Imperial College London

Science and Technology Facilities Council

Implementing a Relativistic Mean Field Theory model in the NEUT neutrino event generator

Jake McKean
j.mckean21@imperial.ac.uk

Joint APP, HEPP and NP Conference 2024
The Spine, Liverpool

April 10 2024

1. Overview of neutrino-nucleus interactions

Neutrino-*nucleon* CC interactions

- Neutrino interaction models lead to a large uncertainty for cross-section and neutrino oscillation measurements.
- Neutrinos interact with matter via different channels depending on the incident neutrino energy.

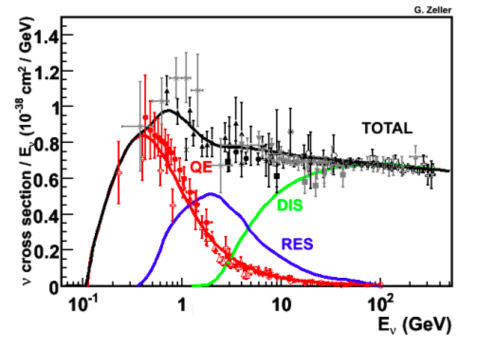


Figure: Figure adapted from G. Zeller

Neutrino-*nucleon* CC interactions

Inelastic scattering off a nucleon

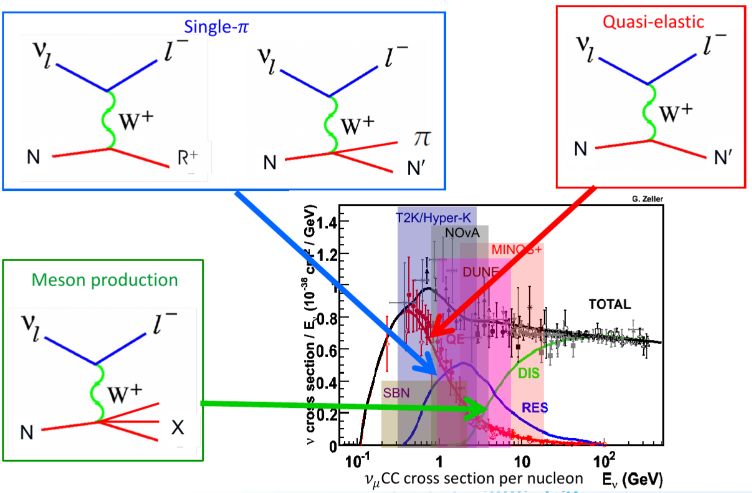
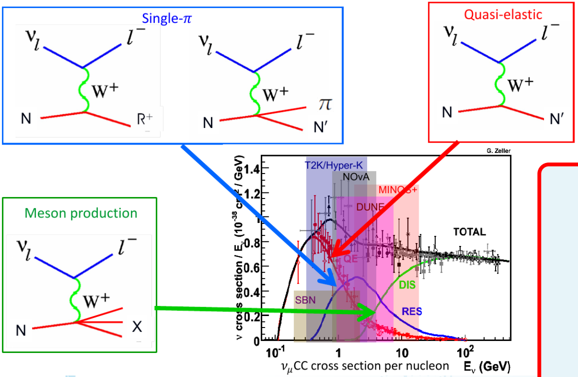


Figure: Figure adapted from G. Zeller & M. Kabirnezhad

Neutrino-*nucleon* CC interactions

Inelastic scattering off a nucleon



For the CCQE case, the neutrino-nucleon interaction model is known and has been used for a long time.

Neutrino-*nucleus* interactions

- Neutrino-**nucleus** interactions are more complex and not well modelled in experimental settings.
- The presence of the nuclear medium means that there are lots of effects that happen within the nucleus. They are often not taken into account in-house but added on at the end.
- There are also secondary interactions that occur only with the final-state particles called final-state interactions (FSI).

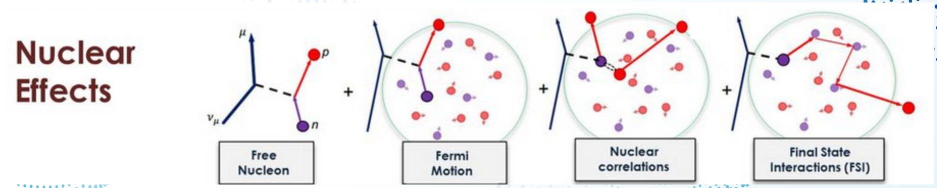
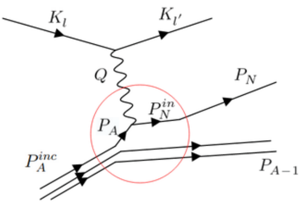


Figure: Figure by S. Dolan

Current approximations used in the field

- Theorists need to employ various approximations in order to reduce the complexity of the problem.
- The following diagram illustrates the Born Approximation and the Impulse Approximation.
- The impulse approximation allows the breakdown of the interaction into three components: elementary vertex, nuclear framework, and the FSI.



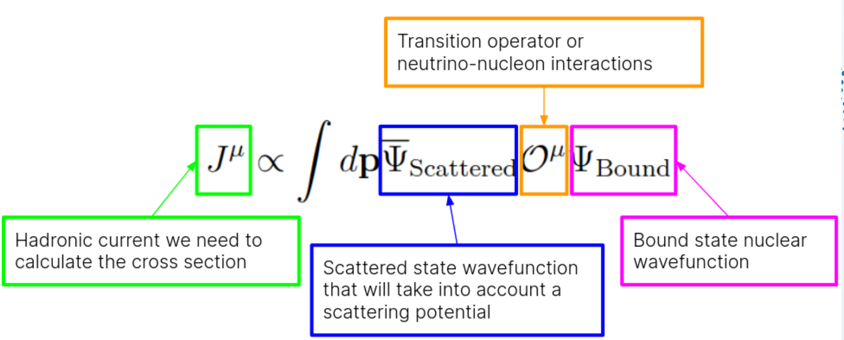
$$J^{\mu}_{\text{hadron}} = \left\langle N, A-1 \middle| \mathcal{O}^{\mu}_{\text{many-body}} \middle| A^{inc} \right\rangle$$

$$J^{\mu}_{\text{hadron}} = \int \mathrm{d}^3\mathbf{p_A} \overline{\Psi}_F(\mathbf{p_N}, \mathbf{q} + \mathbf{p_A}) \mathcal{O}^{\mu}_{\text{one-body}} \Psi_i(\mathbf{p_A})$$

Figure: Figure adapted from R. Gonzalez Jimenez

What is the goal of a calculation?

- We want to calculate the cross section, which is proportional to the contraction of the leptonic and hadronic tensors.
- The hadronic tensor is computationally expensive and is found using the following equation.
- Thanks to the impulse approximation, it breaks down into three simple parts: bound state, operator and scattered state.



2. Model Overview

Model Overview

- Bound state: uses **Relativistic Mean Field** (RMF) Theory in order to obtain the bound state wavefunctions.

Model Overview

- Bound state: uses Relativistic Mean Field (RMF) Theory in order to obtain the bound state wavefunctions.
- Operator: we have the **Elastic/QE operator**.

Model Overview

- Bound state: uses Relativistic Mean Field (RMF) Theory in order to obtain the bound state wavefunctions.
- Operator: we have the Elastic/QE operator.
- Scattered state: **Distorted Wave**.
  Calculates the solution to the Dirac equation with the potential that we supply (Relativistic Optical Potential, RMF, no potential).

Model Overview

- Bound state: uses Relativistic Mean Field (RMF) Theory in order to obtain the bound state wavefunctions.
- Operator: we have the Elastic/QE operator.
- Scattered state: Distorted Wave.
  Calculates the solution to the Dirac equation with the potential that we supply (Relativistic Optical Potential, RMF, no potential).
- Distorted Wave model can incorporate elastic FSI.

3. Relativistic Mean Field Theory

^{12}C bound state potentials

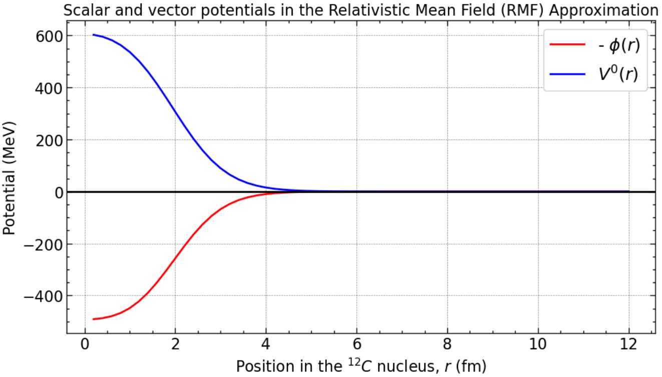
- RMF theory enables one to obtain the bound state wavefunctions and bound state nuclear potentials for the hadronic tensor calculation.

^{12}C bound state potentials

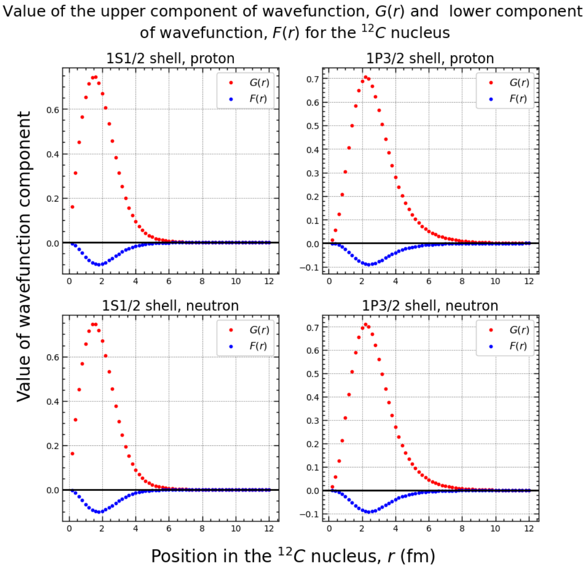
- RMF theory enables one to obtain the bound state wavefunctions and bound state nuclear potentials for the hadronic tensor calculation.
- One can carry these potentials forward to the scattered state to essentially supply an interacting potential. This is currently not done in some neutrino event generators.

^{12}C bound state potentials

- RMF theory enables one to obtain the bound state wavefunctions and bound state nuclear potentials for the hadronic tensor calculation.
- One can carry these potentials forward to the scattered state to essentially supply an interacting potential. This is currently not done in some neutrino event generators.

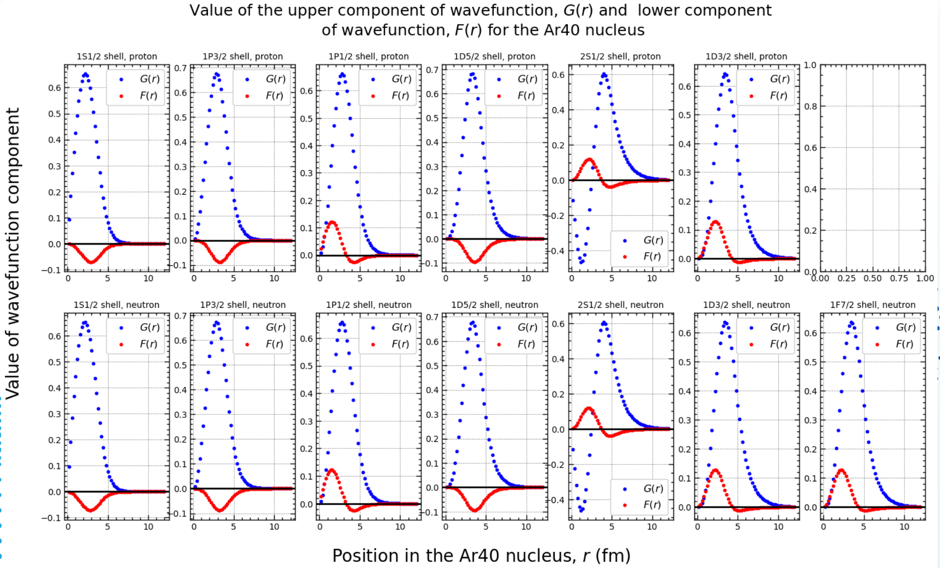


^{12}C bound state wavefunctions



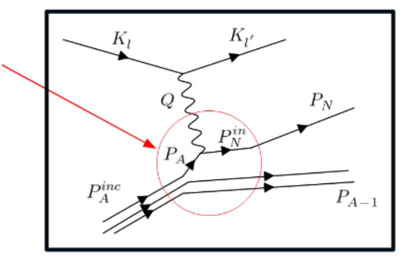
Imperial College London

^{40}Ar bound state wavefunctions



Distorted wave

- The scattered Dirac wavefunction interacts via a scattering potential which can be:
  - Imaginary: such as an optical potential.
  - Real: RMF potential.
- This interaction "distorts" the original shape of the wavefunction and incorporates elastic FSI.



3.1 Current usage of RMF in literature

Current RMF model

- It has not been implemented in event generators as a full model (until now).

Benchmarking intra-nuclear cascade models for neutrino scattering with relativistic optical potentials.

A. Nikolakopoulos,[1,2,*] R. González-Jiménez,[3] N. Jachowicz,[1] K. Niewczas,[1,4] F. Sánchez,[5] and J. M. Udías[3]

[1] *Department of Physics and Astronomy, Ghent University, B-9000 Gent, Belgium*
[2] *Theoretical Physics Department, Fermilab, Batavia, IL 60510, USA*
[3] *Grupo de Física Nuclear, Departamento de Estructura de la Materia, Física Térmica y Electrónica, Facultad de Ciencias Físicas, Universidad Complutense de Madrid and IPARCOS, CEI Moncloa, Madrid 28040, Spain*
[4] *University of Wrocław, Institute of Theoretical Physics, Plac Maxa Borna 9, 50-204 Wrocław, Poland*
[5] *University of Geneva, Section de Physique, DPNC, Geneva, Switzerland*

Background: In neutrino oscillation experiments, the hadrons created in neutrino-nucleus collisions are becoming important observables. The description of final-state interactions (FSI) of hadrons with nuclei in the large phase space probed in these experiments poses a great challenge. In the analysis of neutrino experiments, which operate under semi-inclusive conditions, cascade models are commonly used for this task. The description of FSI under exclusive conditions on the other hand can be treated successfully by using relativistic optical potentials (ROP).

4. Implementation in the NEUT event generator

Implementation progress

- I have been working with the nuclear theory group at **Complutense Universidad de Madrid** (UCM), specifically with the expertise of Dr Raul Gonzalez-Jimenez.

Implementation progress

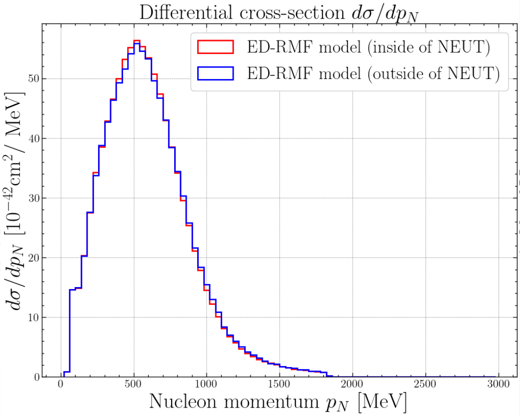
- I have been working with the nuclear theory group at Complutense Universidad de Madrid (UCM), specifically with the expertise of Dr Raul Gonzalez-Jimenez.
- I am currently implementing the nuclear model into the **NEUT**(please see talk by Luke Pickering) neutrino event generator framework.

Implementation progress

- I have been working with the nuclear theory group at Complutense Universidad de Madrid (UCM), specifically with the expertise of Dr Raul Gonzalez-Jimenez.
- I am currently implementing the nuclear model into the NEUT(please see talk by Luke Pickering) neutrino event generator framework.
- The model is now **successfully implemented** in NEUT and has been verified with 1M events.

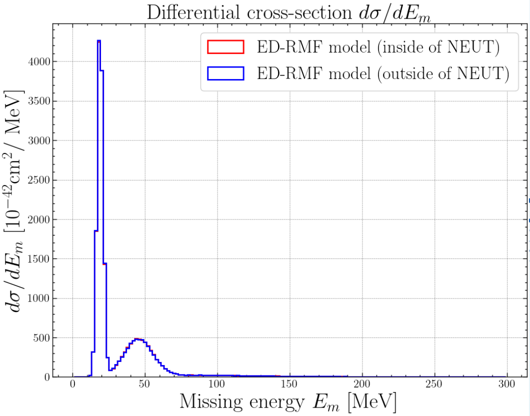
Validation plots

- The following plot shows the model implementation against the model outside of NEUT for the scattered nucleon momentum.
- Good agreement can be seen with 1M events.



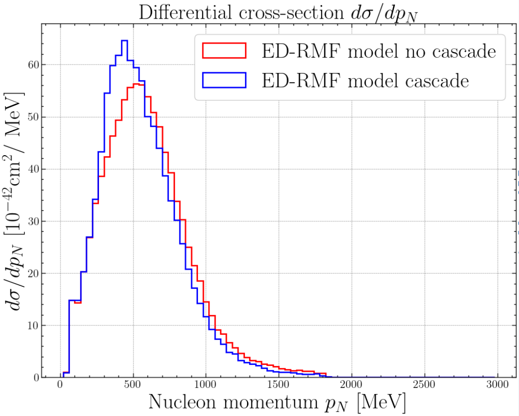
Validation plots

- The following plot shows the missing energy.
- Missing enery is defined as $E_m \stackrel{\text{def}}{=} \omega - T_N - T_B$, where T_B is the kinetic energy of the residual nucleus and ω is the energy transfer.
- Good agreement can be seen with 1M events.



NEUT cascade model

- NEUT has a built in semi-classical cascade model.
- This takes the output of a standard neutrino interaction (such as charged current) and probabilistically produces FSI (such as nucleon rescattering, pion production or pion rescattering).



Why is this model important?

- This is the first macroscopic model that has been implemented into the NEUT event generator.

Why is this model important?

- This is the first macroscopic model that has been implemented into the NEUT event generator.
- It allows one to consistently calculate any elastic FSI effects in-house.

Why is this model important?

- This is the first macroscopic model that has been implemented into the NEUT event generator.
- It allows one to consistently calculate any elastic FSI effects in-house.
- Allows the study of more theory-motivated “dials” that allow for variations to be inserted into a model for systematic error studies.

Why is this model important?

- This is the first macroscopic model that has been implemented into the NEUT event generator.
- It allows one to consistently calculate any elastic FSI effects in-house.
- Allows the study of more theory-motivated “dials” that allow for variations to be inserted into a model for systematic error studies.
- Allows for the inclusion of more complex operators for different processes.
  - One such example is the Kabirnezhad model operator for inelastic pion production.

Summary and outlook

Summary and outlook

- New nuclear model using Relativistic Mean Field Theory has been implemented in NEUT.
- Currently performing studies on the effects of double counting between the elastic part of the NEUT cascade and the inherent elastic FSI of the model.
- This will be a big step for connecting experimental event generators with more up-to-date theory models.
- It will also allow experiments to simulate interactions with more accuracy and can allow for a better route to study systematic theory errors.

Backup

References

[1] J. L. Herraiz et al. "Overview of neutrino-nucleus quasielastic scattering". In: *AIP Conference Proceedings* 1189.1 (Nov. 2009), pp. 125–132. ISSN: 0094-243X. DOI: 10.1063/1.3274142. eprint: https://pubs.aip.org/aip/acp/article-pdf/1189/1/125/11982195/125_1_online.pdf. URL: https://doi.org/10.1063/1.3274142.

[2] S. E. Koonin K. Langanke Joachim A. Maruhn. *Computational Nuclear Physics 1*. Springer Berlin, Heidelberg, 1991. ISBN: 978-3-642-76358-8.

CCQE operator in CC2 form

$$[\Gamma^{\mu}]_{CC2} = F_1^V \gamma^{\mu} + i\frac{F_2^V}{2m_N}\sigma^{\mu\nu}Q_{\nu} + G_A\gamma^{\mu}\gamma^5 + F_P Q^{\mu}\gamma^5 \quad (1)$$

where

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]. \quad (2)$$

Form factors F_1^V and F_2^V are related to the vector component of the hadronic cur- rent, while G_A and F_P respectively correspond to the axial vector and pseudoscalar parts.

NEUT cascade model

- Due to the nature of the RMF model, elastic FSI are already accounted for in the calculation of the cross section.

NEUT cascade model

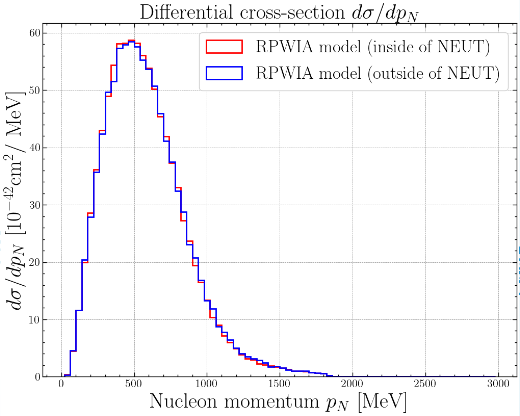
- Due to the nature of the RMF model, elastic FSI are already accounted for in the calculation of the cross section.
- Therefore, this presents a potential double counting of the elastic part of the FSI when using the NEUT semi-classical cascade.

NEUT cascade model

- Due to the nature of the RMF model, elastic FSI are already accounted for in the calculation of the cross section.
- Therefore, this presents a potential double counting of the elastic part of the FSI when using the NEUT semi-classical cascade.
- This is being investigated by looking at different scattered state models that I have implemented in NEUT.

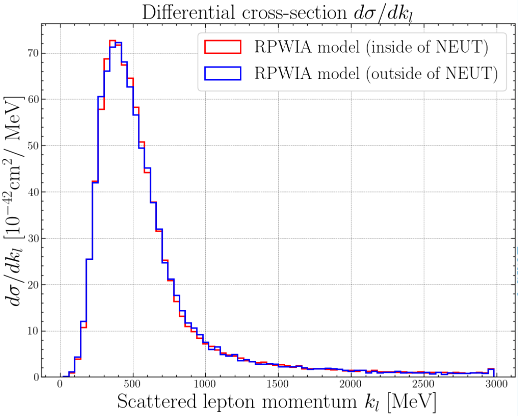
RPWIA model validation

- Relativistic plane wave impulse approximation model validation.
- Performed with 150k events.



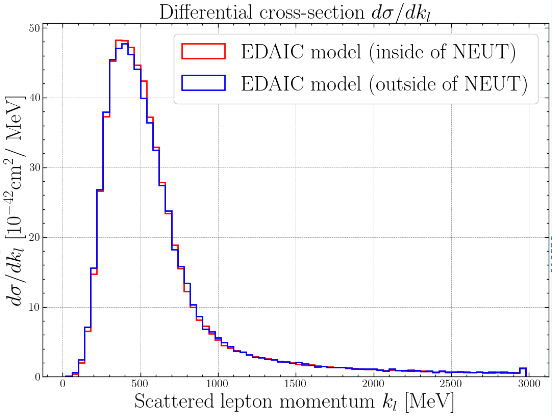
RPWIA model validation

- Relativistic plane wave impulse approximation model validation.
- Performed with 150k events.



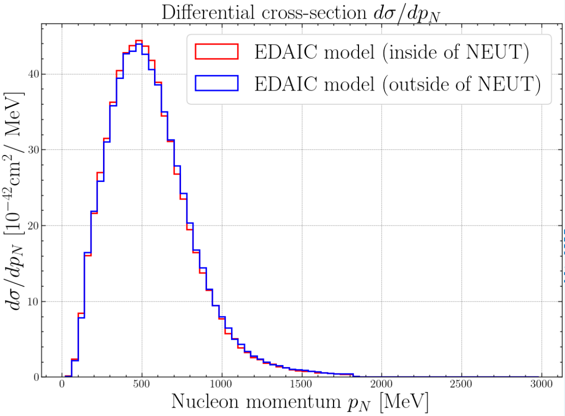
EDAIC model validation

- Relativistic optical potential model validation (energy-dependent A independent).
- Performed with 150k events.



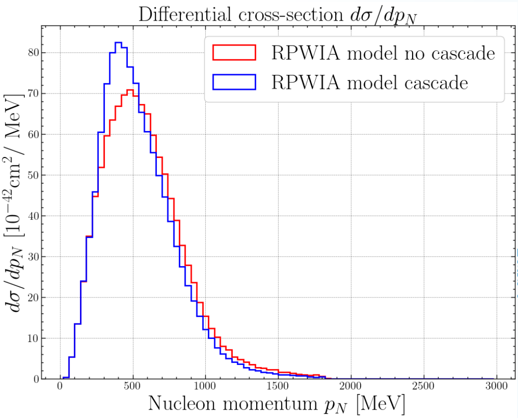
EDAIC model validation

- Relativistic optical potential model validation (energy-dependent A independent).
- Performed with 150k events.



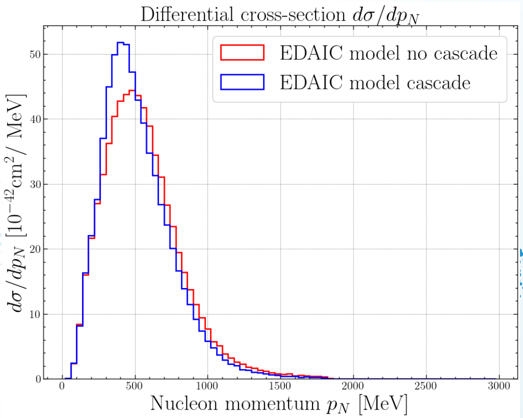
RPWIA with NEUT cascade

- Relativistic plane wave impulse approximation using NEUT cascade
- Performed with 150k events.



EDAIC with NEUT cascade

- Relativistic optical potenial model using NEUT cascade
- Performed with 150k events.



Current approximations used in the field

There are many approximations that are used in order to help theorists evaluate such a complex interaction:

- **Born Approximation (one boson exchange)** $\rightarrow$ **allows lepton and hadron tensor separation** [1].

Current approximations used in the field

There are many approximations that are used in order to help theorists evaluate such a complex interaction:

- Born Approximation (one boson exchange) → allows lepton and hadron tensor separation [1].
- **Independent particle approximation → the nucleons are degrees of freedom (can use shell models)** [2].

Current approximations used in the field

There are many approximations that are used in order to help theorists evaluate such a complex interaction:

- Born Approximation (one boson exchange) $\rightarrow$ allows lepton and hadron tensor separation [1].
- Independent particle approximation $\rightarrow$ the nucleons are degrees of freedom (can use shell models) [2].
- **Impulse approximation** $\rightarrow$ **one body interaction**.

Current approximations used in the field

There are many approximations that are used in order to help theorists evaluate such a complex interaction:

- Born Approximation (one boson exchange) $\rightarrow$ allows lepton and hadron tensor separation [1].
- Independent particle approximation $\rightarrow$ the nucleons are degrees of freedom (can use shell models) [2].
- Impulse approximation $\rightarrow$ one body interaction.
- **Plane wave $\rightarrow$ outgoing wavefunction is plane wave $\rightarrow$ No FSI but can use spectral function approach**.

Current approximations used in the field

There are many approximations that are used in order to help theorists evaluate such a complex interaction:

- Born Approximation (one boson exchange) $\rightarrow$ allows lepton and hadron tensor separation [1].
- Independent particle approximation $\rightarrow$ the nucleons are degrees of freedom (can use shell models) [2].
- Impulse approximation $\rightarrow$ one body interaction.
- Plane wave $\rightarrow$ outgoing wavefunction is plane wave $\rightarrow$ No FSI but can use spectral function approach.

Good overviews for these approximations were given on Tuesday; please see the talks by Raul and Kajetan.

RMF theory: bound state wavefunctions

- The essence of RMF theory is replacing the meson field operators with the expectation value of the field: $\hat{\sigma} \rightarrow \langle \sigma \rangle$.

RMF theory: bound state wavefunctions

- The essence of RMF theory is replacing the meson field operators with the expectation value of the field: $\hat{\sigma} \rightarrow \langle \sigma \rangle$.
- For the bound state nuclear wavefunctions, the Dirac-Hartree equation is solved for scalar (ϕ) and vector (V_0) potential fields.

RMF theory: bound state wavefunctions

- The essence of RMF theory is replacing the meson field operators with the expectation value of the field: $\hat{\sigma} \rightarrow \langle \sigma \rangle$.
- For the bound state nuclear wavefunctions, the Dirac-Hartree equation is solved for scalar (ϕ) and vector (V_0) potential fields.
- The meson field equations are then solved in a self-consistent Hartree approximation method.

$$\{i\gamma_\mu \partial^\mu - g_V \gamma^0 V^0(r) - [M - g_s \phi(r)]\}\psi(x) = 0$$

Vector potential (+ Coulomb): $V^0(r)$

Scalar potential: $\phi(r)$

Baryon wavefunction: $\psi(x)$

RMF theory: bound state wavefunctions

- The Dirac equation can be simplified by considering **spherically symmetric nuclei** (^{12}C, ^{16}O, and ^{40}Ar, etc.) and only the **radial** component of the potential.

Upper Spinor component → $G(r)$

Lower Spinor component → $F(r)$

$$\frac{d}{dr}G(r) + \frac{\kappa}{r}G(r) - \left[E - g_V V^0(r) + M - g_s\phi(r)\right]F(r) = 0.$$

$$\frac{d}{dr}F(r) - \frac{\kappa}{r}F(r) + \left[E - g_V V^0(r) - M + g_s\phi(r)\right]G(r) = 0.$$

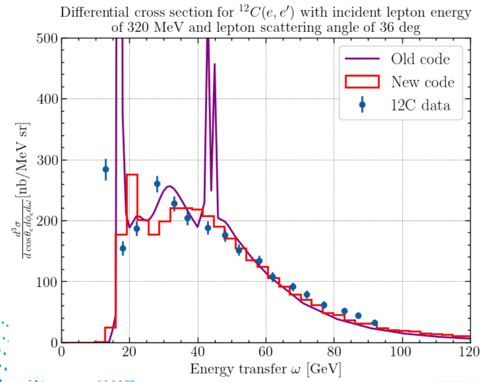
General overview (code)

- The code essentially uses tables of Hadronic tensor values in order to speed up the processing time.
- The tables are made using the full calculation code but only have to be made once.
- The values in the tables are then extrapolated for any intermediate values using a very simple bilinear interpolation regime.
- The model of the interaction is locked into these tables at the time of their creation.

- $H^{\mu\nu} = (J^{\mu})^{*} J^{\nu}$
- $J^{\mu} \propto \int d\mathbf{p} \overline{\Psi}_{\text{Scattered}} \mathcal{O}^{\mu} \Psi_{Bound}$

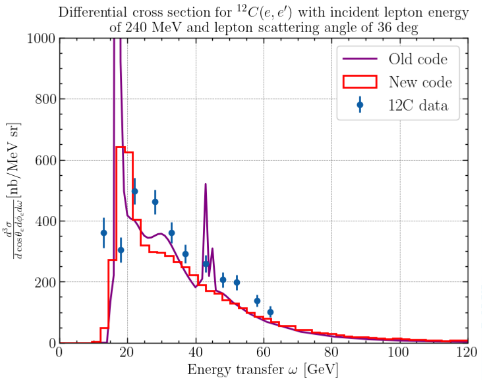
This is calculated using the integral equation for the hadronic current shown in slide 13

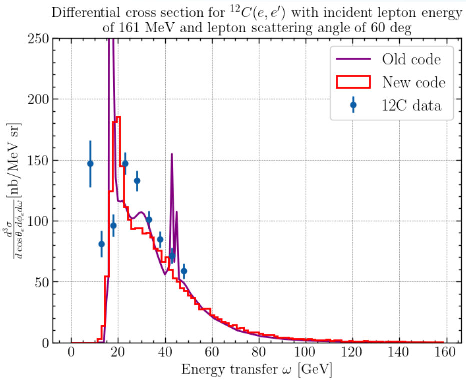
Results for $e^- N$ scattering



- The high and narrow peaks seen at low omega is a resonance effect coming from the final wavefunction being a distorted wave.
- The height and width are due to the pure independent particle shell model.
- They can be “smeared” out by adding correlations between the nucleons.

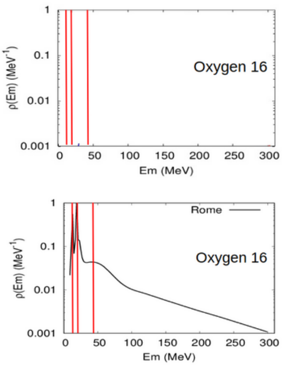
Results for $e^- N$ scattering





Results for $e^- N$ scattering

- One of the key differences in this newer model code is how it treats the missing energy profile.
- In the older model code, Dirac deltas are used ("pure shell model").
- In the new model, Gaussians + background from SRC are taken into account in the tables (with one unoccupied energy level contributing to this)



$$\rho_\kappa(E_m) = \delta(E_m - E_m^\kappa)$$

$$\rho(E_m) = \int d^3\mathbf{p}_m S(E_m, p_m)$$

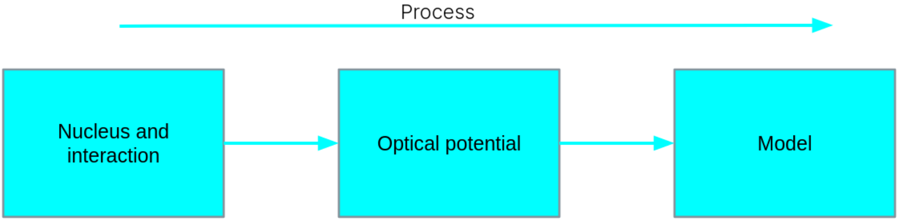
Figure: Figures from Raul Gonzalez-Jimenez (https://indico.fnal.gov/event/56949/attachments/161831/213670 /NuSTEC_talks_RGJ.pdf)

Iteration convergence

- Convergence is defined by delta E which is defined using the Spinors components at the match radius (radius at which Dirac eq is solved up to from 0 and from infinity).
- Discontinuity in F allows us to do this.

$$\left.\begin{matrix}\rho_B \\ \rho_s\end{matrix}\right\} = \sum_{\alpha}^{occ} \left(\frac{2j_\alpha + 1}{4\pi r^2}\right)\left(|G_\alpha(r)|^2 \pm |F_\alpha(r)|^2\right) \quad (3)$$

What does an optical potential give us?



Start with our nucleus and interaction

Optical potential provides us with a mean free path, the probability of something happening. It only tells us that something happened and whether it was elastic or inelastic.

Need to use a model (e.g. cascade model) to describe what “actually” happens when the optical potential says an inelastic interaction happens.