

# RF parameters and longitudinal beam dynamics considerations

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**the European Union**

# Overview

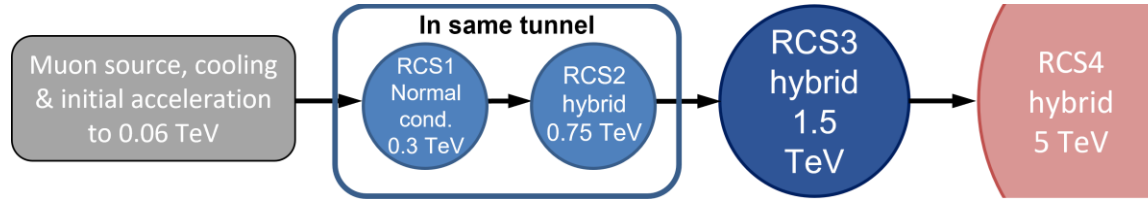
- **Introduction**
- **Trading bending field ramp with RF voltage**
- **RF and longitudinal beam parameters during the acceleration cycle**
- **Baseline RCS parameter assumptions**
- **Summary and outlook**

# Overview

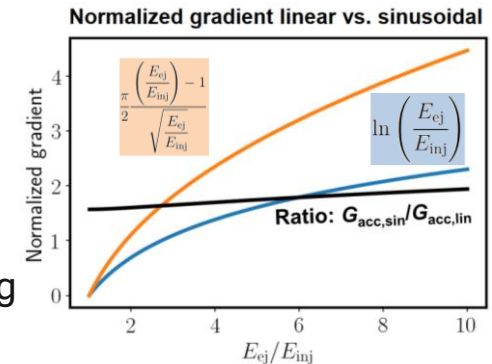
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# The Rapid Cycling Synchrotrons (RCSs)

- Working hypothesis: 60 GeV → 310 GeV → 750 GeV → 1.5 TeV → 5 TeV

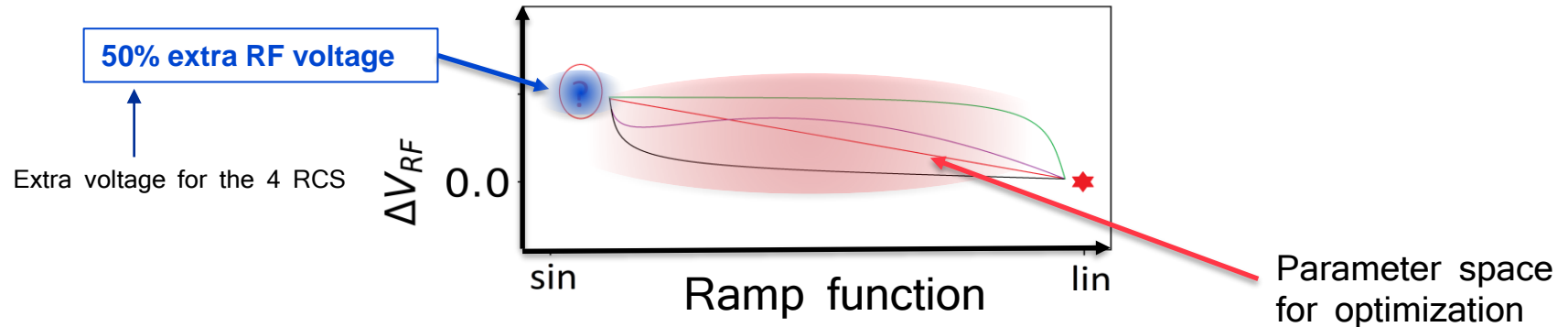


- Optimization problem for ramp function between **sinusoidal optimum for magnet powering** and **linear optimum for RF**
- Linear ramp** → constant  $V_{RF}$ , ideal RF solution  
→ unfavorable for magnet powering
- Non-linear ramp** → reduced peak power **and** magnet powering costs, but required more RF voltage



# The optimization problem

- **Goal:** cost-optimized compromise with quasi-linear magnet and RF ramping



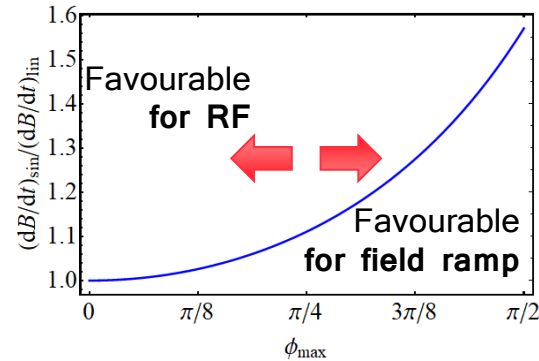
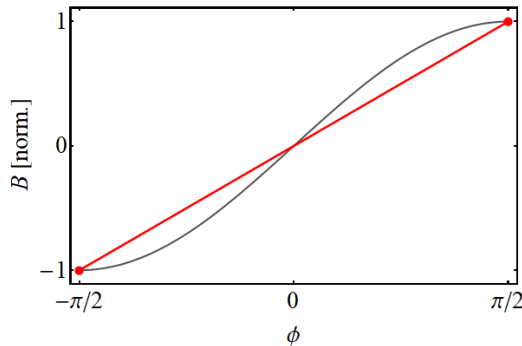
- The required extra voltage is the sum
  - Gradient reduction due to nonlinear ramping
  - Change in the synchronous phase

Mini-workshops on RCS ramp shape

- 20/09/23: <https://indico.cern.ch/event/1309172/>
- 17/01/24: <https://indico.cern.ch/event/1356899/>

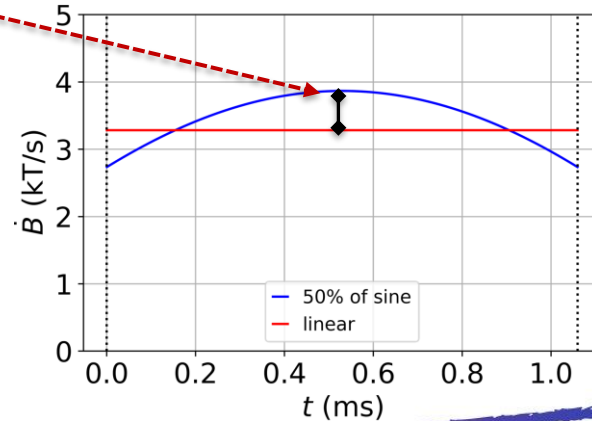
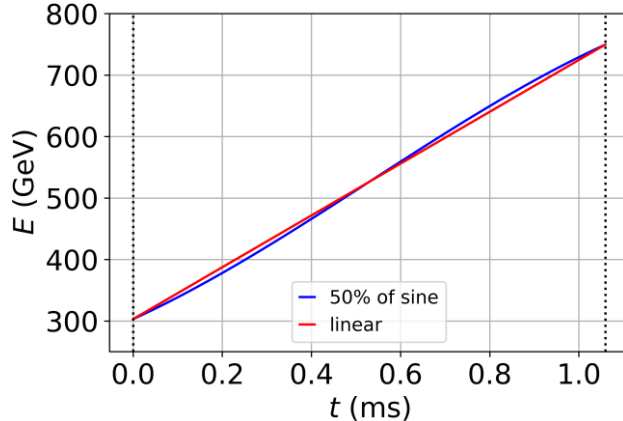
# Reminders

- Initial focus on RCS2 as test case → highest ramp rate  $\frac{dB}{dt}$  of the hybrid RCS
- Ramp shape assumption: fraction of sinusoidal function



# Reminders

- Initial focus on RCS2 as test case → highest ramp rate  $\dot{B}$  of the hybrid RCS
- Ramp shape assumption: fraction of sinusoidal
- Example of central half of sine → almost 20% extra RF voltage required
- That maxima defines the extra costs as the magnet ramp shape defines the voltage shape / RF requirements!



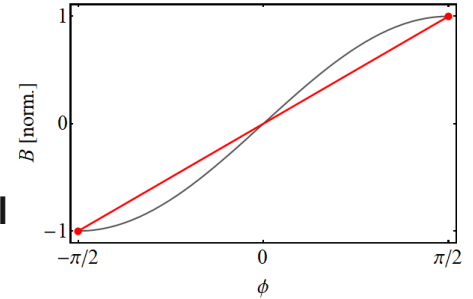


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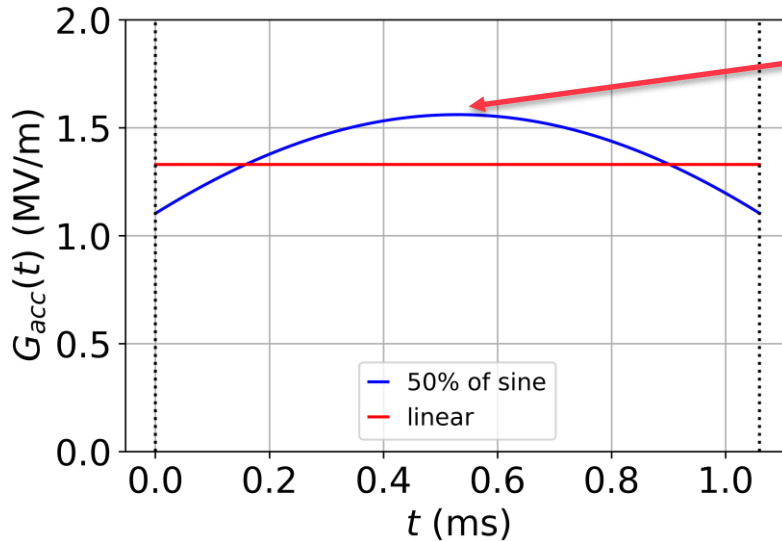
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# Accelerating gradient



- $G_{acc}(t)$  and  $V_{acc}(t)$  follow the shape of  $B(t)$  and define the additional gradient / voltage / number of cavities to achieve the same  $\tau_{acc}$



Maxima that defines extra RF costs!

Average gradient for linear ramp and 90% survival

$$G_{harm}(t) = \frac{(\gamma_{ej} - \gamma_{inj})}{2} \cdot \frac{m_{\mu}}{c} \left( \frac{\dot{B}_{harm}(t)}{B_{ej}} \right)$$

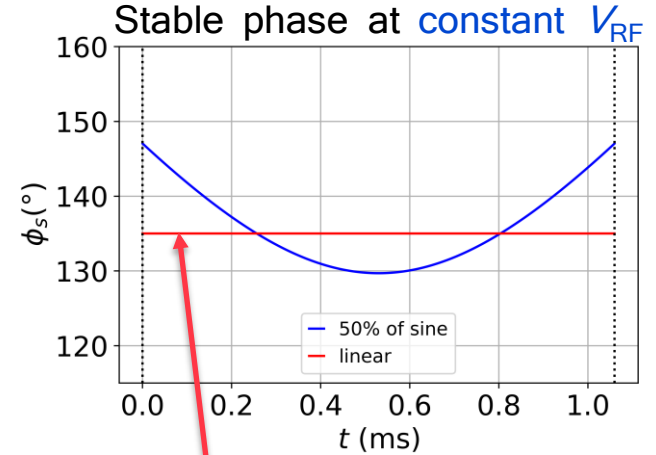
# Synchronous phase

- Additional voltage requirement can be partly compensated by a larger synchronous phase
- Results in reduction in bucket area
- Adjust the energy gain by changing the synchronous phase during acceleration

$$\Delta E(t) = e V_{RF} \cdot \sin[\phi_s(t)] \propto dB/dt$$

and 
$$\phi_s(t) = \arcsin \left( \frac{\dot{B}_{harm}(t)}{\dot{B}_{lin}(t)} \cdot \sin \phi_{s,0} \right)$$

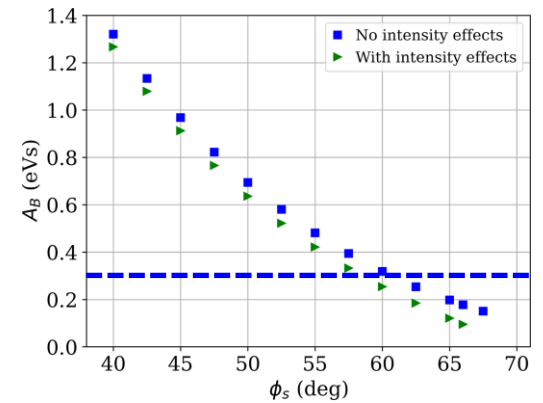
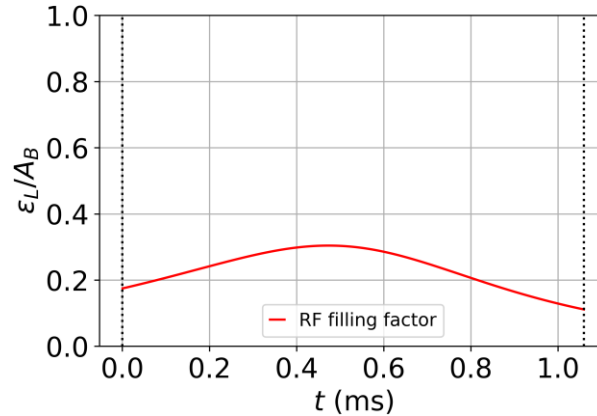
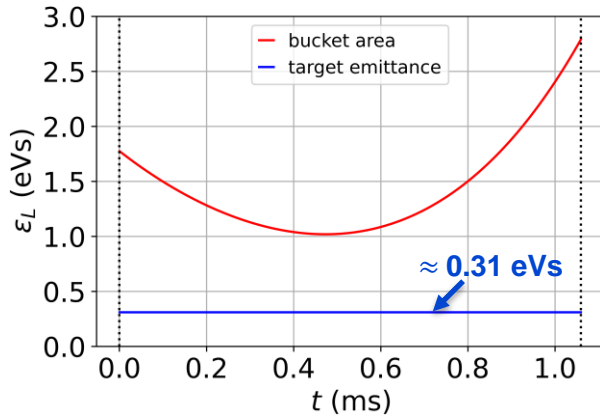
- Time scale of RF voltage changes is given by cavity filling time  $2Q_L/\omega = 0.5$  ms



$\phi_{s,0} = \pi - 45^\circ$  for linear, above transition

# Bucket area and RF filling factor

→  $V(t)$  and  $\phi_s(t)$  are the most important parameters that define available bucket area  $A_B(t)$  and RF filling factor → Important for optimization efforts



Example of  $\phi_s = 60^\circ$ : low filling factor, not ideal, room for optimization → Too large

# Example for RF calculations

## Overall flow:

$$\gamma_{harm}(t) = \gamma_{inj} + (\gamma_{ej} - \gamma_{inj}) \cdot \frac{1}{2} \left( \frac{B_{harm}(t)}{B_{ej}} + 1 \right)$$

$$G_{harm}(t) = \frac{(\gamma_{ej} - \gamma_{inj})}{2} \cdot \frac{m_{\mu}}{c} \left( \frac{\dot{B}_{harm}(t)}{B_{ej}} \right)$$

$$\phi_s(t) = \arcsin \left( \frac{\dot{B}_{harm}(t)}{\dot{B}_{lin}(t)} \cdot \sin \phi_{s,0} \right)$$

$B(t), \dot{B}(t)$  (and higher-level parameters)



$\gamma(t), E(t), G_{acc}(t)$



Max. gradient & number of cavities for costs



$\phi_s(t), V_{acc}(t)$

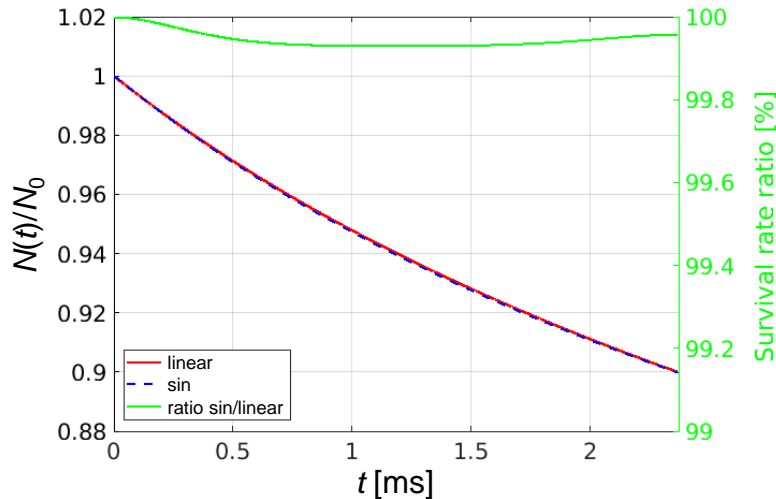


Bucket area  $A_B(t)$ ,  
RF filling factor and  
other criteria

→ Details in next talk  
by L. Thiele

# Survival rate

- The survival rate is not affected by quasi-linear ramping as long as the deviations from linear are not too large. Example for 11% extra voltage requirement:



$$\frac{N(t)}{N_0} = \exp \left( -\frac{1}{\tau_\mu} \int_0^{\tau_{acc}} \frac{dt}{\gamma(t)} \right)$$



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# RCS1/2/3 optimization parameters

## Parameters to be optimized:

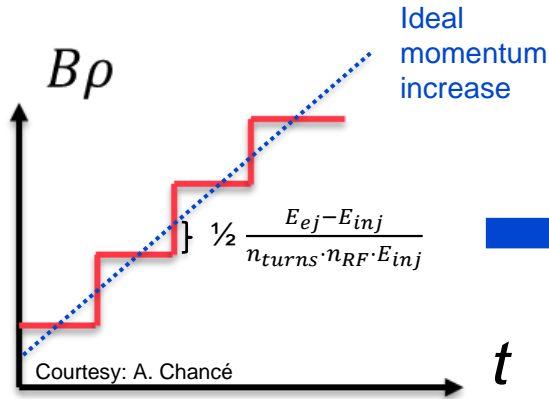
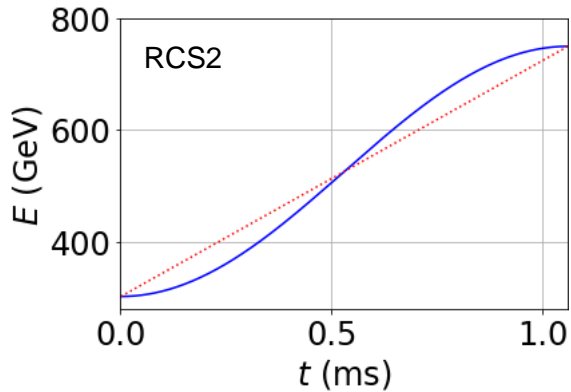
- Circumference
- Survival rate and acceleration time
- Number of RF stations,  $n_{RF}$
- Synchronous phase
- Minimal required bucket area and RF filling factor
- Inj. and ejection  $B$  fields
- **Maximum ramp rate,  $dB/dt$ ?**

	RCS1→314 GeV	RCS2→750GeV	RCS3→1.5TeV
Circumference, $2\pi R$ [m]	5990	5990	10700
Bunch population [ $10^{12}$ ]	<2.7	<2.43	<2.2
<b>Survival rate per ring [%]</b>	<b>~90</b>	<b>~90</b>	<b>~90</b>
Acceleration time, $t_{acc}$ [ms]	0.34	1.04	2.37
Number of turns	17	55	66
Energy gain per turn [GeV]	14.8	7.9	11.4
Avg. acc. gradient [MV/m]	2.4	1.3	1.1
Acc. field in RF cavity [MV/m]	30 (45 optimistically)	30	30
Synchronous phase [°]	45	45	45
Minimal bucket area [eVs]	0.6?	0.6?	0.8?
Injection B field [T] (warm m.)	0.36	-1.8	-1.8
Ejection B field [T] (warm m.)	1.8	1.8	1.8

# Energy and magnetic field synchronization

2<sup>nd</sup> order impact on optimization

- Continuous magnet ramp and discrete energy steps causes a relative field variation / error **for both linear and nonlinear ramping**



RCS1	< 0.36%
RCS2	< 0.13%
RCS3	< 0.09%
RCS	< 0.21%

- $\mu^+$  and  $\mu^-$  are propagating in opposite directions: impact of this variation to be discussed
- **Novel issue** for muon RCS





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# Summary and outlook

- Close interaction RF voltage and bending field
- Fraction of sinusoidal function for linearization, ideally well below 50%
- RF power requirement ( $P \propto V_{RF}^2$ ) given by maximum RF voltage during cycle
- **Outlook**
  - Complete calculations for entire RCS chain
  - Optimize transfer energies between accelerators

