Bubble wall velocity in cosmological phase transitions: electroweak baryogenesis and gravitational waves

Marek Lewicki

University of Warsaw

CERN Cosmo Coffee seminar, 22 V 2024

POLSKIE POWROTY
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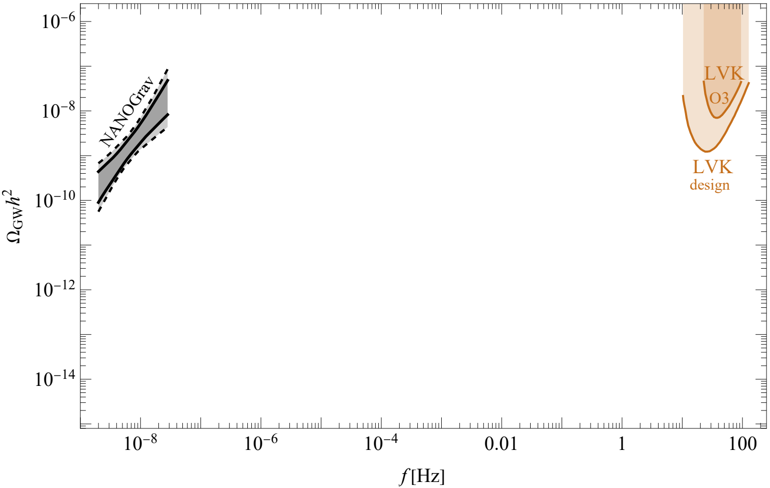

10^{-6}
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$\Omega_{\rm GW}h^2$
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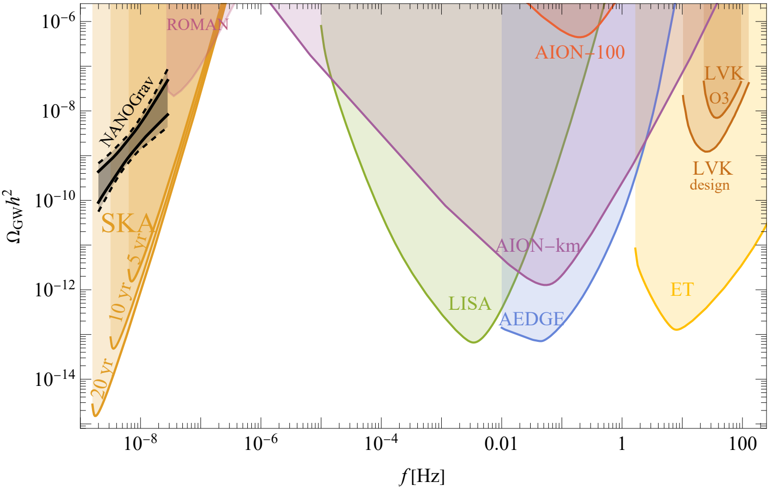
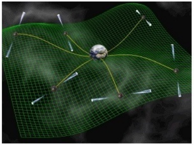
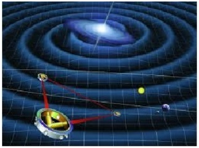




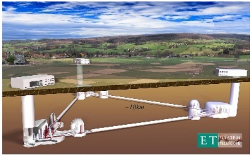
Pulsar Timing

[David Champion/NASA/JPL]



LISA

wiki/Laser_Interferometer_Space_Antenna



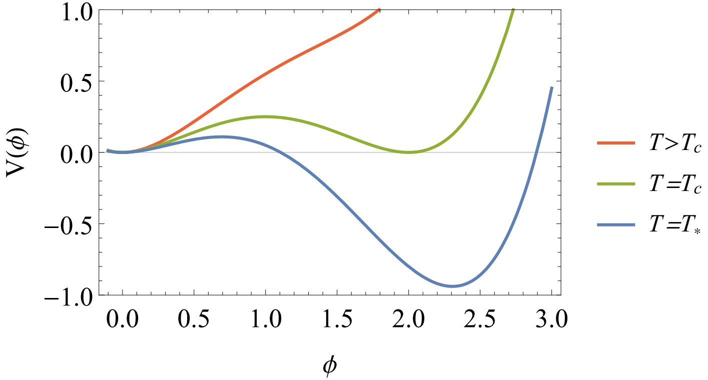
Einstein Telescope

www.et-gw.eu

First Order Phase Transition

- Simple high temperature expansion

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4\,, \quad T_0^2 > 0$$



First Order Phase Transition: bubble nucleation

- Temperature corrections to the potential

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4$$

- EOM $\rightarrow$ bubble profile

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial\phi} = 0,$$

$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

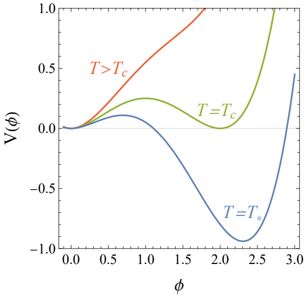
- $\mathcal{O}(3)$ symmetric action

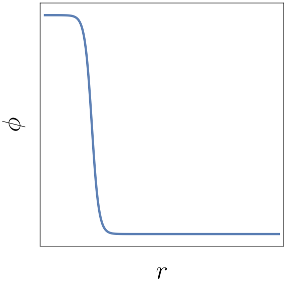
$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- nucleation temperature

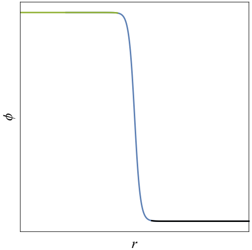
$$\frac{\Gamma}{H^4} \approx \left(\frac{T}{H}\right)^4 \exp\left(-\frac{S_3(T)}{T}\right) \approx 1$$

Linde ’81 ’83

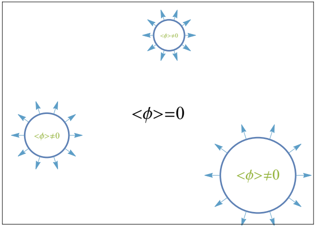




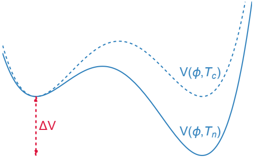
Phase transition parameters



$\Longrightarrow$



- Strength of the transition



$$\alpha \approx \left. \frac{\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}}{\rho_R} \right|_{T=T_*} , \qquad \Delta V = V_f - V_t$$

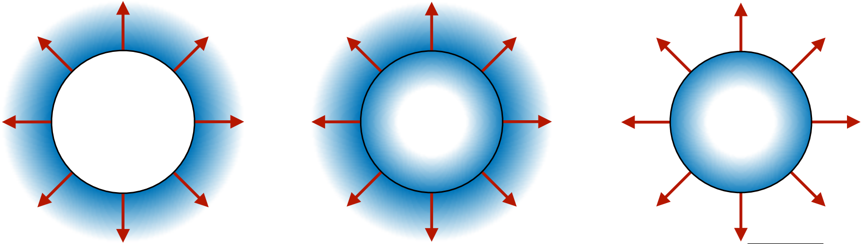
- Characteristic scale

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta\,(t-t_0)} \Longrightarrow \frac{\beta}{H} = T\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\Big|_{T=T_*} = \frac{(8\pi)^{\frac{1}{3}}}{H R_*}$$

- Bubble wall velocity: v_w

Expansion of bubbles

deflagration $v_w < c_s$ hybrid $c_s < v_w < c_J$ detonation $c_J < v_w$



with speed of sound $c_s = \frac{1}{\sqrt{3}}$ and the Jouget velocity $c_J = \frac{1}{\sqrt{3}} \frac{1+\sqrt{3\alpha^2+2\alpha}}{1+\alpha}$.

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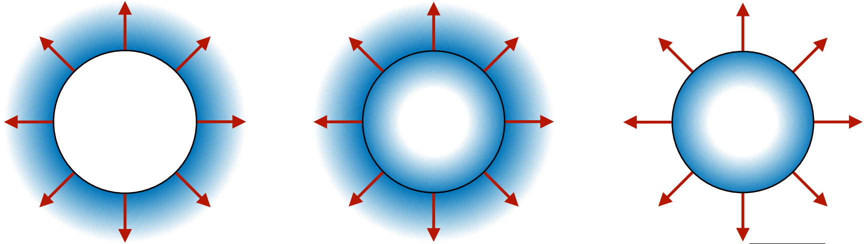

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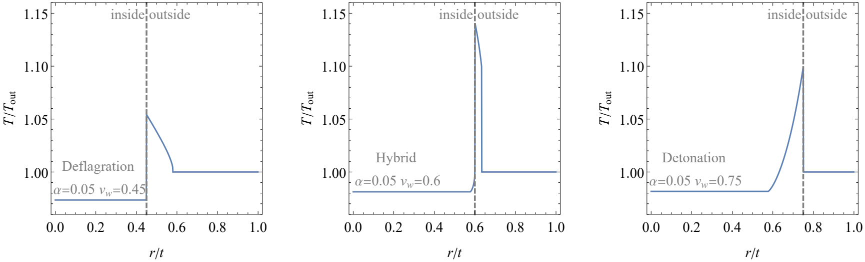


Sound Waves

- Simulation of a scalar coupled to the plasma

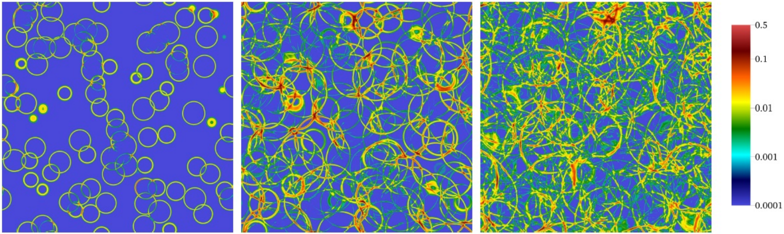


FIG. 4. Slices of fluid kinetic energy density E/T_c^4 at $t = 500\,T_c^{-1}$, $t = 1000\,T_c^{-1}$ and $t = 1500\,T_c^{-1}$ respectively, for the $\eta/T_c = 0.15$, $N_b = 988$ simulation.

- Fit to the GW spectrum

$$\Omega_{\rm gw} \propto \left(\frac{f}{f_p}\right)^3 \left(\frac{7}{4 + 3\,(f/f_p)^2}\right)^{\frac{7}{2}}$$

Hindmarsh, Huber, Rummukainen, Weir, arXiv: 1504.03291, 1704.05871

- Sound shell model

arXiv: 1608.04735 1909.10040, 2106.05984, 2308.12916, 2308.12943

Sound Waves

- Higgsless simulation of the plasma

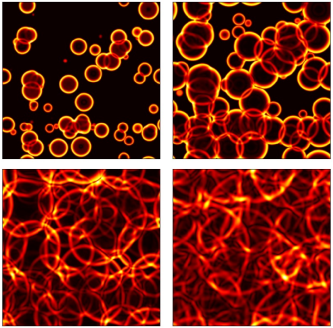
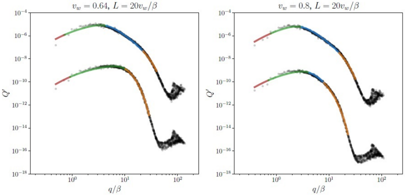


Figure 4: Kinetic energy v^2 in different simulation snapshots: $t = 2.7/\beta$ (top left), $5.4/\beta$ (top right), $10.8/\beta$ (bottom left) and $20.1/\beta$ (bottom right). We use box size $L = 40v_w/\beta$, weak transitions and $v_w = 0.8$.

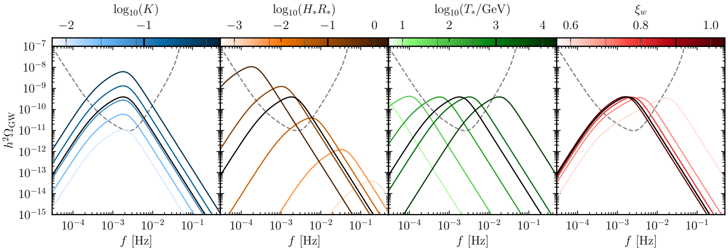


- Fit to the GW spectrum

$$\Omega_{\rm gw} \propto \frac{(f/f_1)^3}{1 + (f/f_1)^2[1 + (f/f_2)^4]}, \qquad f_2/f_1 \approx 1/\xi_{\rm shell}$$

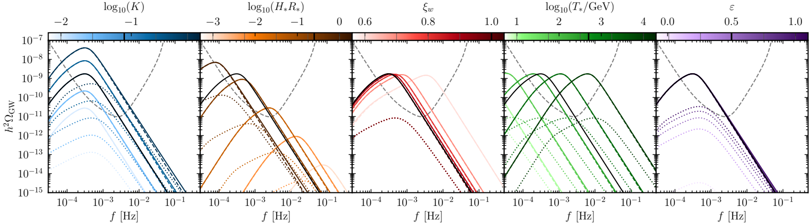
Jinno, Konstandin, Rubira, Stomberg, arXiv: 2209.04369

- sound waves



black line: $K = \left(\frac{\kappa\alpha}{\alpha+1}\right) = 0.1$, $H_* R_* = 0.1$, $\xi_w = 0.9$, $T_* = 1\mathrm{TeV}$

- sound waves (solid) + turbulence (dotted)



black line: $K = \left(\frac{\kappa\alpha}{\alpha+1}\right) = 0.08$, $H_* R_* = 0.25$, $\xi_w = 1$, $T_* = 500\mathrm{GeV}$, $\varepsilon = 0.5$

Caprini et al, arXiv: 2403.03723

Dynamics of the steady state expansion

Integrated EoM of the growing bubble:

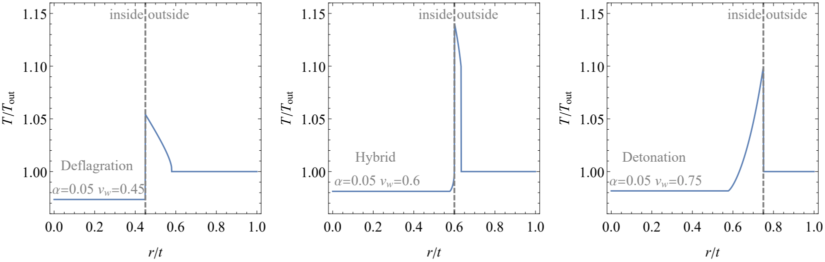
$$\int \mathrm{d}z \frac{\mathrm{d}\phi}{\mathrm{d}z} \left(\Box\phi + \frac{\partial V_{\mathrm{eff}}}{\partial \phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_i} \delta f_i(p,x) \right) = 0$$

$$\Big\downarrow \; \frac{\mathrm{d}\phi}{\mathrm{d}z}\frac{\partial V_{\mathrm{eff}}}{\partial \phi} = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}z} - \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}$$

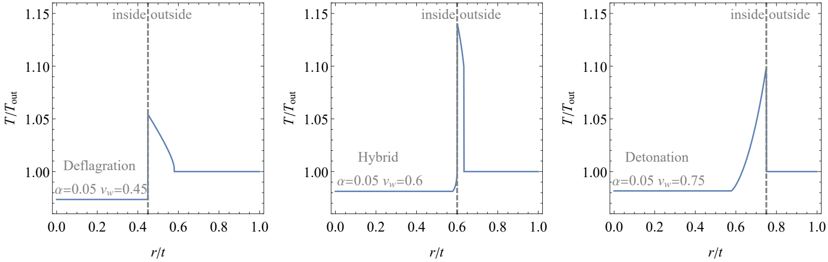
$$\Delta V_{\mathrm{eff}} = \boxed{\int \mathrm{d}z \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}} - \boxed{\sum_i \int \mathrm{d}\phi \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E_i} \delta f_i(p,x)}$$

driving force = hydrodynamic backreaction + non-equilibrium friction

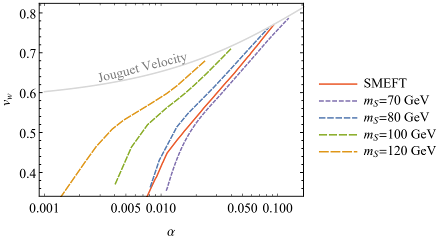
Wall Velocity



Wall Velocity



- No solutions found beyond $c_J = \frac{1}{\sqrt{3}} \frac{1+\sqrt{3\alpha^2+2\alpha}}{1+\alpha}$.

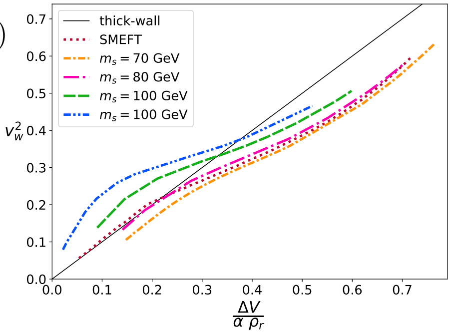


ML, Marco Merchand, Mateusz Zych, JHEP **02** (2022) 017, arXiv: 2111.02393

Wall Velocity analytic approximation

$$v_w = \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_R}} & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_R}} < v_J(\alpha) \\ 1 & \text{for} \quad \sqrt{\frac{\Delta V}{\alpha \rho_R}} \geq v_J(\alpha) \end{cases}$$

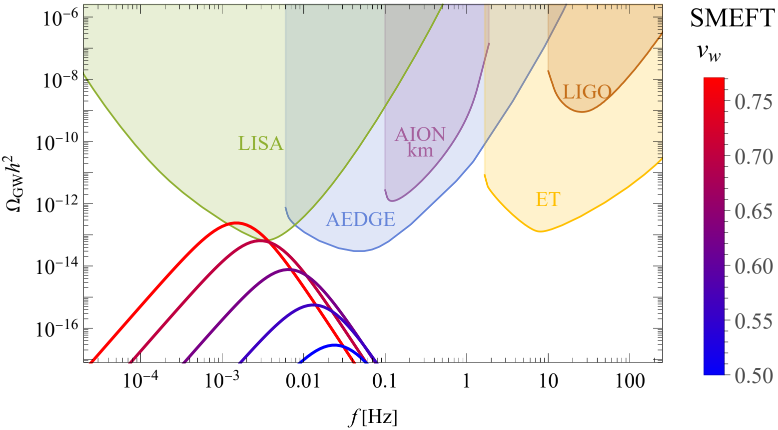
- Here: $\alpha = \frac{1}{\rho_R}\left(\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}\right)$
- Formula does not require solving transport equations
- Only the form of the potential is important



ML, Marco Merchand, Mateusz Zych, JHEP **02** (2022) 017, arXiv: 2111.02393

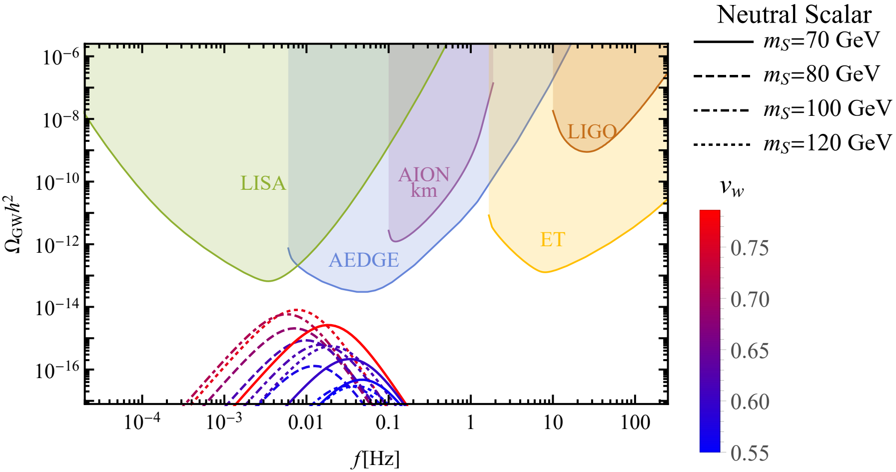
John Ellis, ML, Marco Merchand, José Miguel No, Mateusz Zych arXiv:2210.16305

Gravitational wave signals



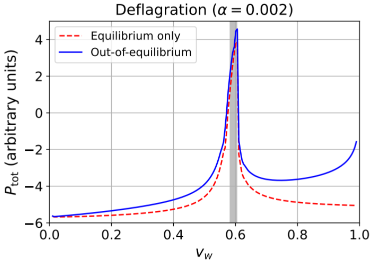
ML, Marco Merchand, Mateusz Zych arXiv: 2111.02393

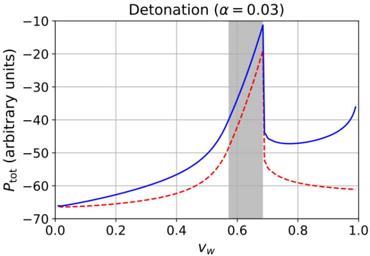
Gravitational wave signals



ML, Marco Merchand, Mateusz Zych arXiv: 2111.02393

- Impact of out-of-equilibrium effects is small





Benoit Laurent, James Cline arXiv:2204.13120

- LTE approximation $(\delta f = 0)$

$$v_w = \left(\left| \frac{3\alpha + \Psi - 1}{2\left(2 - 3\Psi + \Psi^3\right)} \right|^{\frac{p}{2}} + \left| v_{\mathrm{CJ}} \left(1 - a\frac{(1-\Psi)^b}{\alpha} \right) \right|^{\frac{p}{2}} \right)^{\frac{1}{p}}$$

with $a = 0.2233,\ b = 1.704,\ p = -3.433$ and $\Psi = \frac{w_t}{w_f}$

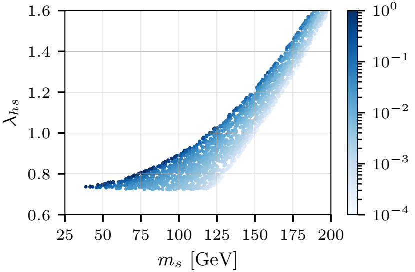
Wen-Yuan Ai, Benoit Laurent, Jorinde van de Vis arXiv:2303.10171

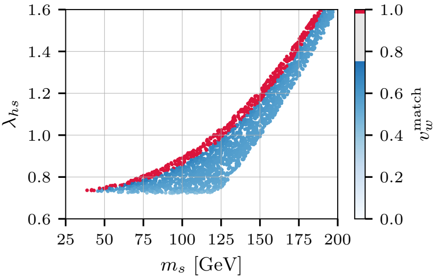
Scalar singlet extension

- Standard Model with an additional singlet scalar

$$V(H,s) = -\mu_h^2|H|^2 + \lambda|H|^4 + \frac{\lambda_{hs}}{2}S^2|H|^2 + \left({m_s}^2 - \frac{\lambda_{hs}v^2}{2}\right)\frac{s^2}{2} + \frac{\lambda_s}{4}S^4$$

- Scan of the parameter space with λ_s =1.
- Wall velocity determined analytically with matching conditions assuming LTE.





Tomasz Krajewski, ML, Mateusz Zych arXiv:2402.15408

Lattice realisation

- The energy-momentum tensor for the field and the fluid:

$$T^{\mu\nu}_{\text{field}} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left(\frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi \right)$$
$$T^{\mu\nu}_{\text{fluid}} = w u^\mu u^\nu + g^{\mu\nu} p$$

- effective coupling of the fluid and scalar:

$$\nabla_\mu T^{\mu\nu}_{\text{field}} = -\nabla_\mu T^{\mu\nu}_{\text{fluid}} = \frac{\partial V(\phi, T)}{\partial \phi} \partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

- Equation of state

$$p(\phi, T) = -V(\phi, T),$$
$$e(\phi, T) = V(\phi, T) - T \frac{\mathrm{d}V(\phi, T)}{\mathrm{d}T},$$
$$w(\phi, T) = -T \frac{\mathrm{d}V(\phi, T)}{\mathrm{d}T}.$$

Lattice realisation: EoM

- EoM for scalar fields assuming LTE $\eta = 0$

$$-\partial_t^2 h + \frac{1}{r^2}\partial_r(r^2\partial_r h) - \frac{\partial V}{\partial h} = 0$$

$$-\partial_t^2 s + \frac{1}{r^2}\partial_r(r^2\partial_r s) - \frac{\partial V}{\partial s} = 0$$

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$$-\partial_t^2 s + \frac{1}{r^2}\partial_r(r^2\partial_r s) - \frac{\partial V}{\partial s} = 0$$

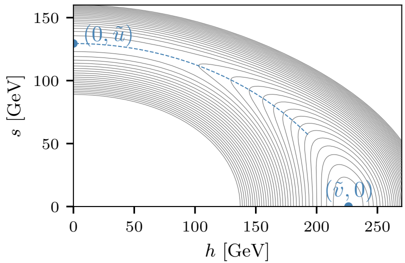
- EoM for plasma assuming LTE $\eta = 0$

$$\partial_t \tau + \frac{1}{r^2}\partial_r(r^2(\tau + p)v) = \frac{\partial V_{\text{eff}}}{\partial h}\partial_t h + \frac{\partial V_{\text{eff}}}{\partial s}\partial_t s$$
$$\partial_t Z + \frac{1}{r^2}\partial_r\left(r^2 Z v\right) + \partial_r p = -\frac{\partial V_{\text{eff}}}{\partial h}\partial_r h - \frac{\partial V_{\text{eff}}}{\partial s}\partial_r s$$

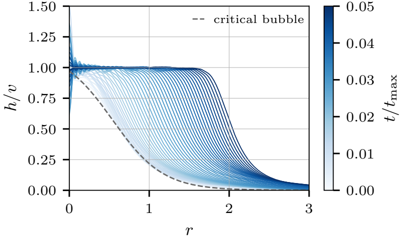
where $Z := w\gamma^2 v$ and $\tau := w\gamma^2 - p$

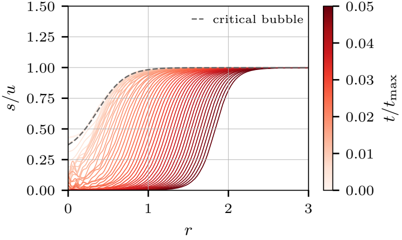
Evolution: early stages

- Initial $h(r)$ and $s(r)$ profiles corresponding to the critical bubble.



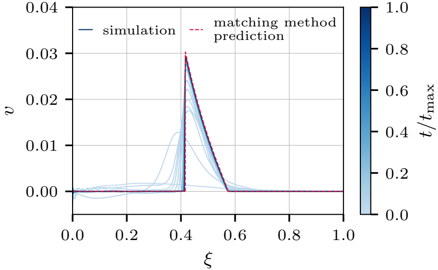
- Plasma initially at rest $v(r) = 0$ with nucleation temperature $T(r) = T_n$





Evolution: late stages

- Self-similar profiles: $\xi = r/t$
- Two possible scenarios for the growing bubble in LTE:



- evolution toward a stationary state predicted by matching conditions

Evolution: late stages

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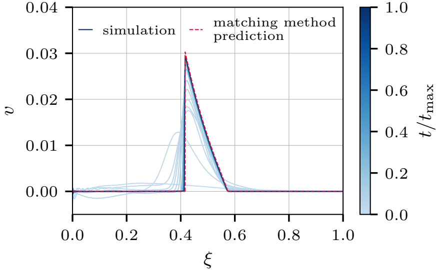
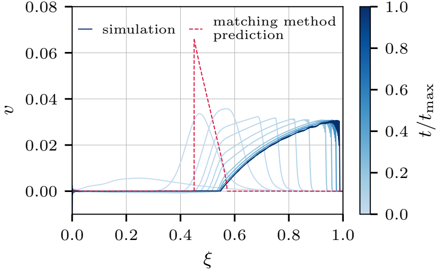
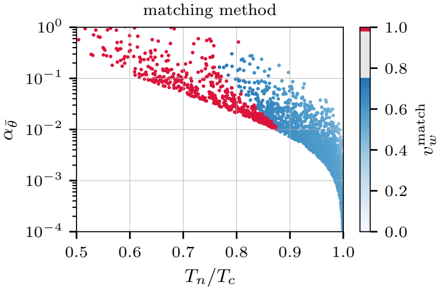


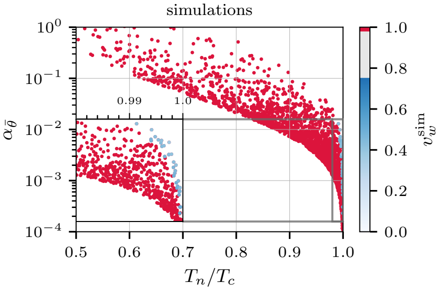


- evolution toward a stationary state predicted by matching conditions
- rapid expansion beyond Chapman-Jouguet velocity leading to a runaway scenario

Tomasz Krajewski, ML, Mateusz Zych arXiv:2402.15408

Analytical treatment vs real-time simulations

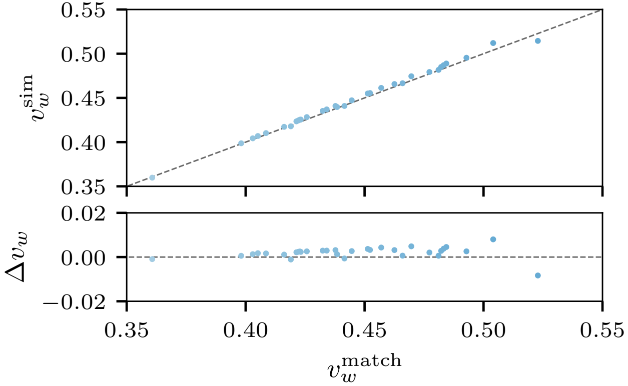




- Matching equations predict significant number of stationary deflagrations and hybrids.
- Real-time simulations predict only a few points evolve towards a stationary state.

Tomasz Krajewski, ML, Mateusz Zych arXiv:2402.15408

Analytical treatment vs real-time simulations



If the stationary state is achieved for a given model, bubble-wall velocity is very accurately predicted by the matching equations.

Conclusions

- GW within the reach of upcoming experiments are typically produced in transitions with very relativistic wall velocities $v_w \approx 1$. This produces a tension with electroweak baryogenesis which requires slower walls.
- Lattice simulations in the absence of non-equilibrium friction predict bubbles to generically expand as runaways even in cases where matching conditions predict a slower wall.
- Stationary profiles are dynamically achieved only for tiny supercooling ($T_n/T_c \lesssim 1$).

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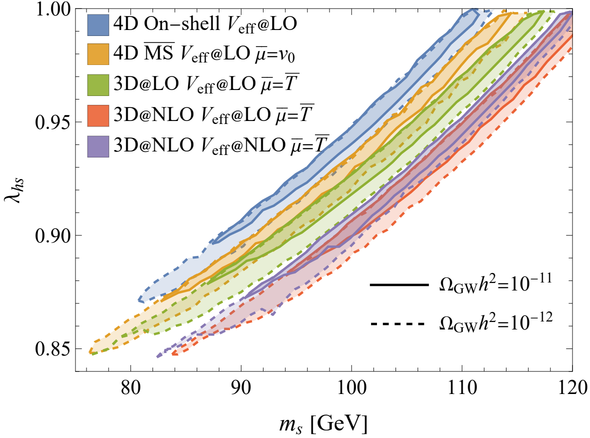
$\boxed{\text{Thank you for your attention!}}$

Backup Slides

Theoretical uncertainty on the parameter space

- Standard Model with an additional singlet scalar

$$V(H,s) = -\mu_h^2|H|^2 + \lambda|H|^4 + \frac{\lambda_{hs}}{2}S^2|H|^2 + \left({m_s}^2 - \frac{\lambda_{hs}v^2}{2}\right)\frac{s^2}{2} + \frac{\lambda_s}{4}S^4$$

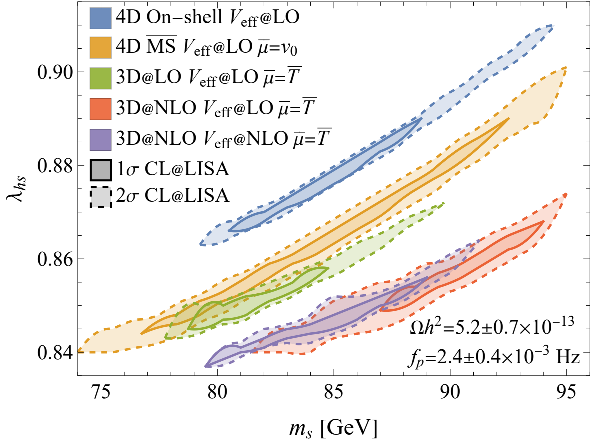


ML, Marco Merchand, Laura Sagunski, Philipp Schicho, Daniel Schmitt arXiv:2403.03769

Theoretical uncertainty on the parameter reconstruction

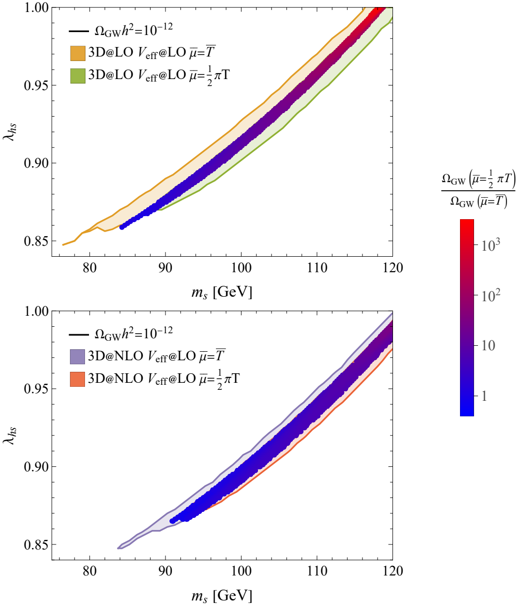
- Standard Model with an additional singlet scalar

$$V(H,s) = -\mu_h^2|H|^2 + \lambda|H|^4 + \frac{\lambda_{hs}}{2}S^2|H|^2 + \left({m_s}^2 - \frac{\lambda_{hs}v^2}{2}\right)\frac{s^2}{2} + \frac{\lambda_s}{4}S^4$$



ML, Marco Merchand, Laura Sagunski, Philipp Schicho, Daniel Schmitt arXiv:2403.03769

Theoretical uncertainty on the parameter space



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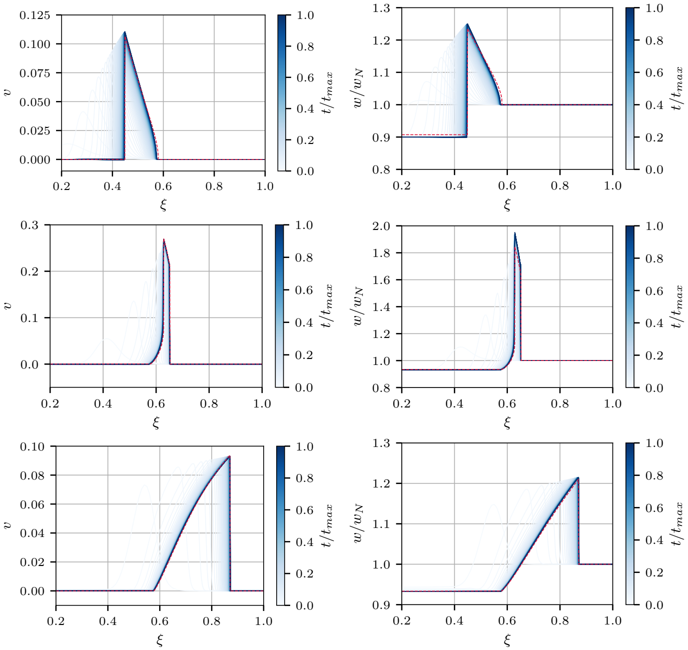
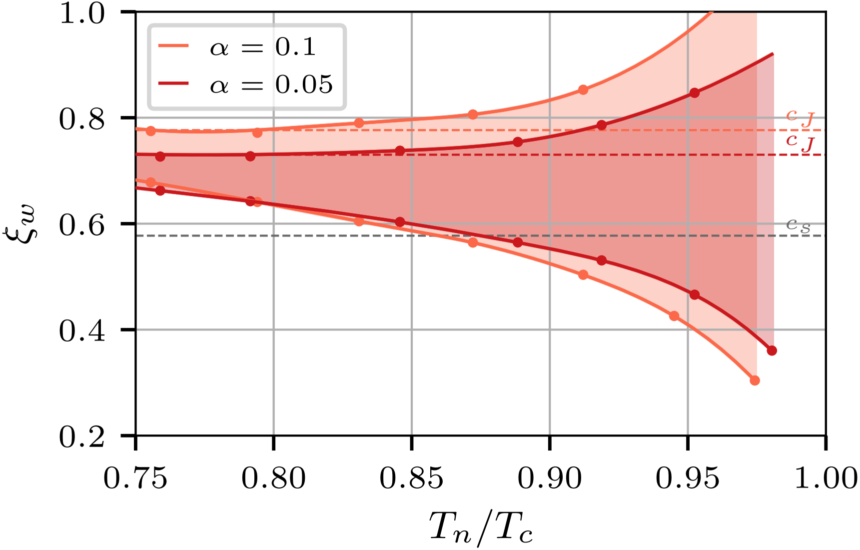
Conclusions II

- Large errors on the GW spectra for individual parameter points corresponds to small $O(1\%)$ error on the reconstructed model parameters
- These small reconstruction errors would still dominate the experimental uncertainties for any detectable spectra



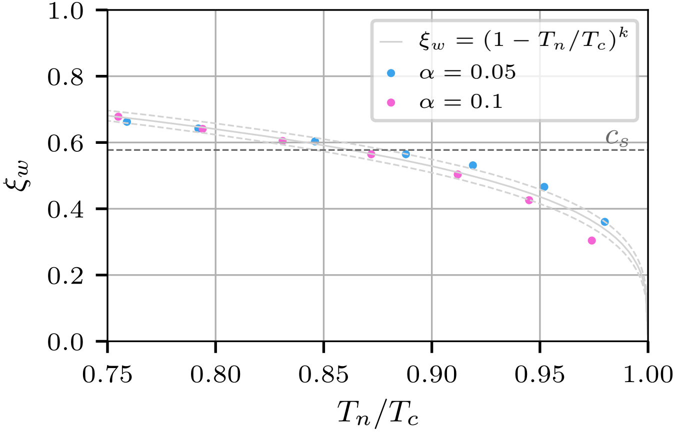
Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

Hydrodynamical obstruction: can all v_w be realised?



Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

Hydrodynamical obstruction: numerical fit



- Simple numerical fit accurate for relatively strong PTs

$$v_w = \left(1 - \frac{T_n}{T_c}\right)^k, \quad \text{with} \quad k = 0.2768 \pm 0.0055$$

Tomasz Krajewski, ML, Mateusz Zych arXiv:2304.18216

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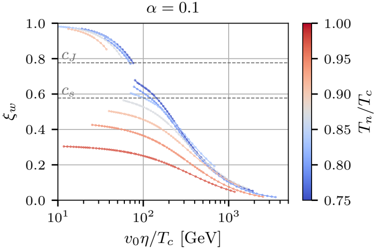

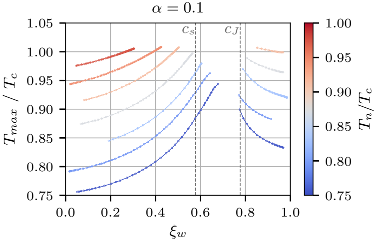

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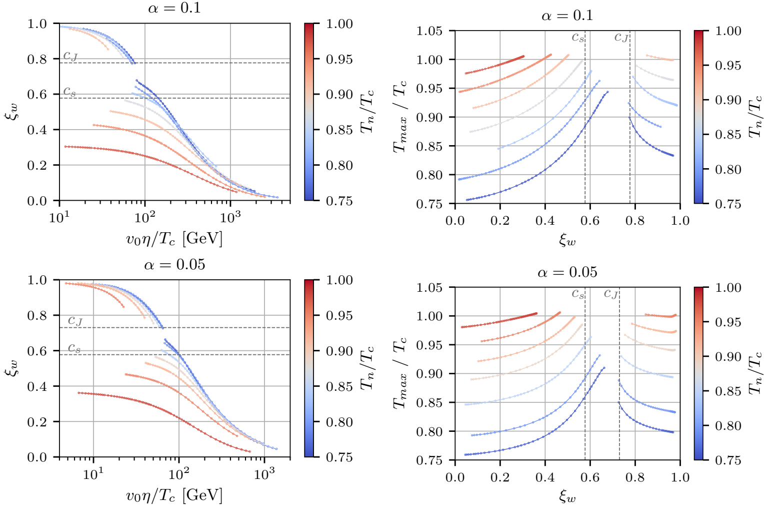
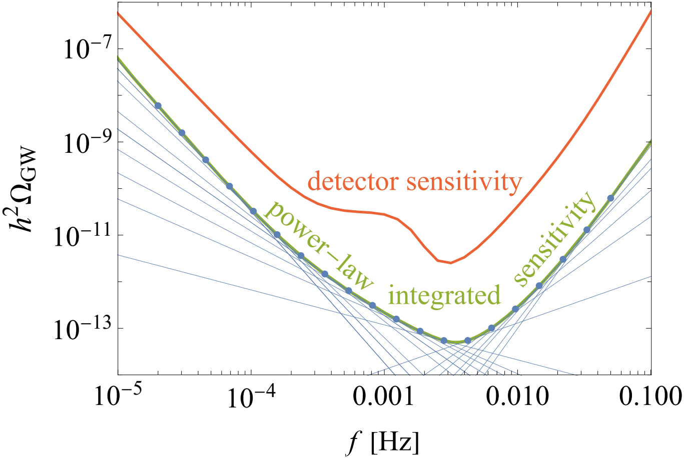
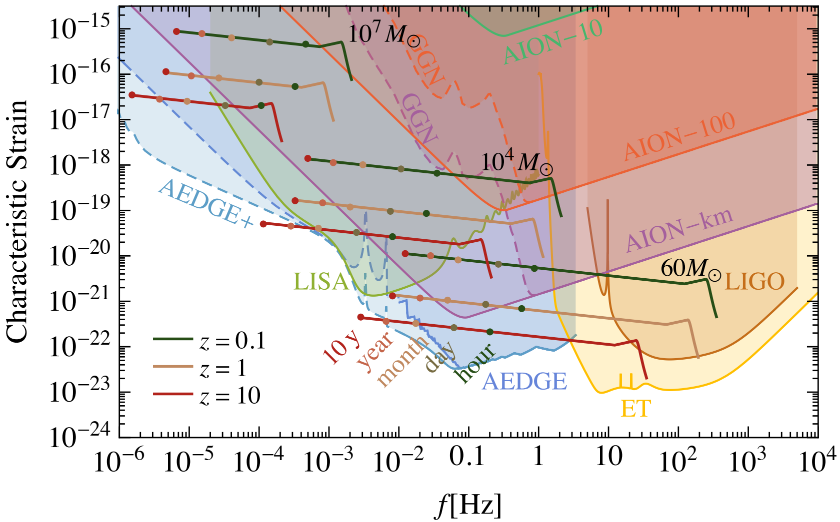


Power-law integrated sensitivity

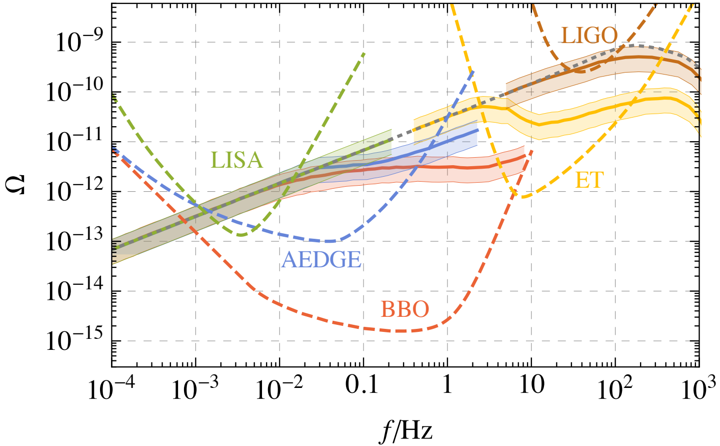
$$\Omega_{\rm GW}^{\rm noise} = \frac{2\pi}{3}\frac{f^3 h_c^2}{H_0^2}, \qquad \mathrm{SNR} = \sqrt{\mathcal{T}\int df \left(\frac{\Omega_{\rm GW}^{\rm signal}}{\Omega_{\rm GW}^{\rm noise}}\right)^2}, \qquad \Omega_{\rm GW}^{\rm signal} = \Omega\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$$



Sensitivity to binary mergers



Foreground from LIGO-Virgo binaries



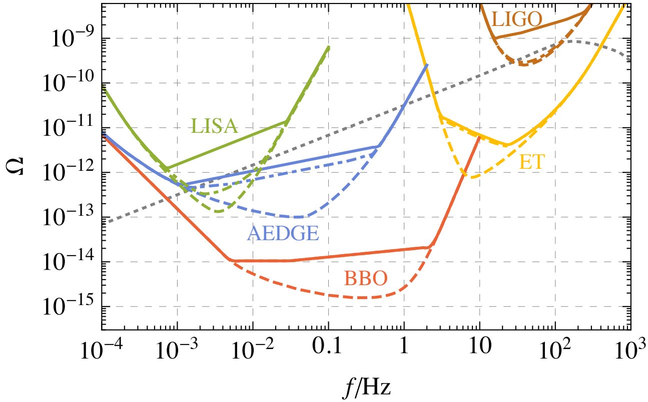
- Dashed gray line: total foreground from LIGO-Virgo binaries
- Thick lines: foreground without individually observable binaries

ML, Ville Vaskonen arXiv:2111.05847

Improved sensitivities from Fisher analysis

- assuming power-law signal as in PI sensitivity

$$\Omega_{\rm GW}(f) = \Omega \left(\frac{f}{f_{\rm ref}}\right)^{\alpha} + A\langle\Omega_{\rm BBH}(f)\rangle + \Omega_{\rm BWD}(f) + \Omega_{\rm instr}(f)$$



ML, Ville Vaskonen arXiv:2111.05847

Above the Jouguet velocity. Can the walls run away?

- Energy of the bubble

$$\mathcal{E} = 4\pi R^2 \sigma \gamma - \frac{4\pi}{3} R^3 p \,, \qquad \gamma = \frac{1}{\sqrt{1-\dot{R}^2}}$$

- Vacuum pressure on the wall
  Coleman '73

$$p_0 = \Delta V$$

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- Next-To-Leading order plasma contribution
Bodeker '17 Gouttenoire '21

$$p = \Delta V - \Delta P_{\rm LO} - \gamma \Delta P_{\rm NLO} \approx \Delta V - \frac{\Delta m^2 T^2}{24} - \gamma\, g^2 \Delta m_V T^3\,.$$

- Next-To-Leading order plasma contribution with resummation
Hoche '20

$$P = \Delta V - P_{1\to 1} - \gamma^2 P_{1\to N} \approx \Delta V - 0.04\Delta m^2 T^2 - 0.005 g^2 \gamma^2 T^4\,.$$