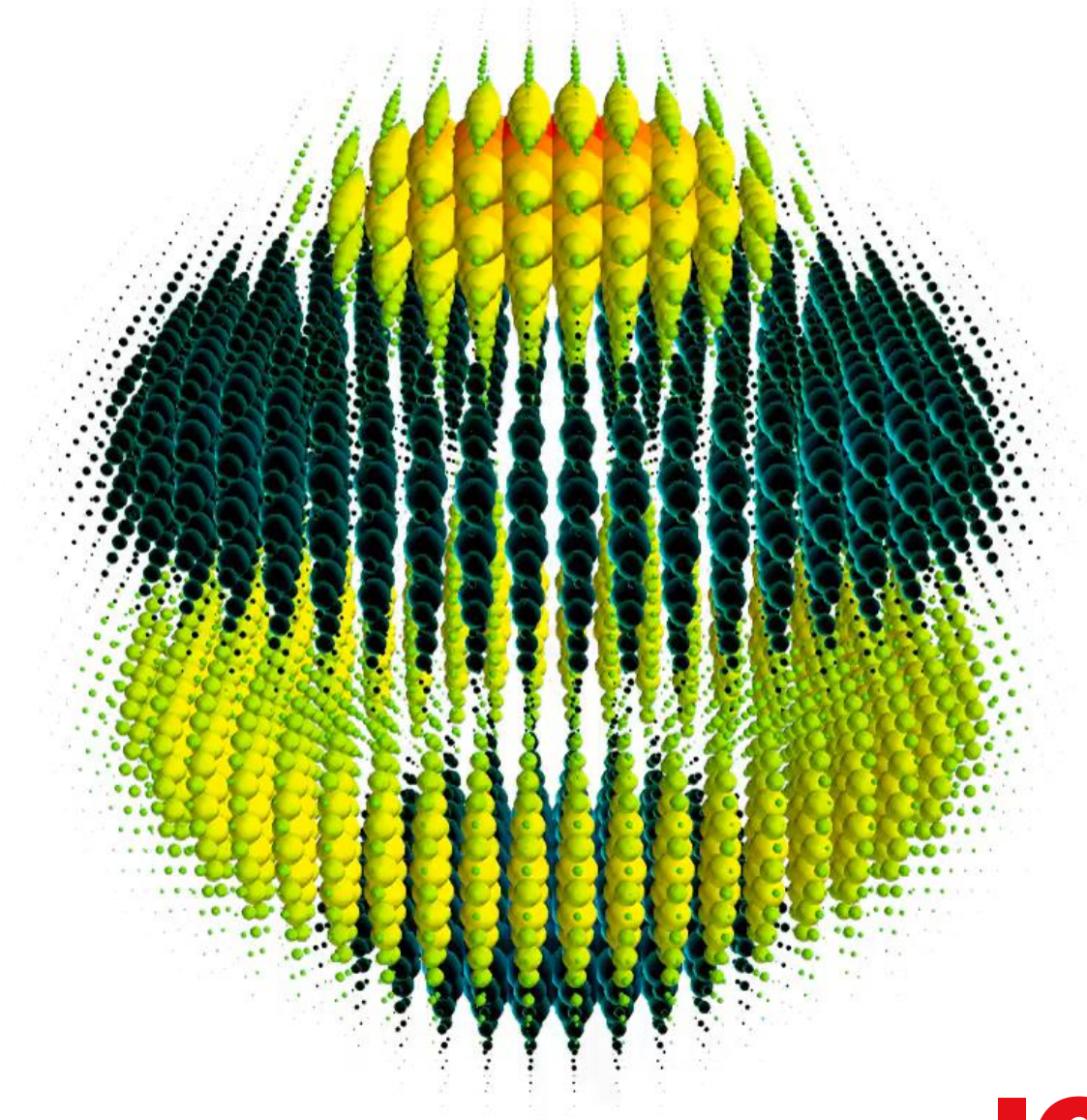


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# QUADRUPOLE AND OCTUPOLE STATES IN ZIRCONIUM CHAIN USING TDHF & QRPA

Abhishek, P. D. Stevenson, and E. Yüksel



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# OUTLINE

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What is our Model (TDHF)

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How it works

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Comparison with QRPA

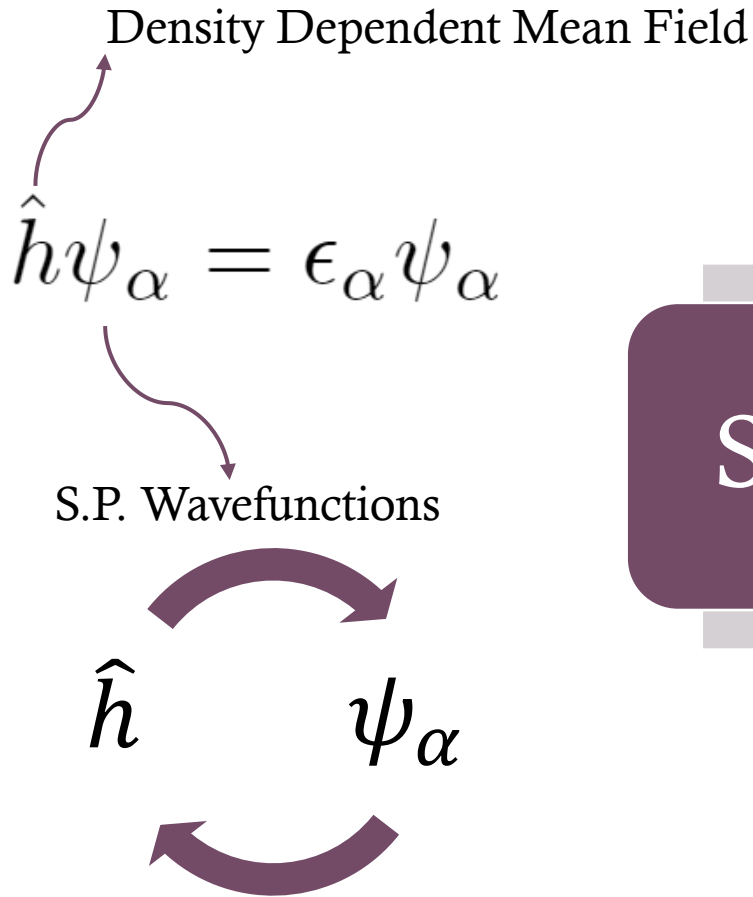
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Results for Zr isotopes

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Conclusion

# TIME DEPENDENT HARTREE FOCK (TDHF) MODEL



Static HF

Time Dependent

$$i\partial_t\psi_\alpha = \hat{h}\psi_\alpha$$

$$\psi_\alpha(t + \Delta t) = \hat{U}(t, t + \Delta t)\psi_\alpha(t)$$
$$\hat{U}(t, t + \Delta t) = \hat{T} \exp\left(\frac{-i}{\hbar} \int_t^{t+\Delta t} \hat{h}(t') dt'\right)$$

# COLLECTIVE EXCITATIONS

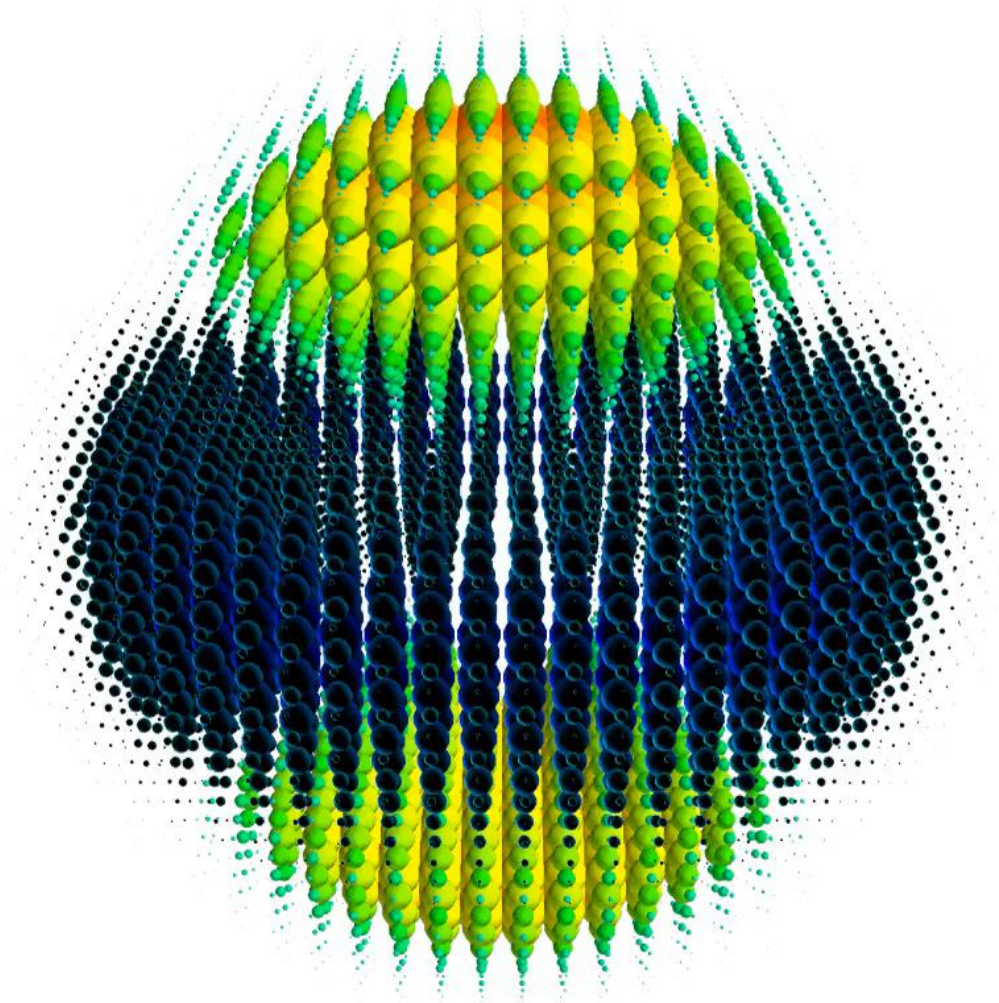
$$\hat{h}_q \rightarrow \hat{h}_q + \eta f(t) F_q(\vec{r})$$

$$F_q(\vec{r}) = \sqrt{2L + 1} r^L Y_{LM}$$

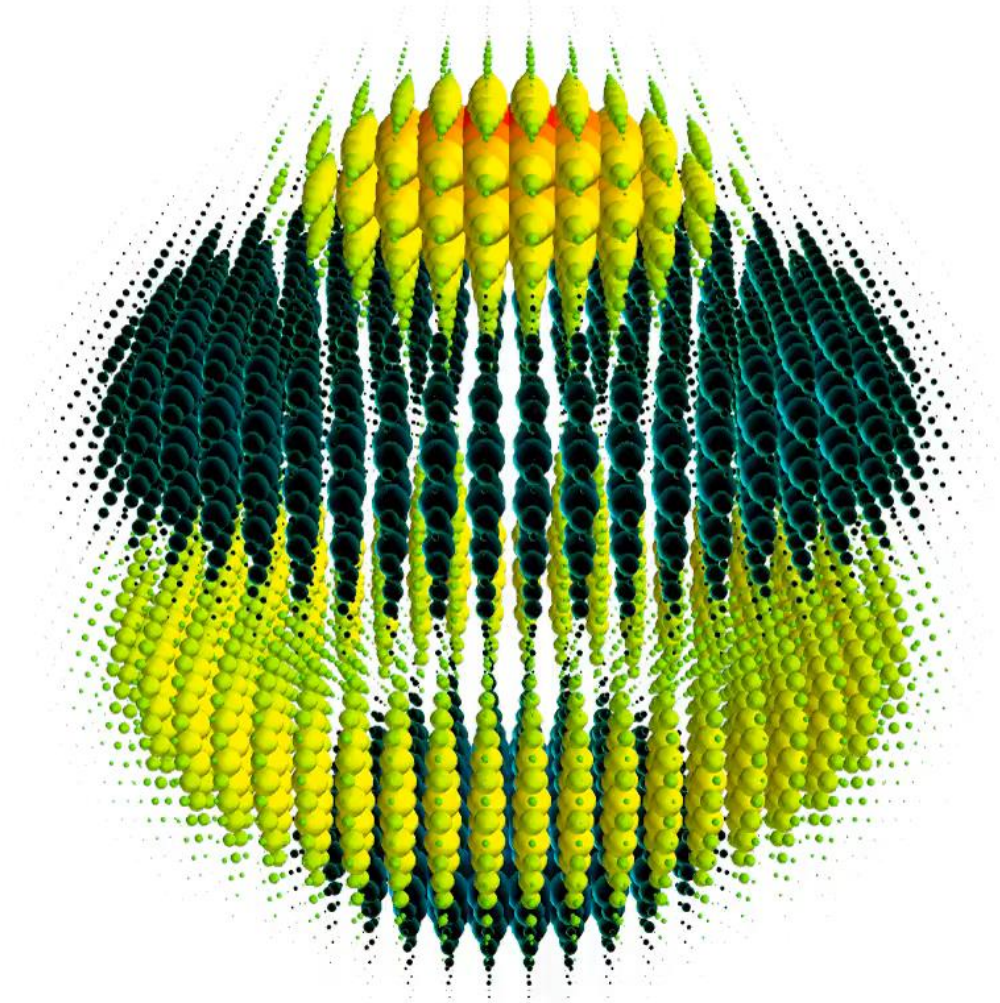
$$F_q(\vec{r}) \rightarrow \frac{F_q(\vec{r})}{1 + e^{(r-r_0)/\Delta r}}$$

L=2 (Quadrupole)  
L=3 (Octupole)

$q$  denotes the isospin

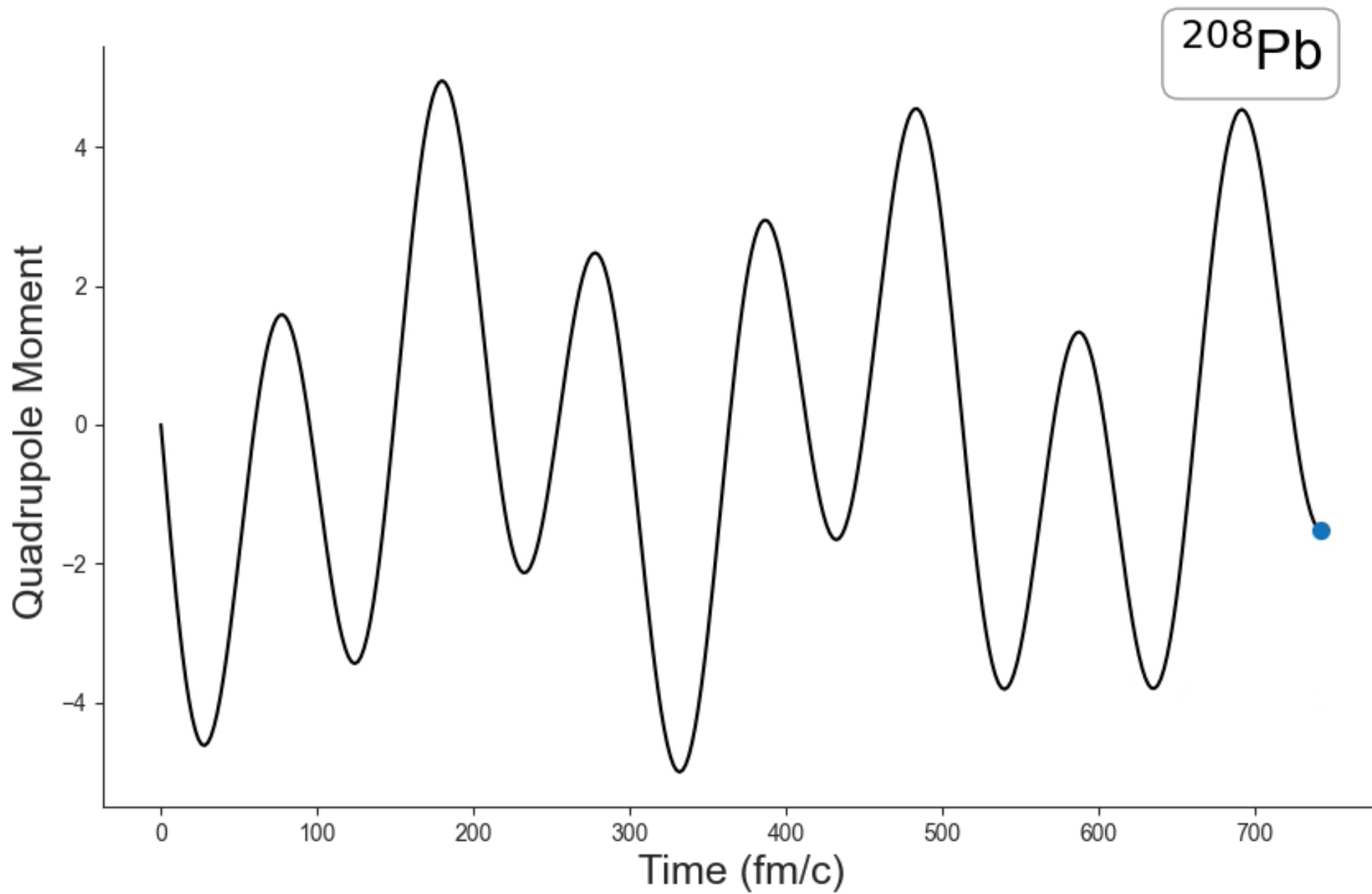


$$r^2 Y_2^0$$



$$r^3 Y_3^0$$

Sly5 | Quadrupole boost



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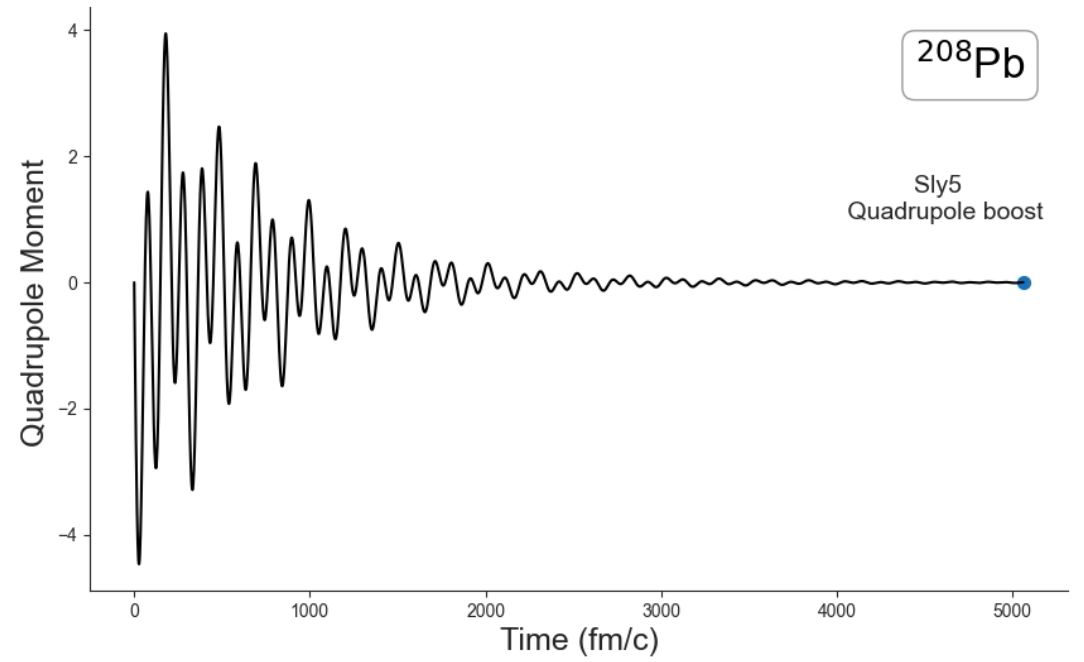
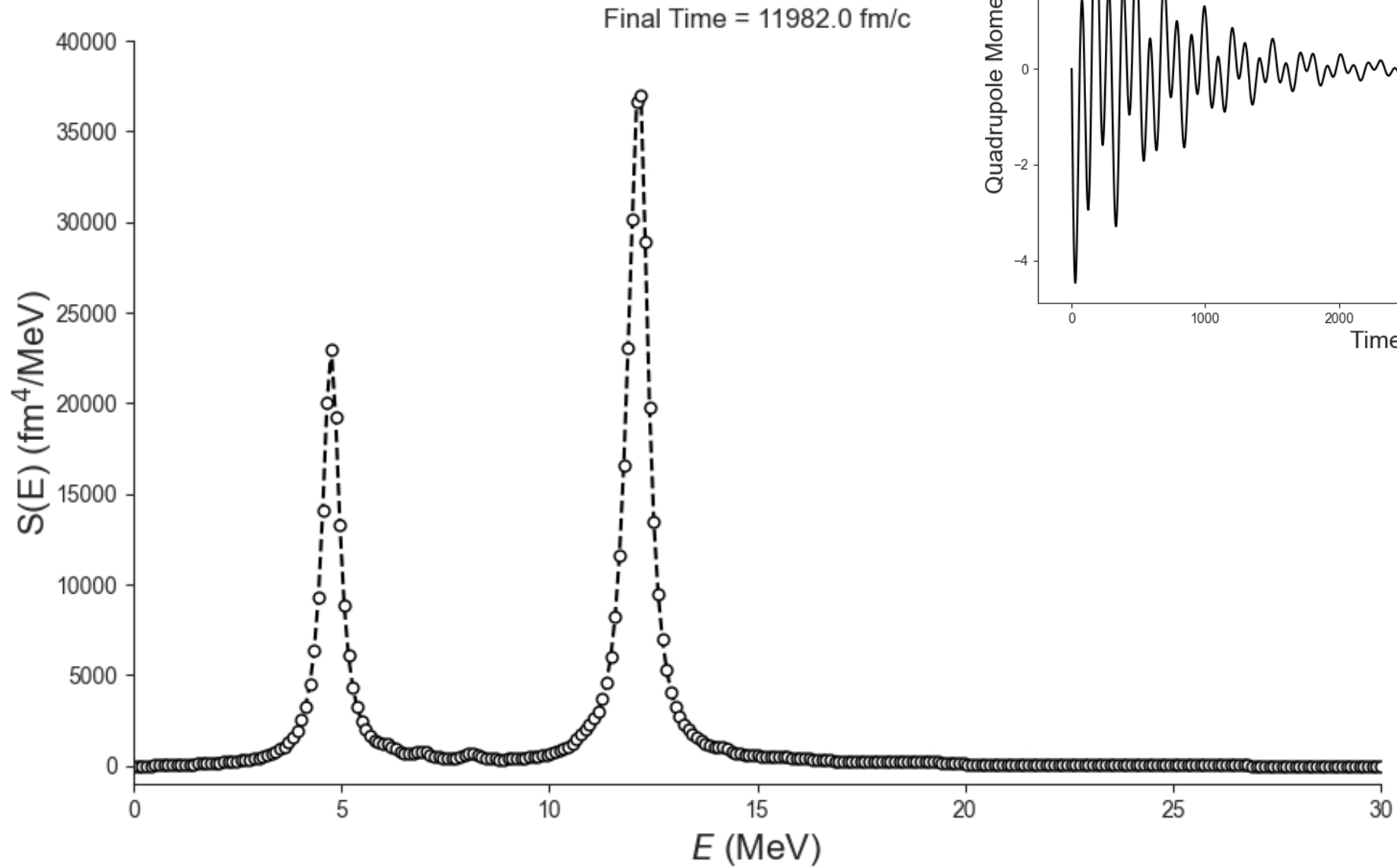
# STRENGTH FUNCTION

The boost gives some weight to all the system's excitation frequencies

$$\hat{F}_q(t) = \langle \Phi(t) | \hat{F}_q | \Phi(t) \rangle$$

$$S(E) = \frac{1}{\eta \hbar \pi} \text{im}[\hat{F}_q(\omega)]$$

$$\hat{F}_q(t) \rightarrow \hat{F}_{q\text{fil}}(t) = \hat{F}_q(t) e^{\frac{-\Gamma t}{2\hbar}},$$





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# COMPARISON WITH QRPA

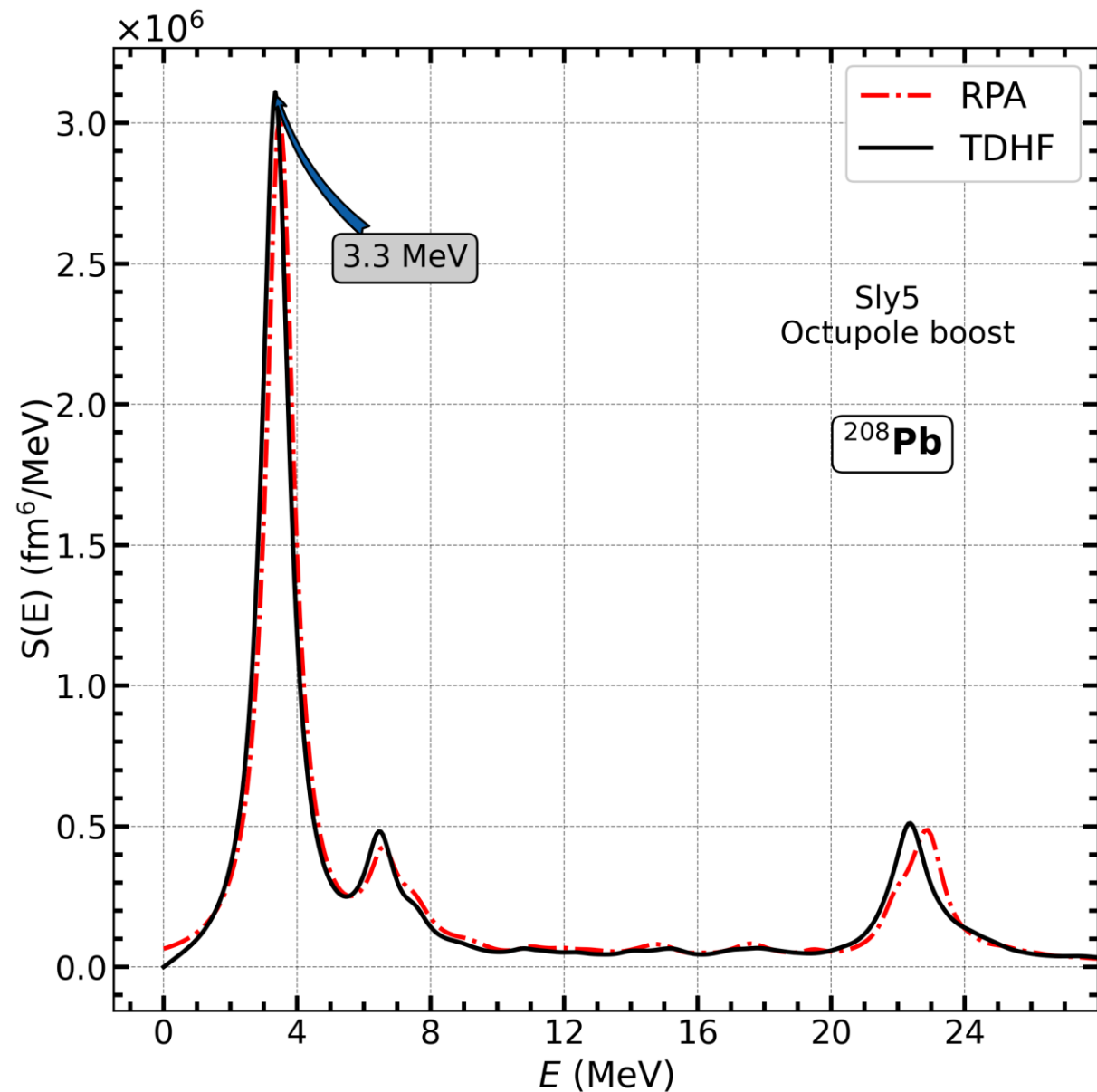
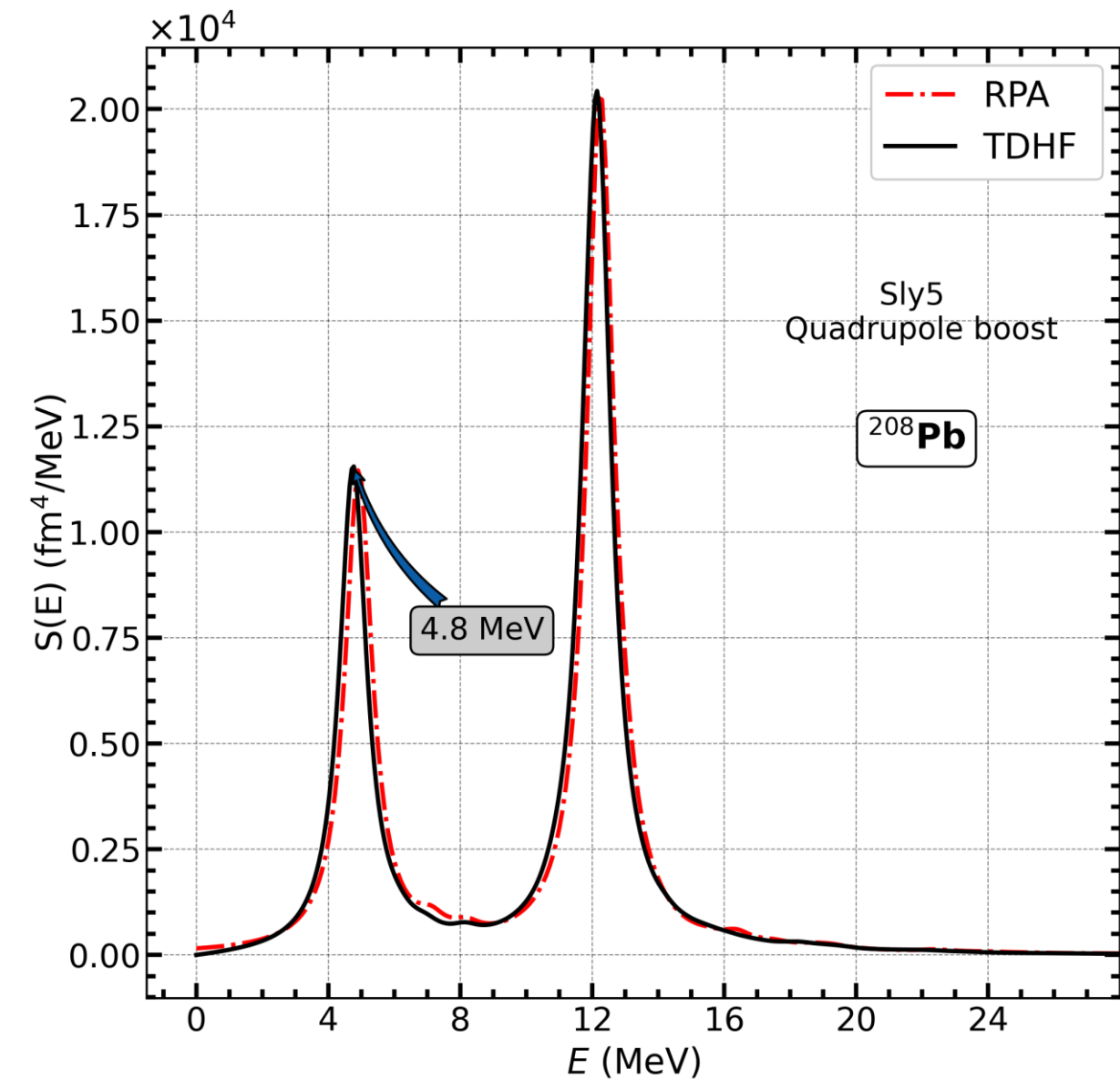
## User guide for the `hfbcscs-qrpa (v1)` code

G. Colò\* and X. Roca-Maza†

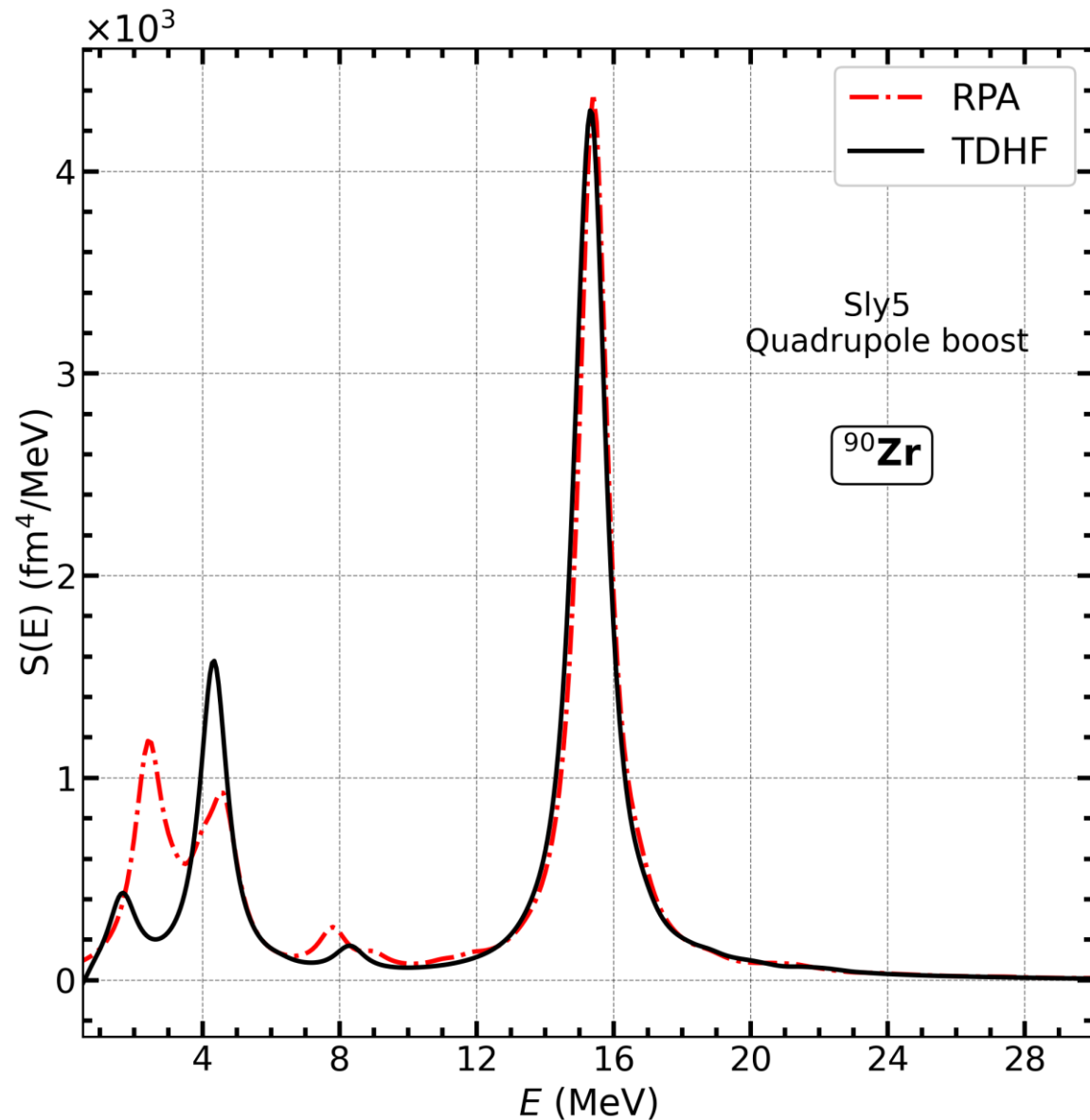
*Dipartimento di Fisica, Università degli Studi di Milano and INFN,  
Sezione di Milano, Via Celoria 16, 20133 Milano, Italy.*

(Dated: February 15, 2021)

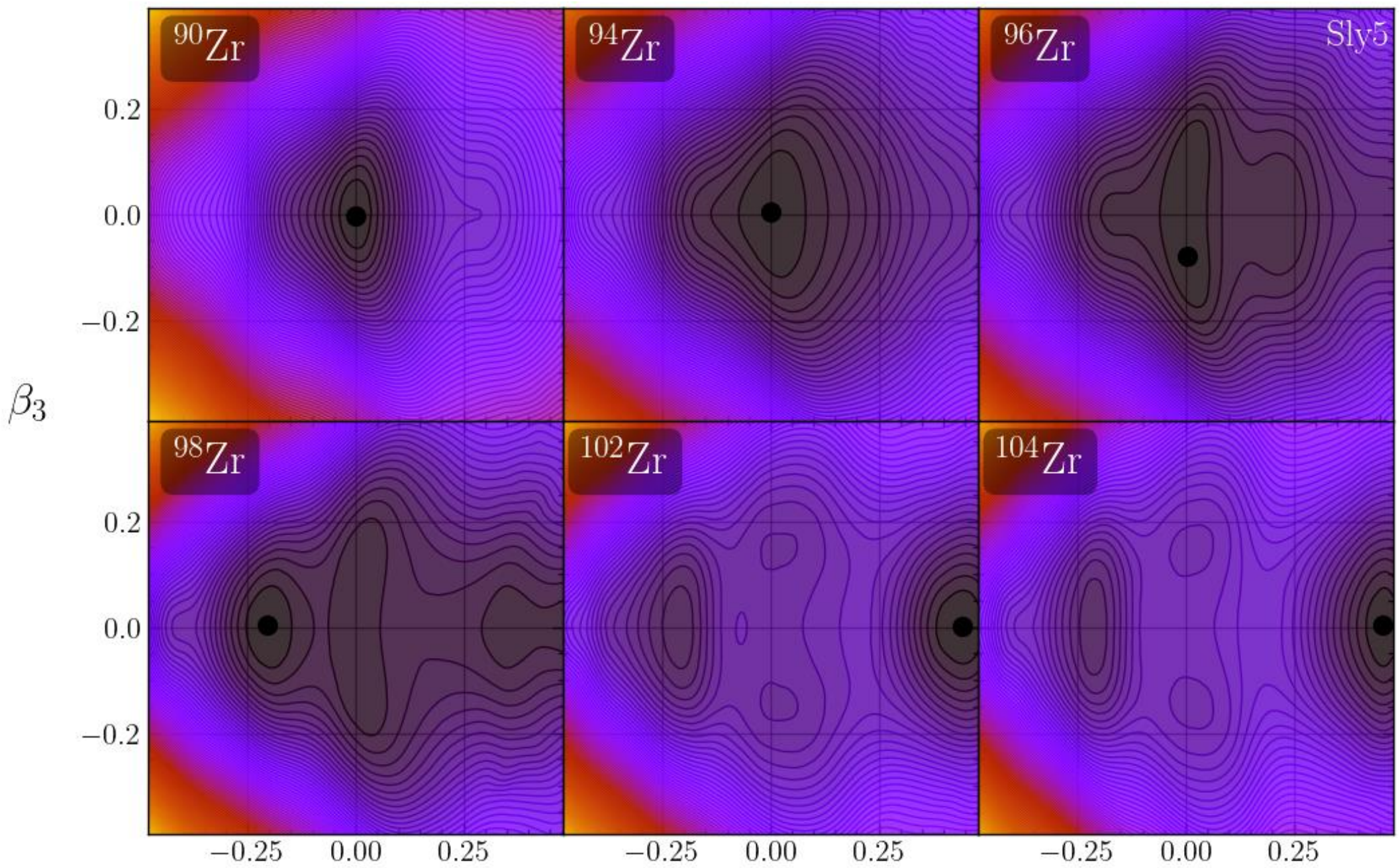
We briefly explain the structure of the Hartree-Fock-Bardeen-Cooper-Schrieffer (HFBCS) and Quasi-particle Random Phase Approximation (QRPA) code `hfbcscs-qrpa`, highlighting only the differences with the code `skyrme_rpa` that has been previously published [1]. The code deals with open shell spherical systems and non-spin flip, non-charge-exchange and natural parity  $(-1)^l$  excitations.



# COMPARISON WITH QRPA

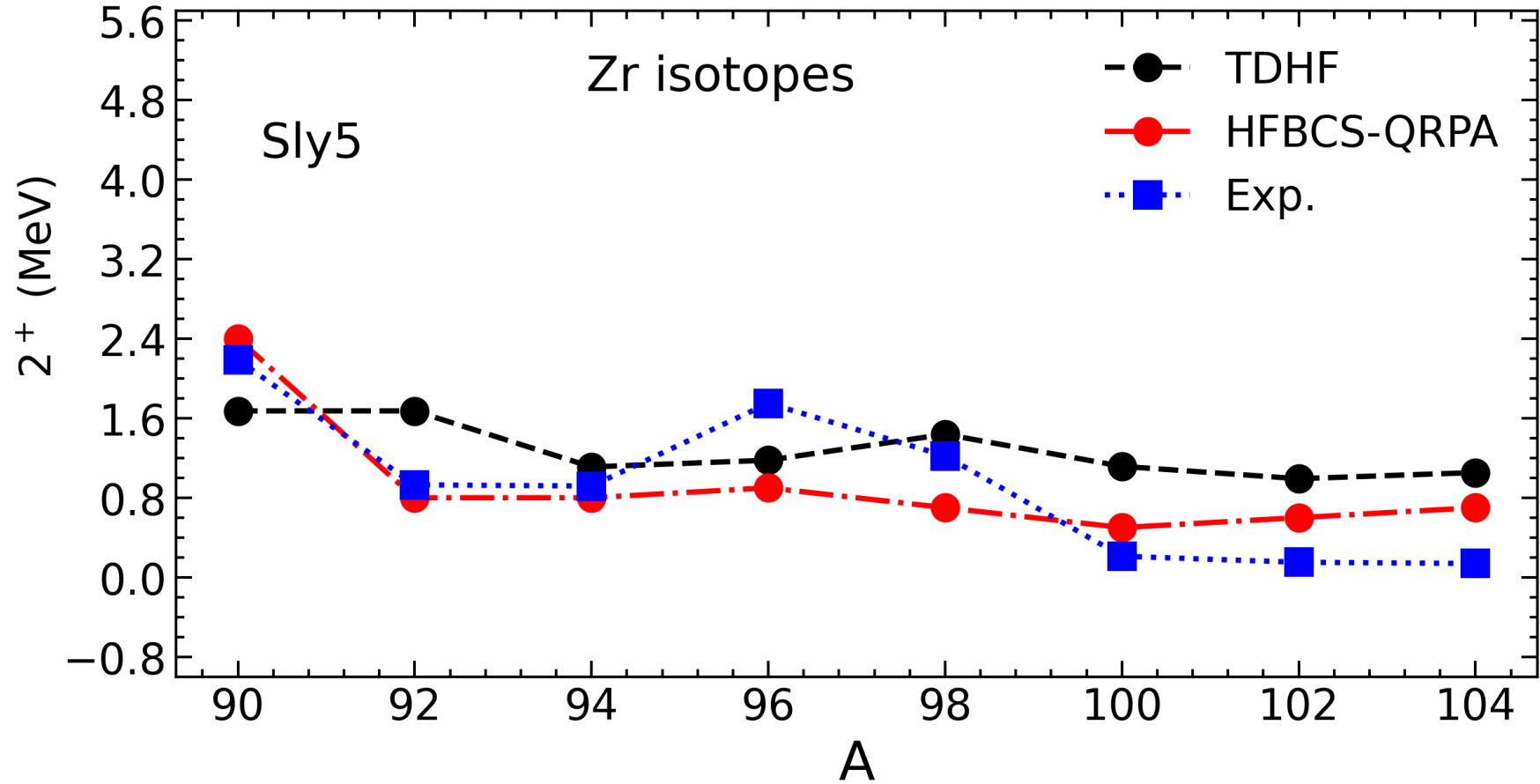


- Differences in lower energy peaks
- Experimental data available
- Comparison of first quadrupole and octupole energy peaks



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# ZR ISOTOPES



# QRPA COLLAPSE

## Octupole deformation instability in atomic nuclei

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*Center for Exotic Nuclear Studies, Institute for Basic Science (IBS), Daejeon 34126, Korea.*

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*Department of Physics, Ho Chi Minh City University of Education,  
280 An Duong Vuong, District 5, Ho Chi Minh City, Vietnam.*

*Department of Theoretical Physics, Faculty of Physics and Engineering Physics,  
University of Science, Ho Chi Minh City, Vietnam and  
Vietnam National University, Ho Chi Minh City, Vietnam.*

Panagiota Papakonstantinou<sup>‡</sup>

*Rare Isotope Science Project, Institute for Basic Science, Daejeon 34047, Korea*

Naftali Auerbach<sup>§</sup>

*School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel*

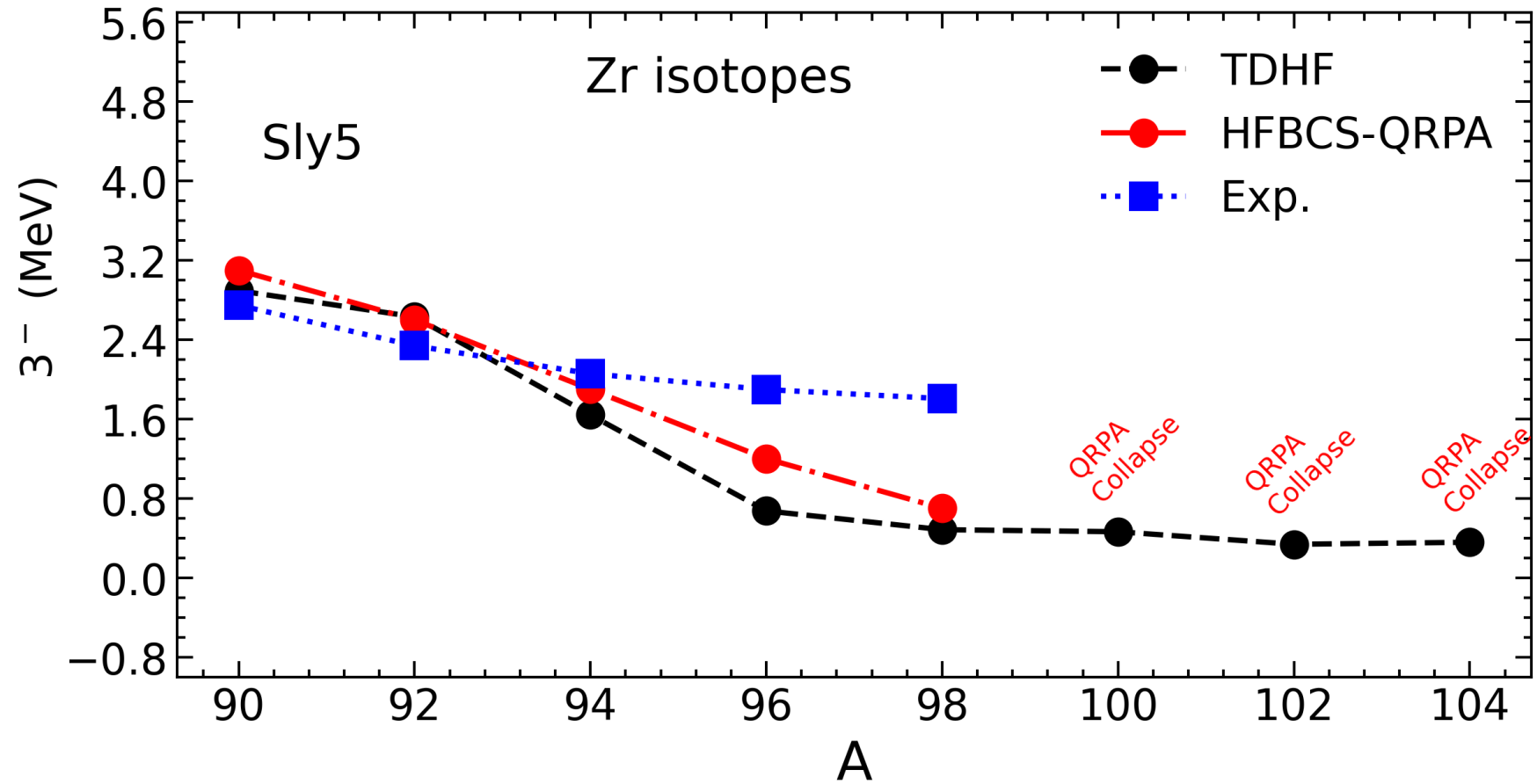
(Dated: March 21, 2023)

Recent high-energy heavy-ion collision experiments have revealed that some atomic nuclei exhibit unusual softness and significant shape fluctuations. In this Letter, we use the fully self-consistent mean-field theory to identify all even-even nuclei that are unstable or soft against octupole deformation. All exceptional cases of enhanced octupole transition strengths in stable even-even nuclei throughout the nuclide chart are resolved. These results represent a significant advance in our understanding of the underlying mechanisms of nuclear octupole deformation and have implications for further experimental and theoretical studies.

TABLE I. The HFBCS-QRPA results: contrast to the results of  $^{96}\text{Ru}$ . The  $E(3_1^-)$  and  $\alpha$  are in MeV, W.u., and W.u./1

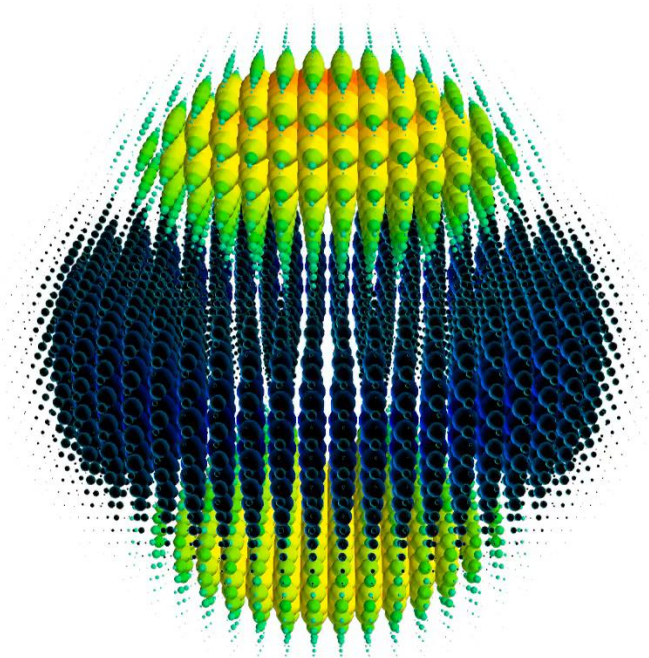
Force	$^{96}\text{Zr}$		
	$E(3_1^-)$	$B(E3)$	$\alpha$
SIII	collapse		
SkM*	collapse		
SLy4	0.961	96.4	12.6
SLy5	1.698	51.9	4.0
Exp.	1.897	53	

# ZR ISOTOPES

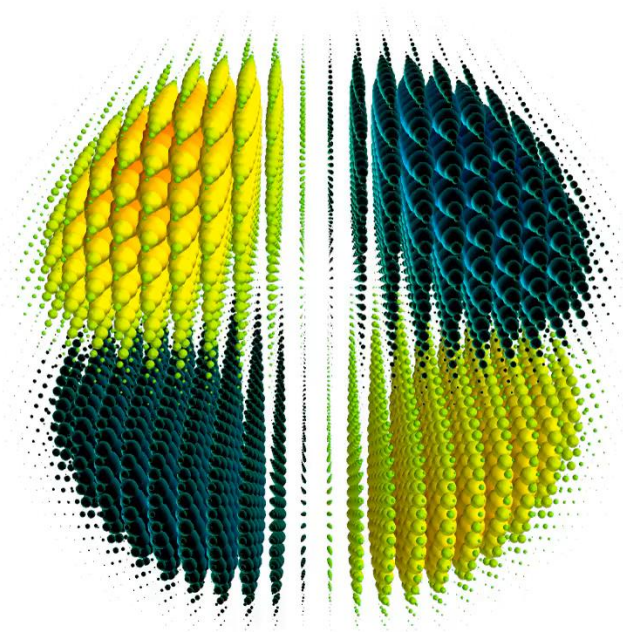


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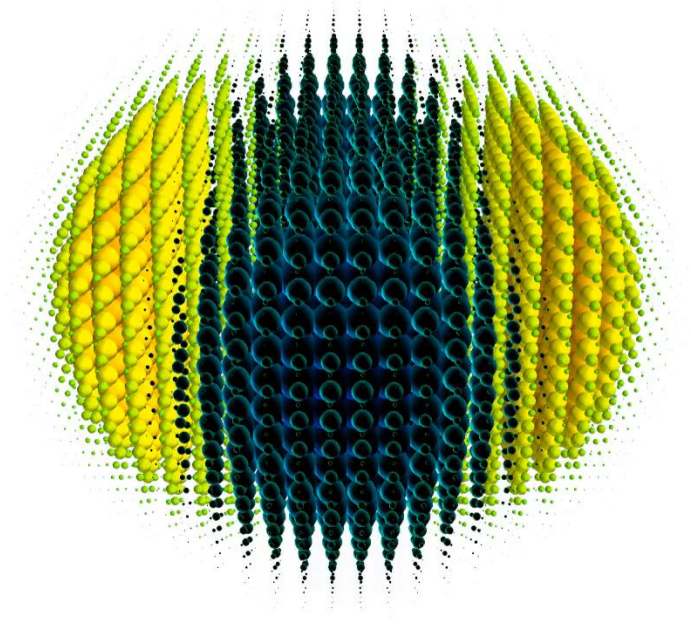
# PROBING ASYMMETRY (QUADRUPOLE)



$$r^2 Y_2^0$$
$$K = 0$$



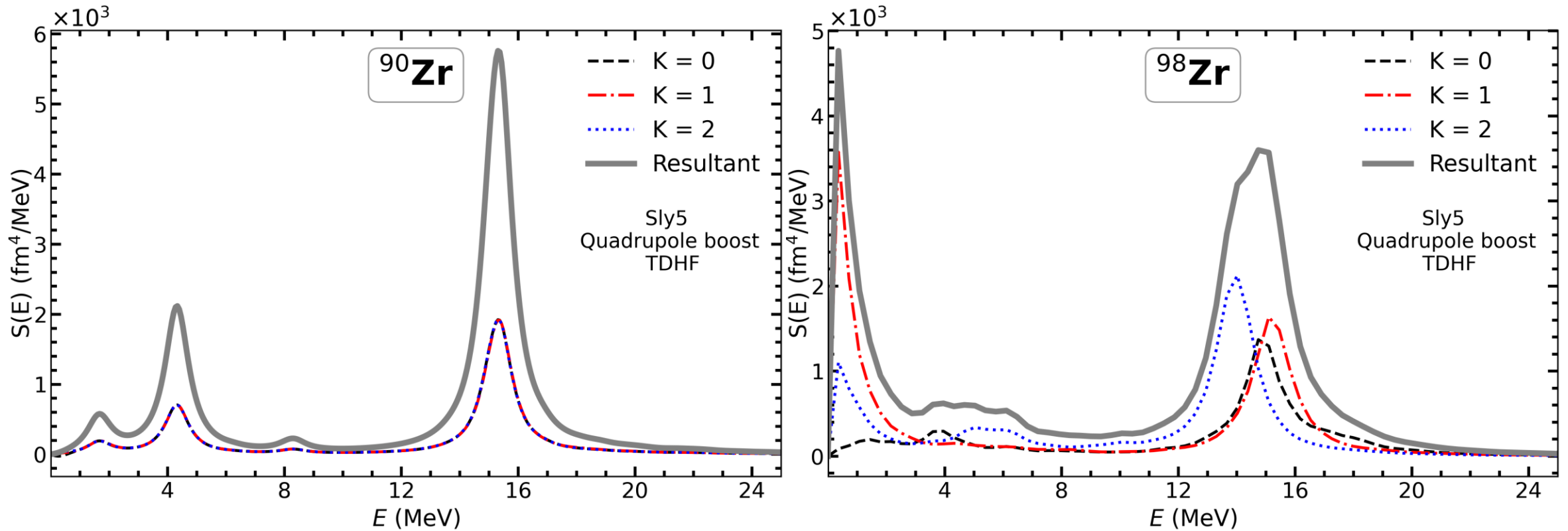
$$r^2 Y_2^1$$
$$K = 1$$



$$r^2 Y_2^2$$
$$K = 2$$

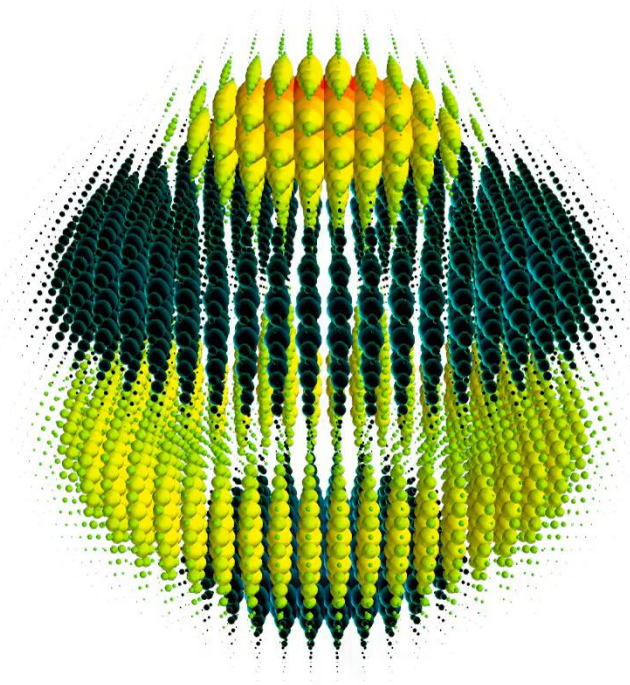


# PROBING ASYMMETRY (QUADRUPOLE BOOST)

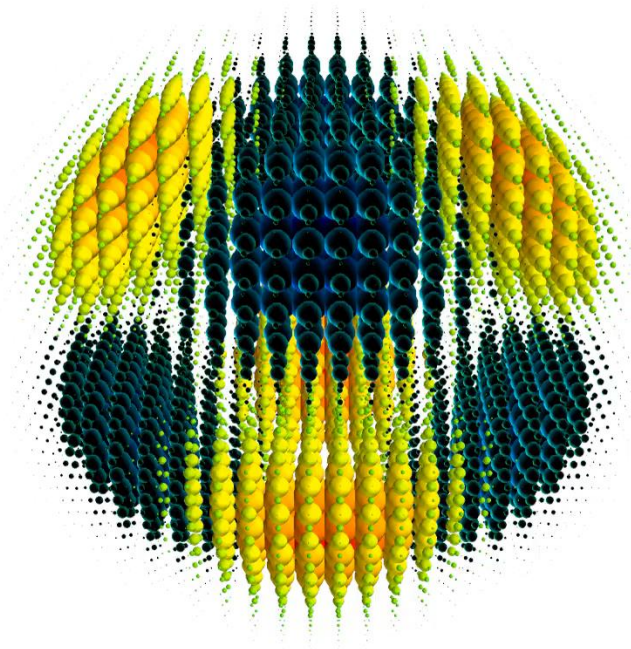


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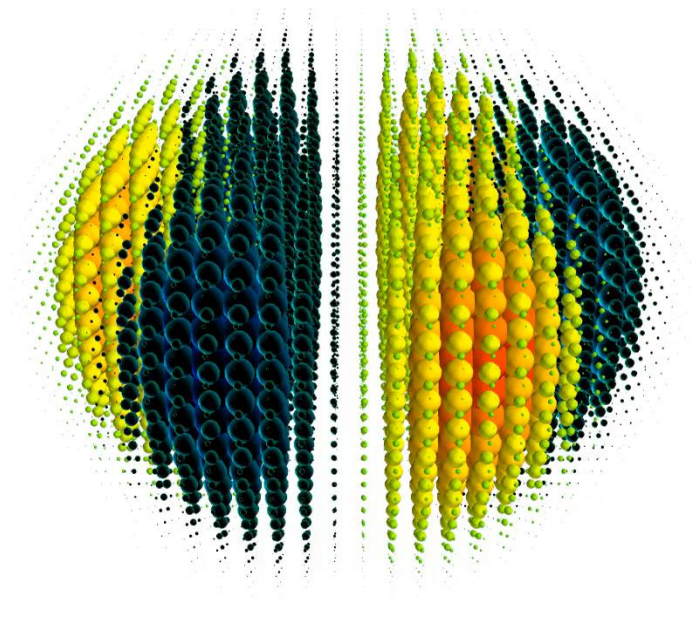
# PROBING ASYMMETRY (OCTUPOLE)



$$r^3 Y_3^0$$
$$K = 0$$

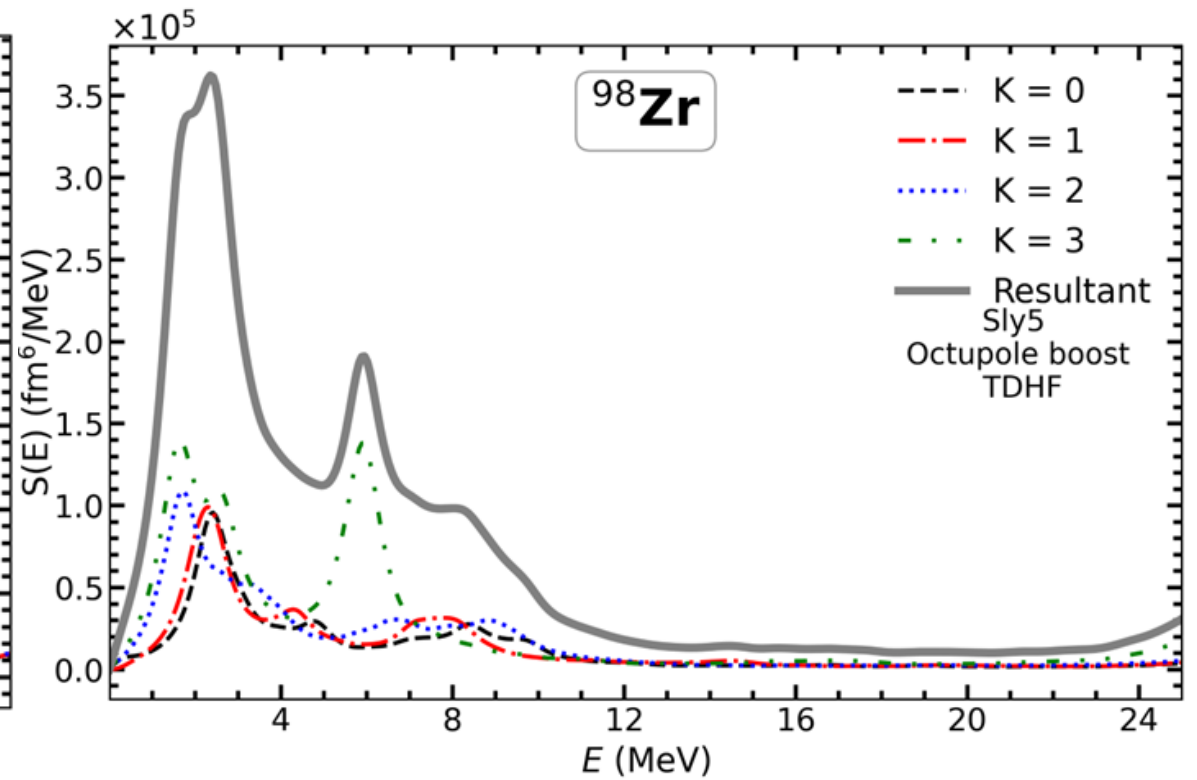
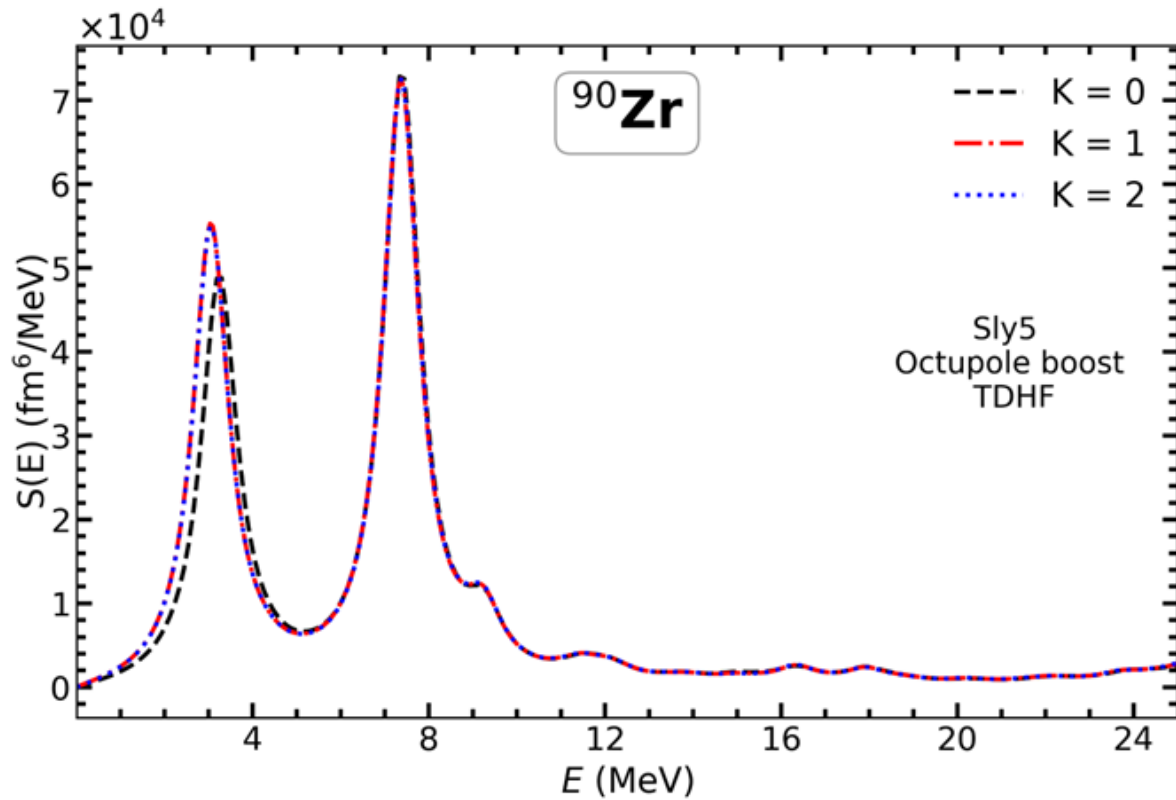


$$r^3 Y_3^2$$
$$K = 2$$



$$r^3 Y_3^3$$
$$K = 3$$

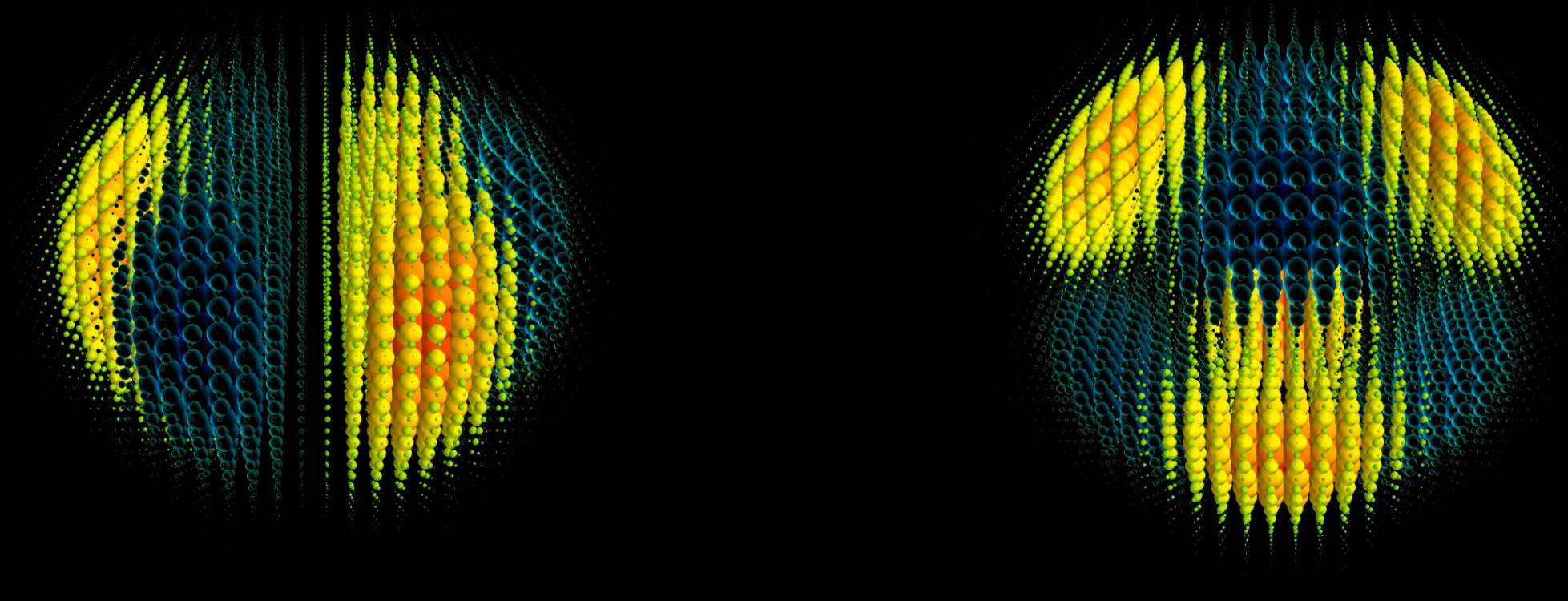
# PROBING ASYMMETRY (OCTUPOLE BOOST)



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# CONCLUSIONS

- Good Agreement within the two models in a doubly magic nucleus (Pb).
- Differences when there is softness along quadrupole and octupole degrees of freedom.
- TDHF can be a viable tool in extremely soft nuclei like Zr isotopes.
- Extreme softness leads to collapse of QRPA but not in TDHF.



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