



Department of Theoretical Physics

# The path to NNLL accurate parton showers

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Joint INFN-UNIMI-UNIMIB Pheno Seminar

Based on

JHEP 03 (2023) 224 [K. Hamilton, AK, G. P. Salam, L. Scyboz, R. Verheyen]  
Phys.Rev.Lett. 131 (2023) 16 [S. Ferrario Ravasio, K. Hamilton, AK, G. P. Salam, L. Scyboz, G. Soyez]  
2406.02661 [eid. + M. v. Beekveld, M. Dasgupta, B. K. El-Menoufi, J. Helliwell, P. F. Monni, A. Soto-Ontoso]

+

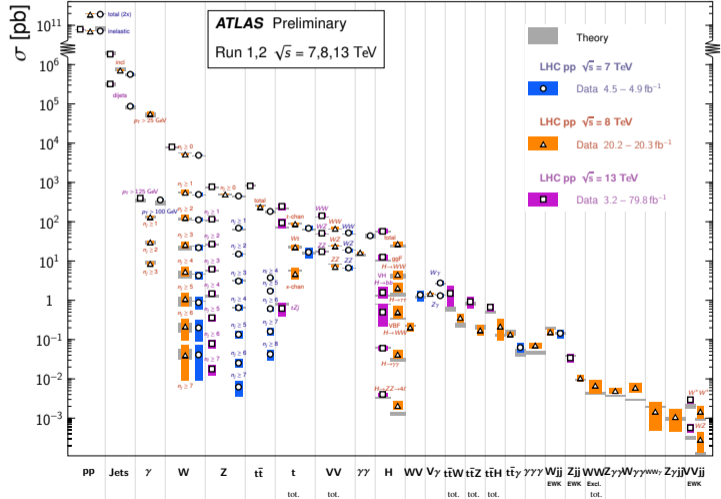
using analytic understanding developed in

JHEP 01 (2019) 083 [A. Banfi, B. K. El-Menoufi, P. F. Monni]  
JHEP 12 (2021) 158 [M. Dasgupta, B. K. El-Menoufi]  
JHEP 05 (2024) 09 [eid. + M. v. Beekveld, J. Helliwell, P. F. Monni]

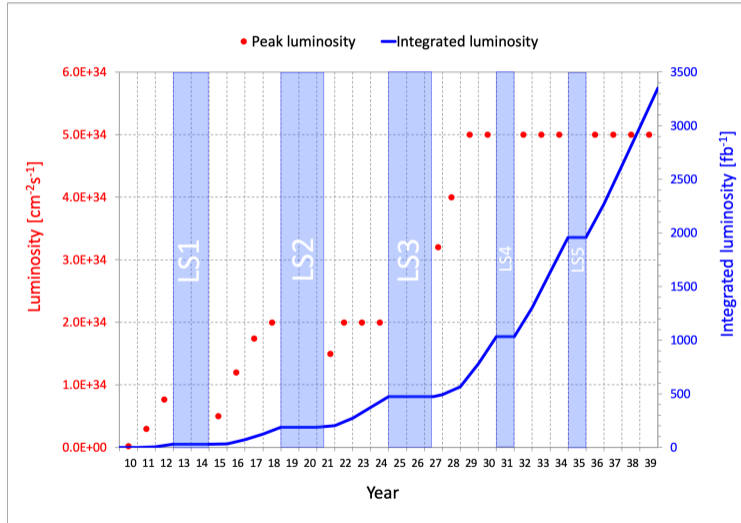
# The precision era of the LHC

Standard Model Production Cross Section Measurements

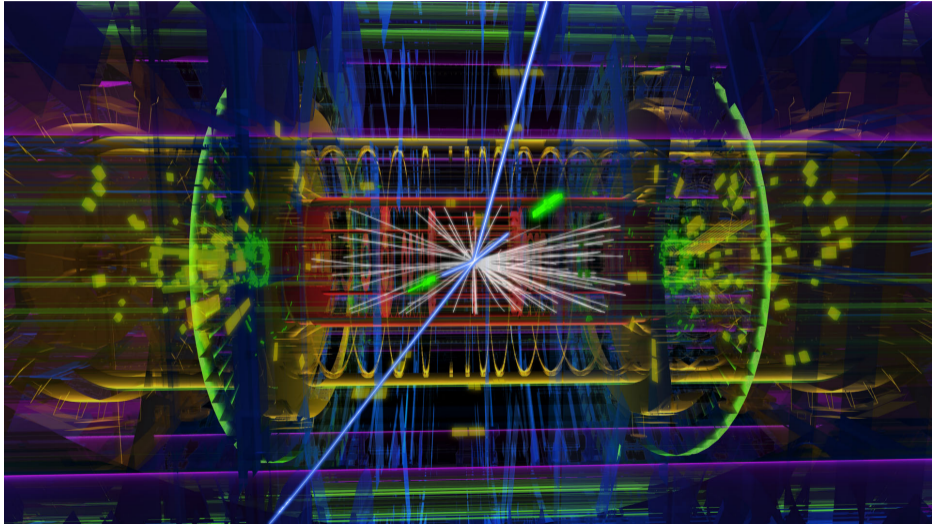
Status: July 2018



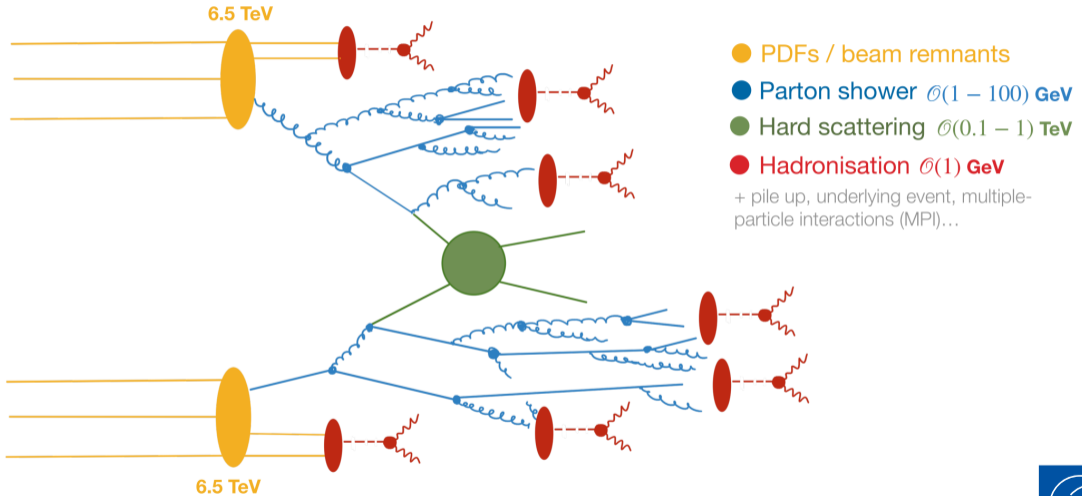
# The precision era of the LHC



# The LHC: A messy environment



# Anatomy of an LHC collision



courtesy M. van Beekveld



# The ubiquitous Parton Shower



## Pythia 8

### An introduction to PYTHIA 8.2

Torbjörn Sjöstrand (Lund U., Dept. Theor. Phys.), Stefan Ask (Cambridge U.), Jesper R. Christiansen (Lund U., Dept. Theor. Phys.), Richard Corke (Lund U., Dept. Theor. Phys.), Nishita Desai (U. Heidelberg, ITP) et al. (Oct 11, 2014)

Published in: *Comput.Phys.Commun.* 191 (2015) 159-177 • e-Print: [1410.3012](https://arxiv.org/abs/1410.3012) [hep-ph]

[pdf](#) [links](#) [DOI](#) [cite](#)

⇒ 4,050 citations



## Herwig 7

### Herwig++ Physics and Manual

M. Bahr (Karlsruhe U., ITP), S. Gieseke (Karlsruhe U., ITP), M.A. Gigg (Durham U., IPPP), D. Grellscheid (Durham U., IPPP), K. Hamilton (Louvain U.) et al. (Mar, 2008)

Published in: *Eur.Phys.J.C* 58 (2008) 639-707 • e-Print: [0803.0883](https://arxiv.org/abs/0803.0883) [hep-ph]

[pdf](#) [links](#) [DOI](#) [cite](#)

⇒ 2,644 citations



## Sherpa

### Event generation with SHERPA 1.1

T. Gleisberg (SLAC), Stefan. Hoeche (Zurich U.), F. Krauss (Durham U., IPPP), M. Schonherr (Dresden, Tech. U.), S. Schumann (Edinburgh U.) et al. (Nov, 2008)

Published in: *JHEP* 02 (2009) 007 • e-Print: [0811.4622](https://arxiv.org/abs/0811.4622) [hep-ph]

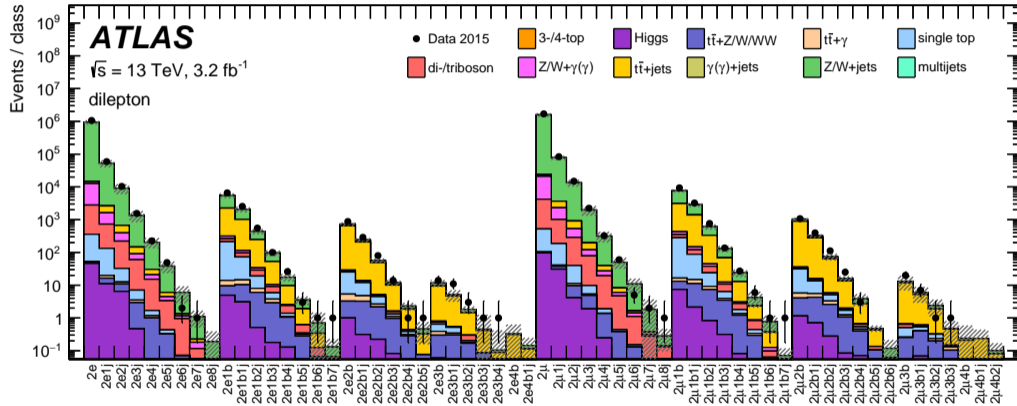
[pdf](#) [links](#) [DOI](#) [cite](#)

⇒ 3,386 citations

Parton Showers enter one way or another in almost 95% of all ATLAS and CMS analyses. Collider physics would not be the same without them.



# The ubiquitous Parton Shower



ATLAS [1807.07447]



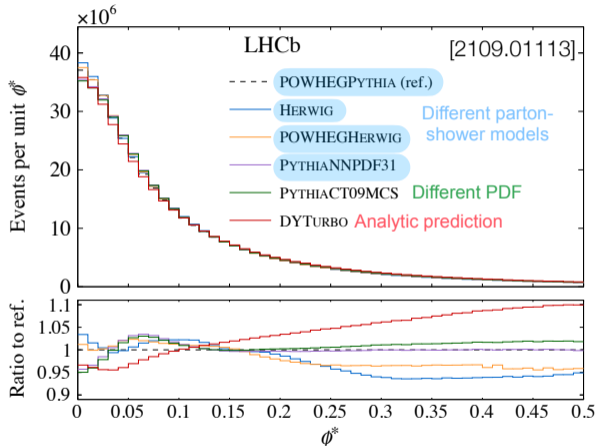
# But differences matter...

Consider measurement of W boson mass

Measurements of  $p_T^Z$  in  
 $Z/\gamma^* \rightarrow l^+l^-$  decays used to  
 validate the MC predictions for  $p_T^W$

The envelope of shifts in  $m_W$   
 originating from differences in these  
 shower predictions is the dominant  
 theory uncertainty (11 MeV)

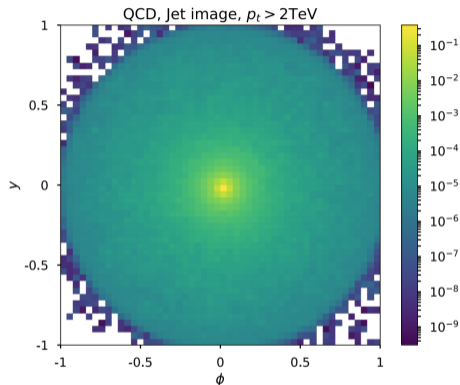
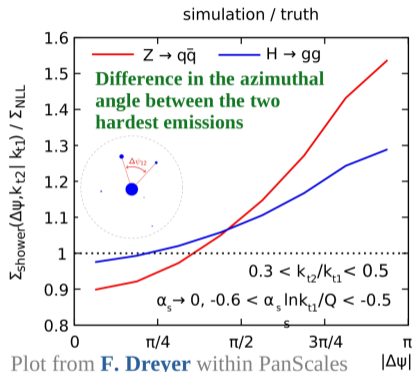
$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$



$$\phi^* = \frac{\tan((\pi - \Delta\phi)/2)}{\cosh(\Delta\eta/2)} \sim \frac{p_T^Z}{m_{ll}} [1009.1580]$$



# Machine learning and jet sub-structure



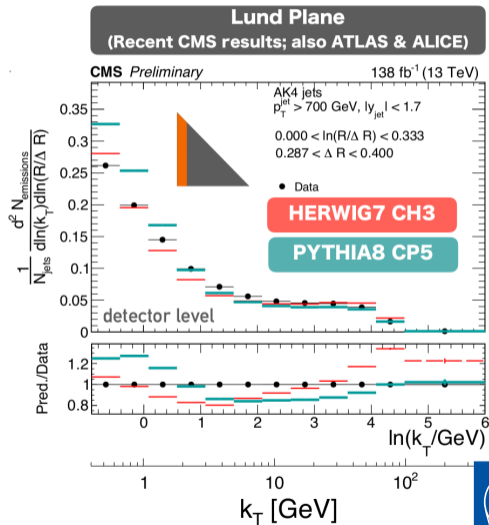
de Oliveira, Kagan, Mackey, Nachmann, Schwartzman [1511.05190]

Machine learning might learn **un-physical “features”** from MC → can significantly impact the potential of new physics searches.

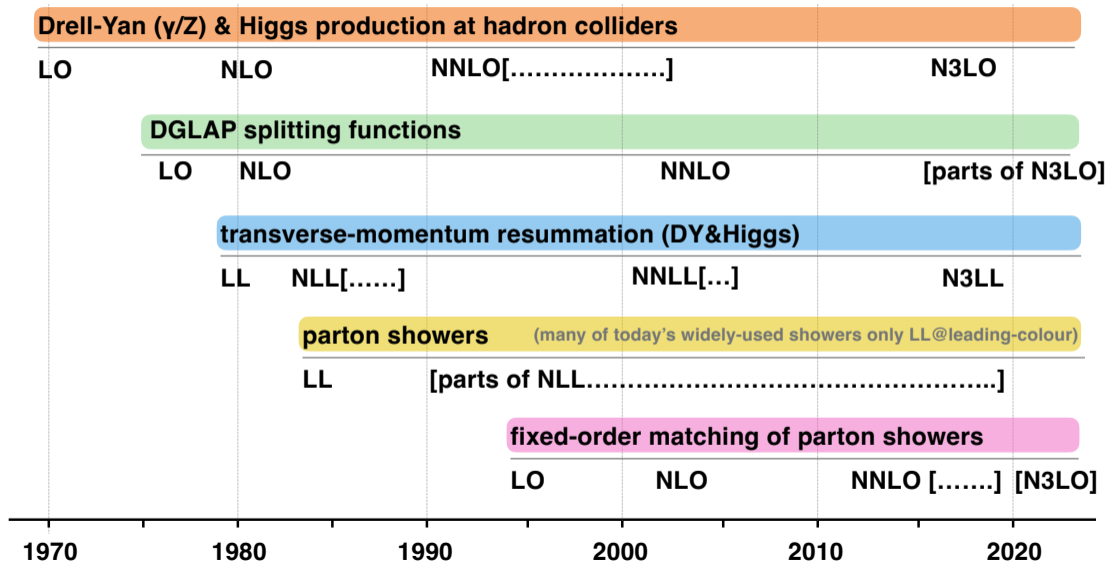


# Lund Plane measurements

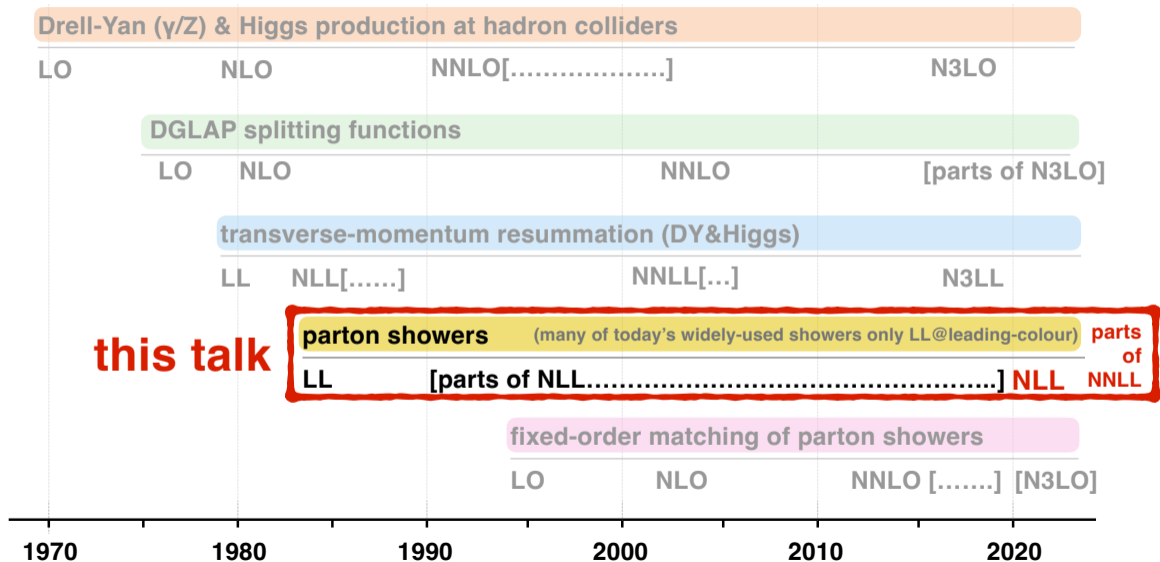
- Despite common showers doing an amazing job at the LHC, there are still places where **big differences** are seen
- In particular as we zoom into very differential phase space regions of jets, these differences can easily reach **10–30%**
- The region shown here is particularly sensitive to **soft emissions**
- This is a region where some of the developments discussed later are relevant
- See also [CMS \[2312.16343\]](#)



# selected collider-QCD accuracy milestones



# selected collider-QCD accuracy milestones



## Why are we talking about logarithmic accuracy?

Parton showers **evolve** hard states  $Q \sim \sqrt{\hat{s}}$  down to the scale where hadronisation takes place  $\Lambda \sim 1 \text{ GeV}$

This evolution **generates logarithms** of the form  $L \sim \ln \frac{Q}{\Lambda} \gg 1$ ,  $(g_X(\alpha_S L) \sim \alpha_S L)$

$$\Sigma(\Theta < e^{-L}) = \exp \left[ -L g_{LL}(\alpha_S L) + g_{NLL}(\alpha_S L) + \alpha_S g_{NNLL}(\alpha_S L) + \dots \right]$$

	$Q = M_Z$	$Q = 1 \text{ TeV}$
$ L g_{LL}  \sim \alpha_S L^2$	2	4
$ g_{NLL}  \sim \alpha_S L$	0.5	0.6 $\leftarrow \mathcal{O}(100\%)$
$ \alpha_S g_{NNLL}  \sim \alpha_S^2 L$	0.06	0.05 $\leftarrow \mathcal{O}(10\%)$

NNLL **crucial** to reach **percent-level** accuracy!

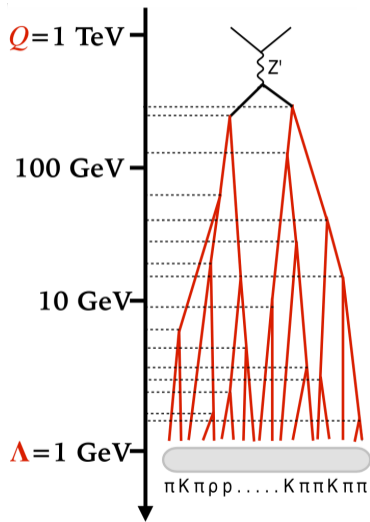


Figure by S. Ferrario Ravasio

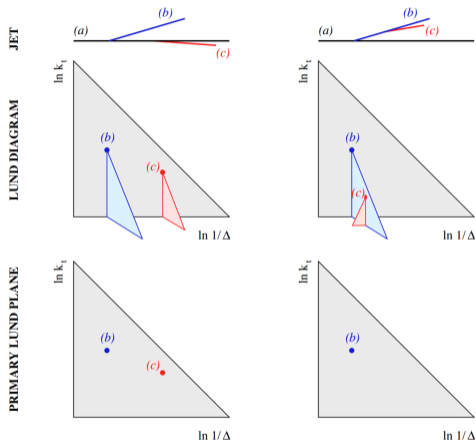


## Current status of parton showers

- The most widely-used event generators at the LHC, Pythia, Herwig, and Sherpa, are **all limited to LL** (some exceptions where NLL can be reached, cf. Bewick, Ferrario Ravasio, Richardson, Seymour [1904.11866])
- Although there has been significant progress in improving the hard matrix elements of event generators with **NNLO matching** and **NLO multi-jet merging**, the logarithmic accuracy has been limited to LL for a very long time
- For this reason, there has been a concerted effort in taking parton showers from **LL**→**NLL** in the last couple of years
- This has been achieved by several groups including PanScales [1805.09327], [2002.11114], [2011.10054], [2103.16526], [2111.01161], [2205.02237], [2207.09467], [2305.08645], [2312.13275], ALARIC Herren, Höche, Krauss, Reichelt, Schoenherr [2208.06057], [2404.14360], APOLLO Preuss [2403.19452], DEDUCTOR Nagy, Soper [2011.04773], and Forshaw-Holguin-Plätzer [2003.06400]
- Very recently we have taken significant steps towards general **NNLL** (focus of this talk)



# The Lund Plane

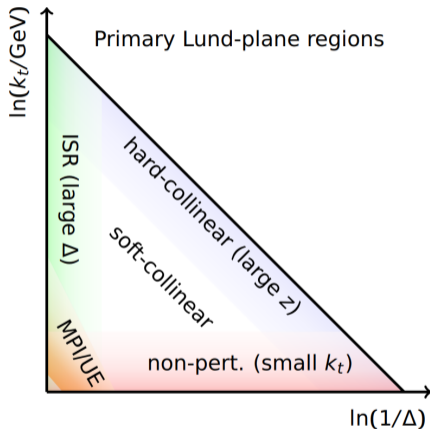


Dreyer, Salam, Soyez [1807.04758]

- To better make the connections between parton showers and their logarithmic accuracy we need to introduce the **Lund Plane**:
- **Cluster the event** with the Cambridge/Aachen algorithm, producing an angular ordered clustering sequence.
- **Decouple** the last clustering and record the **transverse momentum** and the **opening angle** of the declustering (plus other kinematics).
- Iterate along the **hardest branch** after each declustering to produce the **primary** Lund Plane.
- Following the softer branch produces the secondary, tertiary, etc Lund Plane.
- One can impose cuts easily on the declusterings (e.g. that they satisfy  $z > z_{\text{cut}}$ )



# Logarithms in the Lund Plane



Dreyer, Salam, Soyez [1807.04758]

- The emission probability in the Lund Plane is then
 
$$d\rho \sim \alpha_s d \ln k_T d \ln \theta$$
- Hence emissions that are well-separated in **both** directions are associated with **double logarithms** of the form  $\alpha_s^n L^{2n}$
- Emissions separated along one direction are associated with **single logarithms** of the form  $\alpha_s^n L^n$
- Emissions that are **close in the Lund Plane** are associated with a factor  $\alpha_s^n$
- We are now ready to state the **PanScales NLL criteria** for Parton Showers

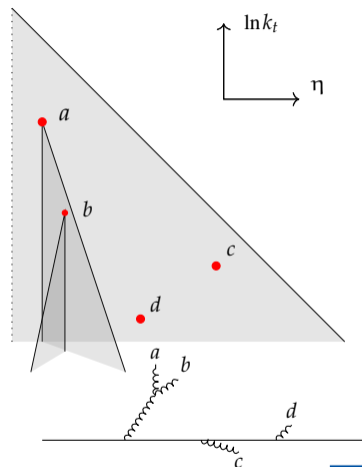




# NLL showers in a nutshell

- A necessary condition for a shower to be NLL is that it correctly describes configurations where **all** emissions are well-separated in a Lund plane [Dasgupta, Dreyer, Hamilton, Monni, Salam \[1805.09327\]](#)
- A core principle in this picture is that two emissions that are well-separated, should **not** influence each other (e.g. emission *d* cannot change the kinematics of *c*)<sup>a</sup>.
- This principle is **violated** by most standard dipole-showers, due to the way the recoil is distributed after an emission First observed by Andersson, Gustafson, Sjogren '92
- For NLL 2-loop running coupling in the CMW scheme is also required
- For full NLL one also needs to include **spin-correlations** and sub-leading **colour** corrections

<sup>a</sup>Spin-correlations are an exception in this context as they introduce long-range azimuthal correlations at NLL. Collinear spin understood in angular ordered showers for decades due to work of Collins '88 and Knowles '88. Extension to dipole showers studied in [Richardson, Webster \[1807.01955\]](#). Both **collinear** and **soft** spin-correlations are included in PanScales at NLL.



**PanLocal** $k_t\sqrt{\theta}$  ordered**Recoil** $\perp$ : local

+: local

-: local

**Dipole partition**  
event CoM**PanGlobal** $k_t$  or  $k_t\sqrt{\theta}$  ordered**Recoil** $\perp$ : global

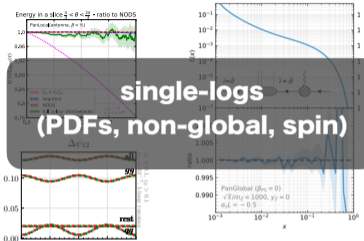
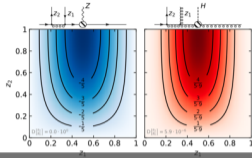
+: local

-: local

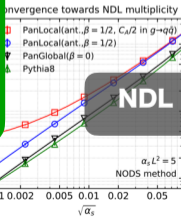
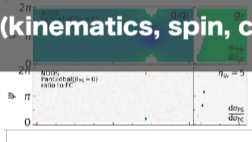
**Dipole partition**  
event CoM**Colour**nested ordered  
double soft  
(NODS)Designed to  
ensure LL are  
full colour  
(also gets many  
NLL at full  
colour)Hamilton, Medves,  
Salam, Scyboz, Soyez  
[2011.10054]**Spin**for correct  
azimuthal  
structure in  
collinear and  
soft  $\rightarrow$  collinear[Collins-Knowles  
extended to soft  
sector]AK, Salam, Scyboz, Verheyen  
[2103.16526],  
eid. + Hamilton [2111.01161] $e^+e^-$ : Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez  
[2002.11114];  $pp$  (w/spin+colour): van Beekveld, Ferrario  
Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]; +  
 $pp$  tests: eid. + Hamilton [2207.09467]; DIS+VBF: van Beekveld,  
Ferrario Ravasio [2305.08645]

# a selection of the logarithmic accuracy tests

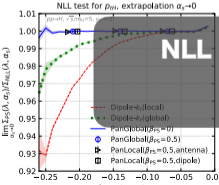
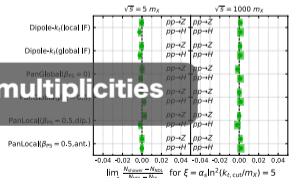
TESTS



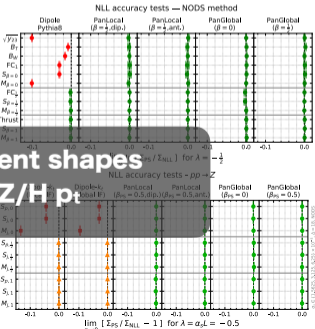
fixed order (kinematics, spin, colour)



NDL multiplicities



NLL event shapes & Z/H  $p_{\perp}$



## Oxford



Gavin Salam



Jack Helliwell



Silvia Zanoli

## NIKHEF



Melissa van Beekveld

## Manchester



Mrinal Dasgupta

## UCL



Keith Hamilton

## Monash



Basem El-Menoufi



Ludo Scyboz

## IPhT



Gregory Soyez

## CERN



AK



Silvia Ferrario Ravasio



Pier Monni

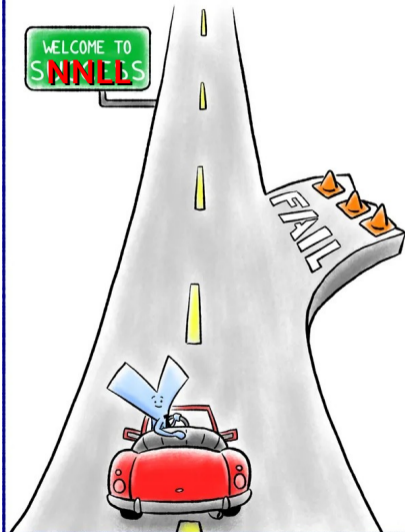


Alba Soto-Ontoso

## PanScales current members

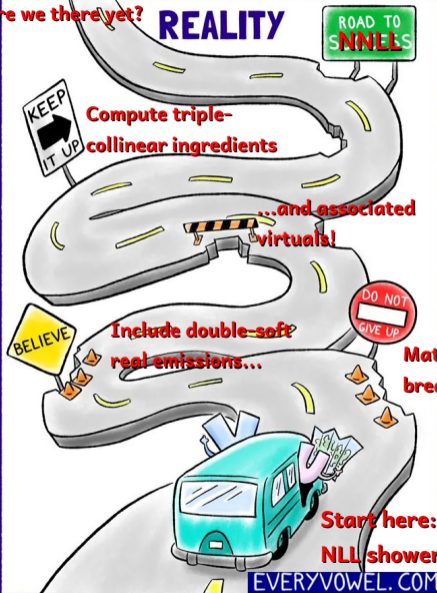
A project to bring logarithmic understanding and accuracy to parton showers

# EXPECTATION



Are we there yet?

# REALITY



Start here:  
NLL shower

EVERYVOWEL.COM



## Analytic structure beyond NLL

Taking an event shape,  $\mathcal{O}$ , to be less than some value  $e^{-|L|}$  we have at **NNLL** (focusing for now on  $e^+e^-$  only)

$$\Sigma(\mathcal{O} < e^{-|L|}) = (1 + \alpha_s C_1 + \dots) \exp \left[ \frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots \right] \quad (1)$$

where  $g_1$  accounts for LL terms,  $g_2$  for NLL terms, and  $g_3$  and  $C_1$  for NNLL terms<sup>1</sup>. Whereas an analytic resummation in principle retains only the terms that are put in (i.e.  $g_1$  and  $g_2$  at NLL) the shower will instead generate spurious higher order terms

$$\Sigma(\mathcal{O} < e^{-|L|}) = (1 + \alpha_s \tilde{C}_1 + \dots) \exp \left[ \frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s \tilde{g}_3(\alpha_s L) + \dots \right] \quad (2)$$

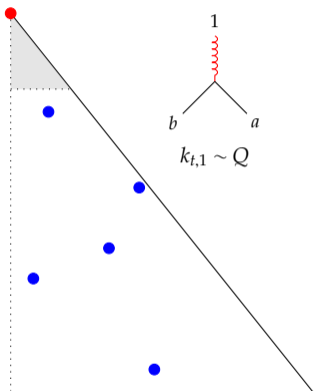
When thinking about going beyond NLL we need to address two things: 1) what are the necessary **analytic ingredients** from resummation and 2) how do we **compensate** the NNLL terms already present in the shower?

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<sup>1</sup>In the language of  $q_T$  resummation  $A_1$  is responsible for LL terms,  $A_2$  and  $B_1$  for NLL terms and  $A_3$  and  $B_2$  for NNLL terms (together with the hard coefficient function  $C_1(z)$ ).

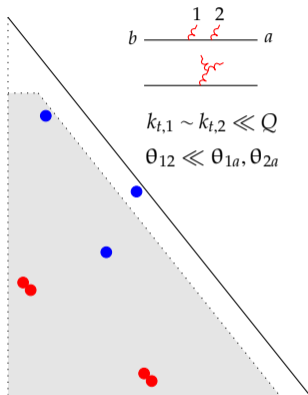


## Lund plane picture



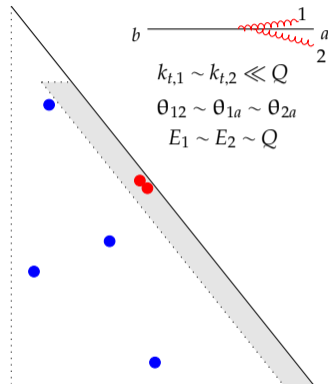
**hard matching**  $\rightarrow$

$\alpha_S$  correct for first emission



**double-soft**  $\rightarrow$

get any pair of soft commensurate energy/angle right

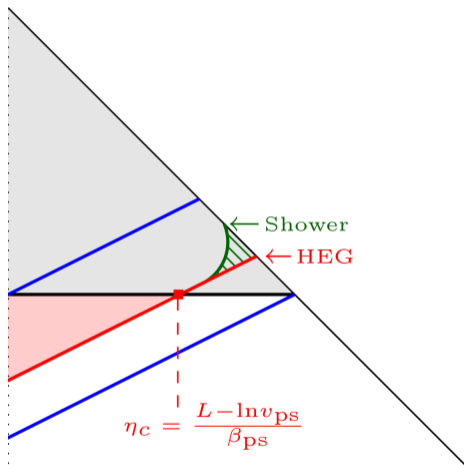


**triple-collinear**  $\rightarrow$

account for genuine  $2 \rightarrow 4$  collinear splittings



# Match without breaking NLL

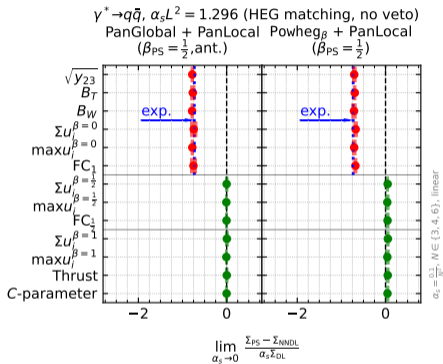


- We have so far explored the two-body decays  $\gamma \rightarrow q\bar{q}$  and  $h \rightarrow gg$  @ NLO
- For matching schemes that rely on the shower to generate the first emission (such as MC@NLO, KrkNLO, and MAcNLOPS) **the matching works more or less out of the box.**
- For POWHEG style matchings (including MiNNLO and GENEVA) **log accuracy is more subtle to maintain.**
- Main concern related to kinematic mismatch between **shower** and **hardest emission generator** (in general they are only guaranteed to agree in the soft-collinear region). This issue has been studied in the past [Corke, Sjöstrand \[1003.2384\]](#) but logarithmic understanding is new.

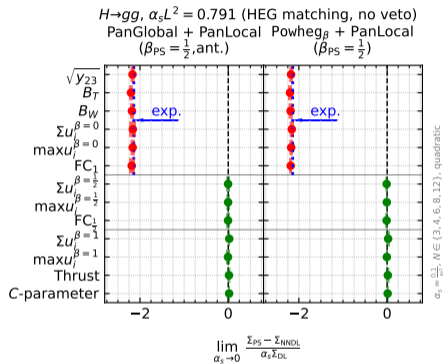




# HEG without a veto is not NN DL ( $\alpha_s^n L^{2n-2}$ ) accurate



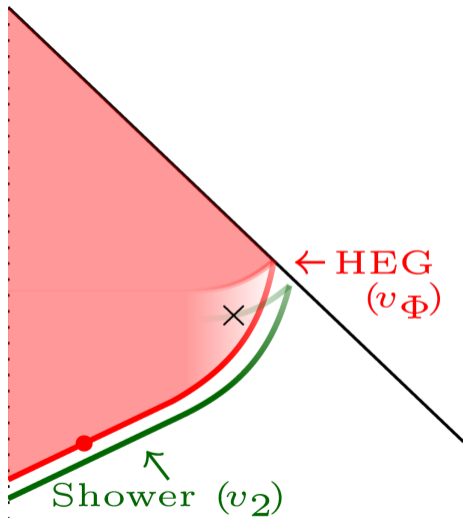
$$\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{PS} - \Sigma_{NN DL}}{\alpha_s \Sigma_{DL}} \Big|_{\text{fixed } \alpha_s L^2}$$



Without a veto NLL accurate showers **fail** our NN DL ( $\alpha_s^n L^{2n-2}$ ) event shape tests. The failure is  $\mathcal{O}(1)$ , and hence phenomenologically relevant. The **dashed blue** line indicates the analytically calculated expected value.



## Further subtleties



- Even when the contours are fully aligned there are issues associated with how dipole showers **partition** the  $g \rightarrow gg(q\bar{q})$  splitting function.
- In PanScales we use

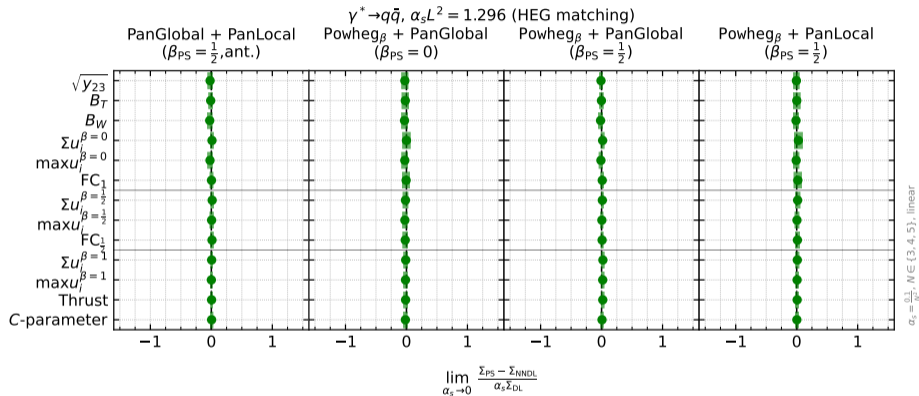
$$\frac{1}{2!} P_{gg}^{\text{asym}}(\zeta) = C_A \left[ \frac{1 + \zeta^3}{1 - \zeta} + (2\zeta - 1) w_{gg} \right],$$

such that  $P_{gg}^{\text{asym}}(\zeta) + P_{gg}^{\text{asym}}(1 - \zeta) = 2P_{gg}(\zeta)$

- This partitioning takes place to isolate the two soft divergences in the splitting function ( $\zeta \rightarrow 0$  and  $\zeta \rightarrow 1$ ), but there is some freedom in how one handles the **non-singular part**.
- The HEG needs to partition in **exactly** the same way. Not clear how easy this is in general, in particular in the soft-large angle region.



# Proper HEG achieves NNDL ( $\alpha_s^n L^{2n-2}$ ) accuracy



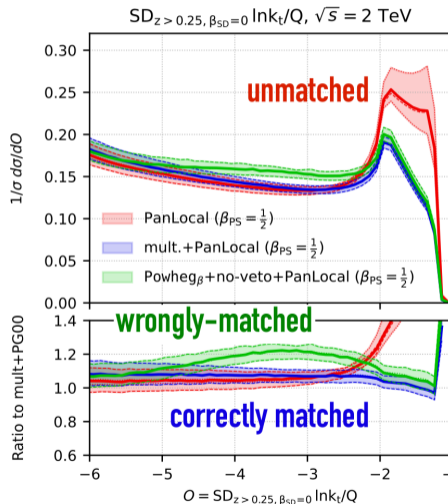
This can be **achieved** through a standard kinematic veto, as long as shower partitioning **matches** the exact matrix element. A veto however **complicates** the inclusion of double-soft emissions, since it effectively alters the **second emission**, complicating the path to further logarithmic enhancement.



# Phenomenological impact

- Contour mismatch by area  $\alpha\Delta$  leads to **breaking** of NLL and exponentiation
- Correct matching on the other hand **augments** the shower from NLL to NLL+NNDL for event shapes.
- Impact of NLL breaking terms vary - for SoftDrop they have a **big impact** due to the single-logarithmic nature of the observable. In particular the breaking manifests as terms with **super-leading** logs

$$\partial_L \Sigma_{SD}(L) = \bar{\alpha} c e^{\bar{\alpha} c L - \bar{\alpha} \Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2} (1 - e^{-\bar{\alpha} \Delta})$$

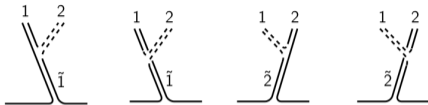


## Include double-soft real emissions

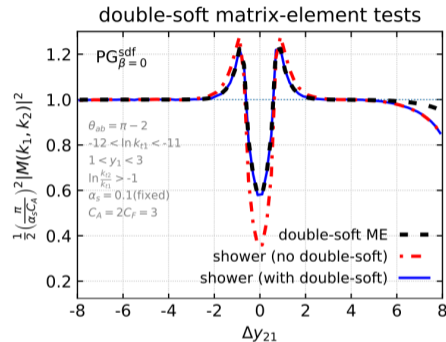
- NLO matching is a necessary ingredient for going beyond NLL, but to some extent NLO matching is a **solved problem**
- **Until recently** the inclusion of double-soft emissions in an NLL shower was still an **open question**
- To get them right we must ensure that **any pair** of soft emissions with commensurate energy and angles should be produced with the **correct ME**
- Any additional soft radiation off that pair must also come with the correct ME
- In addition must get the single-soft emission rate right at NLO (CMW-scheme)
- This should achieve **NNDL accuracy** for multiplicities, i.e. terms  $\alpha_s^n L^{2n}$ ,  $\alpha_s^n L^{2n-1}$  **and**  $\alpha_s^n L^{2n-2}$
- and next-to-single-log (**NSL**) accuracy for non-global logarithms, for instance the energy in a rapidity slice,  $\alpha_s^n L^n$  and  $\alpha_s^n L^{n-1}$  (albeit only at leading- $N_C$  for now)



# The double-soft ME



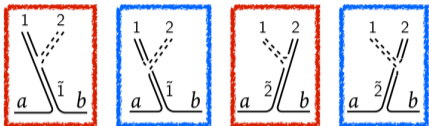
- For now we have focused on PanGlobal
- Any two-emission configuration in a dipole-shower comes with up to **four histories** (for PanLocal this would in fact be eight)
- We accept any such configuration with the true ME divided by the shower's **effective double-soft ME** summed over all histories that could have lead to that configuration.



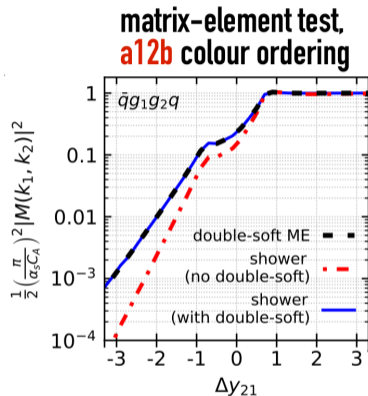
$$P_{\text{accept}} = \frac{|M_{\text{DS}}^2|}{\sum_h |M_{\text{shower},h}^2|}$$



# Correcting the colour-ordering



- We have two distinct colour orderings  $a12b$  and  $a21b$
- We need to get the relative fractions  $F^{(12)}$  and  $F^{(21)}$  right in order to ensure that any further emissions are also correct.
- In practice we **accept** a colour ordering if the shower generates too little of it, and **swap** them if the shower generates too much (and similarly for  $q\bar{q}$  vs  $gg$  branchings).



$$P_{\text{swap}} = \frac{F_{\text{shower}}^{(12)} - F_{\text{DS}}^{(12)}}{F_{\text{shower}}^{(12)}}$$

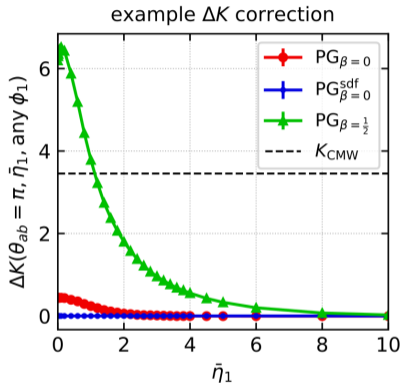


## ...and associated virtuals!

- The PanScales showers have **correct** soft emission intensity at NLO in the **soft-collinear** (sc) region due to the use of the CMW-coupling

$$\alpha_s \rightarrow \alpha_s + \alpha_s^2 K_1 / 2\pi$$

- This in general is not enough to get to soft wide-angle region right and we need to add a  $\Delta K_1$  which depends on the rapidity of the single soft emission
- This is related to the fact, that the shower organises its phase space in such a way, that the rapidity of soft pair,  $y_{12}$ , **does not coincide** with the parent rapidity,  $y_{\bar{1}}$ .

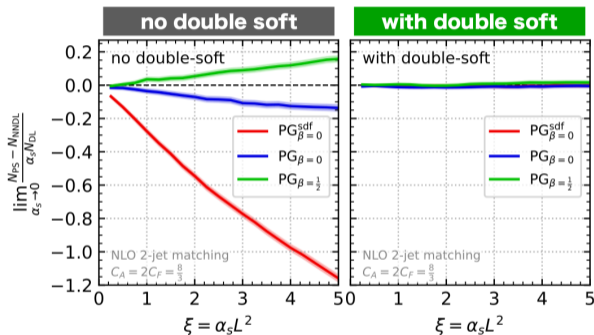


$$\Delta K_1 = \int d\Phi_{12/\bar{1}}^{(PS)} |M_{12/\bar{1}}^{(PS)}|^2 - \int d\Phi_{12/\bar{1}_{sc}}^{(PS)} |M_{12/\bar{1}_{sc}}^{(PS)}|^2.$$





# Lund Multiplicities at NNDL ( $\alpha_s^n L^{2n-2}$ )

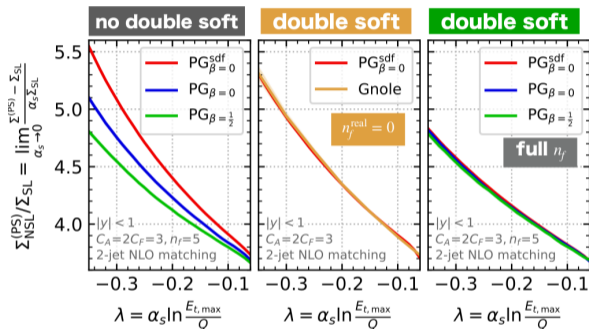


$$\lim_{\alpha_s \rightarrow 0} \frac{N_{(PS)} - N_{NNDL}}{\alpha_s N_{DL}} \Big|_{\text{fixed } \alpha_s L^2}$$

- Reference NNDL analytic result from Medves, Soto-Ontoso, Soyez [2205.02861]
- We take  $\alpha_s \rightarrow 0$  limit to isolate NNDL terms. This is **significantly more challenging** than at NDL due to presence of  $1/\alpha_s$  in denominator.
- Showers without double-soft corrections show **clear differences** from reference (and each other).
- Adding the double-soft corrections brings **NNDL agreement**.



# Energy in a slice at NSL ( $\alpha_s^n L^{n-1}$ )

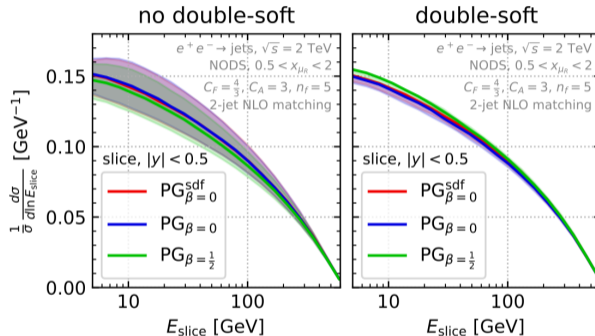


$$\lim_{\alpha_s \rightarrow 0} \frac{\Sigma^{(\text{PS})} - \Sigma_{\text{SL}}}{\alpha_s} \Big|_{\text{fixed } \alpha_s L}$$

- Reference NSL from Gnole Banfi, Dreyer, Monni [2111.02413] (see also Becher, Schalch, Xu [2307.02283]).
- We did this test **semi-blind**: only compared to Gnole after we had agreement between the three PanGlobal variants.
- We have **NSL agreement with Gnole** (using  $n_f^{\text{real}} = 0$ ) and agreement between all showers with full- $n_f$  dependence (first calculation of this kind as a by-product!)



# What about pheno?



- We studied energy flow between two hard (1 TeV) jets as a **preliminary** pheno case
- The three PanGlobal variants are remarkably close without double-soft corrections, but have **large uncertainties**
- With double-soft corrections we see a small shift in central values but a **significant reduction in uncertainties**.



# Compute triple-collinear ingredients

- Double-soft corrections are **not** by themselves enough to reach NNLL accuracy for event shapes. We also need triple-collinear ingredients (cf. Dasgupta, El-Menoufi [2109.07496], eid. + van Beekveld, Helliwell, Monni [2307.15734], eid. + AK [2402.05170] for work in this direction)
- However, it turns out that with the inclusion of real double-soft emissions, only the **Sudakov form factor** needs to be modified to reach NNLL for event shapes, i.e. we do not need the fully differential triple-collinear structure
- Taking

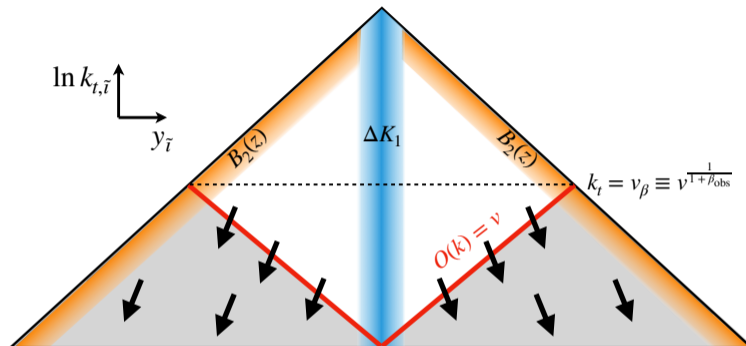
$$\alpha_{\text{eff}} = \alpha_s \left[ 1 + \frac{\alpha_s}{2\pi} (K_1 + \Delta K_1(y) + B_2(z)) + \frac{\alpha_s^2}{4\pi^2} K_2 \right]$$

there are two pieces missing -  $B_2$  which is of triple-collinear origin [2109.07496], [2307.15734] and  $K_2$  ( $A_3$ ) which is known Banfi, El-Menoufi, Monni [1807.11487], Catani, De Florian, Grazzini [1904.10365]

- NB: NLL showers generate spurious  $\tilde{B}_2$  and  $\tilde{K}_2 \rightarrow$  must be **compensated**



# An intuitive picture



Imagine an emission,  $\tilde{I}$ , sitting anywhere right at the observable boundary (red line). The key observation is that whenever the shower splits  $\tilde{I} \rightarrow 12$ , the kinematic variables  $(y_{12}, k_{t,12}, z_{12})$  of the resulting pair, do not agree with that of the parent  $(y_{\tilde{I}}, k_{t,\tilde{I}}, z_{\tilde{I}})$ . Since the Sudakov was computed assuming conserved kinematics of  $\tilde{I}$ , and the observable is computed with the actual kinematics of  $(12)$ , we have generated a mismatch. We can compute these drifts!



## Relation between shower and resummation ingredients

It is fairly straightforward to see that at NNLL we **only depend** on  $\Delta K_1$  and  $B_2$  through their respective **integrals**

$$\Delta K_1^{\text{int}} \equiv \int_{-\infty}^{\infty} dy \Delta K_1(y), \quad B_2^{\text{int}} \equiv \int_0^1 dz \frac{P_{gq}(z)}{2C_F} B_2(z).$$

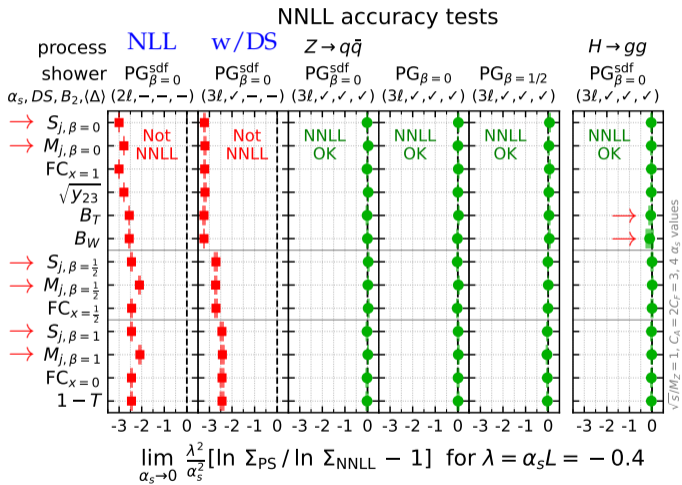
These (and  $K_2$ ) can be related to the **drifts** in  $y$  ( $\langle \Delta_y \rangle$ ),  $\ln z$  ( $\langle \Delta_{\ln z} \rangle$ ), and  $\ln k_t$  ( $\langle \Delta_{\ln k_t} \rangle$ ) and analytical resummation through

$$\Delta K_1^{\text{int,PS}} = 2\langle \Delta_y \rangle, \quad B_2^{\text{int,PS}} = B_2^{\text{int,NLO}} - \langle \Delta_{\ln z} \rangle, \quad K_2^{\text{PS}} = K_2^{\text{resum}} - 4\beta_0 \langle \Delta_{\ln k_t} \rangle.$$

Using these relations and taking  $B_2^{\text{int,NLO}}$  from [2109.07496], [2307.15734] and  $K_2^{\text{resum}}$  from [1807.11487] one can **prove** that our showers are **NNLL accurate for event-shape observables**.



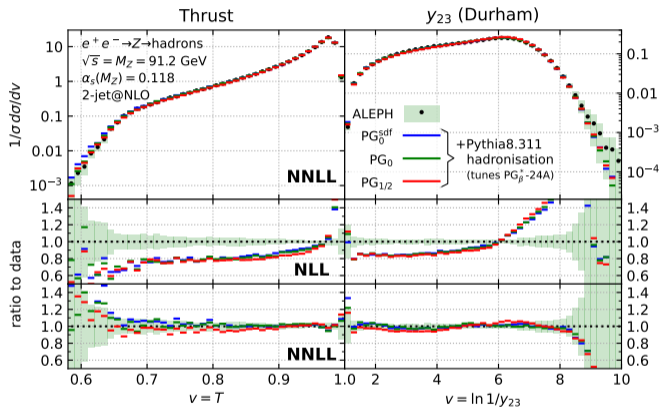
# Are we there yet?



- : New analytic results, not available in literature van Beekveld, Buonocore, El-Menoufi, Ferrario Ravasio, Monni, Soto-Ontoso, Soyez [in preparation]
- With no NNLL improvements, the coefficient of NNLL difference is significant,  $\mathcal{O}(2-3)$ , indicating importance of getting NNLL right
- With the inclusion of double-soft, observables with the same  $\beta_{\text{obs}}$  align but do still not agree with the analytics
- After inclusion of shifts and  $B_2$  and  $K_2$  we have perfect agreement



# Not far now...



Long-standing **tension** between LEP data and Pythia8 unless using an **anomalously** large value of  $\alpha_s(M_Z) = 0.137$  Skands, Carrazza, Rojo [1404.5630] (also present for PanScales showers)

Inclusion of NNLL brings **large** corrections with respect to NLL

**Agreement** with data achieved **without** anomalously large value of  $\alpha_s$

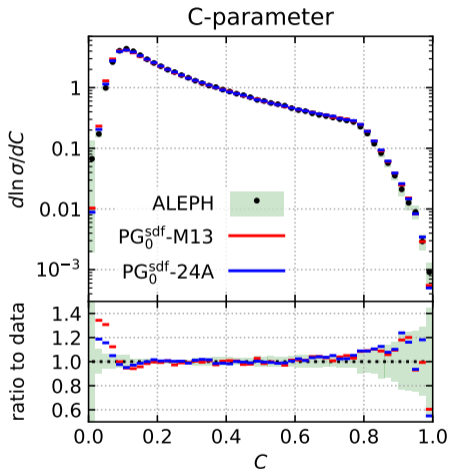
**Beware:** no 3j@NLO which is known to be relevant in the hard regions

Residual uncertainties still need to be worked out





# What about tuning?



Improved agreement with data across a large range of event shapes

Tuning here still rough

→ We start from the Monash tune (see ref. above) but fix  $\alpha_s(M_Z) = 0.118$  (M13)

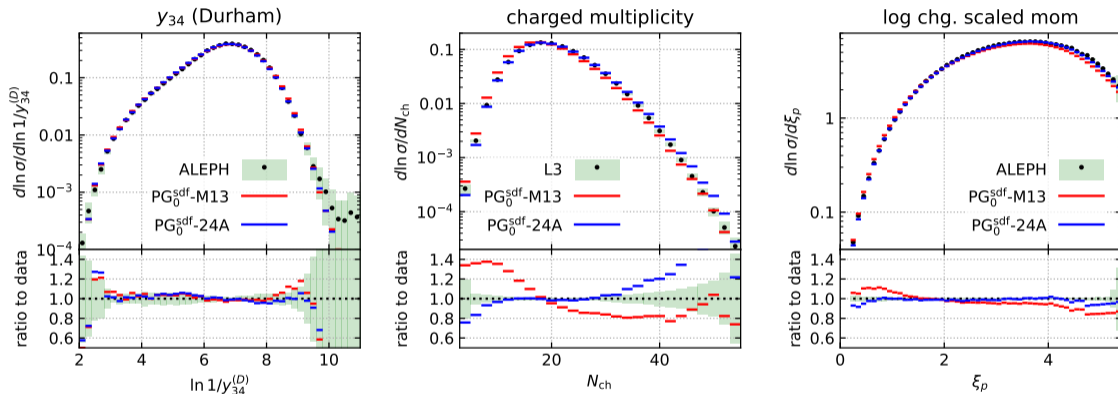
For our NLL showers this is the tune we use

For the NNLL showers we tune a number of parameters in the string model semi-automatically (24A)

Full tuning exercise still to be done!



# What about tuning?



Impact of tune **very minor** on infrared safe observables, even those that are only NLL accurate

Impact on unsafe observables **much larger**, bringing good agreement with ALEPH data.



# Conclusions and outlook

- As the experiments at the LHC record more and more data, it will become increasingly more important to **improve on the accuracy** of event generators
- NLL accurate showers have now been established by several groups
- First steps towards general NNLL accuracy was taken recently with the inclusion of double-soft corrections in the PanGlobal family of showers
- With these corrections we have reached **NNDL accuracy** for multiplicity and **NSL accuracy** for non-global observables
- The next natural step is to get **NNLL right for event shapes**
- This can be achieved using known ingredients from resummation together with an understanding of how the showers differ from analytic resummation through mainly recoil
- This we have **achieved** very recently
- The associated NNLL code has been made public in a the 0.2 release of the **PanScales code**
- Naturally we now are thinking about how to bring these advances to hadron-collisions
- For full general NNLL the shower needs to also correctly reproduce triple-collinear kinematics (e.g. for fragmentation functions)
- Work in that direction is also ongoing

