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# Electroweak corrections to double Higgs production at the LHC

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Based on: Phys. Rev. Lett. 132 (2024), 231802 (arXiv: 2311.16963)

Huan-Yu Bi, Li-Hong Huang, Rui-Jun Huang, Yan-Qing Ma, Huai-Min Yu

JHEP 11 (2024) 040, (arXiv: 2407.04653)

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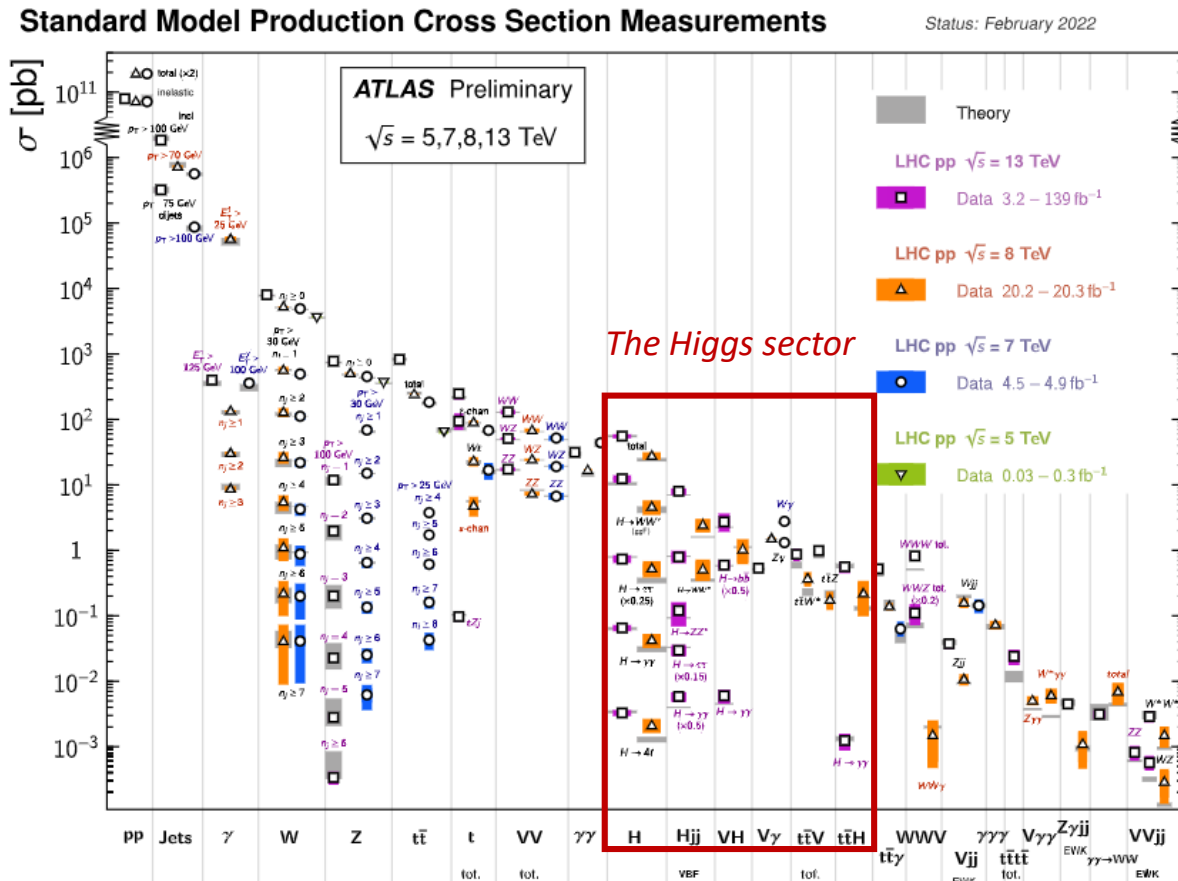
Gudrun Heinrich, Stephen Jones, Matthias Kerner, Thomas Stone, Augustin Vestner

# Introduction to Higgs



- Discovery of Higgs boson(2012,LHC): the last found elementary particle in SM.
- Experiments at the ATLAS and CMS: consistent with the results predicted by SM.

Azzurri: *Int.J.Mod.Phys.A* 38 (2023) 09n10, 23300077

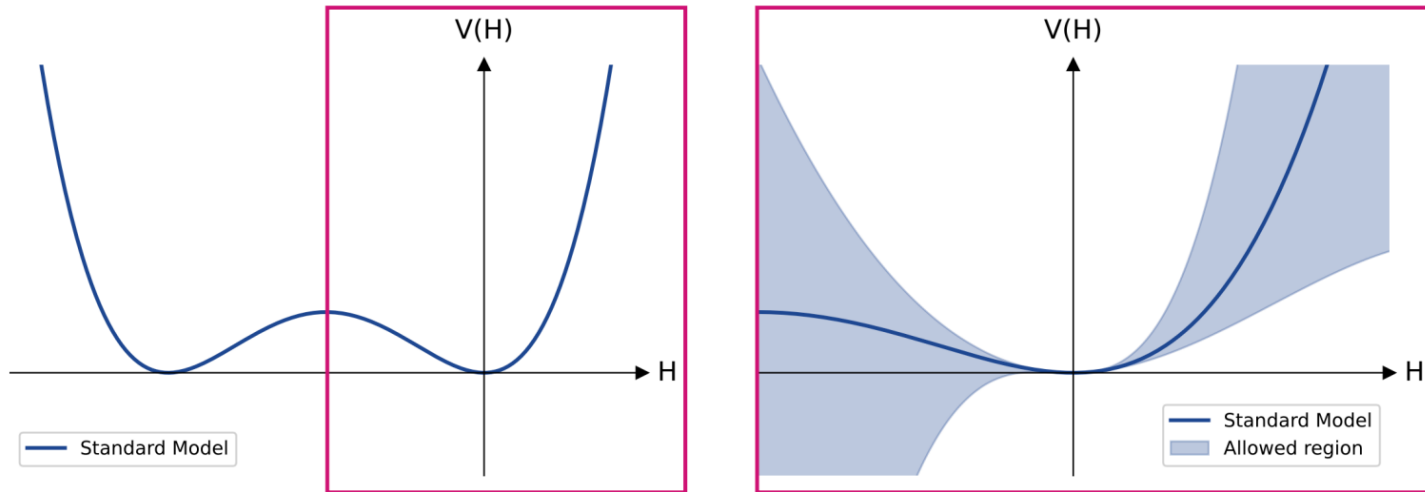


# Higgs Potential



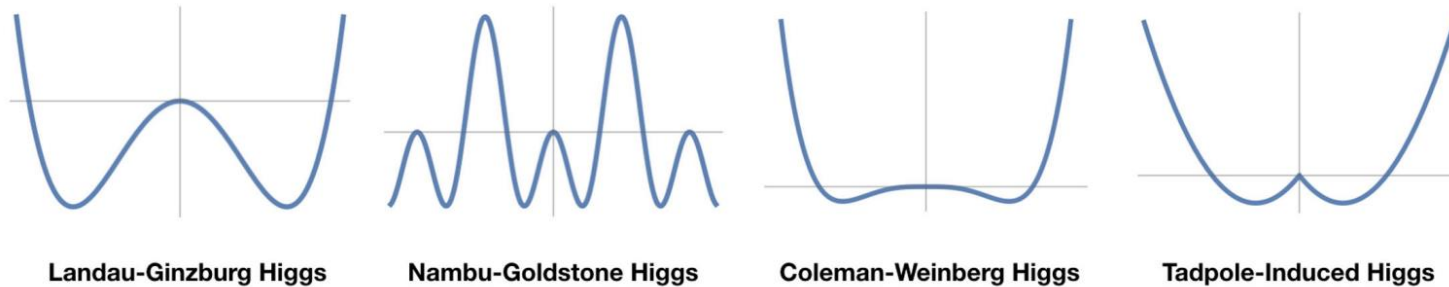
- **Problems not clear:** shape of Higgs potential, new physics beyond SM...

Plot taken from B. Moser: [Higgs 2023](#)



[using current ATLAS limits @ 95% CL]

[Agrawal et al: 1907.02078](#)



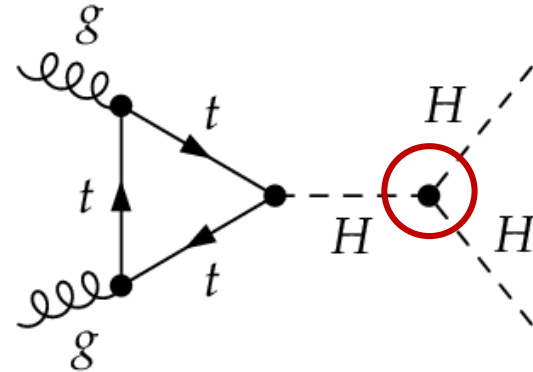
Higgs potential predicted by other BSM theories.

# Higgs trilinear coupling



- Higgs potential is probed through determining the strength of Higgs boson self-interactions in searches for HH production. ( $\lambda^{SM} \approx 1/8$ )

$$V(h) = \frac{m_h^2}{2} h^2 + \lambda^{SM} v h^3 + \frac{1}{4} \lambda^{SM} h^4$$



- Experiment constraints on Higgs boson self-interactions:

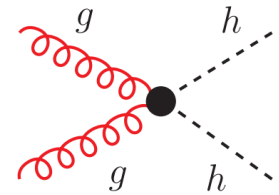
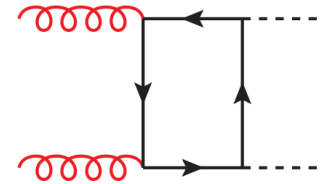
ATLAS: 2007.02873

CMS: 2202.09617

Jones: LHEP 2023 (2023) 442

- Current:  $-1.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 6.7$  for ATLAS,  $-2.3 < \lambda_{hhh}^{EX}/\lambda^{SM} < 9.4$  for CMS.
- Future: a limit of  $-0.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 1.5$  will be achieved.

# QCD corrections status



Ajjath et al:2209.03914

- corrections **with full top quark mass dependence**
  - NLO corrections keeping top quark mass, [Borowka et al:1604.06447](#)
  - NLO corrections matched to parton shower, [Heinrich et al:1703.09252](#)
  - NLO corrections with soft-gluon resummation, [Ferrera et al: 1609.01691](#)
  - Approximations for top-mass effects at NNLO, [Grazzini et al: 1803.02463](#)
- corrections **in heavy top limit (HTL) approximation**
  - N<sup>2</sup>LO in HTL, [Florian et al:1305.5206](#)
  - N<sup>3</sup>LO in HTL, [Chen et al:1909.06808](#)
  - N<sup>2</sup>LO in HTL+ NLO with full top-quark mass dependence, [Florian et al:2106.14050](#)
  - N<sup>3</sup>LO in HTL include the top-quark mass effects, [Chen et al:1912.13001](#)
  - N<sup>2</sup>LO in HTL matched to parton shower, [Alioli et al: 2212.10489](#)
  - N<sup>3</sup>LO in HTL+ NLO with full top-quark mass dependence + soft-gluon resummation, [Ajjath et al:2209.03914](#)
- Top quark mass scheme uncertainties: **pole mass versus  $\overline{MS}$  mass.** [Baglio et al: 1811.05692](#)
- Current QCD corrections uncertainties:  **$\mathcal{O}(1\%)$**  [Jones: LHEP 2023 \(2023\) 442](#)

# Why EW corrections

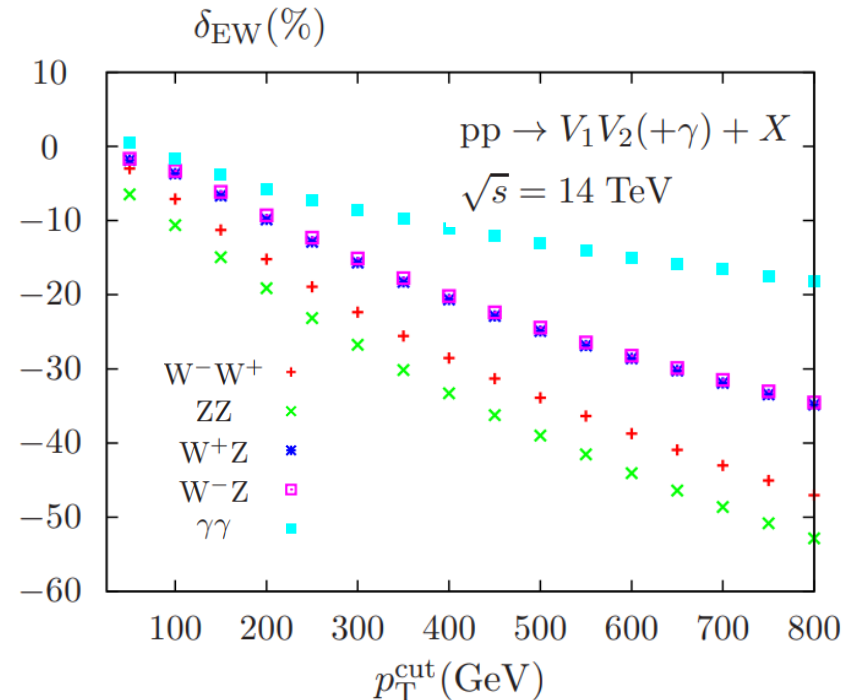
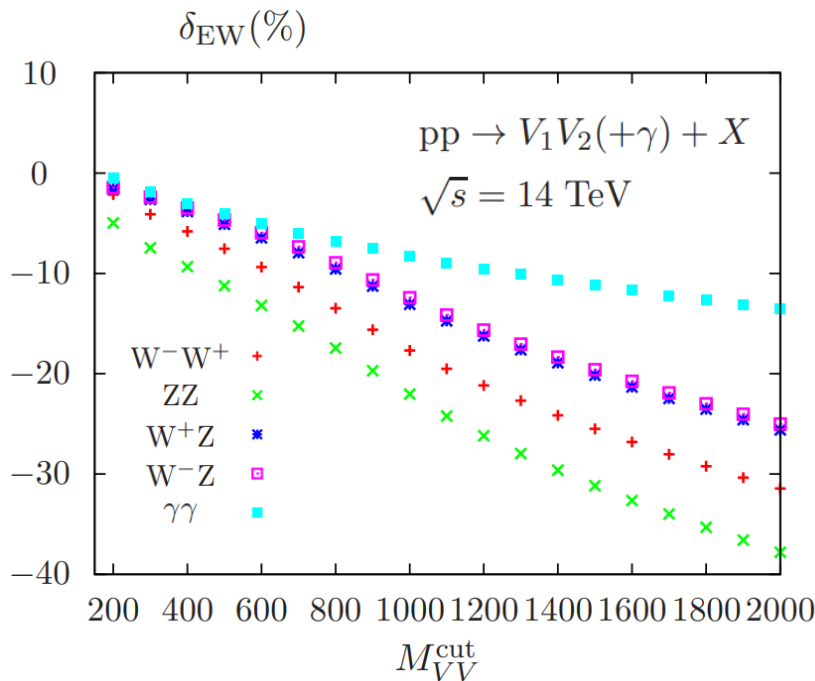


- EW corrections

- $\alpha \sim \mathcal{O}(1\%)$ , the biggest uncertainty from perturbative corrections!
- Sudakov enhancement,  $\mathcal{O}(10\% \sim 30\%)$  corrections in high energy region.

[Bierweiler et al:1305.5402](#)

[Anderson et al:1605.04692](#)



- NLO EW corrections are crucial**, a focal point in 2015, 2017, 2019 and 2021 Les Houches precision wish lists.

[Les Houches 2017:1803.07977](#)

[Amoroso et al:2003.01700](#)

[Huss et al:2207.02122](#)

# EW corrections status



- **Multiple scales** make reduction of scalar integrals and calculation of master integrals difficult.
- Four mass scales  $m_H, m_t, m_W, m_Z$ , two Mandelstam variables  $\hat{s}, \hat{t}$ .
- Recent developments on this topic :
  - Higgs self-coupling corrections in SMEFT, [Borowka et al: 1811.12366](#)
  - two-loop box diagrams, [Davies et al:2207.02587](#)
  - top-quark Yukawa corrections, [Muhlleitner et al:2207.02524](#)
  - NLO EW corrections in large- $m_t$  limit, [Davies et al:2308. 01355](#)
  - Yukawa and Higgs self-coupling corrections, [Heinrich et al:2407. 04653](#), [Li et al:2407, 14716](#)
  - Comprehensive NLO EW corrections, [Bi et al:2311. 16963](#)



This talk

# Production Rate



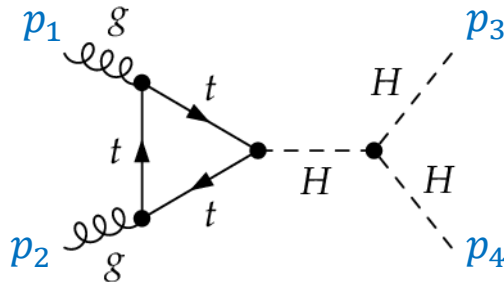
- Di-Higgs production cross section:

$$\sigma(pp \rightarrow HH) = \int dx_1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}_{gg \rightarrow HH}(\hat{s}, m^2)$$

gluon PDF

- Multiple mass scales, analytic result for  $\hat{\sigma}$  is challenging. ❌
- Monte Carlo integration method can be adopted. ✅

$$\int dx_1 dx_2 f_g(x_1) f_g(x_2) d\hat{t} \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, \hat{t}) = \sum_{i,j} \Delta_{i,j} \times \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}_i, \hat{t}_j)$$



$$\begin{cases} \hat{s} = (p_1 + p_2)^2 \\ \hat{t} = (p_1 - p_3)^2 \end{cases}$$

- Lots of numerical results for  $d\hat{\sigma}/d\hat{t}$  at different phase space points are required.



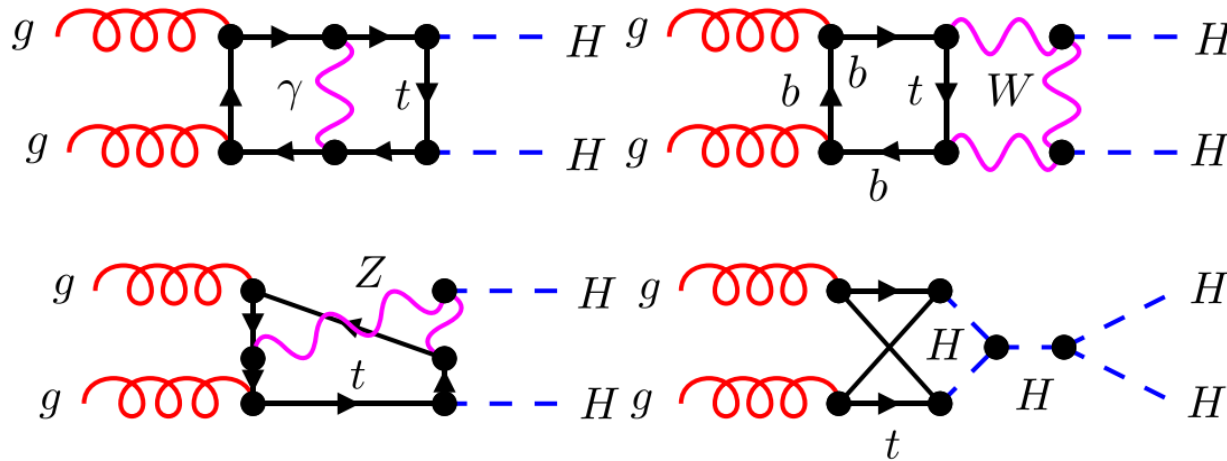
# Feynman Diagrams



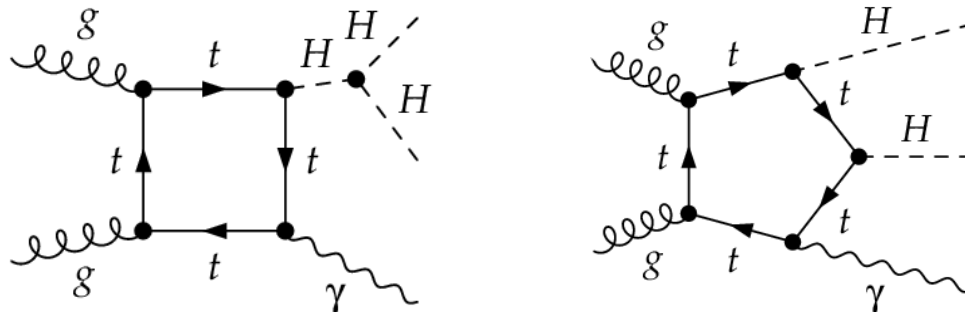
T. Hahn:0012260

- Generate Feynman diagrams and amplitudes using FeynArts Package.

- 2020 NLO virtual correction diagrams, some typical Feynman diagrams.



- NLO **real corrections** are **forbidden** due to Furry theorem.



# Manipulate amplitudes




- Amplitudes for  $g(p_1)g(p_2) \rightarrow H(p_3)H(p_4)$ ,  $M_{ab} = \delta_{ab}\epsilon_1^\mu\epsilon_2^\nu M_{\mu\nu}$

- Decomposition to form factor:

$$M_{\mu\nu} = F_1(\hat{s}, \hat{t}, m^2)T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m^2)T_2^{\mu\nu}$$

- Decomposition to scalar integrals:

$$F_i(\hat{s}, \hat{t}, m^2) = \sum_j C_{i,j}(\hat{s}, \hat{t}, m^2) \times \boxed{FI_{i,j}(\hat{s}, \hat{t}, m^2)}$$


To be reduced 

- Reduction to master integrals using integration-by-part identity, implemented by Blade package.

Guan et al:2405. 14621

<https://gitlab.com/multiloop-pku>


$$FI_{i,j}(\hat{s}, \hat{t}, m^2) = \sum_k P_{i,j,k}(\hat{s}, \hat{t}, m^2) \times \boxed{I_{i,j,k}(\hat{s}, \hat{t}, m^2)}$$

**Key point !** 

To be calculated, numerical results for  $\mathcal{O}(10^4)$   $(\hat{s}_i, \hat{t}_i)$  are required.

# Calculate integrals

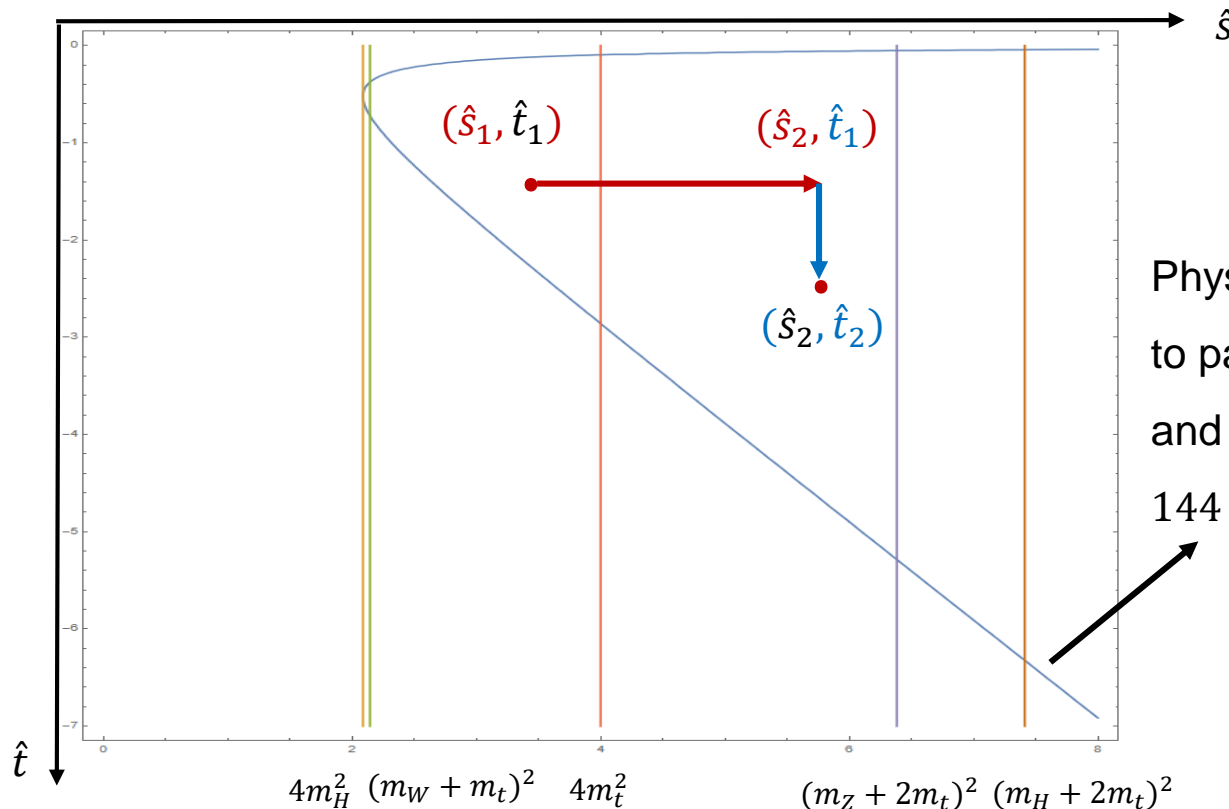


- Calculate integrals for a phase space point:
  - Two dimensional regulator  $\epsilon$  points ( $\epsilon = \pm \frac{1}{1000}$ ) are sufficient to confirm the cancellation of divergences.
  - 3000 cpu.h run time using AMFlow program package. <https://gitlab.com/multiloop-pku>  
Liu et al:2201.11636
- Calculate integrals for 30000 phase space points:
  - ~~$\mathcal{O}(10^8)$~~  cpu.h run time with all points calculated by AMFlow.
- Calculate integrals using differential equation:
  - Analytical differential equation  $\frac{\partial}{\partial \hat{s}} \vec{I} = A_{\hat{s}} \vec{I}$ ,  $\frac{\partial}{\partial \hat{t}} \vec{I} = A_{\hat{t}} \vec{I}$  provided by Blade.
  - Numerical boundary  $\vec{I}(\hat{s}_0, \hat{t}_0)$  provided by AMFlow.
  -  Differential equation running method required  $\mathcal{O}(10^5)$  cpu.h run time.

# DE running



- Physical singularities occur only in  $\hat{s}$  direction:  $4m_H^2, (m_W + m_t)^2, 4m_t^2, (m_Z + 2m_t)^2, (m_Z + 2m_t)^2$ .
- Continuation direction for  $\hat{s}$  is  $+i\delta$ .
- Asymptotic expansion at singularities is required in the  $\hat{s}$  direction and Taylor expansion is performed in the  $\hat{t}$  direction.



Physical boundary corresponds to parallelization between  $p_1$  and  $p_3$ .

$$144 - 552\hat{t} + 529\hat{s}\hat{t} + 529\hat{t}^2 = 0$$

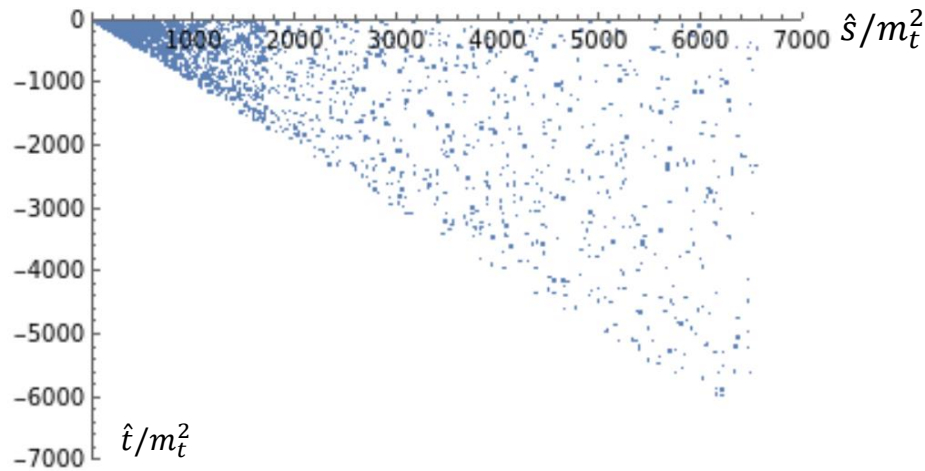
# MC samples



- LO production cross section:

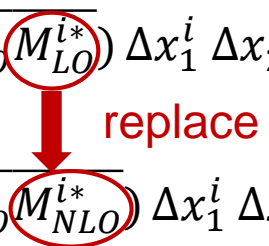
$$\sigma_{LO} = \sum_{i=1}^N \frac{1}{\text{flux}_i} f_g(x_1^i, \mu) f_g(x_2^i, \mu) |\overline{M_{LO}^i}(\hat{s}, \hat{t})|^2 \Delta x_1^i \Delta x_2^i \Delta \Phi_2^i$$

- Samples are sparse at high energy region due to the **suppression of gluon PDFs**.



- **Reweight:**  $\sigma_{LO} = \sum_{i=1}^N \frac{1}{\text{flux}_i} f_{g/p}(x_1^i, \mu) f_{g/p}(x_2^i, \mu) 2\text{Re}(\overline{M_{LO}^i} M_{LO}^{i*}) \Delta x_1^i \Delta x_2^i \Delta \Phi_2^i$

$$\sigma_{NLO} = \sum_{i=1}^N \frac{1}{\text{flux}_i} f_{g/p}(x_1^i, \mu) f_{g/p}(x_2^i, \mu) 2\text{Re}(\overline{M_{LO}^i} M_{NLO}^{i*}) \Delta x_1^i \Delta x_2^i \Delta \Phi_2^i$$



# Total cross sections



- Yukawa and Higgs self-coupling part, 7000 events.

$\sqrt{s}$	13 TeV	13.6 TeV	14 TeV
LO [fb]	16.45	18.26	19.52
NLO <sup>EW</sup> [fb]	16.69	18.52	19.79
NLO <sup>EW</sup> /LO	1.01	1.01	1.01

- PDF4LHC21\_40 PDF set,  $\mu_r = \mu_f = M_{HH}/2$ .
- NNPDF3.1 PDF set,  $\mu_r = \mu_f = M_{HH}/2$ . NLO<sup>EW</sup> cross section is **20.19 fb**, which is a **1%** enhancement compared to the LO.
- Full NLO electroweak cross section:  $1.8 \times 10^4$  events.

$\mu$	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	$m_H$
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$ -factor	0.958(1)	0.957(1)	0.954(1)

- NNPDF3.1 PDF set,  $\mu_r = \mu_f = \mu$ ,  $\sqrt{s} = 14$  TeV.
- **-4.2%** NLO EW corrections.
- The **discrepancy** between **1%** and **-4.2%** suggests that the gauge boson contribution dominates the corrections and has the opposite sign.

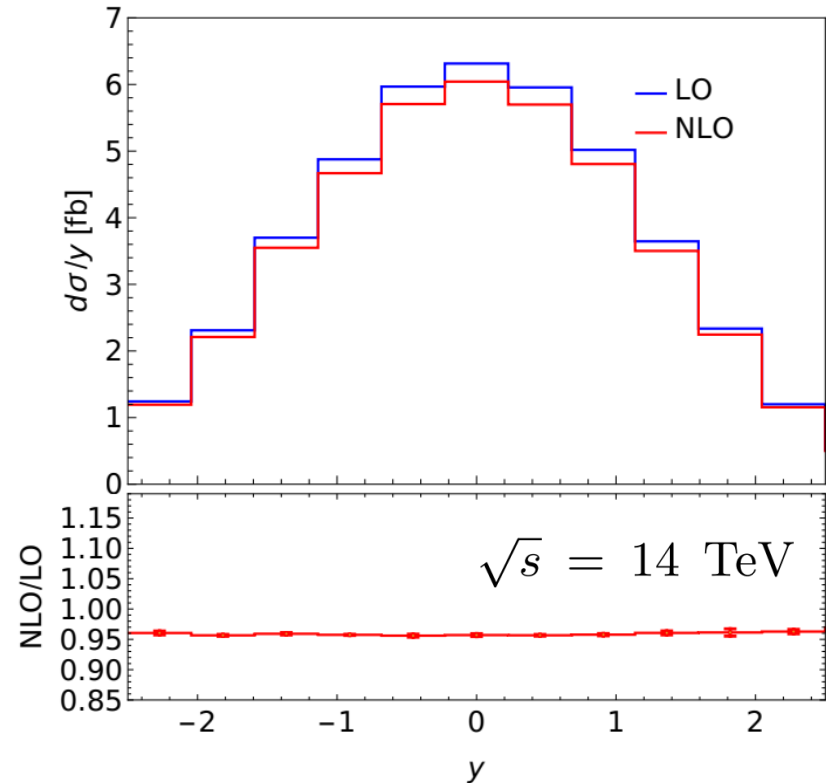
# y differential distribution



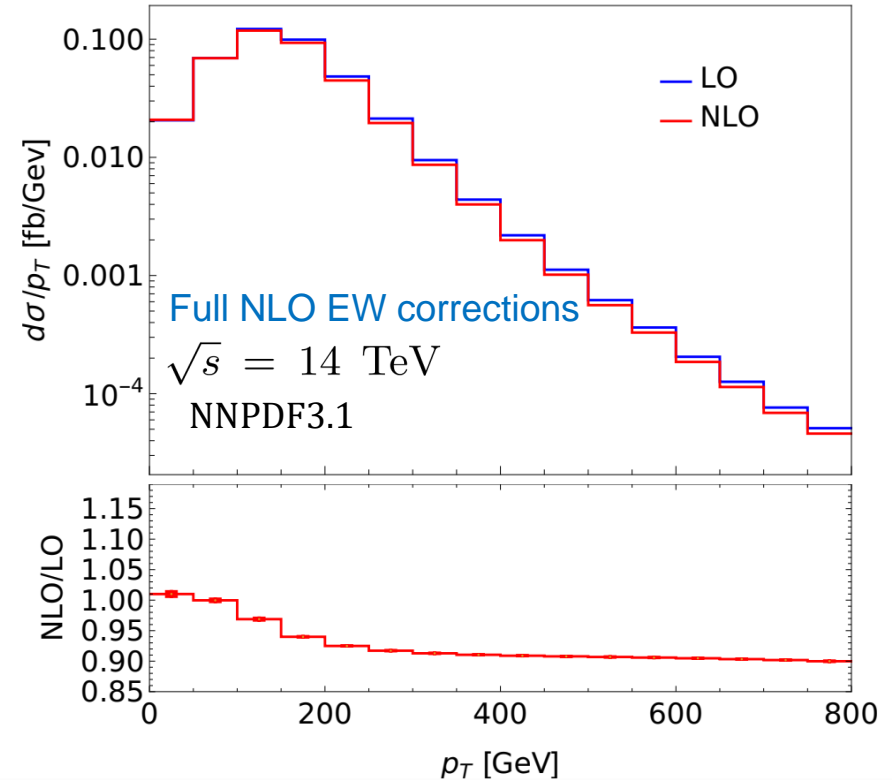
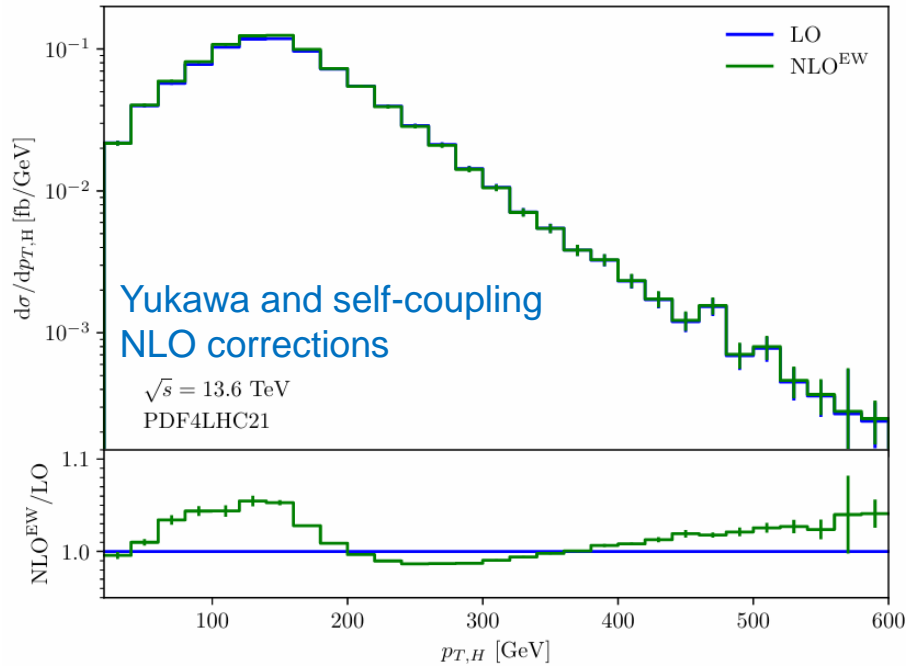
- The differential K factor can get a controllable error with far fewer events

$$\Delta\sigma^{\text{NLO}} = \Delta\sigma^{\text{LO}} \times \Delta K$$

- $\Delta\sigma^{\text{LO}}$  uses  $3 \times 10^5$  events.
- $\Delta K$  uses  $1.8 \times 10^4$  events for  $\sigma$  and additional 400 events for each bin.
- Up to NLO,  $K \approx 0.96$ .



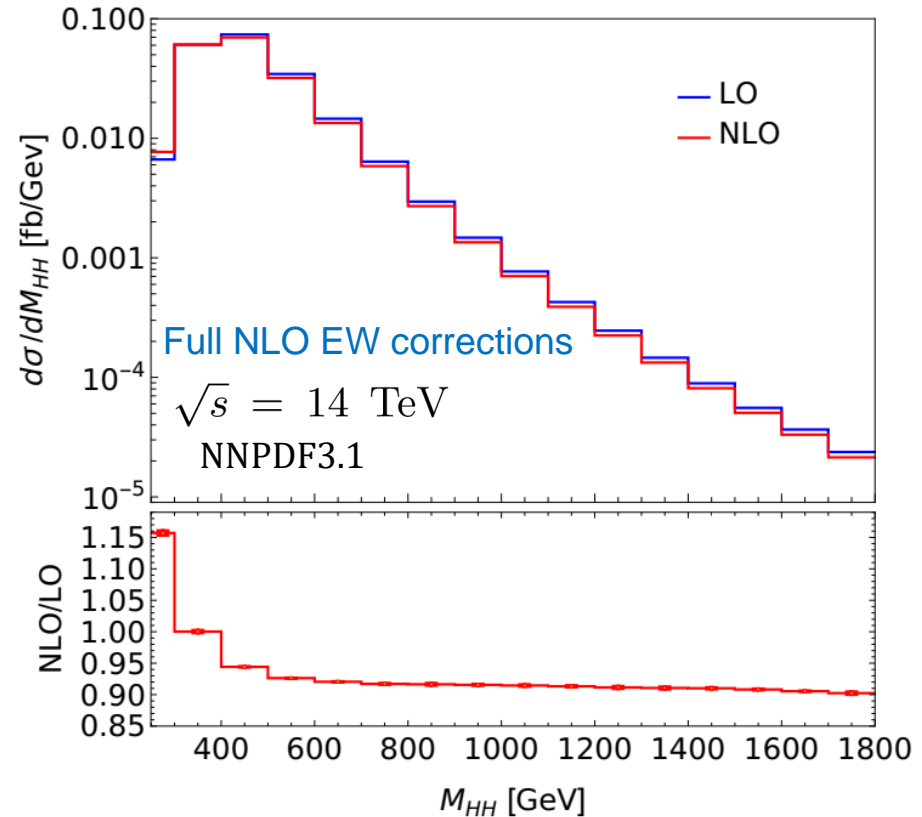
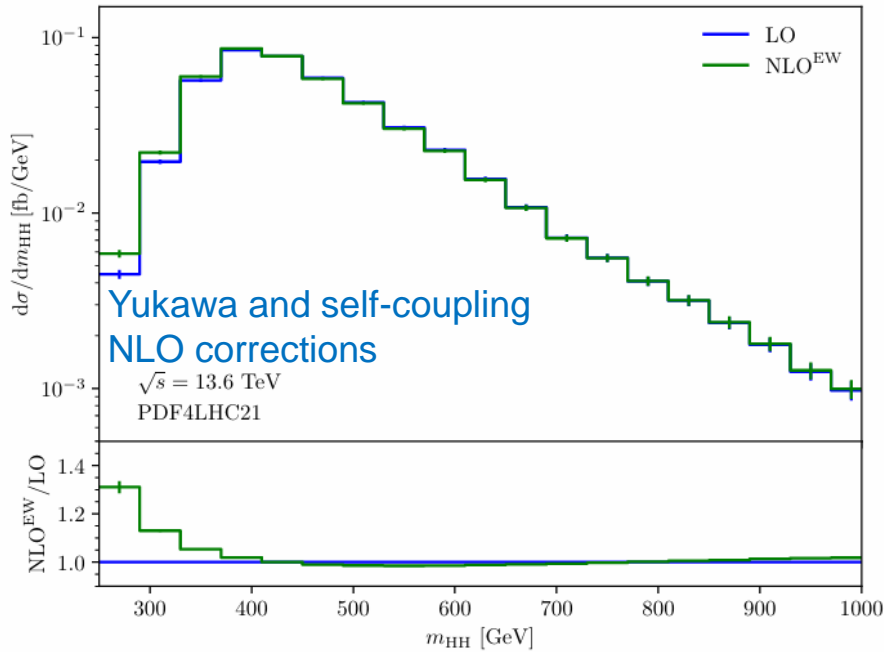
# $p_T$ differential distribution



- Yukawa and self-coupling NLO corrections: **5% enhancement** near 170 GeV. At high- $p_T$  region, **2% suppression** around 250 GeV.
- Full NLO electroweak corrections : **-10%** corrections at the tail, **Sudakov enhancement**



# $M_{HH}$ differential distribution



- Yukawa and self-coupling NLO corrections: **+25% enhancement** near Higgs pair production threshold.
- Full NLO electroweak corrections : **+15%** NLO corrections at the beginning of spectrum, **-10%** corrections at the tail, **Sudakov enhancement**.

# Summary



- **Higgs trilinear coupling** is important.
- Full NLO EW correction to total cross sections is about  $-4\%$ . Yukawa and self-coupling correction is about  $1\%$ .
- $-4\%$  NLO EW corrections to rapidity distribution.
- Full NLO EW corrections at the beginning of spectrum for the  $M_{HH}$  is  $+15\%$ . Yukawa and self-coupling correction near this region is  $+25\%$ .
- **Sudakov effect** was observed for both  $p_T$  and  $M_{HH}$  distribution when the full NLO EW corrections were considered.
- **Sufficient precision** from current QCD corrections and NLO EW corrections for measurements at the HL-LHC.

Thanks for your attention!