

Multi-Higgs production from VBS in HEFT

Javier Martínez-Martín¹

¹In collaboration with Rafael L. Delgado, Raquel Gómez-Ambrosio, Alexandre Salas-Bernárdez and Juan J. Sanz-Cillero - [2311.04280v2](#) - [JHEP 03 \(2024\) 037](#)

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Outline

1 Introduction

- Motivation
- Framework
- Higgs Effective Field Theory (HEFT)
- Relation to SMEFT

2 Amplitudes and cross sections calculations

- $!!! \rightarrow 2h$
- $!!! \rightarrow 3h$
- $!!! \rightarrow 4h$

3 Cross section phenomenology

- CMS and ATLAS data
- Benchmark points
- BP study

4 State of the art and next steps

5 Conclusions

Motivation

- Understanding the SM as the leading order of an EFT, we would like to find new terms.
- At the current energies, both SMEFT and HEFT are valid descriptions of the currently available LHC data.
- Multi-Higgs measurements at HL-LHC and beyond are crucial to check their validity range.

Framework

- We used the Equivalence theorem approximation where $W_L^a \rightarrow \frac{1}{f} \partial^a \phi$.
- Assuming $m_h^2 \ll m_W^2 \ll s \ll \Lambda^2$.
- Using only derivative terms of the Lagrangian up to 2 derivatives (which scale with the energy).
- As a first approximation, only one SMEFT operator is needed (at each EFT dimension).

Higgs Effective Field Theory

Canonical form

HEFT Lagrangian¹

[Appelquist et al. - Phys. Rev. D 22 (1980) 200, Longhitano et al. - Phys. Rev. D 22 (1980) 1166]

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} F(h) (\partial_\mu \phi^a)^2 + \mathcal{O}(\phi^4)$$

Flare function²

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2204.01762]

$$F(h) = 1 + a_1 \frac{h}{v} + a_2 \frac{h^2}{v^2} + a_3 \frac{h^3}{v^3} + a_4 \frac{h^4}{v^4} + \mathcal{O}(h^5)$$

$$a = \frac{a_1}{2} ; a_2 = b \quad \text{with} \quad a_{1,\text{SM}} = 2 ; a_{2,\text{SM}} = 1 ; a_{3,\text{SM}} = 0 ; a_{4,\text{SM}} = 0$$

¹We only focus on the EW sector. In the conditions where, under the Goldstone equivalence theorem, longitudinal VBS is approximated by Goldstone scattering.

²Where a_n is the effective coupling of $\phi\phi$ with nh . This can be done similarly for the Yukawa sector through the study of $t\bar{t} \rightarrow n h$ processes, see:

Englert et al. - 2308.11722, Gómez-Ambrosio et al. - 2207.09848

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Englert et al. - 2308.11722 , Gómez-Ambrosio et al. - 2207.09848

Higgs Effective Field Theory

Check: Redefined form

Calculations have also been checked with:

Redefined HEFT Lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} \hat{F}(h) \partial_\mu \phi^a \partial^\mu \phi^a + \mathcal{O}(h^4)$$

Redefined Flare function³

$$\hat{F}(h) = 1 + \hat{a}_2 \frac{h^2}{v^2} + \hat{a}_3 \frac{h^3}{v^3} + \hat{a}_4 \frac{h^4}{v^4} + \mathcal{O}(h^5)$$

$$\hat{a}_2 = b - a^2; \quad \hat{a}_3 = a_3 - \frac{4a}{3} b - a^2; \quad \hat{a}_4 = a_4 - \frac{3}{2} a a_3 + \frac{5}{3} a^2 b - a^2$$

³This redefinition gives a more direct interpretation of the results, see:

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Relation to SMEFT

SMEFT

SMEFT lagrangian

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$L_{\text{SMEFT}} = L_{\text{SM}} + \sum_{n=5}^{\infty} \sum_i \frac{c_i^{(n)}}{\Lambda^{n-4}} O_i^{(n)}$$

O_H operator

$$O_H^{(6)} = (H^\dagger H) (H^\dagger H); \quad O_H^{(8)} = (H^\dagger H)^2 (H^\dagger H); \quad @^2$$

SMEFT parameters

$$d = \frac{2v^2 c_H^{(6)}}{\Lambda^2}; \quad = \frac{c_H^{(8)}}{2(c_H^{(6)})^2}$$

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Relation to SMEFT

Relation between parameters

Relation with canonical parameters⁴

[Gómez-Ambrosio, Llanes-Estrada, Salas-Bernárdez and Sanz-Cillero - 2207.09848]

$$\begin{aligned}
 a_1 &= a = 1 + \frac{d}{2} + \frac{d^2}{2} \frac{3}{4} + \dots + O(d^3) \\
 a_2 &= b = 1 + 2d + 3d^2(1 + \dots) + O(d^3) \\
 a_3 &= \frac{4}{3}d + d^2 \frac{14}{3} + 4\dots + O(d^3) \\
 a_4 &= \frac{1}{3}d + d^2 \frac{11}{3} + 3\dots + O(d^3)
 \end{aligned}$$

⁴ a_5 and a_6 can be found in L. Delgado et al. 2311.04280v2.
 a_n for $n \geq 7$ vanishes at order 1 = ⁴.

!!! 2h

The following results in this section are calculated in the massless limit.

Amplitude⁵

$$T_{\omega\omega\rightarrow 2h} \stackrel{\text{HEFT}}{=} \frac{\hat{a}_2 s}{v^2} = \stackrel{\text{SMEFT}}{=} \frac{s}{v^2} d + 2d^2(1 + \dots) + \mathcal{O}(d^3)$$

Cross section

$$\sigma_{\omega\omega\rightarrow 2h} \stackrel{\text{HEFT}}{=} \frac{8}{s} \frac{\hat{a}_2^2}{16 v^2} s^2 = \stackrel{\text{SMEFT}}{=} \frac{8}{s} d^2 + 4d^3(1 + \dots) \frac{s}{16 v^2} s^2 + \mathcal{O}(d^4)$$

⁵Compatible with previous analysis in e.g. Arganda et al. - [1807.09763](#), Dobado et al. - [1711.10310](#).

!!! 3h

Amplitude⁶

$$T_{!!! 3h}^{\text{HEFT}} = \frac{3\hat{a}_3 s}{v^3} =$$

$$\text{SMEFT} \frac{4s}{v^3} d^2 (1 + \dots) + \mathcal{O}(d^3)$$

Cross section

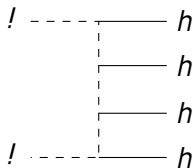
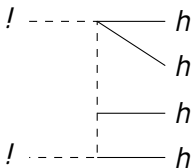
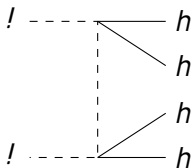
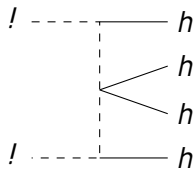
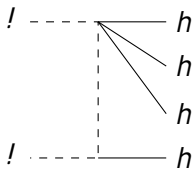
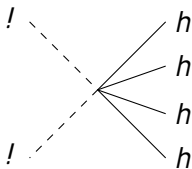
$$\sigma_{!!! 3h}^{\text{HEFT}} = \frac{12}{s} \frac{\hat{a}_3^2}{16 v^2} s^3 =$$

$$\text{SMEFT} \frac{64}{3s} d^4 (1 + \dots)^2 \frac{s}{16 v^2} s^3 + \mathcal{O}(d^5)$$

⁶Previous analysis with modifications in e.g. Gonzalez-Lopez et al. - [2011.13915](#),
Chen et al. - [2105.11500](#).

!!! 4h

Contributing diagrams



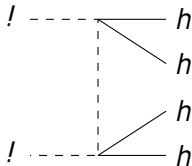
With permutations of external particles, there are a total of 75 diagrams

!!! 4h

Amplitude

$$T_{!!! 4h}^{\text{HEFT}} = \frac{4s}{v^4} (3\hat{a}_4 + \hat{a}_2^2 (B-1)) =$$

$$\stackrel{\text{SMEFT}}{=} \frac{4s}{v^4} d^2 (1 + \dots + B) + O(d^3)$$



Cross section

$$\sigma_{!!! 4h}^{\text{HEFT}} = \frac{8}{9s} \frac{s}{16 v^2} \left(3\hat{a}_4^2 + \hat{a}_2^2 + 2 \hat{a}_4 \hat{a}_2 + \hat{a}_2^2 + \hat{a}_2^2 + \hat{a}_2^4 \right) =$$

$$\stackrel{\text{SMEFT}}{=} \frac{8}{9s} \frac{s}{16 v^2} d^4 \left((1 + \dots)^2 + 2(1 + \dots) + \dots + O(d^5) \right)$$

!!! 4h

Parameters

Definitions⁷

•

<

$$B = f_1 f_2 f_3 f_4 \quad B_{1234} + B_{1324} + B_{1423} + B_{2314} + B_{2413} + B_{3412}$$

$$B_{ijk} = \frac{z_{ij} z_{k\bar{}}}{2 f_i f_j z_{ij} f_i z_i f_j z_j} ; \quad z_{ij} = z_{ji} = \frac{q^2 (p_i p_j)}{(q p_i)(q p_j)} \stackrel{\text{CM}}{=} 2 \sin^2(\theta_{ij=2})$$

$$f_i = \frac{q p_i}{q^2} \stackrel{\text{CM}}{=} k p_i k = p_{\bar{s}} ; \quad z_i = \frac{2 k_1 p_i}{q p_i} \stackrel{\text{CM}}{=} 2 \sin^2(\theta_{i=2})$$

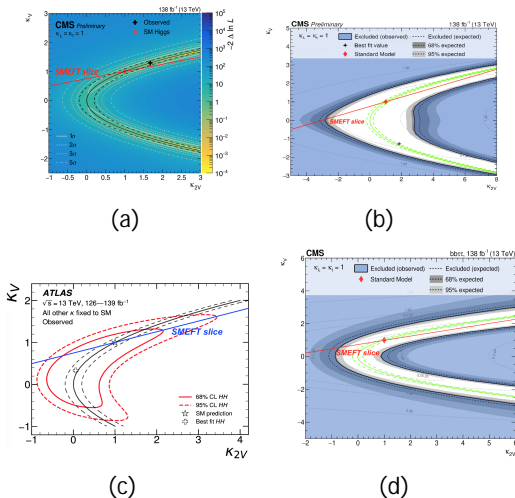
$$q = k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

Z

$$n = \bigvee_4^1 \quad d\Pi_4 B^n ; \quad \alpha_1 = 0.124984(10) ; \quad \alpha_2 = 0.0193760(16)$$

⁷Numerical values computed with a personal code, see:
<https://github.com/mamupaxs/mamupaxs>

Previous CMS and ATLAS data



- a_1 is relatively well known from Higgs decays at LHC. Close to the SM, up to $O(10\%)$.

- $$v = a = \frac{a_1}{2}$$

$$2v = a_2$$

- Superimposed SMEFT correlation:

$$a_2 = 2a_1 \quad 3$$

- Plotted parabolas:

$$\hat{a}_2 \quad a_2 \quad \frac{a_1^2}{4} = 0$$

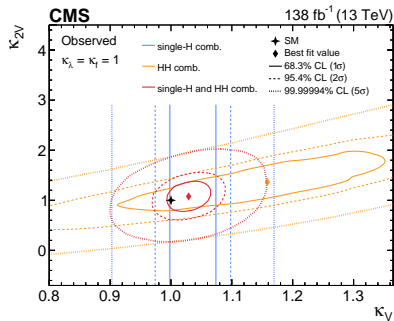
$$\hat{a}_2 = 0.2$$

Figure: (a) Phys. Rev. Lett. 131 (2023) 041803 [2205.06667]

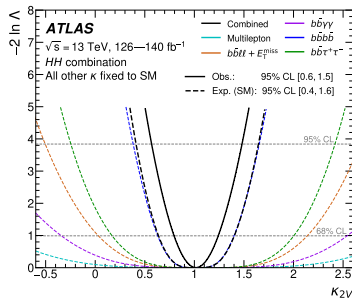
(b) CMS-PAS-HIG-21-005 (c) ATLAS-CONF-2022-050

(d) Phys. Lett. B 842 (2023) 137531 [2206.09401].

Latest CMS and ATLAS data



(a)



(b)

Figure: (a) Submitted to Phys. Lett. B [2407.13554] (b) Phys. Rev. Lett. 133 (2024) 101801 [2406.09971].

- The constraint on a_2 is driven by the HH categories enriched in VBF HH events.
- The observed (expected) 95% CL interval: $0.6 < a_2 < 1.5$ ($0.4 < a_2 < 1.6$).

Benchmark points

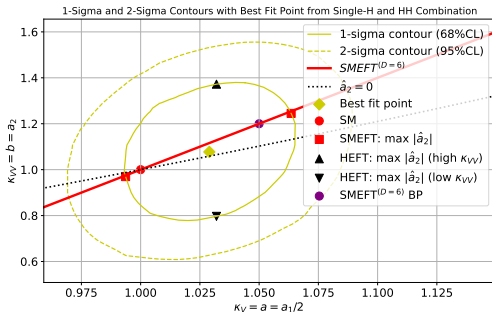


Figure: <https://www.hepdata.net/record/ins2808928>.

SMEFT^(D=6) BP

$$d = 0:1$$

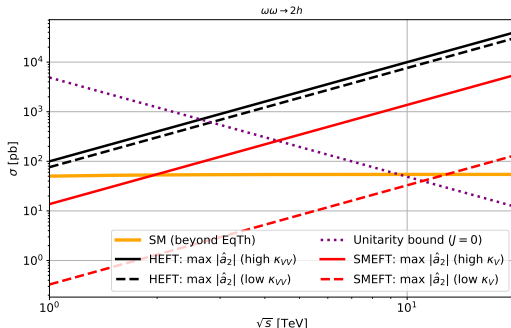
$$a = a_1 = 2 = 1.05; \quad b = a_2 = 1.20$$

$$a_3 = 0.19; \quad a_4 = 0.09$$

- d is compatible with the SM deviation range of ATLAS and CMS, $\Delta a = a - 1 = 0.05$ with $a = 1 + d = 2$.
- d is crucial for the convergence of the expansion.
- $a_{1,\text{SM}} = 2; a_{2,\text{SM}} = 1$
 $a_{3,\text{SM}} = 0; a_{4,\text{SM}} = 0$

BP study

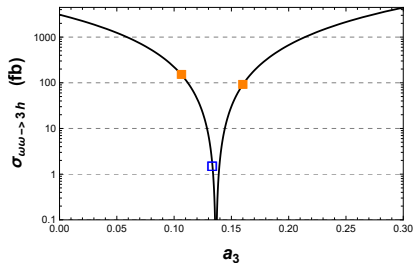
!!! 2h (Preliminary)



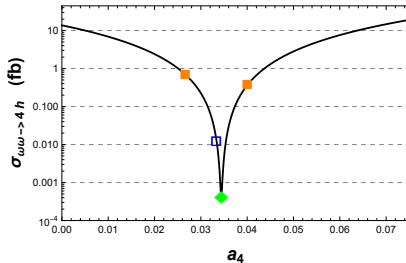
- SMEFT is suppressed by 1 order of magnitude.
- All of the SM needs to be considered.
- Unitarity studied for pure s-wave.

BP study

!!! 3h & !!! 4h



- a_1 and a_2 are set to their SMEFT^(D=6) values. $\bar{s} = 1$ TeV.
- SMEFT is a particular value (blue).
- A 20% variation on a_3 varies the XS by 2 orders of magnitude (SM: $a_3 = 0$).
- The XS vanishes for a particular value.



- a_1 , a_2 and a_3 are set to their SMEFT^(D=6) values. $\bar{s} = 1$ TeV.
- SMEFT is a particular value (blue).
- A 20% variation on a_4 varies the XS by 2 orders of magnitude (SM: $a_4 = 0$).
- The XS doesn't vanish.

State of the art

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State of the art

Our next steps

Work in progress

- Include the complete SM.

- Calculating processes with W_L and masses.

- Adding the PDFs for the full processes.

- Partial waves and unitarity study.

Possible future works

- Higgs self-interactions.

- Final-state particles decay.

- New-physics models for these couplings.

- Beyond VBS processes.

Experimentalists' next steps. HEFT Wishlist

Experimentalists' next steps. HEFT Wishlist

Experimentalists' next steps. HEFT Wishlist

Conclusions

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We studied longitudinal VBS using the Goldstone equivalence theorem, the massless approximation, and $\mathcal{O}(\alpha_s^2)$.

We computed the analytic expressions for $1, 2h$ and $4h$ production at tree level for SMEFT and HEFT.

We found multi-Higgs production is very suppressed in SMEFT but not in general HEFT scenarios.

There are lots of steps left to take.

This is a very hot topic being worked on nowadays.

THANK YOU SO MUCH!

6 Back up

Back up 1. Redefinition of HEFT

Back up 2. HEFT-SMEFT parameters relation

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$|a| \rightarrow |a + g(h)|; \quad h \rightarrow h + N(1 + g(h)) \frac{|a|^2}{v}$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$h^a \rightarrow h^a + g(h) \frac{h^a h^a}{v}$$

HEFT lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} F(h) \frac{h^a h^a}{v^2} + \mathcal{O}(h^4)$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$h \rightarrow h + N(1 + g(h)) \frac{|\phi|^2}{v}$$

Redefined HEFT lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} F^A(h) \partial_\mu \phi^A \partial^\mu \phi^A + \mathcal{O}(|\phi|^4)$$

Back up 1. Redefinition of HEFT

Fields redefinition

Fields redefinition

$$h \rightarrow h + N(1 + g(h)) \frac{|a|^2}{v}$$

Redefined HEFT Lagrangian

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} F^A(h) \partial_\mu a \partial^\mu a + \mathcal{O}(|a|^4)$$

Redefined bare function

$$F^A(h) = F(h) \left(1 + g(h) \right)^2$$

Appendix 1. Redefinition of HEFT

Flare function redefinition

Normalization

$$g(h) = a \frac{h}{v} + a^2 \frac{h^2}{v^2} + \frac{1}{3} a (b - 4a^2) \frac{h^3}{v^3} + \frac{1}{4} a (a_3 - 4ab + 8a^3) \frac{h^4}{v^4} + O(h^5)$$

$$N = \frac{a}{2}$$

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Flare function

$$F(h) = 1 + a_1 \frac{h}{v} + a_2 \frac{h^2}{v^2} + a_3 \frac{h^3}{v^3} + a_4 \frac{h^4}{v^4} + O(h^5)$$

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Redefinition of flare function

$$F(h) = 1 + a_2 \frac{h^2}{v^2} + a_3 \frac{h^3}{v^3} + a_4 \frac{h^4}{v^4} + O(h^5)$$

Back up 1. Redefinition of HEFT

Parameters redefinition

Redefined parameters

$$a_2 = b - a^2$$

$$a_3 = a_3 - \frac{4a}{3} b - a^2$$

$$a_4 = a_4 - \frac{3}{2} a a_3 + \frac{5}{3} a^2 b - a^2$$

Back up 2. HEFT-SMEFT parameters relation

Parameters

SMEFT parameters

$$d = \frac{2v^2 c_H^{(6)}}{\Lambda^2} \quad ; \quad = \frac{c_H^{(8)}}{2(c_H^{(6)})^2}$$

Back up 2. HEFT-SMEFT parameters relation

Relation with canonical parameters

Relation with canonical parameters

$$\begin{aligned}
 a_{1=2} &= a = 1 + \frac{d}{2} + \frac{d^2}{2} \left(\frac{3}{4} + \frac{1}{2} \right) + \mathcal{O}(d^3) \\
 a_2 &= b = 1 + 2d + 3d^2 \left(1 + \frac{1}{2} \right) + \mathcal{O}(d^3) \\
 a_3 &= \frac{4}{3}d + d^2 \left(\frac{14}{3} + 4 \right) + \mathcal{O}(d^3) \\
 a_4 &= \frac{1}{3}d + d^2 \left(\frac{11}{3} + 3 \right) + \mathcal{O}(d^3)
 \end{aligned}$$

Back up 2. HEFT-SMEFT parameters relation

Relation with redefined parameters

Relation with redefined parameters

$$\hat{a}_2 = d + 2d^2(1 + \epsilon) + \mathcal{O}(d^3)$$

$$\hat{a}_3 = \frac{4}{3}d^2(1 + \epsilon) + \mathcal{O}(d^3)$$

$$\hat{a}_4 = \frac{1}{3}d^2(1 + \epsilon) + \mathcal{O}(d^3)$$