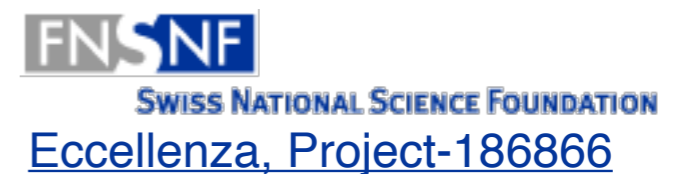


# Topological Portal to the Dark Sector

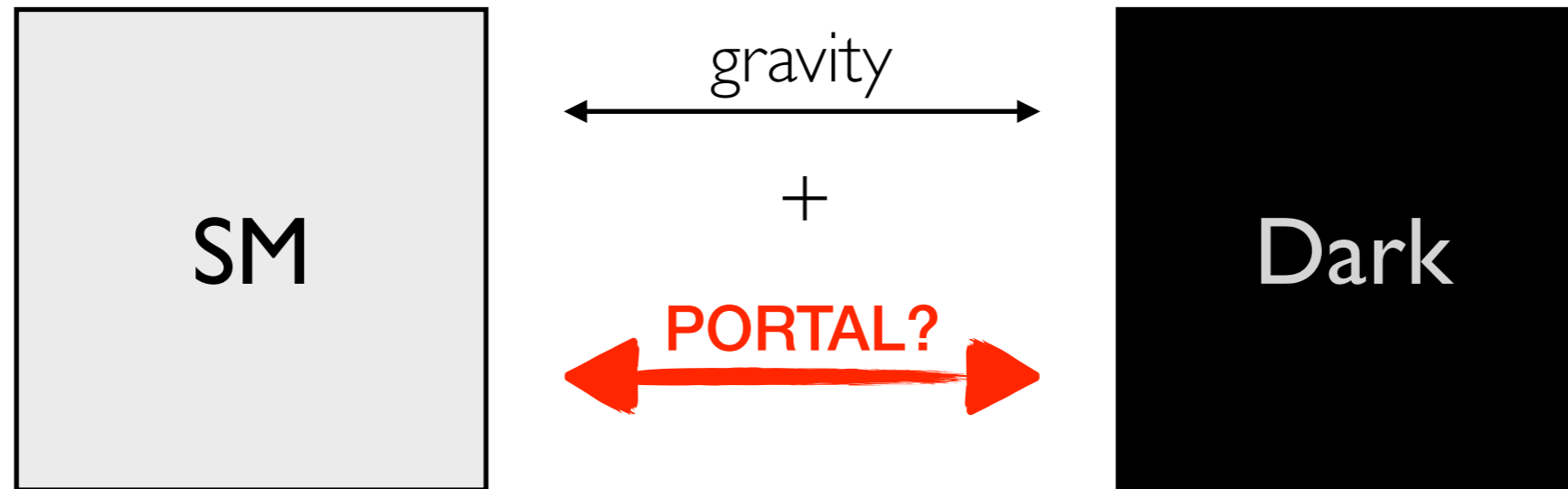
Davighi, AG, Selimovic; [2401.09528](#)

Admir Greljo



16.05.2024, CERN TH BSM Forum

# Motivation



- Dark matter revealed its presence through gravitational interactions.
- Portals to the dark sector are critical to probing its nature.
- They drive:
  - Cosmological history (Relic abundance),
  - Direct / Indirect detection,
  - Collider searches.
- Known portals are scarce.
  - Renormalizable:  $H^\dagger H S^\dagger S$ ,  $B_{\mu\nu} X^{\mu\nu}$ ,  $\bar{L} H N$ .
  - Higher-dimensional:  $aG \wedge G$ , ...

# Today



**hep-th**

A novel portal topological interaction between QCD and Dark Pions!

**hep-ph**

Elegant realization of the light thermal inelastic DM scenario.

**hep-ex**

Novel signatures at Belle-II

# Introduction

- $\chi PT$
- Chiral anomalies
- Dirac quantisation
- The WZW term

# Chiral Lagrangian

- Low-energy limit of QCD
- Chiral symmetry breaking

$$G = SU(3)_L \times SU(3)_R$$

$$m_{u,d,s} < \Lambda \quad \downarrow \quad \langle \bar{q}_L^i q_R^j \rangle \approx \Lambda^3 \delta^{ij}$$

$$\mathcal{H} = SU(3)_{L+R}$$

- Coset space = vacuum manifold

$$\langle \rangle_{ij} \rightarrow (L^\dagger R)_{ij} \quad X = \frac{G}{\mathcal{H}} = SU(3)$$

- Goldstone (Pion) matrix

$$U(x) : M^4 \rightarrow SU(3)$$

$$G : U(x) \rightarrow L^\dagger U(x) R$$

# Chiral Lagrangian

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$$U(x) : M^4 \rightarrow SU(3)$$

$$G : U(x) \rightarrow L^\dagger U(x) R$$

- $\chi PT$  (non-linear effective action)

Weinberg '68, CCWZ '69

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{tr} \left( D_\mu U^\dagger D^\mu U \right) + \mathcal{O}(D_\mu^4)$$

$$U(x) = \exp \left( \frac{2i}{f_\pi} \pi(x) \right)$$

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

- Symmetries

1.  $P_0 : \vec{x} \rightarrow -\vec{x}$

2.  $(-1)^{N_\pi} : U \rightarrow U^\dagger \implies \pi^a \rightarrow -\pi^a$

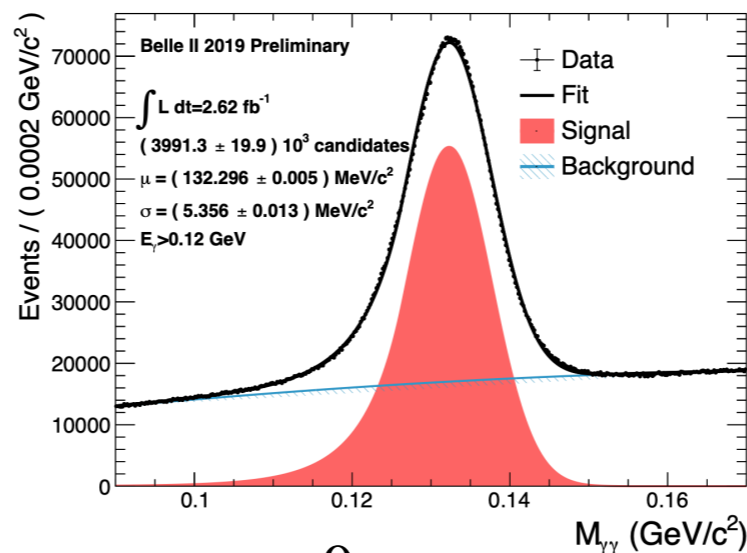
- CCWZ terms are invariant under both.

- QCD preserves only  $P = P_0 (-1)^{N_\pi}$   
e.g.  $K^+ K^- \rightarrow (\phi) \rightarrow \pi^+ \pi^- \pi^0$

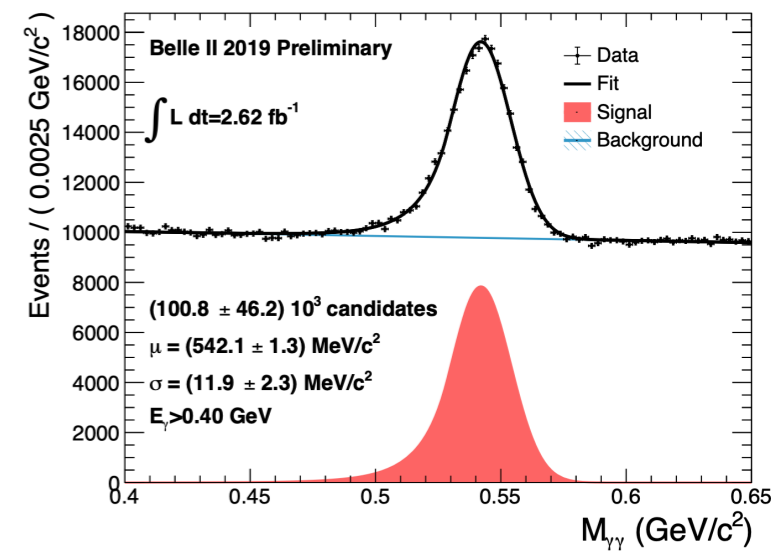
# Chiral anomalies

## Omnipresent

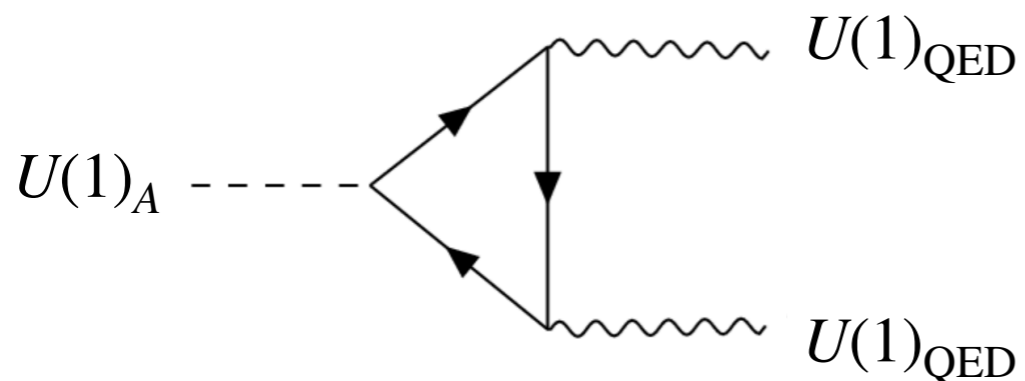
- Violates  $P_0$  and  $(-1)^{N_\pi}$  keeping  $P$



$$\pi^0 \rightarrow \gamma\gamma$$



$$\eta \rightarrow \gamma\gamma$$



Mixed (ABJ) anomaly

Adler; Bell, Jackiw '69

$$\partial^\mu J_{A\mu}^3 = \frac{N_c}{96\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

Anomaly matching ('t Hooft): Deep IR

**Q:** How does  $\mathcal{L} = \frac{N_c e^2}{96\pi^2 f_\pi} \pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$  arise in a  $G$ -invariant EFT?

Wess, Zumino '71

Witten '83

## Invariant action?

- An attempt to construct a  $G$ -invariant Lagrangian:

$$\epsilon^{\mu\nu\rho\sigma} \text{tr} \left( U^\dagger (\partial_\mu U) U^\dagger (\partial_\nu U) U^\dagger (\partial_\rho U) U^\dagger (\partial_\sigma U) \right) = 0$$

$\swarrow$   
 $P_0$ -odd

No  $G$ -invariant term in 4d violates  $P_0$

- But 4d covariant EOM exist: [Witten '83](#)

$$\frac{1}{2} f_\pi^2 \partial_\mu (U^\dagger \partial^\mu U) = \frac{k}{48\pi^2} \epsilon^{\mu\nu\rho\sigma} U^\dagger (\partial_\mu U) U^\dagger (\partial_\nu U) U^\dagger (\partial_\rho U) U^\dagger (\partial_\sigma U)$$

$\swarrow$   
 $P_0$ -odd

- How can we build an invariant action?



# Invariant action?

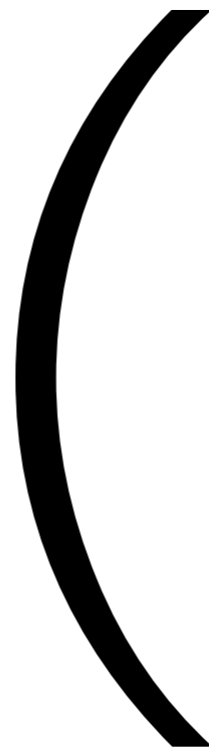


Witten '83: Global aspects of current algebra

- The WZW action:
  - Extend  $U(x)$  to a 5d bulk  $D$  whose boundary is a 4d spacetime
  - Action  $S \sim \int_D \omega_5$  where  $\omega_5 \sim \text{tr}(U^{-1}dU)^5$  is a 5-form

Properties of  $\omega_5$ :

- $G$ -invariant
- Closed:  $d\omega_5 = 0$
- Integral: The path integral phase  $e^{iS}$  independent of  $D$  (Topological)



# Dirac quantisation

Consider a charge  $e$  moving on a unit sphere  $S_2$  around a monopole  $g$

$$\text{EOM: } m \frac{d\mathbf{v}}{dt} = e\mathbf{v} \times \mathbf{B}$$

$$\mathbf{B} = \frac{g\hat{r}}{4\pi r^2} \implies \int d\mathbf{S} \cdot \mathbf{B} = g$$

Symmetries of  $\ddot{x}_i = \epsilon_{ijk} x_j \dot{x}_k$

- $SO(3)$  rotations
- LHS:  $P$  and  $T$
- RHS:  $PT$

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$$\nabla \times \mathbf{A} = \mathbf{B} = \frac{g\hat{r}}{4\pi r^2} \implies \int d\mathbf{S} \cdot \mathbf{B} = g$$

$$A_\phi^N = \frac{g(1 - \cos\theta)}{4\pi r \sin\theta}$$

## Dirac string

Rotational symmetry violated, thus

$$S = \int dt \left( \frac{1}{2} m \mathbf{v}^2 + e\mathbf{v} \cdot \mathbf{A} \right)$$

is not manifestly symmetric.

Symmetries of  $\ddot{x}_i = \epsilon_{ijk} x_j \dot{x}_k$

- $SO(3)$  rotations
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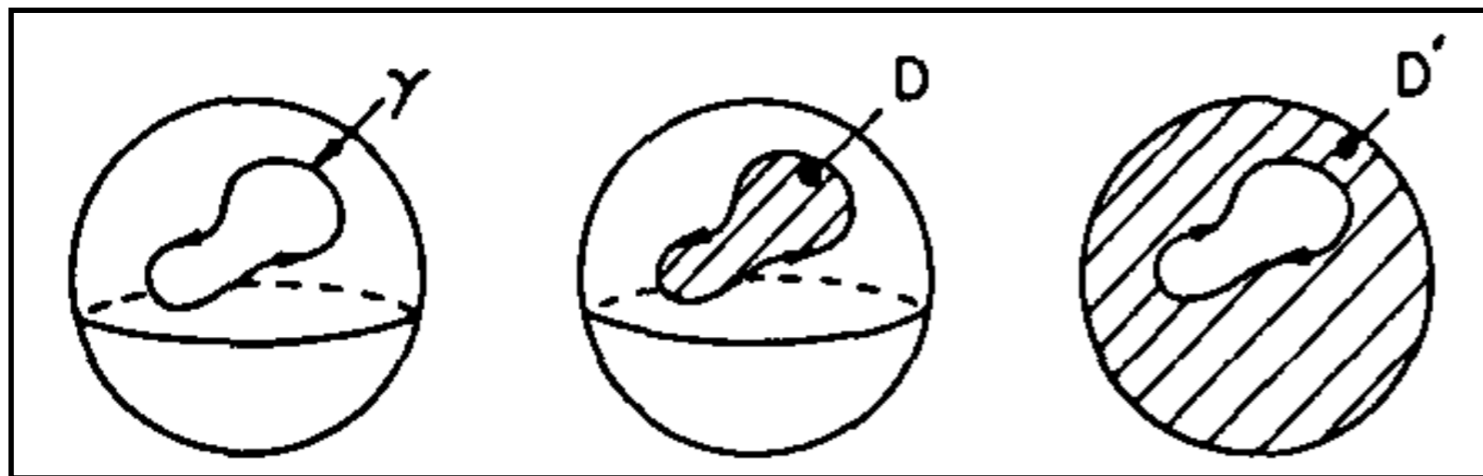
# Dirac quantisation

Consider a charge  $e$  moving on a unit sphere  $S_2$  around a monopole  $g$

In QM, a phase of a closed orbit

$$\alpha = \oint_{\gamma} \mathbf{A} \cdot d\mathbf{x} = \int_D d\mathbf{S} \cdot \mathbf{B}$$

Witten '83



$$\pi_1(S^2) = 0$$

- $SO(3)$ -invariant 2-form  
 $F_{ij} \sim \epsilon_{ijk} x^k / |x|^3$   
 but  $D$  is not unique.

Thus,  $e^{ie\alpha} = e^{ie\alpha'}$ , implies  
 $eg = 2\pi n, \quad n \in \mathbb{Z}$

“integrality condition”

$$\pi_2(S^2) = \mathbb{Z}$$



# The WZW action

- Compactify the spacetime to a large 4d sphere  $S^4$  (fields asymptote at infinity) Witten '83
- The goldstone matrix  $U(x) : S^4 \rightarrow SU(3)$

$$S = \frac{f_\pi^2}{4} \int d^4x \operatorname{tr}(\partial_\mu U^\dagger \partial^\mu U) + \boxed{k \int_D d^5y \omega} \quad \partial D = S^4$$

- $U(y) : S^5 \rightarrow SU(3)$ , unique 5-form:

$$\omega = -\frac{i}{240\pi^2} \epsilon^{\mu\nu\rho\sigma\tau} \operatorname{tr} \left( U^\dagger \frac{\partial U}{\partial y^\mu} U^\dagger \frac{\partial U}{\partial y^\nu} U^\dagger \frac{\partial U}{\partial y^\rho} U^\dagger \frac{\partial U}{\partial y^\sigma} U^\dagger \frac{\partial U}{\partial y^\tau} \right)$$

Tong, Gauge Theory

1.  $G$ -invariant
2. Closed:  $d\omega = 0$

# The WZW action

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Tong, Gauge Theory

- |                          |
|--------------------------|
| 1. $G$ -invariant        |
| 2. Closed: $d\omega = 0$ |

- $\pi_4(SU(3)) = 0, \partial D = S^4$ .

- **Ambiguity** in the choice of  $D$ . Phase independence of  $D$

$$\implies \exp \left( ik \int_D d^5y \omega \right) = \exp \left( \underset{\substack{\downarrow \\ \text{opposite orientation}}}{-ik} \int_{D'} d^5y \omega \right)$$

$$\int_{S^5} d^5y \omega = 2\pi n \quad \text{winding } n \in \mathbf{Z}$$

$$\pi_5(SU(3)) = \mathbf{Z}$$

$$D \cup D' = S^5 \quad \exp \left( ik \int_{S^5} d^5y \omega \right) = 1 \quad 16$$

$$k \in \mathbf{Z}$$

3. Integral



# The WZW action

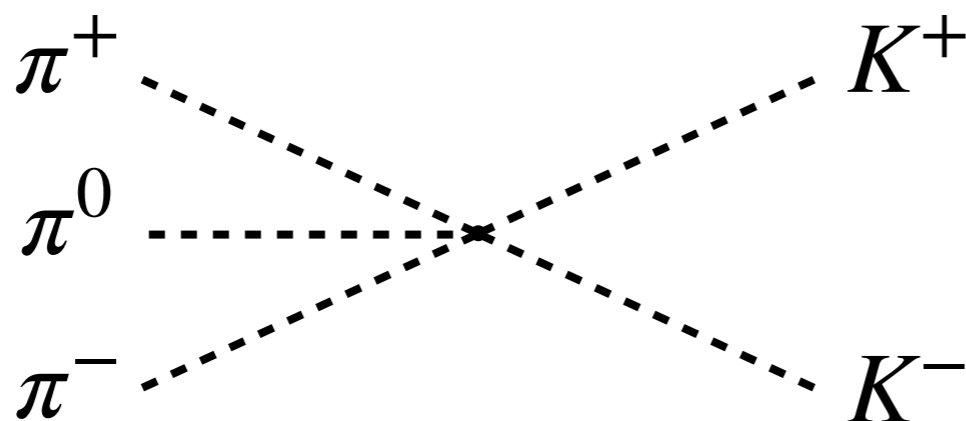
Witten '83

- Expand  $U^\dagger \partial_\mu U = \frac{2i}{f_\pi} \partial_\mu \pi + \mathcal{O}(\pi^2)$

$$\int_D d^5 y \omega = \frac{2}{15\pi^2 f_\pi^5} \int_D d^5 y \epsilon^{\mu\nu\rho\sigma\tau} \partial_\mu \text{tr} \left( \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \partial_\tau \pi \right) + \mathcal{O}(\pi^6)$$

$$= \frac{2}{15\pi^2 f_\pi^5} \int_{\mathbf{S}^4} d^4 x \epsilon^{\nu\rho\sigma\tau} \text{tr} \left( \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \partial_\tau \pi \right) + \mathcal{O}(\pi^6)$$

- Use Stoke's theorem  $\implies$



- Violates  $(-1)^{N_\pi}$
- Phenomenology dominated by gauging, next slide

## Gauging the WZW

Witten '83

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

- Gauge  $U(1)_{\text{QED}} \supset SU(3)_{L+R}$
- $U \rightarrow U + i\epsilon[Q, U]$

- Gauging must take place in 4d where no manifest  $G$ -invariance
- Non-trivial derivation using Noether method ( $\delta S \sim \int \partial_\mu \epsilon J^\mu$ ).

# Gauging the WZW

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- 
- After trial and error  $J^\mu = \frac{1}{48\pi^2} \epsilon^{\mu\rho\sigma\tau} \text{tr} \left( \{Q, U^\dagger\} \partial_\rho U U^\dagger \partial_\sigma U U^\dagger \partial_\lambda U \right)$

$$S_{WZW} = k \left[ \int_D d^5x \omega - e \int d^4x A_\mu(x) J^\mu + \frac{ie^2}{24\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} (\partial_\mu A_\nu) A_\rho \text{tr} \left( \{Q^2, U^\dagger\} \partial_\sigma U + U^\dagger Q U Q U^\dagger \partial_\sigma U \right) \right]$$

# Gauging the WZW

Witten '83

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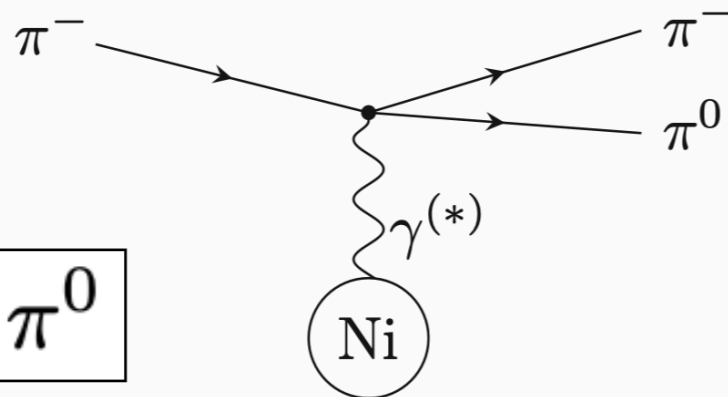
$$\mathcal{L} = \frac{ke^2}{96\pi^2 f_\pi} \pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

$$k = N_c$$

# Gauging the WZW

Witten '83

- COMPAS @ CERN 2310.09138



$$\pi^- \gamma \rightarrow \pi^- \pi^0$$

$$F_{3\pi}^{\text{theory}} = \frac{eN_c}{12\pi^2 F_\pi^3} = (9.78 \pm 0.04) \text{ GeV}^{-3}$$

$$F_{3\pi}^{\text{preliminary}} = (10.3 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}}) \text{ GeV}^{-3}$$

- VAAA anomaly:  $\gamma \pi^+ \pi^- \pi^0$

$$S_{WZW} = k \left[ \int_D d^5x \omega - e \int d^4x A_\mu(x) \frac{1}{48\pi^2} \epsilon^{\mu\rho\sigma\tau} \text{tr} \left( \{Q, U^\dagger\} \partial_\rho U U^\dagger \partial_\sigma U U^\dagger \partial_\lambda U \right) + \frac{ie^2}{24\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} (\partial_\mu A_\nu) A_\rho \text{tr} \left( \{Q^2, U^\dagger\} \partial_\sigma U + U^\dagger Q U Q U^\dagger \partial_\sigma U \right) \right]$$

$$\mathcal{L} = \frac{ke^2}{96\pi^2 f_\pi} \pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

$$k = N_c$$

## Invariant forms?

$$\begin{array}{c}
 G = SU(3)_L \times SU(3)_R \\
 \\
 X = \frac{G}{\mathcal{H}} = SU(3) \quad \downarrow \\
 \\
 \mathcal{H} = SU(3)_{L+R}
 \end{array}$$

The only two  $G$ -invariant forms on  $X$

$$\omega_5 \sim \text{tr}(U^{-1}dU)^5$$

WZW

$$\omega_3 \sim \text{tr}(U^{-1}dU)^3$$

What can we do with this?

# The 3-form

$$\omega_3 \sim \text{tr}(U^{-1}dU)^3$$

- It does not appear in the QCD action.
- However, it does appear in the topologically conserved current, **the baryon number!** Goldstone, Wilczek '81

Consider a static field configuration:  $U(\mathbf{x}) : \mathbf{S}^3 \mapsto SU(N_f)$

$$B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr} (U^\dagger (\partial_\nu U) U^\dagger (\partial_\rho U) U^\dagger \partial_\sigma U)$$

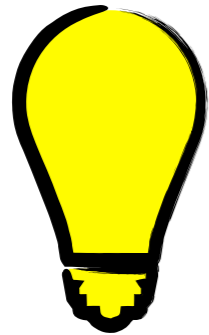
$$\partial_\mu B^\mu = 0 \quad B = \int d^3x B^0 \quad \text{Conserved winding no: } B \in \mathbf{Z}$$

(antisymmetry)  $\pi_3(SU(3)) = \mathbf{Z}$

The global symmetry of QCD:  $G \times U(1)_V$

The only candidate symmetry is  $U(1)_V$ !

What else can we do with this?



$$\omega_5^{\text{PORTAL}} = \omega_3^{\text{QCD}} \wedge \omega_2^{\text{DARK}}$$

Davighi, AG, Selimovic; [2401.09528](#)

## The topological portal

- Formulation
- Relic abundance
- Direct/Indirect detection
- Collider searches




## The setup

- We formulate a low-energy EFT for pions and dark pions

$$X = \frac{SU(3)_L \times SU(3)_R \times K}{SU(3)_{L+R} \times H} \cong SU(3) \times \frac{K}{H}$$

Dark sector coset



## The setup

- We formulate a low-energy EFT for pions and dark pions

$$X = \frac{SU(3)_L \times SU(3)_R \times K}{SU(3)_{L+R} \times H} \cong SU(3) \times \frac{K}{H}$$

Dark sector coset

- Properties of  $\omega_5^{\text{PORTAL}} = \omega_3^{\text{QCD}} \wedge \omega_2^{\text{DARK}}$ 
  - Closed:**  $d\omega_5^{\text{PORTAL}} = 0$  implies  $d\omega_2^{\text{DARK}} = 0$  since  $d\omega_3^{\text{QCD}} = 0$
  - G-invariance:** Product structure implies  $\omega_2^{\text{DARK}}$  is  $K$ -invariant
  - Integrality:** Cycles factorize; normalize  $\omega_2^{\text{QCD}}$  and  $\omega_2^{\text{DARK}}$  separately

Which dark coset fits?

# Cosetology

Searching for a closed, invariant, and integral  $K/H$  2-form

- Consider following cosets (motivated by QCD-like theories):

$$\frac{K}{H} = \left\{ \frac{SU(N)_L \times SU(N)_R}{SU(N)_{L+R}}, \frac{SU(N)}{SO(N)}, \frac{SU(2N)}{Sp(2N)} \right\}$$

- All these are *symmetric spaces*  $\implies$

$K$ -invariant forms on  $K/H$  are in 1-to-1 with cohomology classes

see e.g. Davighi, Gripaos, Randal-Williams, 2011.05768

The portal  $\exists$  iff  $H^2(K/H) \neq 0$

de Rham cohomology  $H^k(M)$  - the set of closed modulo exact  $k$ -forms on  $M$

# Cosetology

- Unique choice of the dark coset!

Davighi, AG, Selimovic; [2401.09528](#)

$p$	1	Portal 2	3	4	SIMP 5
$H^p(SU(2))$	0	0	$\mathbb{R}$	—	—
$H^p(SU(n)), n \geq 3$	0	0	$\mathbb{R}$	0	$\mathbb{R}$
$H^p(SU(2)/SO(2))$	0	$\mathbb{R}$	—	—	—
$H^p(SU(3)/SO(3))$	0	0	0	0	$\mathbb{R}$
$H^p(SU(4)/SO(4))$	0	0	0	$\mathbb{R}$	$\mathbb{R}$
$H^p(SU(n)/SO(n)), n \geq 5$	0	0	0	0	$\mathbb{R}$
$H^p(SU(2n)/Sp(2n)), n \geq 2$	0	0	0	0	$\mathbb{R}$

QCD

The SIMP mechanism ( $3 \rightarrow 2$ )

[Hochberg et al, 1402.5143, 1411.3727](#)

(mutually exclusive)

## The portal

- Take  $K/H = SU(2)/SO(2) \cong S^2$       Two dark pions

$$\Omega_2 = \epsilon_{ij} d\chi_i \wedge d\chi_j / (8\pi f_D^2)$$

- the volume form on  $S^2$   
 $\chi_1/f_D \in [0, \pi], \chi_2/f_D \in [0, 2\pi)$

- The portal

$$\omega_{\text{portal}} = \frac{n}{24\pi^2} \text{Tr} (U^{-1} dU)^3 \wedge \Omega_2, \quad n \in \mathbb{Z}$$

## The portal

- Take  $K/H = SU(2)/SO(2) \cong S^2$  Two dark pions

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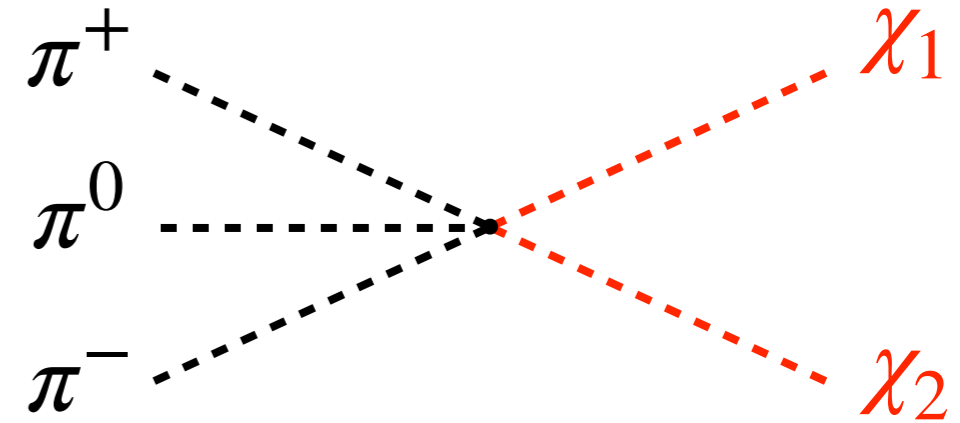
- Expanding  $U^{-1} dU \implies \omega_{\text{portal}} = \frac{n}{96\pi^3 f_\pi^3 f_D^2} f_{abc} \epsilon_{ij} d\pi_a d\pi_b d\pi_c d\chi_i d\chi_j$

- Use Stoke's theorem

$$\mathcal{L}_{\text{portal}}^{e=0} = \frac{in\epsilon^{\mu\nu\rho\sigma}}{48\pi^2 f_\pi^3 f_D^2} f_{abc} \epsilon_{ij} \pi_a \partial_\mu \pi_b \partial_\nu \pi_c \partial_\rho \chi_i \partial_\sigma \chi_j$$

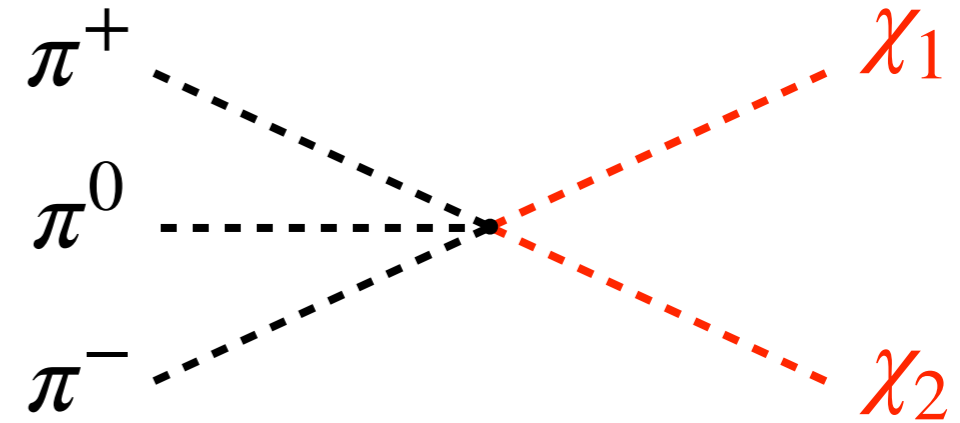
# The portal

$$\mathcal{L}_{\text{portal}}^{e=0} = \frac{i n \epsilon^{\mu\nu\rho\sigma}}{48\pi^2 f_\pi^3 f_D^2} f_{abc} \epsilon_{ij} \pi_a \partial_\mu \pi_b \partial_\nu \pi_c \partial_\rho \chi_i \partial_\sigma \chi_j$$



# The portal

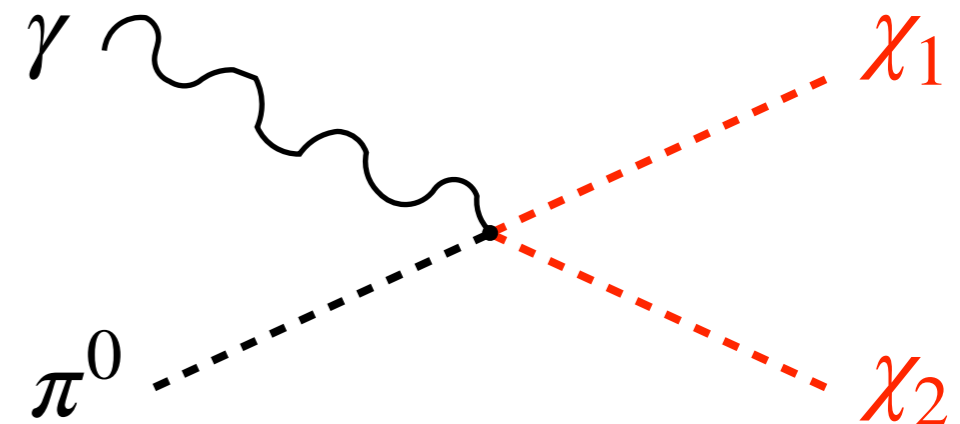
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- As before, QED gauging drives the phenomenology

$$U(1)_Q \subset SU(3)_{L+R}$$

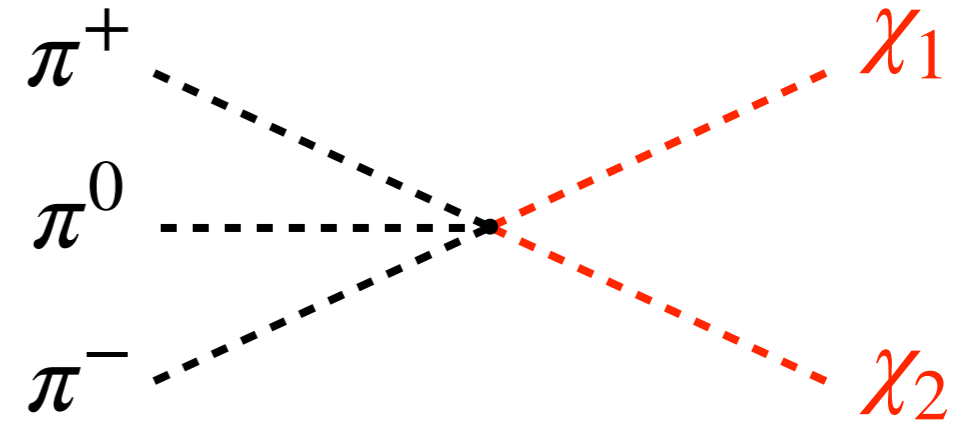
$$\frac{1}{f_\pi^2} \partial_\mu \pi^+ \partial^\mu \pi^- \rightsquigarrow e F^{\mu\nu}$$





# The portal

$$\mathcal{L}_{\text{portal}}^{e=0} = \frac{in\epsilon^{\mu\nu\rho\sigma}}{48\pi^2 f_\pi^3 f_D^2} f_{abc} \epsilon_{ij} \pi_a \partial_\mu \pi_b \partial_\nu \pi_c \partial_\rho \chi_i \partial_\sigma \chi_j$$



- As before, QED gauging drives the phenomenology

$$U(1)_Q \subset SU(3)_{L+R}$$

$$\frac{1}{f_\pi^2} \partial_\mu \pi^+ \partial^\mu \pi^- \rightsquigarrow e F^{\mu\nu}$$

- Prescription

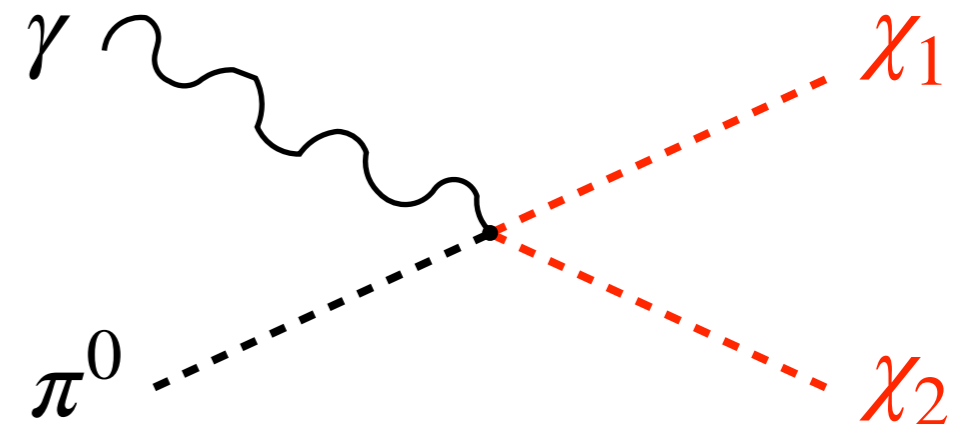
[Yonekura, 2009.04692](#)

Shift  $\omega_3^{\text{QCD}}$  by

$$\frac{-e}{4\pi^2} F \wedge \text{Tr} (Qg^{-1} dg)$$



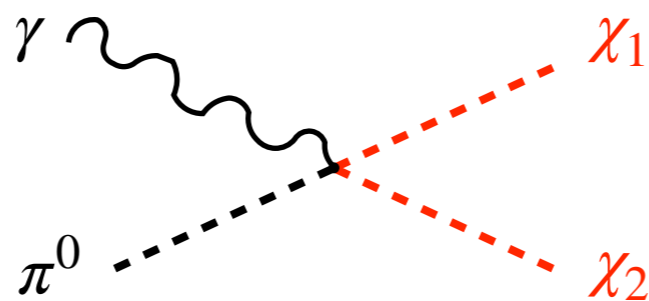
$$\mathcal{L} = \frac{ne \epsilon^{\mu\nu\rho\sigma}}{16\pi^2 f_\pi f_D^2} \left( \pi^0 + \frac{\eta}{\sqrt{3}} \right) F_{\mu\nu} \partial_\rho \chi_1 \partial_\sigma \chi_2$$



The leading portal in the EFT power counting. The next term

$$\frac{1}{f_\pi^2 f_D^2} (D_\mu \pi_a D^\mu \pi^a) (\partial_\nu \chi_i \partial^\nu \chi^i)$$

# DM relic abundance



Davighi, AG, Selimovic; [2401.09528](#)

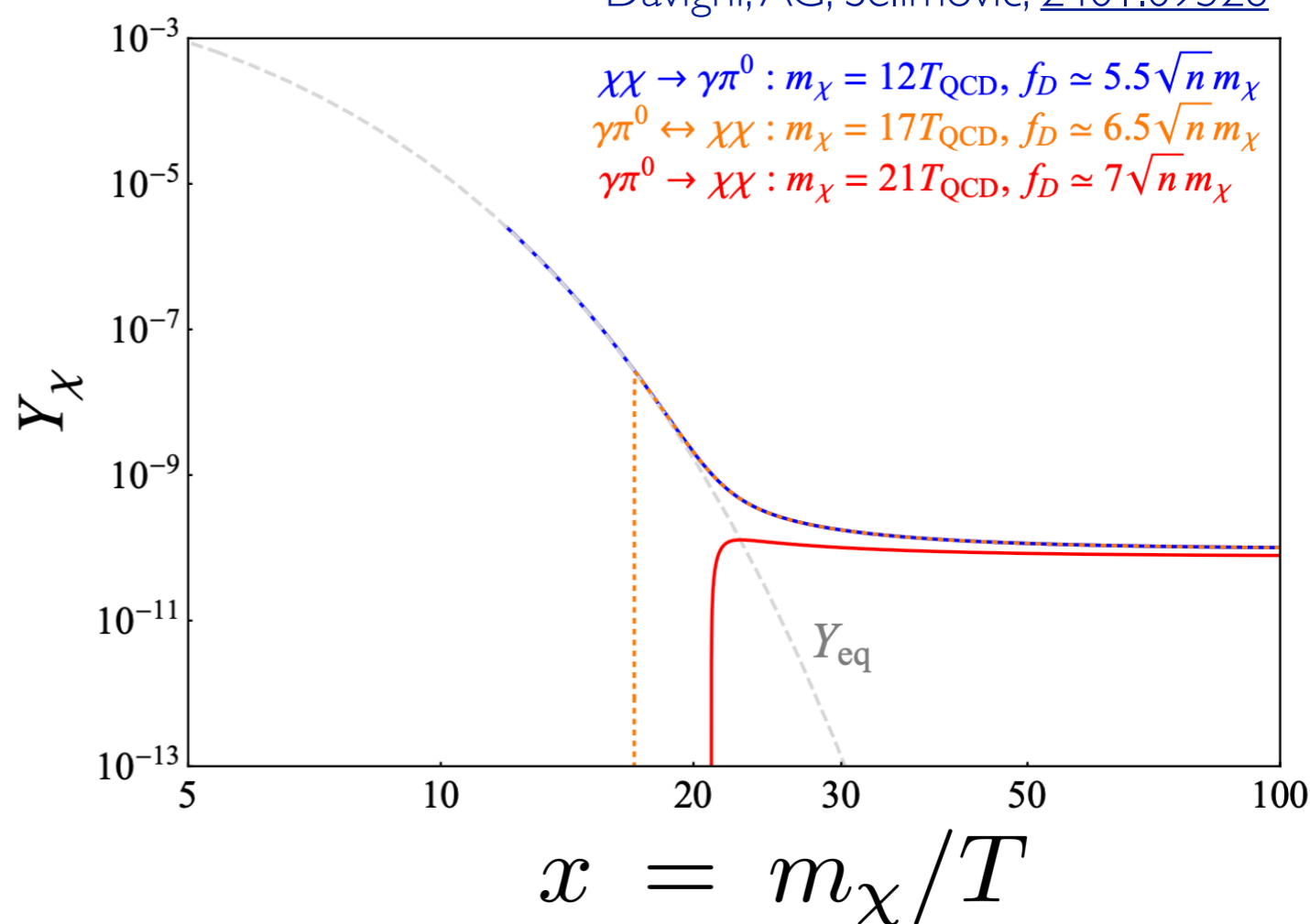
- Solving Boltzmann eqs. numerically

$$m_{\chi_1} = m_{\chi_2} \quad \chi_1 + \chi_2 \text{ yield}$$

$$\frac{dY_\chi}{dx} = -\sqrt{\frac{\pi g_*}{45}} \frac{M_{\text{P}} m_\chi}{x^2} \langle \sigma v \rangle (Y_\chi^2 - Y_{\text{eq}}^2)$$

$$\langle \sigma v \rangle = \frac{\int_{4m_\chi^2}^{\infty} \sigma \sqrt{s} (s - 4m_\chi^2) K_1(\sqrt{s}/T) ds}{8m_\chi^4 T K_2^2(m_\chi/T)}$$

$$\sigma = \frac{n^2}{1536 \pi^4} \frac{\alpha_Q}{f_\pi^2 f_D^4} s^{3/2} \sqrt{s - 4m_\chi^2}$$



- Quick thermalization — all we need is the theory at the freeze-out!
- Our EFT applies when the freeze-out occurs after the QCD phase transition

## DM relic abundance

- Consistent EFT description:  $m_\chi \lesssim 23T_{\text{QCD}} \approx 3.7 \text{ GeV}$
- Relic abundance fits for  $f_D \sim \mathcal{O}(5.5 - 7) \times \sqrt{n} m_\chi$   
 $\implies \chi_{1,2}$  indeed the lightest dark states  
 Dark number conservation ( $\mathbb{Z}_2$  symmetry)  
 ensures the stability of the lightest dark pion.

- 
- A small explicit breaking of  $\mathbf{K}$  needed to give  $m_\chi \neq 0$ , as in QCD
  - For a small mass splitting — **co-annihilations**

$$\Delta := \frac{\Delta m_\chi}{m_{\chi_1}} \implies f_D(\Delta) \approx f_D(0) e^{-\frac{x_{\text{max}} \Delta}{4}} \quad (\text{Boltzmann suppression})$$

# Direct & Indirect DM detection

- $\chi_2 \rightarrow \chi_1 \gamma \pi^0$  before the onset of BBN when  $\Delta m_\chi > m_\pi$
- Dark matter is composed only of  $\chi_1$ 
  - **Indirect detection:**
    - 2-form antisymmetrization:  $\chi_1 \wedge \chi_2$
    - $\chi_1 \chi_1$  annihilations highly suppressed
    - Evades limits from CMB anisotropies on elastic s-wave scattering, [Planck, 1807.06209](#)
  - **Direct detection:**
    - $\chi_1 \rightarrow \chi_2$  inelastic up-scattering kinematically forbidden
    - $\chi_1 \rightarrow \chi_1$  highly suppressed

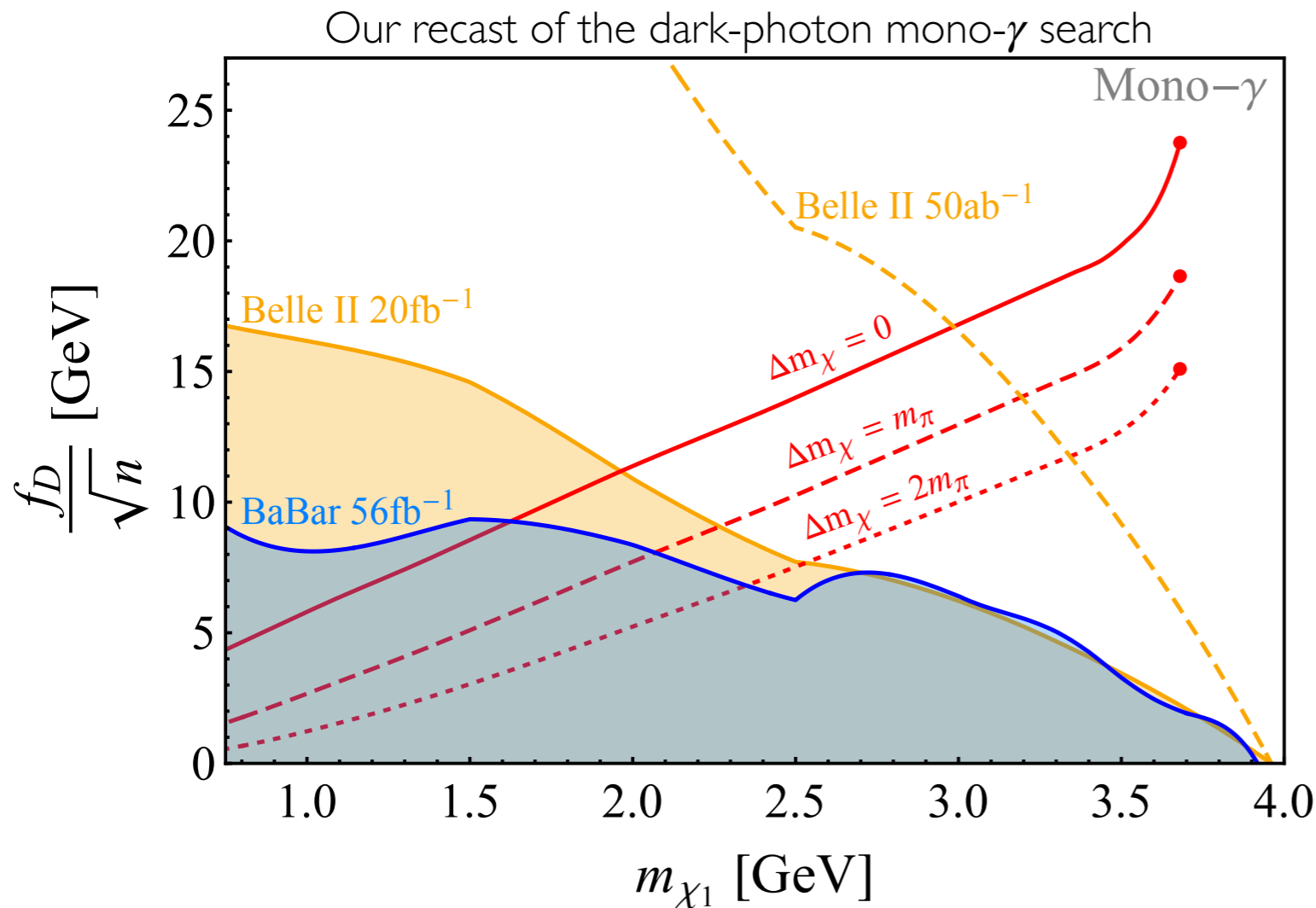
The **antisymmetrization** inherent to forms provides an elegant and natural realization of the *light thermal inelastic DM* scenario!

# Collider searches

- Production  $e^+e^- \rightarrow \gamma^* \rightarrow \chi_1\chi_2\pi^0$

- Signatures

$\Delta m_\chi$	$\lesssim 1.7m_{\pi^0}$	$\gtrsim 1.7m_{\pi^0}$
Signature	$\pi^0 + \cancel{E}_T$	$\pi^0 + \cancel{E}_T + \text{DV}(\pi^0\gamma\cancel{E}_T)$



- Novel signatures yet to be explored by the experiment
- New analysis is needed, in particular for the displaced vertex
- Excellent prospects at Belle-II
- If the signal is found, look for  $\eta$

## Outlook

- A well-rounded EFT tale

$$\omega_5^{\text{PORTAL}} = \omega_3^{\text{QCD}} \wedge \omega_2^{\text{DARK}}$$

hep-th

*Light thermal  
inelastic DM*

hep-ph

*Belle-II*

hep-ex

- The open question:

UV completion?

Alhambra of Granada



***Thank you***



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