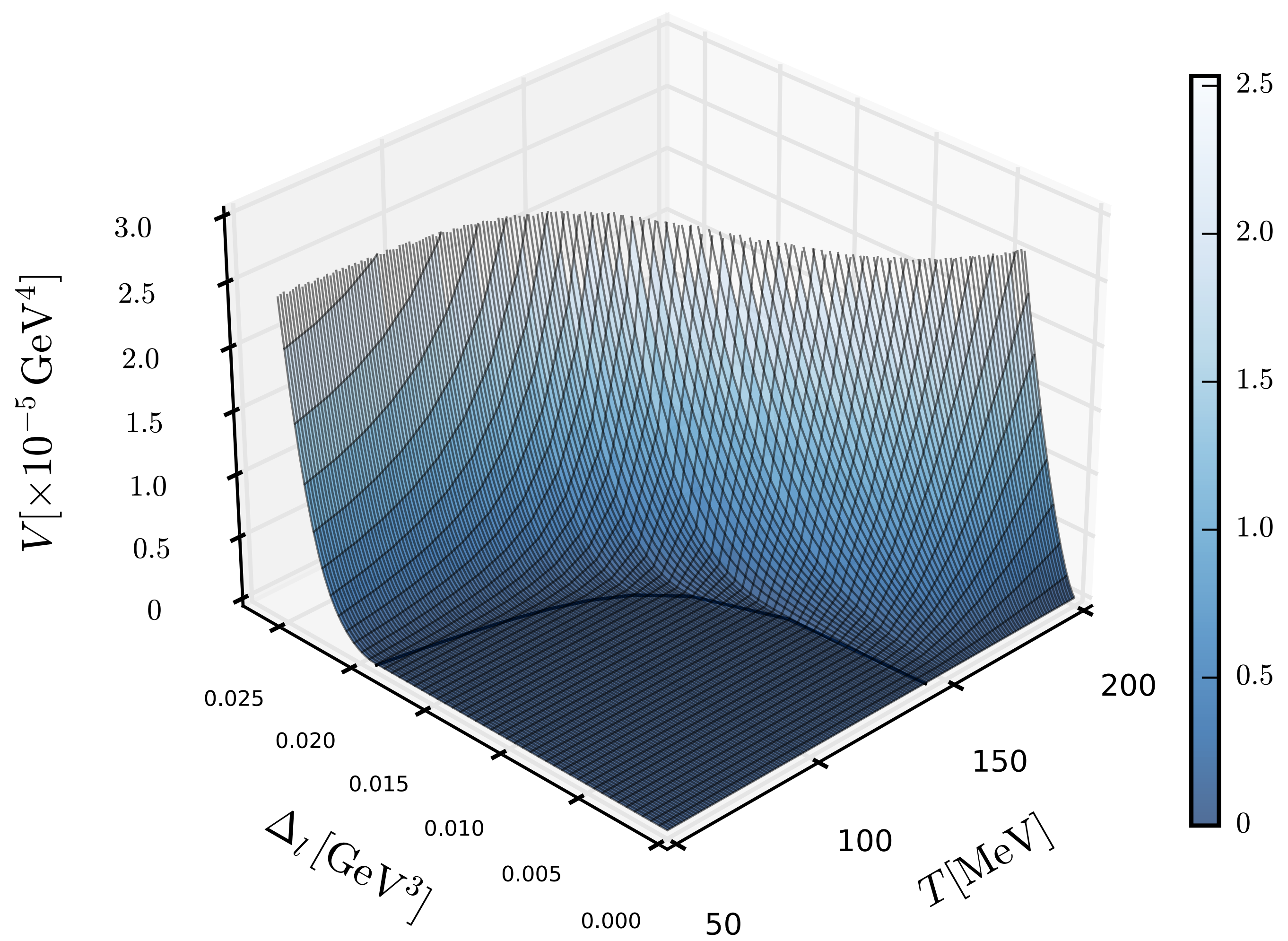


Order parameter potential & scaling



$$V_\chi \approx \Delta_l^n \qquad \Delta_l(H) \propto H^{\frac{1}{n-1}}$$

$$V_\chi^{(\text{fRG})} \approx V_\chi^{(\text{DSE})}$$

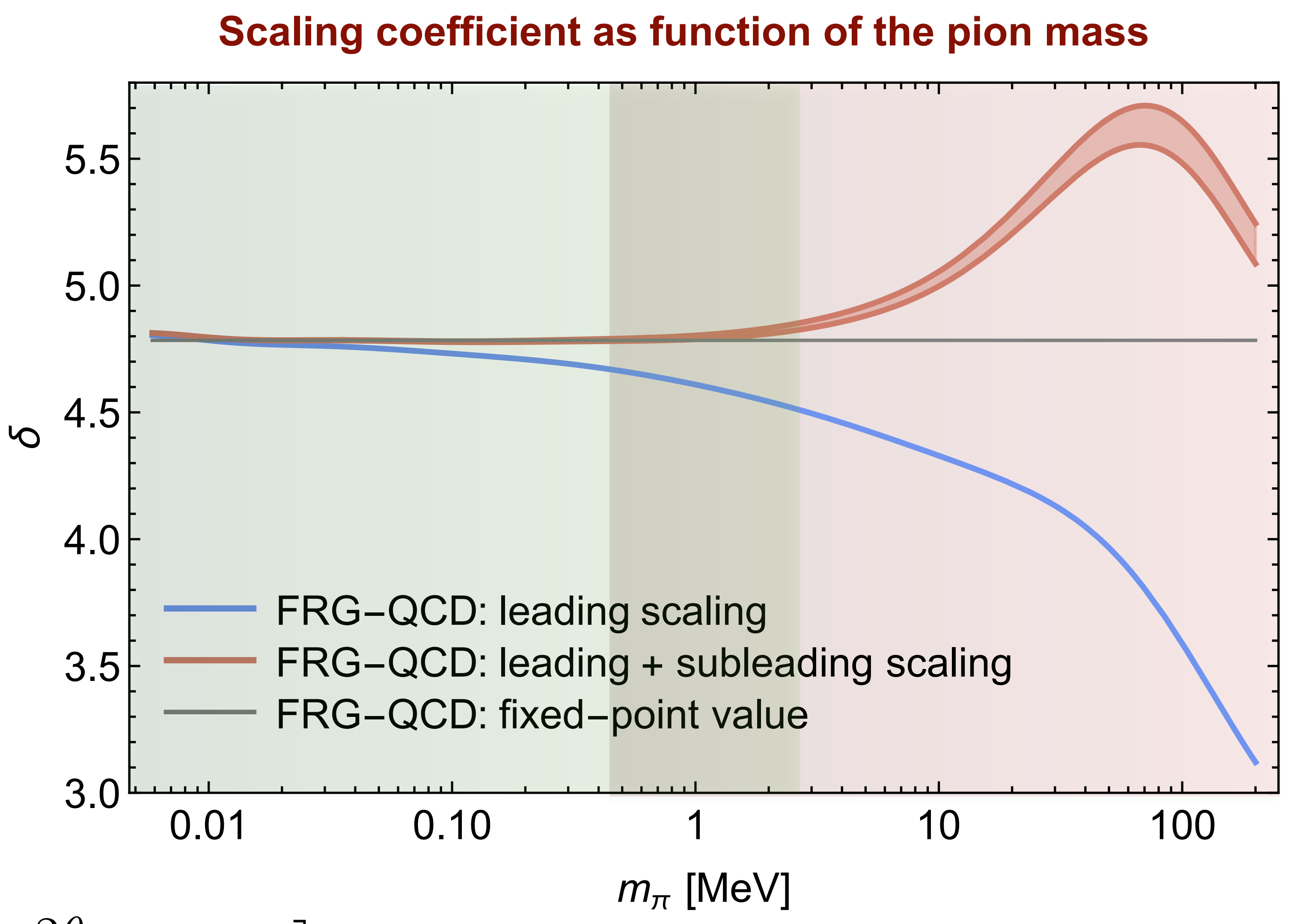
(Critical) exponent: $\dfrac{1}{\delta} = \dfrac{1}{n-1}$

Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes



Critical O(4) scaling

'chiral scaling'

Trivial $\Delta_l^{1+\delta}$ scaling

QM: Chen, Wen, WF, PRD 104 (2021) 054009

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H} + \cdots\right]$$

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes

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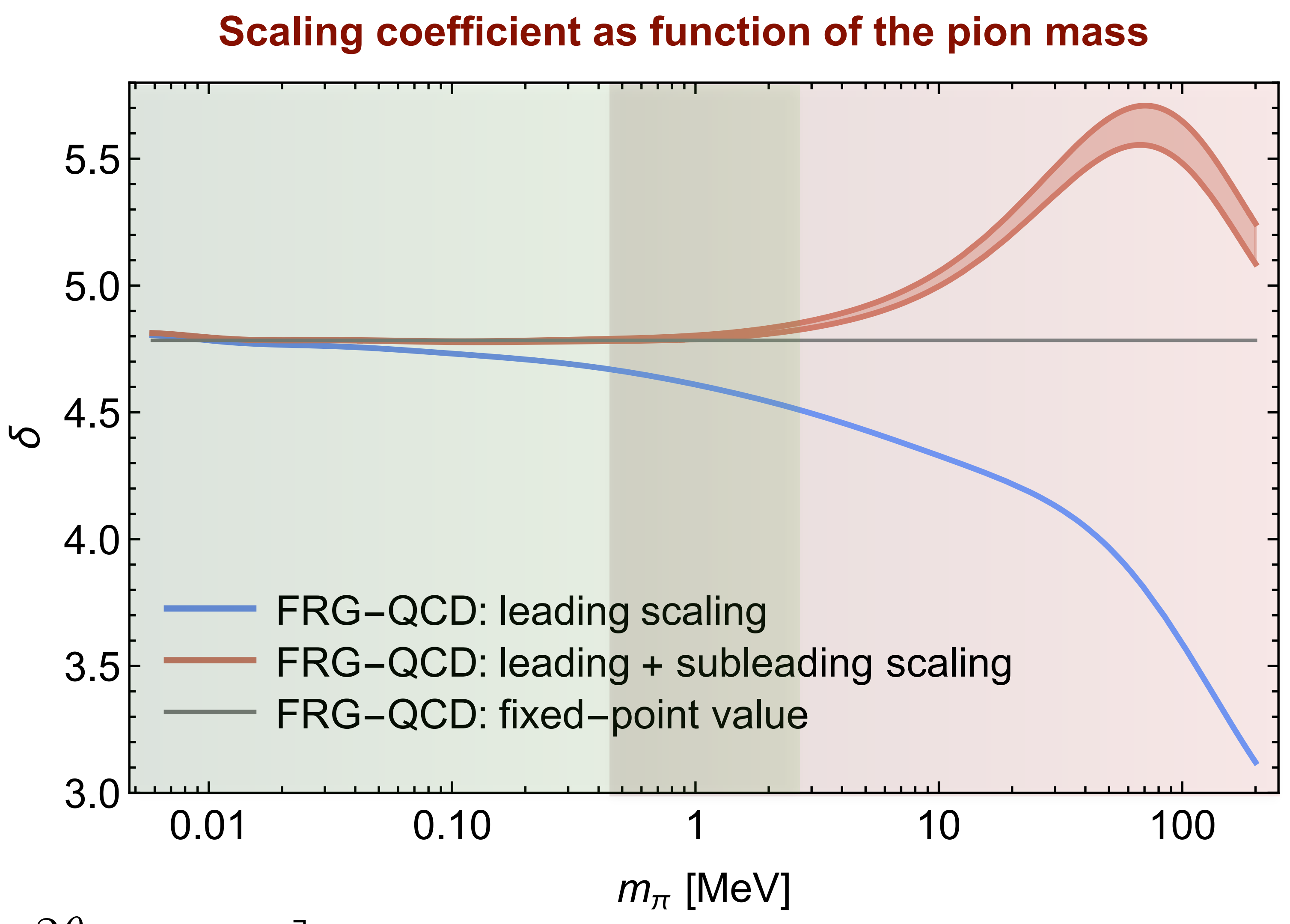

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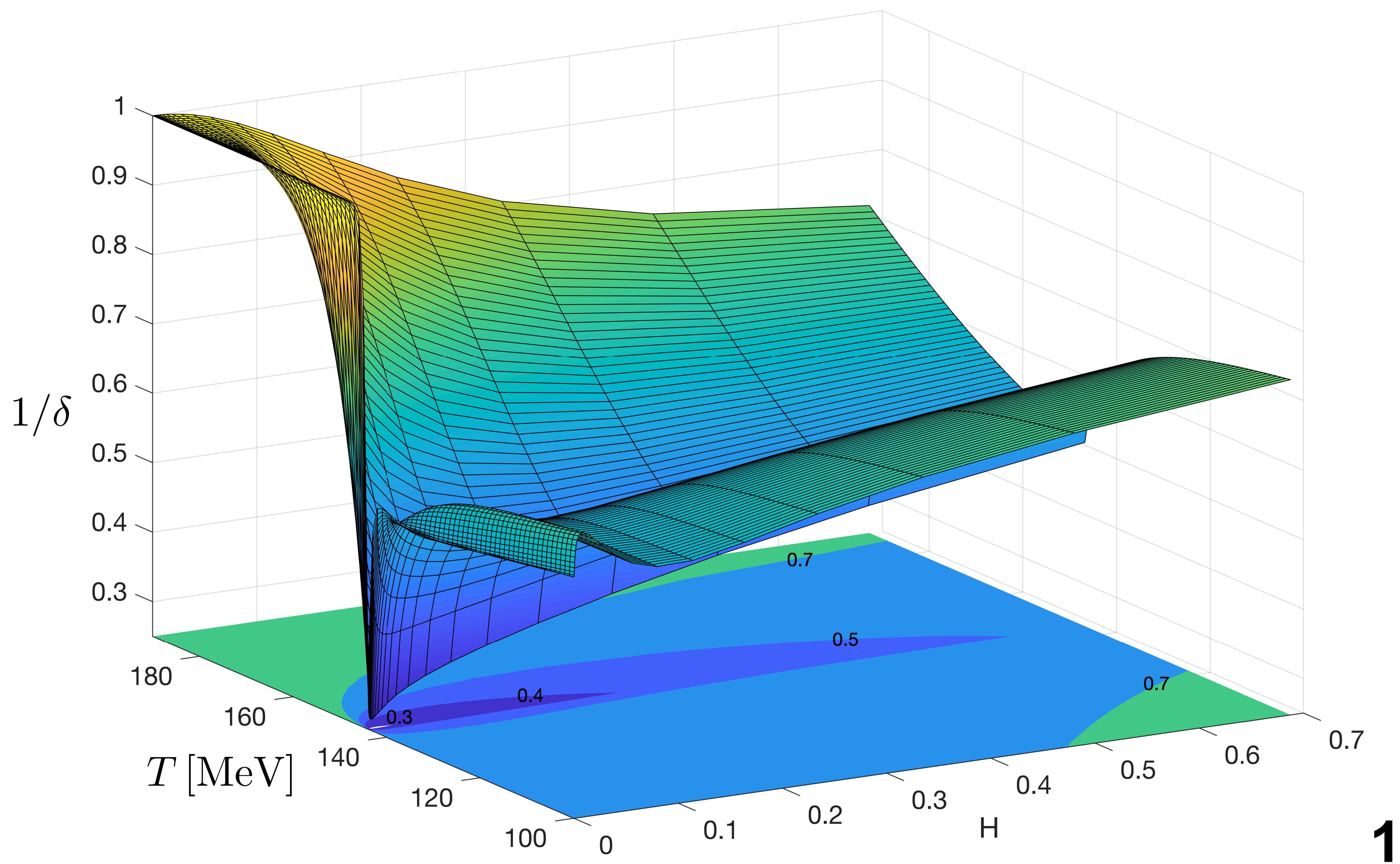
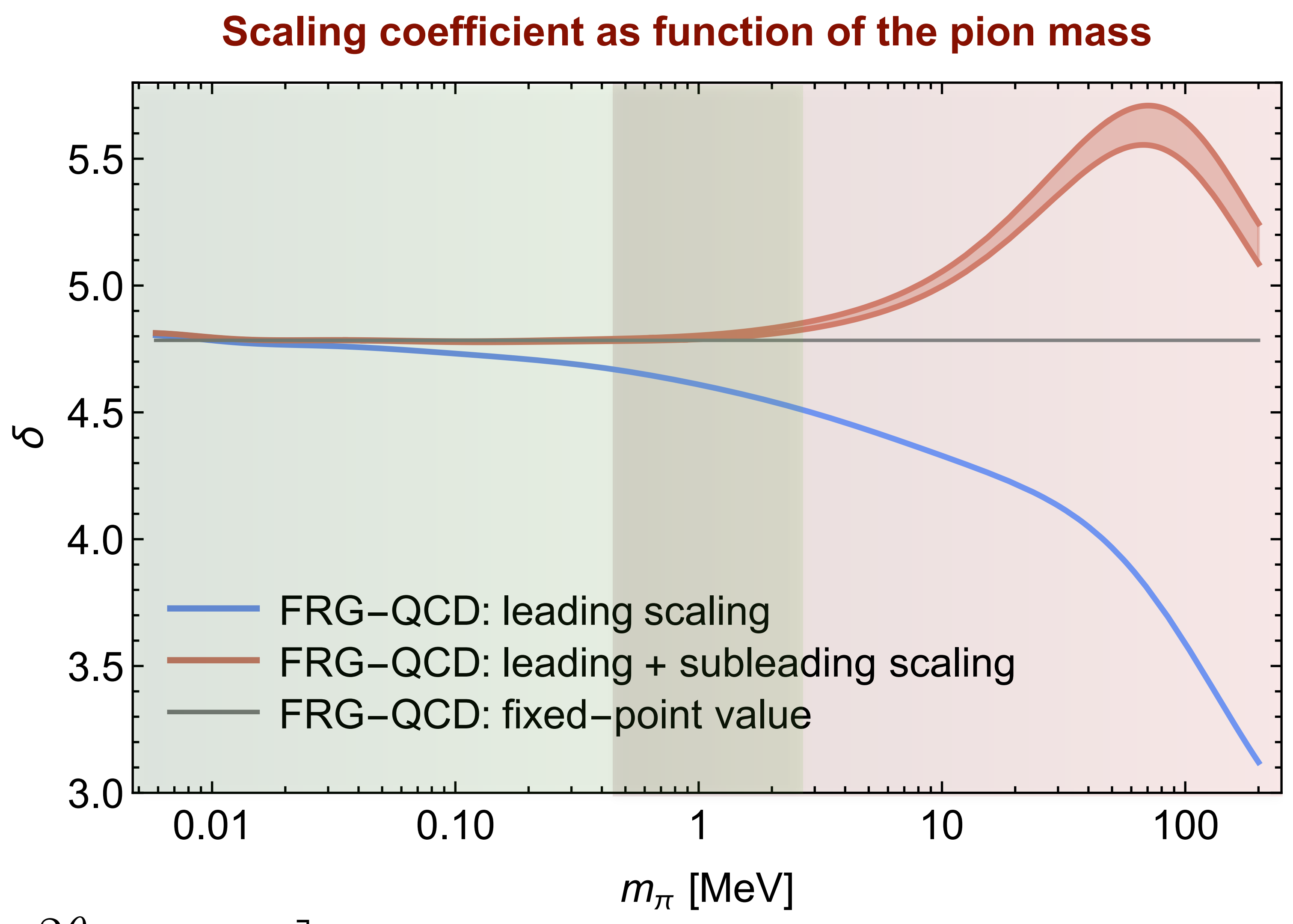


Small chiral scaling regime

↓

Small critical regime around pot. CEP

Chiral dynamics & quasi-massless modes



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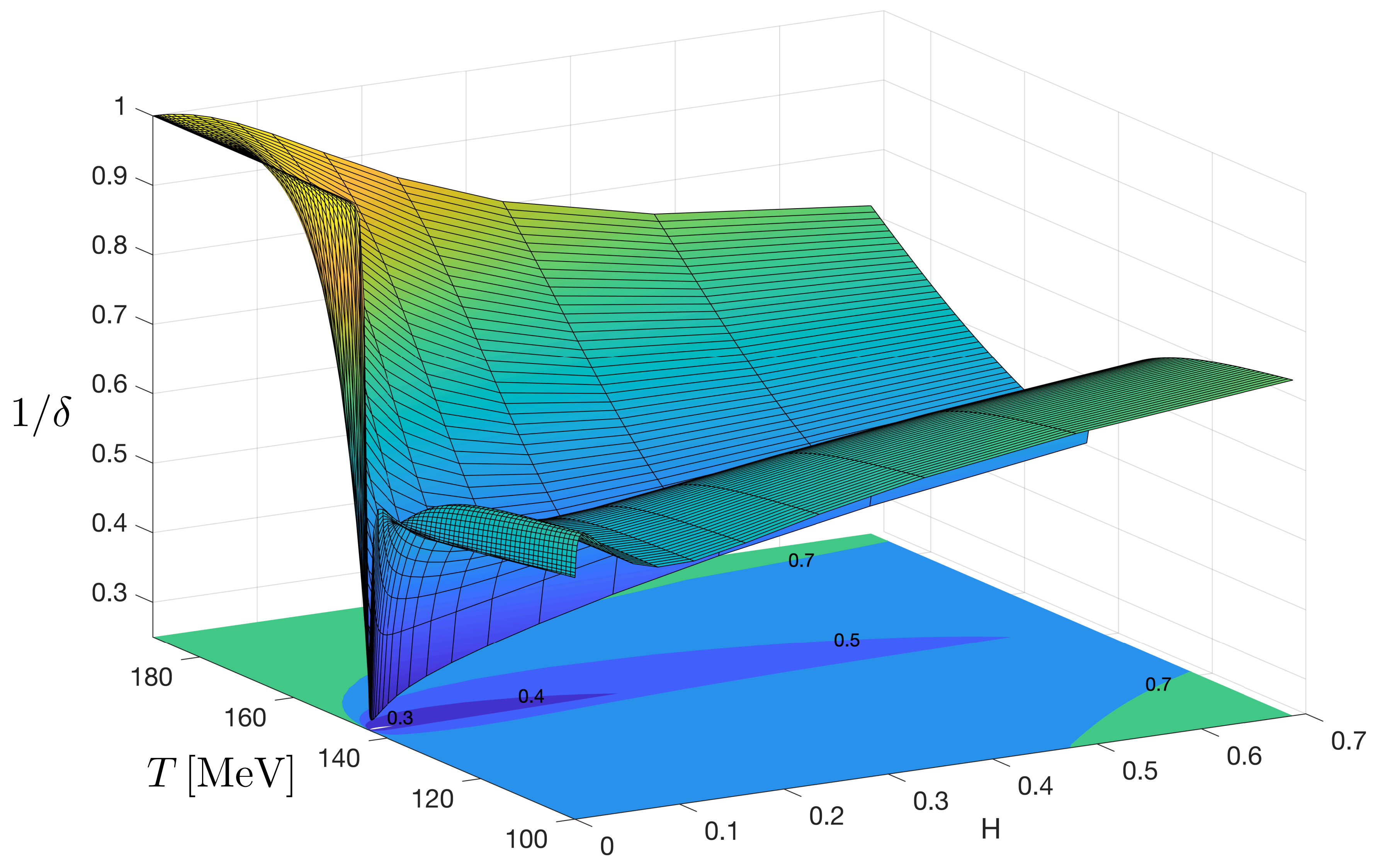


Small chiral scaling regime

↓

Small critical regime around pot. CEP

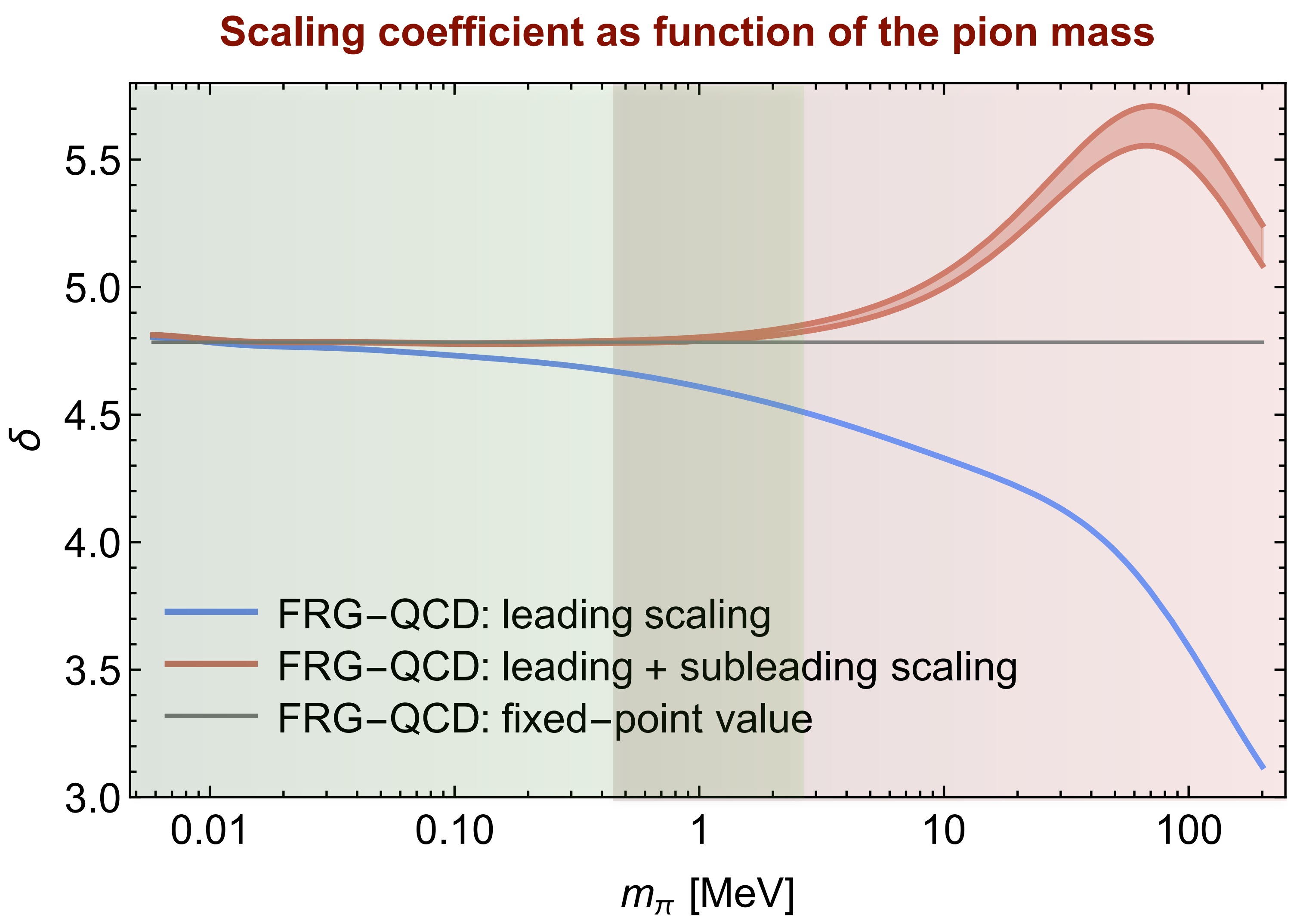
!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

Chiral dynamics & quasi-massless modes

fQCD

Critical O(4) scaling



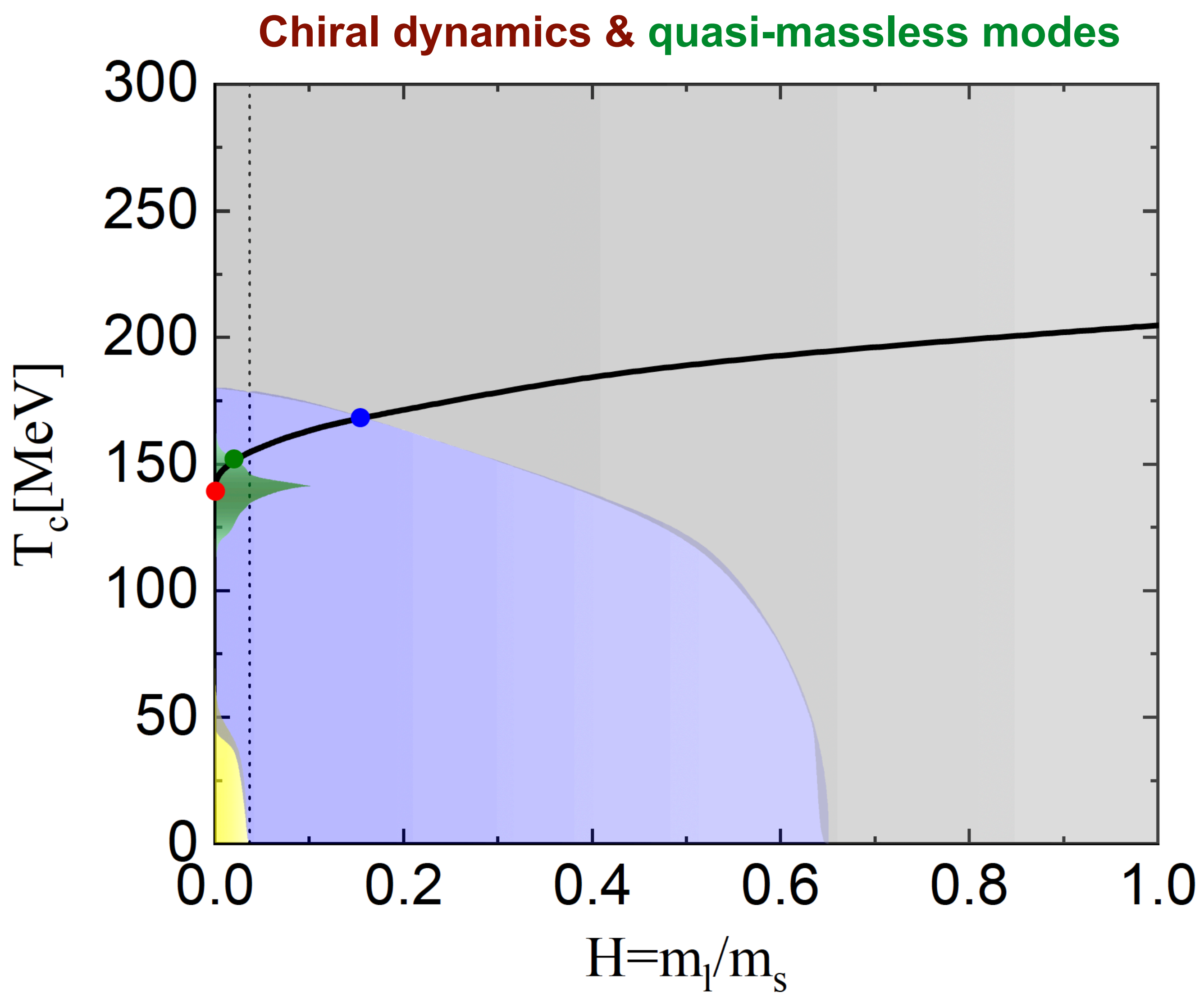
'chiral scaling'

Trivial $\Delta_l^{1+\delta}$ scaling

fQCD collaboration, in preparation

QM: Chen, Wen, WF, PRD 104 (2021) 054009

Critical scaling



Gao, JMP, PRD 105 (2022) 094020

'Non-critical chiral scaling'

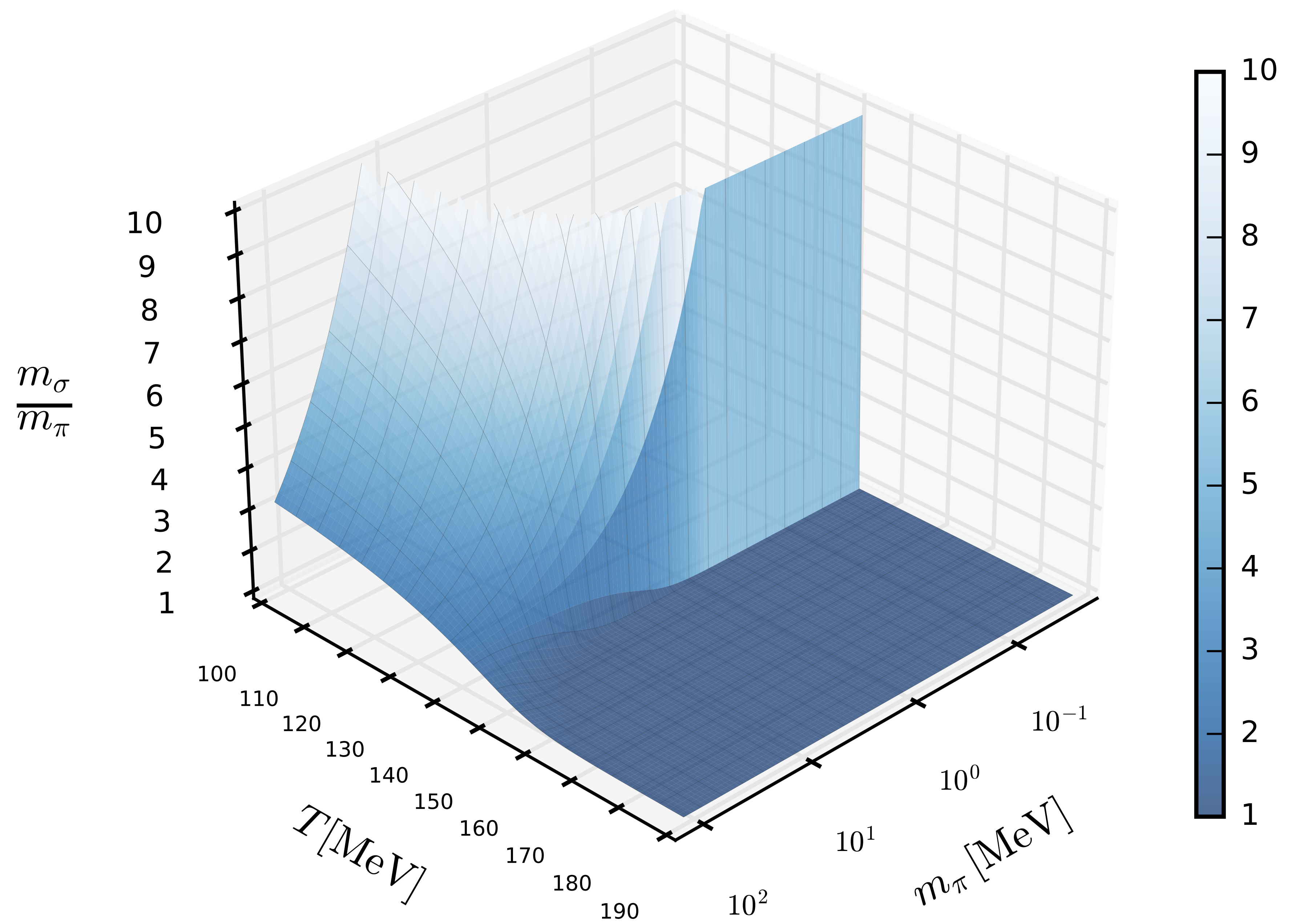
Far away from the critical regime for $m_\pi \gtrsim 1\,\mathrm{MeV}$

$$\Delta_l(T,H) \approx \Delta_{l,\chi}(0)\left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H\right)$$

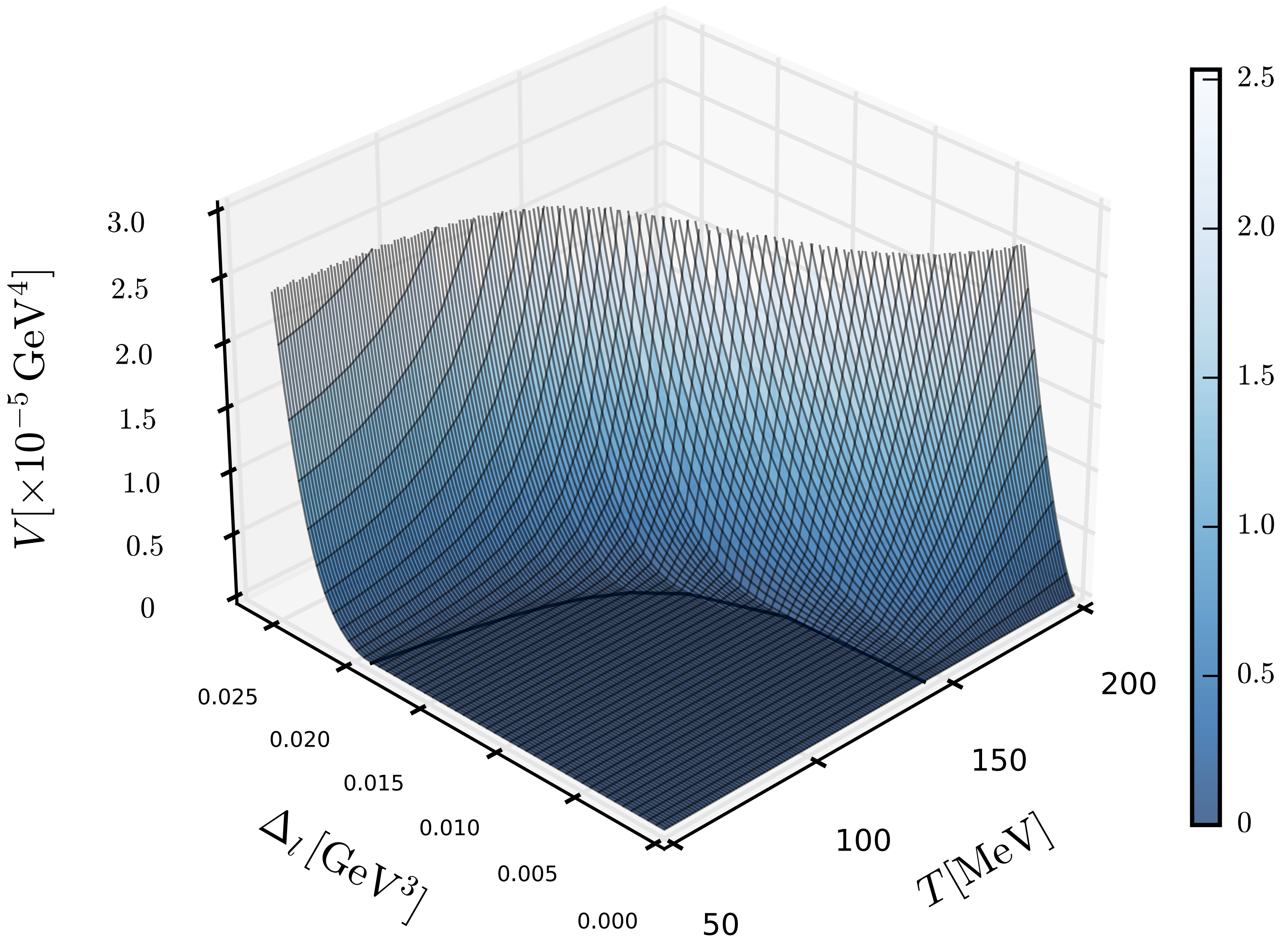
$V_\chi(\Delta_l) \propto$ Δ_l^6 Δ_l^4 Δ_l^2

Full order parameter potential

Measure: correlation length



Use for chiral dynamics in heavy ion collisions

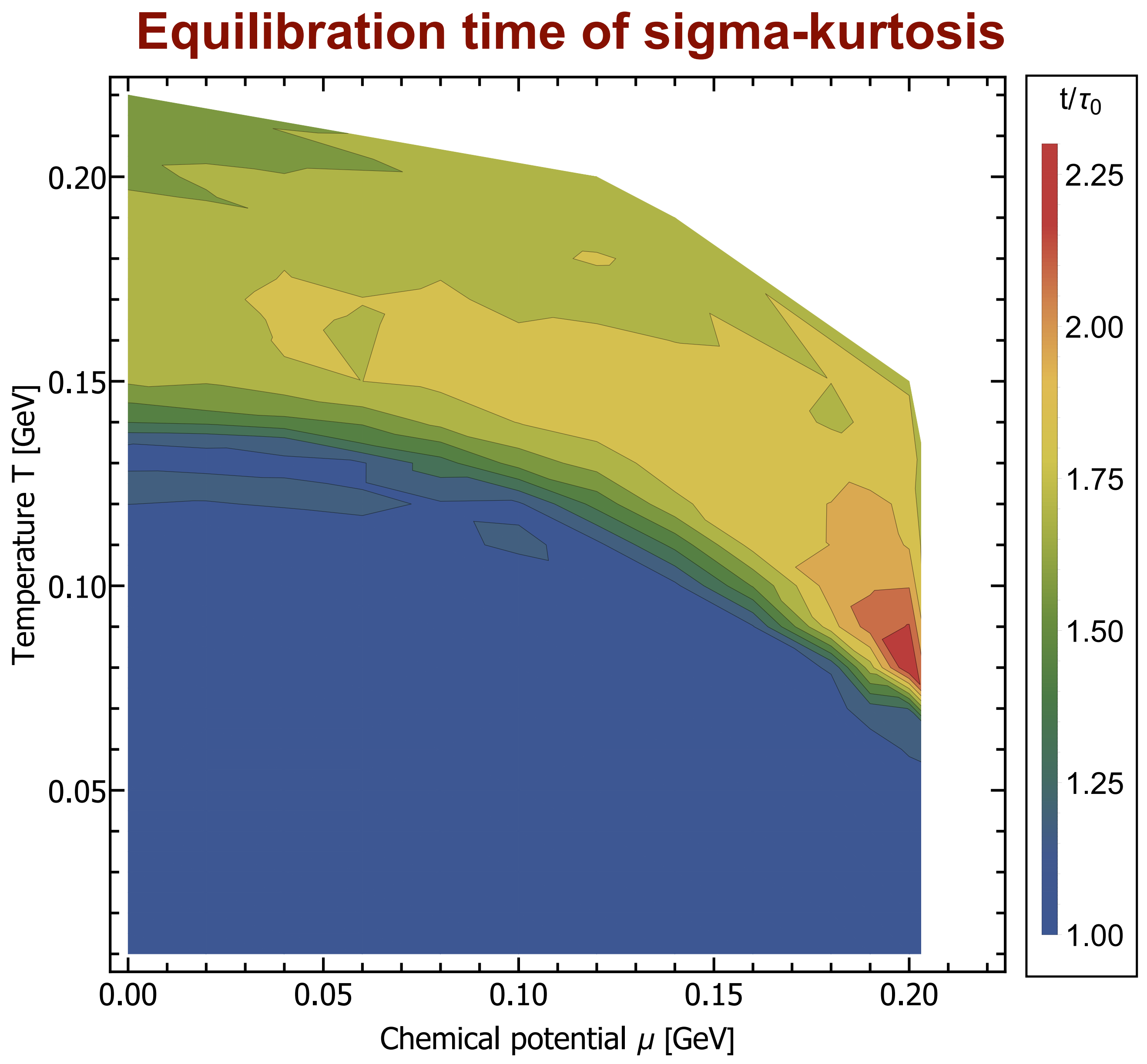


QCD: fQCD collaboration, in preparation

Dynamics and the size of the critical regime

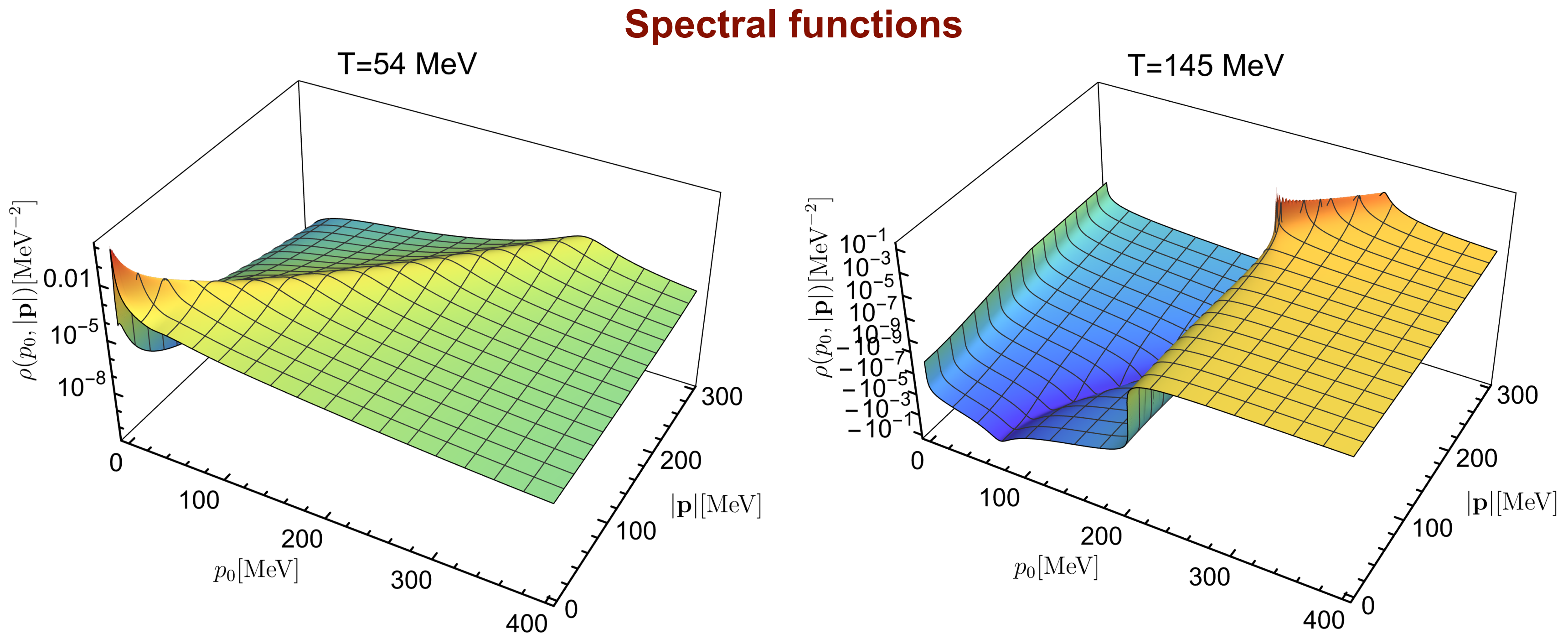
Showcases in linear sigma models

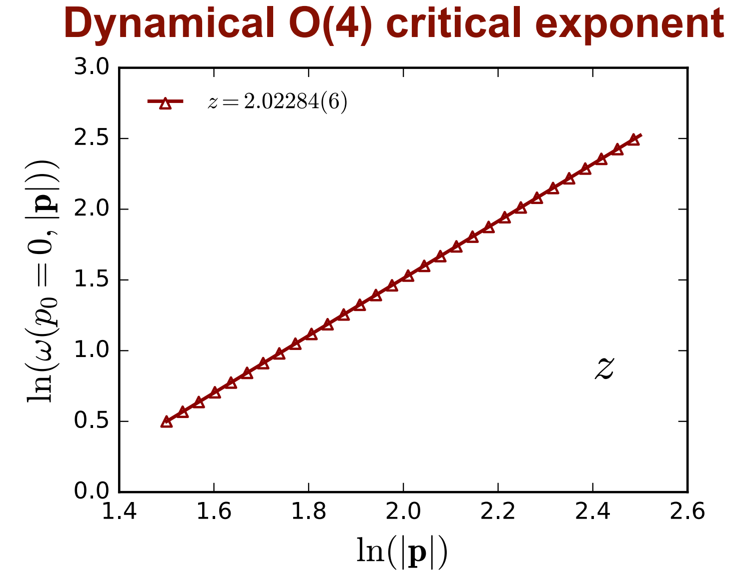
Transport with fRG spectral functions & effective potential



Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, NPA 982

Dynamical universality





$$\omega \propto (\vec{p}^{\,2})^{\frac{z}{2}}$$

Tan, Chen, Fu, SciPost Phys. 12 (2022) 026

QM: Roth, Schweitzer, Rieke, von Smekal , PRD 105 (2022)

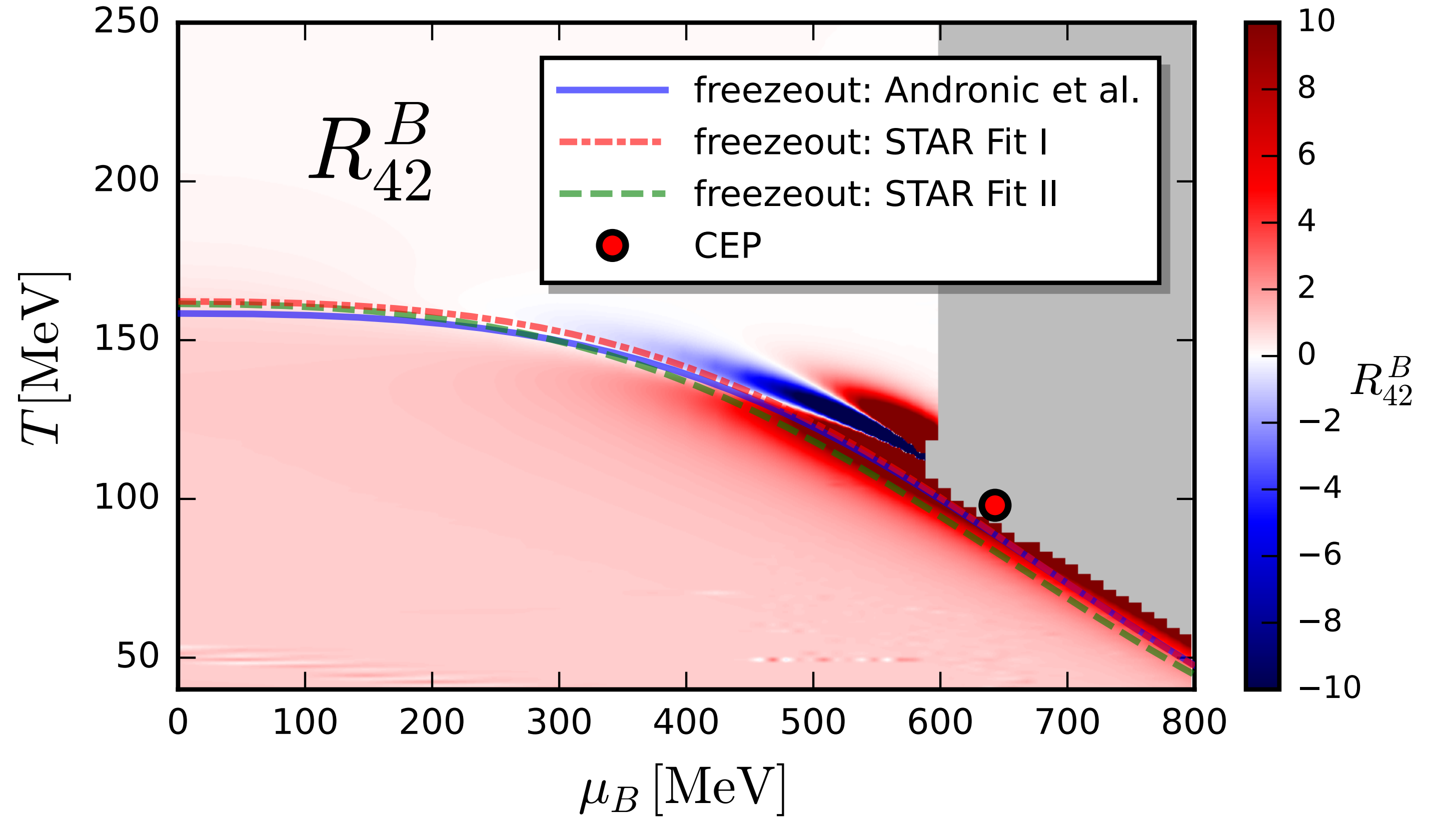
Fluctuations of conserved charges: Ripples of the critical end point

or

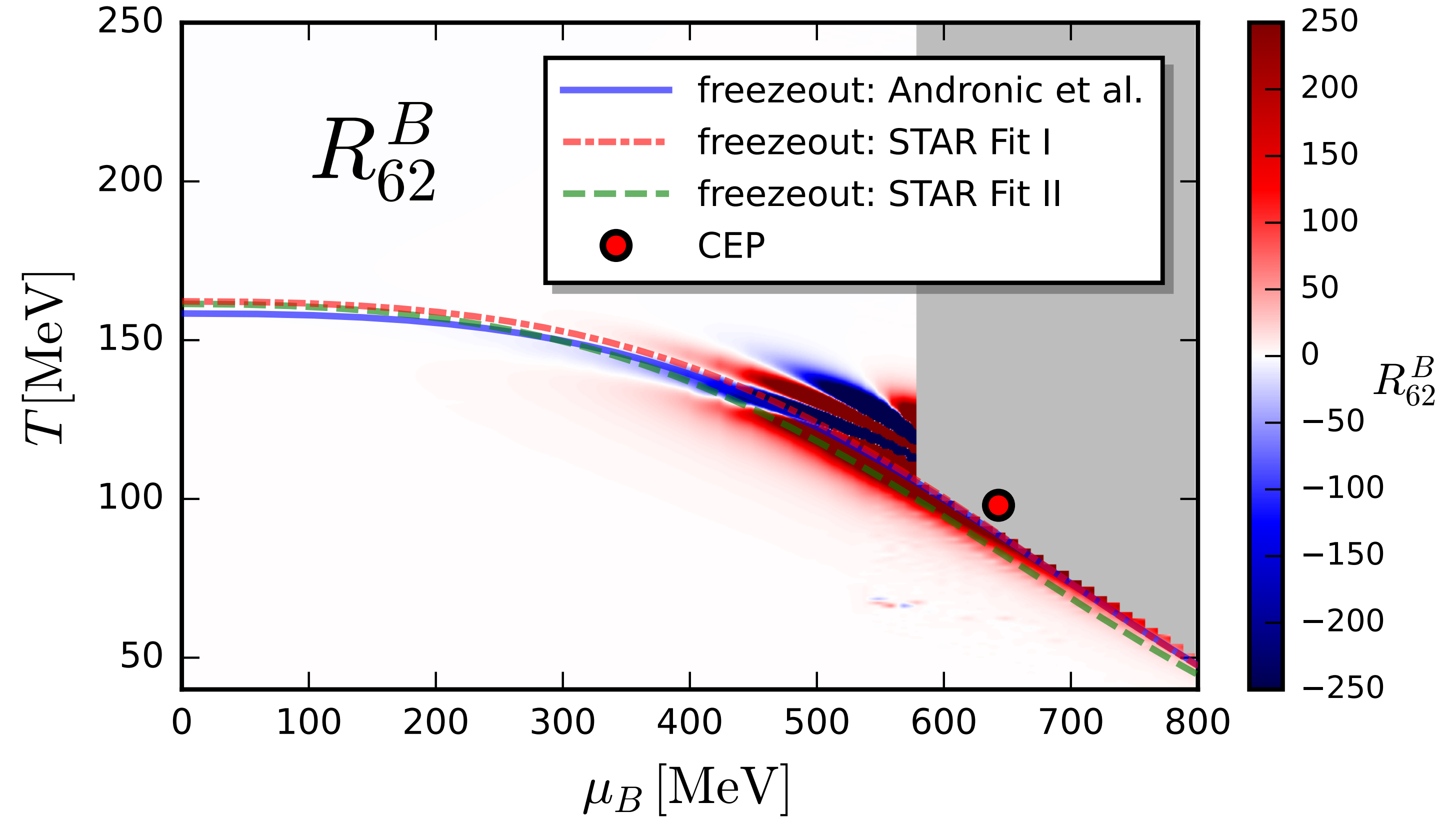
the LEGO® principle at work

Ripples of the critical point

Baryon number fluctuations in the phase structure



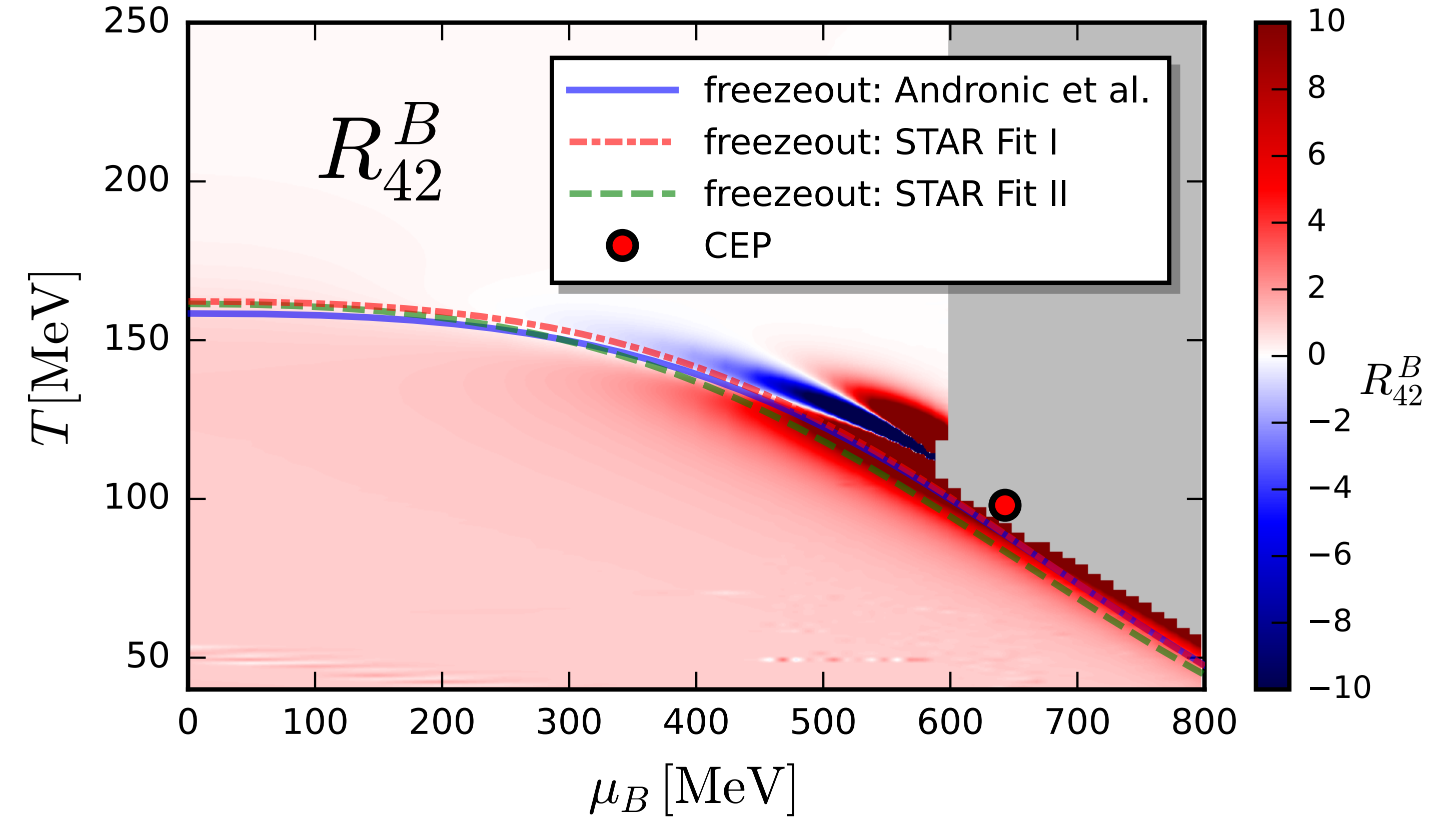
$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (98, 643)\,\mathrm{MeV}$$



Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Ripples of the critical point

Baryon number fluctuations in the phase structure



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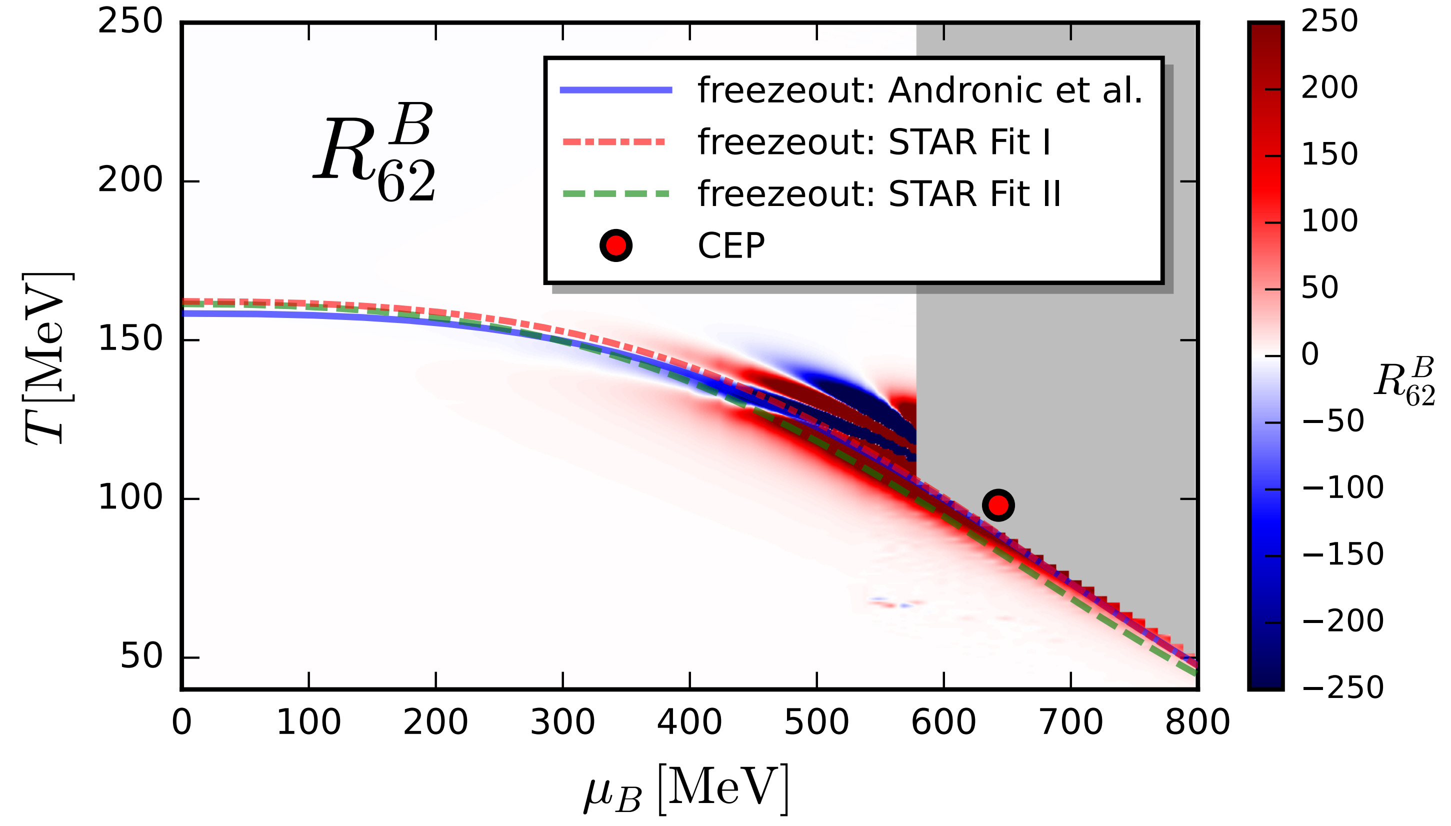
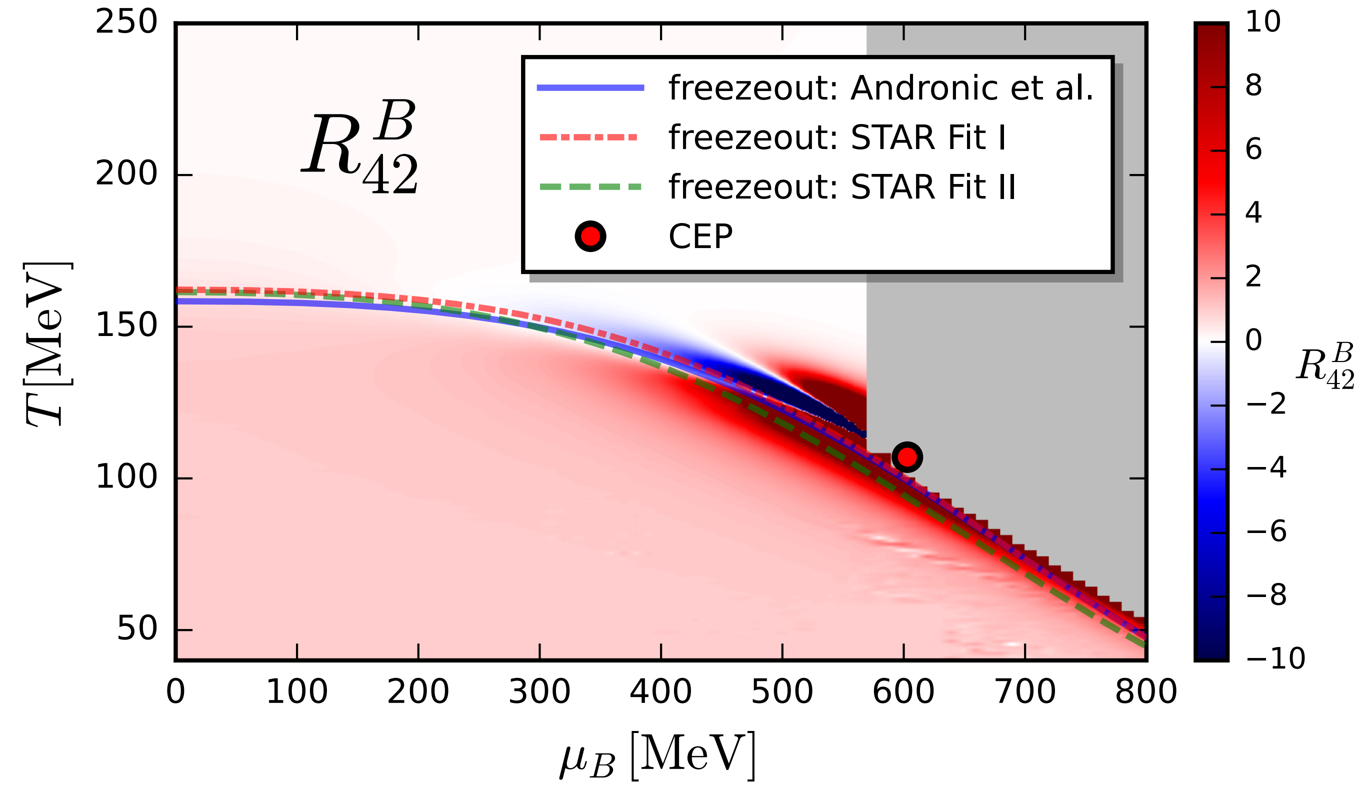
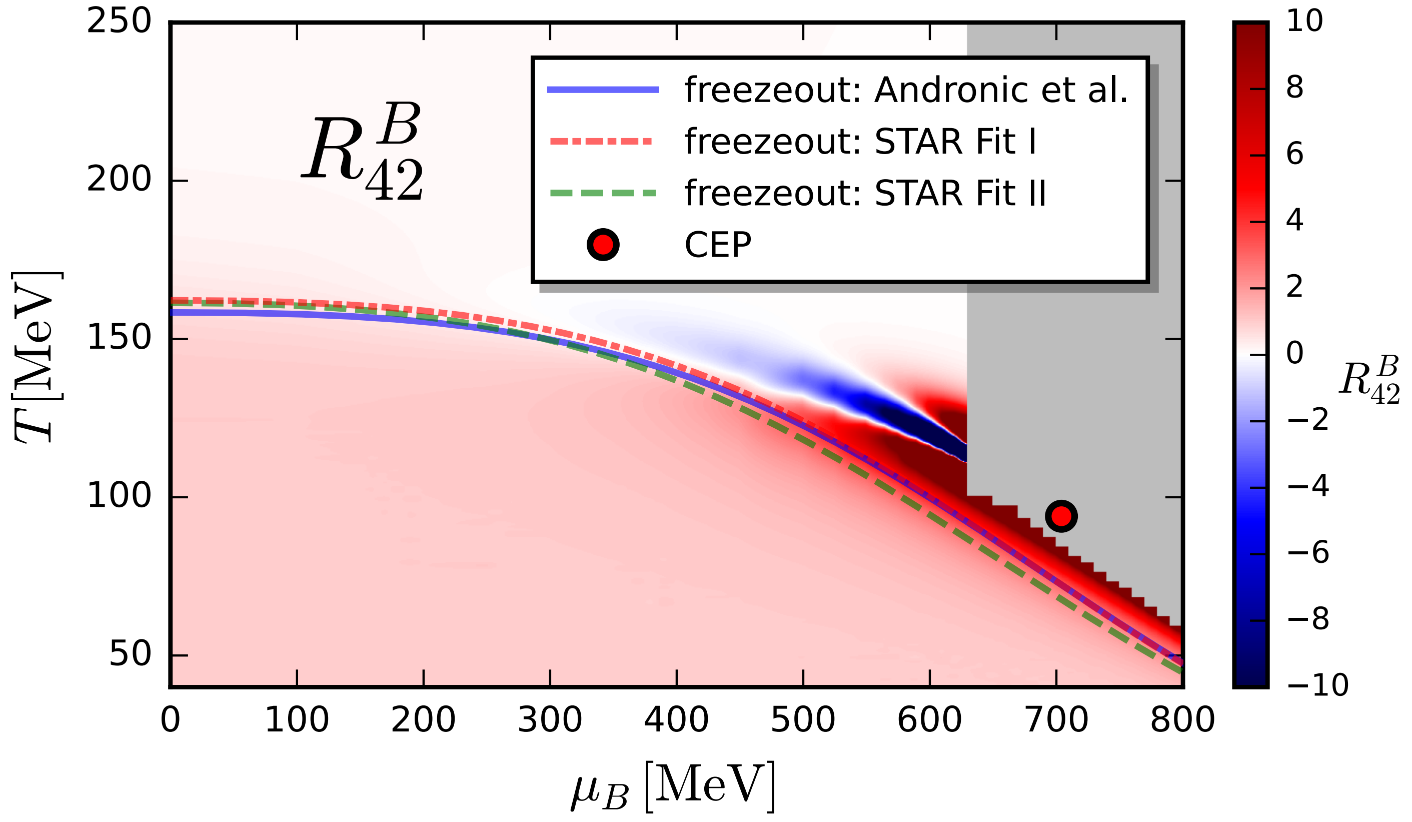


Variations of the CEP



$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (108, 604)\,\mathrm{MeV}$$



$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}}) = (94, 704)\,\mathrm{MeV}$$

Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Ripples of the critical point

Canonical corrections via subensemble acceptance method

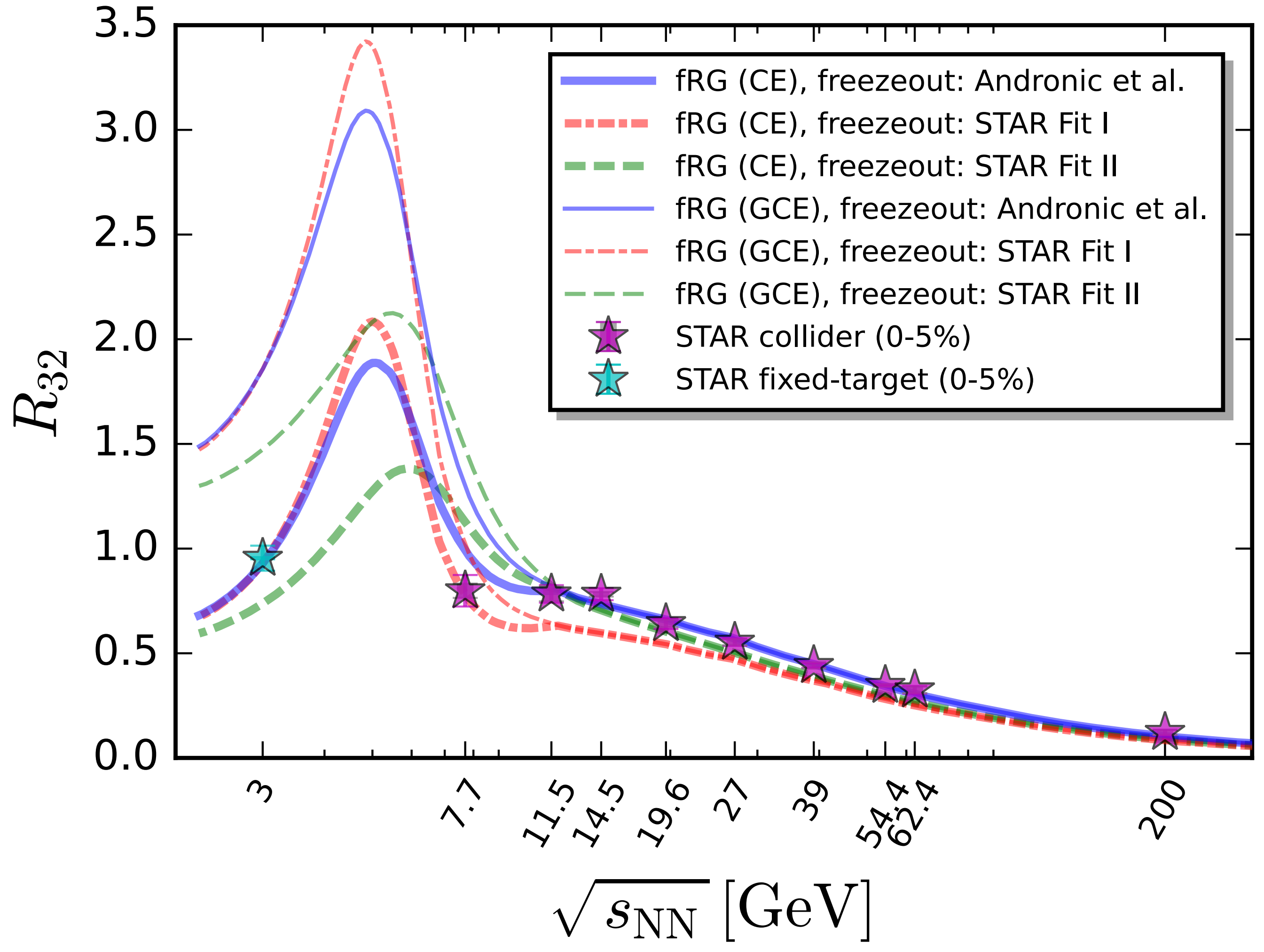
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume **system volume**

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha) R_{32}^B$$



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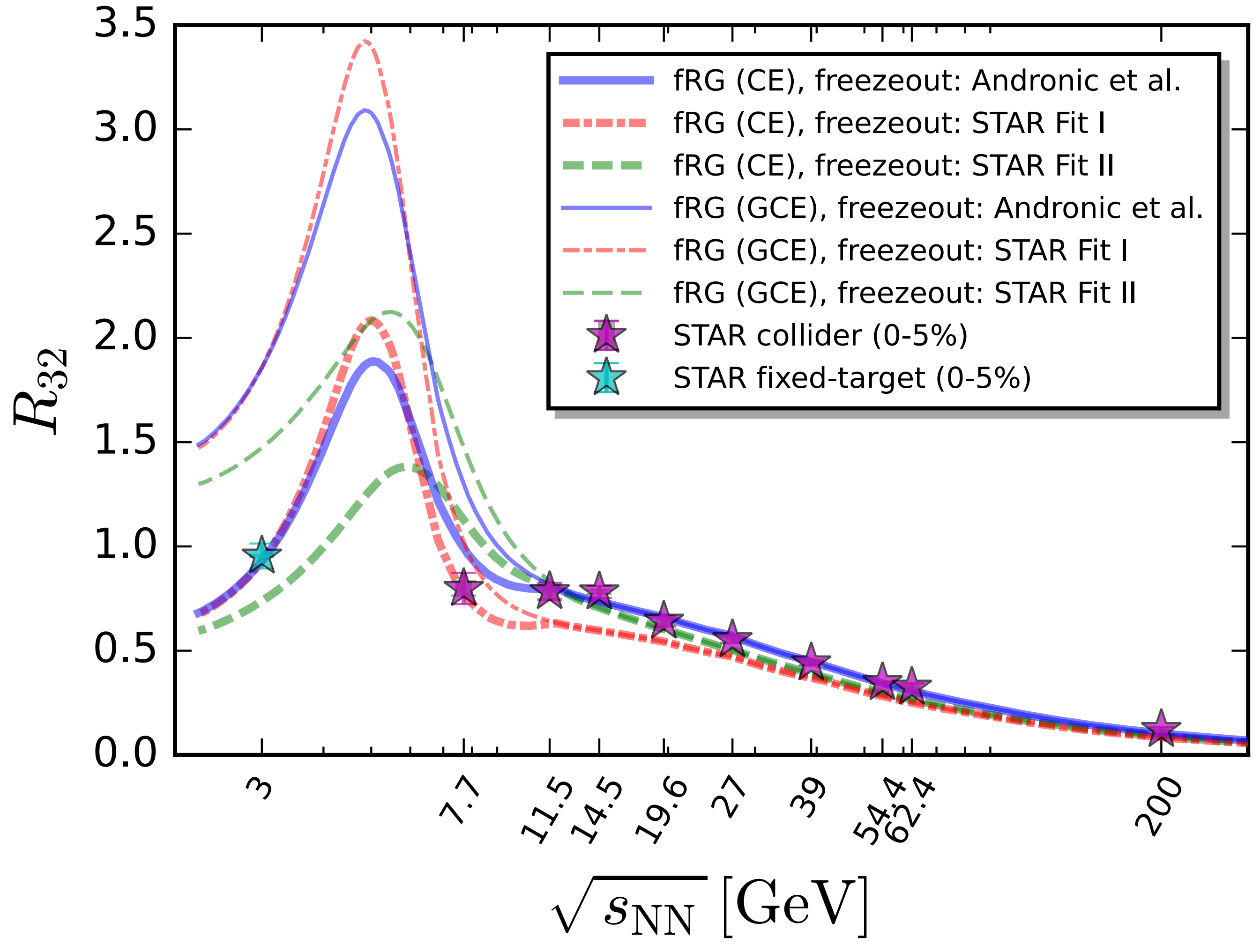

qualitative adjustment

$$\alpha(\bar{s}) = a\left(1-\sqrt{\bar{s}}\right)\theta\left(1-\bar{s}\right)$$

$$a = 0.33 \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,{\rm GeV}}$$

Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

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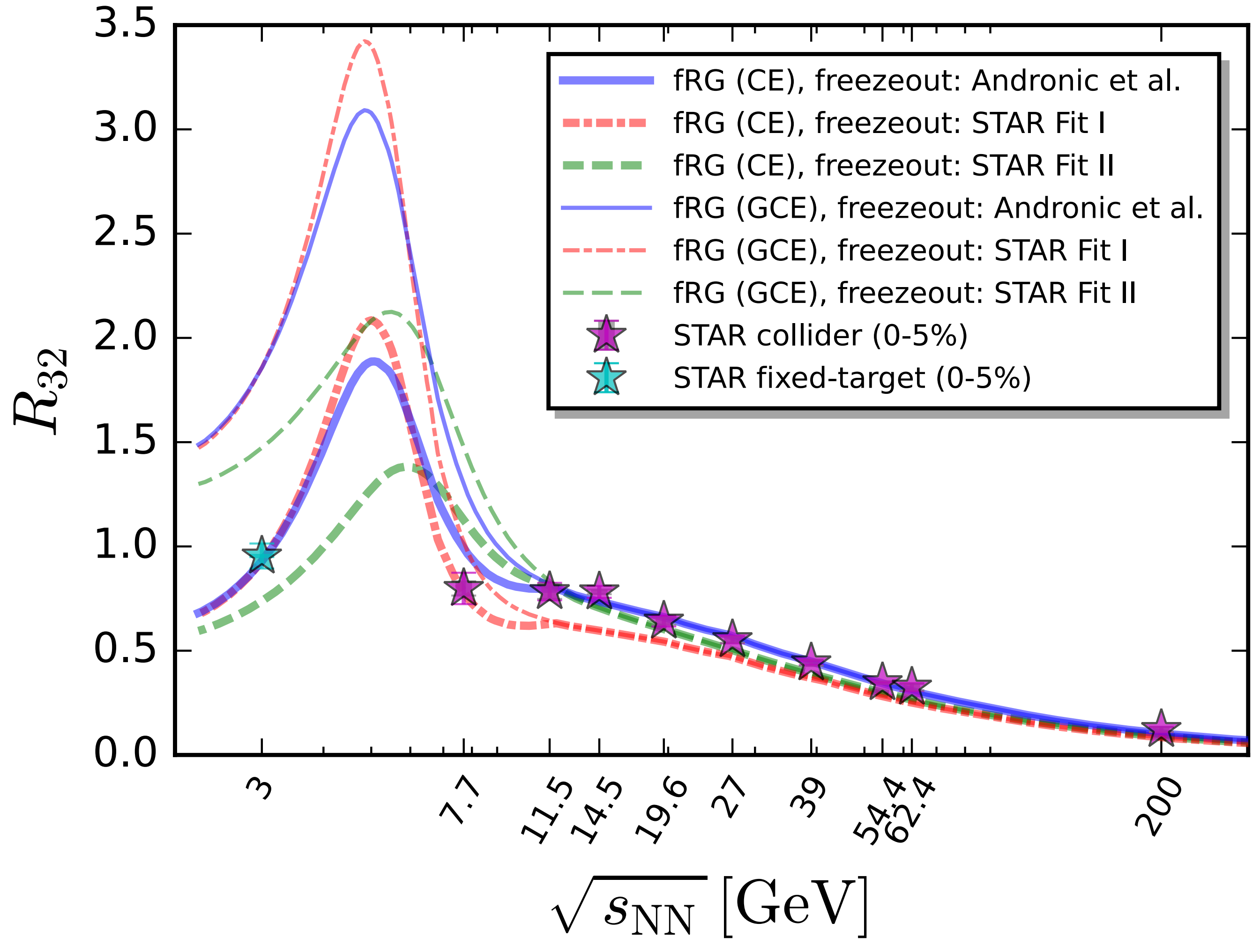

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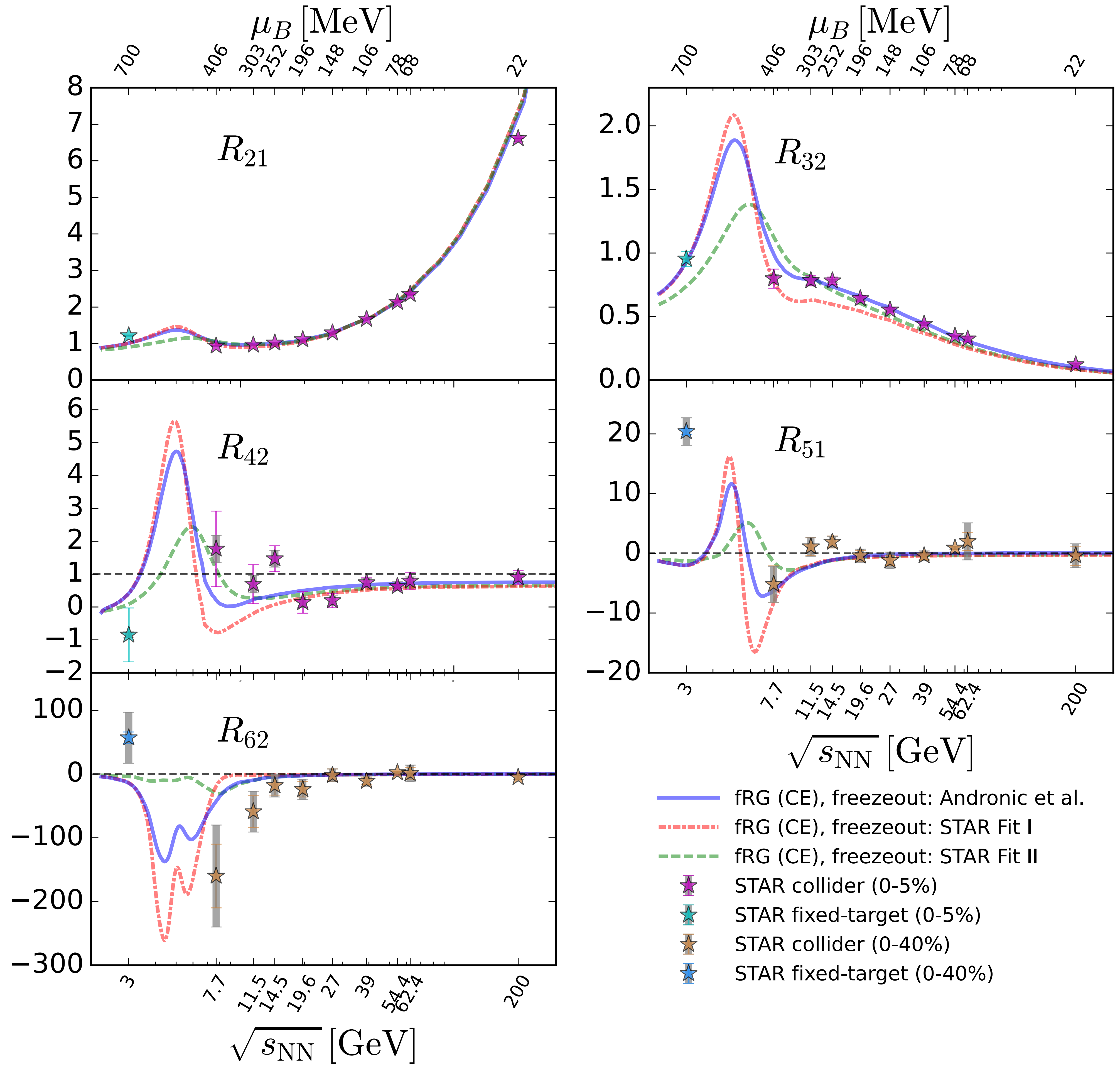
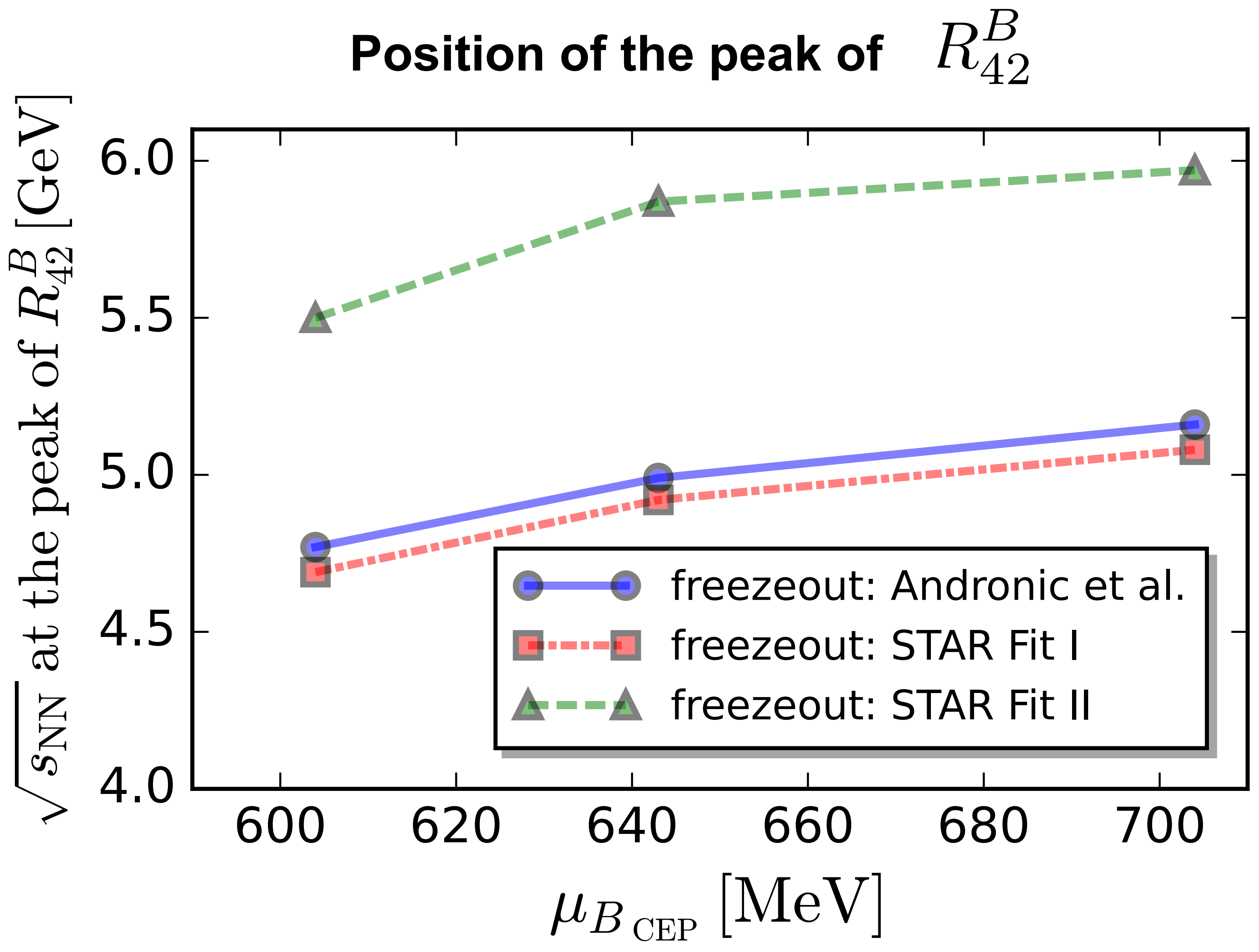
baryon & proton number fluctuations

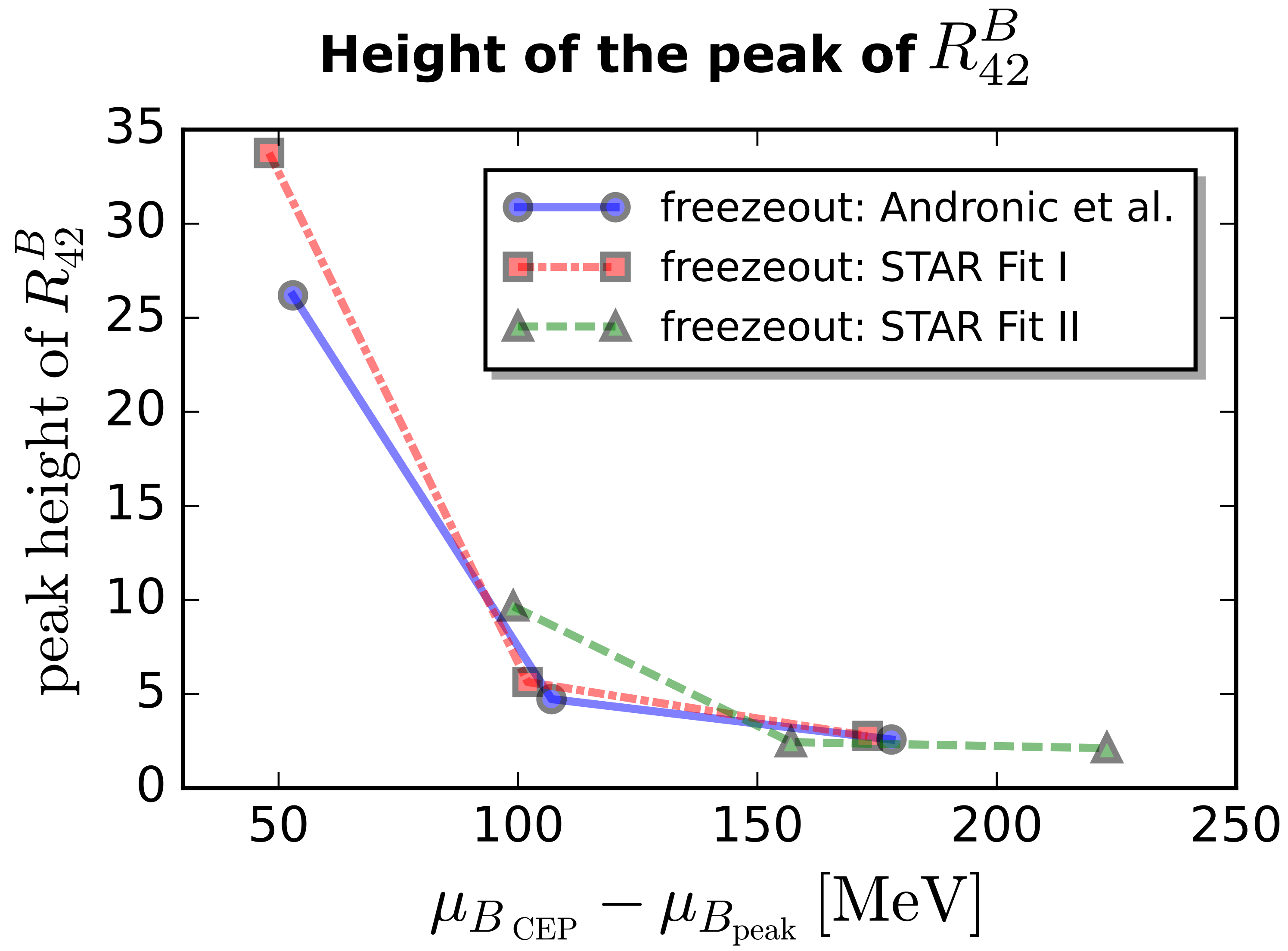


Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Ripples of the critical point

Reconstructing the CEP

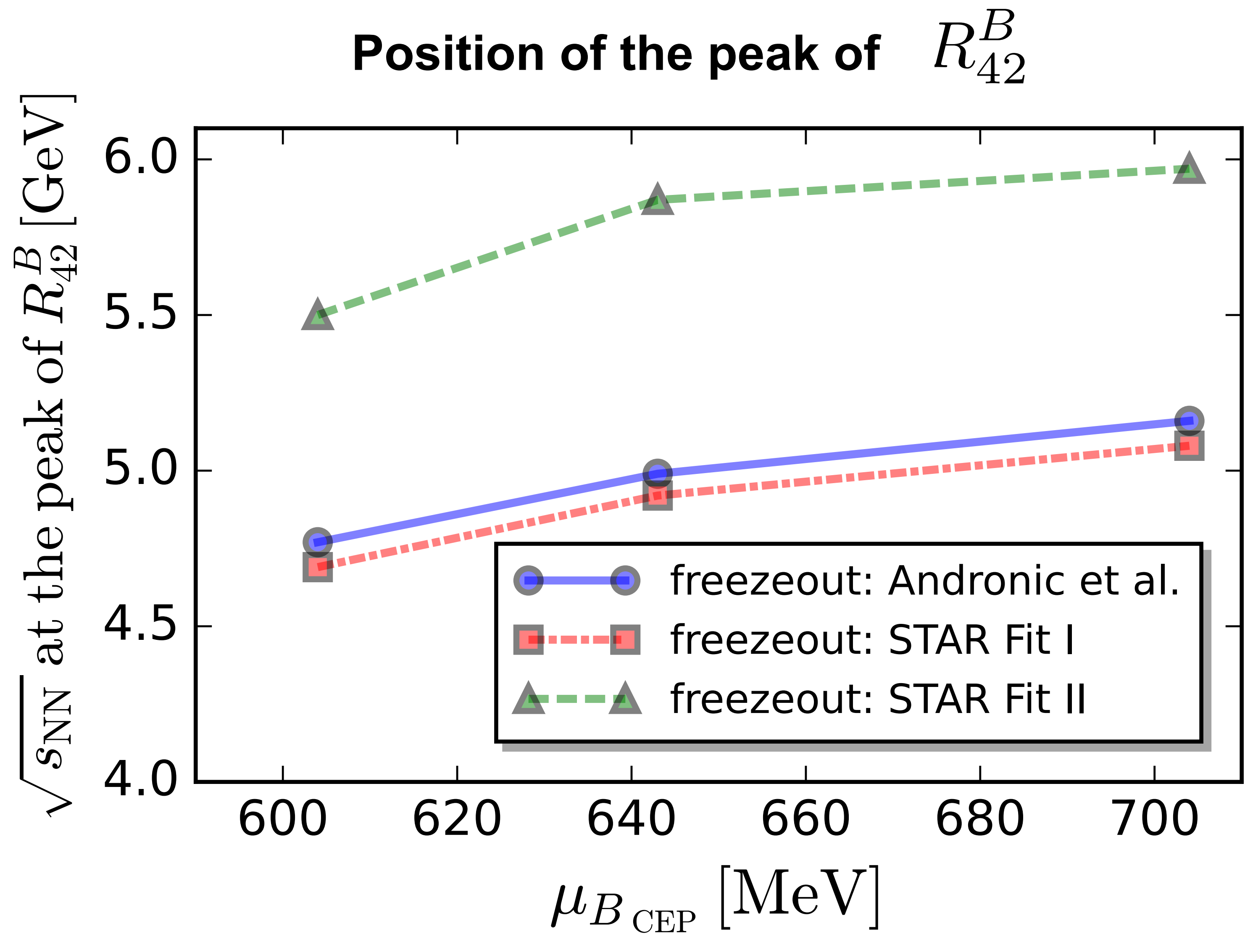


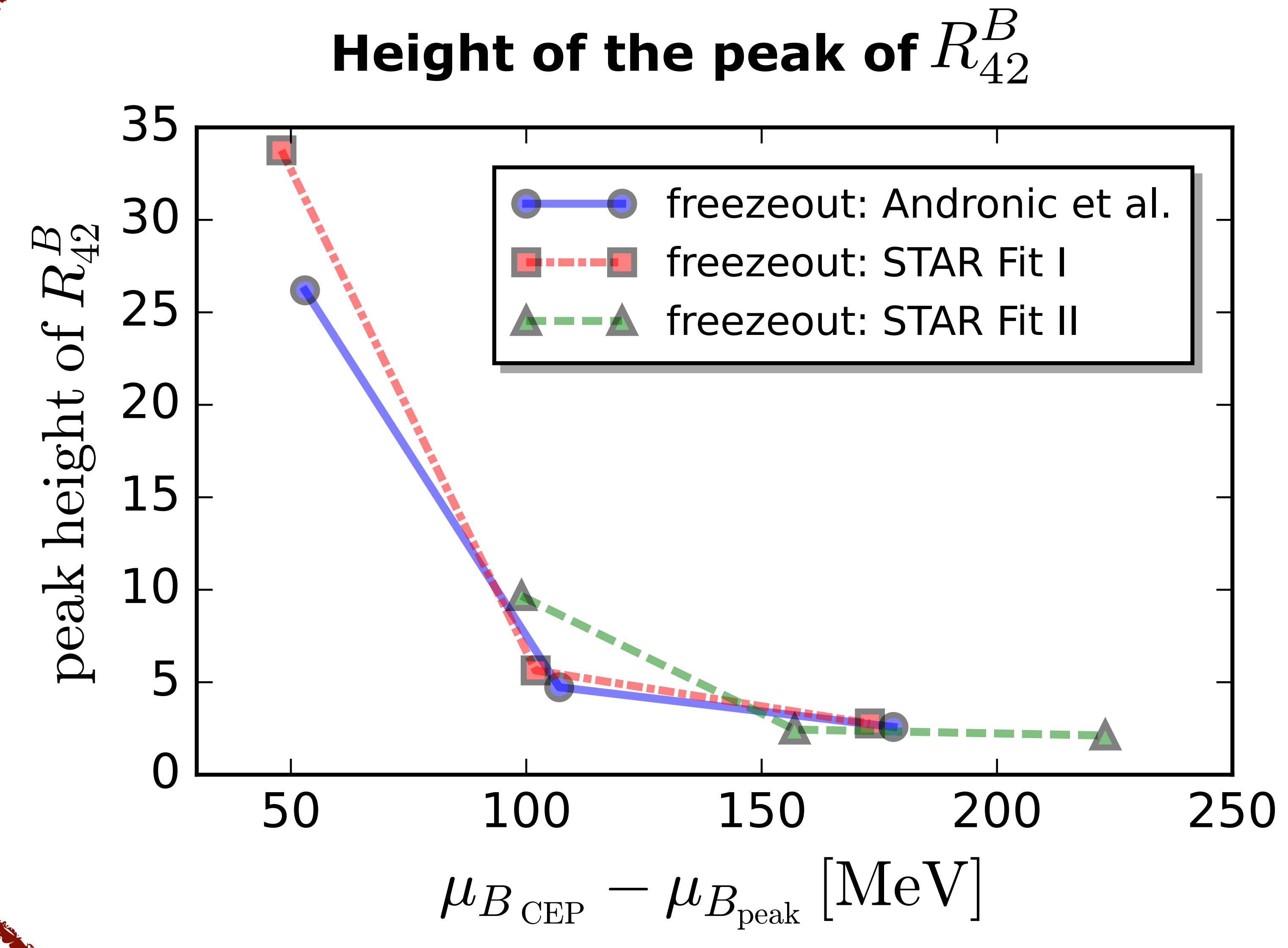


Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Ripples of the critical point

Reconstructing the CEP





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Ripples of the critical point

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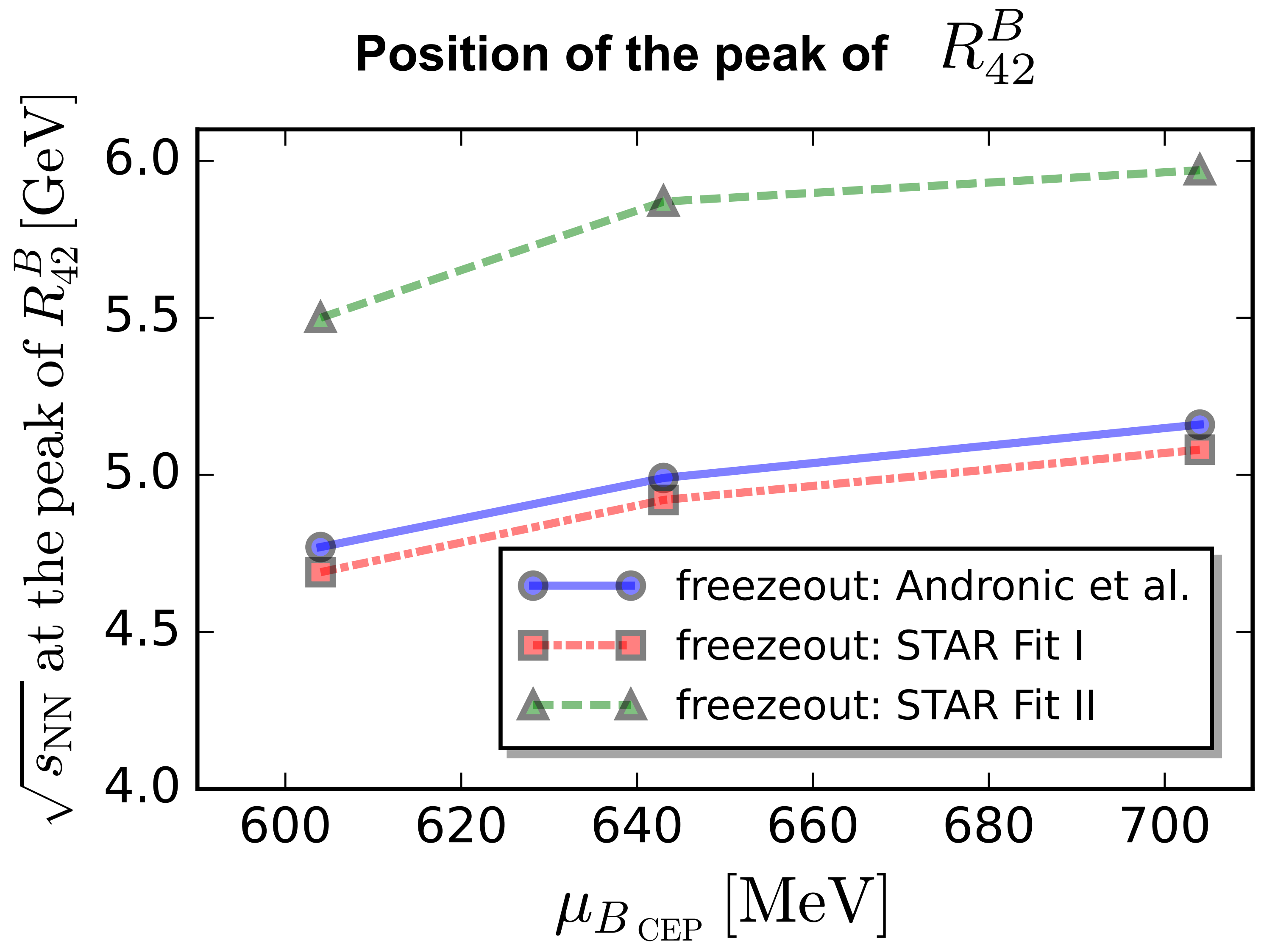

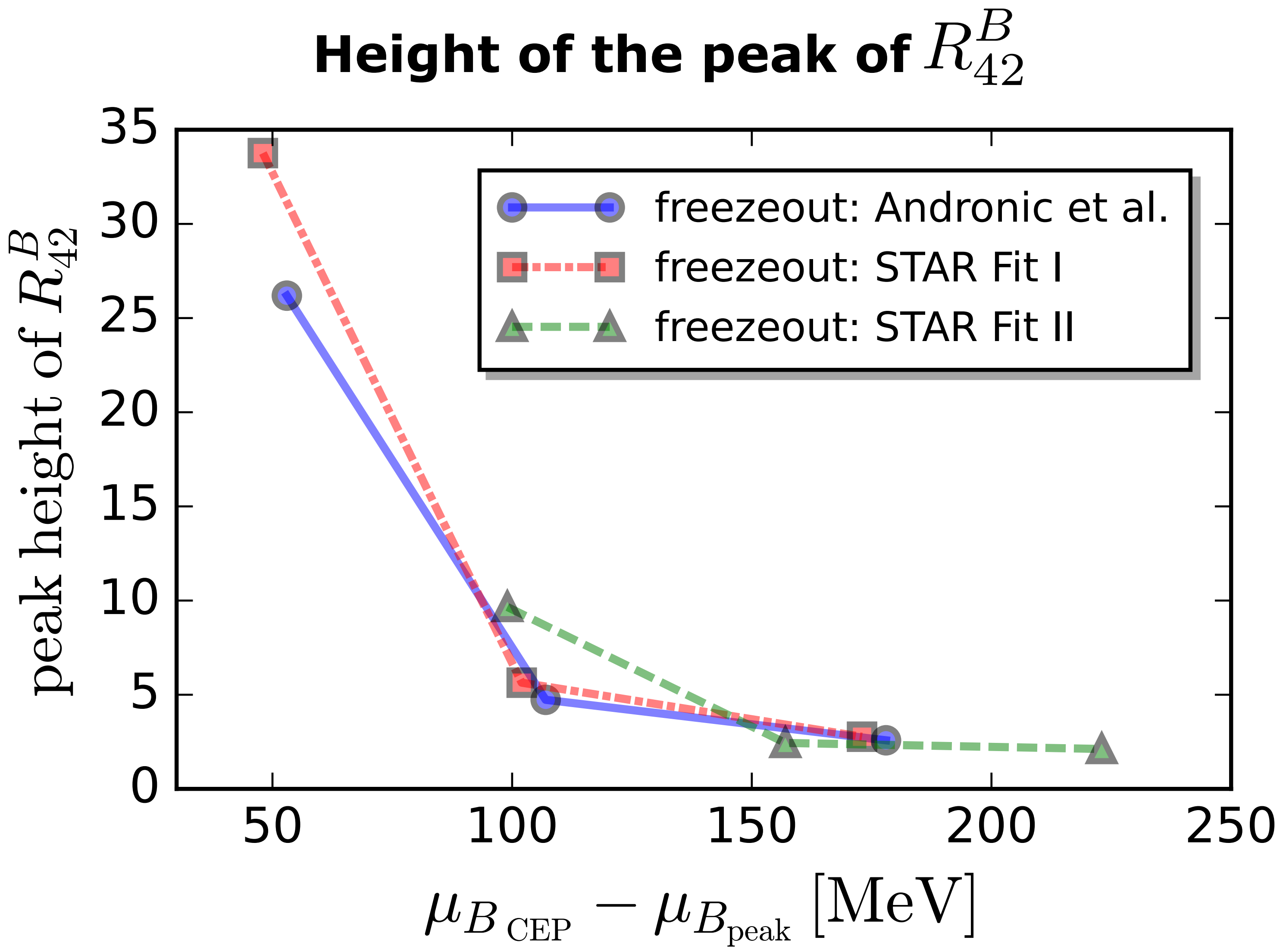

Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fu, Luo, JMP, Rennecke, Wen, Yin, arXiv:2308.15508

Summary

(I) Functional Renormalisation Group for QCD

- The renormalisation group is a one-loop exact functional approach
  - consistent RG-scaling ✓
  - systematic expansion schemes & error control ✓
  - compatibility with other functional approaches ✓
- Resonances via dynamical hadronisation
  - hadrons as exchange fields of quarks scattering vertices
  - BSE wave function ⟷ quark-hadron vertex
  - Stable dynamical emergence of low energy effective theories
- Quark-gluon-meson correlation functions
  - Self-consistent results: all correlation functions computed are fed back
  - Dynamical chiral symmetry breaking & confinement
  - Quantitative agreement with lattice results

(II) Functional QCD and the QCD phase structure

- **QCD at finite temperature and density**
  - all available benchmarks in the vacuum passed
  - confinement-deconfinement phase transition
  - compatibility with other functional approaches

- **Locating the QCD phase structure and the critical end point**
  - Quantitative predictions for $\mu_B/T \lesssim 4$, estimates for $\mu_B/T \lesssim 800\mathrm{MeV}$
  - Estimate for the location of the CEP: $(\mu_B\,,\,T)_{\mathrm{CEP}} \sim (600-650\,,\,105-115)\ \mathrm{MeV}$ 
  - Diquark domination for $\mu_B/T \gtrsim 8$ 

- **Fluctuations of conserved charges: Ripples of the critical end point**
  - QCD-assisted low energy effective theory with the phase structure of QCD
  - Quantitative agreement of the fluctuations of conserved charges with lattice results
  - Qualitative accounting for canonical effects with the sub-ensemble method
  - Remarkable compatibility with the new STAR data (baryon vs proton fluctuations, finite volume effects, …)