


Double- and Triple- parton scattering in p-A collisions

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Outline

 Introduction to double parton scattering (DPS) and hadronic Physics

 Nuclear DPS (pA)

 Triple Parton Scattering (TPS)

 Nuclear TPS (pA)

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 Introduction to double parton scattering (DPS) and hadronic Physics

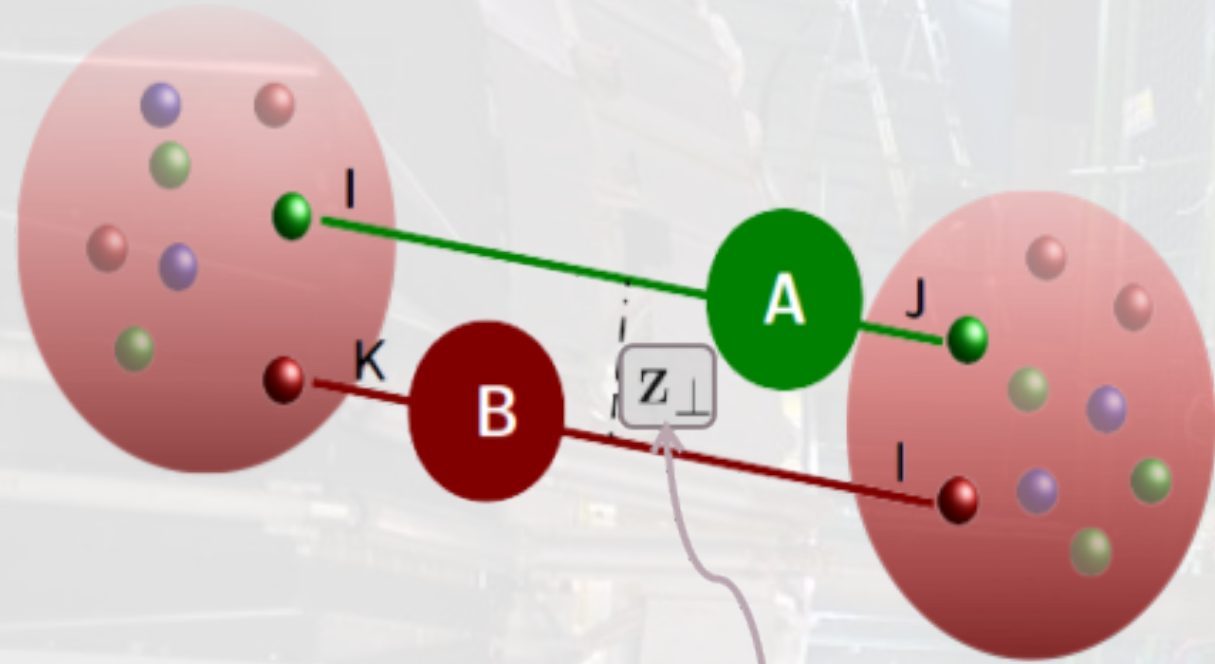
 Nuclear DPS (pA)

 Triple Parton Scattering (TPS)

 Nuclear TPS (pA)

Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$d\sigma \propto \int d^2z_{\perp} \underbrace{F_{ij}(x_1, x_2, z_{\perp}, \mu_A, \mu_B) F_{kl}(x_3, x_4, z_{\perp}, \mu_A, \mu_B)}$$

Double Parton Distribution (DPD)

N. Paver and D. Treleani, *Nuovo Cimento* 70A, 215 (1982)

Mekhfi, *PRD* 32 (1985) 2371

M. Diehl et al, *JHEP* 03 (2012) 089

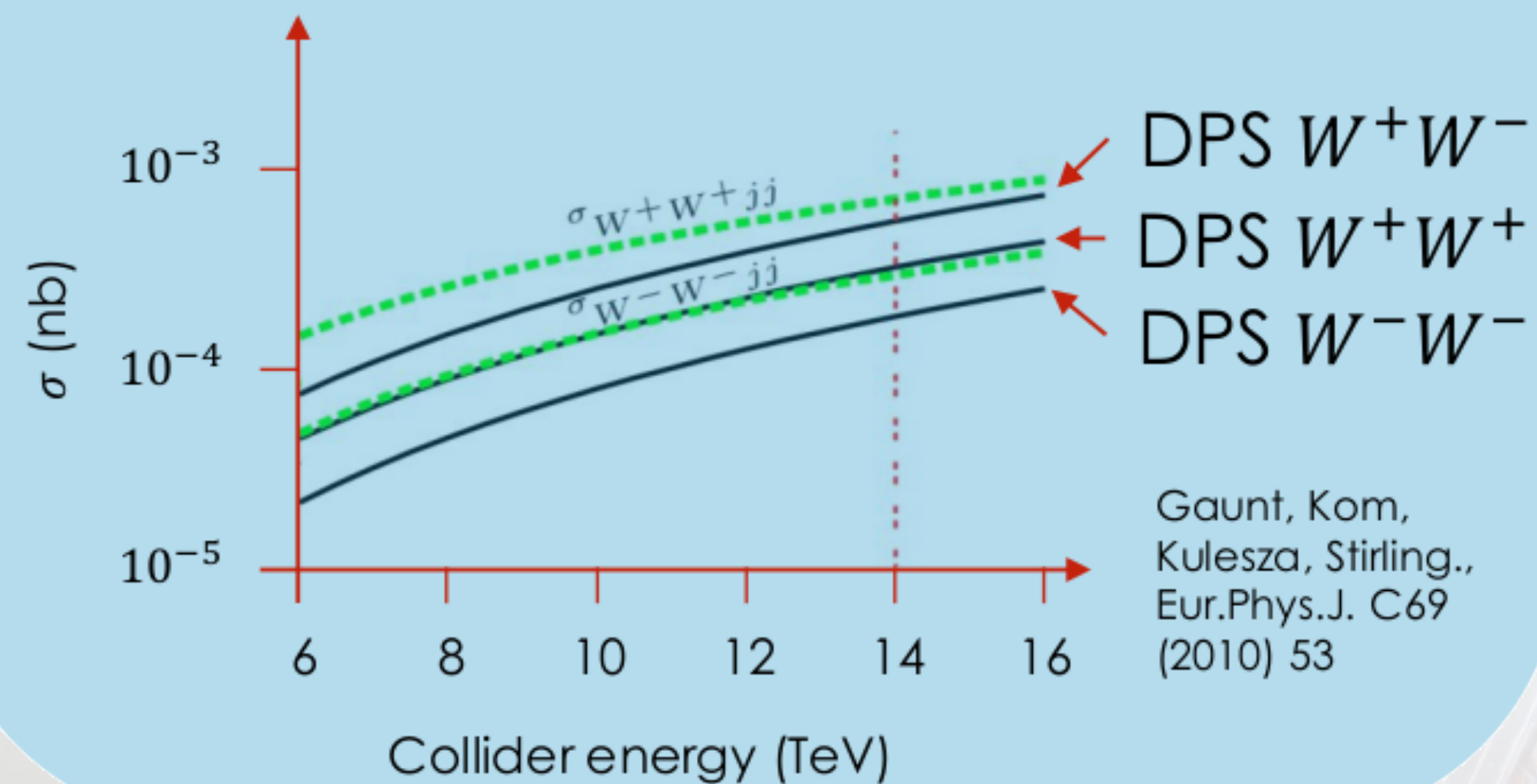
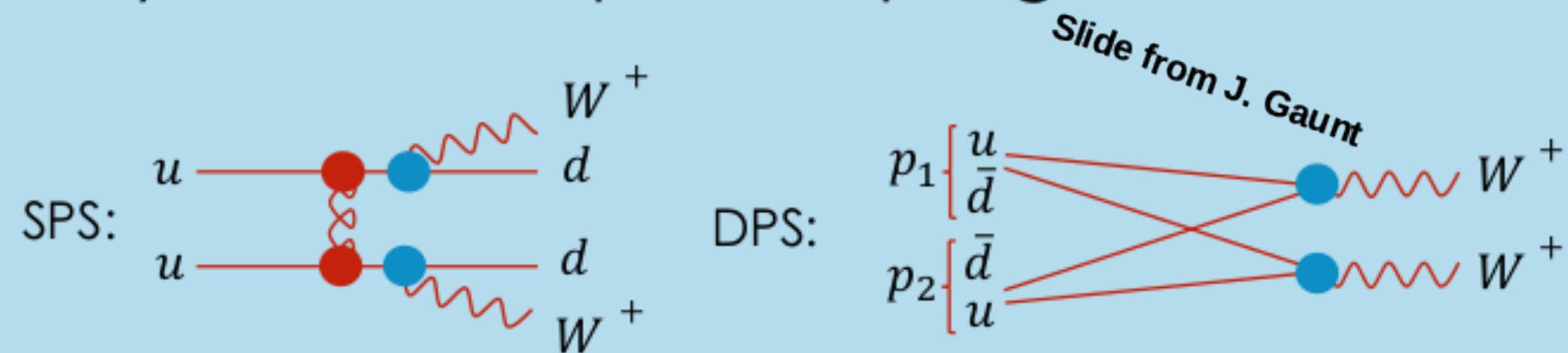
$$F_{ij}^{\lambda_1, \lambda_2}(x_1, x_2, \vec{k}_{\perp}) = (-8\pi P^+) \frac{1}{2} \sum_{\lambda} \int d\vec{z}_{\perp} e^{i\vec{z}_{\perp} \cdot \vec{k}_{\perp}} \\ \times \int \left[\prod_l^3 \frac{dz_l^-}{4\pi} \right] e^{ix_1 P^+ z_1^- / 2} e^{ix_2 P^+ z_2^- / 2} e^{-ix_1 P^+ z_3^- / 2} \\ \times \langle \lambda, \vec{P} = \vec{0} | \hat{O}_i^1 \left(z_1^- \frac{\vec{n}}{2}, z_3^- \frac{\vec{n}}{2} + \vec{z}_{\perp} \right) \hat{O}_j^2 \left(z_2^- \frac{\vec{n}}{2} + \vec{z}_{\perp}, 0 \right) | \vec{P} = \vec{0}, \lambda \rangle$$

$$\hat{O}_i^k(z, z') = \bar{q}_i(z) \hat{O}(\lambda_k) q_i(z')$$

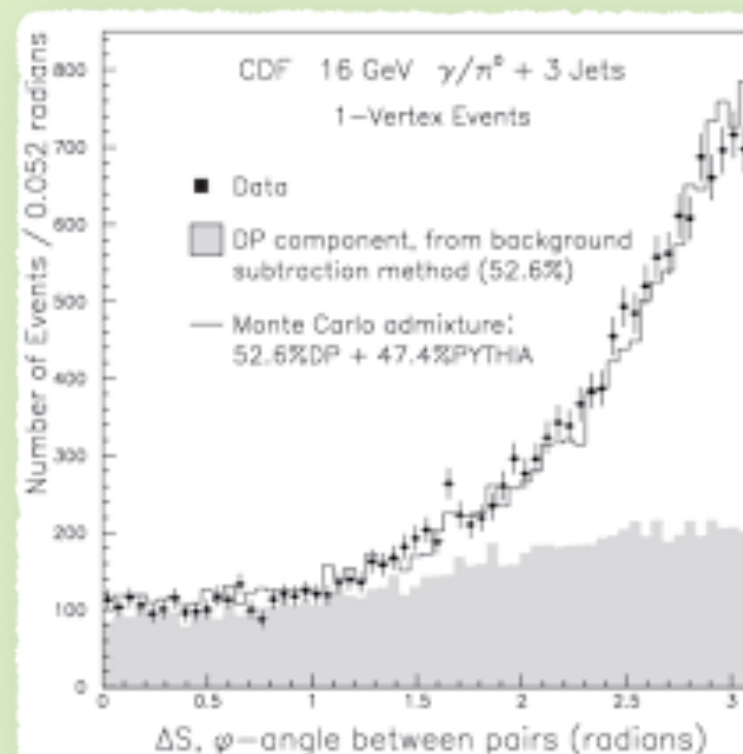
$$\hat{O}(\lambda_k) = \frac{\not{n}}{2} \frac{1 + \lambda_k \gamma_5}{2} .$$

Where and Why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

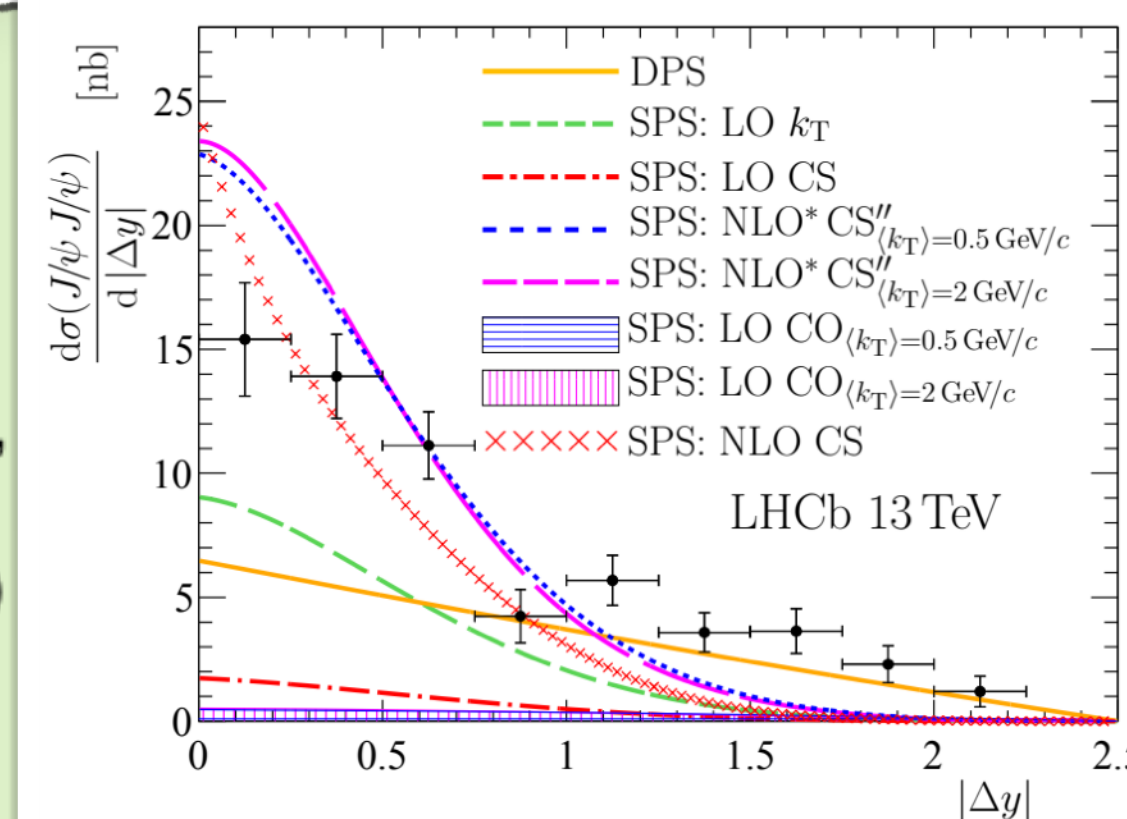


...or in certain phase space regions

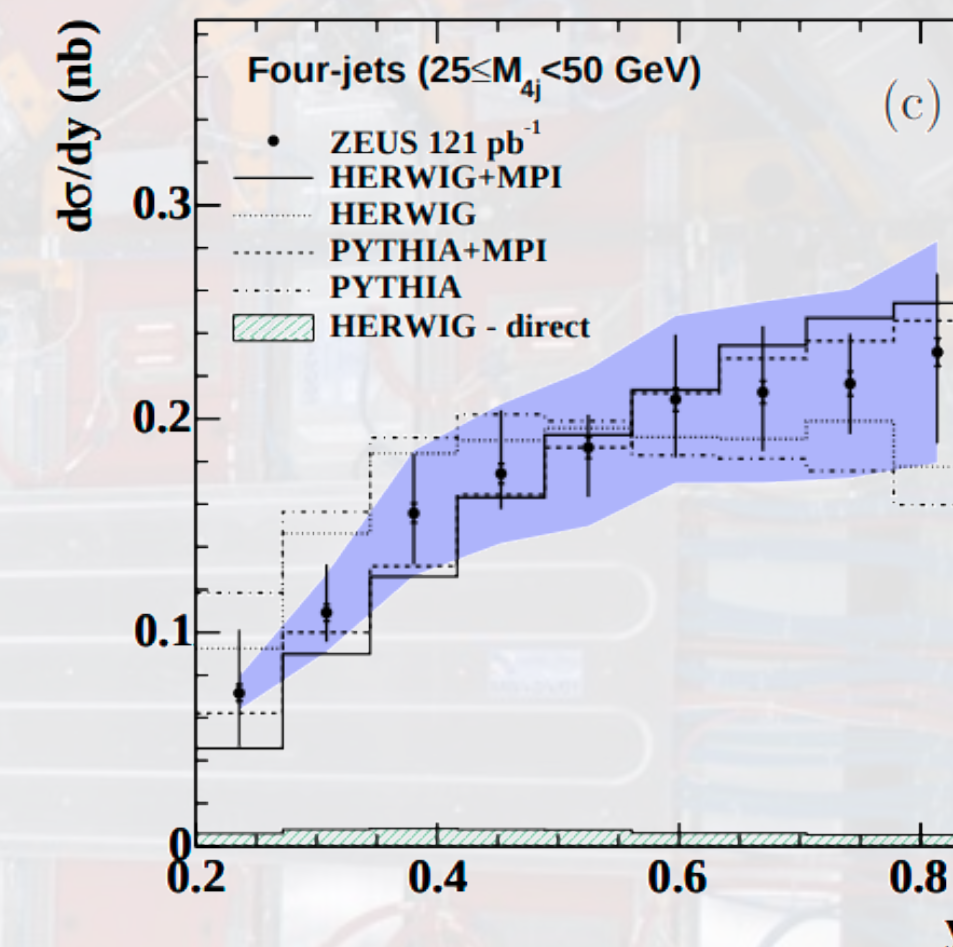


CDF, $\gamma + 3j$, Phys.Rev. D56 (1997) 3811-3832

LHCb, double J/ψ , JHEP 06, 047, (2017)



in ep Colliders?

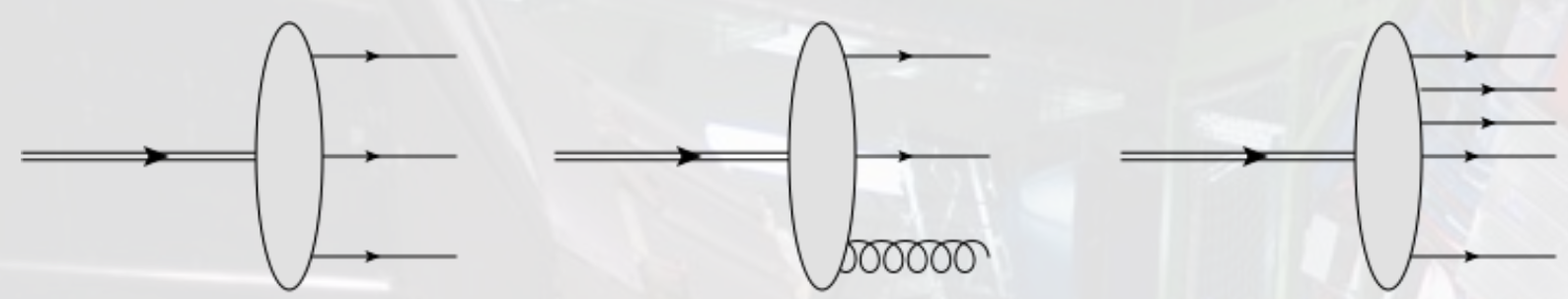


HERA data, ZEUS coll, Nucl. Phys. B 729, 1 (2008)

Access to:
 - double parton correlations
 - the transverse distance distribution of partons!!

all UNKNOWN

Multidimensional picture of hadrons

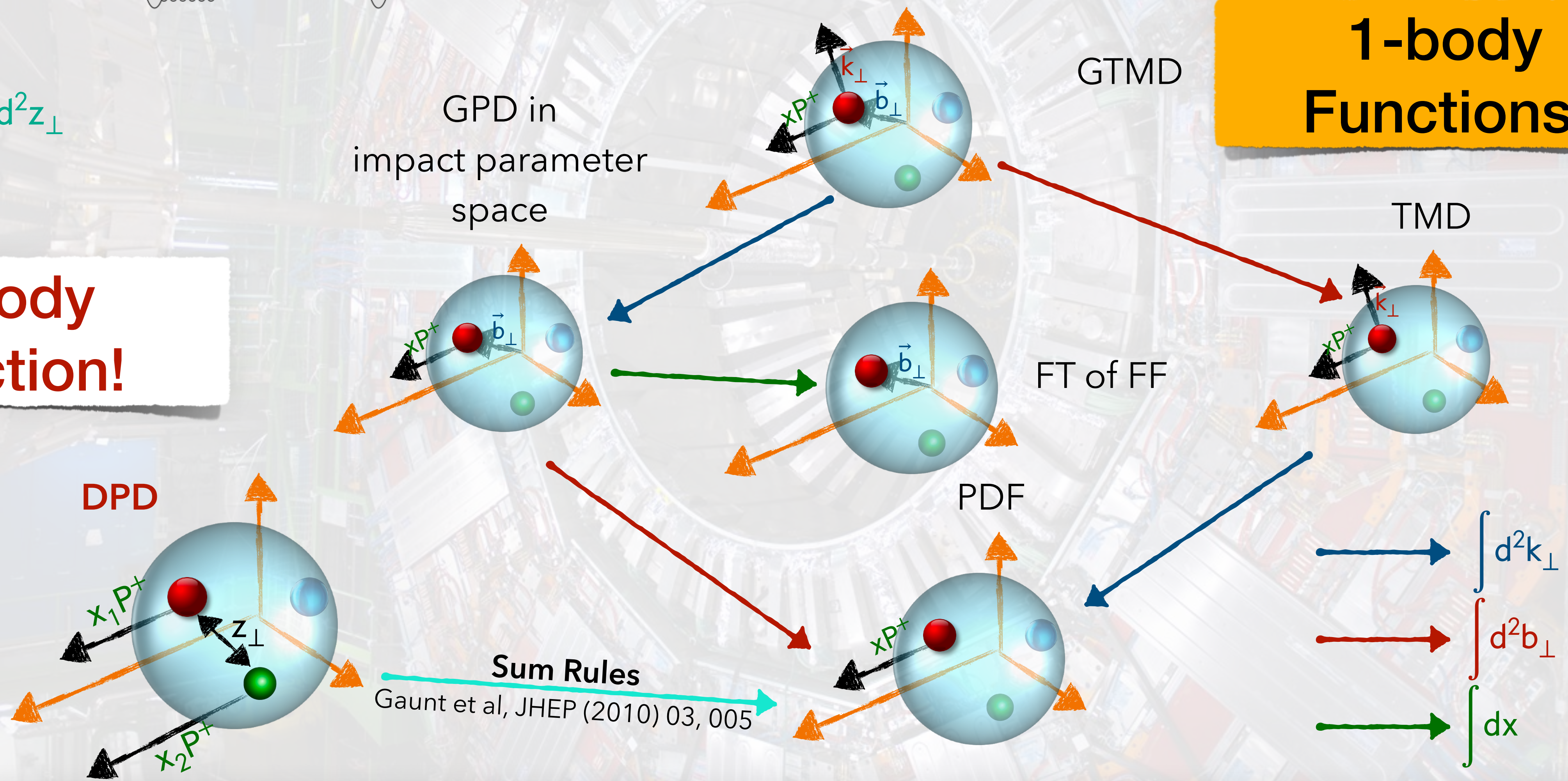


Light-Front wave-function

$\int d^2z_{\perp}$

2-body Function!

1-body Functions!



Sum Rules
Gaunt et al, JHEP (2010) 03, 005

$\int d^2k_{\perp}$
 $\int d^2b_{\perp}$
 $\int dx$

How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

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uncorrelated scenario:

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \boxed{g(x_1, x_2)} \tilde{T}(\vec{z}_\perp)$$

double PDF

Sum Rules

pQCD evolution

PDF(x_1)*PDF(x_2)
uncorrelated scenario

$$\frac{\alpha_s(t)\Delta t}{2\pi} P_{j' \rightarrow j_1 j_2} \left(\frac{x_1}{x_1+x_2}, \frac{\delta x_1}{x_1+x_2} \right) + \sum_{j'} D_h^{j'}(x_1+x_2; t) \delta x_2$$

The diagram shows two types of parton evolution. The first is a splitting of a parton j' into two partons j_1 and j_2 . The second is a parton h splitting into two partons j_1 and j_2 . The diagram is associated with the evolution equation shown above it.

How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

uncorrelated scenario:

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \boxed{g(x_1, x_2)} \tilde{T}(\vec{z}_\perp)$$

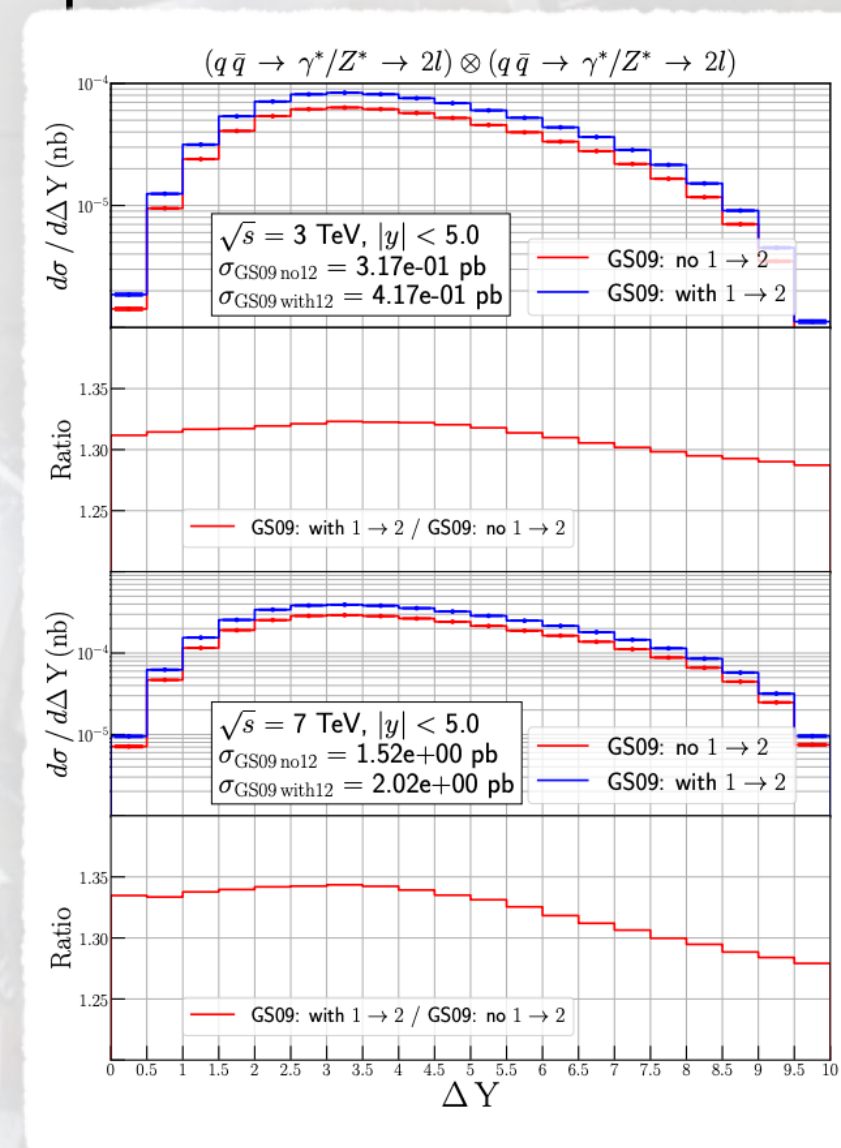
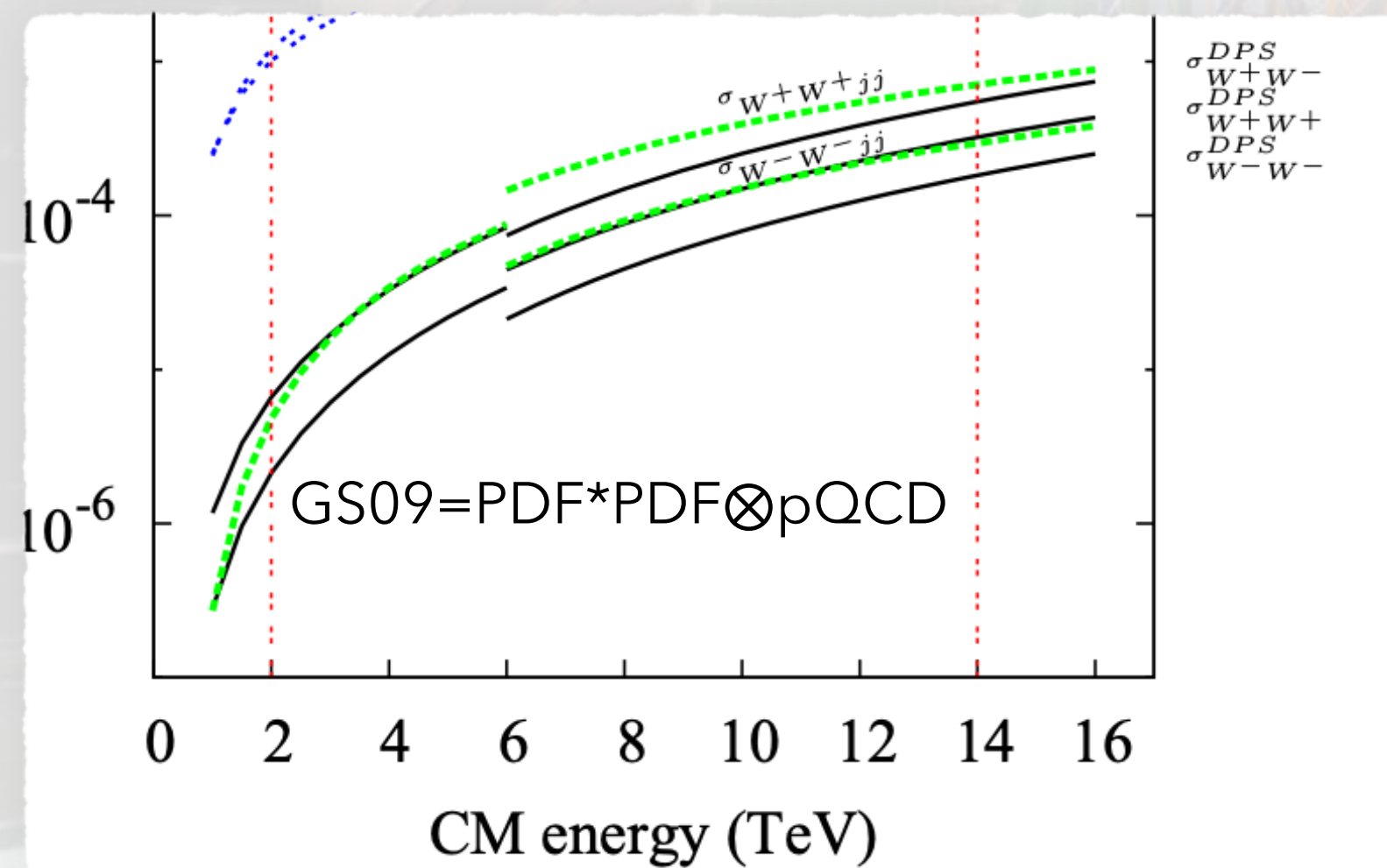
double PDF

Sum Rules

pQCD evolution

PDF(x₁)*PDF(x₂)
uncorrelated scenario

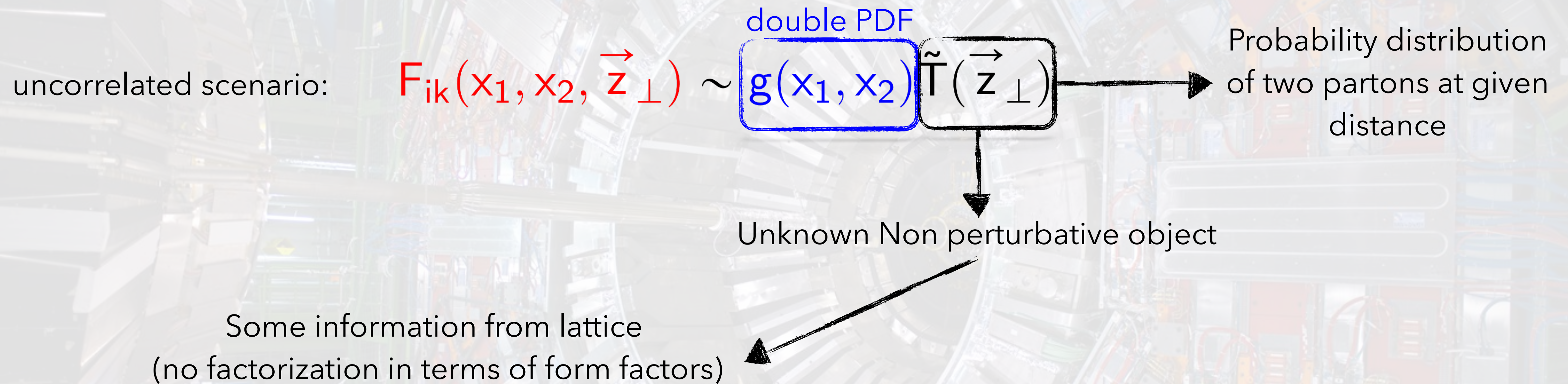
J. R. Gaunt et al, EPJC 69 (2010) 54-65



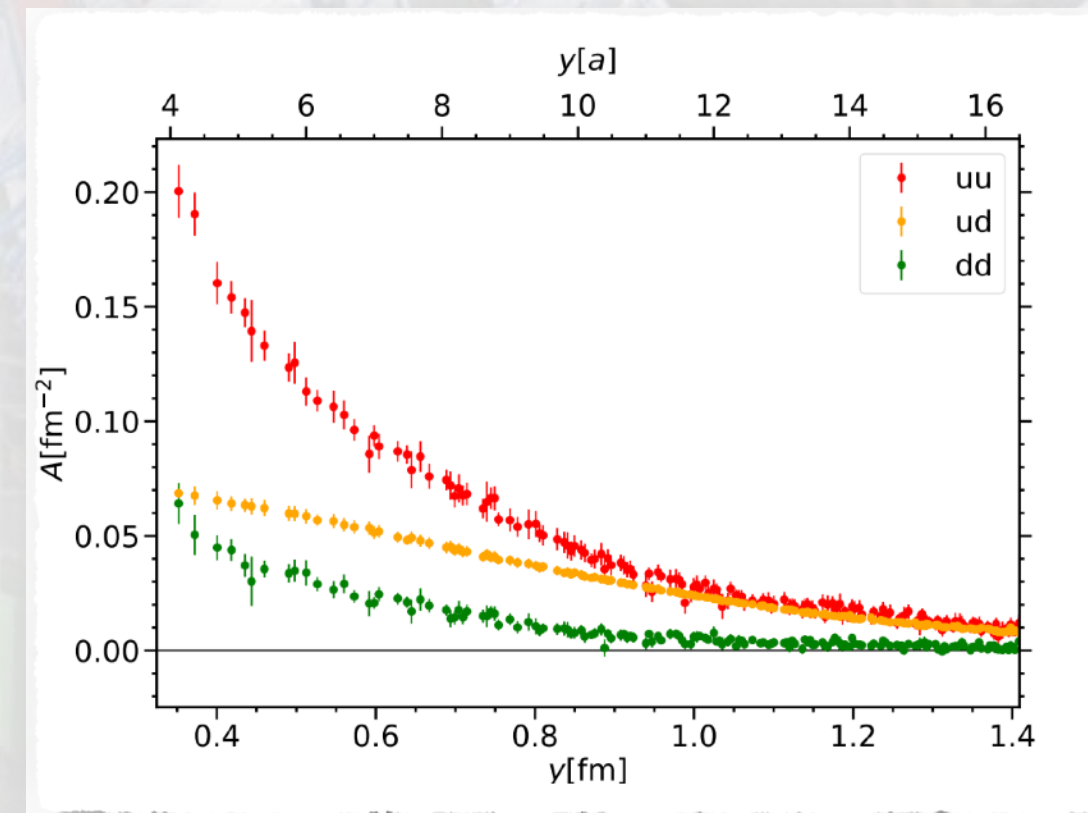
O. Fedkevych, J. R. Gaunt, JHEP 02 (2023) 090

How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. For phenomenology @LHC kinematics (small x and many partons produced)



G. S. Bali et al, JHEP 09 (2021) 121



How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

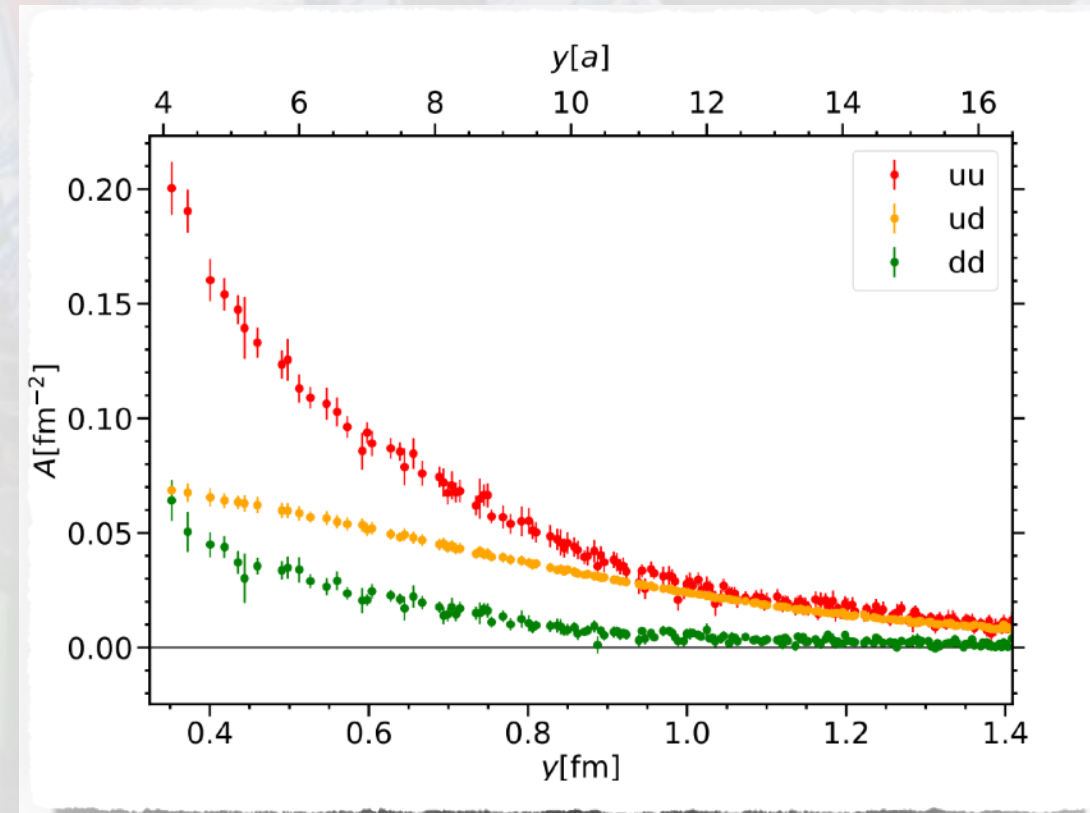
uncorrelated scenario: $F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \underbrace{g(x_1, x_2)}_{\text{double PDF}} \tilde{T}(\vec{z}_\perp)$ → Probability distribution of two partons at given distance

Unknown Non perturbative object

Some information from lattice
(no factorization in terms of form factors)

Some constraints from data

Some Constraints from general properties



$$\sigma_{\text{eff}}^{-1} = \int d^2z_\perp \tilde{T}(z_\perp)^2$$

G. S. Bali et al, JHEP 09 (2021) 121

Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_A^{\text{pp}} \sigma_B^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

POCKET FORMULA

Differential X-section single parton scattering for the process: $pp \rightarrow A(B) + X$

Differential X-section double parton scattering for the process: $pp \rightarrow A + B + X$

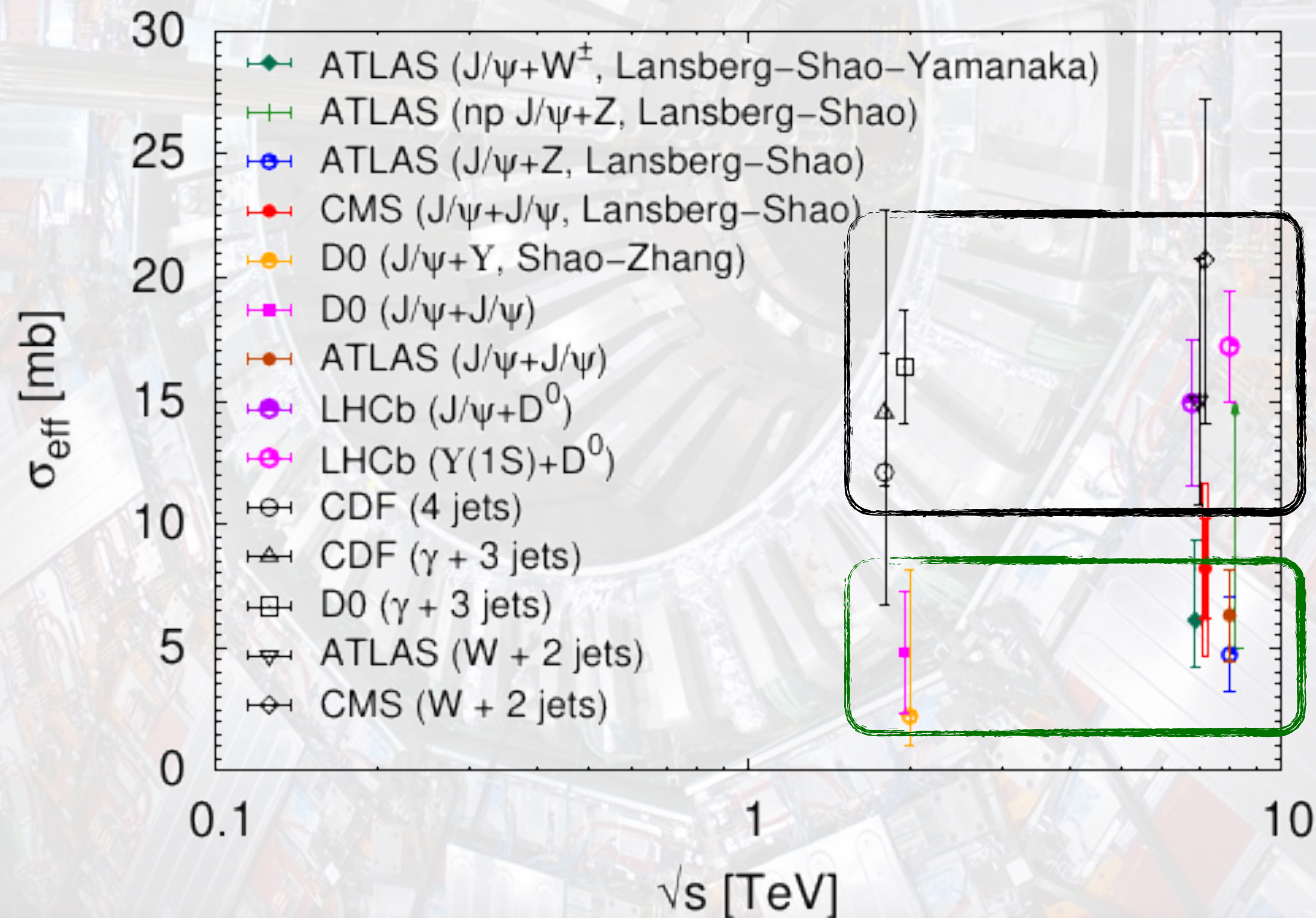
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→ Differential X-section single parton scattering for the process: $pp \rightarrow A(B) + X$
→ Differential X-section double parton scattering for the process: $pp \rightarrow A + B + X$

POCKET FORMULA

- Results for W, Jet productions...
- Results for quarkonium productions



First observation of same sign WW via DPS:

$$\sigma_{\text{eff}} = 12.2^{+2.9}_{-2.2} \text{ mb}$$

[CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\text{DPS}} \sim 6.28 \text{ fb}$$

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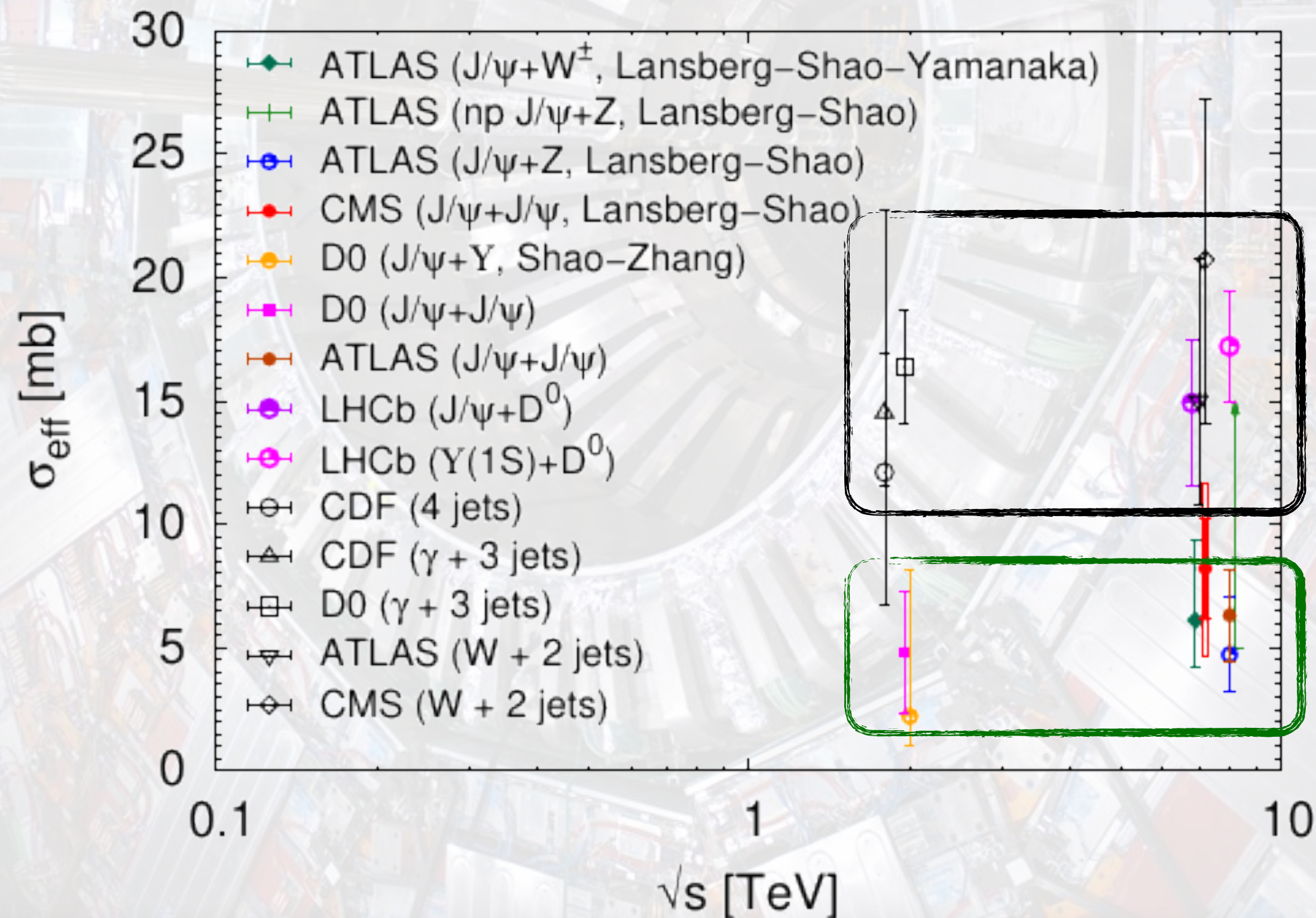
- 1) Process dependent?
- 2) Sensitive to correlations
- 3) Sensitive to the inner structure?

predicted by all models!

M.R. et al PLB 752,40 (2016)

M. Traini, M. R. et al, PLB 768, 270 (2017)

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



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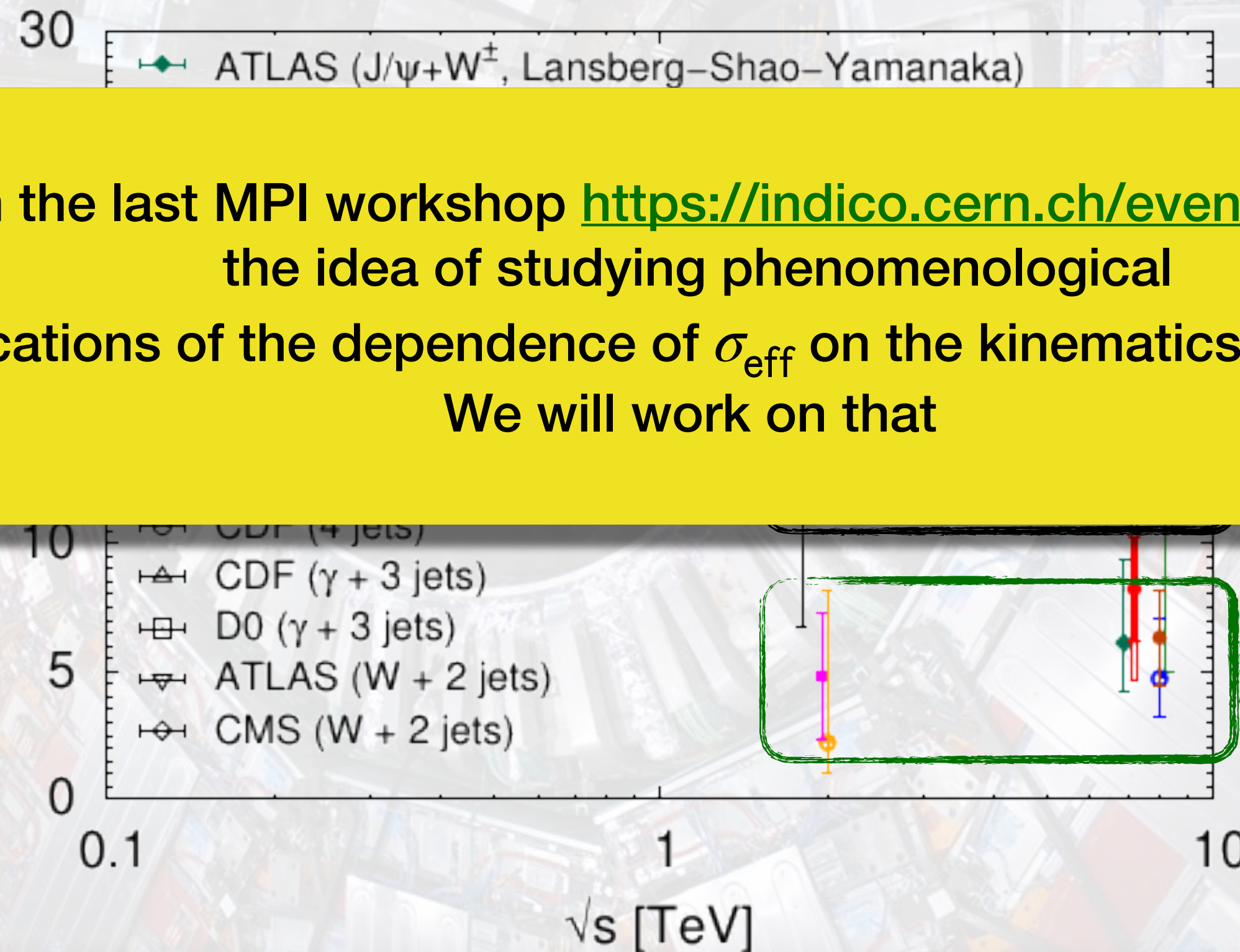
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From the last MPI workshop <https://indico.cern.ch/event/1281679/> the idea of studying phenomenological implications of the dependence of σ_{eff} on the kinematics came out!! We will work on that



First observation of WW via DPS:

$$2.2^{+2.9}_{-2.2} \text{ mb}$$

PRL 131 (2023) 091803

$$\sigma_{\text{eff}} \sim 6.28 \text{ fb}$$

Effective Cross Section and proton structure

If DPDs factorize in terms of PDFs then

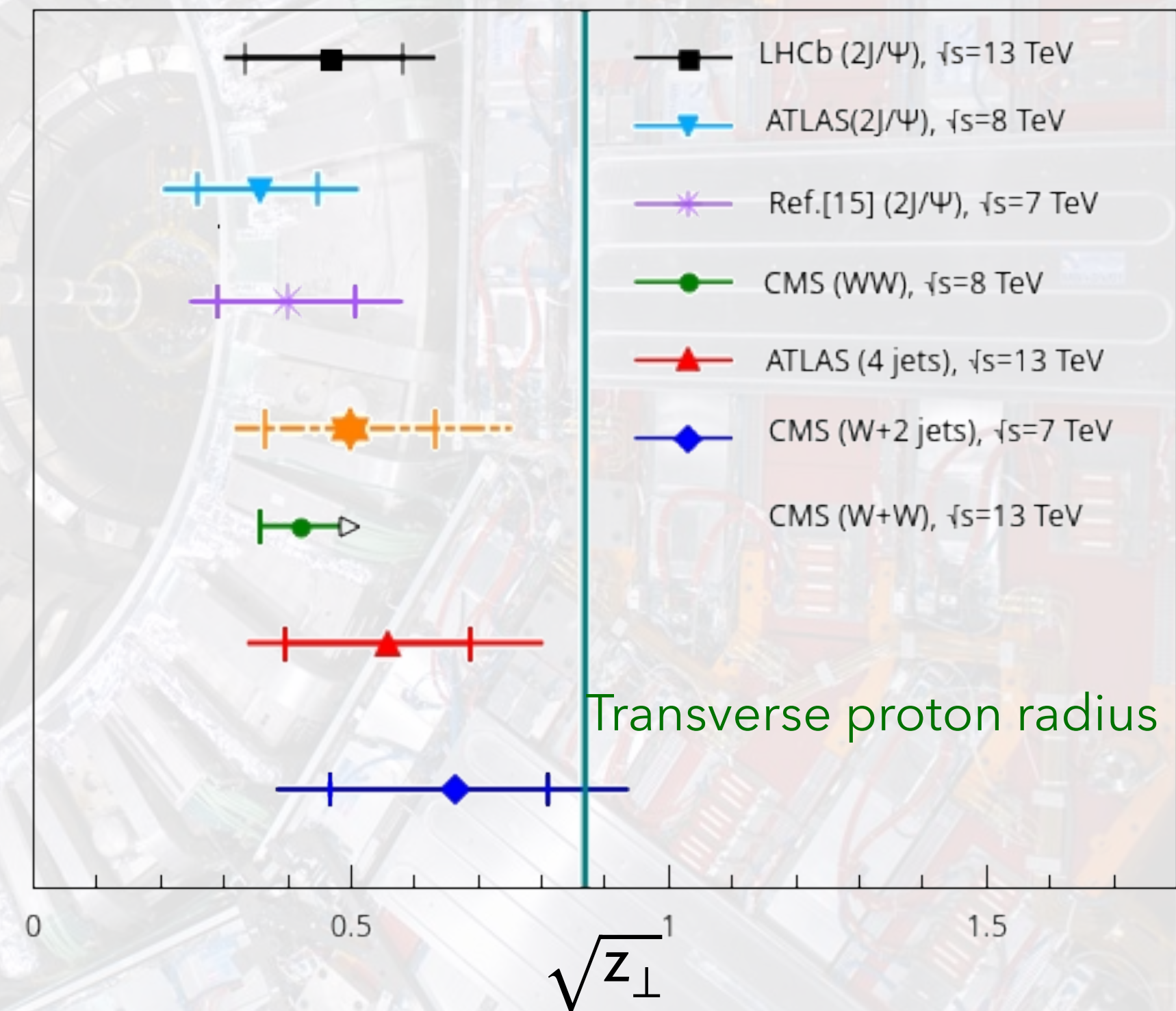
$$\sigma_{\text{eff}}^{-1} = \int d^2z_{\perp} \tilde{T}(z_{\perp})^2 = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$$

As for the standard FF:

$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_{\perp}^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$



M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

DPS in pA collisions - why?

- 1) Increase the DPS cross-section
- 2) easier to extract $\sigma_{eff,pp}$
- 3) in the future we can extract information on **NEUTRON** DPDs
- 4) Nuclear effects in DPDs!!
- 5) Are DPDs of free proton the same of those for bound protons?

DPS in pA collisions

- 1) Same-sign WW production in proton-nucleus collisions at the LHC as a signal for double parton scattering
D. d'E. & A. Snigirev, PLB 718 (2013) 1395-1400
- 2) Enhanced $J/\psi/\Psi$ -pair production from double parton scatterings in nucleus-nucleus collisions at the Large Hadron Collider
D. d'E. & A. Snigirev, PLB 727 (2013) 157-162
- 3) Pair production of quarkonia and electroweak bosons from double-parton scatterings in nuclear collisions at the LHC
D. d'E. & A. Snigirev, Nucl. Phys. A 931 (2014) 303-308
- 4) **B. Blok et al, EPJC (2013) 73:2433**
- 5) **B. Blok and F. A. Ceccopieri, PRD 101, 094029 (2020)**
- 6) **B. Blok and F. A. Ceccopieri, EPJC (2020) 80:278**
- 7) **D. Treleani and G. Calucci, PRD 86, 036003 (2012)**
- 8) **M. Strickman and D. Treleani, PRL 88, 031801 (2002)**
- 9) **E. Cattaruzza, A. del Fabbro and D. Treleani, PRD 70, 034022 (2004)**

DPS in pA collisions

$$F_{a_1 a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-) p^+} \\ \times \langle A | \mathcal{O}_{a_2}(0, z_2) \mathcal{O}_{a_1}(y, z_1) | A \rangle$$

In this case we have two mechanisms that contribute:

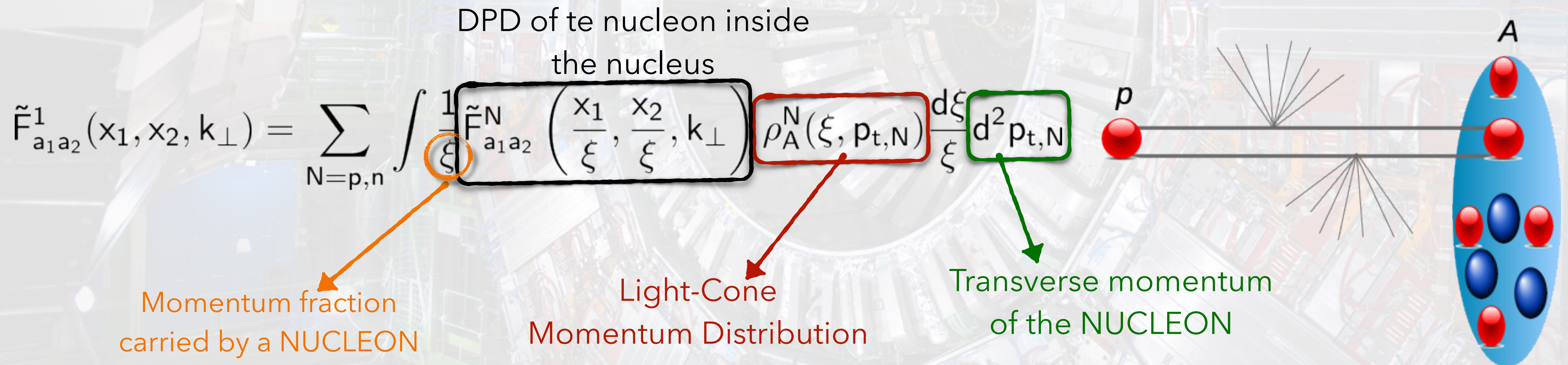
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B. Blok et al, EPJC (2013) 73:2422

DPS 1: The two partons belong to the SAME nucleon in the nucleus!



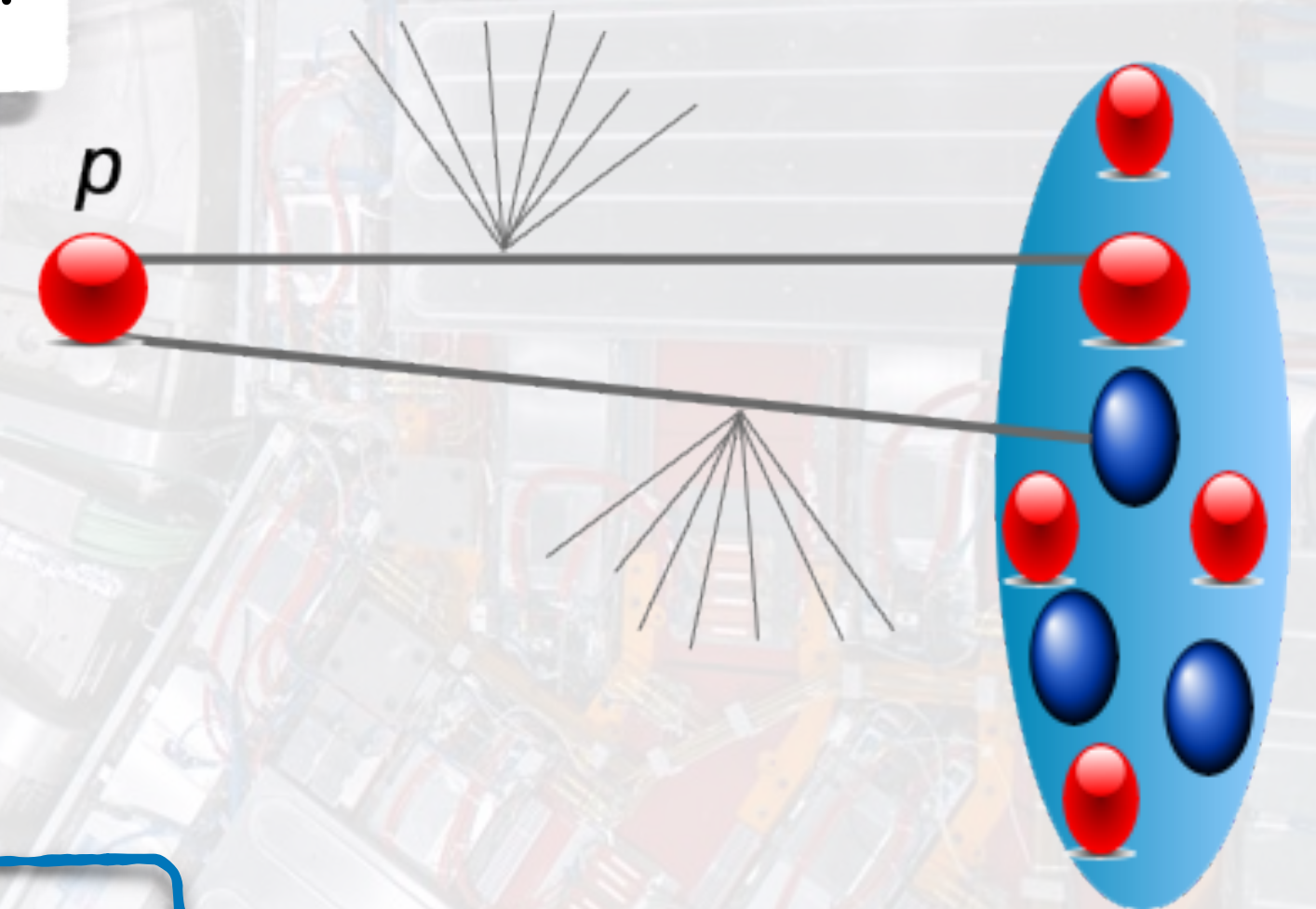
DPS in pA collisions

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In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422

DPS 2: The two partons belong to the DIFFERENT nucleons in the nucleus!



$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}, \dots) \times \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp, \dots) G_{a_1}^{N_1}(x_1/\xi_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2/\xi_2, |\vec{k}_\perp|)$$

Nucleus wf

Nucleon GPD

DPS in pA collisions

D. d'Enterria and A. Snigirev, PLB 718 (2013)

One can generalize the "Pocket formula":

$$\sigma_{\text{pA}}^{\text{DPS}} = \sigma_{\text{pA}}^{\text{DPS},1} + \sigma_{\text{pA}}^{\text{DPS},2}$$

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\downarrow

$$A \sigma_{\text{pp}}^{\text{DPS}}$$

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\downarrow \downarrow

$$A\sigma_{\text{pp}}^{\text{DPS}} \qquad \sigma_{\text{pp}}^{\text{DPS}} \cdot \sigma_{\text{eff,pp}} \cdot F_{\text{pA}}$$

$$F_{\text{pA}}$$


Related to the transverse nuclear distribution. Can be evaluated with some models for heavy ions and realistically for light-nuclei

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$$A\sigma_{\text{pp}}^{\text{DPS}} + \sigma_{\text{pp}}^{\text{DPS}} \cdot \sigma_{\text{eff,pp}} \cdot F_{\text{pA}}$$

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}}{A + \sigma_{\text{eff,pp}} F_{\text{pA}}}$$

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\downarrow
 \downarrow

$$A\sigma_{pp}^{\text{DPS}} \quad \sigma_{pp}^{\text{DPS}} \cdot \sigma_{\text{eff},pp} \cdot \boxed{F_{pA}}$$

$$\sigma_{\text{eff},pA} = \frac{\sigma_{\text{eff},pp}}{A + \sigma_{\text{eff},pp} F_{pA}}$$

For pPb: $\left\{ \begin{array}{l} \text{Wood-Saxon density} \\ \sigma_{\text{eff},pp} \sim 13 \text{ mb} \end{array} \right.$

$$\sigma_{\text{eff},pPb} \sim 22 \mu\text{b}$$

DPS in pA collisions

D. d'Enterria and A. Snigirev, PLB 718 (2013)

One can generalize the "Pocket formula":

$$\sigma_{pA}^{\text{DPS}} = \sigma_{pA}^{\text{DPS},1} + \sigma_{pA}^{\text{DPS},2}$$

↓ ↓

$$A\sigma_{pp}^{\text{DPS}} \quad \sigma_{pp}^{\text{DPS}} \cdot \sigma_{\text{eff},pp} \cdot \boxed{F_{pA}}$$

$$\sigma_{\text{eff},pA} = \frac{\sigma_{\text{eff},pp}}{A + \sigma_{\text{eff},pp} F_{pA}}$$

If $\sigma_{\text{eff},pA}$ is experimentally obtained then
we can extract $\sigma_{\text{eff},pp}$

DPS in pA collisions



Some examples of predictions:

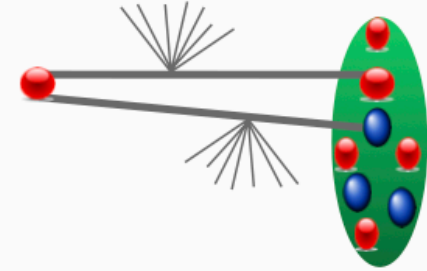
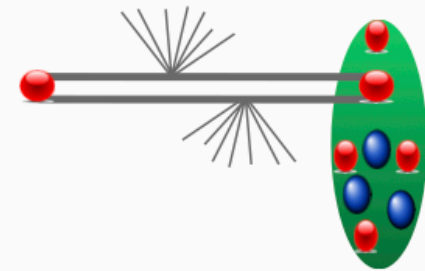
W+di-jets

B. Blok and F. A. Ceccopieri EPJC (2020) 80, 278

Same sign WW

D. D'Enterria and Snigirev, PLB 718 (2013) 1395-1400

DPS in pA collisions

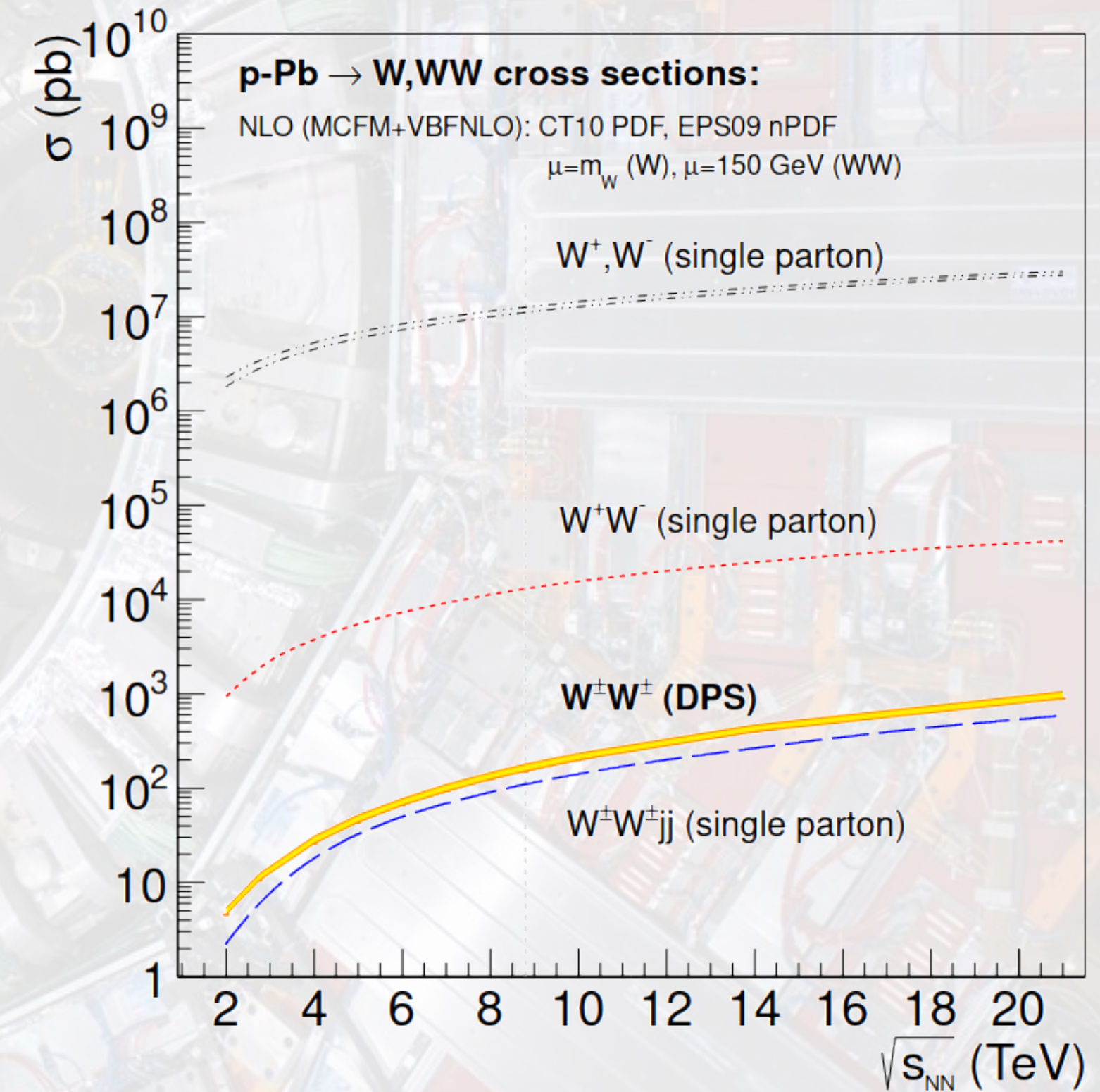
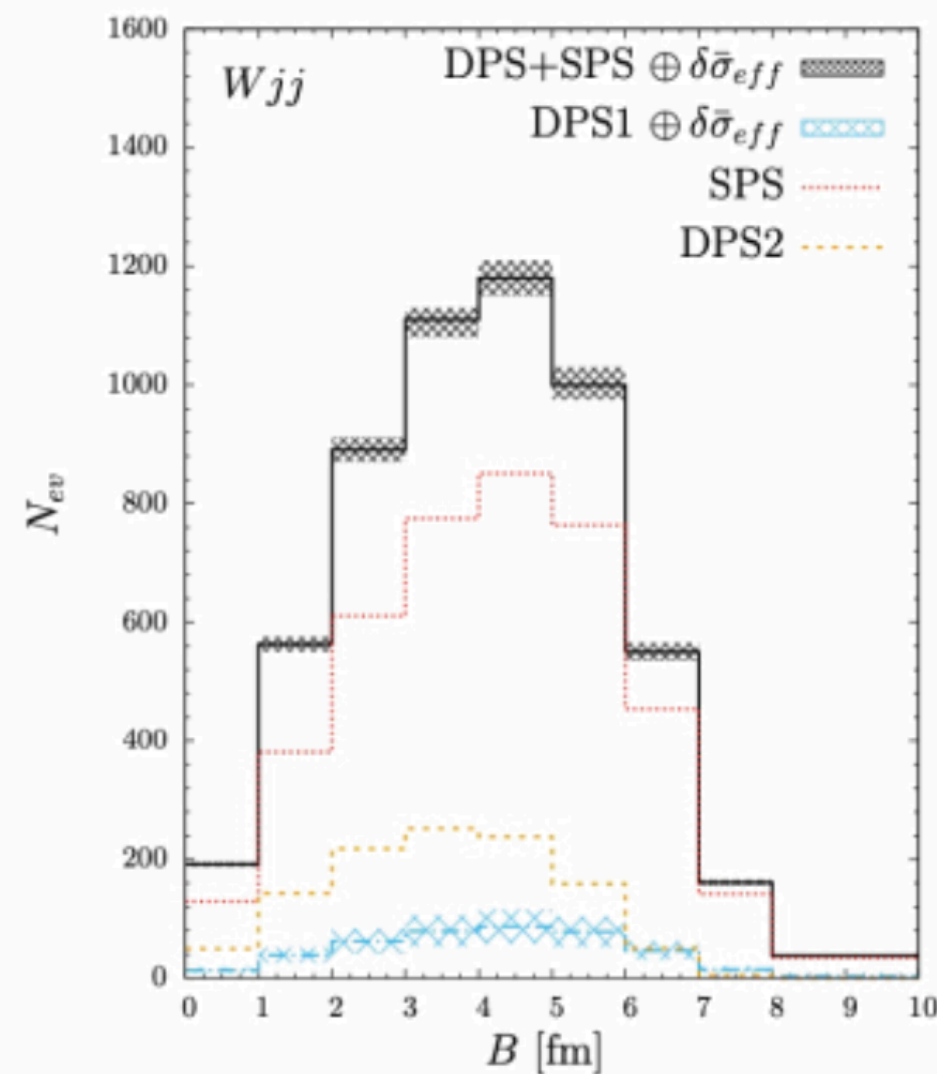


A lot of effort (slides from):
 - Boris Blok
 - Federico Alberto Ceccopieri
 - Mark Strikman
 - Massimiliano Alvioli
 - Daniele Treleani

W+di-jets

σ^{Wjj}	$p_T^j > 20$ GeV [nb]	$p_T^j > 25$ GeV [nb]	$p_T^j > 30$ GeV [nb]
DPS1	19 ± 6	8 ± 3	4 ± 2
DPS2	49	22	11
SPS	81	57	41
Tot	149 ± 6	87 ± 3	56 ± 2

- SPS dominant
- DPS2 bigger than DPS1 has expected



DPS in pA collisions - predictions

[DdE, Snigirev, NPA 931 (2014) 303]

- Cross sections & rates for **DPS processes with $J/\psi, \Upsilon$ & W, Z bosons**
[Also V. Goncalves (2018): double- J/ψ ; Paukunen (2019): double-D,...]

pPb (8.8 TeV)	$J/\psi + J/\psi$	$J/\psi + \Upsilon$	$J/\psi + W$	$J/\psi + Z$
$\sigma_{pN \rightarrow a}^{\text{SPS}}, \sigma_{pN \rightarrow b}^{\text{SPS}}$	45 μb ($\times 2$)	45 μb , 2.6 μb	45 μb , 60 nb	45 μb , 35 nb
$\sigma_{pPb}^{\text{DPS}}$	45 μb	5.2 μb	120 nb	70 nb
$N_{pPb}^{\text{DPS}} (1 \text{ pb}^{-1})$	~ 65	~ 60	~ 15	~ 3
	$\Upsilon + \Upsilon$	$\Upsilon + W$	$\Upsilon + Z$	ss WW
$\sigma_{pN \rightarrow a}^{\text{SPS}}, \sigma_{pN \rightarrow b}^{\text{SPS}}$	2.6 μb ($\times 2$)	2.6 μb , 60 nb	2.6 μb , 35 nb	60 nb ($\times 2$)
$\sigma_{pPb}^{\text{DPS}}$	150 nb	7 nb	4 nb	150 pb
$N_{pPb}^{\text{DPS}} (1 \text{ pb}^{-1})$	~ 15	~ 8	~ 1.5	~ 4

Leptonic final states: $\text{BR}(J/\psi, \Upsilon, W, Z) = 6\%, 2.5\%, 11\%, 3.4\%$

Accept.*Effic. = 1% ($J/\psi, |y|=0,2$), 20% ($\Upsilon, |y|<2.5$), 50% ($W, Z |y|<2.4$)

- **Many double hard scatterings** processes with visible p-Pb x-sections at the LHC. (Note: J/ψ values are per unit- $|y|$).
- Useful **independent extraction of $\sigma_{\text{eff,pp}}$** !

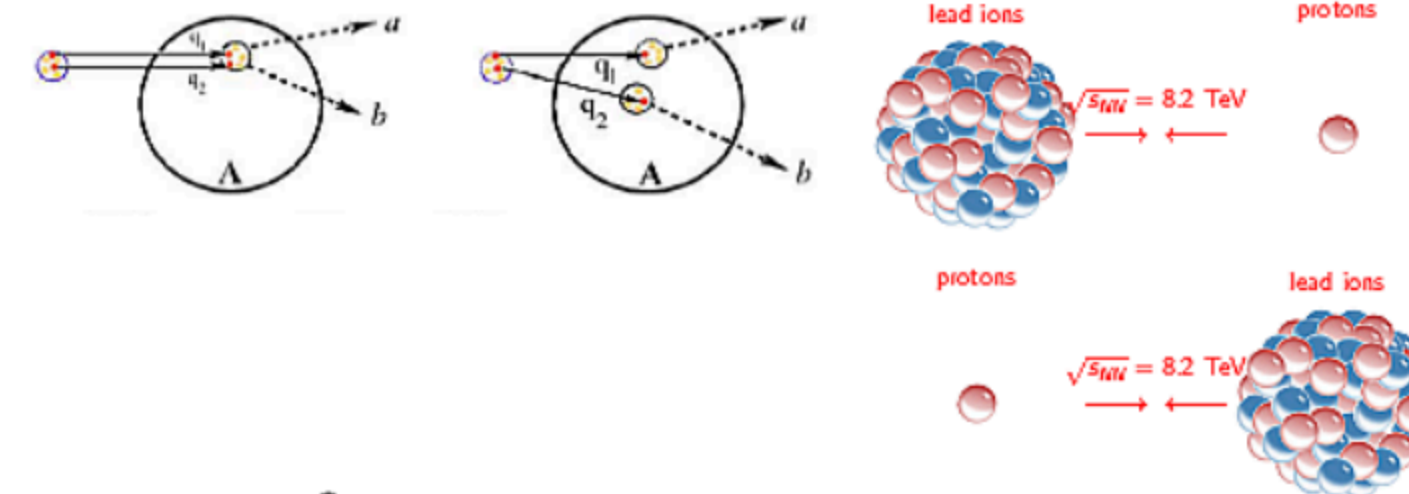
D. d'Enterria's
slide

DPS in pA collisions

[LHCb, PRL 125 (2020) 212001]

■ Double-charm production in p-Pb collisions:

- select pairs of $D^0, \bar{D}^0, D^+, D^-, D_s^+, D_s^-$ and J/ψ
- sort them into pair production and “DPS” categories



$$\sigma_{C_1, C_2} = \alpha \frac{\sigma_{C_1} \sigma_{C_2}}{\sigma_{\text{eff}}}$$

$$R_{\text{forward}}^{D_1 D_2} = \frac{\sigma_{D_1 D_2}}{\sigma_{D_1 \bar{D}_2}} = 0.308 \pm 0.015 \pm 0.010$$

$$R_{\text{backward}}^{D_1 D_2} = 0.391 \pm 0.019 \pm 0.025$$

$$R_{pp}^{D^0 D^0} = 0.109 \pm 0.008$$

Like sign charm fraction tripled!

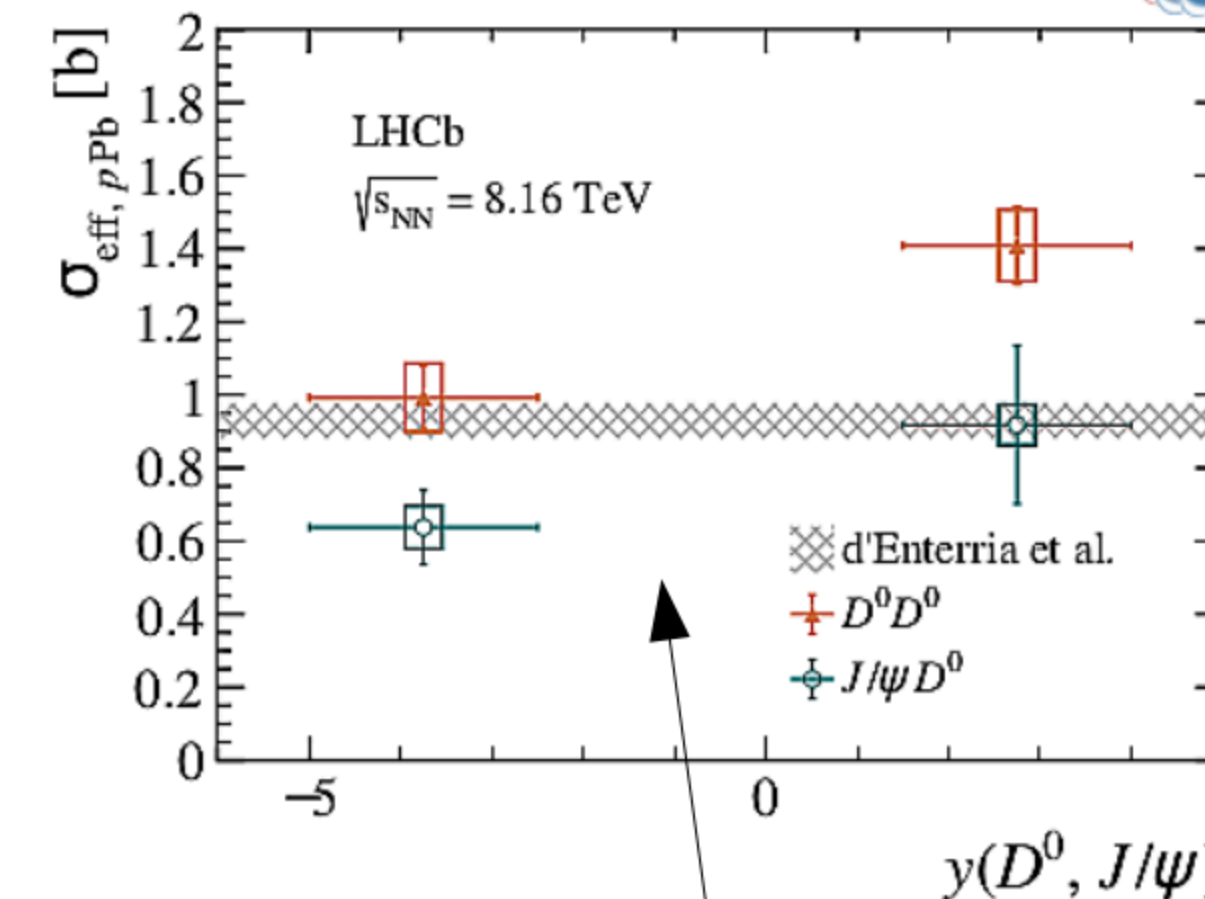
$$\sqrt{s_{NN}} = 8.2 \text{ TeV} \quad \text{Phys. Rev. Lett. 125 (2020) 212001}$$

Albert Bursche

charming DPS

10th October 2021

15 / 17



■ Useful independent extraction of $\sigma_{\text{eff,pp}}$:

nPDF effects visible in -y/+y results.

$$\sigma_{\text{eff,pA}} = \frac{\sigma_{\text{eff,pp}}}{A + \sigma_{\text{eff,pp}} F_{pA}}$$

$$\sigma_{\text{eff,pp}}(D^0 D^0) = 7\text{--}16 \text{ mb}$$

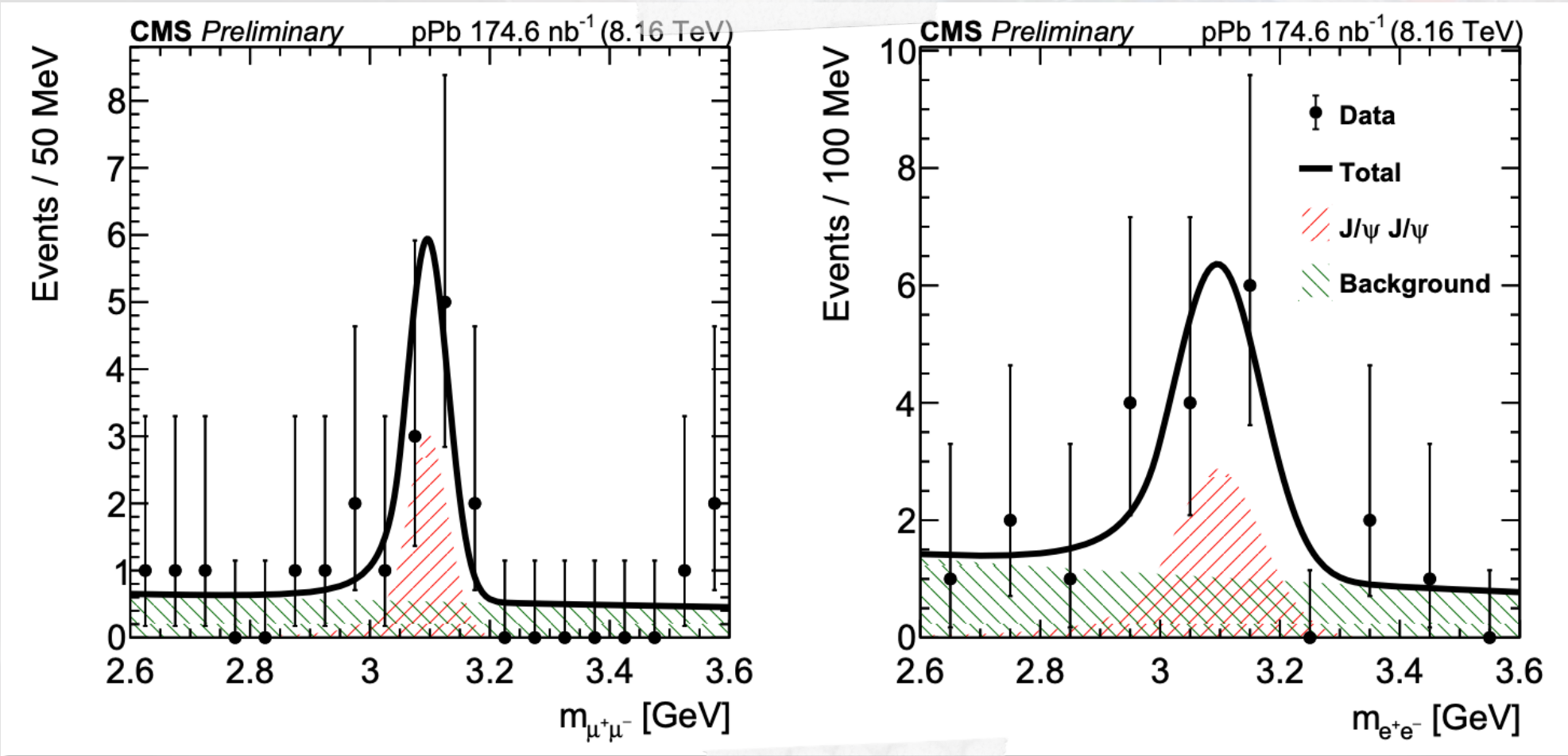
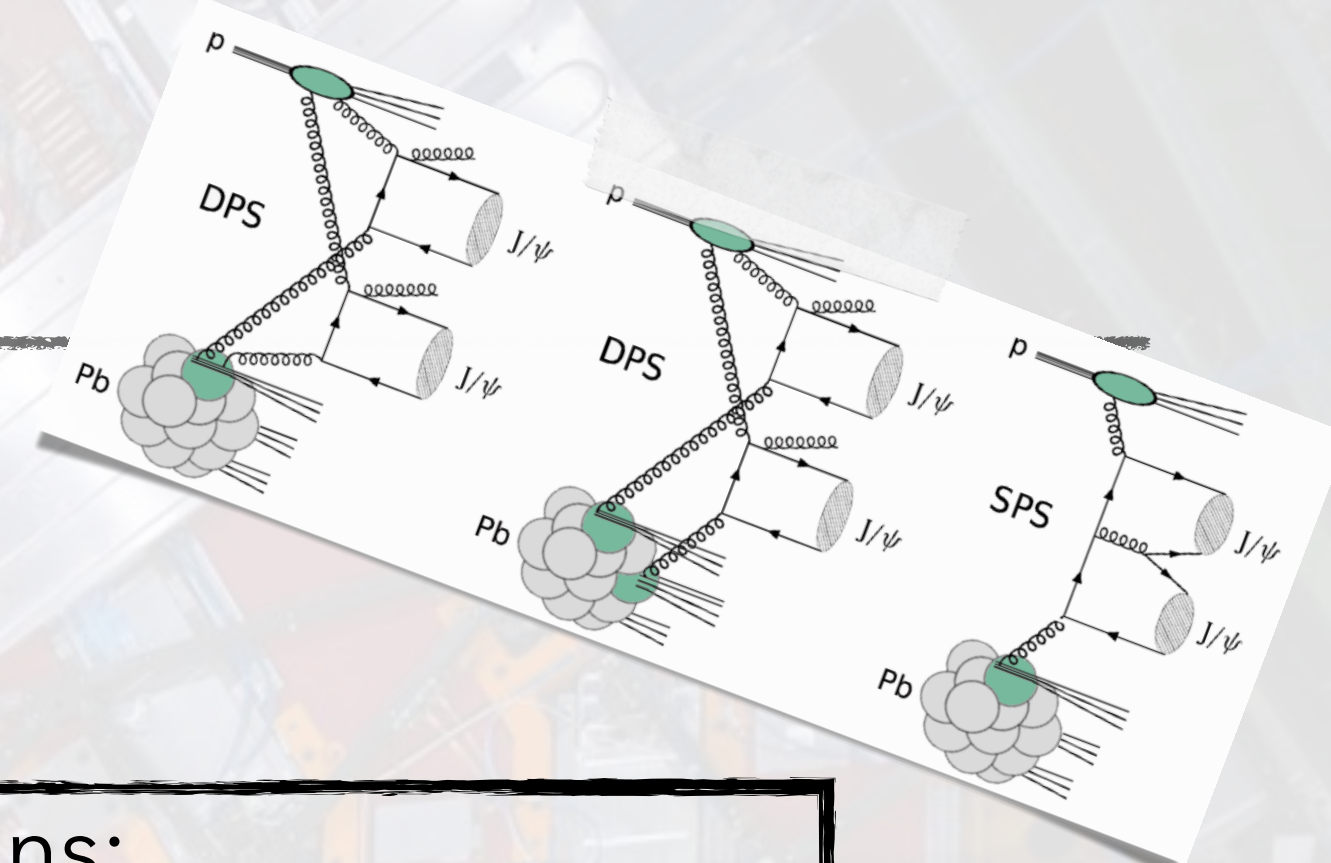
$$\sigma_{\text{eff,pp}}(J/\psi D^0) = 13\text{--}40 \text{ mb}$$

(LHCb should quote the equivalent $\sigma_{\text{eff,pp}}$ values...)

D. d'Enterria's slide

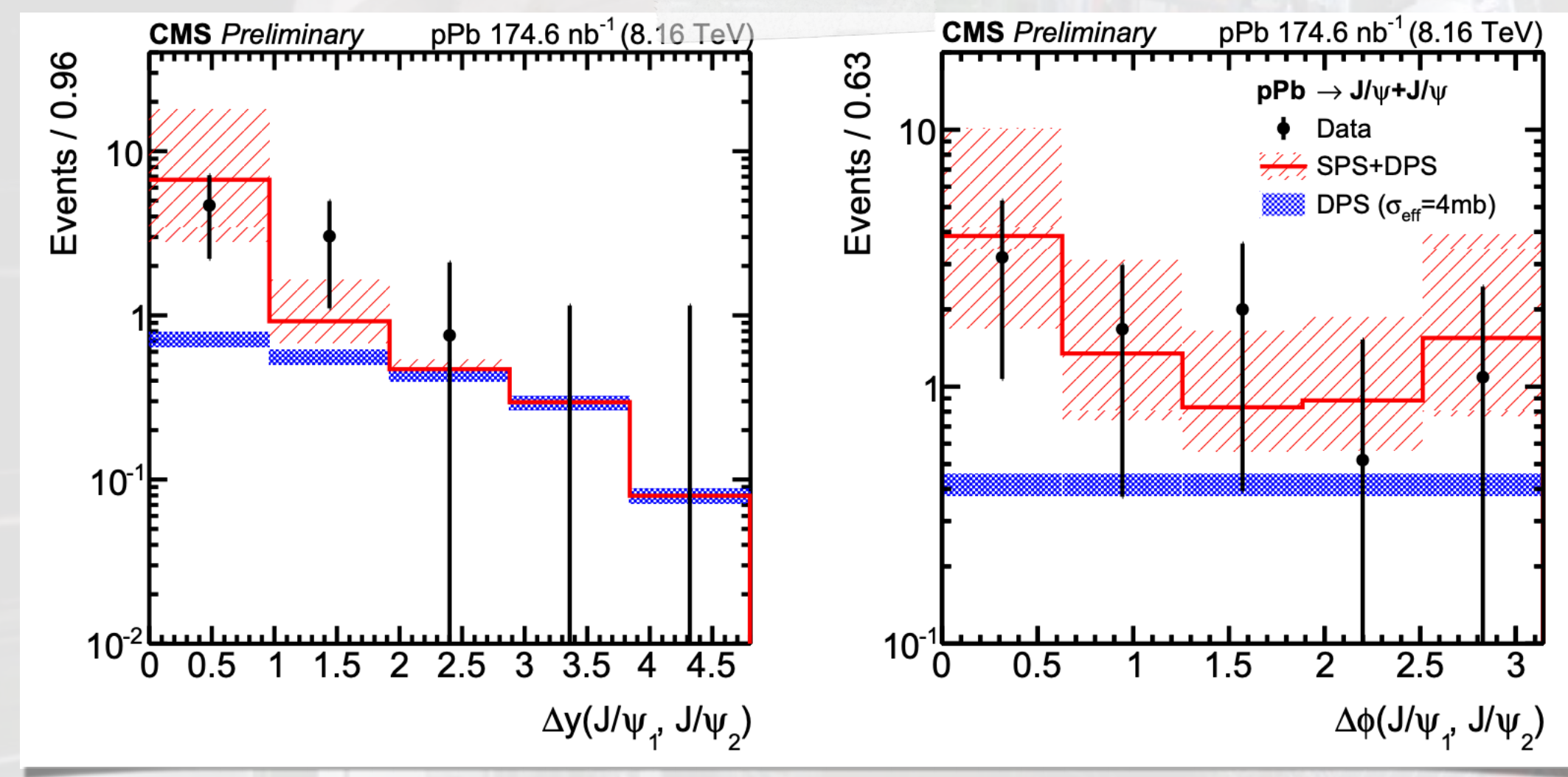
DPS in pA collisions

double- J/ψ meson production in pPb collisions at 8.16 TeV



The relative cross-section contributions:

$$\sigma_{\text{SPS}}^{\text{pPb} \rightarrow J/\psi J/\psi + X} = 16.5 \pm 10.8 \text{ (stat)} \pm 0.1 \text{ (syst) nb}$$

$$\sigma_{\text{DPS}}^{\text{pPb} \rightarrow J/\psi J/\psi + X} = 5.4 \pm 6.2 \text{ (stat)} \pm 0.4 \text{ (syst) nb}$$


The extracted pp effective X-section:

$$\sigma_{\text{eff,pp}} = 4.0_{-1.5}^{+\infty} \text{ mb}$$

Triple Parton Scattering



A pocket formula for Triple parton Scattering (TPS)

$$\sigma_{\text{TPS}} \propto \frac{\sigma_{a_1}^{\text{SPS}} \sigma_{a_2}^{\text{SPS}} \sigma_{a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

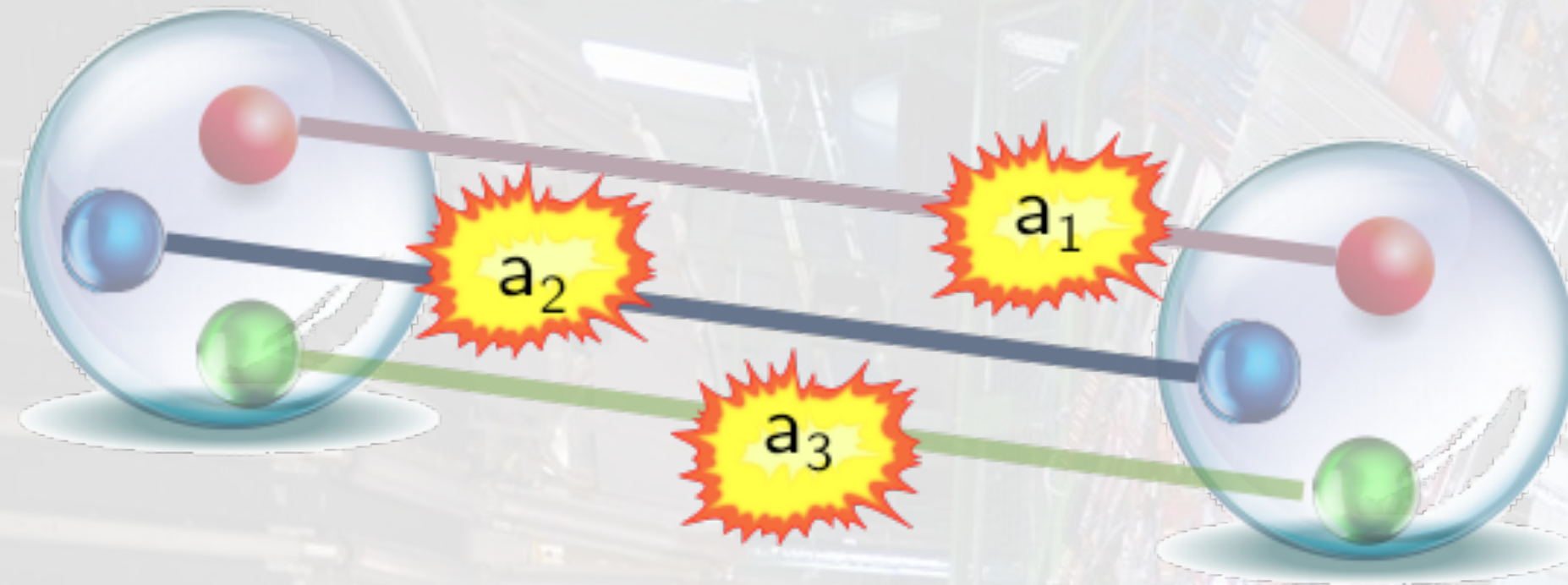
D. d' E. et al PRL 118 (2017) 122001

TPS is a new window to access new information on the hadron structure:

$$\sigma_{\text{eff,TPS}}^2 = \left[\int d^2b T^3(\mathbf{b}) \right]^{-1} \xrightarrow{\text{Model calculations}} \sigma_{\text{eff,TPS}} = k \times \sigma_{\text{eff,DPS}}, \text{ with } k = 0.82 \pm 0.11$$

- 1) $\sigma_{\text{eff,TPS}}$ encodes new details on the geometrical structure
- 2) Triple Parton Distributions (tPDFs) could depend from unknown triple parton correlations!

Triple Parton Scattering



A pocket formula for Triple parton Scattering (TPS)

$$\sigma_{\text{TPS}} \propto \frac{\sigma_{a_1}^{\text{SPS}} \sigma_{a_2}^{\text{SPS}} \sigma_{a_3}^{\text{SPS}}}{\sigma_{\text{eff,TPS}}^2}$$

D. d' E. et al PRL 118 (2017) 122001

SUM rules can be used to build phenomenological distributions: **O. Fedkevych and J. R. Gaunt, JHEP 02, 090 (2023)**

$$\sum_{j_3} \int_0^{1-x_1-x_2} dx_3 x_3 T_{j_1 j_2 j_3}^B(x_1, x_2, x_3) = (1-x_1-x_2) D_{j_1 j_2}^B(x_1, x_2)$$

Momentum Sum Rule

$$\int_0^{1-x_1-x_2} dx_3 \boxed{T_{j_1 j_2 j_3}^B(x_1, x_2, x_3)} = \left(N_{j_3} - \delta_{j_3 j_1} - \delta_{j_3 j_2} + \delta_{\bar{j}_3 j_1} + \delta_{j_3 j_2} \right) \boxed{D_{j_1 j_2}^B(x_1, x_2)}$$

Number Sum Rule

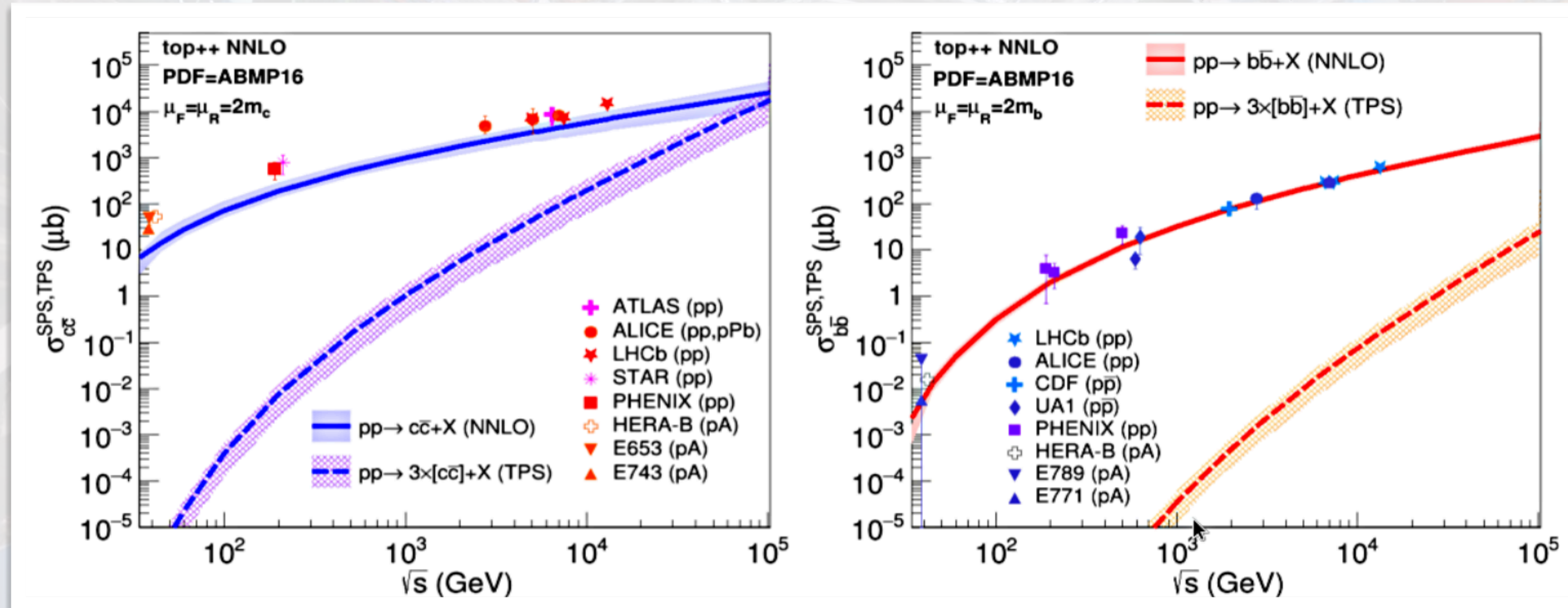
Distribution integrated on transverse dependence

Triple Parton Scattering - where?



Triple Charm and Beauty production

D. d'Enterria and A. M. Snigirev, Phys. Rev. Lett. 118, no.12, 122001 (2017)



- small x-section, but it increases fast with the c.m. energy
- Since triple charm is $> 15\%$ of the inclusive charm production

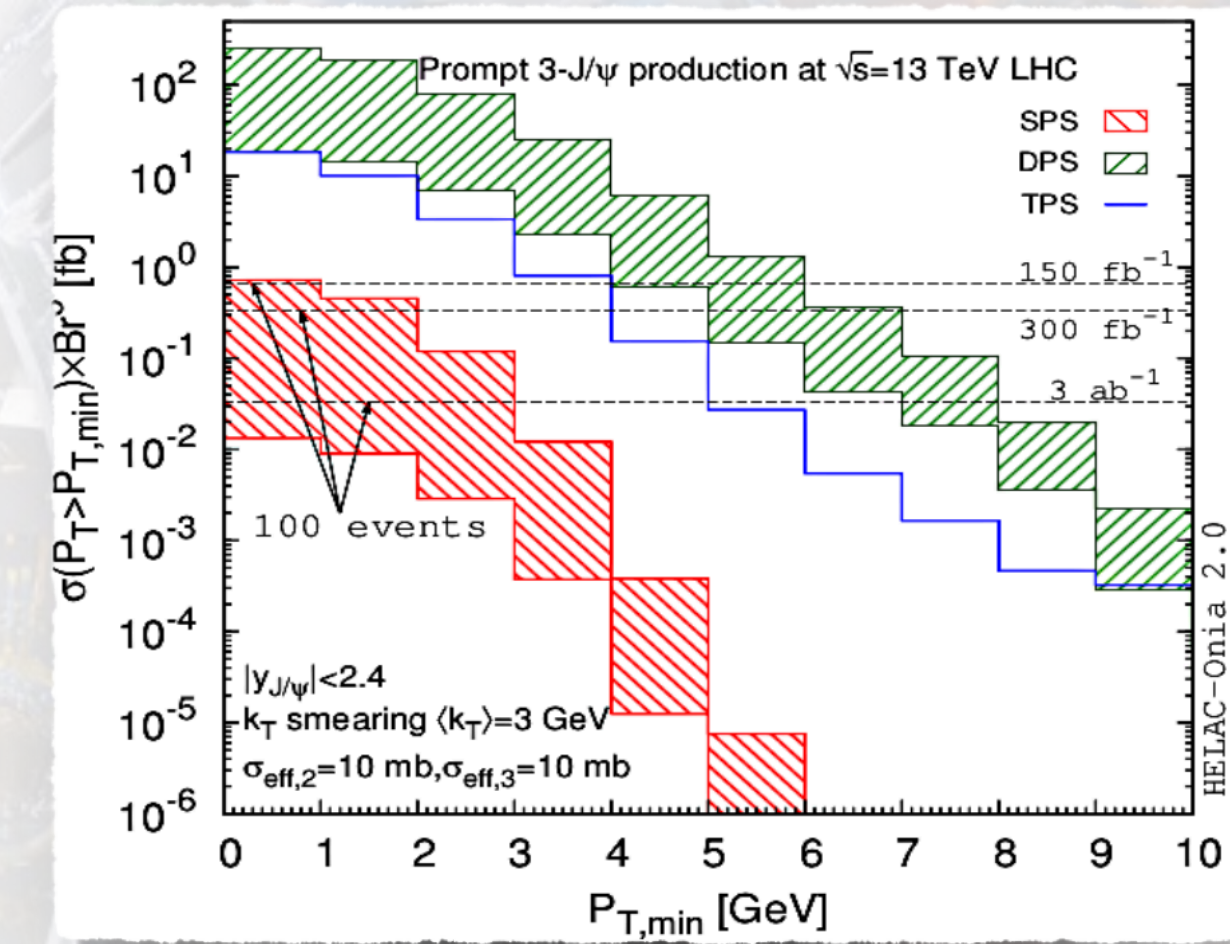
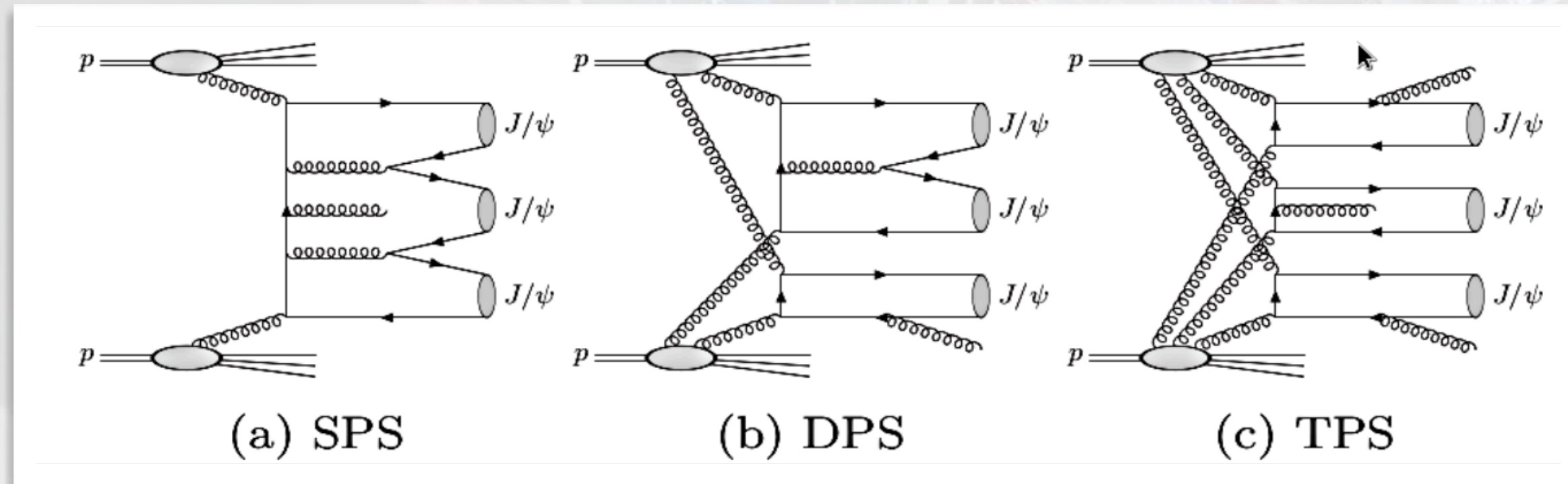


Important channel to extract the DPS and TPS contributions

Triple Parton Scattering - where?



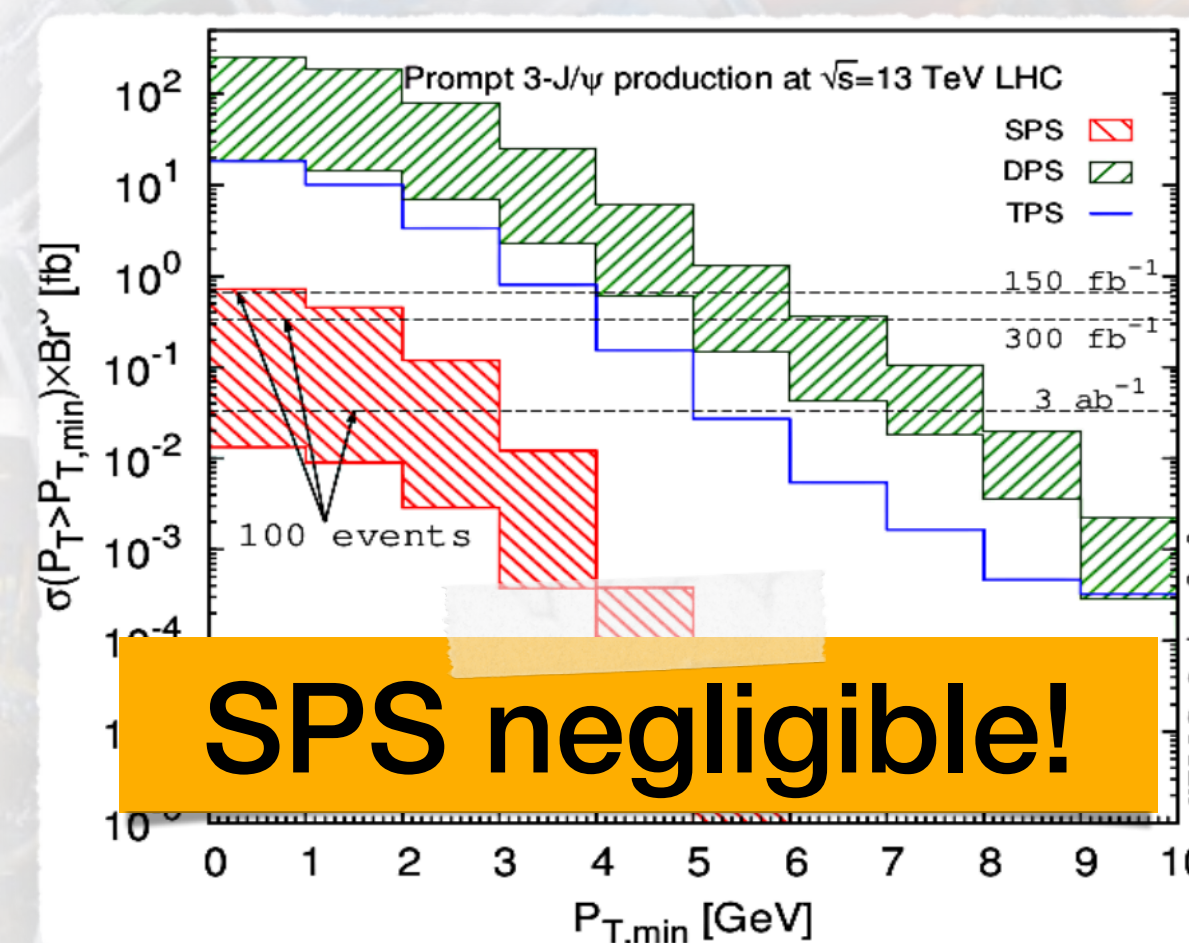
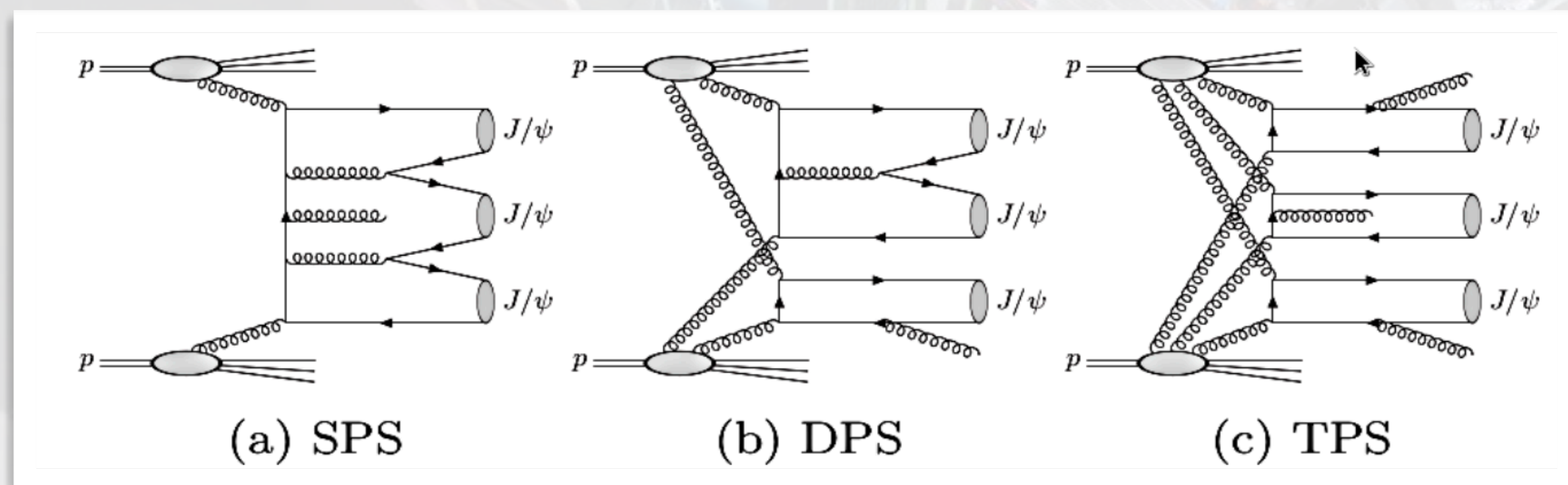
Triple J/ψ production! H. S. Shao and Y. J. Zhang, Phys. Rev. Lett. 122, no.19, 192002 (2019)



Triple Parton Scattering - where?



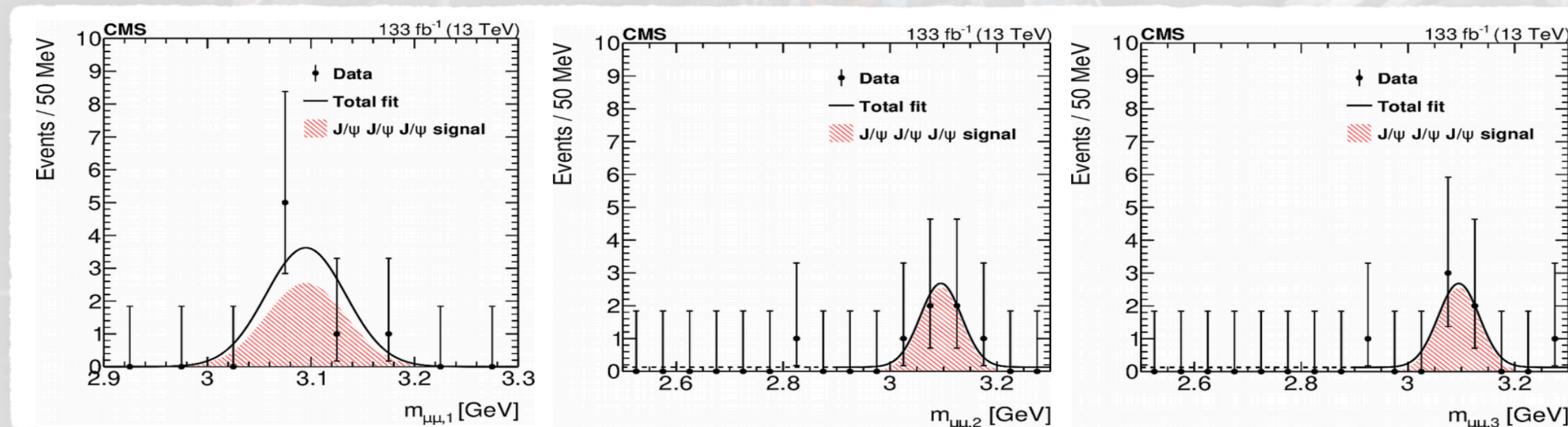
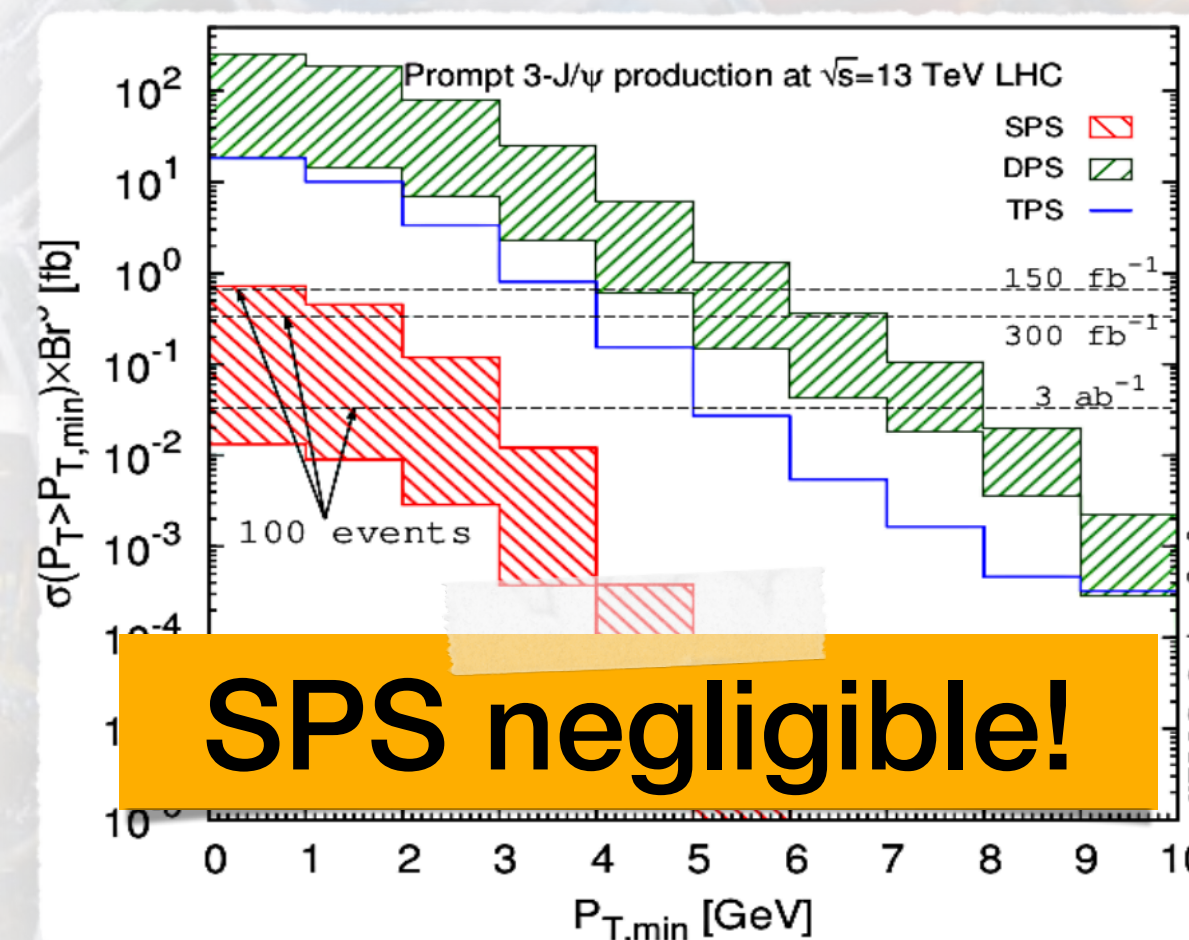
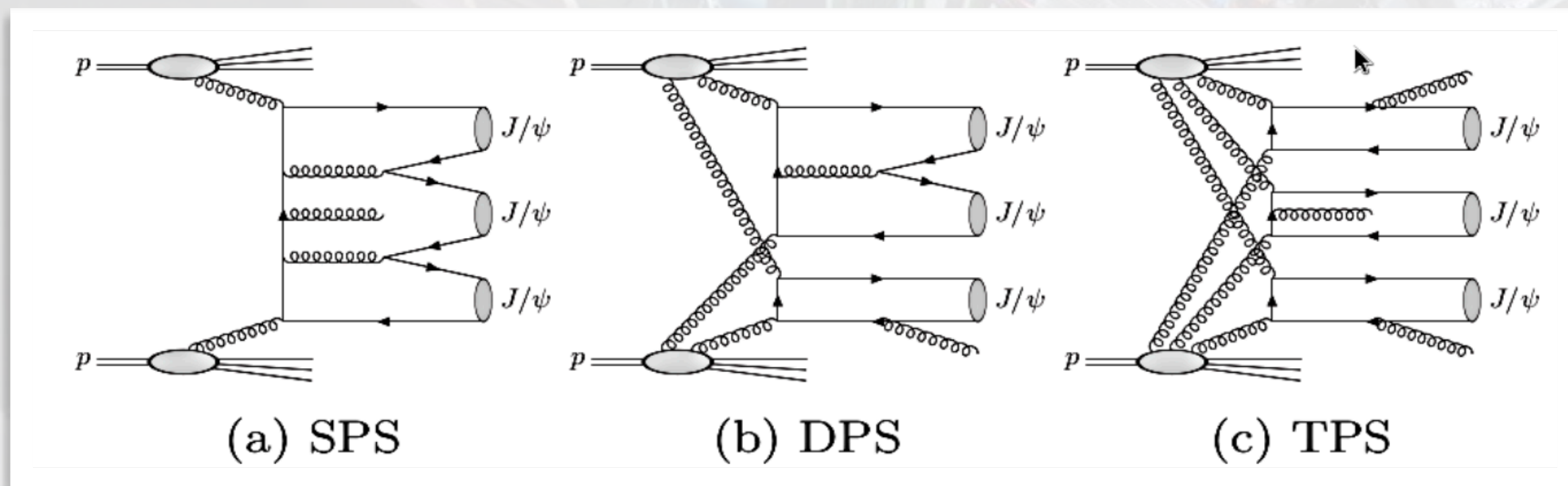
Triple J/Ψ production! H. S. Shao and Y. J. Zhang, Phys. Rev. Lett. 122, no.19, 192002 (2019)



Triple Parton Scattering - where?



Triple J/Ψ production! H. S. Shao and Y. J. Zhang, Phys. Rev. Lett. 122, no.19, 192002 (2019)



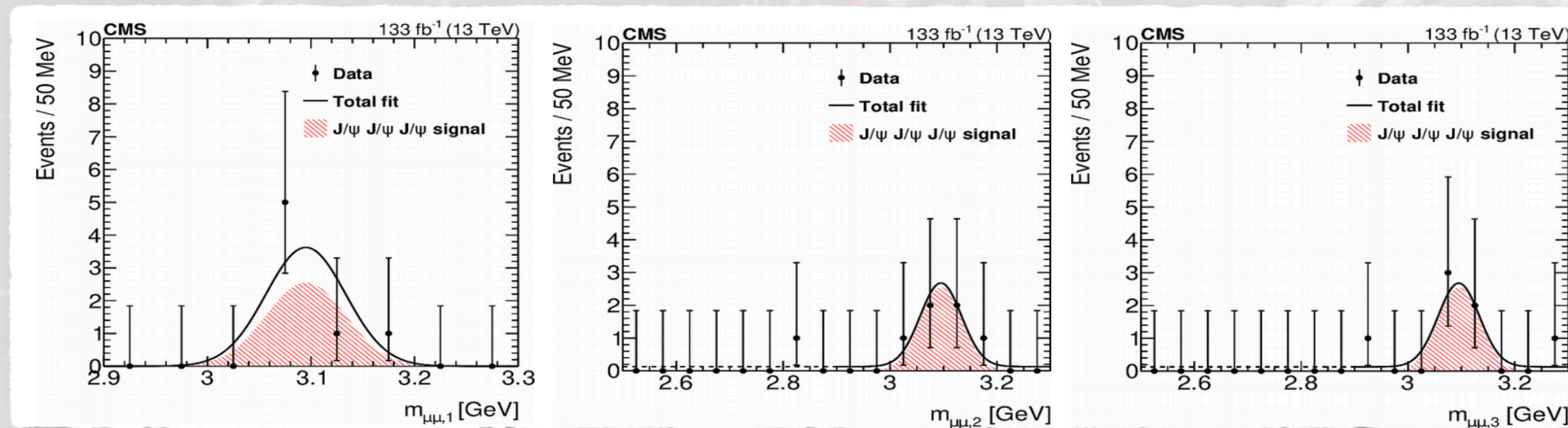
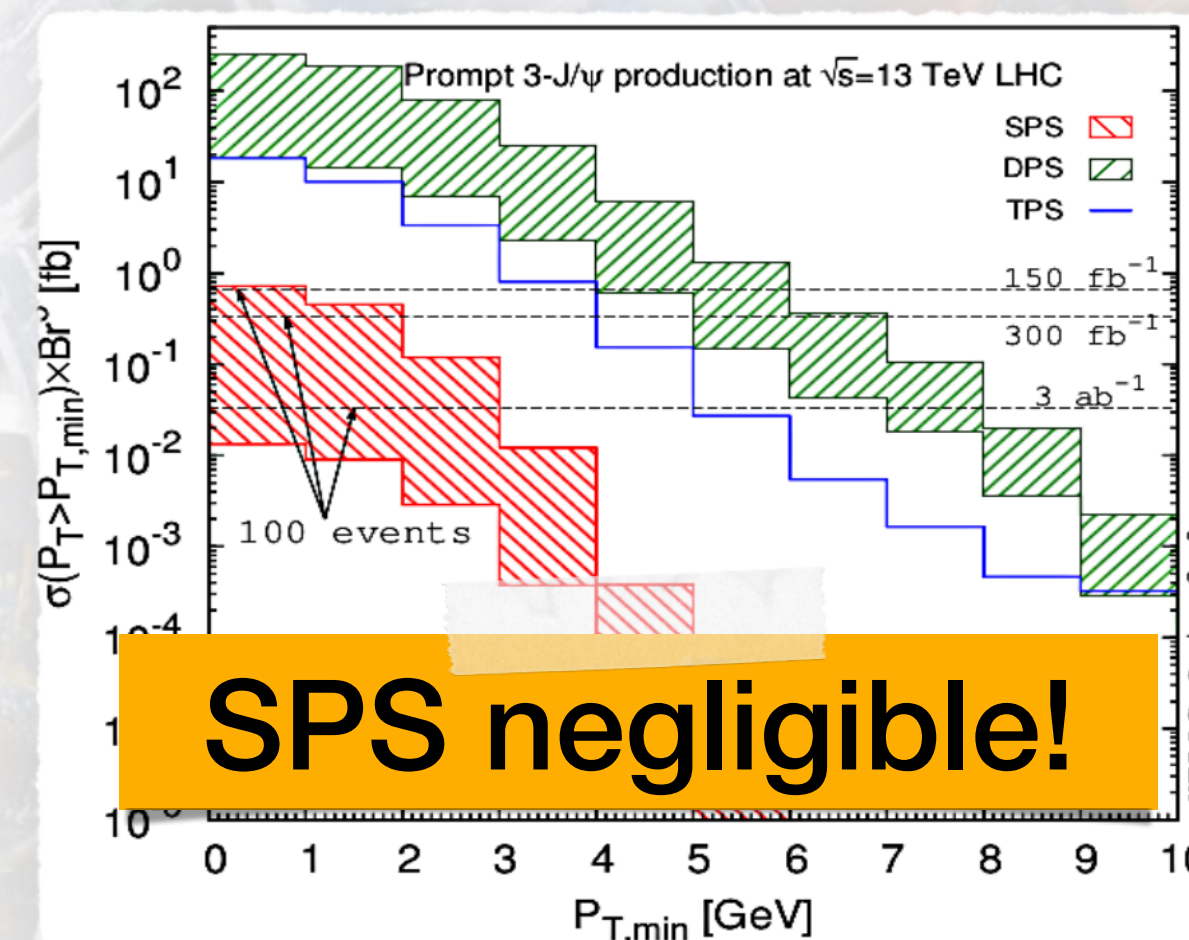
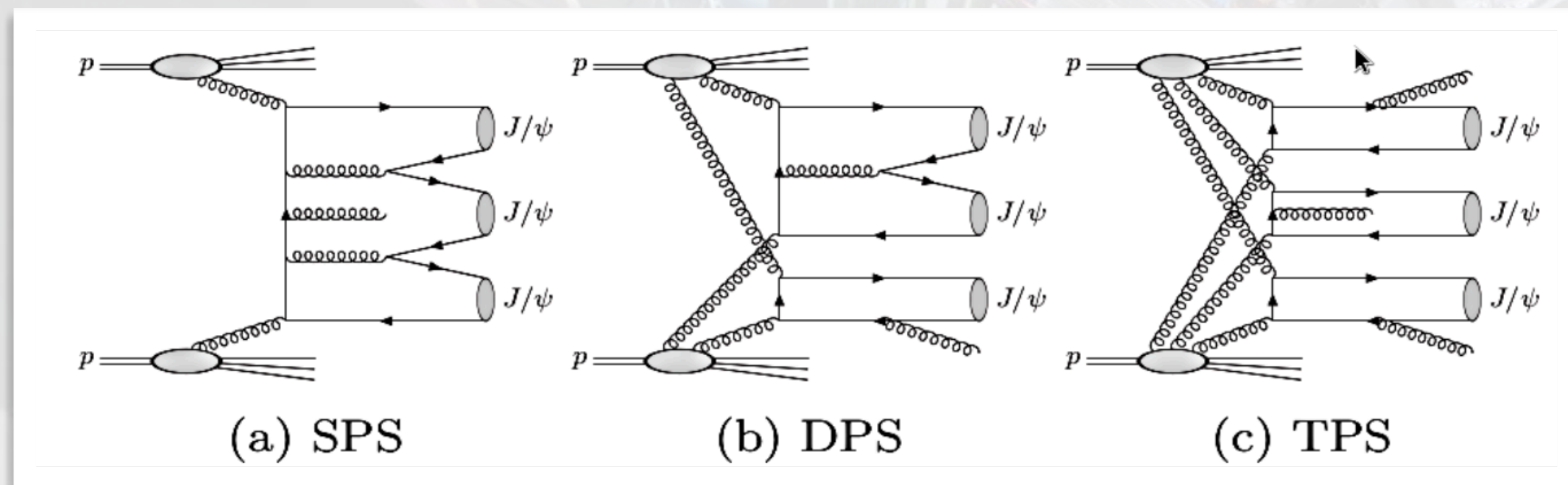
$$\sigma = 272^{+141}_{-104} \text{ (stat)} \pm 17 \text{ (syst) fb}$$

A. Tumasyan et al [CMS], Nature Phys. 19, no.3, 338-350 (2023)

Triple Parton Scattering - where?



Triple J/Ψ production! H. S. Shao and Y. J. Zhang, Phys. Rev. Lett. 122, no.19, 192002 (2019)



$$\sigma = 272_{104}^{+141} \text{ (stat)} \pm 17 \text{ (syst) fb}$$

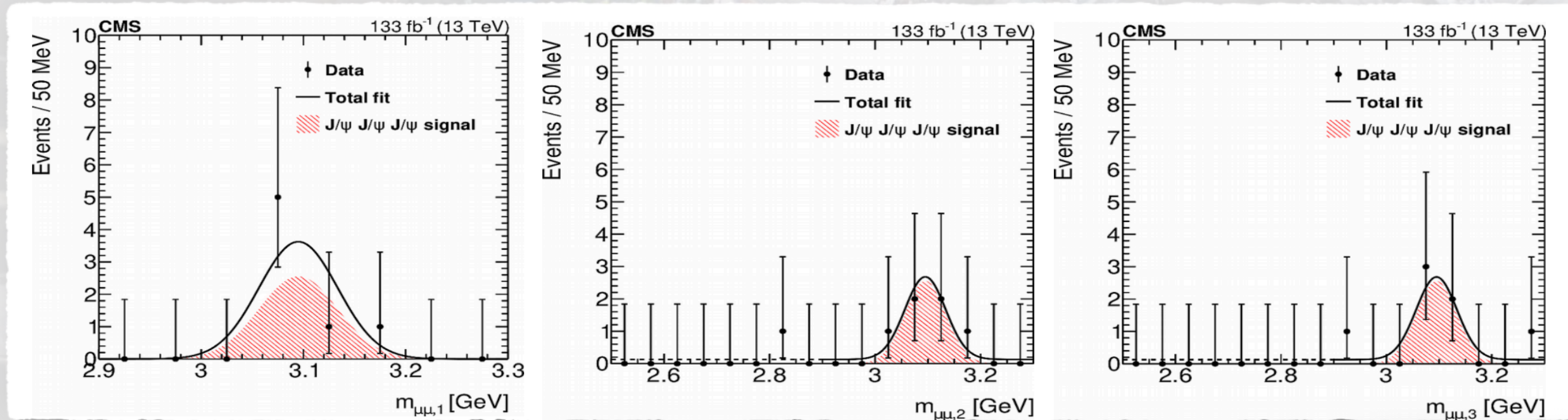
{ SPS -> 6%
 DPS -> 74%
 TPS -> 20%

A. Tumasyan et al [CMS], Nature Phys. 19, no.3, 338-350 (2023)

Triple Parton Scattering - where?



Triple J/ψ production! A. Tumasyan et al [CMS], Nature Phys. 19, no.3, 338-350 (2023)

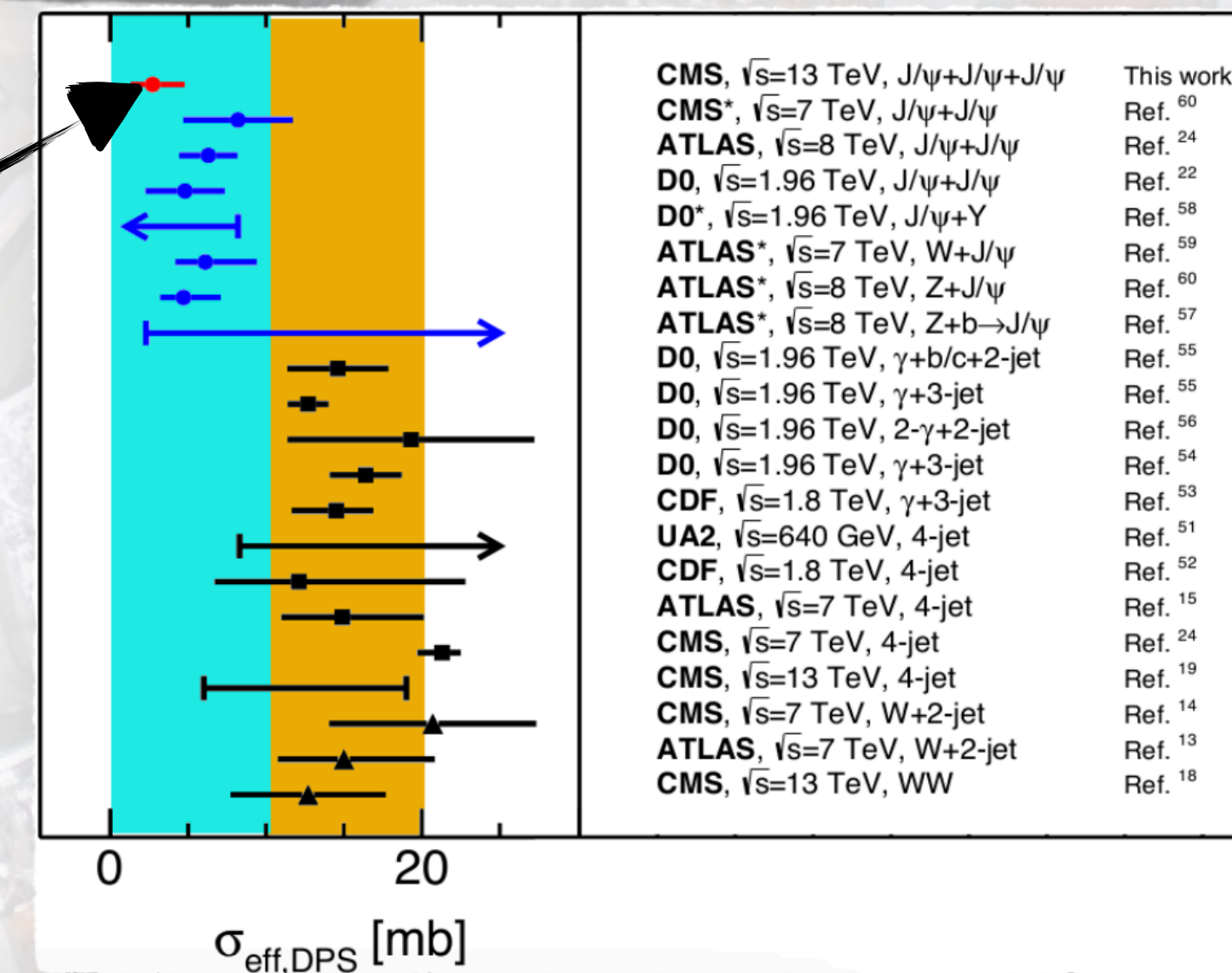


$$\sigma = 272_{104}^{+141} \text{ (stat)} \pm 17 \text{ (syst) fb}$$

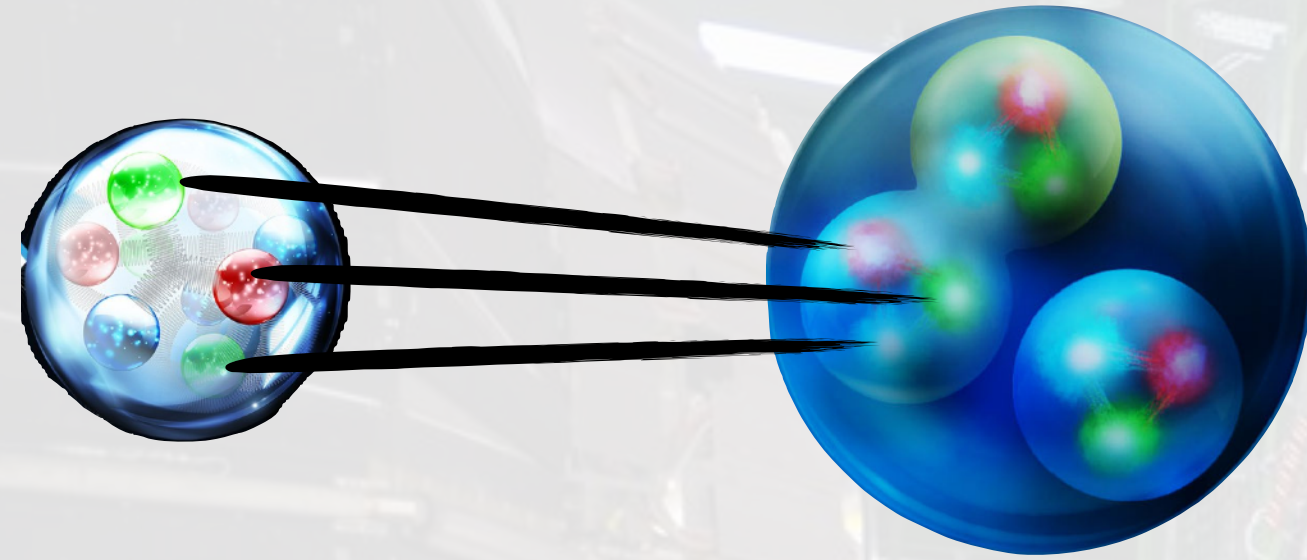
{ SPS -> 6%
 DPS -> 74%
 TPS -> 20%

Novel way to extract the DPS effective cross-section:

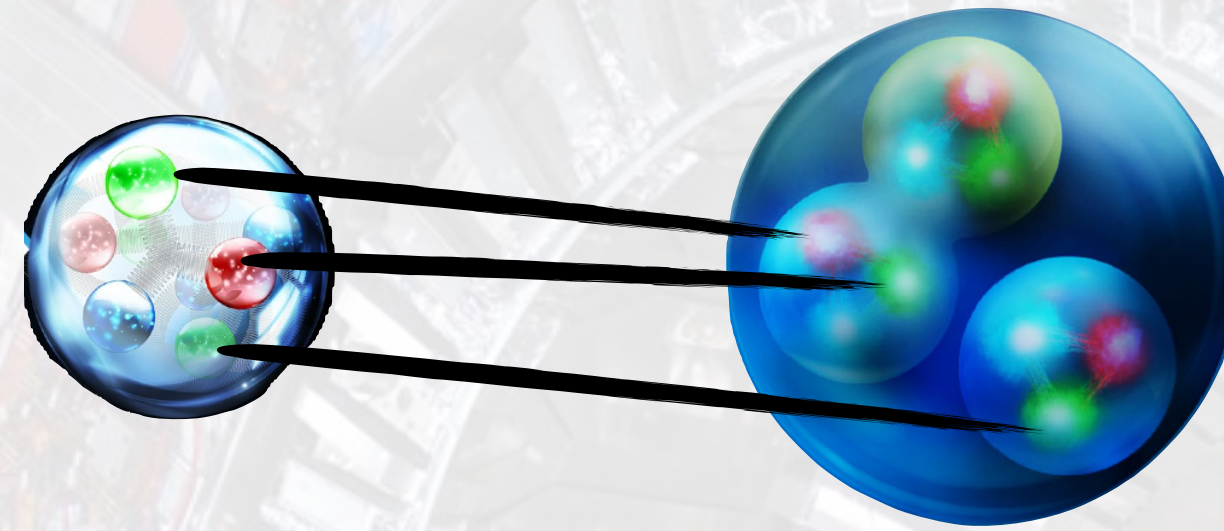
$$\sigma_{\text{eff,DPS}} = 2.7_{-1.0}^{+1.4} \text{ (exp)}_{-1.0}^{+1.5} \text{ (theo) mb}$$



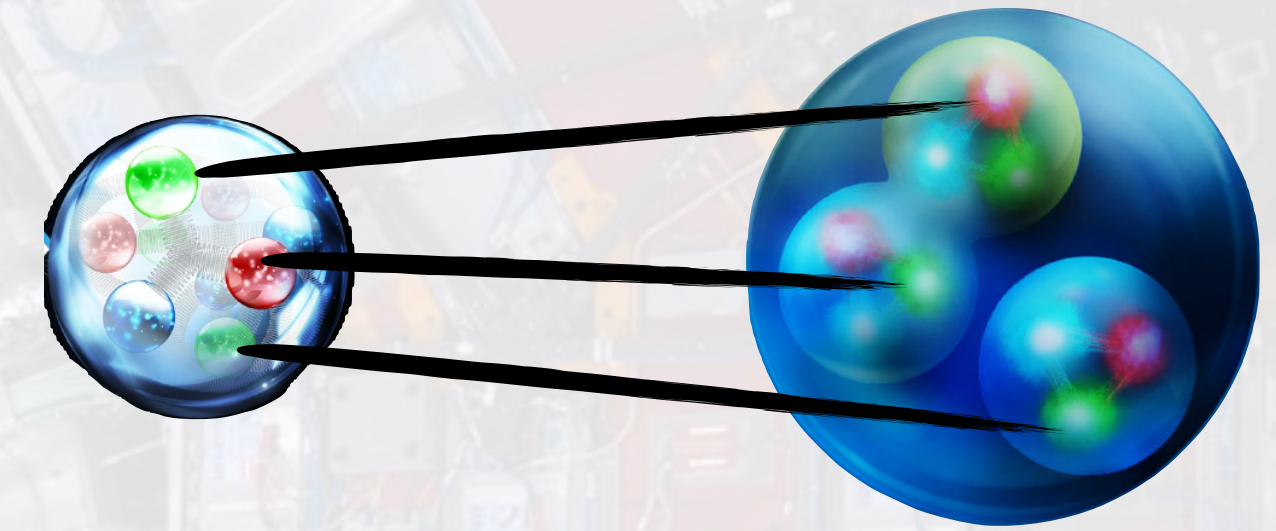
Triple Parton Scattering - pA



TPS1 = TPS



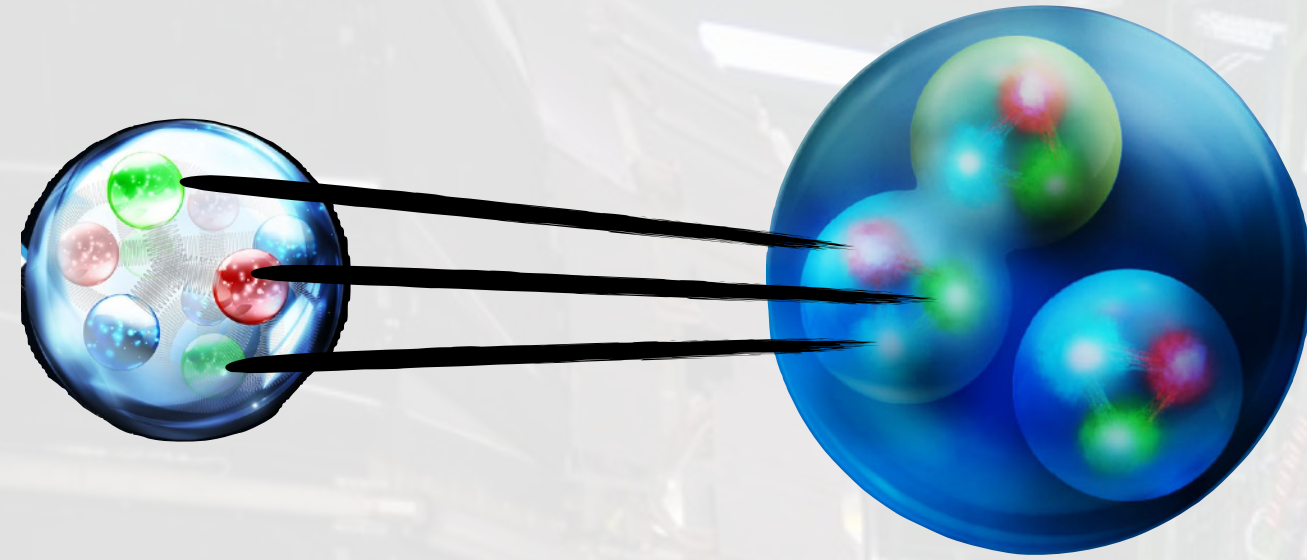
TPS2 = DPS ⊗ SPS



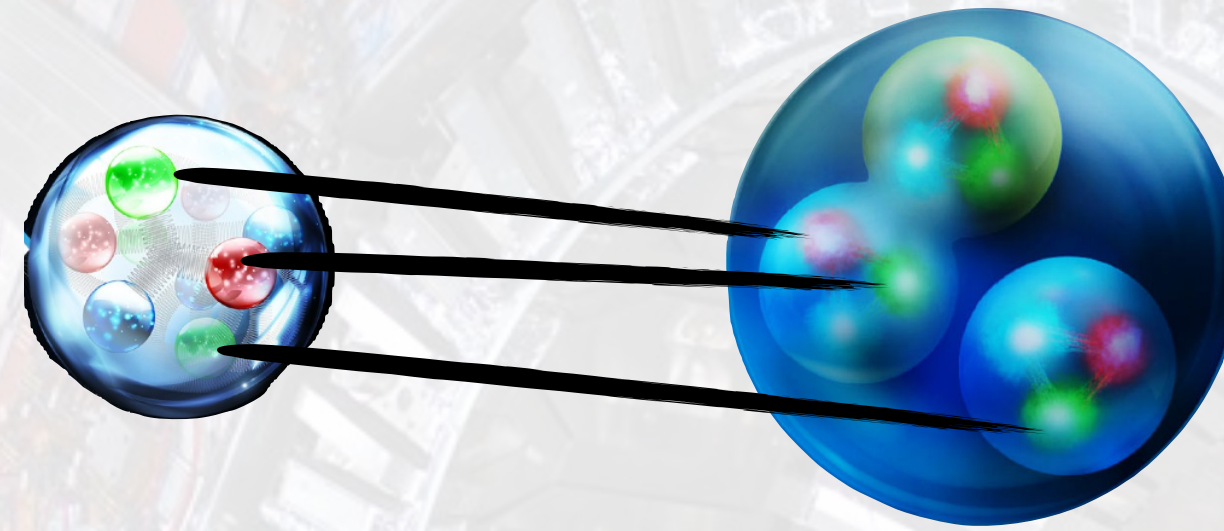
TPS3 = SPS ⊗ SPS ⊗ SPS

Relative size: **1:4.54:3.56** D. d'Enterria et al, EPJC 78 (2018) 359

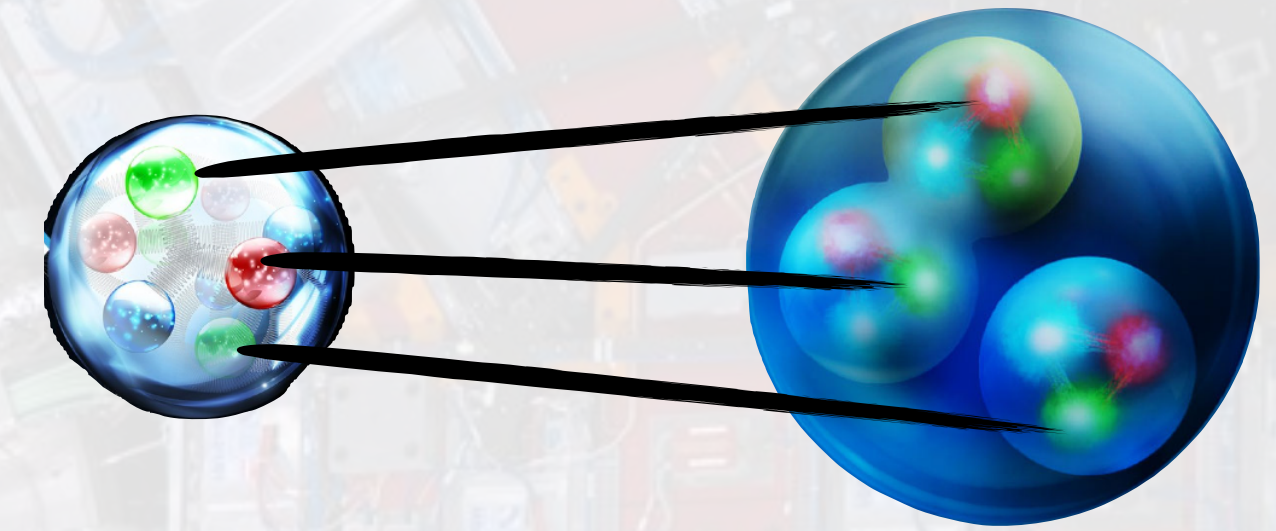
Triple Parton Scattering - pA



TPS1 = TPS



TPS2 = DPS ⊗ SPS



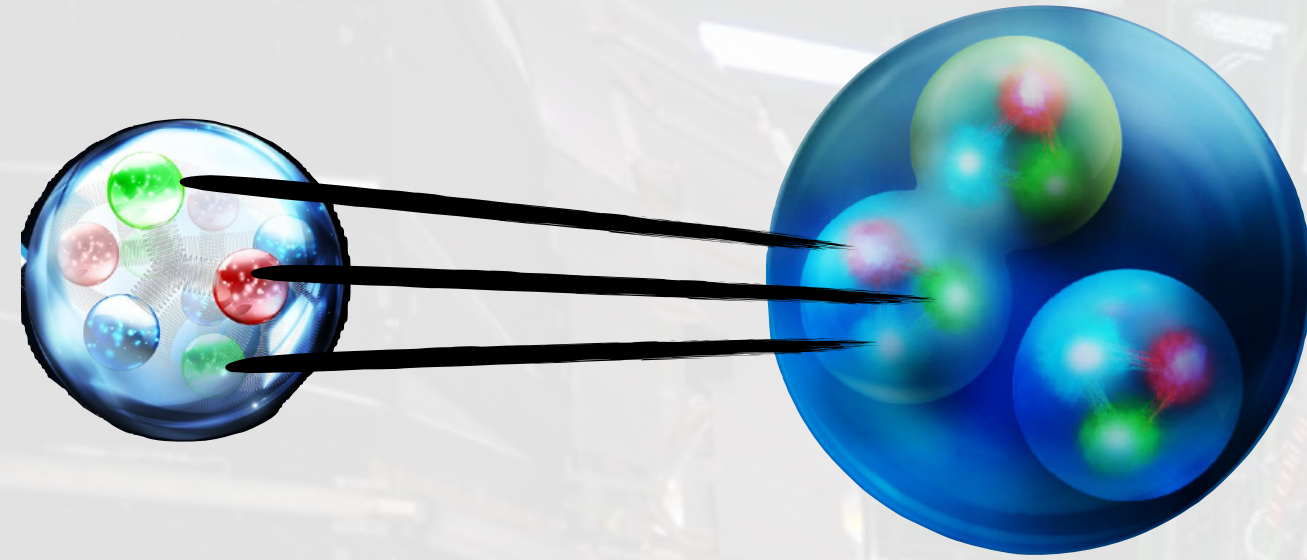
TPS3 = SPS ⊗ SPS ⊗ SPS

Relative size: **1:4.54:3.56** D. d'Enterria et al, EPJC 78 (2018) 359

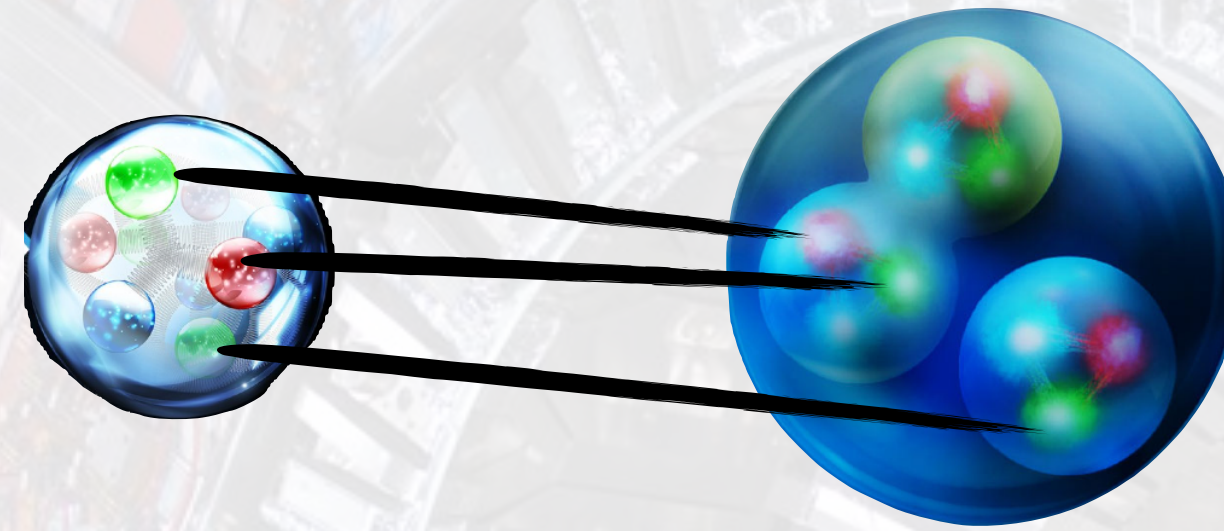
Pocket Formula:

$$\sigma_{pA \rightarrow abc}^{\text{TPS}} = \left(\frac{m}{6}\right) \frac{\sigma_{pN \rightarrow a}^{\text{SPS}} \cdot \sigma_{pN \rightarrow b}^{\text{SPS}} \cdot \sigma_{pN \rightarrow c}^{\text{SPS}}}{\sigma_{\text{eff,TPS,pA}}^2} \rightarrow \sigma_{\text{eff,TPS,pA}} = \left[\frac{A}{\sigma_{\text{eff,TPS}}^2} + \frac{3F_{pA} [\text{mb}^{-1}]}{\sigma_{\text{eff,DPS}}} + C_{pA} [\text{mb}^{-2}] \right]^{-1/2}$$

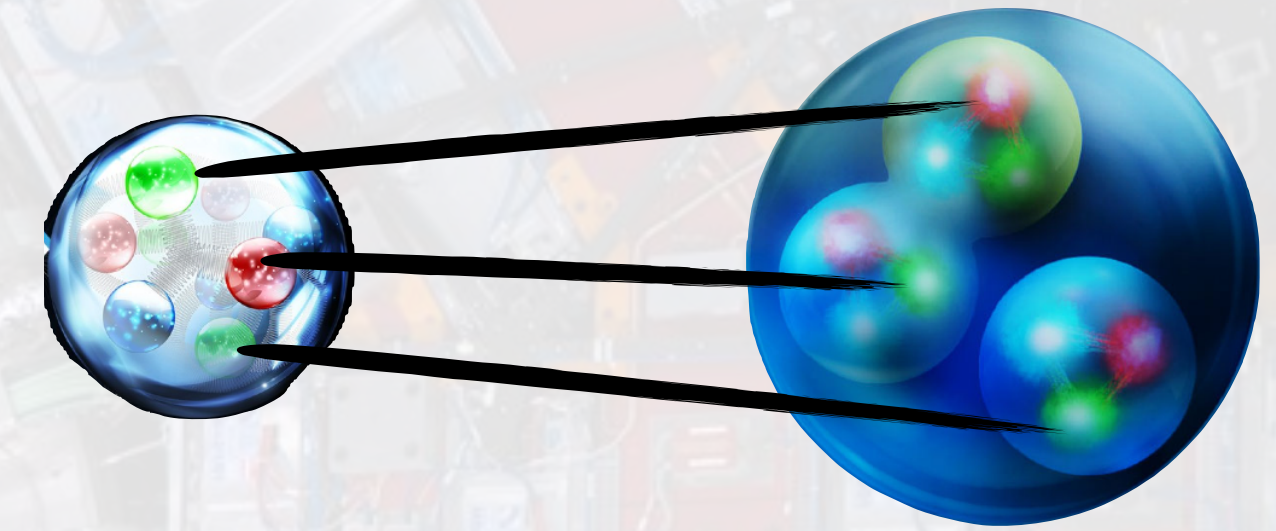
Triple Parton Scattering - pA



TPS1 = TPS



TPS2 = DPS ⊗ SPS



TPS3 = SPS ⊗ SPS ⊗ SPS

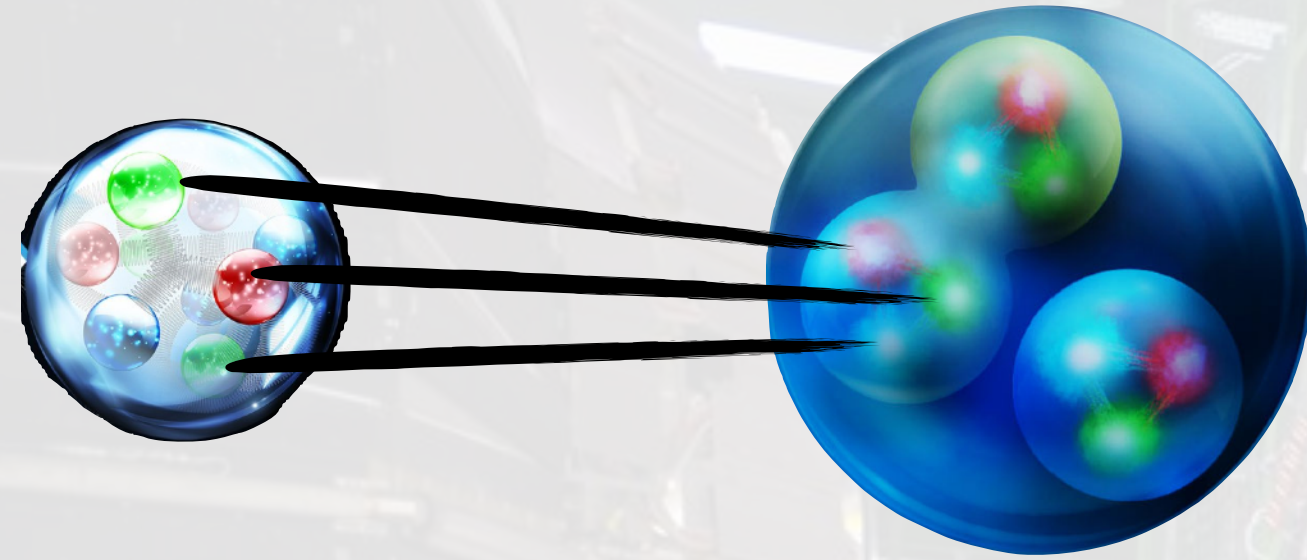
Relative size: **1:4.54:3.56** D. d'Enterria et al, EPJC 78 (2018) 359

Pocket Formula:

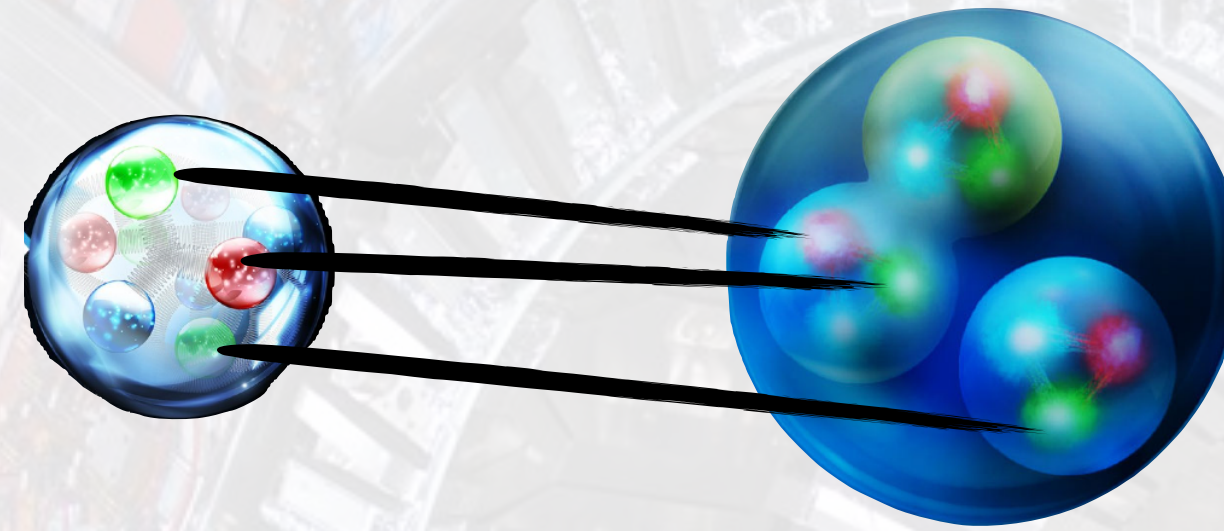
$$\sigma_{pA \rightarrow abc}^{\text{TPS}} = \left(\frac{m}{6}\right) \frac{\sigma_{pN \rightarrow a}^{\text{SPS}} \cdot \sigma_{pN \rightarrow b}^{\text{SPS}} \cdot \sigma_{pN \rightarrow c}^{\text{SPS}}}{\sigma_{\text{eff,TPS,pA}}^2} \quad \sigma_{\text{eff,TPS,pA}} = \left[\frac{A}{\sigma_{\text{eff,TPS}}^2} + \frac{3F_{pA} [\text{mb}^{-1}]}{\sigma_{\text{eff,DPS}}} + C_{pA} [\text{mb}^{-2}] \right]^{-1/2}$$

F_{pA} } Coefficients that should be calculated
 C_{pA} } within a model of the nuclear structure

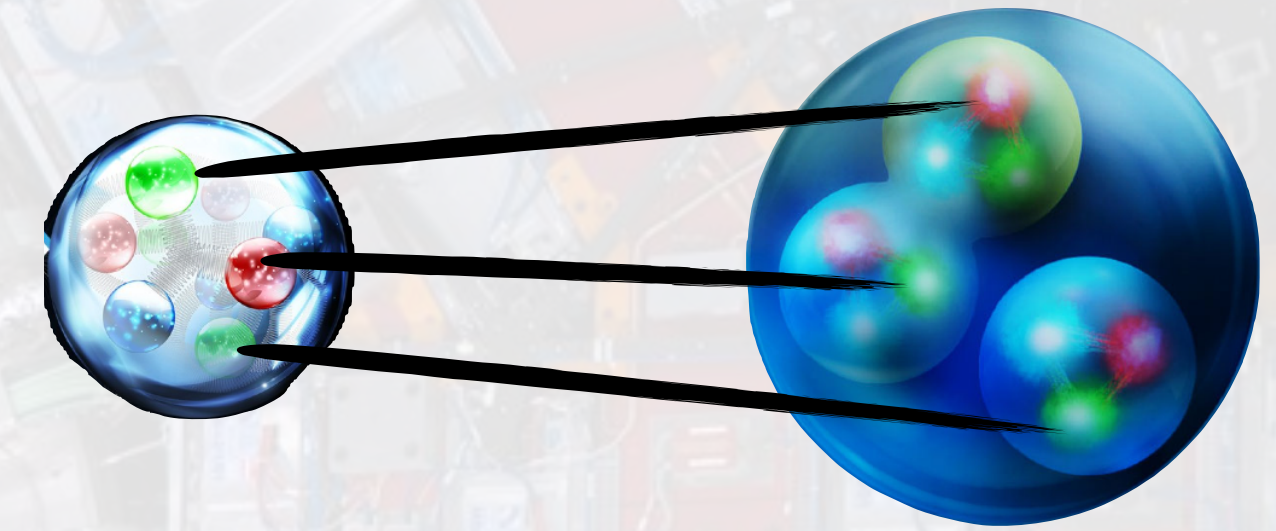
Triple Parton Scattering - pA



TPS1 = TPS



TPS2 = DPS ⊗ SPS



TPS3 = SPS ⊗ SPS ⊗ SPS

Relative size: **1:4.54:3.56**

D. d'Enterria et al, EPJC 78 (2018) 359

$$\sigma_{\text{eff,TPS,pA}} = \left[\frac{A}{\sigma_{\text{eff,TPS}}^2} + \frac{3F_{\text{pA}} [\text{mb}^{-1}]}{\sigma_{\text{eff,DPS}}} + C_{\text{pA}} [\text{mb}^{-2}] \right]^{-1/2}$$

$$\sigma_{\text{eff,TPS}} = 12.5 \text{ mb}$$

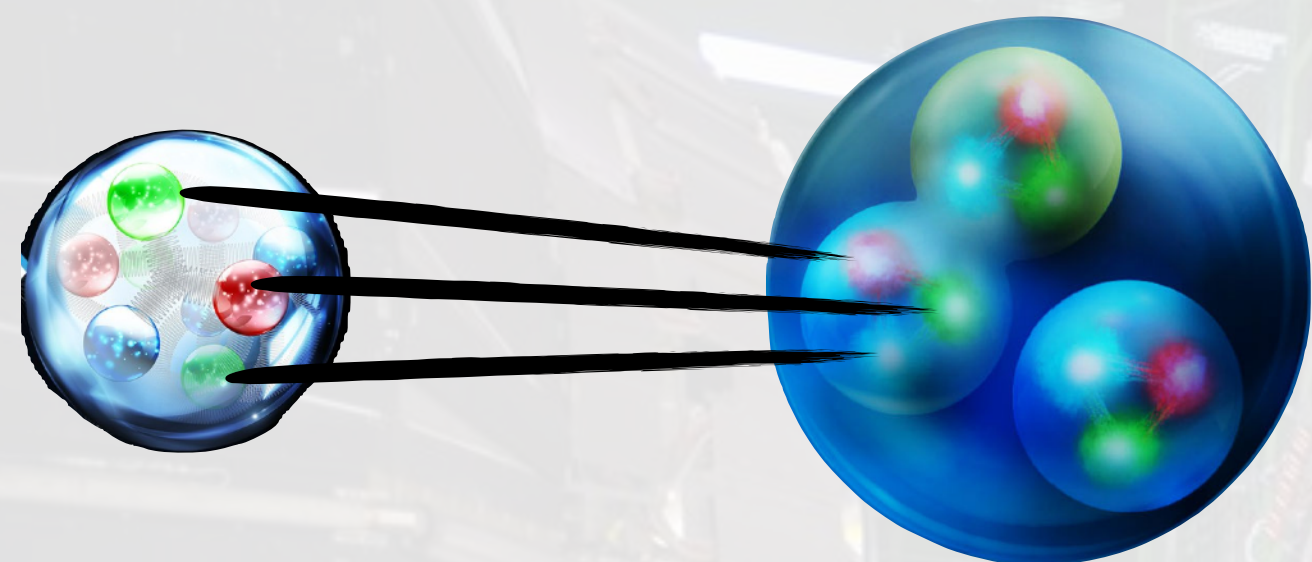
45 times

$$\sigma_{\text{eff,TPS,pPb}} = 0.29 \text{ mb}$$

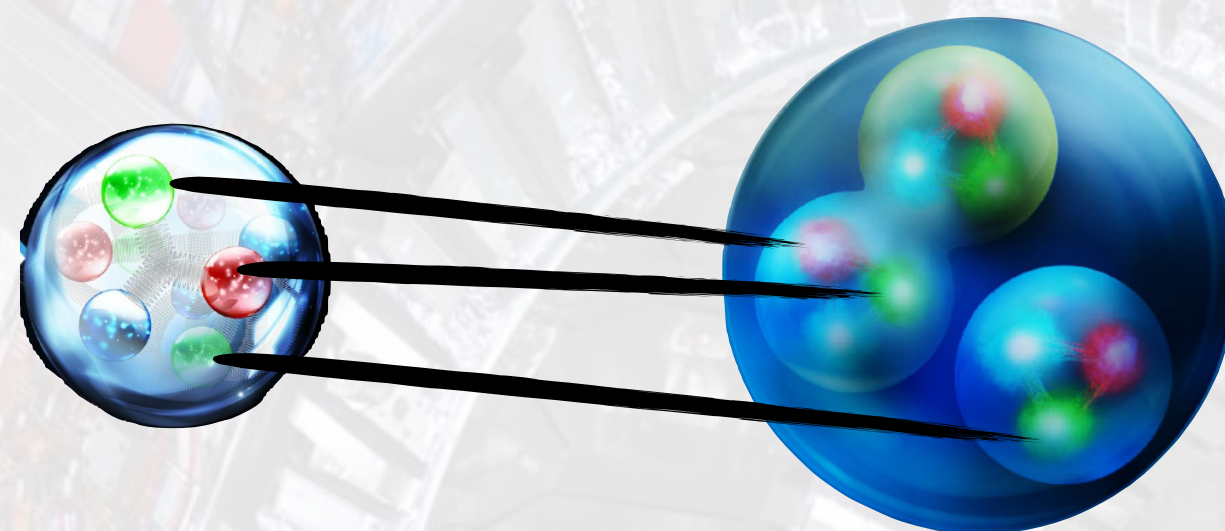


$$\sigma_{\text{TPS,pPb}} \sim 45 \sigma_{\text{TPS,pp}}$$

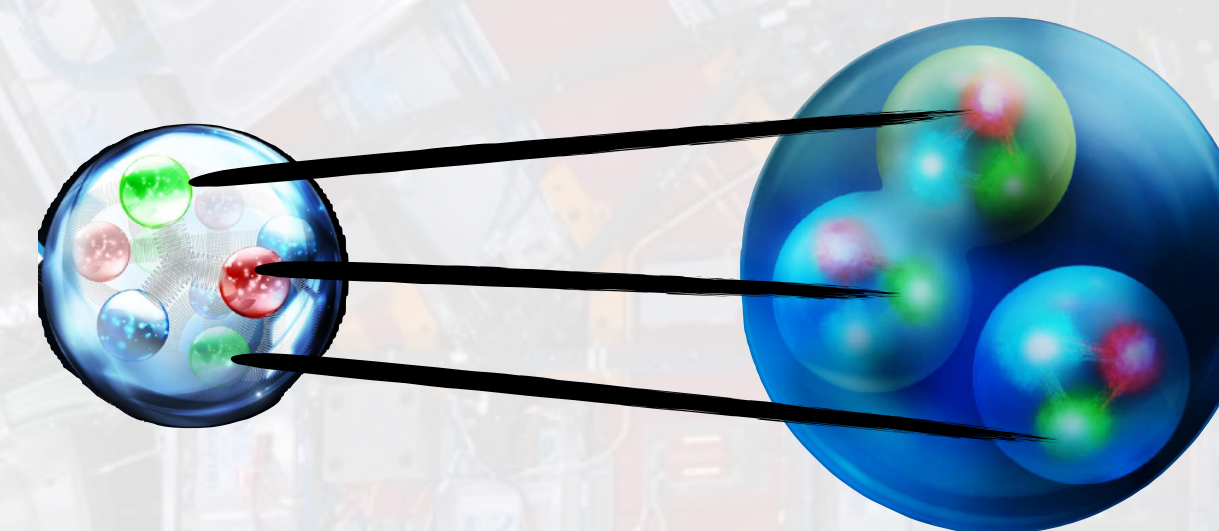
Triple Parton Scattering - pA



TPS1 = TPS



TPS2 = DPS ⊗ SPS



TPS3 = SPS ⊗ SPS ⊗ SPS

Relative size: **1:4.54:3.56**

D. d'Enterria et al, EPJC 78 (2018) 359

$$\sigma_{\text{eff,TPS,pA}} = \left[\frac{A}{\sigma_{\text{eff,TPS}}^2} + \frac{3F_{\text{pA}} [\text{mb}^{-1}]}{\sigma_{\text{eff,DPS}}} + C_{\text{pA}} [\text{mb}^{-2}] \right]^{-1/2}$$

$\sigma_{\text{eff,TPS}} = 12.5 \text{ mb}$

45 times

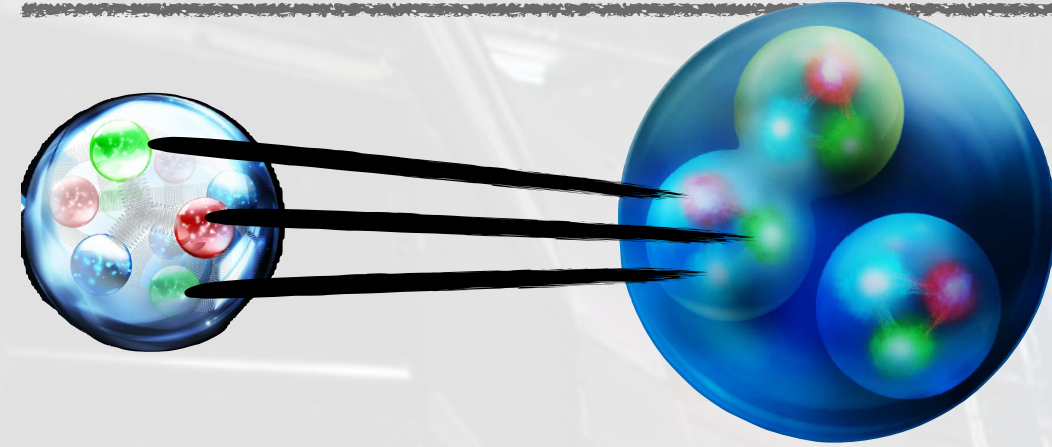
$\sigma_{\text{eff,TPS,pPb}} = 0.29 \text{ mb}$



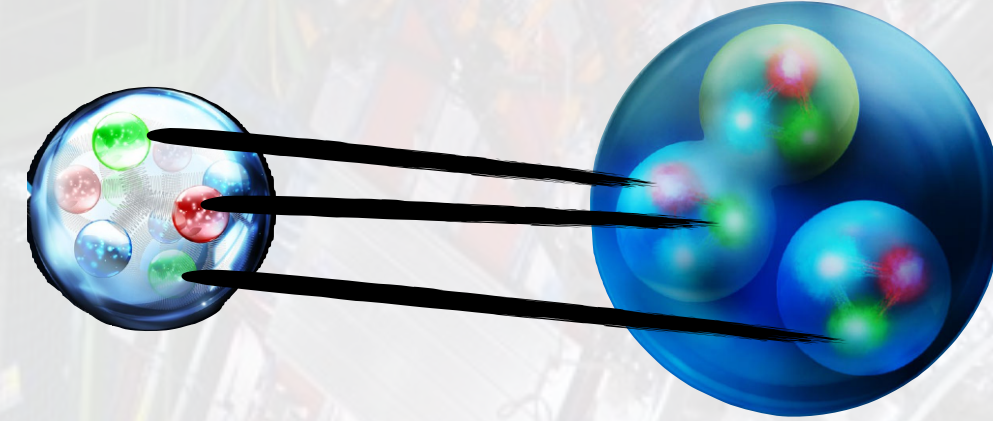
$\sigma_{\text{TPS,pPb}} \sim 45 \sigma_{\text{TPS,pp}}$

Novel way to extract them!!

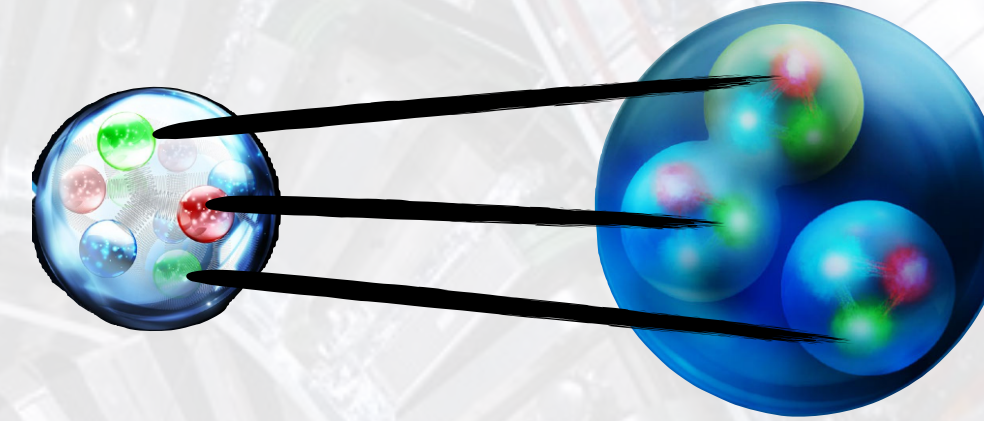
Triple Parton Scattering - pA



TPS1 = TPS



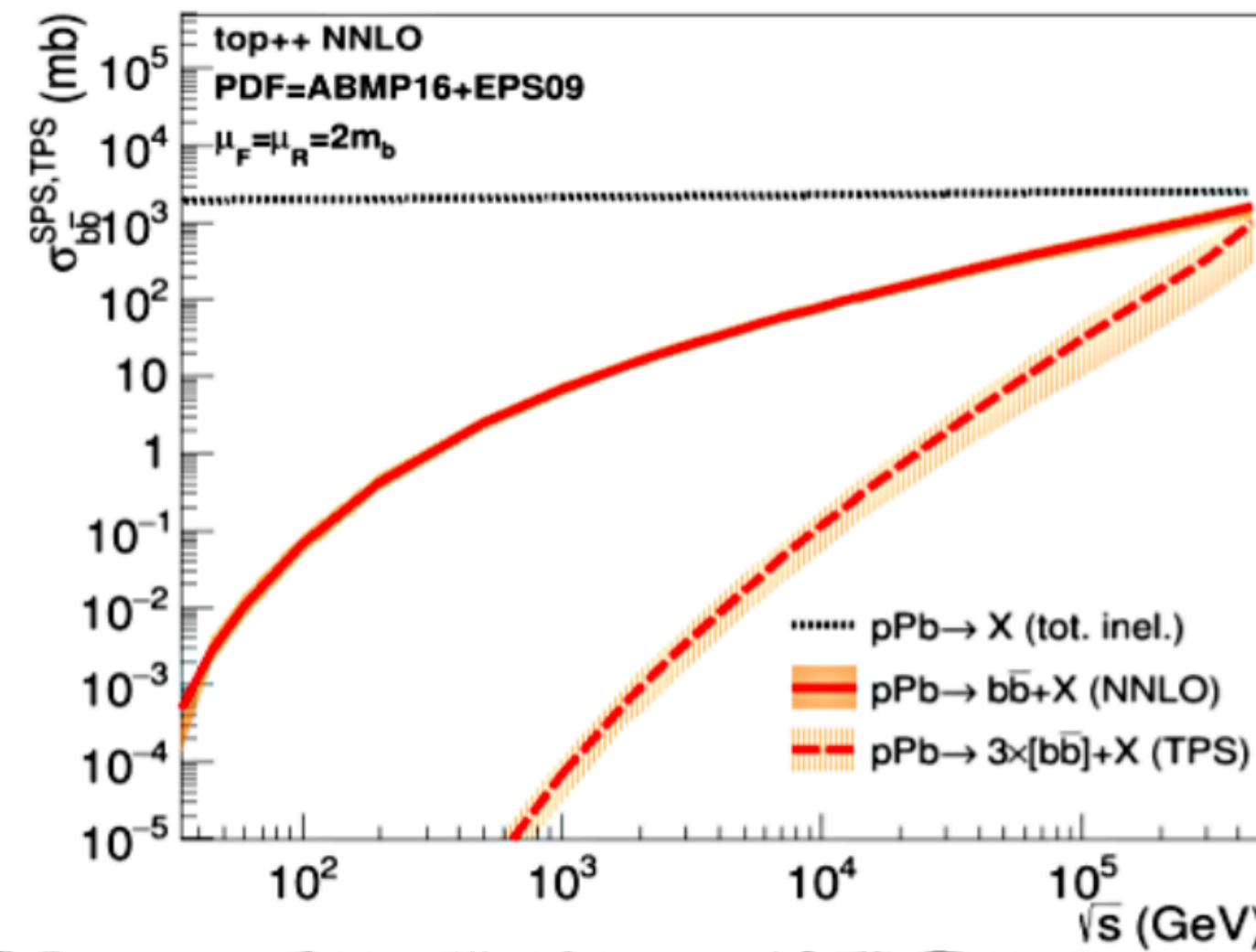
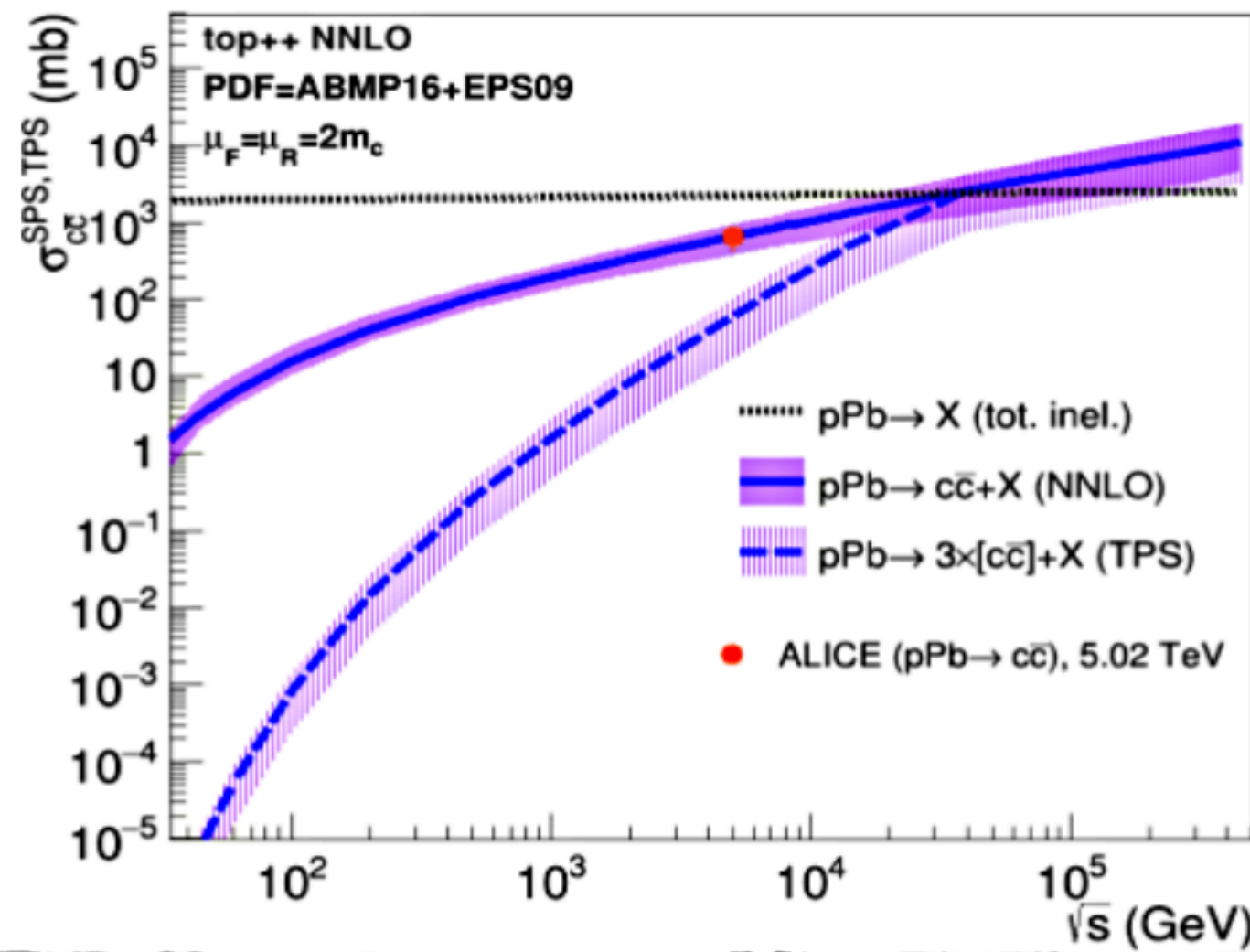
TPS2 = DPS ⊗ SPS



TPS3 = SPS ⊗ SPS ⊗ SPS

Triple charm & beauty production:

D. d'Enterria and A. M. Snigirev, Eur. Phys. J. C 78, no.5, 359 (2018)



Process	pPb(8.8 TeV)	pPb(63 TeV)	p-Air(430 TeV)
σ_{pA}^{inel}	2.2 ± 0.4 b	2.4 ± 0.4 mb	0.61 ± 0.10 b
$\sigma_{c\bar{c}+X}^{SPS}$	$0.96 \pm 0.45_{sc} \pm 0.10_{PDF}$ b	$3.4 \pm 1.9_{sc} \pm 0.4_{PDF}$ b	$0.75 \pm 0.5_{sc} \pm 0.1_{PDF}$ b
$\sigma_{c\bar{c}c\bar{c}+X}^{TPS}$	$200 \pm 140_{tot}$ mb	$8.7^* \pm 6.2_{tot}$ b	$5.0^* \pm 3.6_{tot}$ b
$\sigma_{b\bar{b}+X}^{SPS}$	$72 \pm 12_{sc} \pm 5_{PDF}$ mb	$370 \pm 75_{sc} \pm 30_{PDF}$ mb	$110 \pm 25_{sc} \pm 5_{PDF}$ mb
$\sigma_{b\bar{b}b\bar{b}+X}^{TPS}$	$0.084 \pm 0.045_{tot}$ μ b	$11 \pm 7_{tot}$ μ b	$17 \pm 11_{tot}$ μ b

- Triple charm amounts to ~20% (~100%!) of inclusive charm x-sections at LHC (FCC).
- Large triple J/Ψ production at FCC:
- Triple beauty amounts to ~3% of inclusive beauty x-sections at FCC.

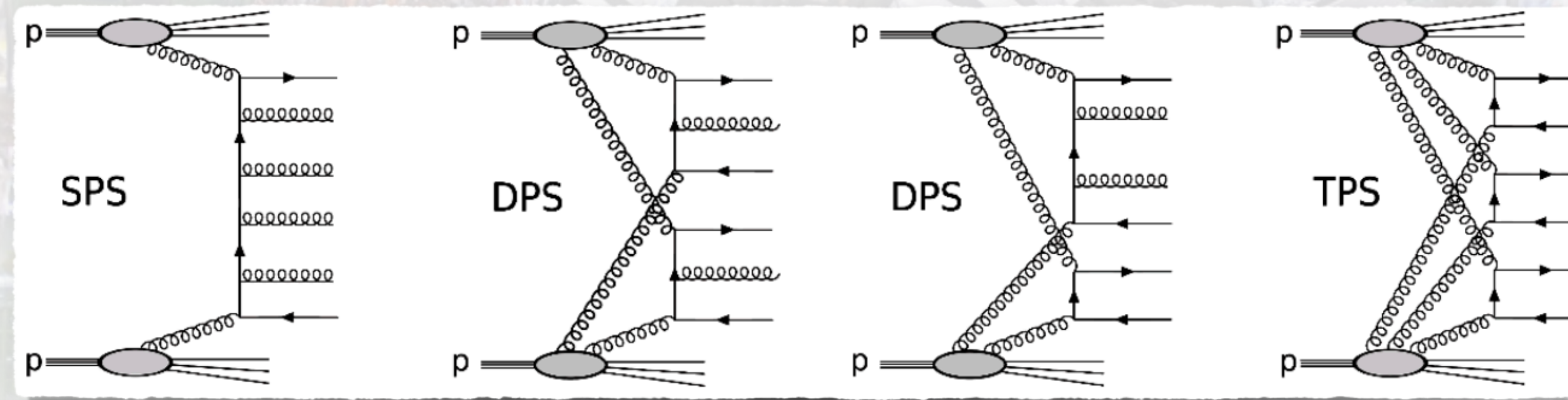
MT thanks D. d'enterria for slides

Triple Parton Scattering - pA



6-jet production in pp (14 TeV) and pPb (8.8 TeV)

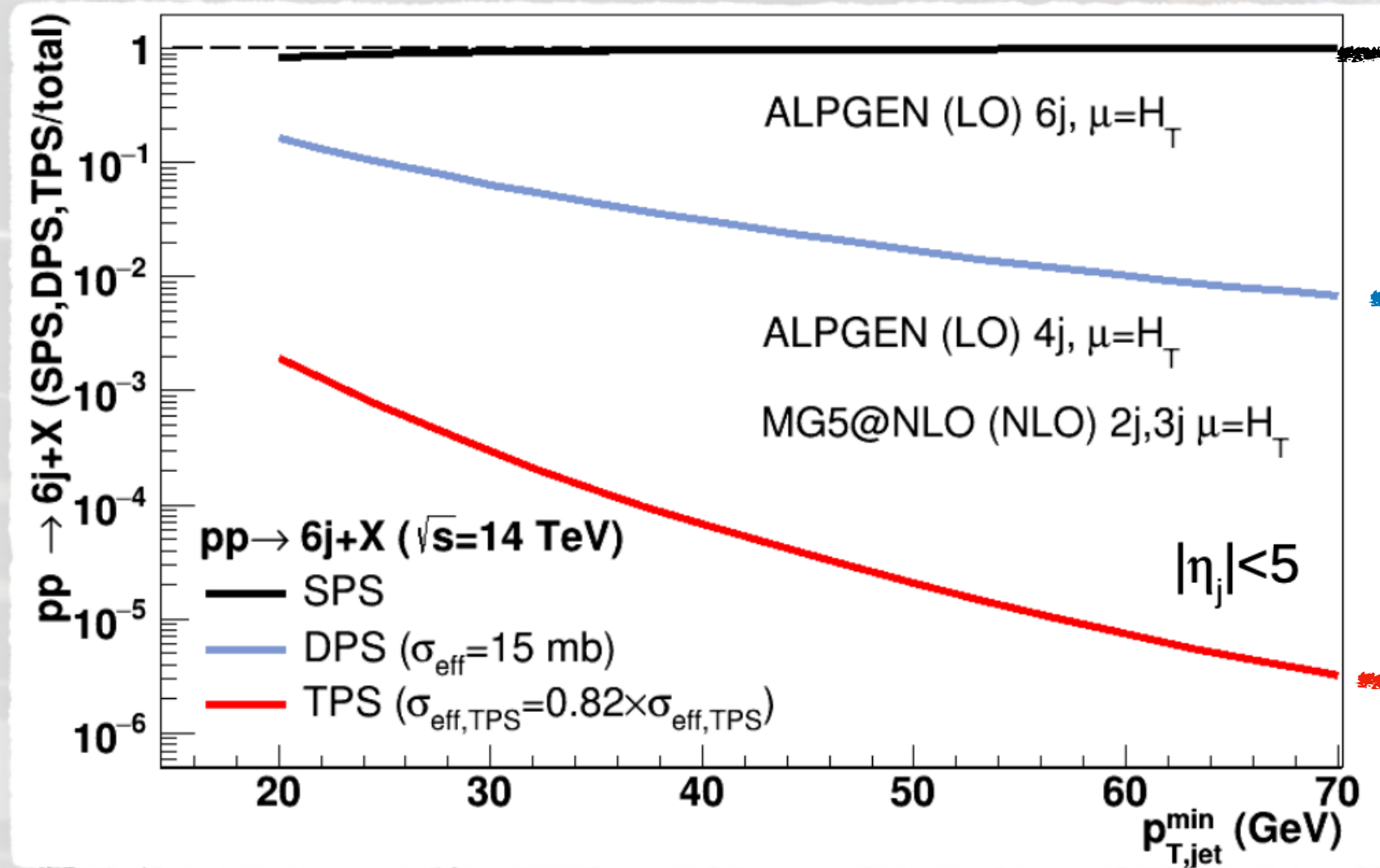
M. Maneyro & D. d'Enterria



Triple Parton Scattering - pA



pp



$$\sigma_{\text{SPS}}(6j) \approx 30 \text{ nb} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

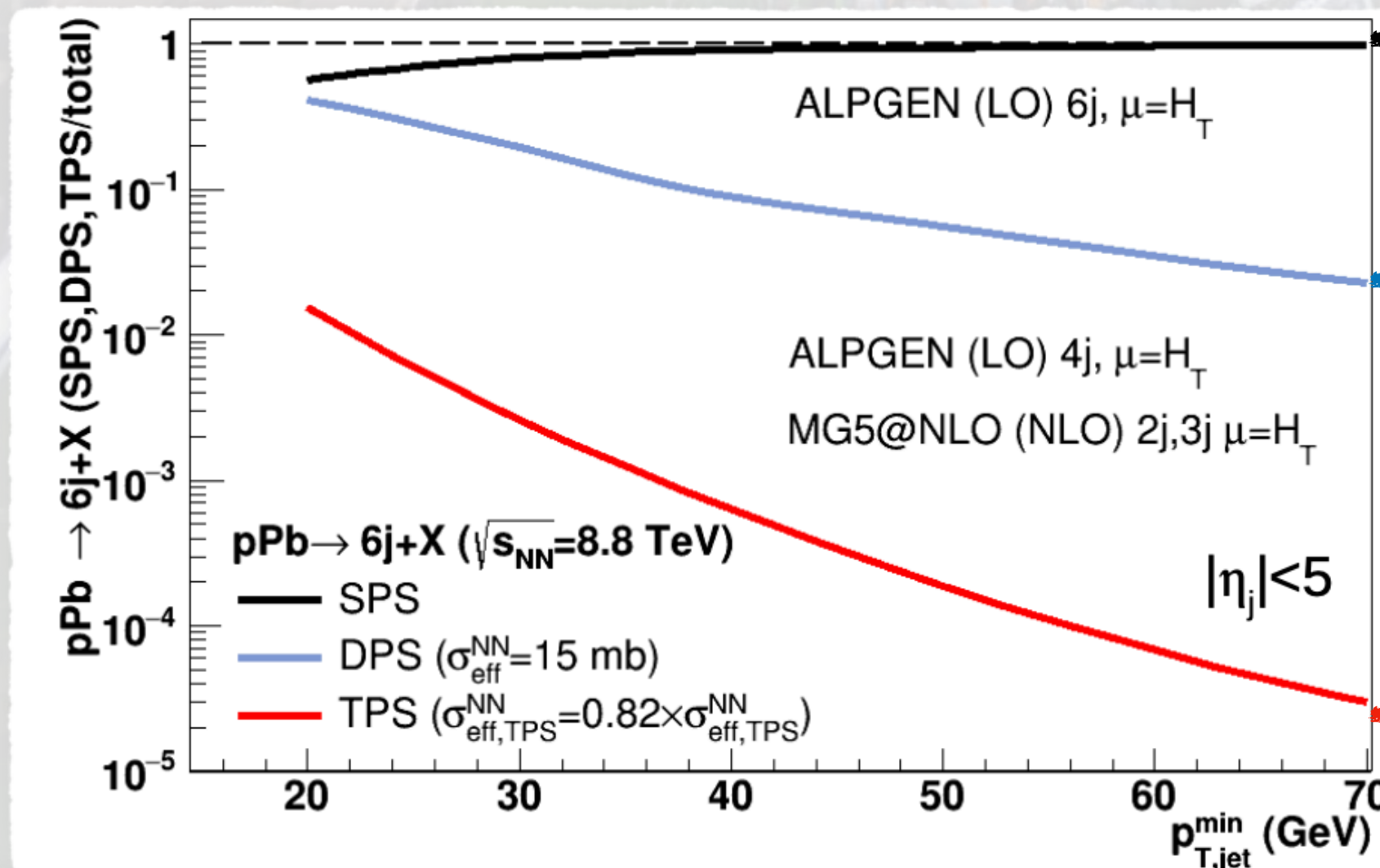
$$\sigma_{\text{DPS}}(6j) \approx 4 \text{ nb} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

DPS \rightarrow 20% – 2%

$$\sigma_{\text{TPS}}(6j) \approx 3 \text{ pb} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

TPS $\leq 10^{-3} \%$

pPb



$$\sigma_{\text{SPS}}(6j) \approx 1.2 \mu\text{b} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

$$\sigma_{\text{DPS}}(6j) \approx 800 \text{ nb} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

DPS \rightarrow 40% – 6%

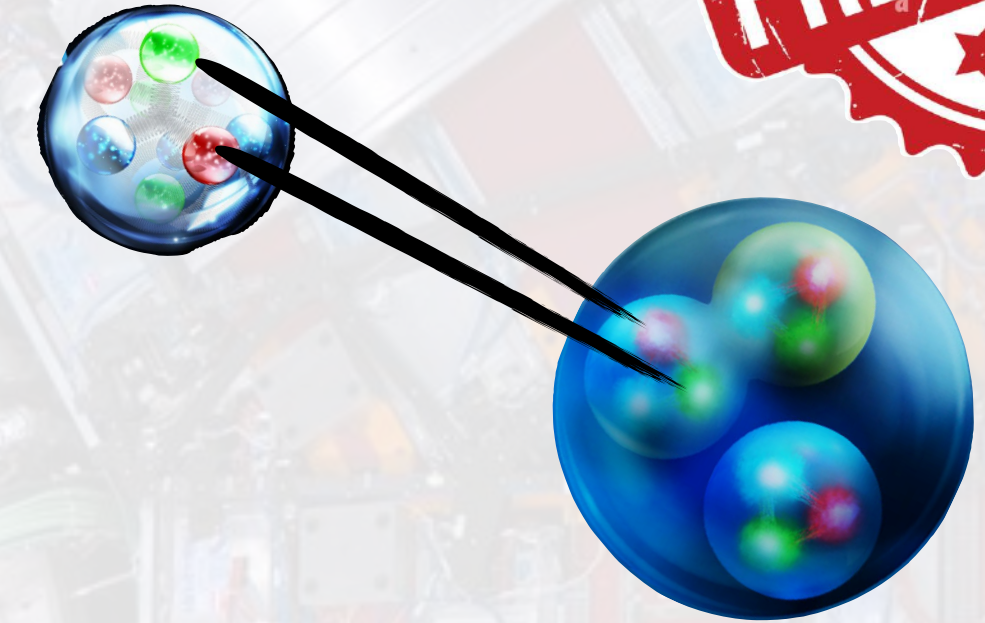
$$\sigma_{\text{TPS}}(6j) \approx 1.2 \text{ nb} \quad (p_T > 35\text{GeV}, |\eta| < 5)$$

TPS $\leq 2 \%$

Nuclear DPS and double EMC effect



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:



$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

The **nuclear light-cone distribution** can be evaluated with realistic wave-function (from Av18 +UIV potential) for light nuclei and modeled for heavy ions.

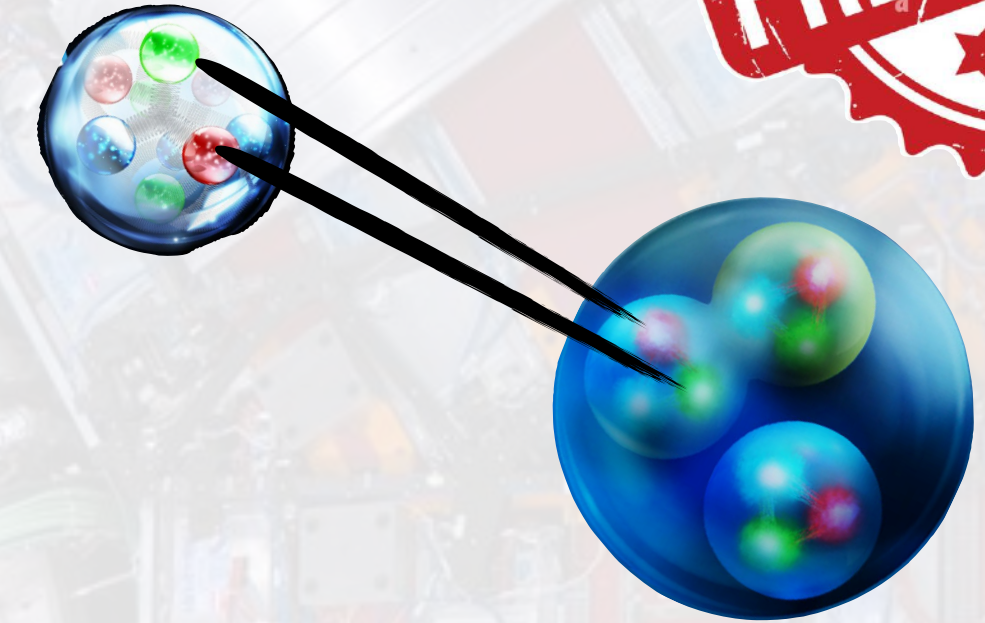
We can define the double structure functions (dSF):

$$F^{2,A}(x_1, x_2) \equiv \sum_{ij} e_i^2 e_j^2 x_1 x_2 \tilde{F}_{ij}^1(x_1, x_2, 0)$$

Nuclear DPS and double EMC effect



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:



$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

The **nuclear light-cone distribution** can be evaluated with realistic wave-function (from Av18 +UIV potential) for light nuclei and modeled for heavy ions.

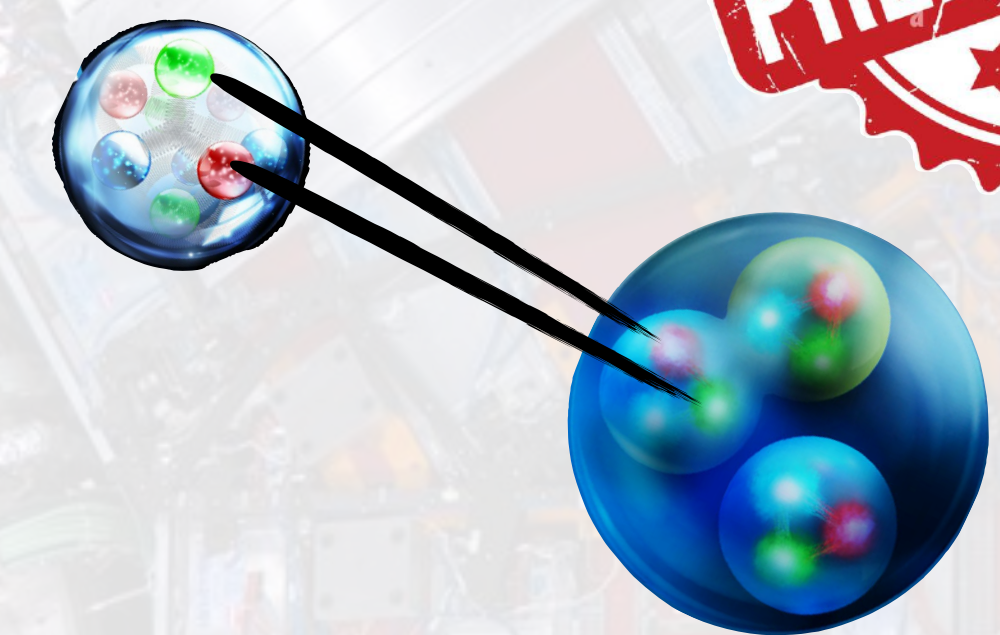
We can define the double structure functions (dSF): $F^{2,A}(x_1, x_2) \equiv \sum_{ij} e_i^2 e_j^2 x_1 x_2 \tilde{F}_{ij}^1(x_1, x_2, 0)$

We can generalize the EMC ratio: $R_{EMC}^A(x) = \frac{F_2^A(x)}{A} \frac{2}{F_2^2(x)}$ \rightarrow $R_{2EMC}^A(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{A} \frac{2}{F^{2,2}(x_1, x_2)}$

Nuclear DPS and double EMC effect



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism:



$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

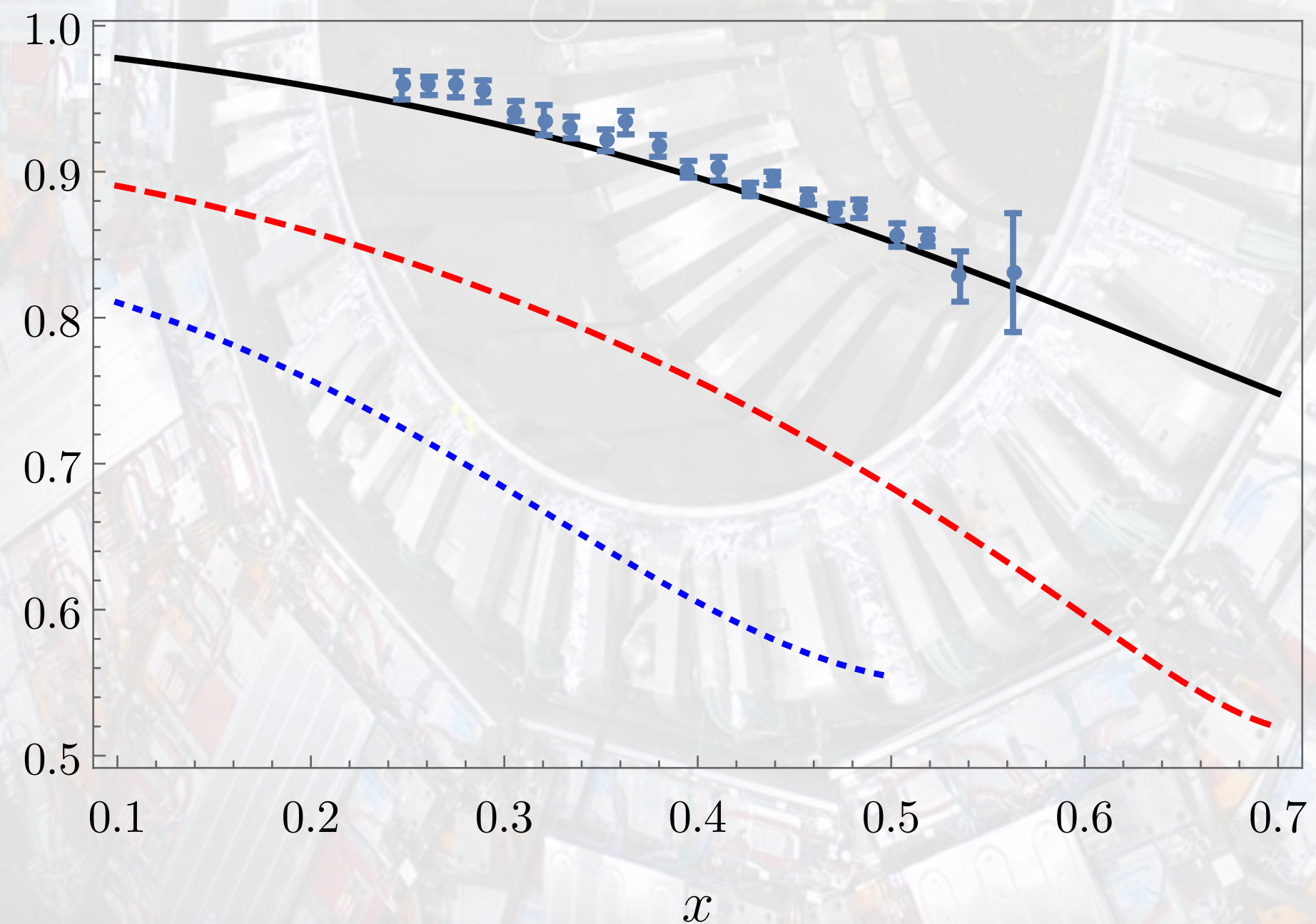
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Pb

We can generalize the EMC ratio:

$$R_{EMC}^A(x) = \frac{F_2^A(x)}{A} \frac{2}{F_2^2(x)}$$

$$R_{2EMC}^A(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{A} \frac{2}{F^{2,2}(x_1, x_2)}$$



Calculations with the model of:
D. N. Kim and G. A. Miller, PRC 106, no.5, 055202 (2022)

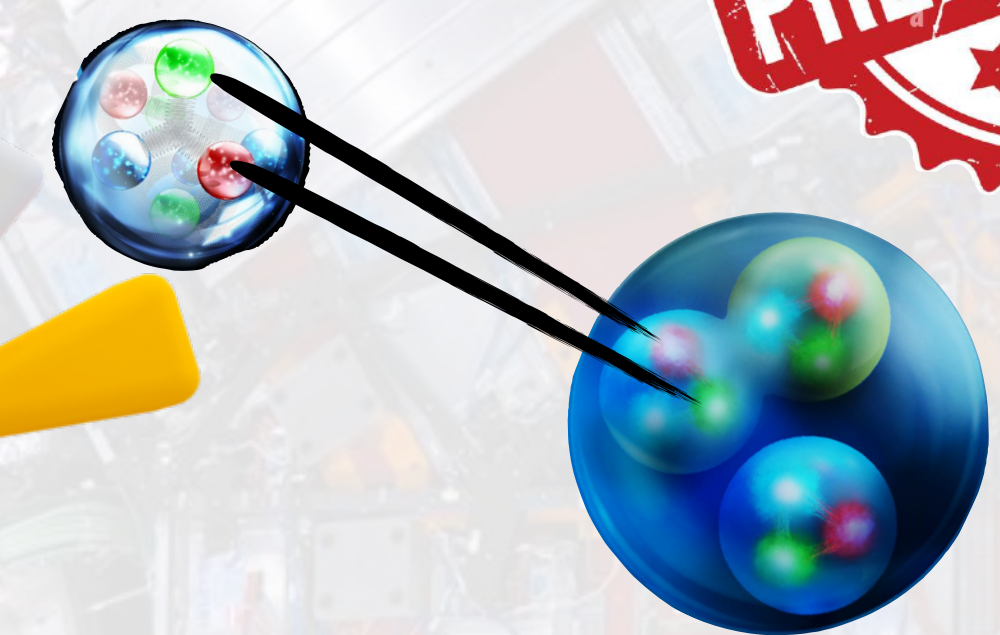
- $R_{EMC}^{Pb}(x)$
- - - $R_{2EMC}^{Pb}(x, 0.3)$
- ⋯ $R_{2EMC}^{Pb}(x, 0.5)$

Nuclear DPS and double EMC effect



Nuclear DPD (in momentum space) corresponding to the DPS1 mechanism

$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) d\xi$$



We can define the double structure function

- 1) EMC like effect more deep!
- 2) A novel way to test our models of the EMC effect! (short-range correlations, ..., off-shellness..)
- 3) Neutron DPDs

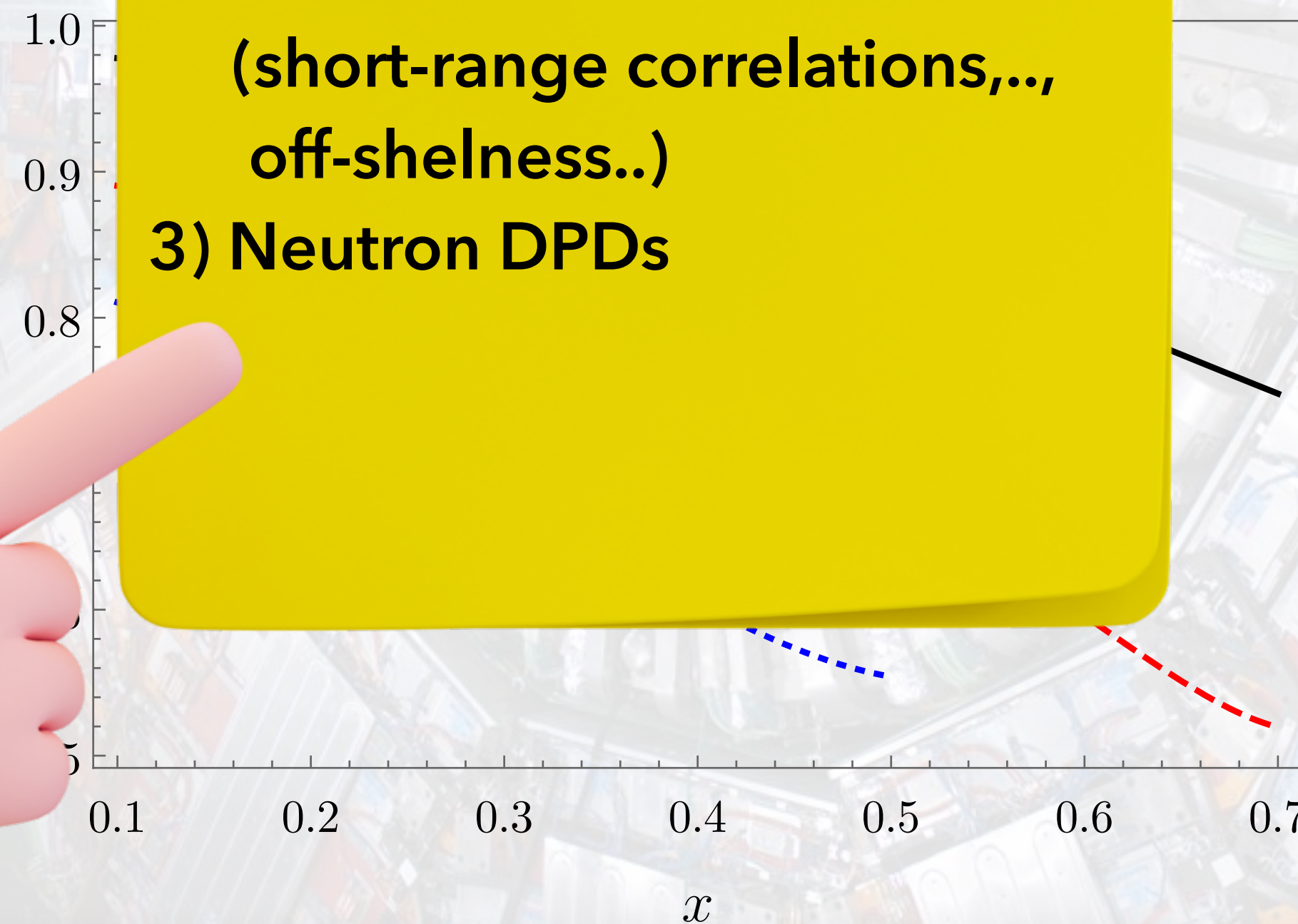
$$x_2 \tilde{F}_{ij}^1(x_1, x_2, 0)$$

We can generalize the EMC ratio:

$$R_{EMC}^A(x) = \frac{F_2^A(x)}{A} \frac{2}{F_2^2(x)}$$

$$R_{2EMC}^A(x_1, x_2) = \frac{F^{2,A}(x_1, x_2)}{2A}$$

Calculations with the model of:
D. N. Kim and G. A. Miller, PRC 106, no.5, 055202 (2022)



- $R_{EMC}^{Pb}(x)$
- - - $R_{2EMC}^{Pb}(x, 0.3)$
- ⋯ $R_{2EMC}^{Pb}(x, 0.5)$

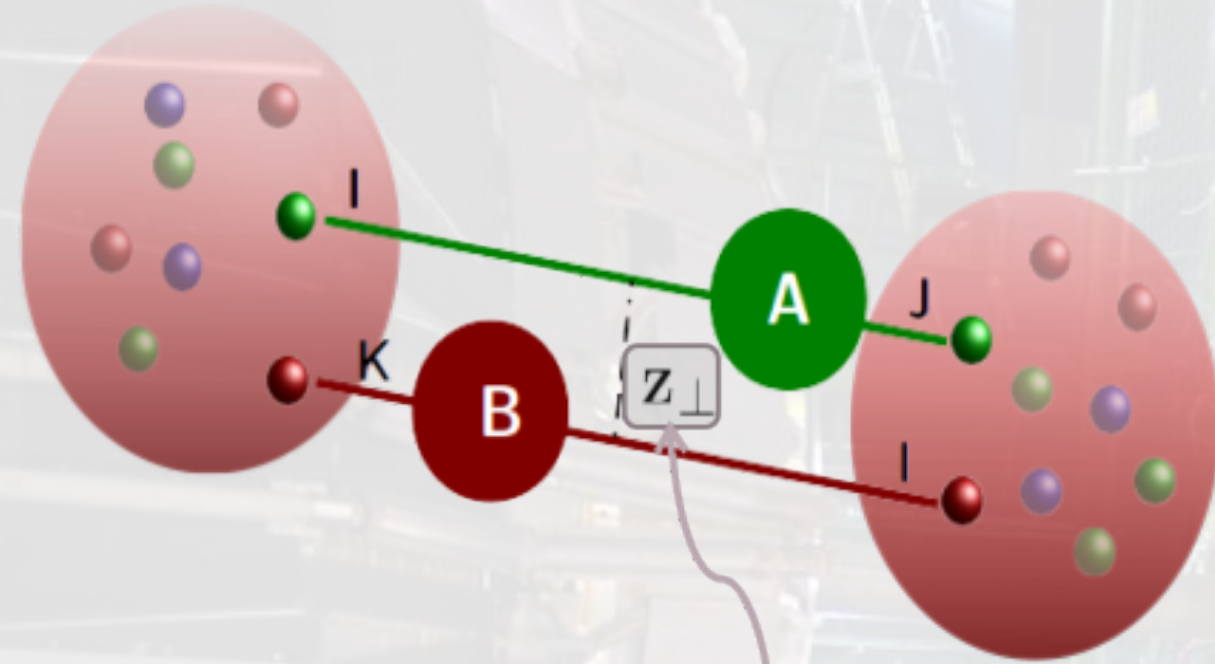


CONCLUSIONS (a world beyond SPS)

- 1) DPS represents a new and unique window towards the inner structure of hadrons
- 2) DPS in pA will be essential to increase the signal and to access the DPS effective cross-section:
 - a) $J/\Psi + J/\Psi$, $J/\Psi + \Upsilon$, $J/\Psi + W$, $J/\Psi + Z$, $\Upsilon + \Upsilon$, $\Upsilon + W$, and $\Upsilon + Z$
 - b) $W + \text{di jets}$, same sign WW , 4-jet, 2jet+2b
- 3) TPS and TPS in pA can be very important for the study of new exciting channels (like triple J/Ψ production) offering unique opportunities:
 - a) Access information beyond 1- and 2- body distributions
 - b) Access Triple Parton Correlations
 - c) properly estimate the background for rare processes
 - d) Help in accessing information related to DPS

Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$d\sigma \propto \int d^2z_{\perp} \underbrace{F_{ij}(x_1, x_2, z_{\perp}, \mu_A, \mu_B) F_{kl}(x_3, x_4, z_{\perp}, \mu_A, \mu_B)}_{\text{Double Parton Distribution (DPD)}}$$

Double Parton Distribution (DPD)

N. Paver and D. Treleani, *Nuovo Cimento* 70A, 215 (1982)

Mekhfi, *PRD* 32 (1985) 2371

M. Diehl et al, *JHEP* 03 (2012) 089

A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

R. Nagar's talk MPI 2021

Diehl et al. *JHEP* 03 (2012) 089, *JHEP* 01 (2016) 076

Vladimirov *JHEP* 04 (2018) 045

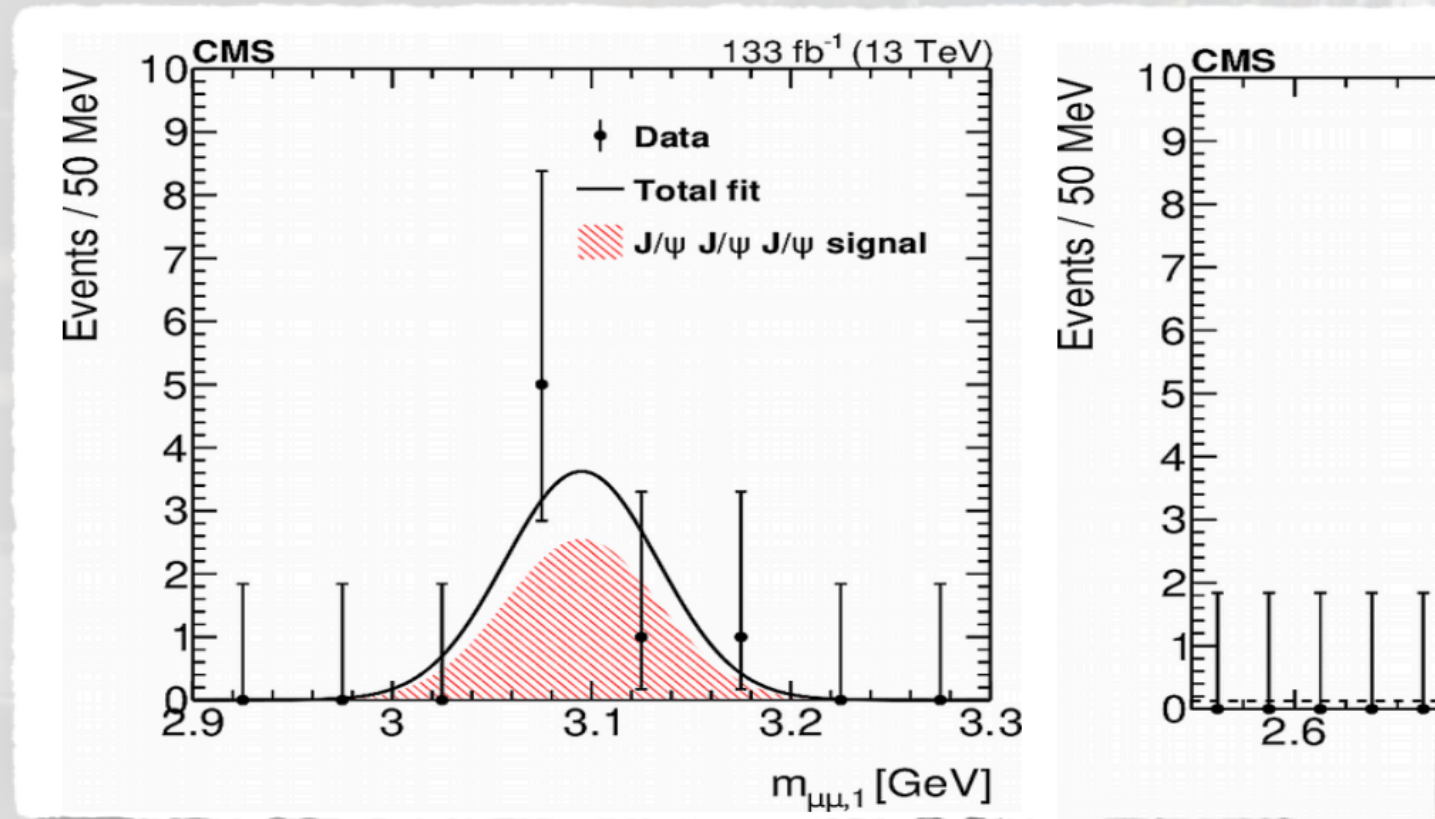
Buffing et al. *JHEP* 01 (2018) 044

Diehl, RN *JHEP* 04 (2019) 124

Triple Parton Scattering - where?

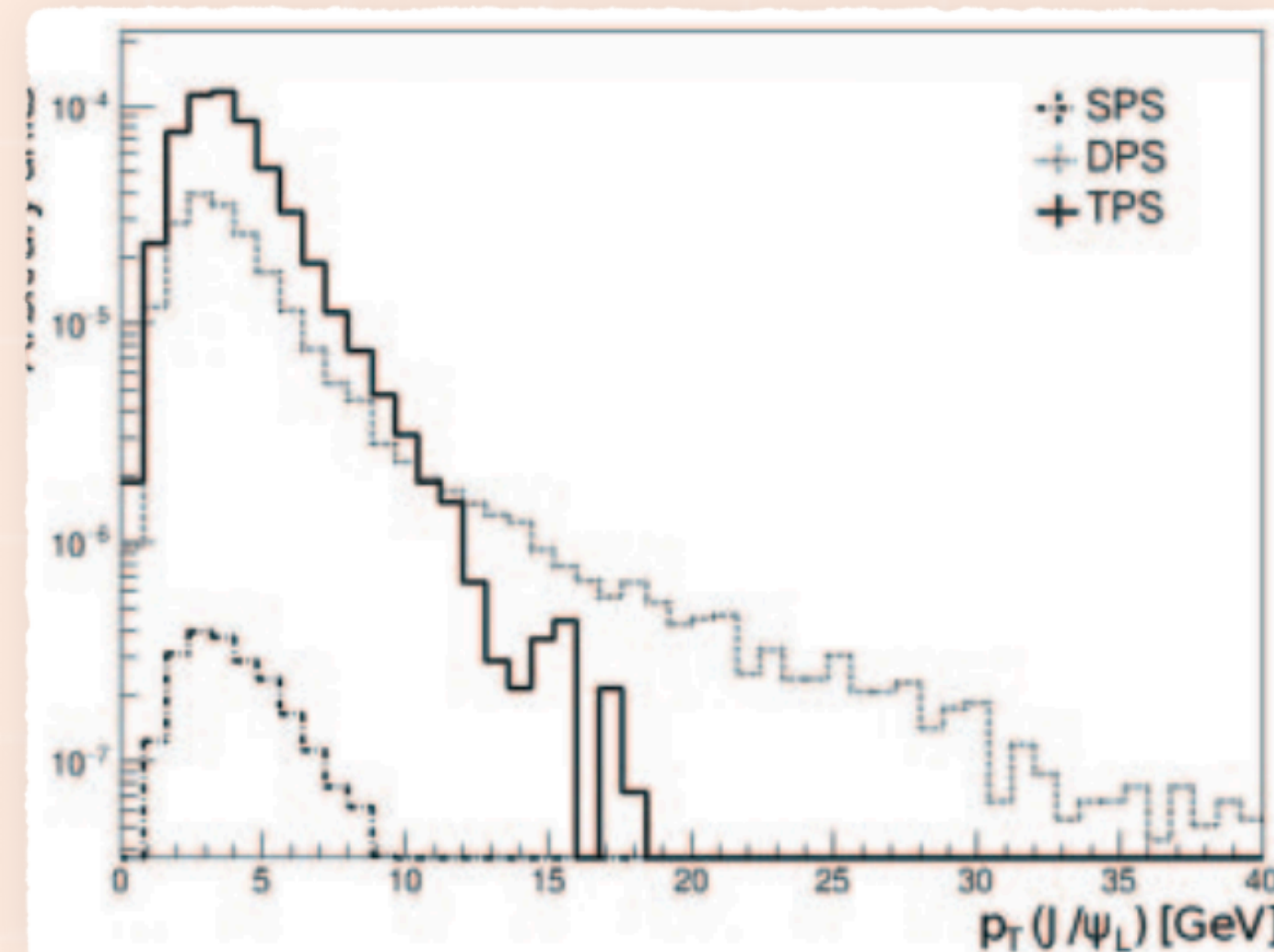


Triple J/ψ production! A. Tumasyan et al [CMS], Nature Phys. 19, no.3, 338-350 (2023)



Promising also the study of TPS for the production of 2 J/ψ and a D*

M. E. Ascioti [CMS], Nuovo Cimento 46 C (2023) 82



$$\sigma = 272_{104}^{+141} \text{ (stat)} \pm 17 \text{ (syst) fb}$$

{ SPS -> 6%
 DPS -> 74%
 TPS -> 20%

Novel way to extract the DPS effect

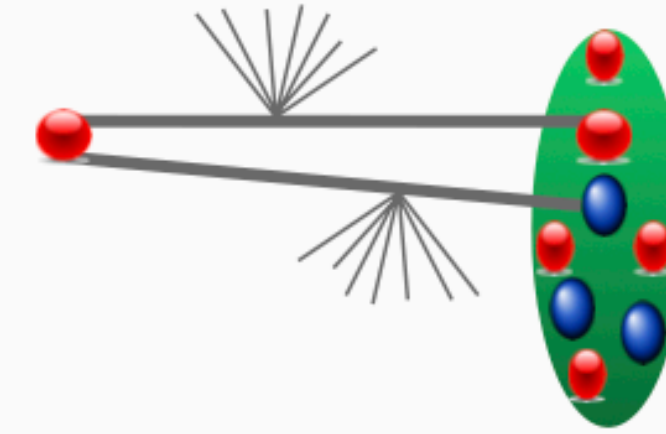
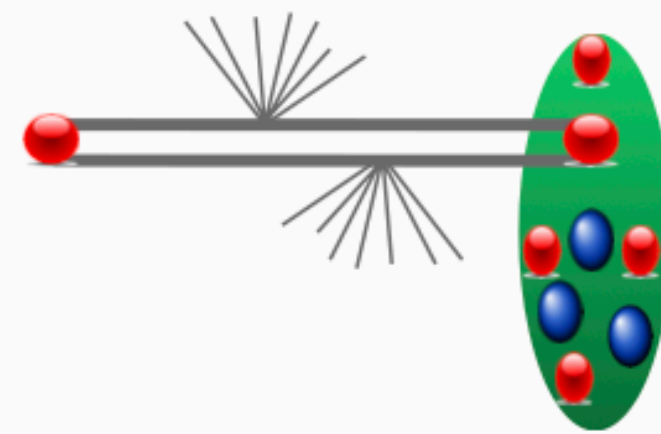
$$\sigma_{\text{eff,DPS}} = 2.7_{-1.0}^{+1.4} \text{ (exp)}_{-1.0}^{+1.5} \text{ (th)}$$

$\sigma_{\text{eff,DPS}}$ [mb]

CMS, $\sqrt{s}=13$ TeV, J/ψ+J/ψ+J/ψ	This work
CMS*, $\sqrt{s}=7$ TeV, J/ψ+J/ψ	Ref. 60
ATLAS, $\sqrt{s}=8$ TeV, J/ψ+J/ψ	Ref. 24
D0, $\sqrt{s}=1.96$ TeV, J/ψ+J/ψ	Ref. 22
D0*, $\sqrt{s}=1.96$ TeV, J/ψ+Y	Ref. 58
ATLAS*, $\sqrt{s}=7$ TeV, W+J/ψ	Ref. 59
ATLAS*, $\sqrt{s}=8$ TeV, Z+J/ψ	Ref. 60
ATLAS*, $\sqrt{s}=8$ TeV, Z+b→J/ψ	Ref. 57
D0, $\sqrt{s}=1.96$ TeV, γ+b/c+2-jet	Ref. 55
D0, $\sqrt{s}=1.96$ TeV, γ+3-jet	Ref. 55
D0, $\sqrt{s}=1.96$ TeV, 2-γ+2-jet	Ref. 56
D0, $\sqrt{s}=1.96$ TeV, γ+3-jet	Ref. 54
CDF, $\sqrt{s}=1.8$ TeV, γ+3-jet	Ref. 53
UA2, $\sqrt{s}=640$ GeV, 4-jet	Ref. 51
CDF, $\sqrt{s}=1.8$ TeV, 4-jet	Ref. 52
ATLAS, $\sqrt{s}=7$ TeV, 4-jet	Ref. 15
CMS, $\sqrt{s}=7$ TeV, 4-jet	Ref. 24
CMS, $\sqrt{s}=13$ TeV, 4-jet	Ref. 19
CMS, $\sqrt{s}=7$ TeV, W+2-jet	Ref. 14
ATLAS, $\sqrt{s}=7$ TeV, W+2-jet	Ref. 13
CMS, $\sqrt{s}=13$ TeV, WW	Ref. 18

DPS in pA collisions - predictions

DPS in pA collisions

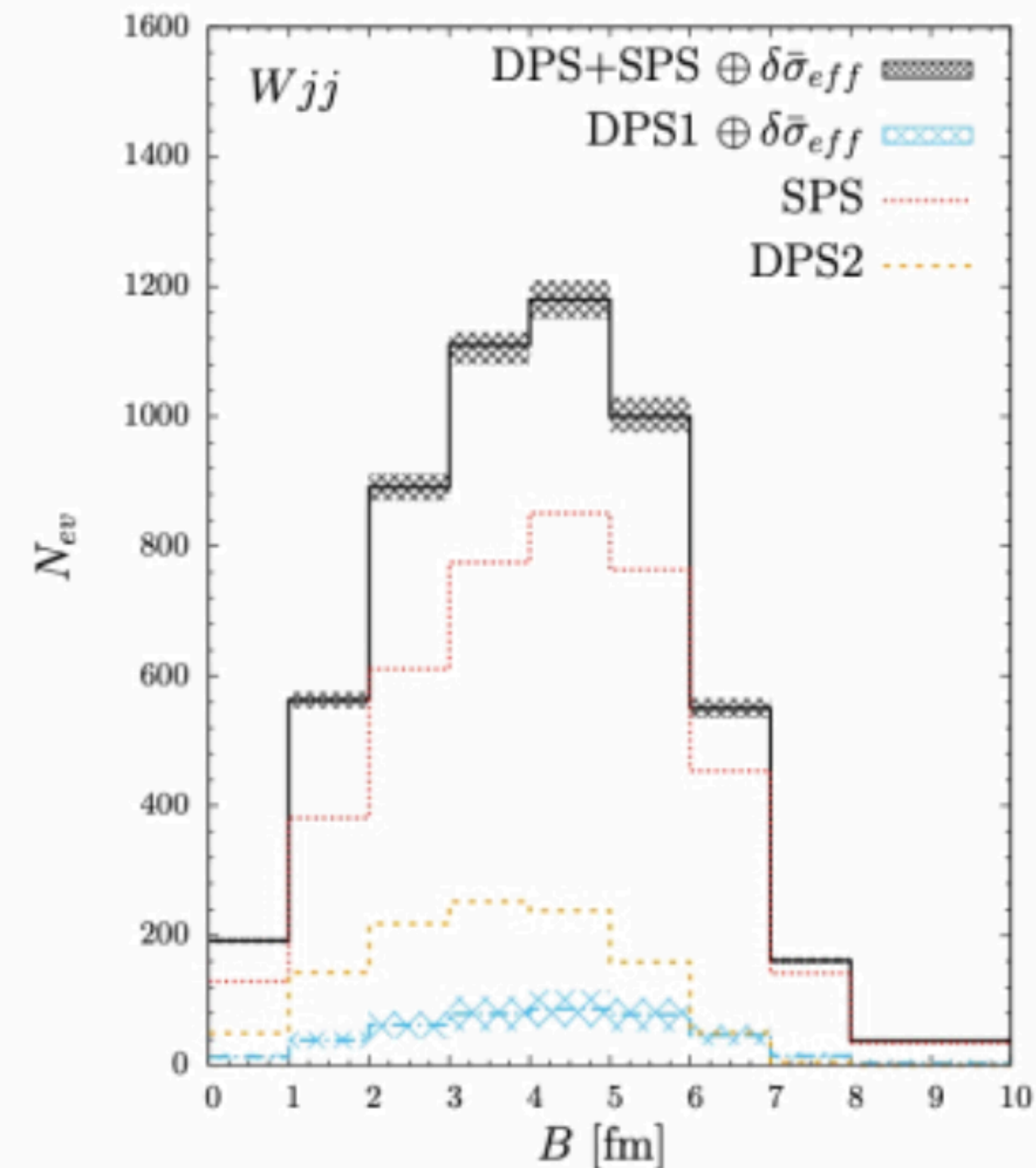


A lot of effort (slides from):
 - Boris Blok
 - Federico Alberto Ceccopi
 - Mark Strikman
 - Massimiliano Alvioli
 - Daniele Treleani

W+di-jets

σ^{Wjj}	$p_T^j > 20$ GeV [nb]	$p_T^j > 25$ GeV [nb]	$p_T^j > 30$ GeV [nb]
DPS1	19 ± 6	8 ± 3	4 ± 2
DPS2	49	22	11
SPS	81	57	41
Tot	149 ± 6	87 ± 3	56 ± 2

- SPS dominant
- DPS2 bigger than DPS1 has expected



Effective Cross Section and proton structure

If DPDs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int d^2z_{\perp} \tilde{T}(z_{\perp})^2 = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$$

As for the standard FF:

$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_{\perp}^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$

Verified in all model calculations:

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)

$$\text{DPD} = \text{GPD} \otimes \text{GPD}$$

Constituent quark models for:
proton

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

Pion

M.R. EPJC 80 (2020) 7, 678

W. Broniovski and E. R. Arriola PRD 101 (2020), 1, 014019

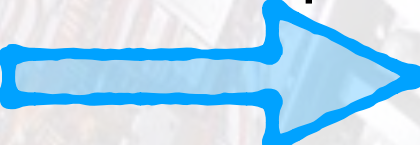
ρ

M.R. EPJC 80 (2020) 7, 678

DPS in γA collisions with light nuclei?

M.R. in progress

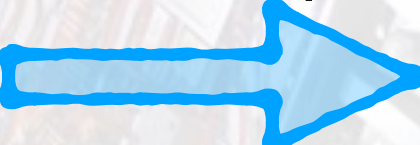
In p-Pb collisions there are some difficulties (personal view):

- 1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important  could be difficult to extract some information on the proton DPD
- 2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

DPS in γA collisions with light nuclei?

M.R. in progress

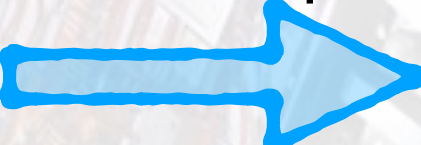
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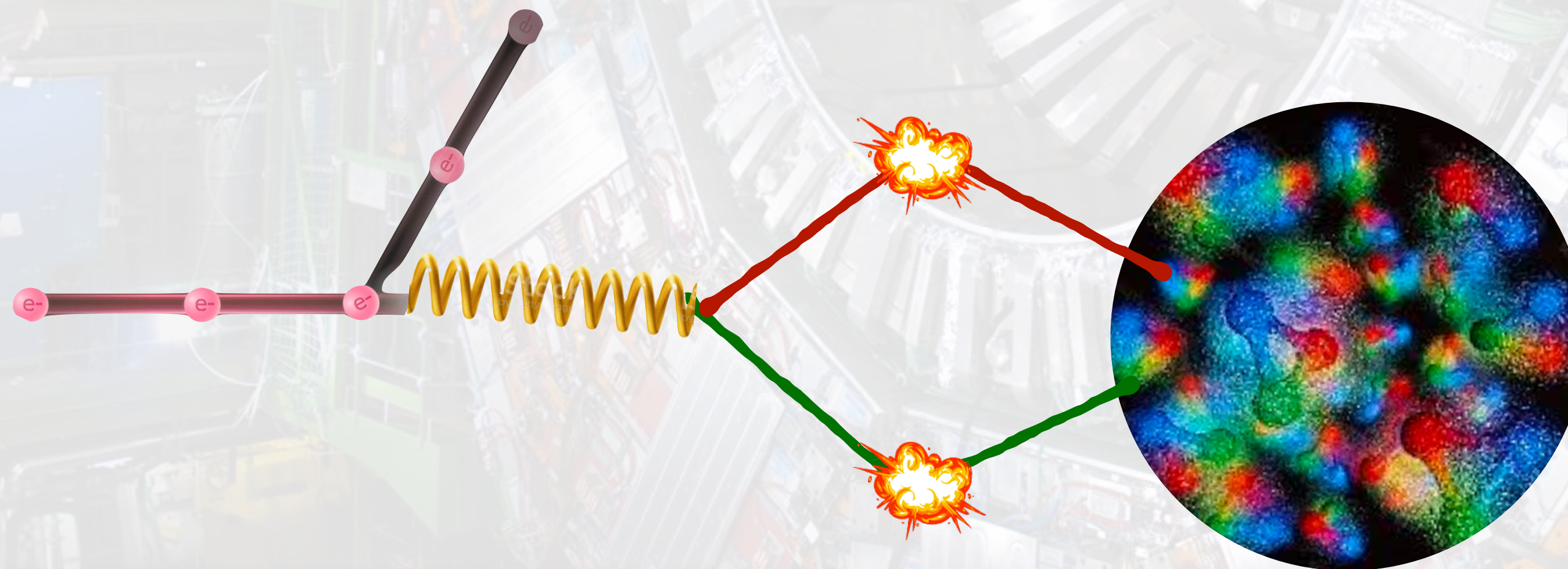
DPS in γA collisions with light nuclei?

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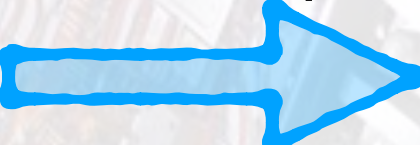
POSSIBLE SOLUTION?




DPS in γA collisions with light nuclei?

M.R. in progress

In p-Pb collisions there are some difficulties (personal view):

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POSSIBLE SOLUTION?

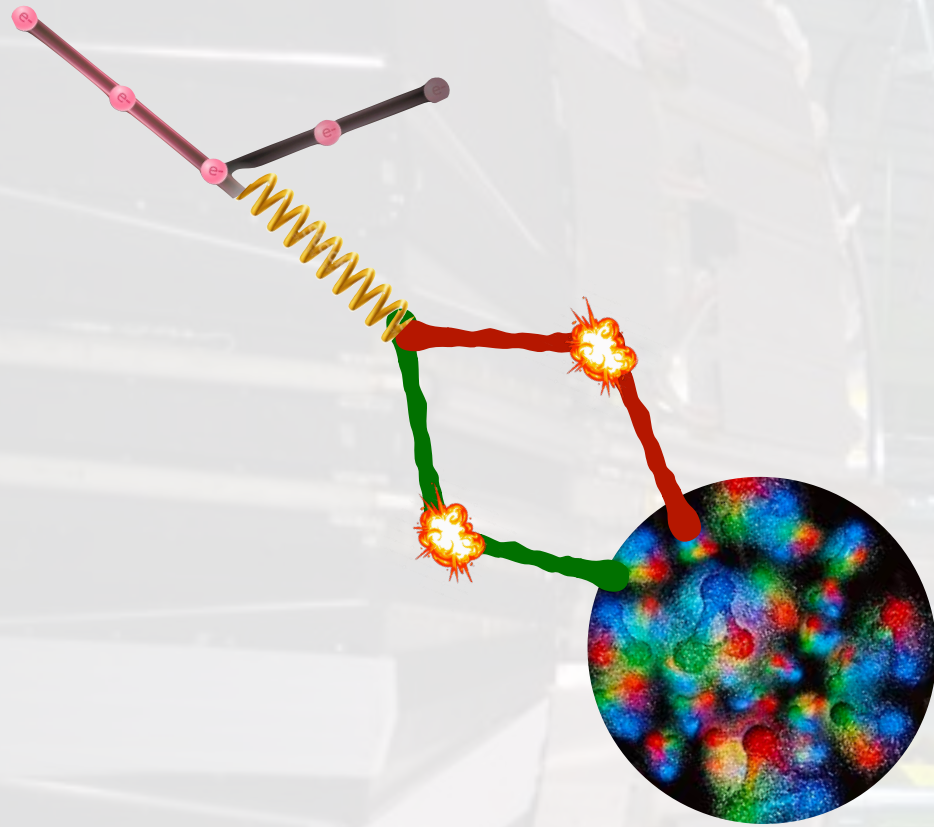
- 1) In γA the DPS2 will not contain any DPD of the proton  this mechanism can now be viewed as a background that can be evaluated if we properly treat the photon (as previously discussed) and the Nuclear geometry
- 2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)!

Could we access the DPD of bound nucleons? Double EMC effect?

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS1:



$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N\left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp\right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

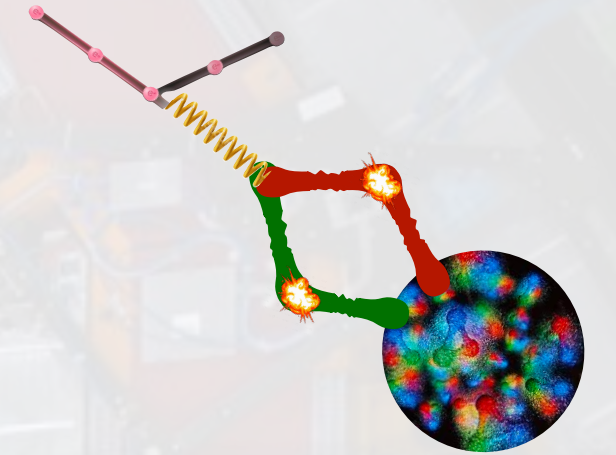
The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) in a fully relativistic and Poincaré covariant approach for:

- 1) H^2 in **E. Pace and G. Salmé, TNPI2000 (2001), arXiv:nucl-th/0106004**
- 2) He^3 in e.g. **A. Del Dotto et al, PRC 95, 014001 (2017), M.R. et al, PLB 839 (2023), 137810**
- 3) He^4 from **F. Fornetti, E. Pace, M. R., G. Salmé, S. Scopetta and M. Viviani, PLB submitted**

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS1:
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Let us check sum rules:

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1} N_{i_2} & \text{for } i_1 \neq i_2 \\ (N_{i_1} - 1) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

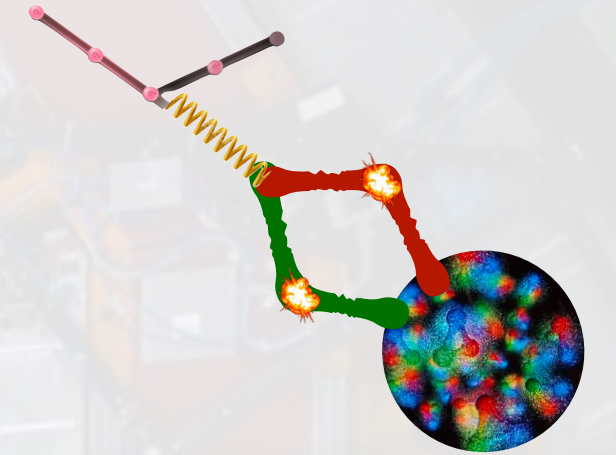
Gaunt's sum rules

**J. R. Gaunt and W. J. Stirling,
JHEP 03, 005 (2010)**

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS1:
$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$



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Gaunt's sum rules

**J. R. Gaunt and W. J. Stirling,
JHEP 03, 005 (2010)**

However for the nuclear case one needs also the DPS2

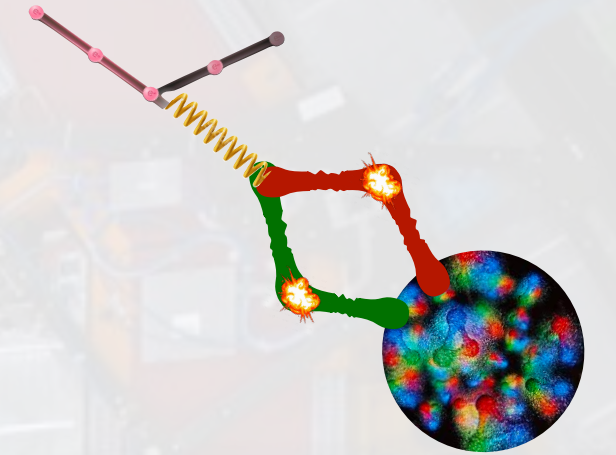


Thus we can introduce approximated partial sum rules (APSR)

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS1:
$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left(\frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$



$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1} N_{i_2} & \text{for } i_1 \neq i_2 \\ (N_{i_1} - 1) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

Gaunt's sum rules

**J. R. Gaunt and W. J. Stirling,
JHEP 03, 005 (2010)**

APSR: Since $f_n^A(\xi) = \int d^2 p_{t,N} \rho_A^N(\xi, p_{t,N})$ is peaked around $1/A$

$$\int_0^A dx_1 \int_0^{A-x_1} dx_2 \tilde{F}_{i_1 i_2}^{A,1}(x_1, x_2, 0) \sim \sum_{n=N,P} \int d\xi \xi f_n^A(\xi)$$

$$\begin{cases} (N_{i_1}^n - 1) N_{i_2}^n & i_1 = i_2 \\ N_{i_1}^n N_{i_2}^n & i_1 \neq i_2 \end{cases}$$

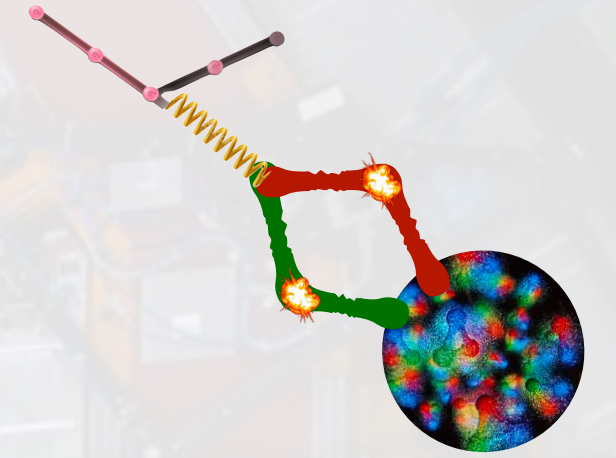
Gaunt's sum rules for the nucleon DPD: numbers of quarks with given flavor i in the nucleon n

Normalized to 1

DPS in γA collisions with light nuclei?

M.R. in progress

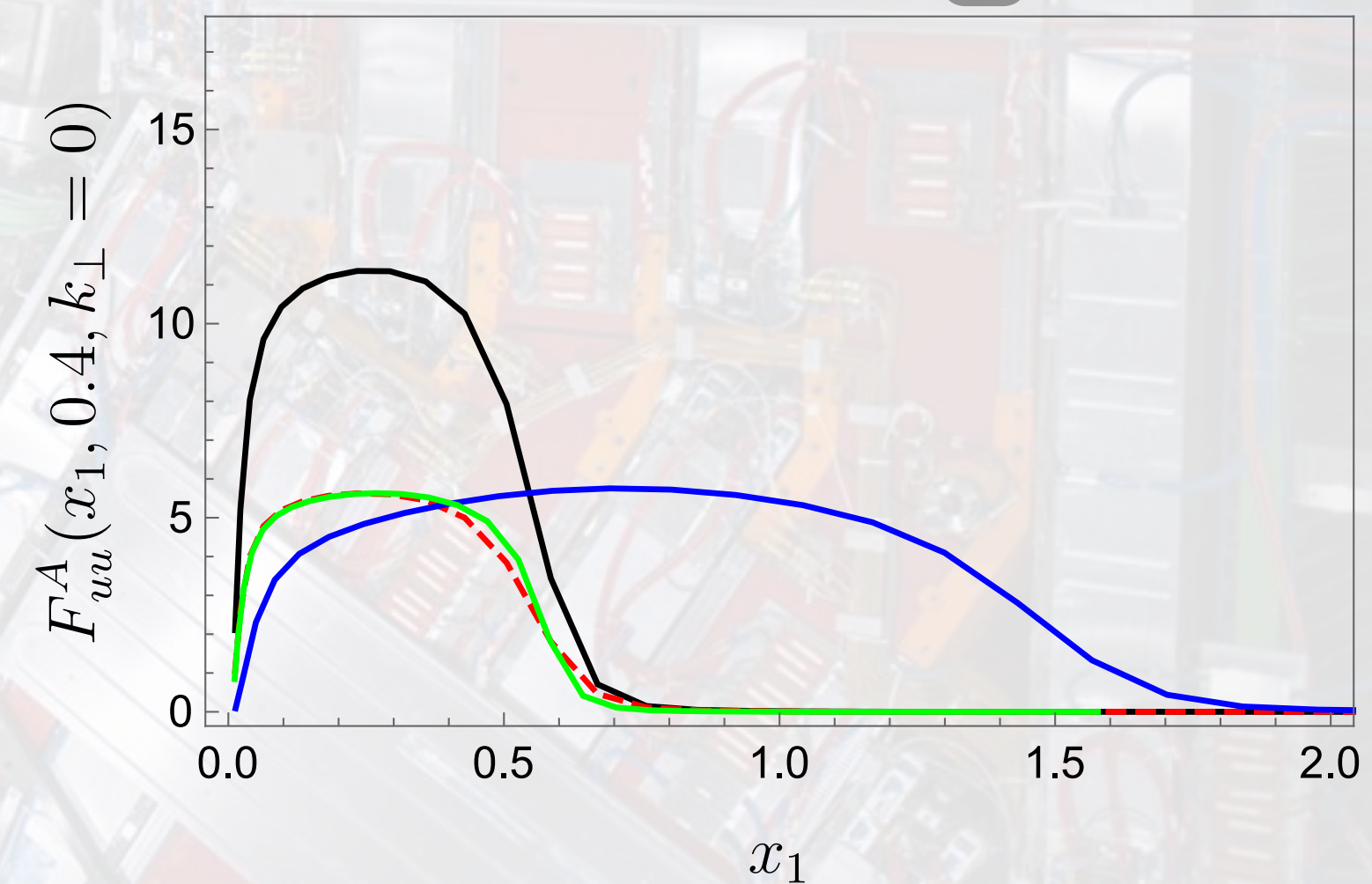
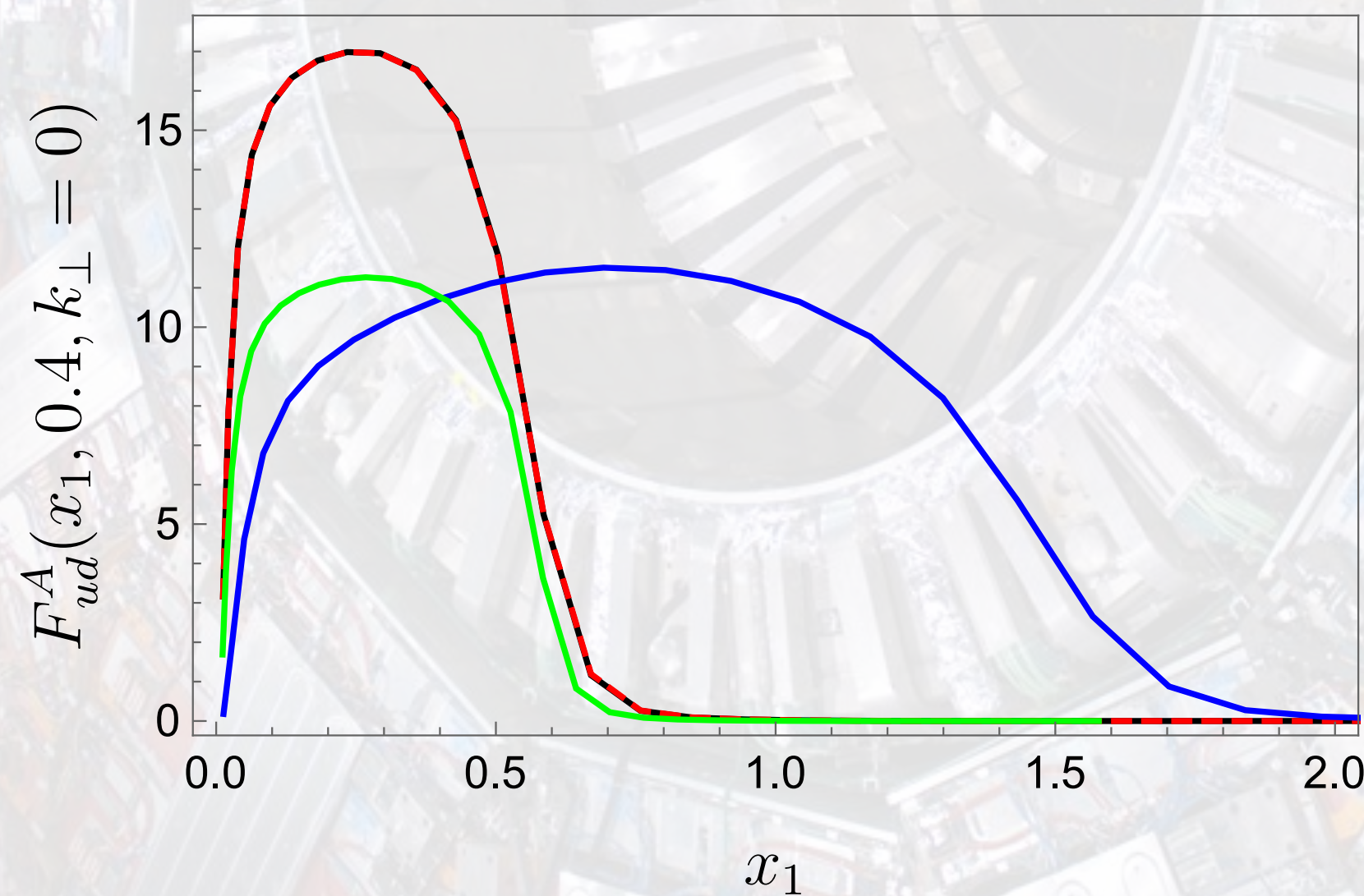
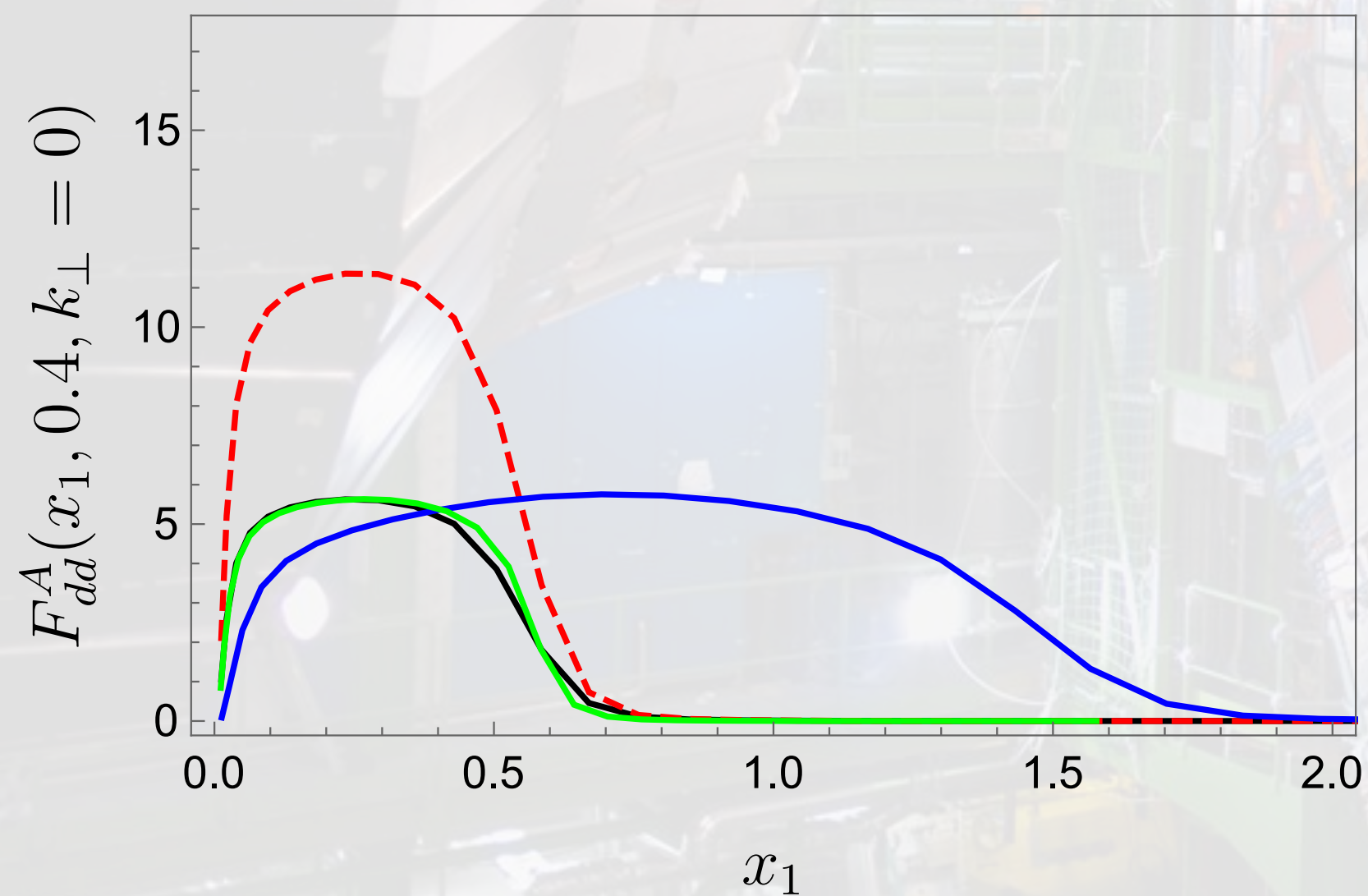
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- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)

$$0 < x_i < A$$

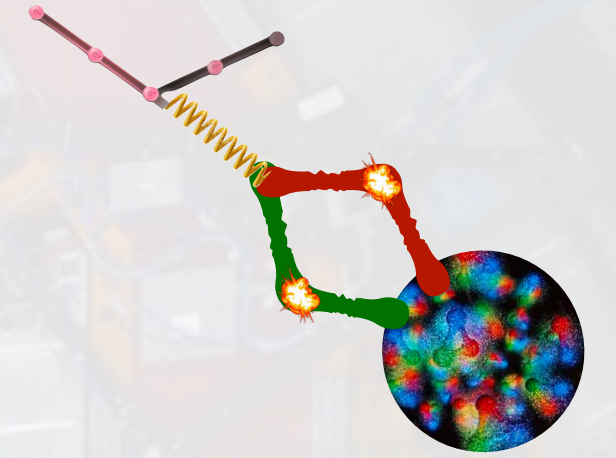
— 3He — 4He - - - 3H — 2H



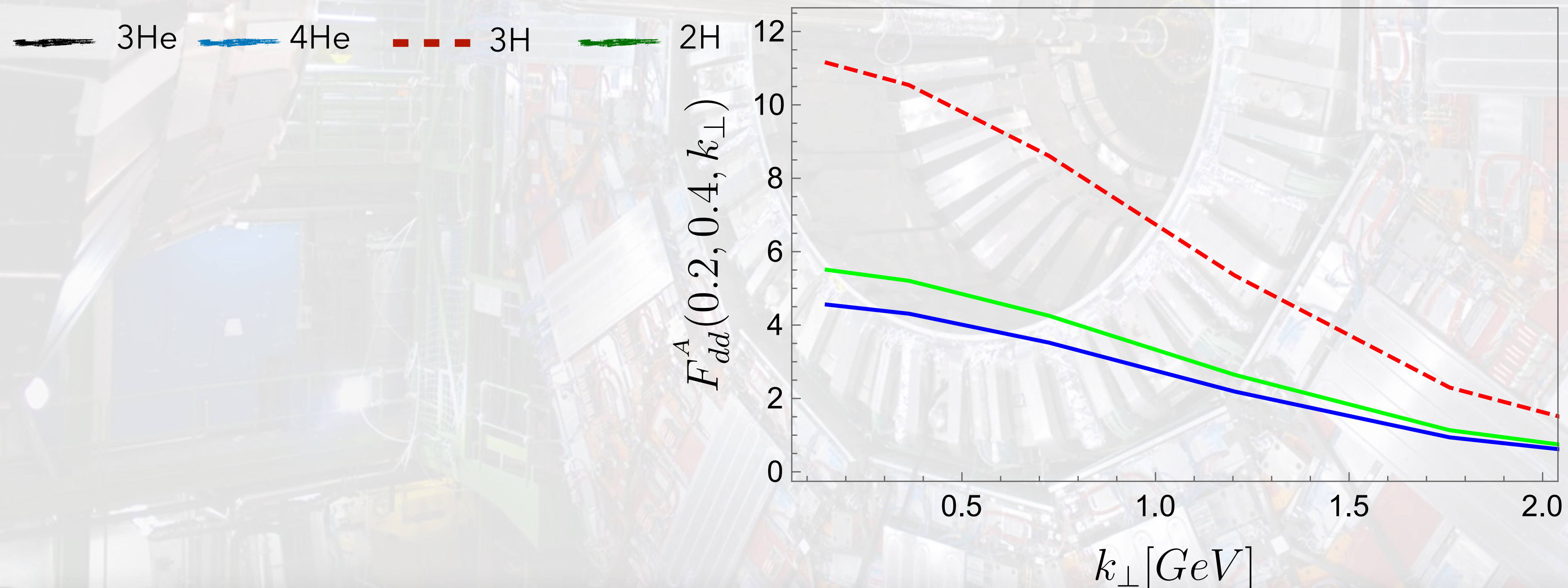
DPS in γA collisions with light nuclei?

M.R. in progress

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- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)



DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp)$$
$$\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right);$$

$$\stackrel{\xi_i \sim 1}{\sim} G_{a_1}^{N_1}(x_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2, |\vec{k}_\perp|)$$

$$\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp) \right]$$

Nuclear 2-body form factor $F_2(\vec{k}_\perp, -\vec{k}_\perp)$

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\begin{aligned} \tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) &\propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A\left(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp\right) \\ &\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right); \\ &\stackrel{\xi_i \sim 1}{\sim} G_{a_1}^{N_1}(x_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2, |\vec{k}_\perp|) \\ &\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A\left(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp\right) \right] \end{aligned}$$

Nuclear 2-body form factor $F_2(\vec{k}_\perp, -\vec{k}_\perp)$

Calculated $F_2(\vec{k}_2, \vec{k}_1)$ for ${}^3\text{He}$ and ${}^4\text{He}$ in:

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent J/ψ electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp)$$

$$\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right)$$

$$\stackrel{\xi_i \sim 1}{\sim} G_{a_1}^{N_1}(x_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2, |\vec{k}_\perp|)$$

$$\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp) \right]$$

**WE HAVE A LINK BETWEEN
2 DIFFERENT PROCESSES!**

Nuclear 2

$\vec{k}_\perp, -\vec{k}_\perp$

Calculated $F_2(\vec{k}_2, \vec{k}_1)$ for ${}^3\text{He}$ and ${}^4\text{He}$

V. Guzey, M.R., S. Scopetta, M. Strikman and ... *Electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing* ... JHEP 129 (2022) 24, 242503

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS2:

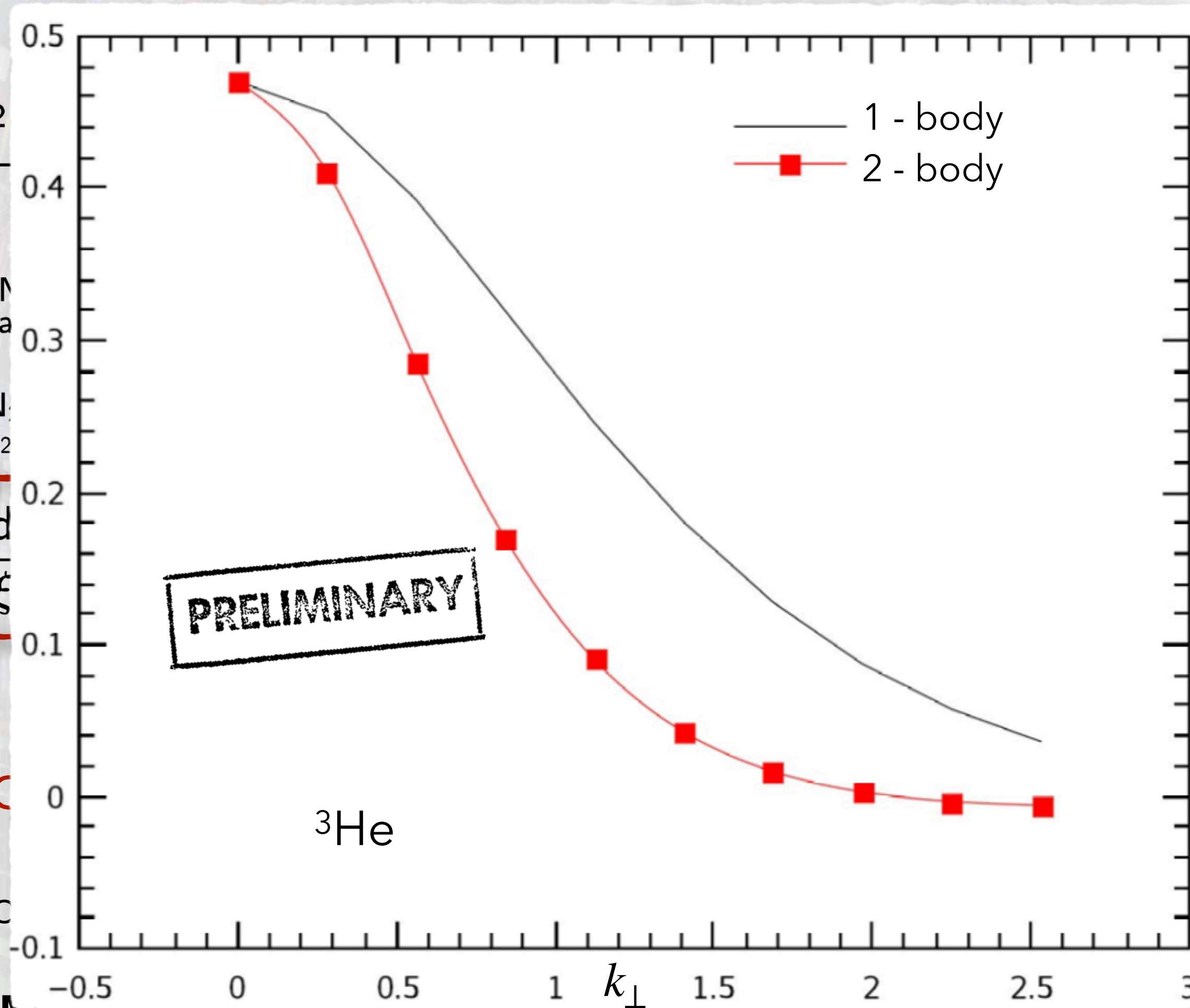
$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \xi_i}{\xi_i} \times G_{a_1}^{N_1} \left(\frac{x_1}{\xi_1}, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left(\frac{x_2}{\xi_2}, |\vec{k}_\perp| \right)$$

$$\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \xi_i}{\xi_i} \times G_{a_1}^{N_1} \left(x_1, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left(x_2, |\vec{k}_\perp| \right) \right]$$

Nuc

Calculated $F_2(\vec{k}_2, \vec{k}_1)$

fc



^3He

$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

(\vec{k}_\perp)

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent J/ψ electroproduction on ^4He and ^3He at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503

DPS in γA collisions with light nuclei?

M.R. in progress

For example in DPS2:

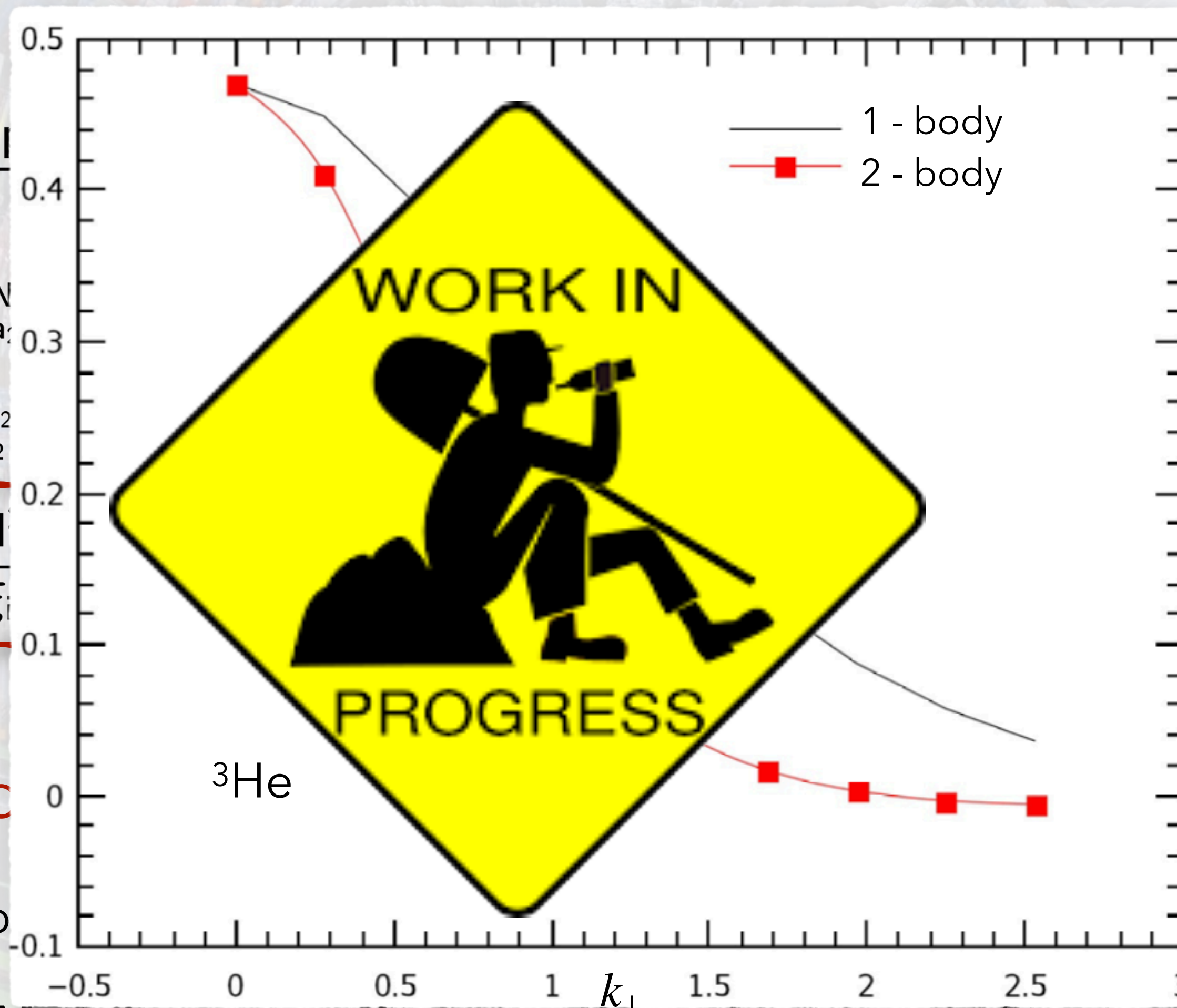
$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \perp_i}{\xi_i} \times G_{a_1}^{N_1} \left(\frac{x_1}{\xi_1}, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left(\frac{x_2}{\xi_2}, |\vec{k}_\perp| \right)$$

$$\times \left[\int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \perp_i}{\xi_i} G_{a_1}^{N_1} \left(x_1, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left(x_2, |\vec{k}_\perp| \right) \right]$$

Nuc

Calculated $F_2(\vec{k}_2, \vec{k}_1)$

for



$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

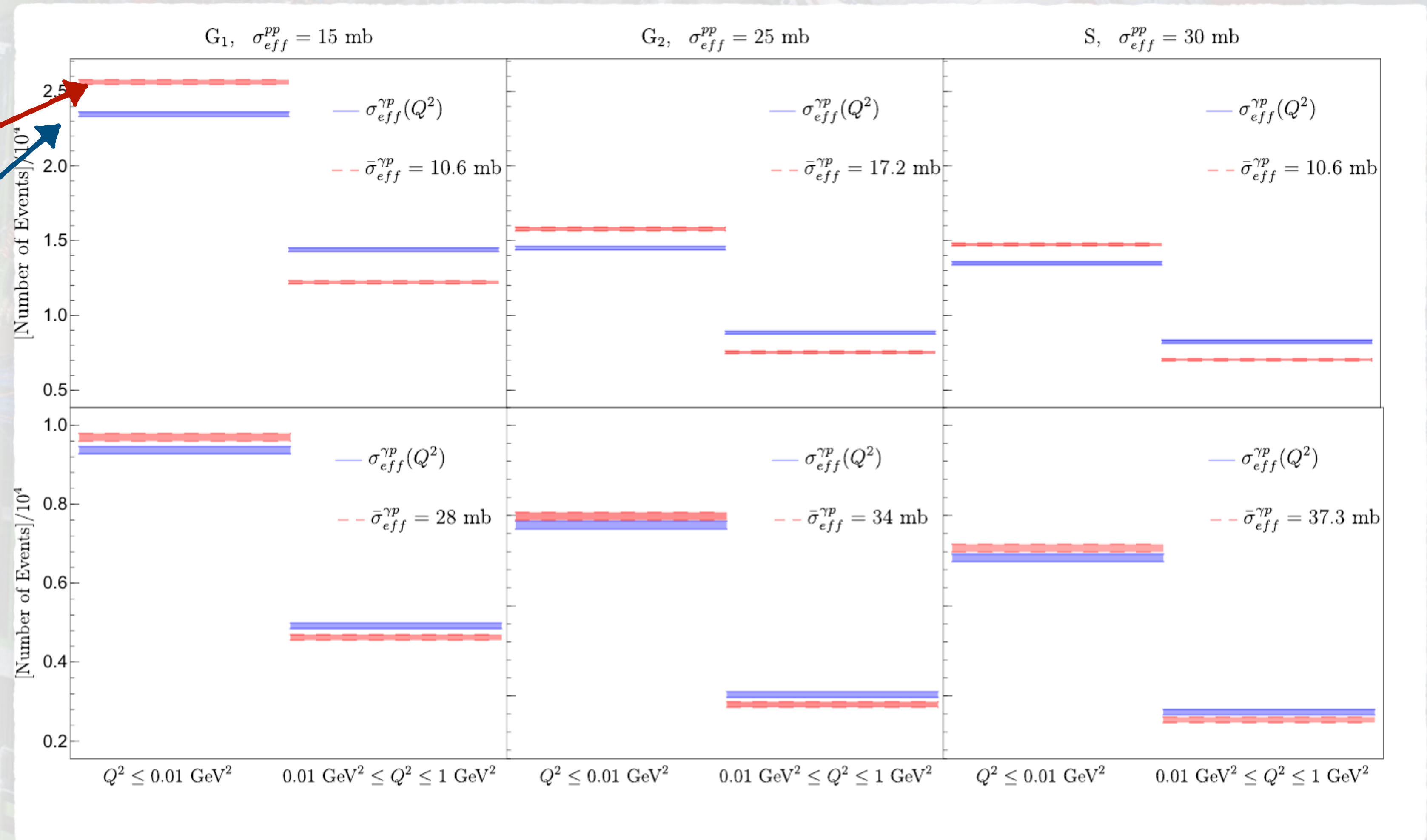
$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

(\vec{k}_\perp)

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent J/ψ electroproduction on He_4 and He_3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503

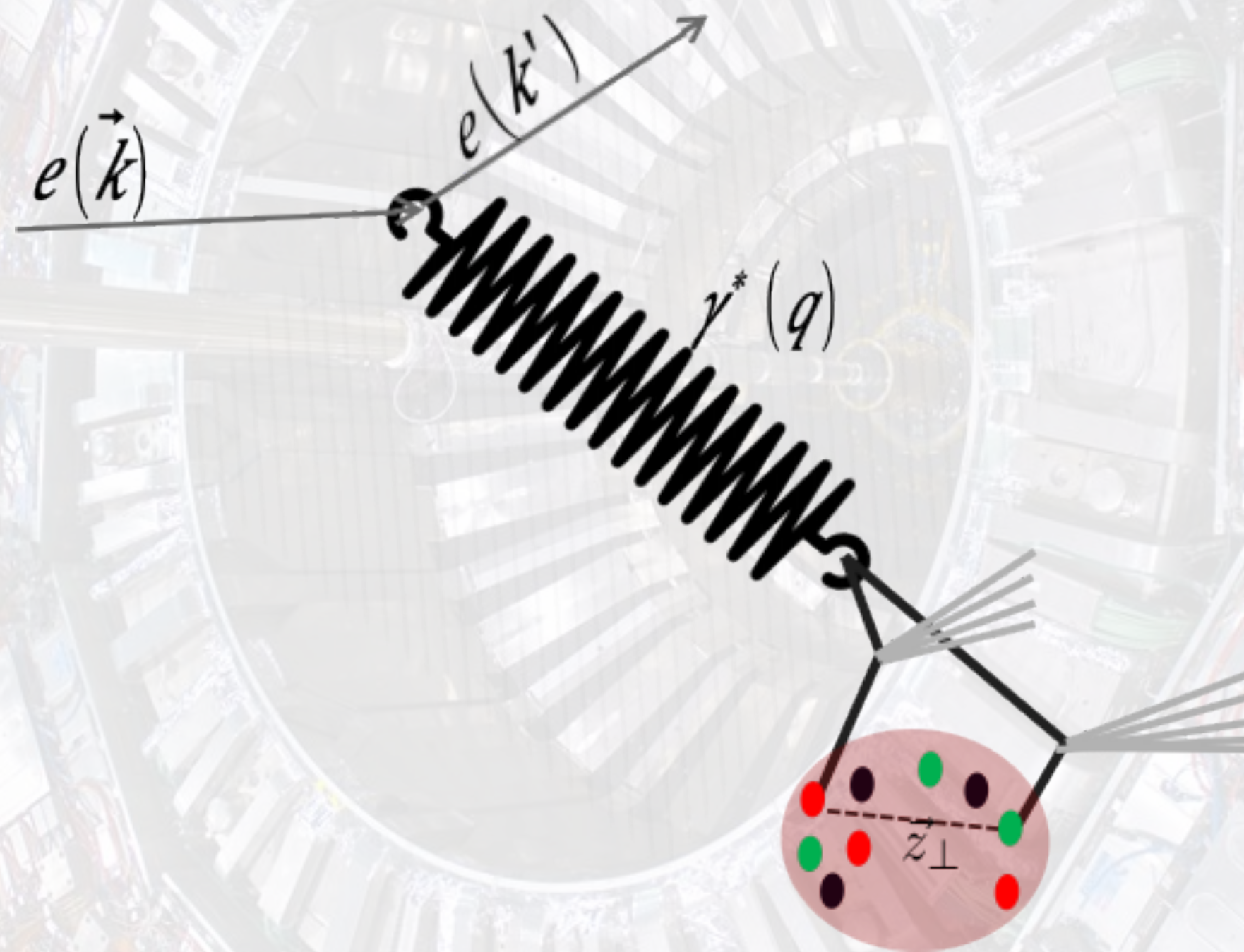
Backup - Luminosity II

With an integrated luminosity of 200 pb⁻¹ we can separate:



DPS in $\gamma - p$ interactions

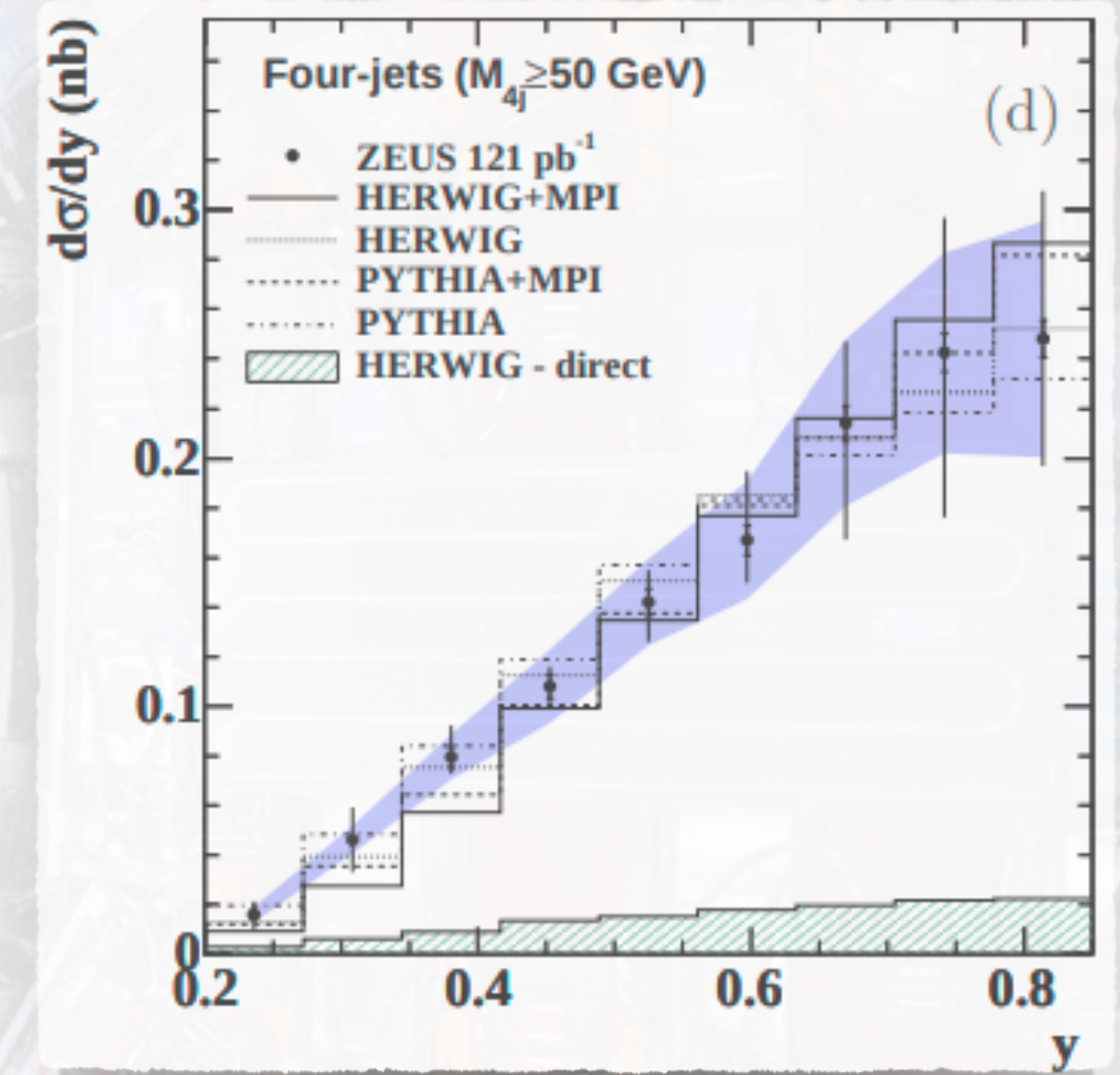
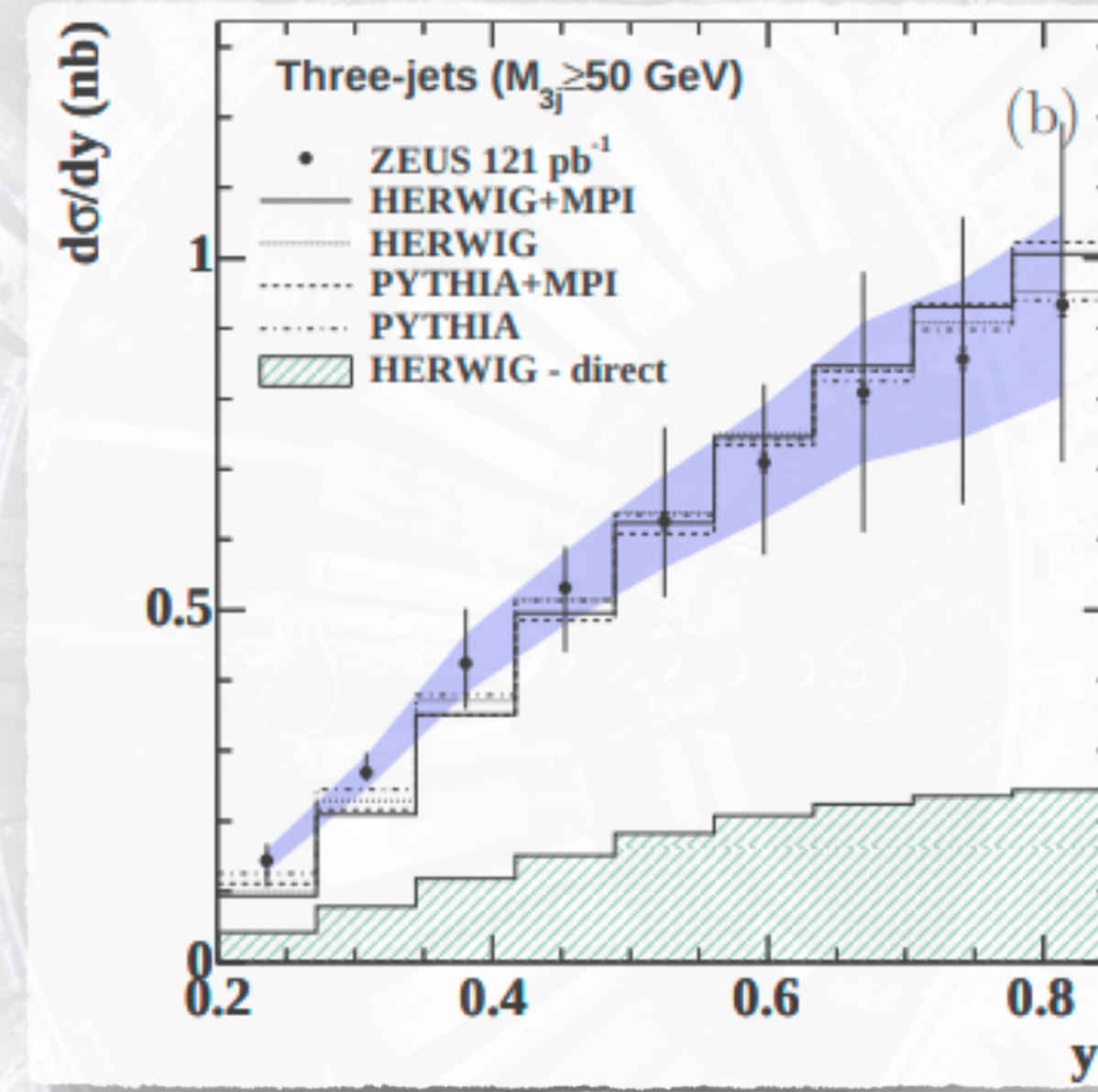
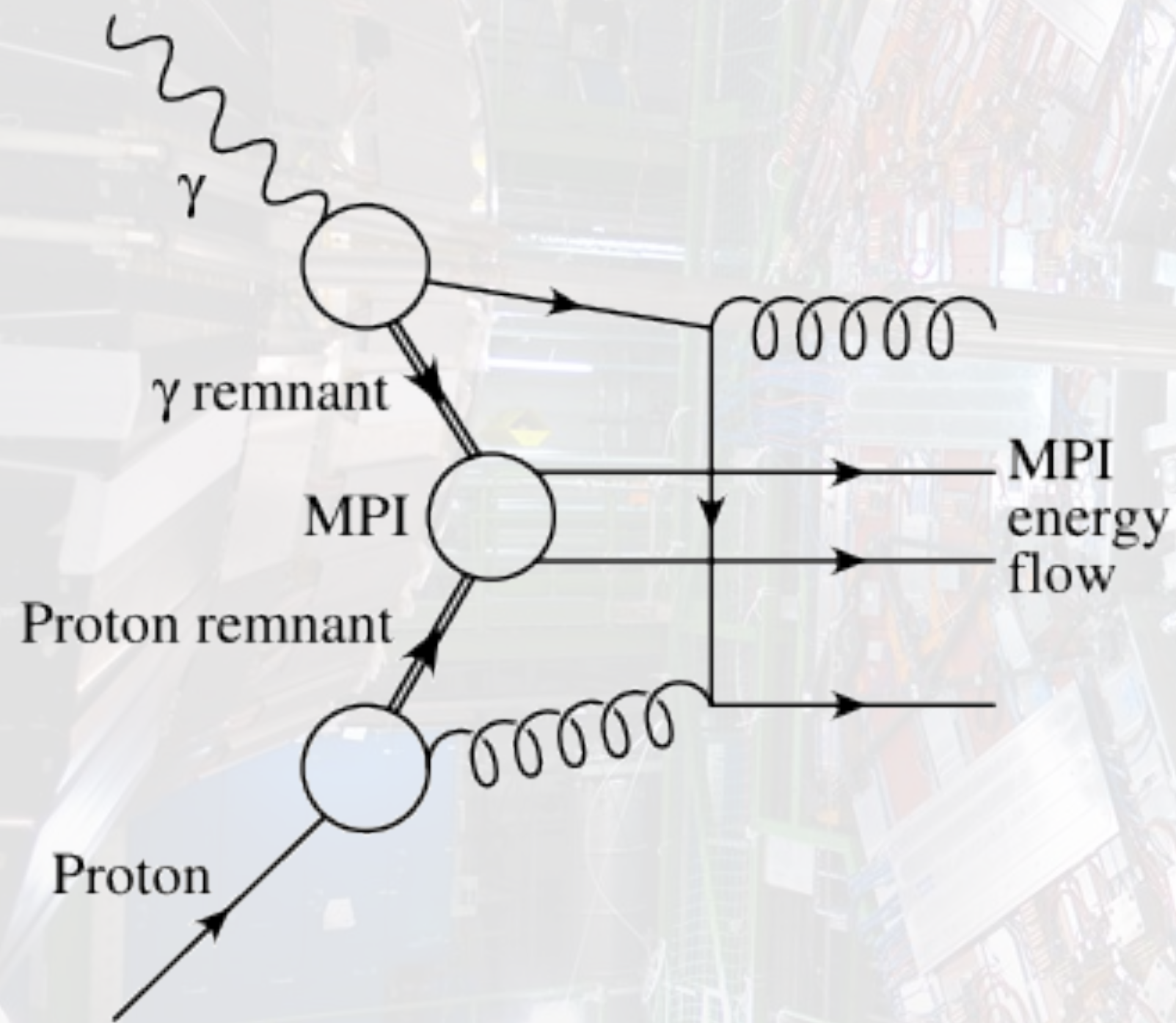
We consider the possibility offered by a DPS process involving a photon FLUCTUATING in a quark-antiquark pair interacting with a proton:



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

DPS in $\gamma - p$ interactions

Already at HERA the importance of MPI for the **3,4 jets photo-production** has been addressed:



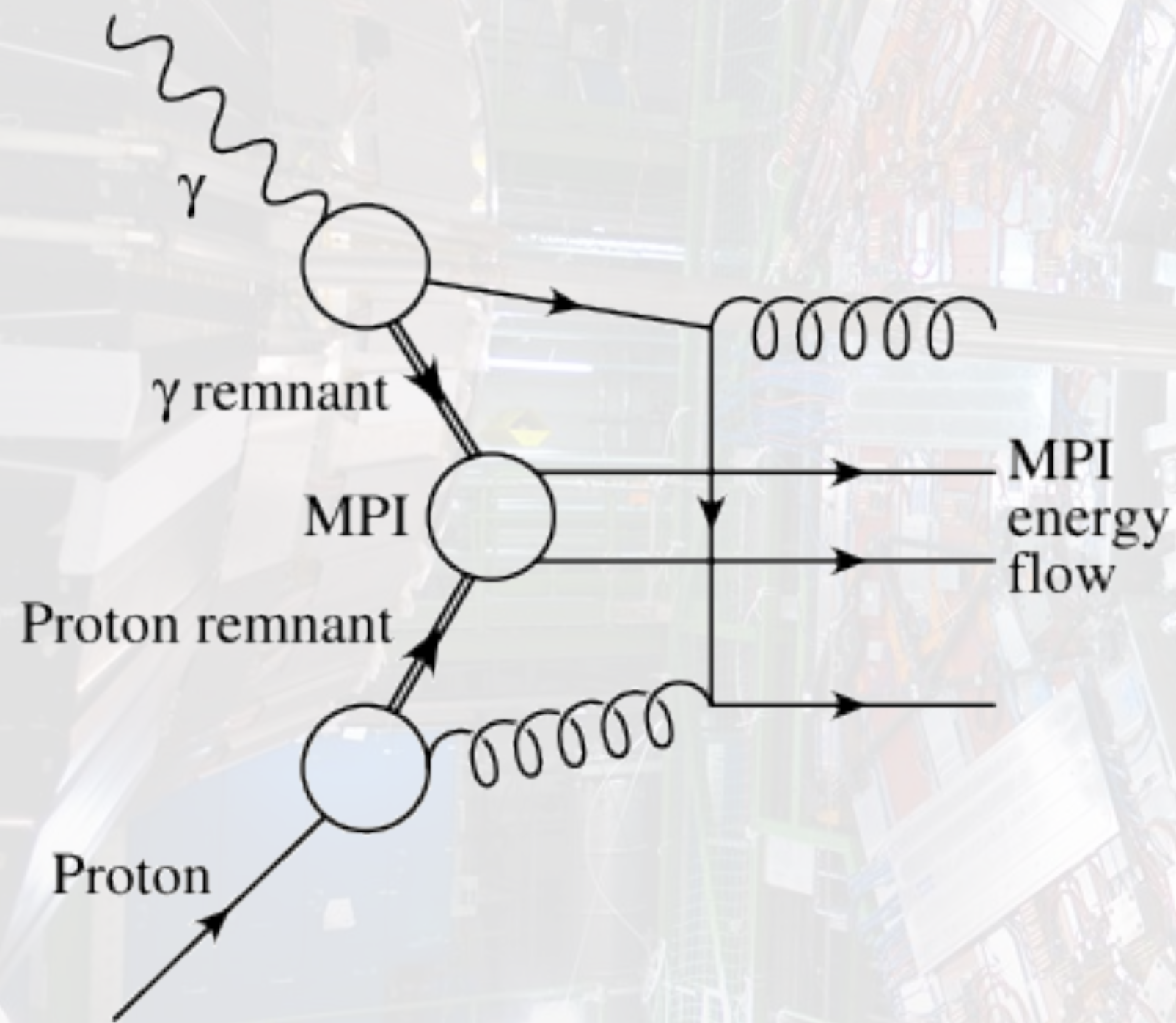
J. R. Forshaw et al, Z phys. C 72, 637

S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)

DPS in $\gamma - p$ interactions

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (**S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**)

For this first investigation, we make use of the **POCKET FORMULA:**



Flux Factor
P. Nason et al, PLB319

$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 f_{\gamma/e}(y, Q^2) \times \sigma_{\text{eff}}^{\gamma P}(Q^2)$$

$$\times \left. \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \right\} \text{SPS}$$

$$\times \left. \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \right\} \text{SPS}$$

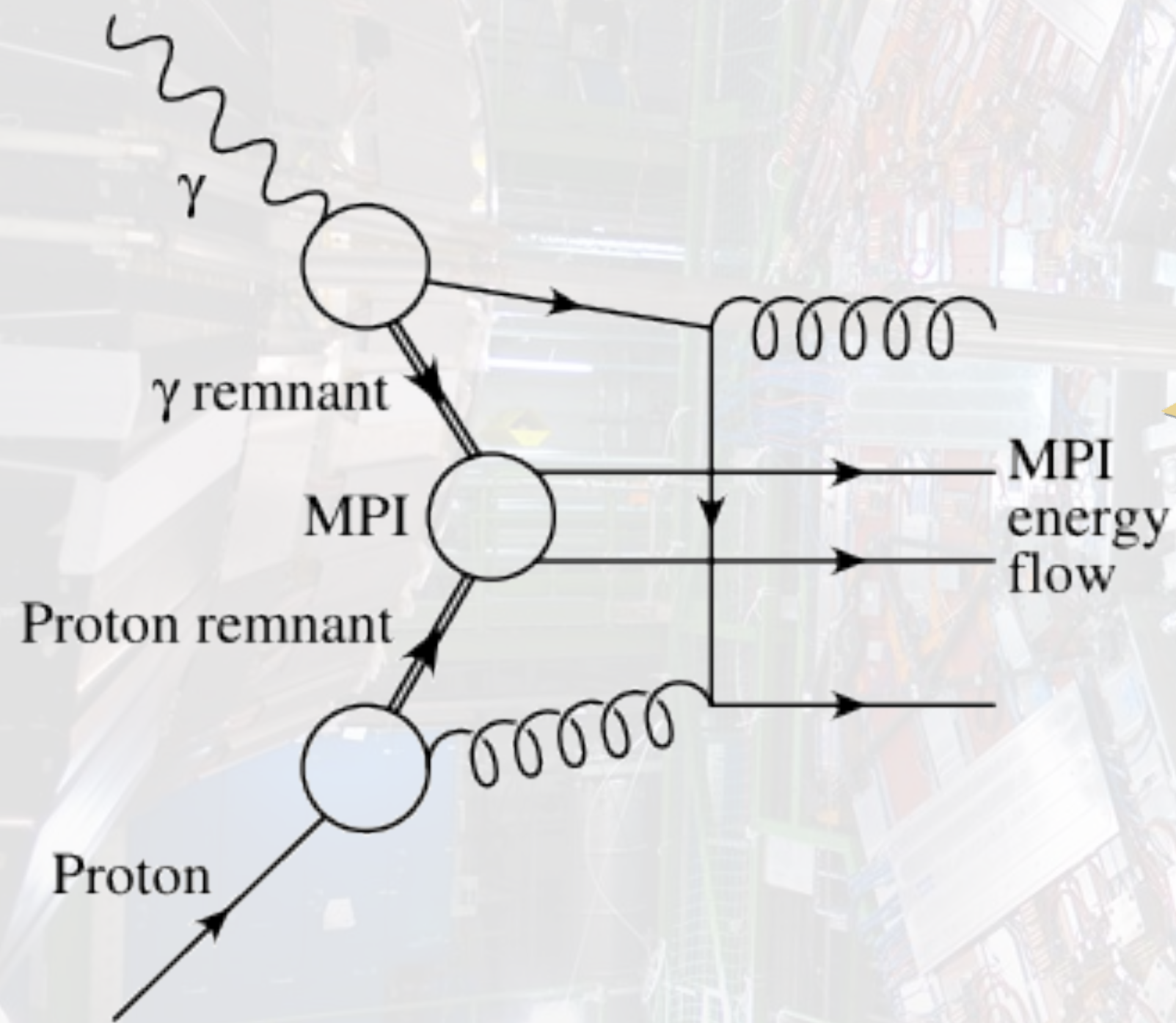
Proton PDF
(J. Pumplin et al. JHEP 07, 012 (2002))

Photon PDF
(M. Gluck et al. PRD46, 1973 (1992))

DPS in $\gamma - p$ interactions

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))

For this first investigation, we make use of the POCKET FORMULA:



Flux Factor
P. Nason et al, PLB319

$$f_{\gamma/e}(y, Q^2) \times \sigma_{\text{eff}}^{\gamma p}(Q^2)$$

The main quantity we have to evaluate is:
 $\sigma_{\text{eff}}^{\gamma p}(Q^2)$

$$\left. \begin{aligned} & (x_{\gamma b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \\ & \gamma(x_{\gamma d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \end{aligned} \right\} \begin{array}{l} \text{SPS} \\ * \\ \text{SPS} \end{array}$$

Photon PDF
(M. Gluck et al. PRD46, 1973 (1992))

(J. Pumplin et al.)

The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

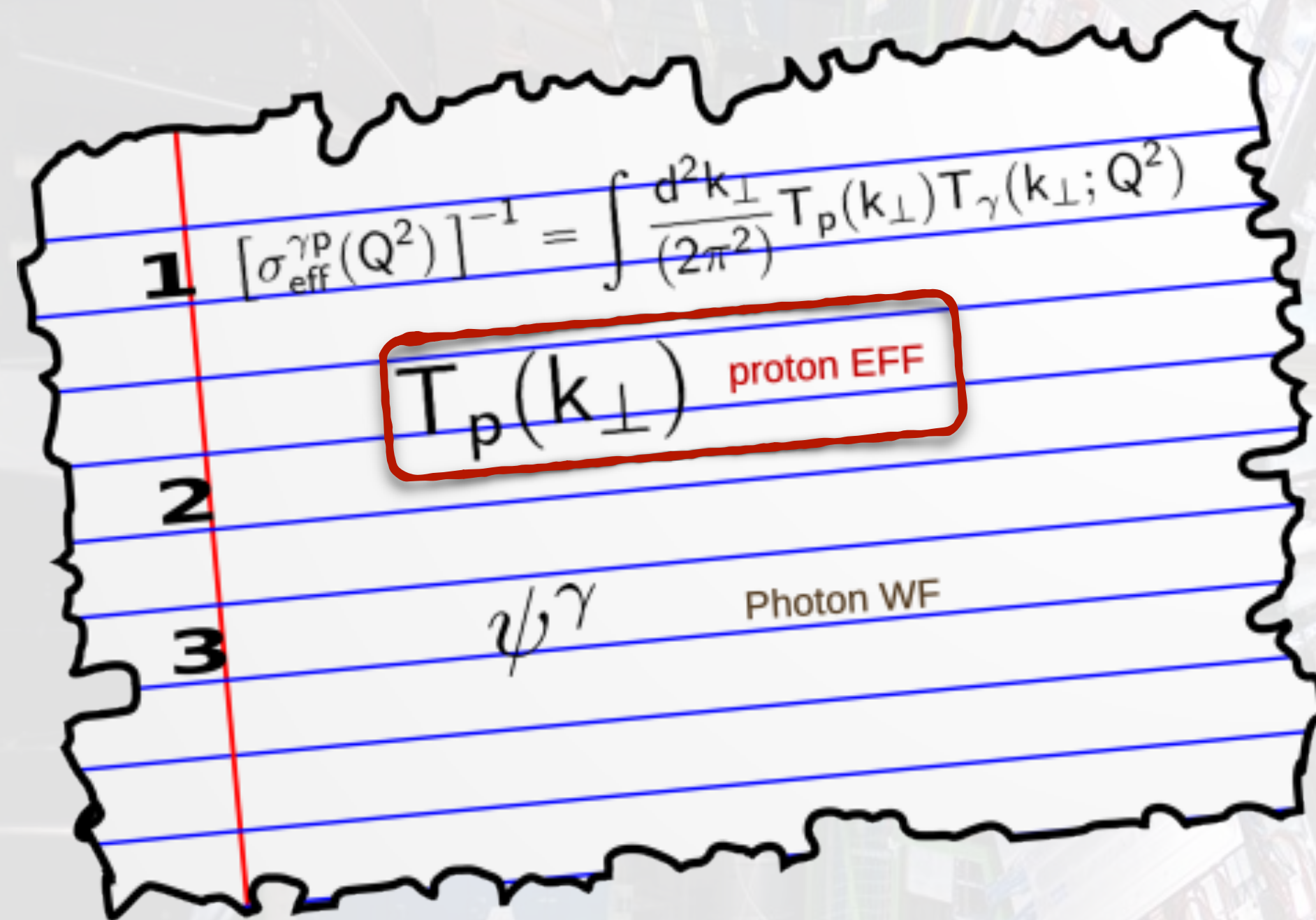
$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} \overset{\text{Proton EFF}}{\boxed{T_p(k_{\perp})}} \overset{\text{Photon EFF}}{\boxed{T_{\gamma}(k_{\perp}; Q^2)}}$$

The full DPS cross section depends on the amplitude of the splitting photon in a $q - \bar{q}$ pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions

The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The main ingredients of the calculations:



1 $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

2 $T_p(k_{\perp})$ proton EFF

3 ψ/γ Photon WF

For the proton EFF use has been made of three choices:

1) G1 $e^{-\alpha_1 k_{\perp}^2}$, $\alpha_1 = 1.53 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 15 \text{ mb}$

2) G2 $e^{-\alpha_2 k_{\perp}^2}$, $\alpha_2 = 2.56 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 25 \text{ mb}$

3) S $\left(1 + \frac{k_{\perp}^2}{m_g^2}\right)^{-4}$, $m_g^2 = 1.1 \text{ GeV}^2 \implies \sigma_{\text{eff}}^{\text{pp}} = 30 \text{ mb}$

The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The main ingredients of the calculations:

1 $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

2 $T_p(k_{\perp})$ proton EFF

3 ψ_{γ} Photon WF

For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q,\bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k) \gamma \cdot \varepsilon^{\lambda} v_{\bar{q}}(q - k)}{\sqrt{x(1-x)} \left[Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)} \right]}$$

2) Non-Perturbative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

$$\psi_A^{\gamma}(x, k_{\perp 1}; Q^2) = \frac{6(1 + Q^2/m_{\rho}^2)}{m_{\rho}^2 \left(1 + 4 \frac{k_{\perp 1}^2 + Q^2 x(1-x)}{m_{\rho}^2} \right)^{5/2}}$$

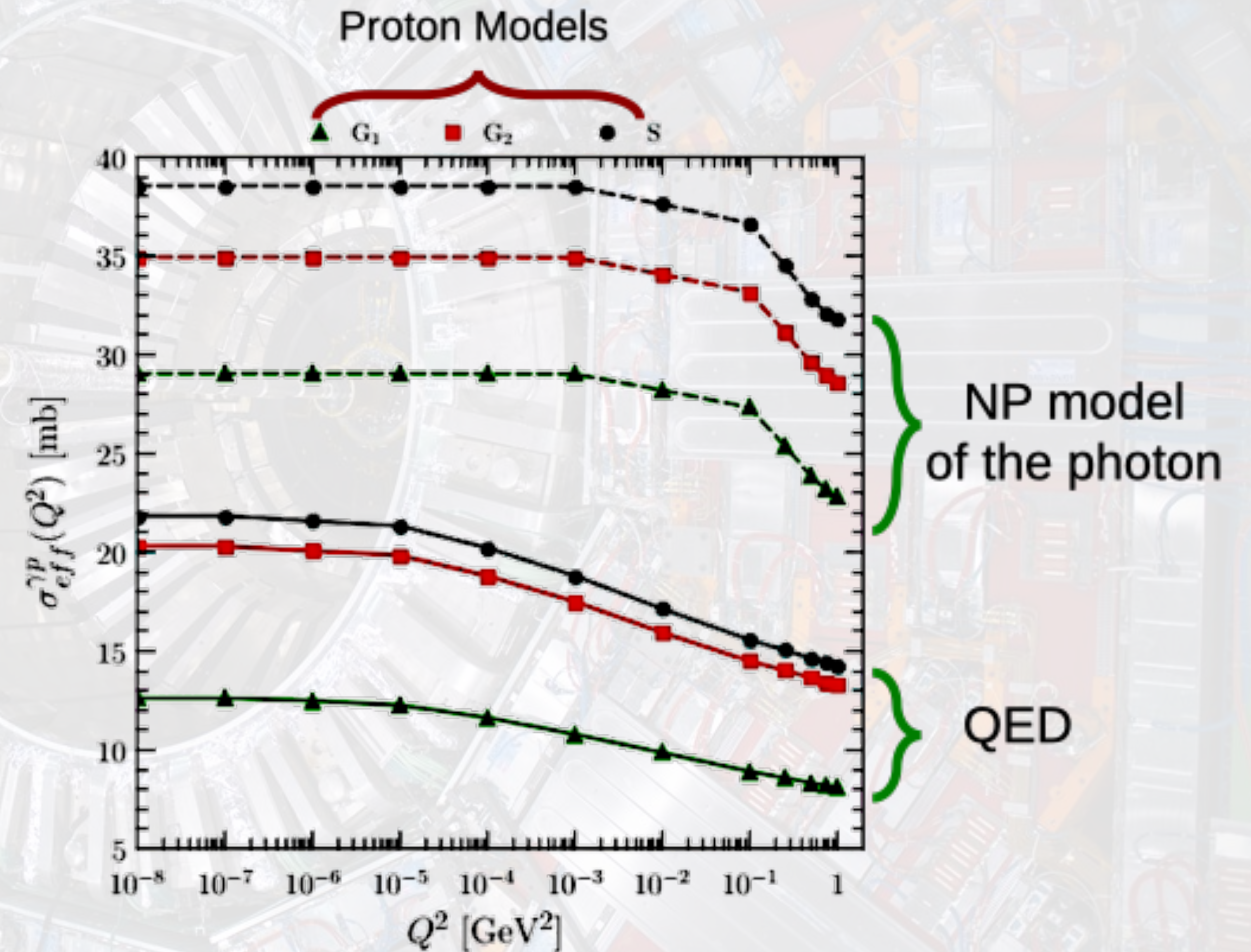
The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$\mathbf{1} \quad [\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

$\mathbf{2} \quad T_p(k_{\perp})$ proton EFF

$\mathbf{3} \quad \psi/\gamma$ Photon WF



The $\gamma - p$ effective cross-section

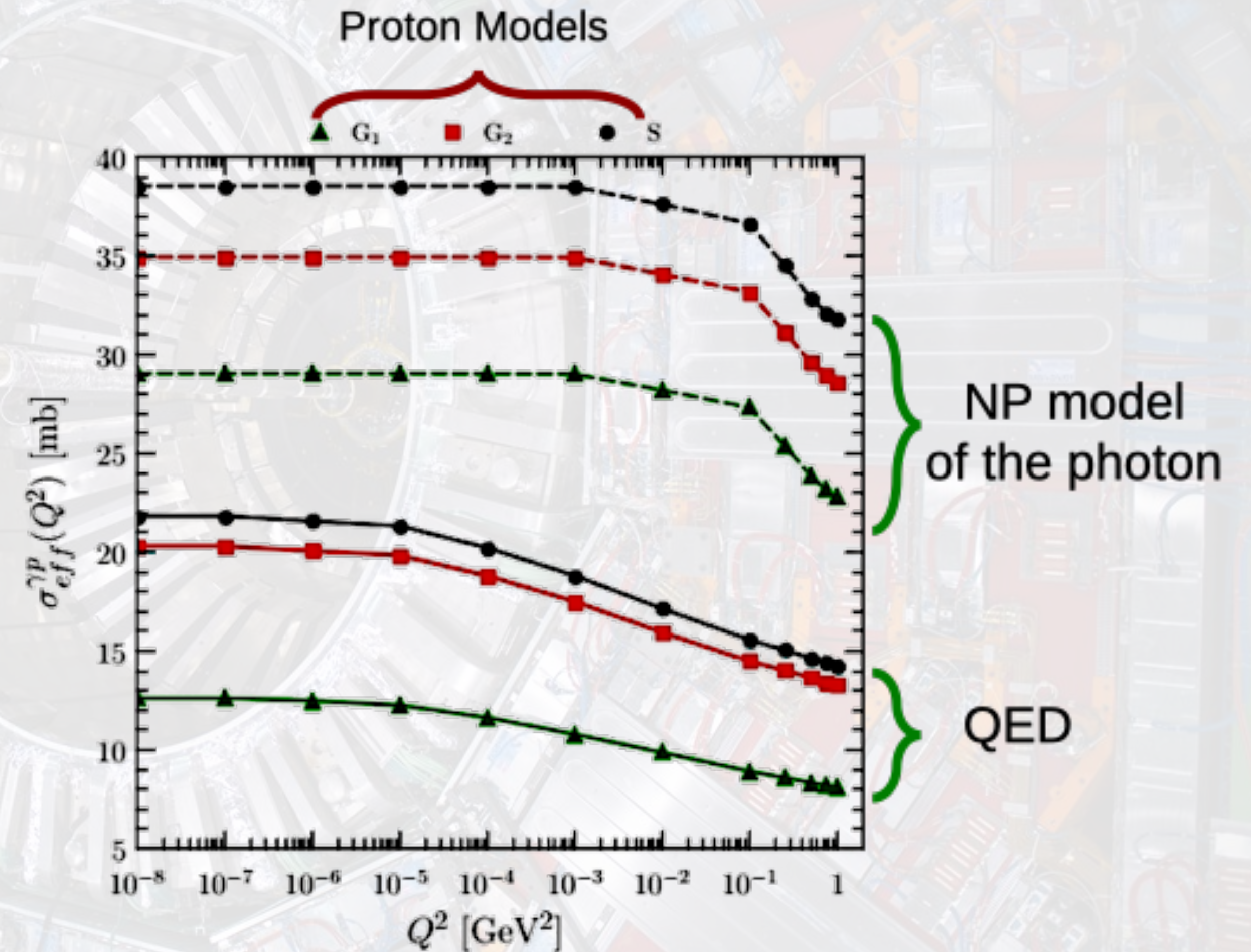
M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$1 \quad [\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

2 $T_p(k_{\perp})$ proton EFF

3 ψ/γ Photon WF

The effective cross-section depends on the photon virtuality! (NEW)



The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 \frac{f_{\gamma/e}(y, Q^2)}{\sigma_{\text{eff}}^{\gamma p}(Q^2)} \times$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b})$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d})$$

KINEMATICS:

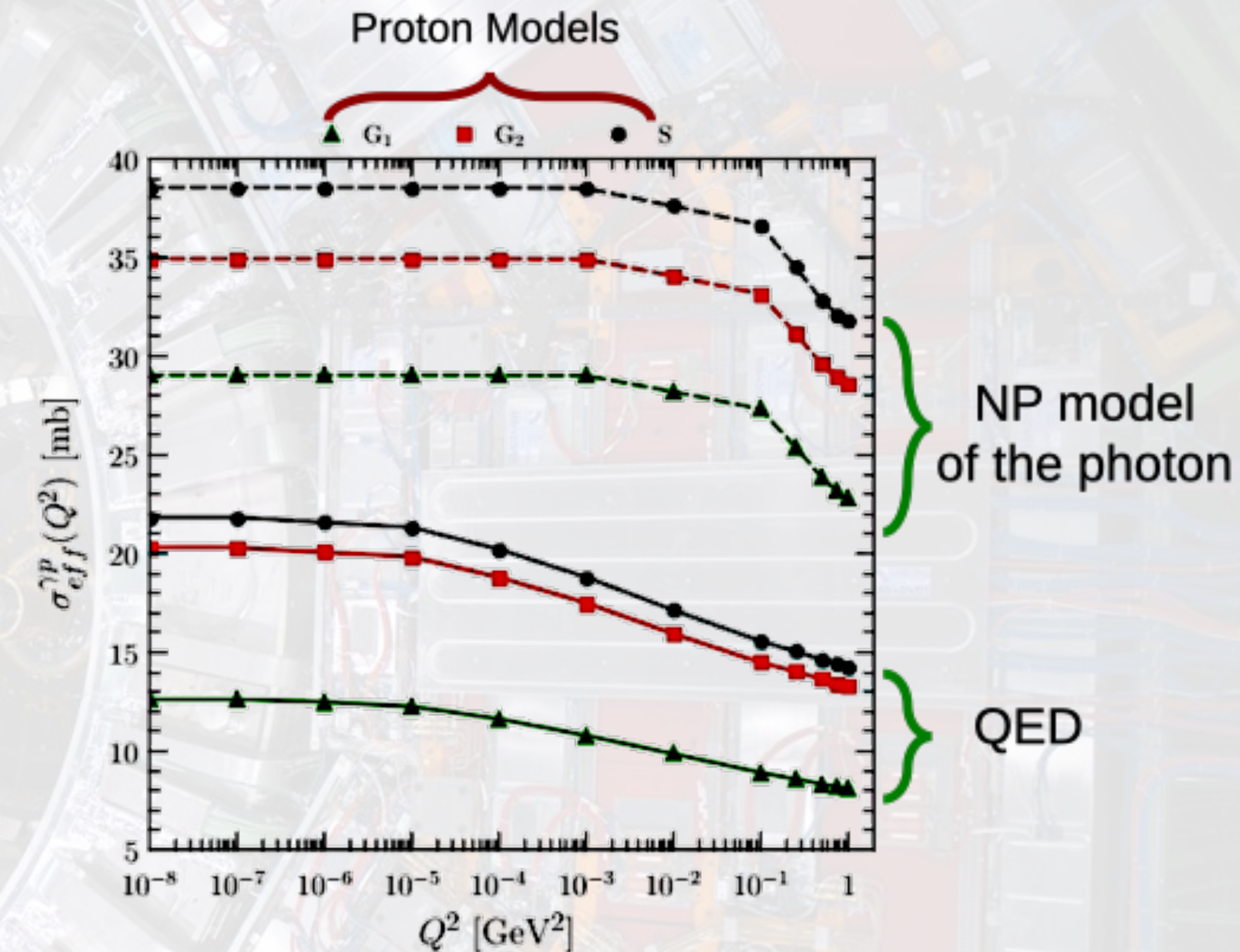
$$E_T^{\text{jet}} > 6 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb
S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)



The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{DPS}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dx_{pa} dx_{\gamma b} f_{a/p} \times \int dx_{pc} dx_{\gamma d} f_{c/p}$$

KINEMATICS:

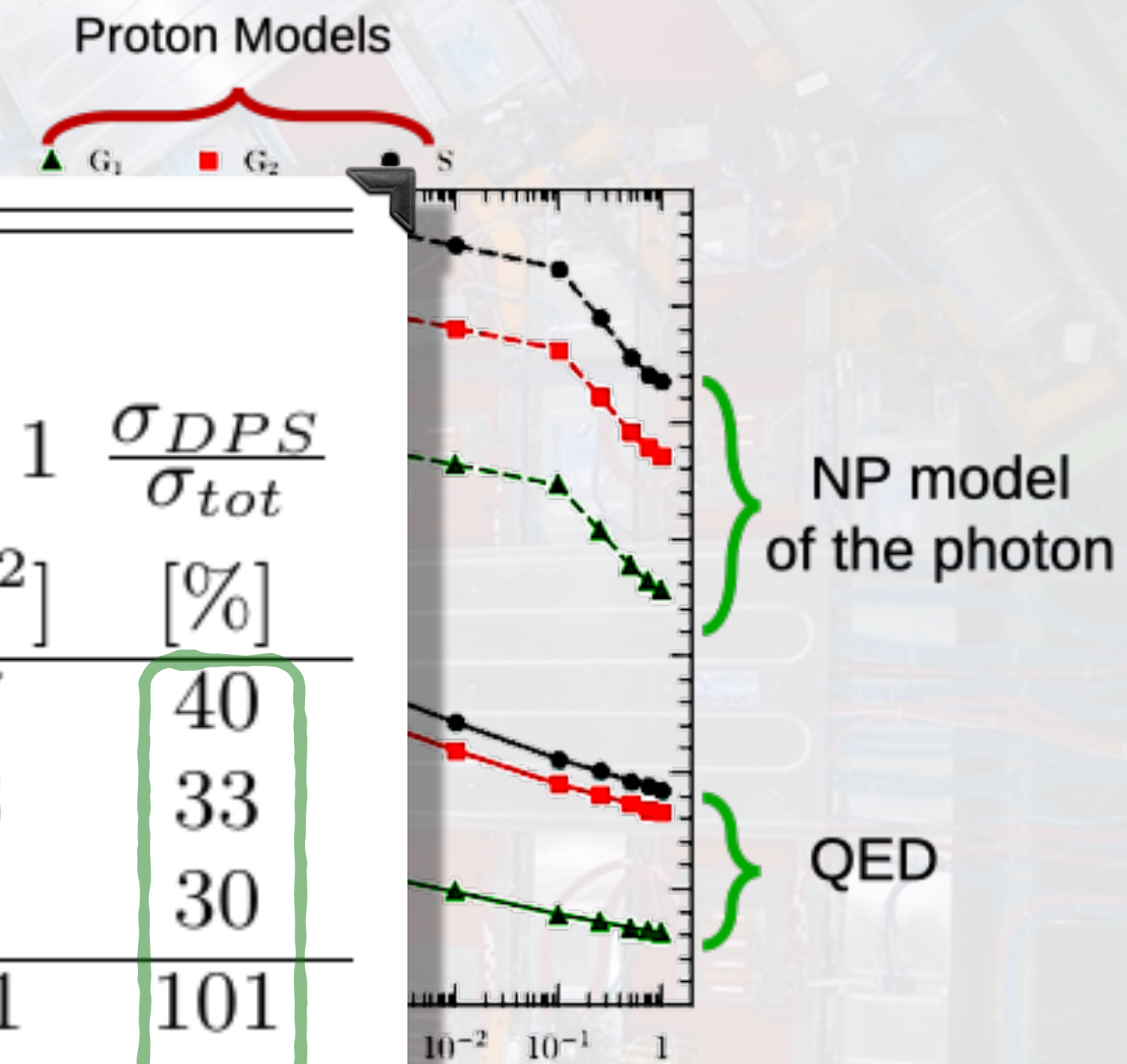
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		σ_{DPS} [pb]			
		$Q^2 \leq 10^{-2}$	$10^{-2} \leq Q^2 \leq 1$	$Q^2 \leq 1$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
		[GeV ²]	[GeV ²]	[GeV ²]	[%]
Proton	G ₁	35.1	18.6	53.7	40
	G ₂	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
Photon	G ₁	87.8	54.3	142.1	101
	G ₂	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60



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S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{DPS}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dx_{pa} dx_{\gamma b} f_{a/p}(x_{pa}, Q^2) \times \int dx_{pc} dx_{\gamma d} f_{c/p}(x_{pc}, Q^2) \times \int d^3p_a d^3p_b d^3p_c d^3p_d \delta^4(p_a + p_b + p_c + p_d - p_{jet1} - p_{jet2} - p_{jet3} - p_{jet4}) \times \mathcal{M}_{ab,cd}^2$$

KINEMATICS:

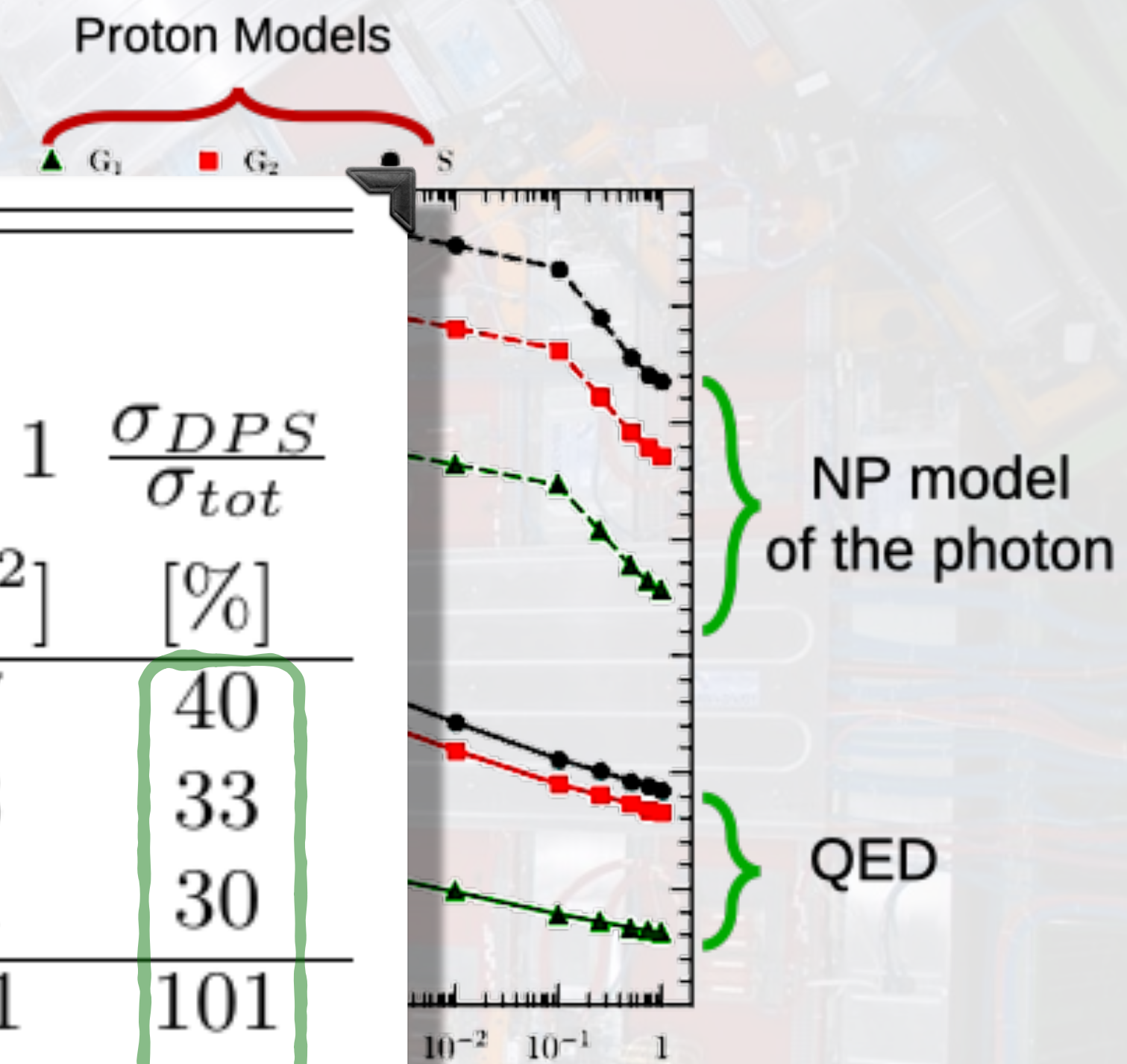
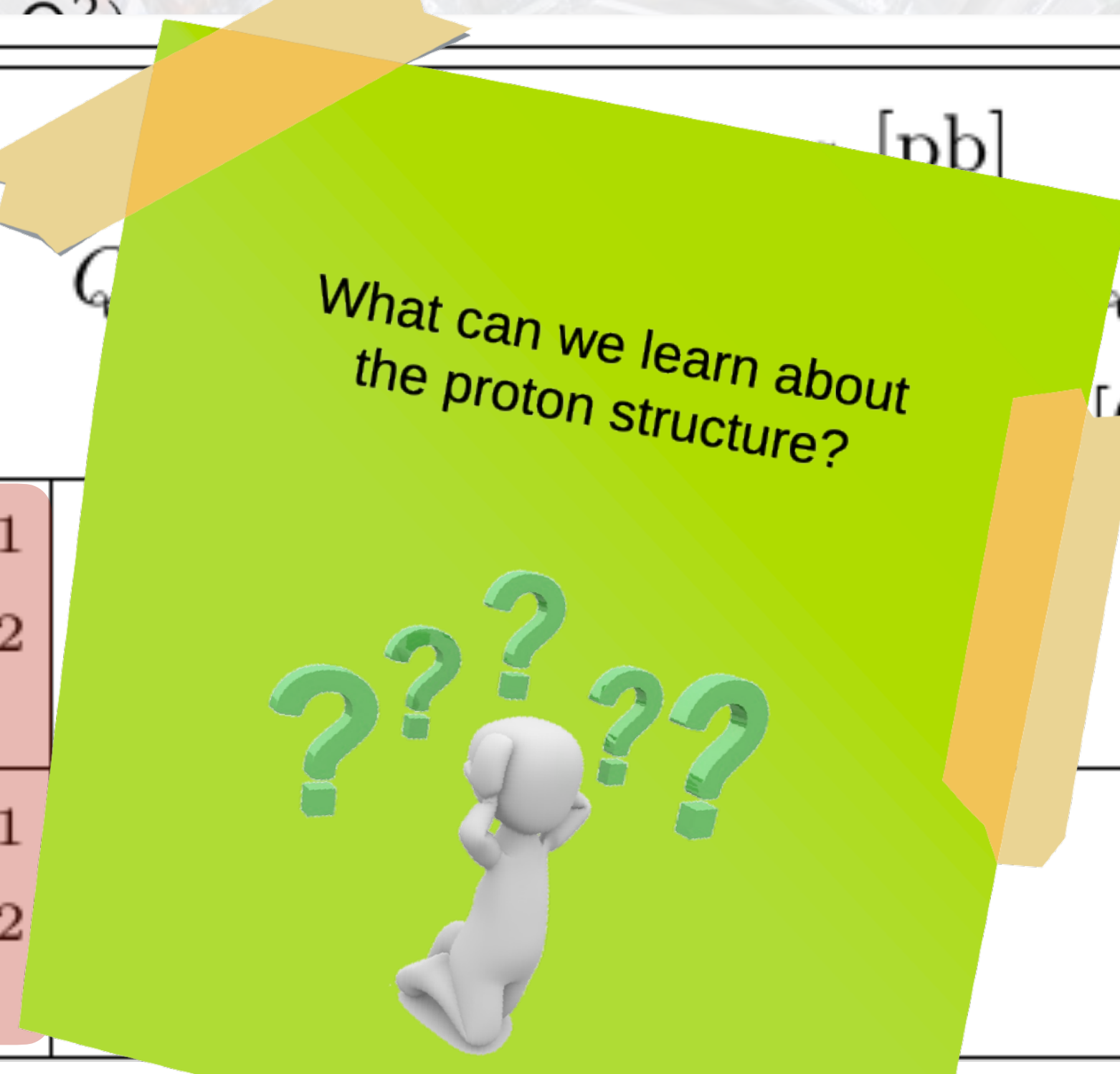
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$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

Proton		Photon		$Q^2 \leq 1 \text{ GeV}^2$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
		NP Model		[nb]	[%]
NP Model	G ₁	G ₁	G ₁	53.7	40
	G ₂	G ₂	G ₂	44.3	33
	S	S	S	40.1	30
QED	G ₁	G ₁	G ₁	142.1	101
	G ₂	G ₂	G ₂	87.7	65
	S	S	S	81.6	60



The ZEUS collaboration quotes a total 4-jet cross section of 136 pb
 S. Checkanov et al. (ZEUS), Nucl. Phys B772, 1 (2008)

A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section can be also written in terms of probability distribution:

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2 z_{\perp} \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

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We can expand the distribution related to the photon:

$$\tilde{F}_2^{\gamma}(z_{\perp}; Q^2) = \sum_n C_n(Q^2) z_{\perp}^n$$

Coefficients determined in a given approach describing the photon structure

A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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Coefficients determined in a given approach describing the photon structure

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \sum_n C_n(Q^2) \langle z_{\perp}^n \rangle_p$$

Mean value of the transverse distance between two partons in the PROTON

A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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Mean value of the transverse distance between two partons in the PROTON

If we could measure $\sigma_{\text{eff}}^{\gamma p}(Q^2)$ we could access NEW INFORMATION ON THE PROTON STRUCTURE

A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2z_{\perp} \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

We can expand $\tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$ in terms of photon:

$\tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$

We estimated that with an integrated luminosity of 200 pb⁻¹ Q² effects can be observed

Coefficients determined in a given approach describing the photon structure

$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2z_{\perp} \tilde{F}_2^p(z_{\perp}) \langle z_{\perp}^n \rangle_p$$

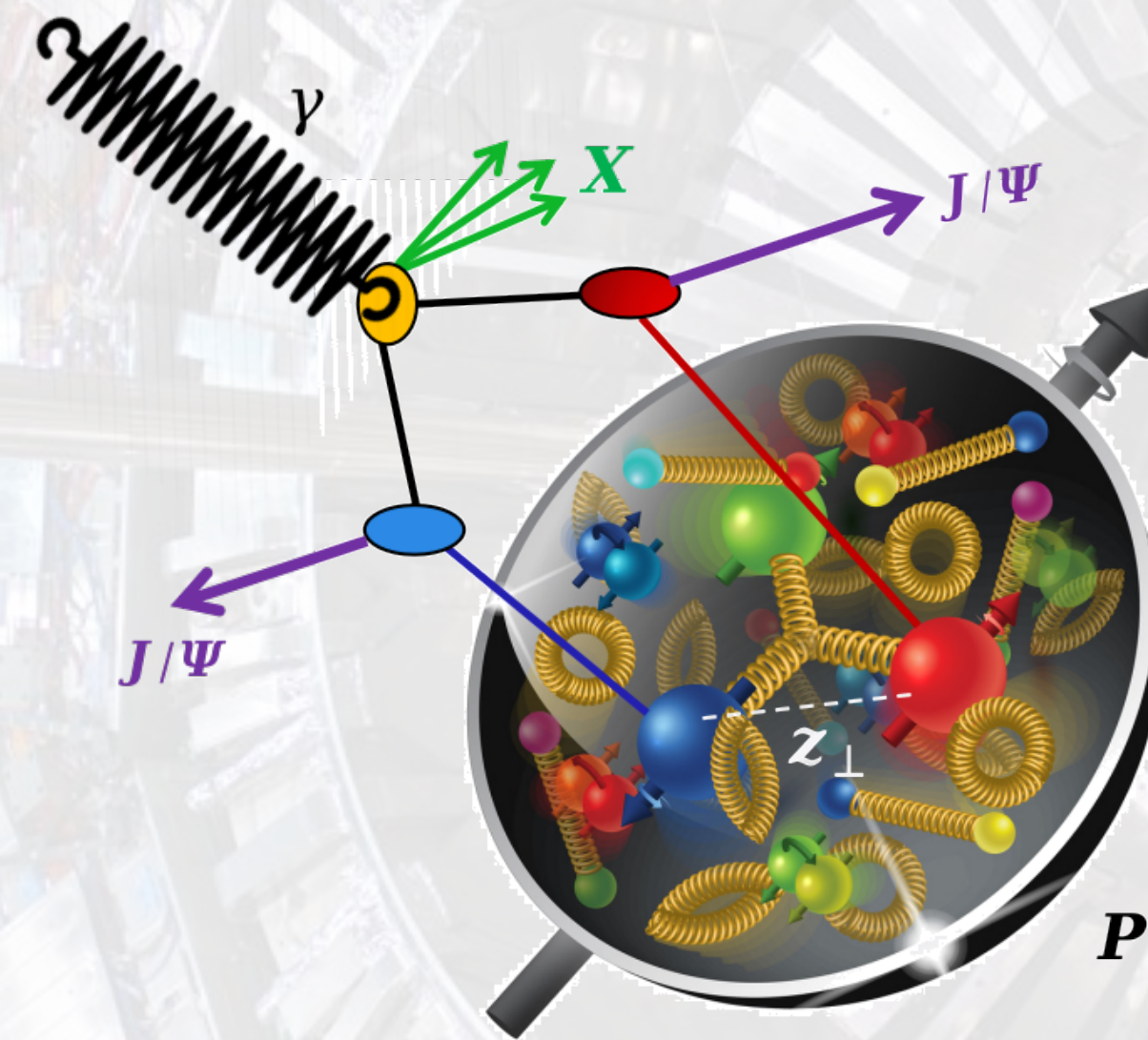
Mean value of the transverse distance between two partons in the PROTON

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Di J/ψ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Illustration of DPS for $\gamma + p \rightarrow J/\psi + J/\psi + X$

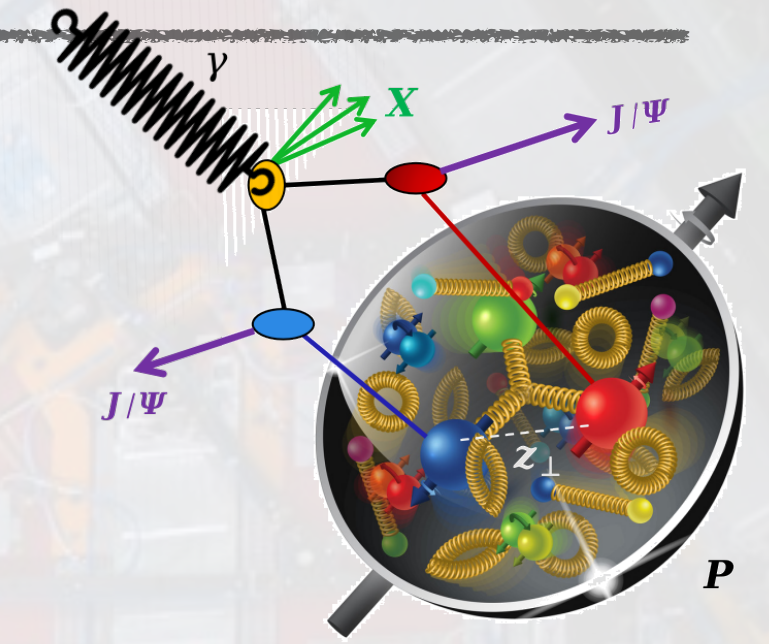


We consider the possibility of **resolved** photon to estimate the DPS cross section in quarkonium-pair photoproduction at the EIC

Di J/ψ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

*Slide from R. Sangem



$$\sigma_{SPS}^{(J/\psi, J/\psi)} \propto \sum_{a=g,q} \int dx_{p_a} f_{a/p}(x_{p_a}, \mu) d\hat{\sigma}^{\gamma a \rightarrow J/\psi + J/\psi + a} \quad \text{unresolved/direct}$$

$$\sigma_{SPS}^{(J/\psi, J/\psi)} \propto \sum_{a,b=g,q} \int dx_{\gamma_a} dx_{p_b} f_{a/\gamma}(x_{\gamma_a}, \mu) f_{b/p}(x_{p_b}, \mu) d\hat{\sigma}^{ab \rightarrow J/\psi + J/\psi} \quad \text{resolved}$$

$$\sigma_{DPS}^{(J/\psi, J/\psi)} \propto \frac{1}{2} \frac{1}{\sigma_{eff}^{\gamma p}} \sum_{a,b,c,d} \int dx_{\gamma_a} dx_{p_b} f_{a/\gamma}(x_{\gamma_a}, \mu) f_{b/p}(x_{p_b}, \mu) d\hat{\sigma}_{SPS}^{ab \rightarrow J/\psi}(x_{\gamma_a}, x_{p_b})$$

$$\times dx_{\gamma_c} dx_{p_d} f_{c/\gamma}(x_{\gamma_c}, \mu) f_{d/p}(x_{p_d}, \mu) d\hat{\sigma}_{SPS}^{cd \rightarrow J/\psi}(x_{\gamma_c}, x_{p_d})$$

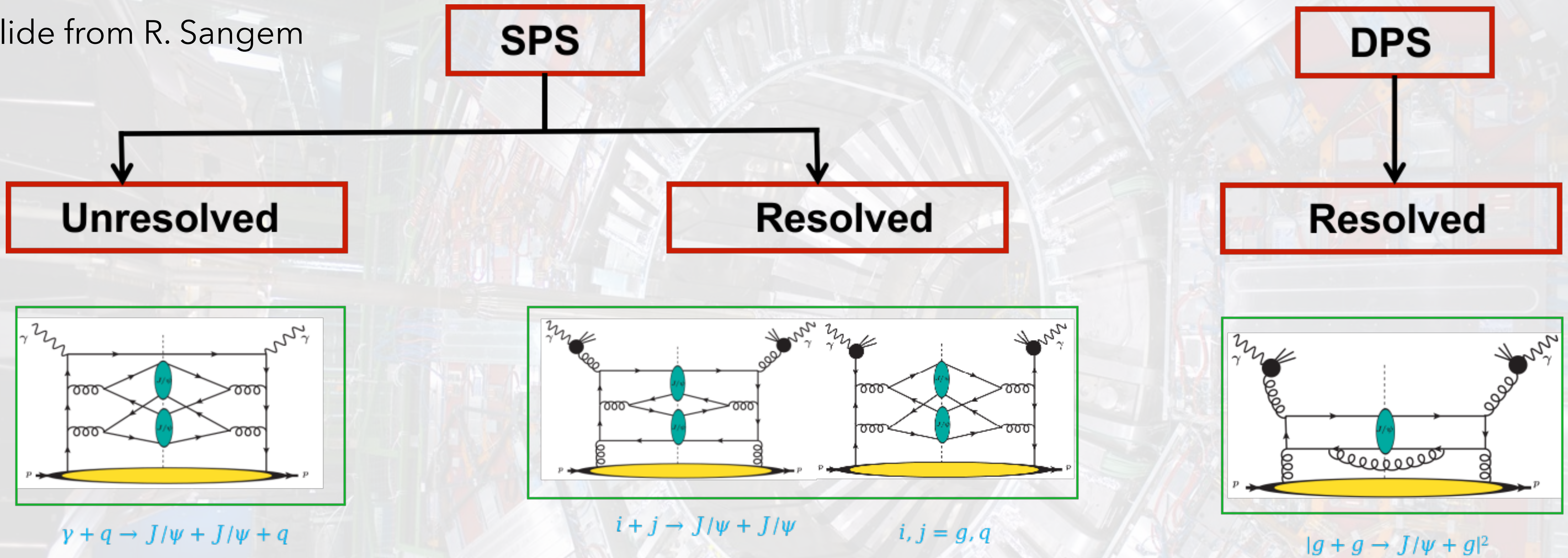
Proton PDF
Photon PDF
Partonic x-sections

Single SPS resolved (namely same partonic cross section as hadroproduction)

Di J/ψ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

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- GRV photon PDF is used [PRD 46, 1973 \(1992\)](#) , while CT18NLO PDF for proton [T.J. Hou et al., PRD 103, 014013 \(2021\)](#)
- HELAC-Onia latest version is used for generating matrix elements [HS Shao, CPC 184, 2562 \(2013\), 198, 238 \(2016\)](#)
- CO LDMEs are taken from [M. Butenschoen and B. A. Kniehl, PRD 84, 051501 \(2011\)](#)
- We expect at least 600 four-muon events with 100 fb^{-1} luminosity

Numerical Results

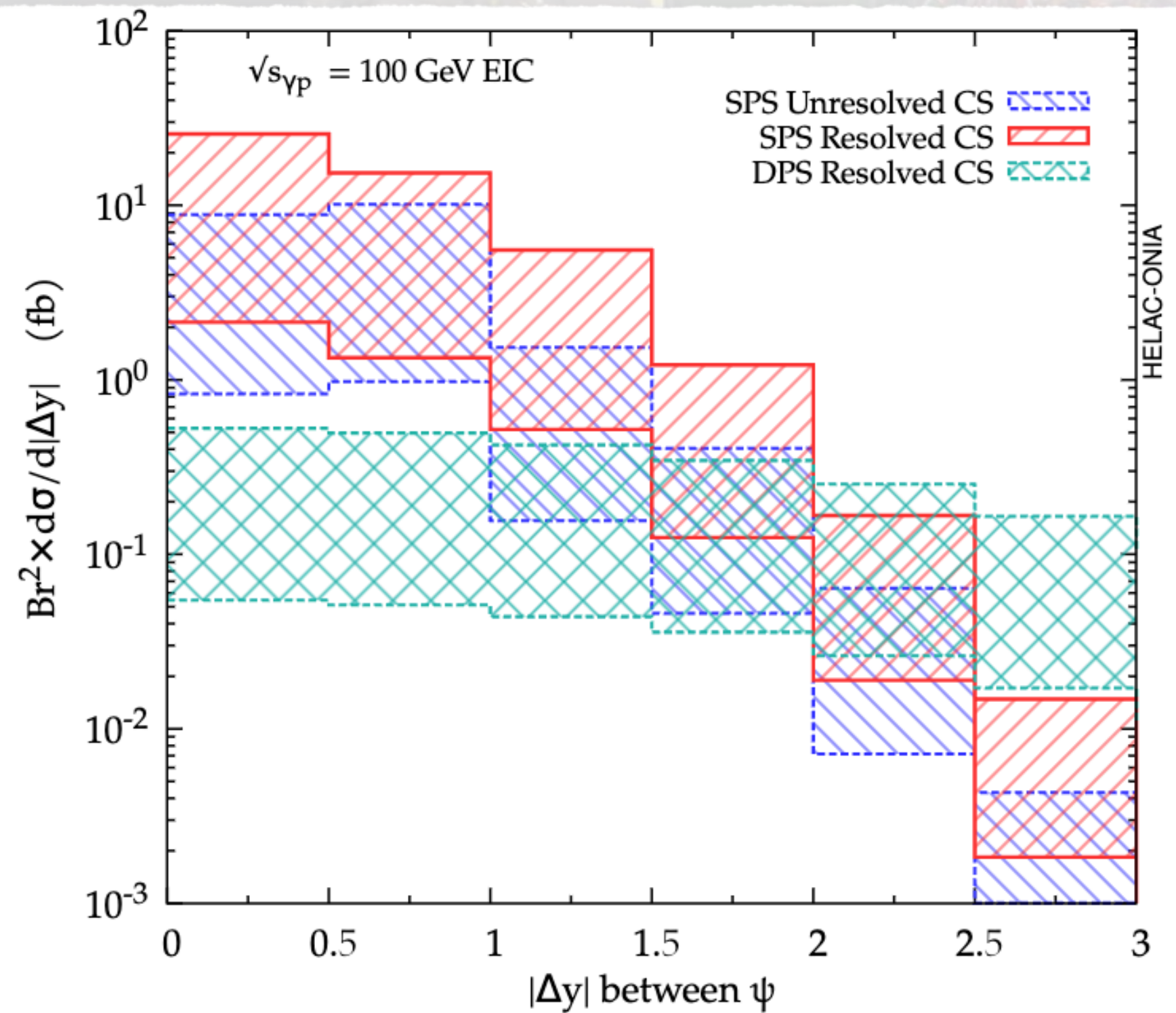
PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

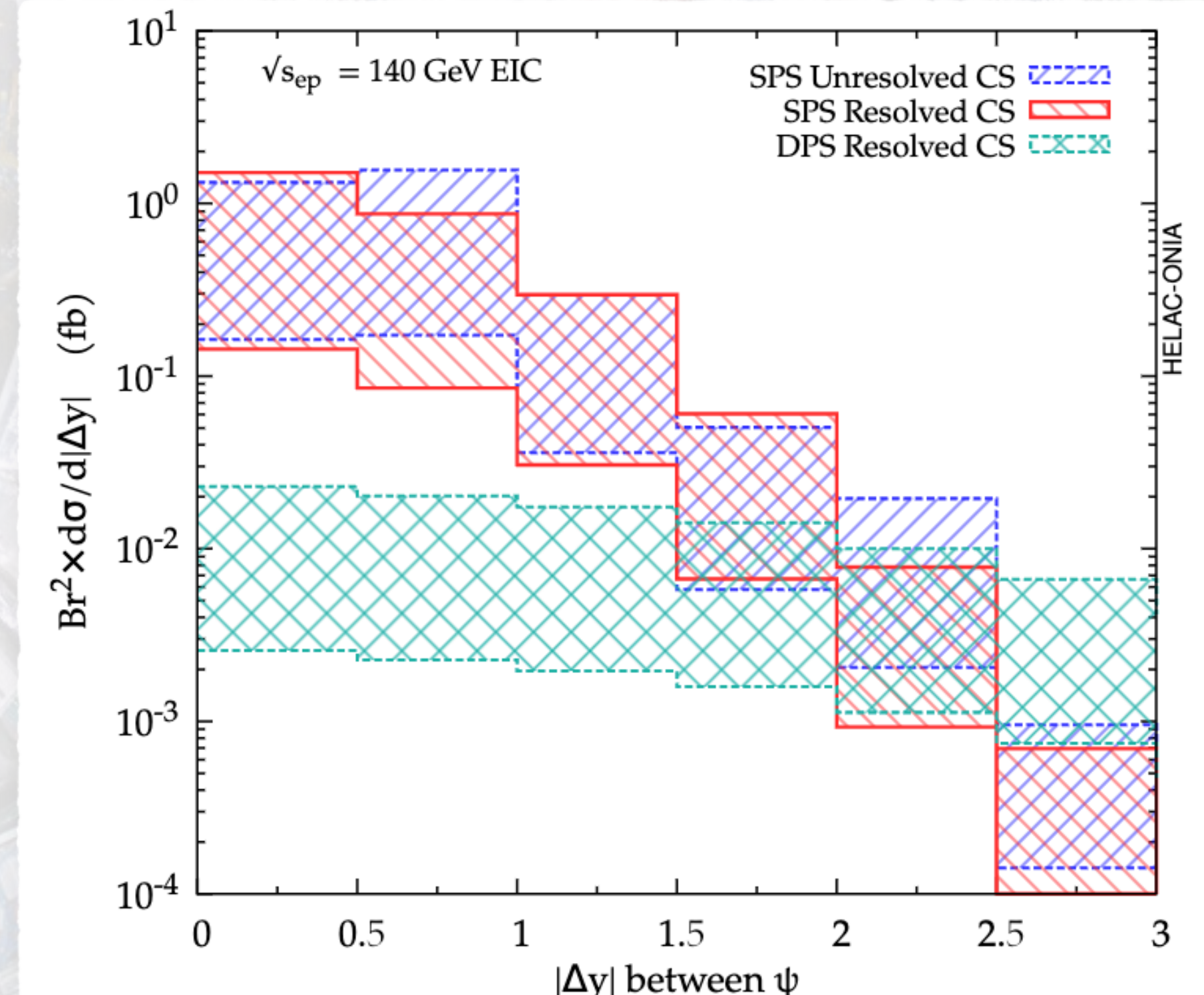
Absolute rapidity difference between the two J/ψ

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$



- DPS dominates at high $|\Delta y|$
- DPS is suppressed at low $|\Delta y|$



Numerical Results

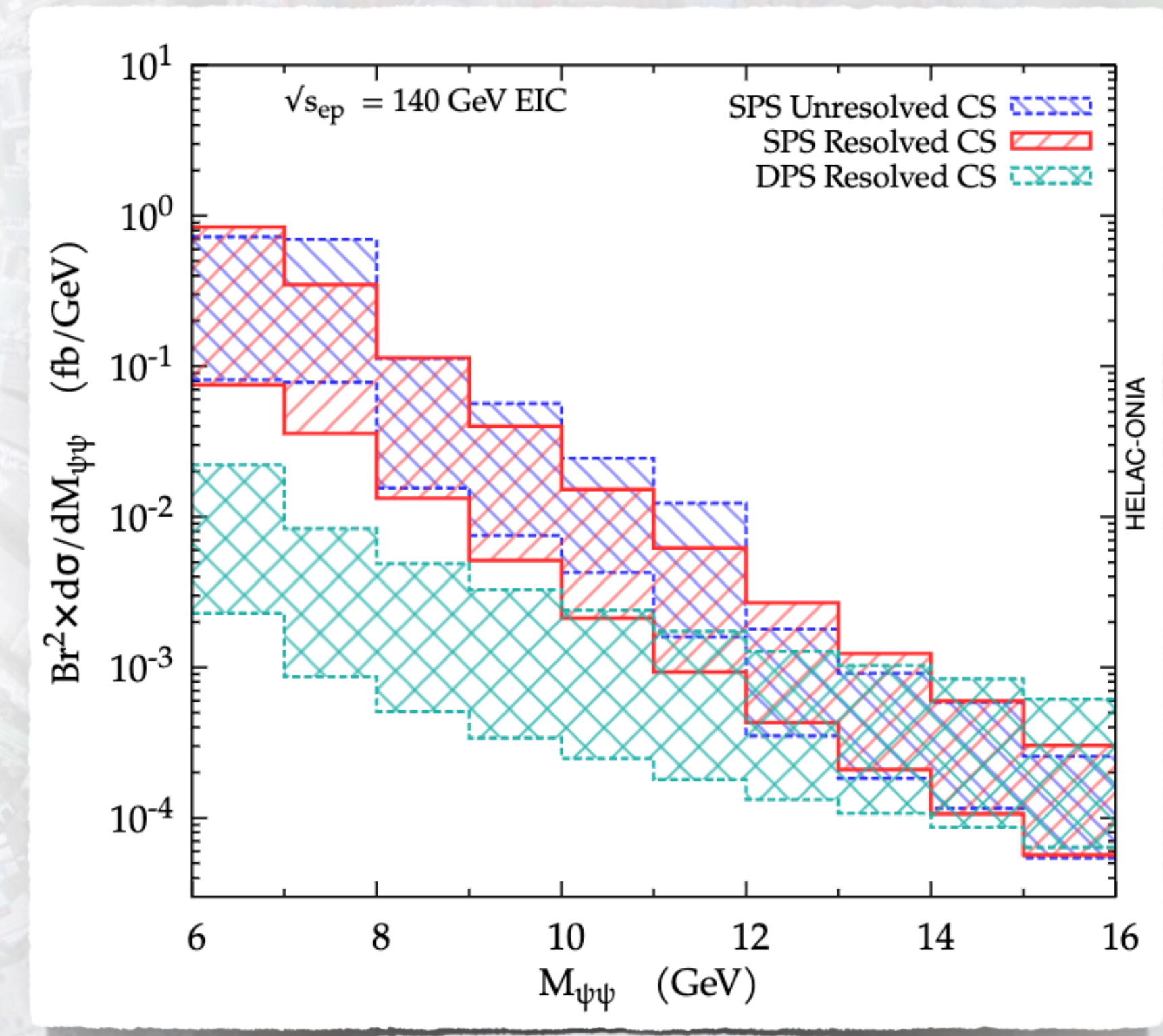
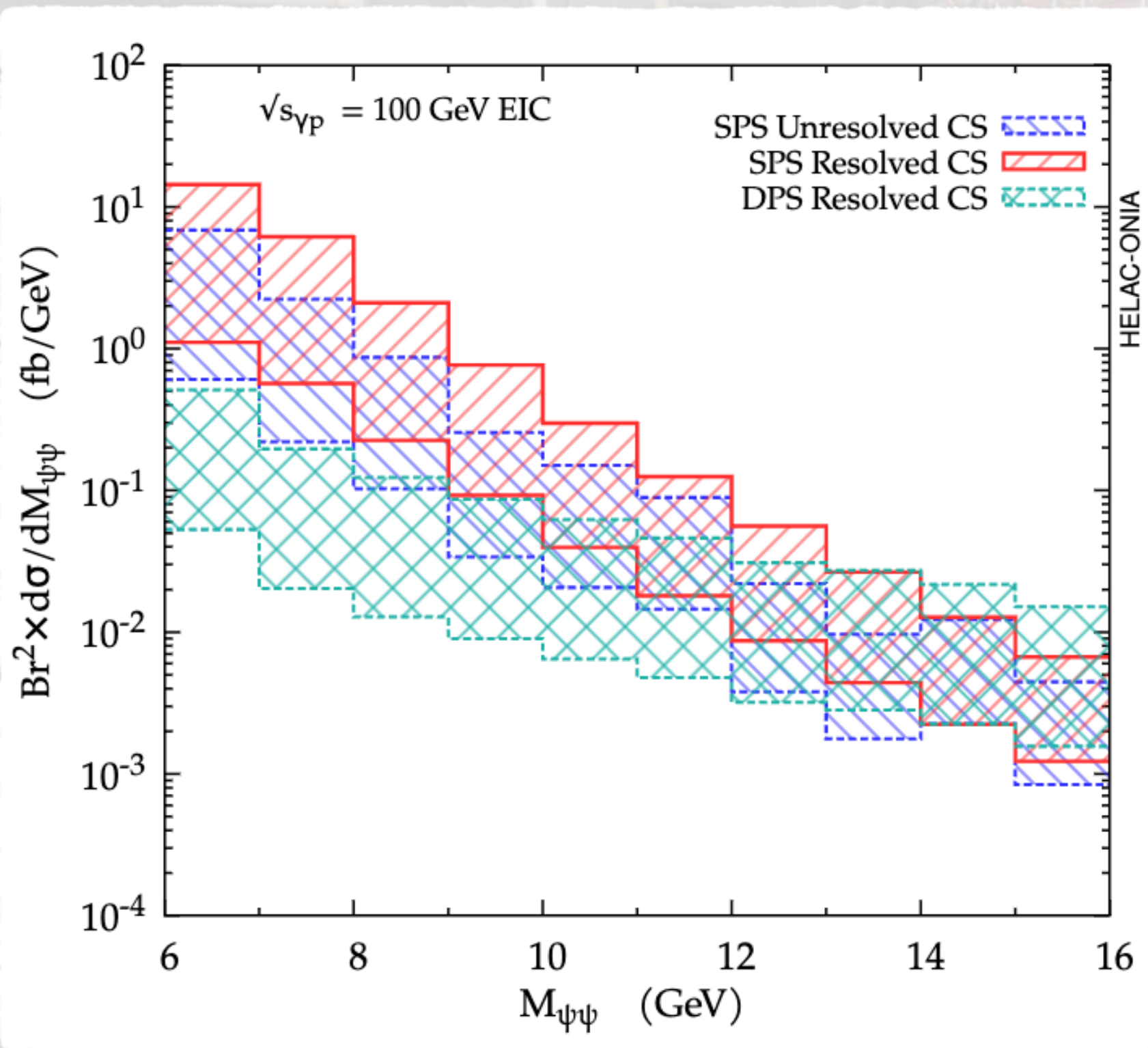
PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Invariant mass of the J/ψ pair

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$



Numerical Results

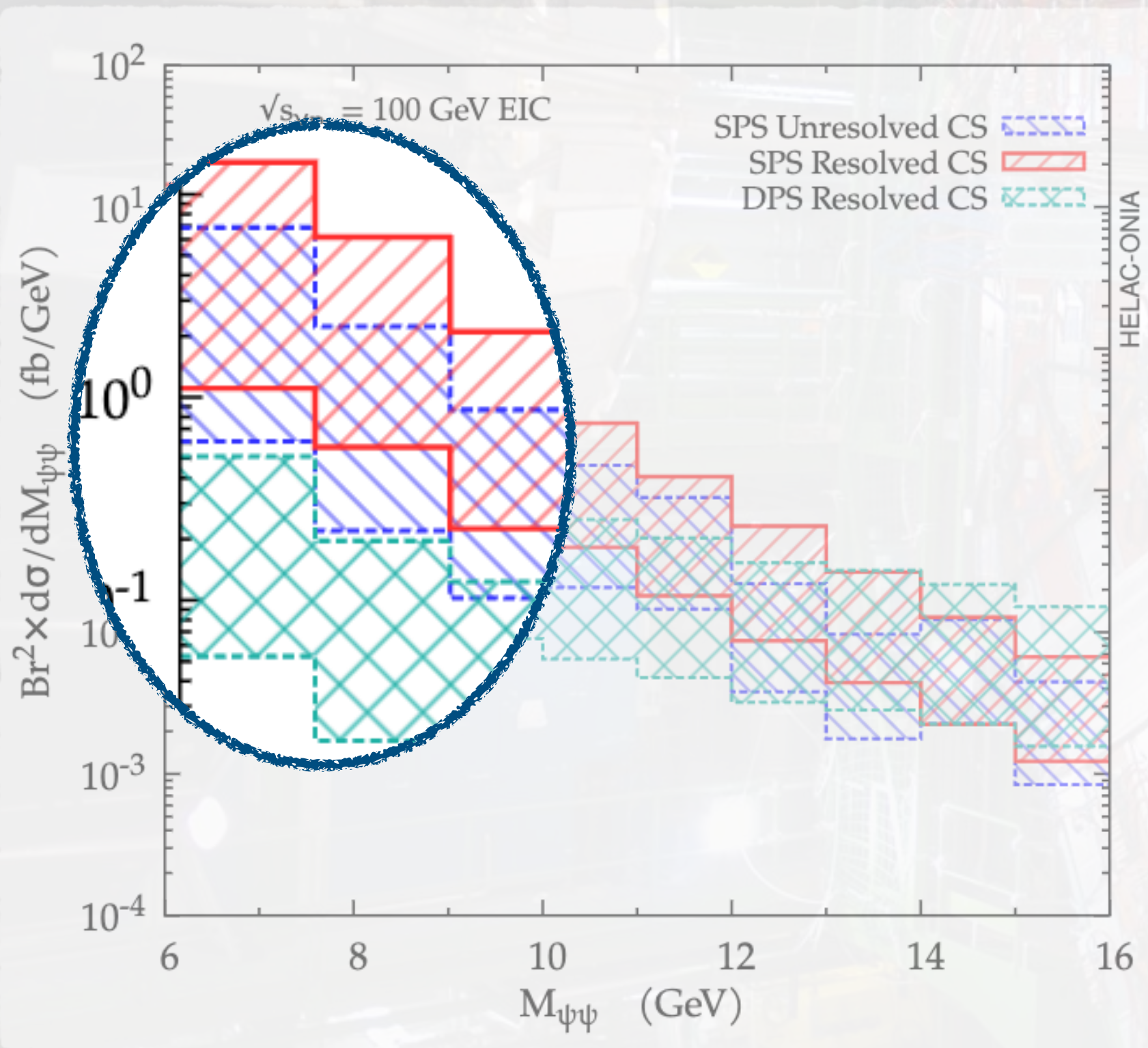
PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Invariant mass of the J/ψ pair

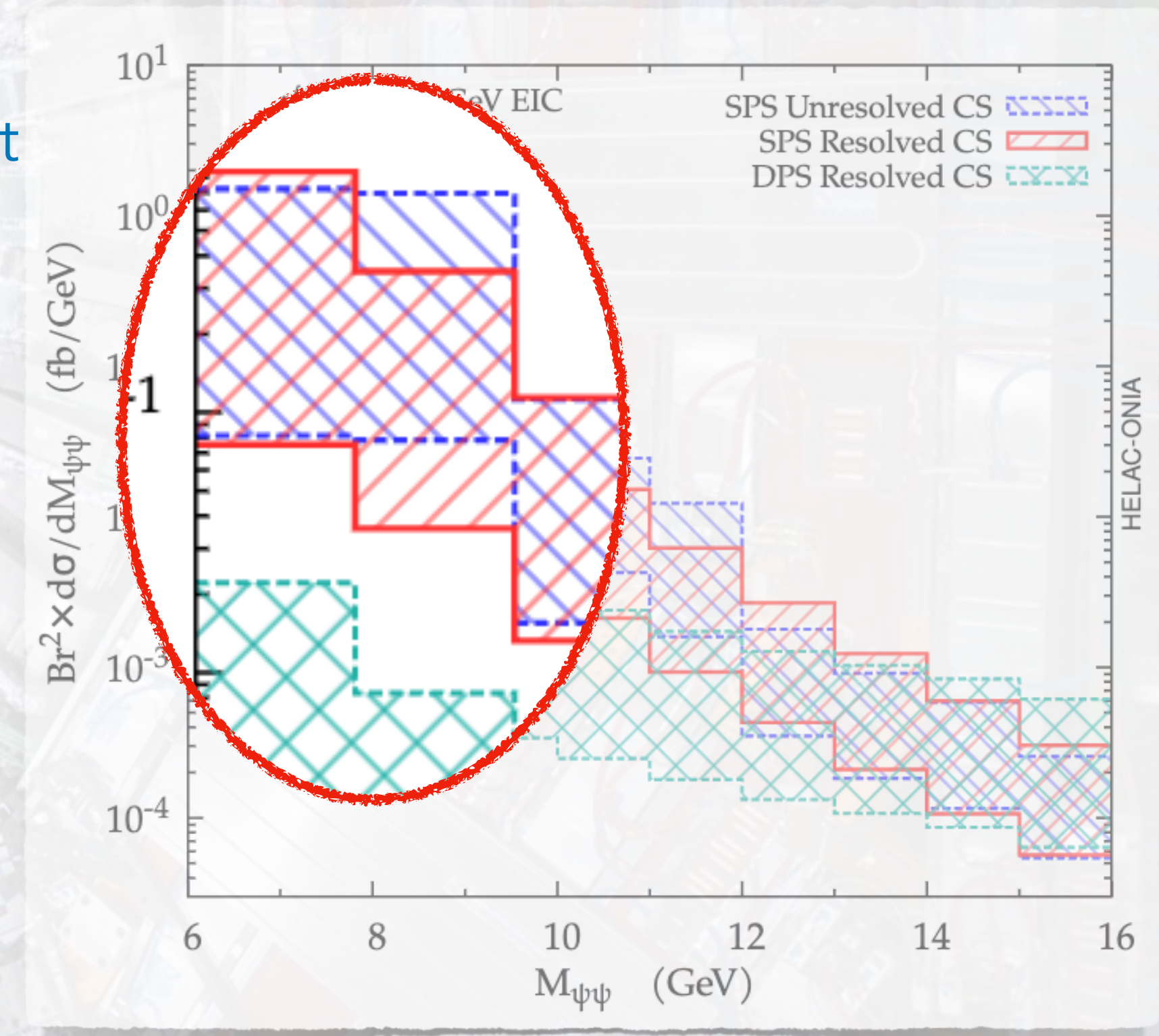
$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$



a) at low invariant mass:

- DPS smaller than SPS, but not negligible
- DPS negligible



Numerical Results

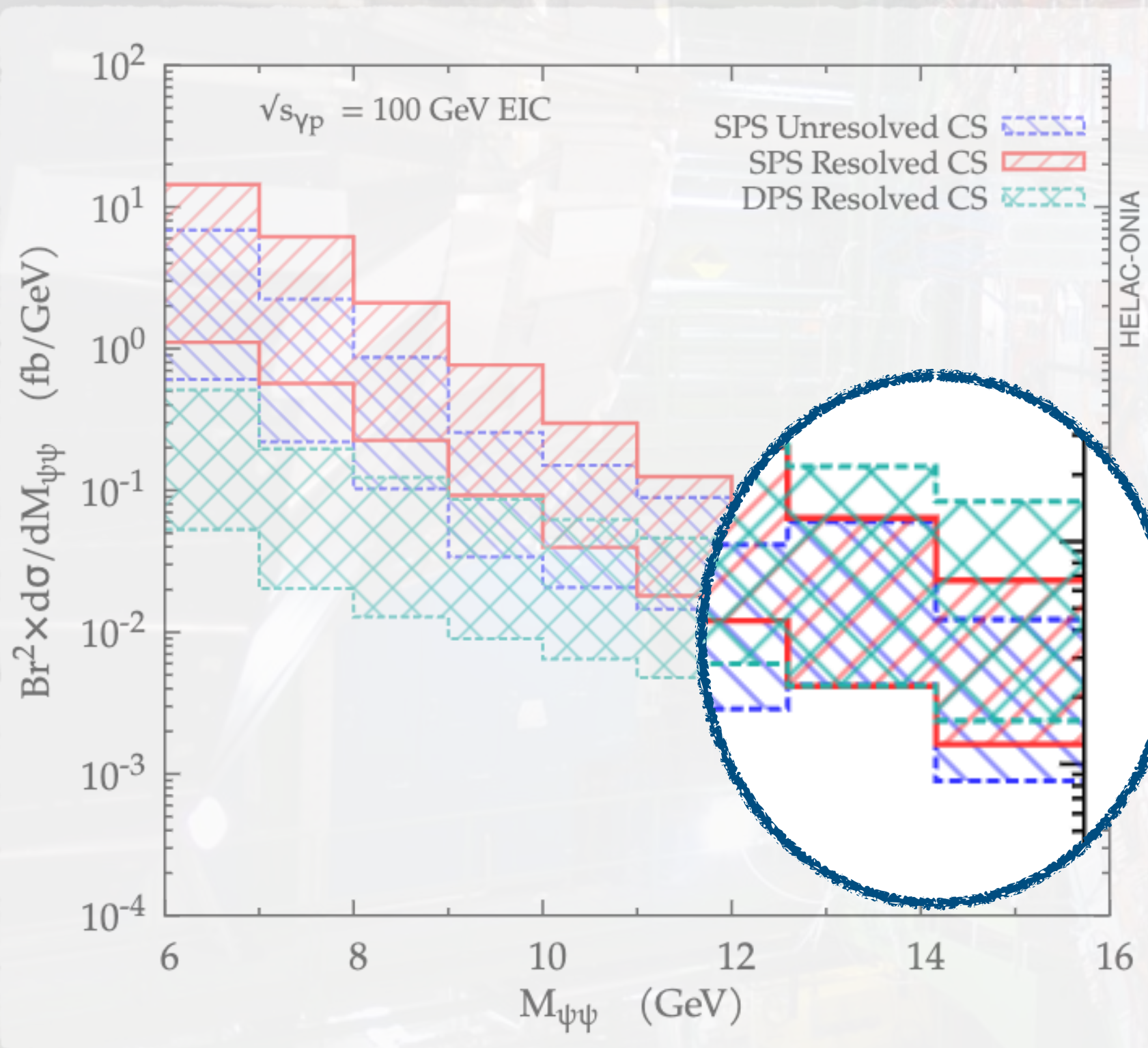
PRELIMINARY

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Invariant mass of the J/ψ pair

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$

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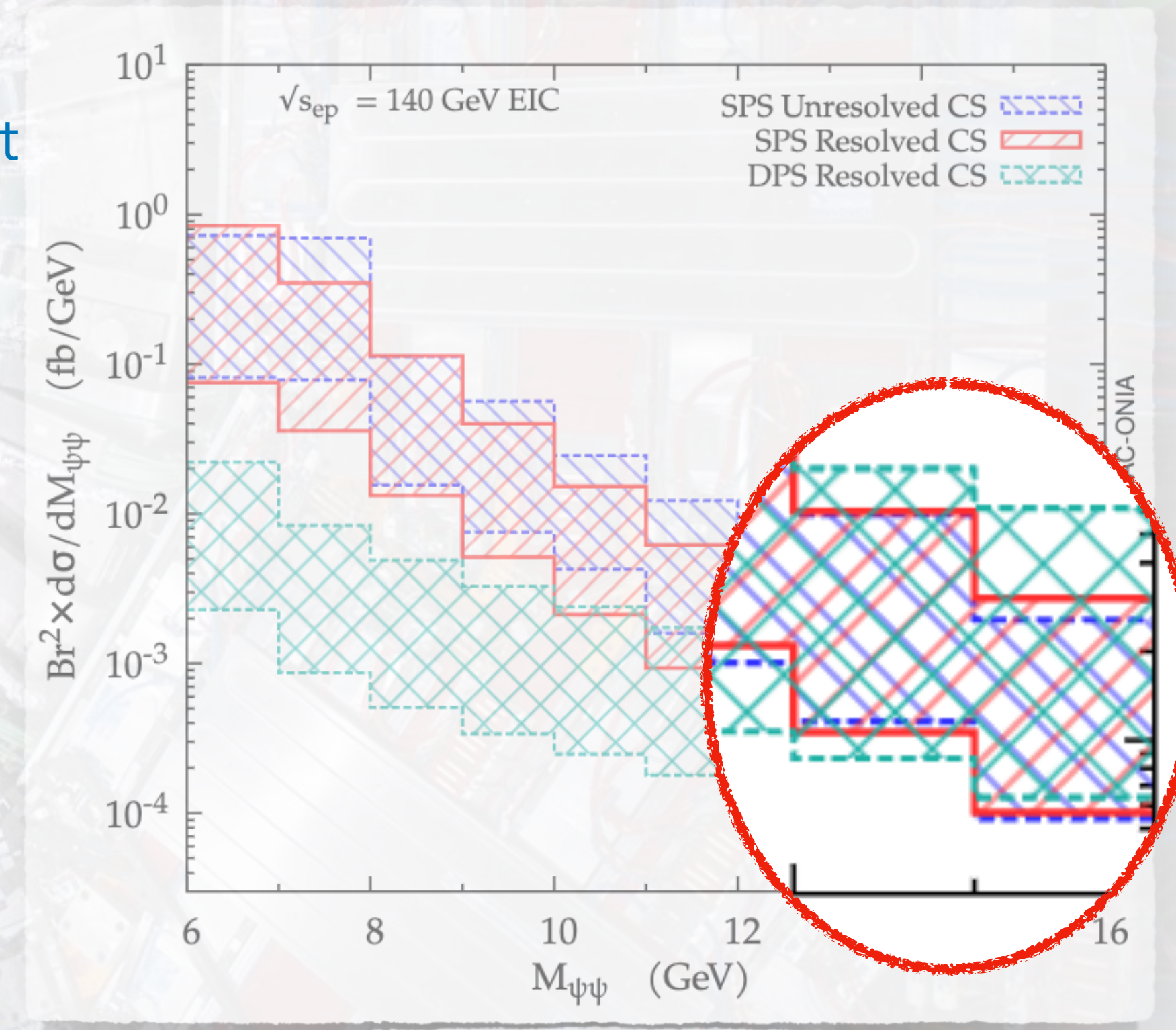


a) at low invariant mass:

- DPS smaller than SPS, but not negligible
- DPS negligible

b) at low invariant mass:

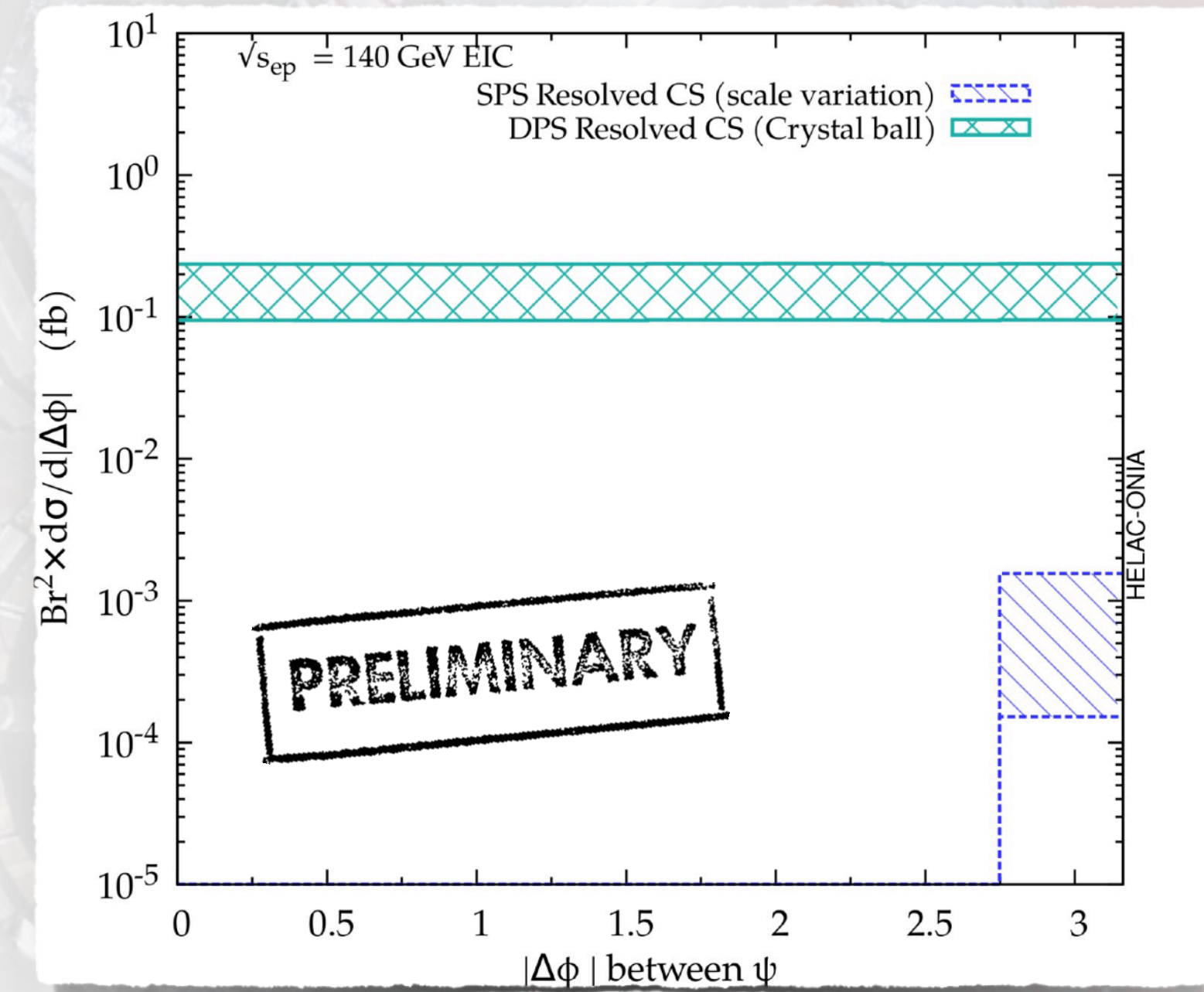
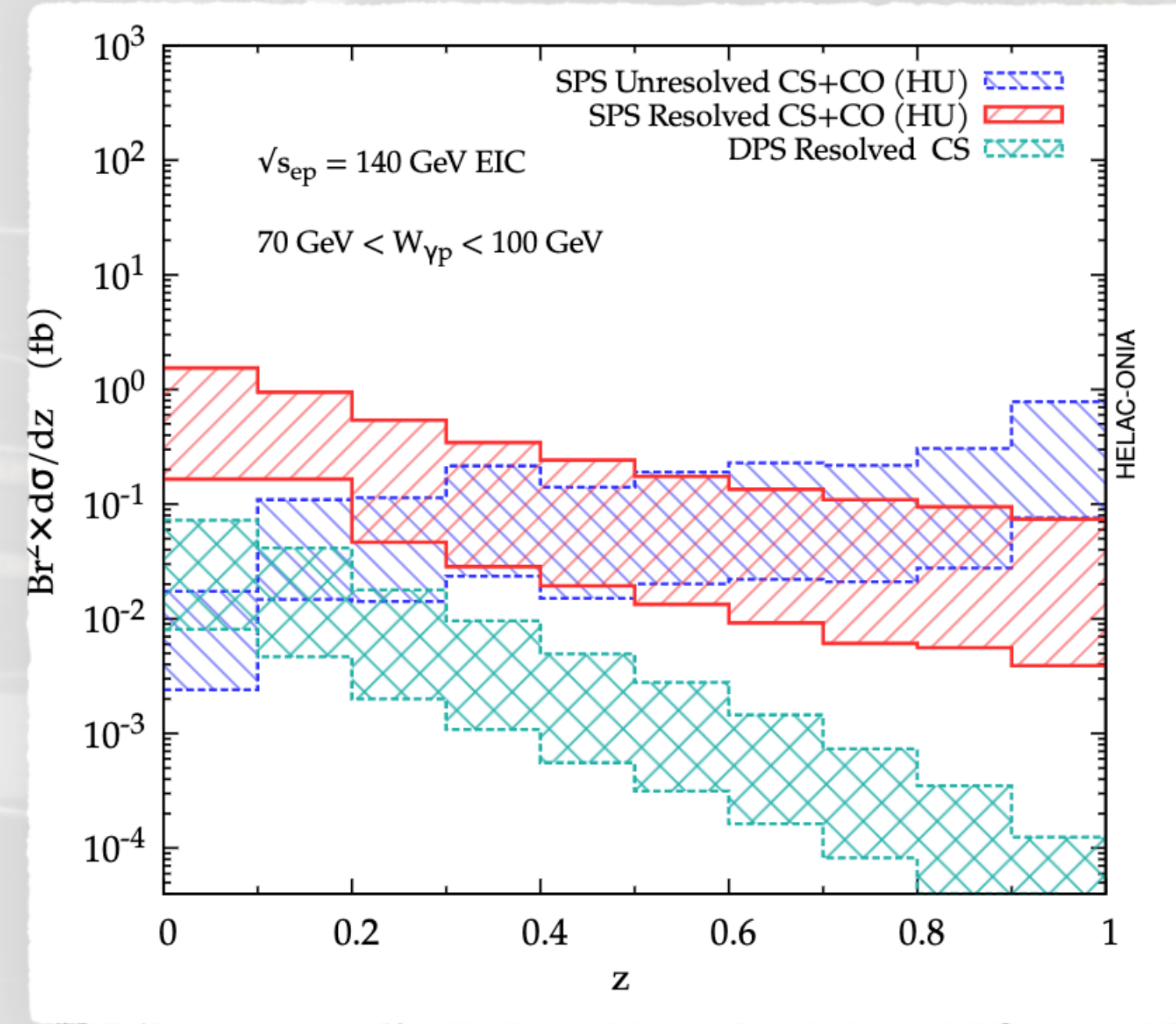
- DPS bigger than SPS
- DPS similar to SPS



Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

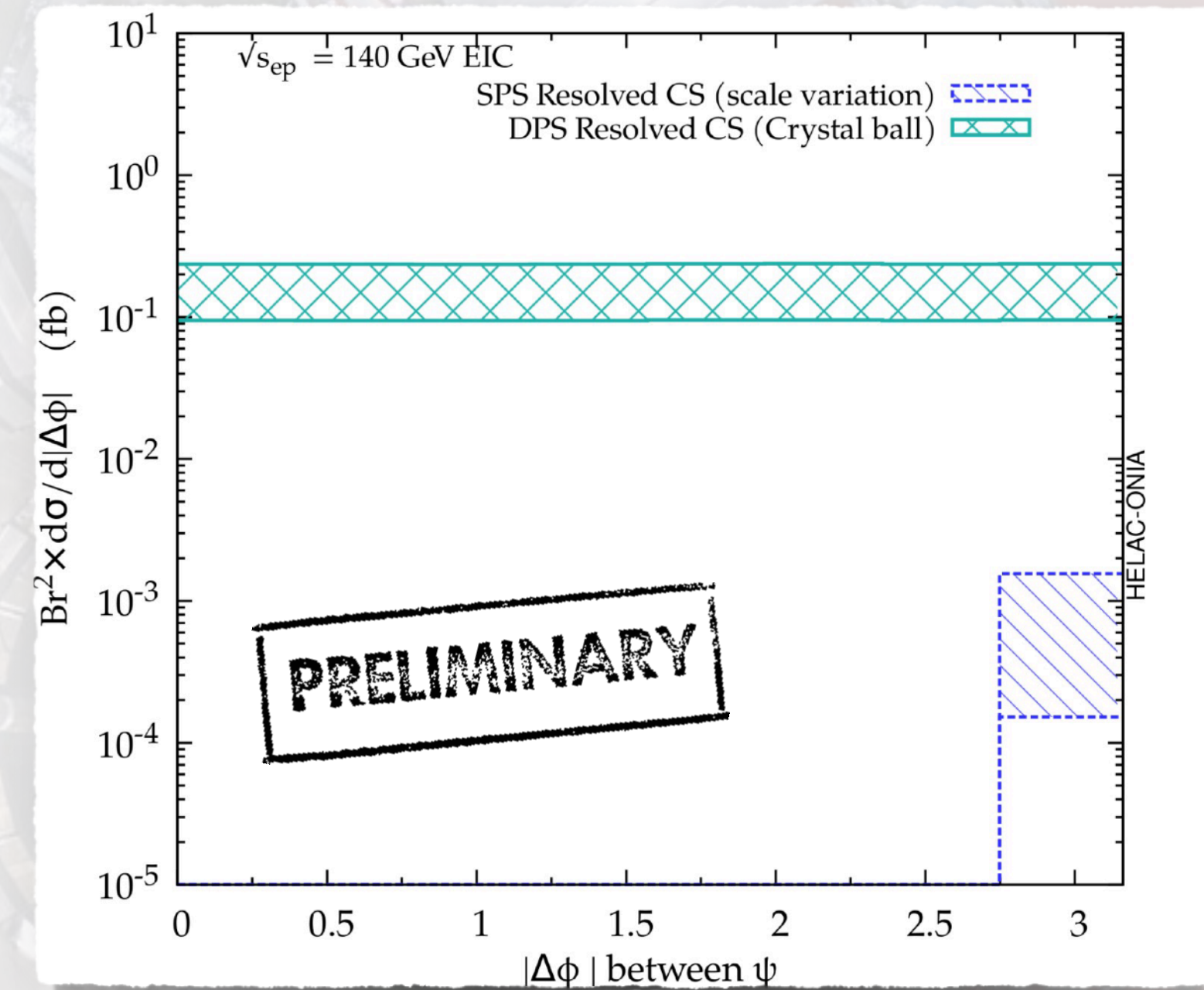
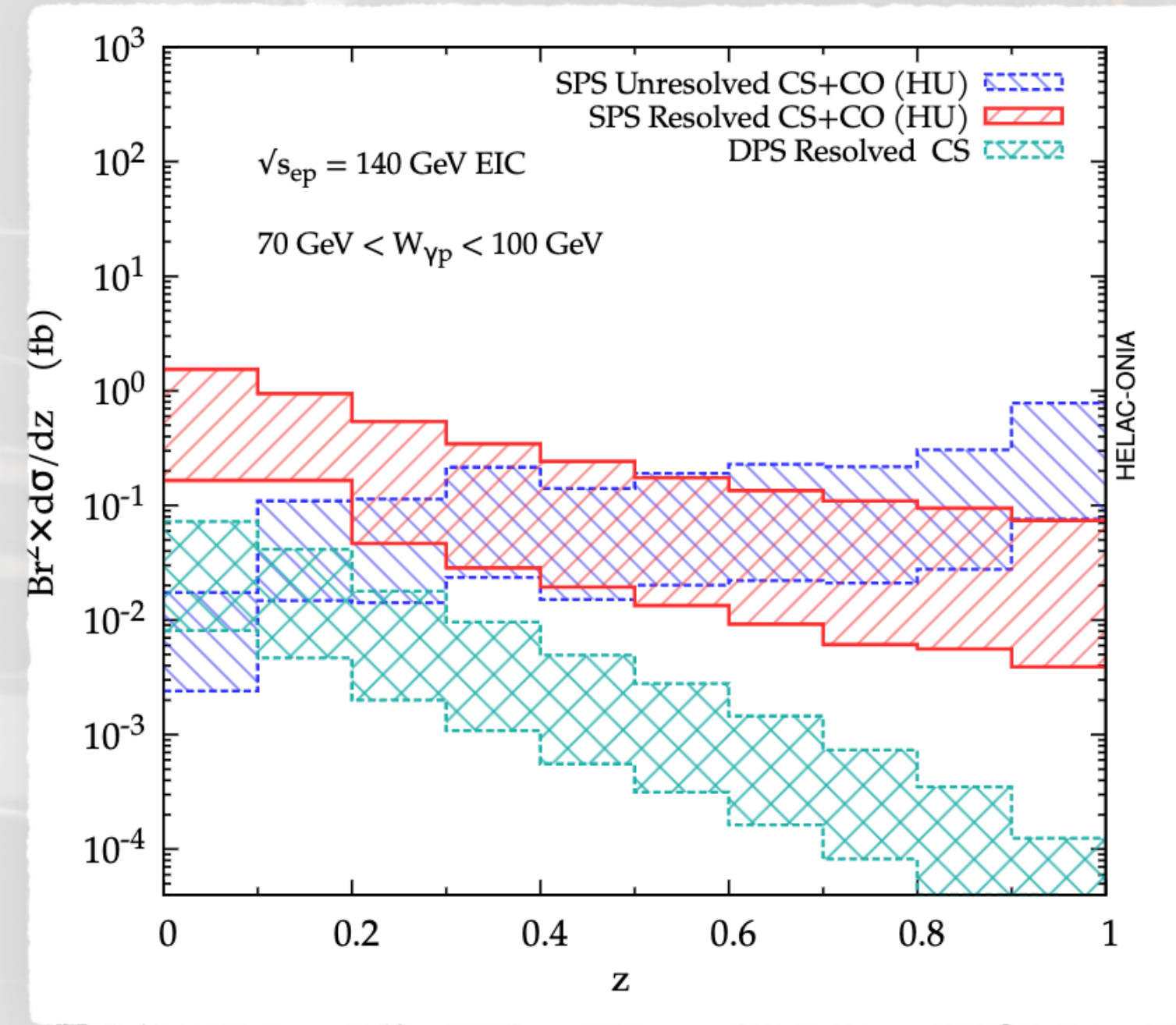


* for $z < 0.1$, SPS resolved dominates \longrightarrow unique opportunity to investigate the PHOTON structure

Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



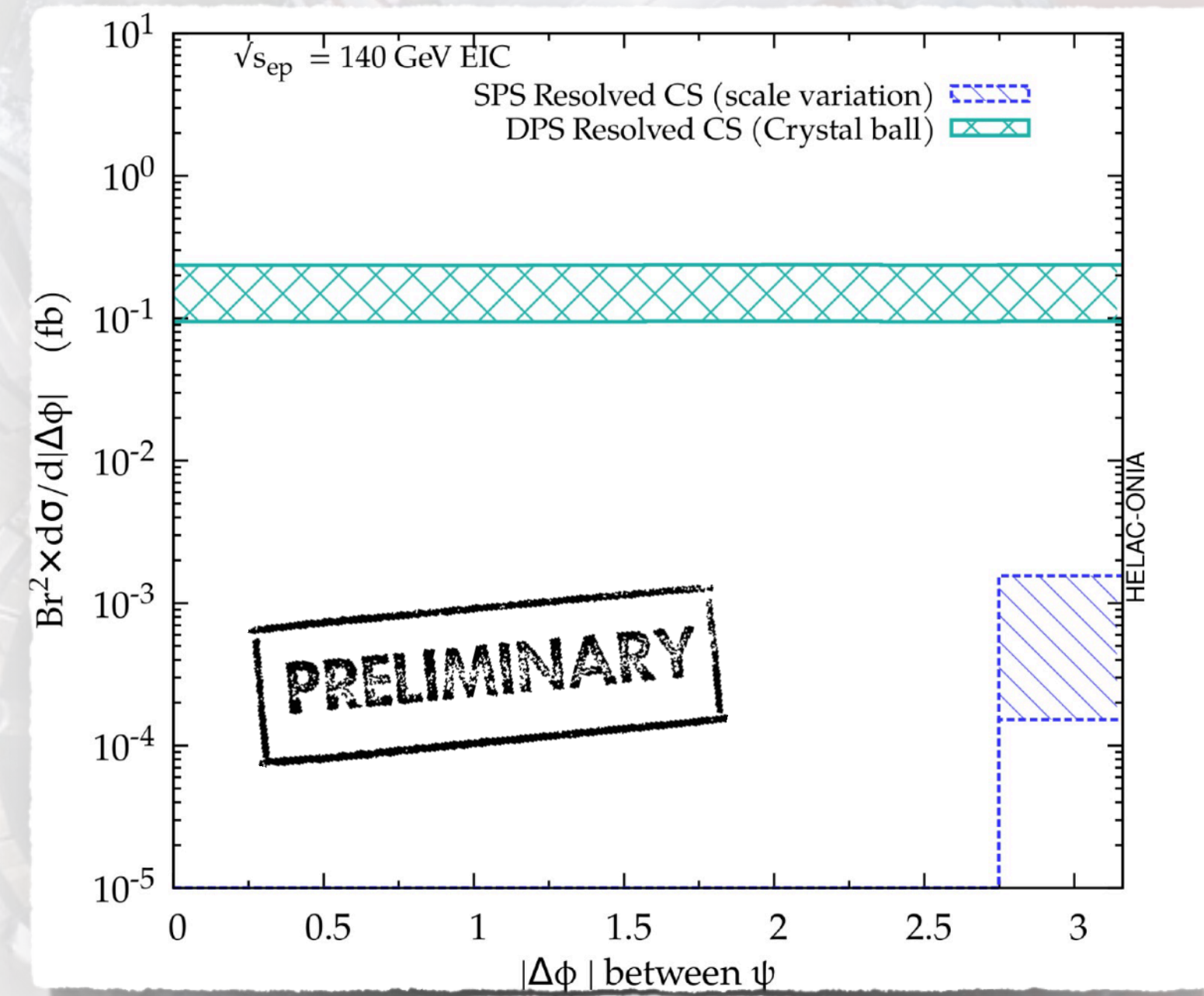
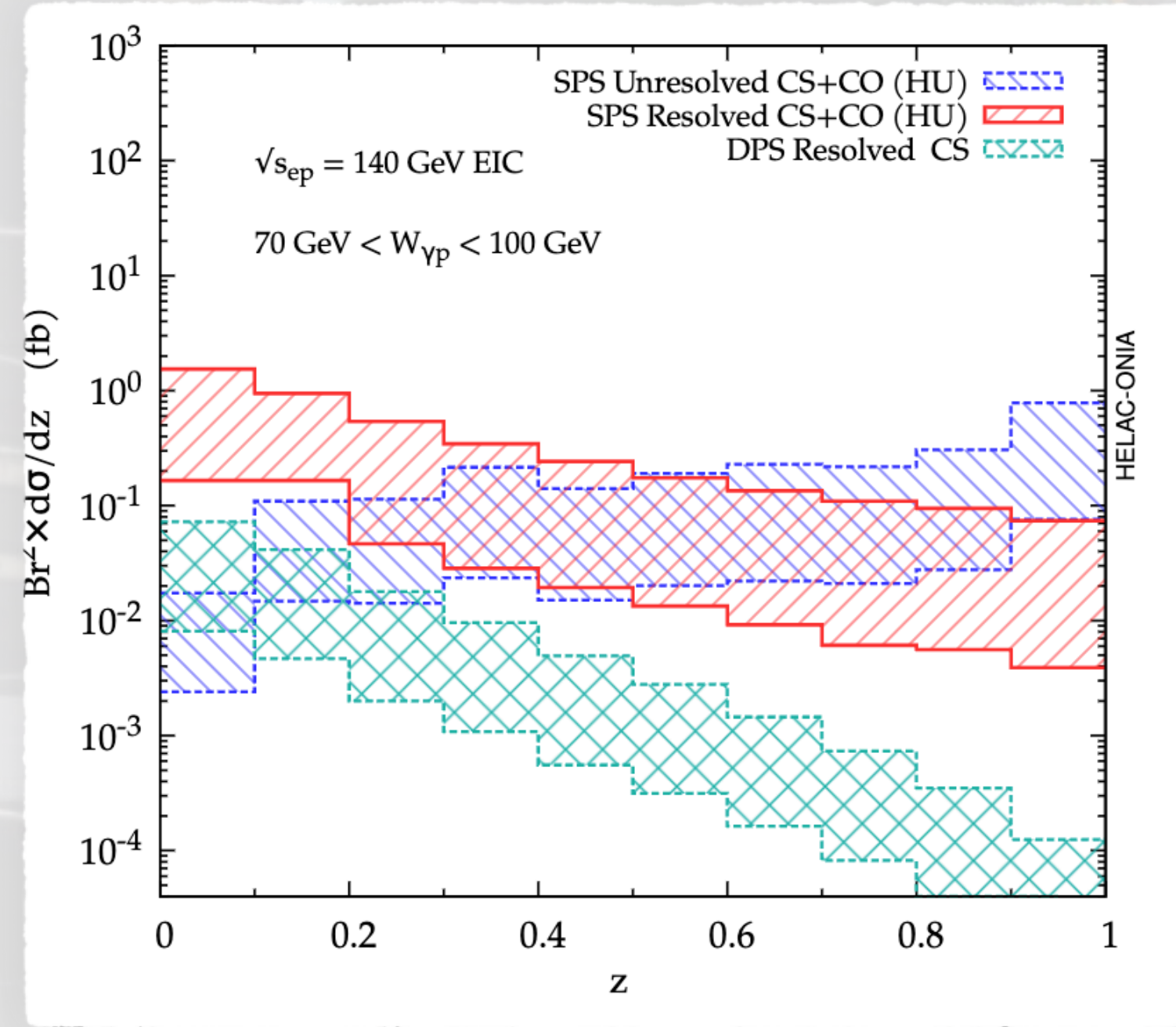
* for $z < 0.1$, SPS resolved dominates \longrightarrow unique opportunity to investigate the PHOTON structure

* for high z , the direct SPS contribution dominates \longrightarrow we test the quarkonia production via direct photoproduction

Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

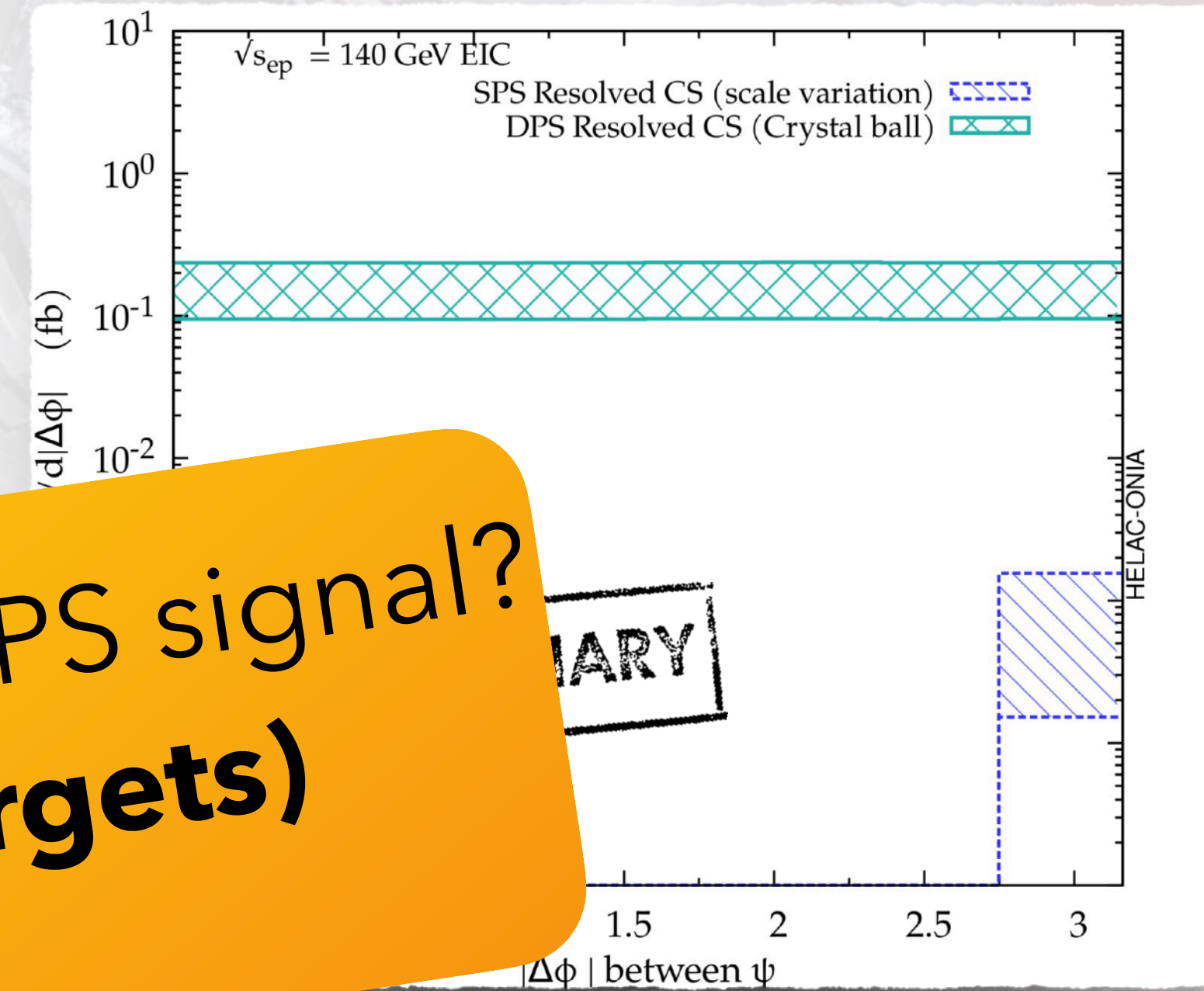
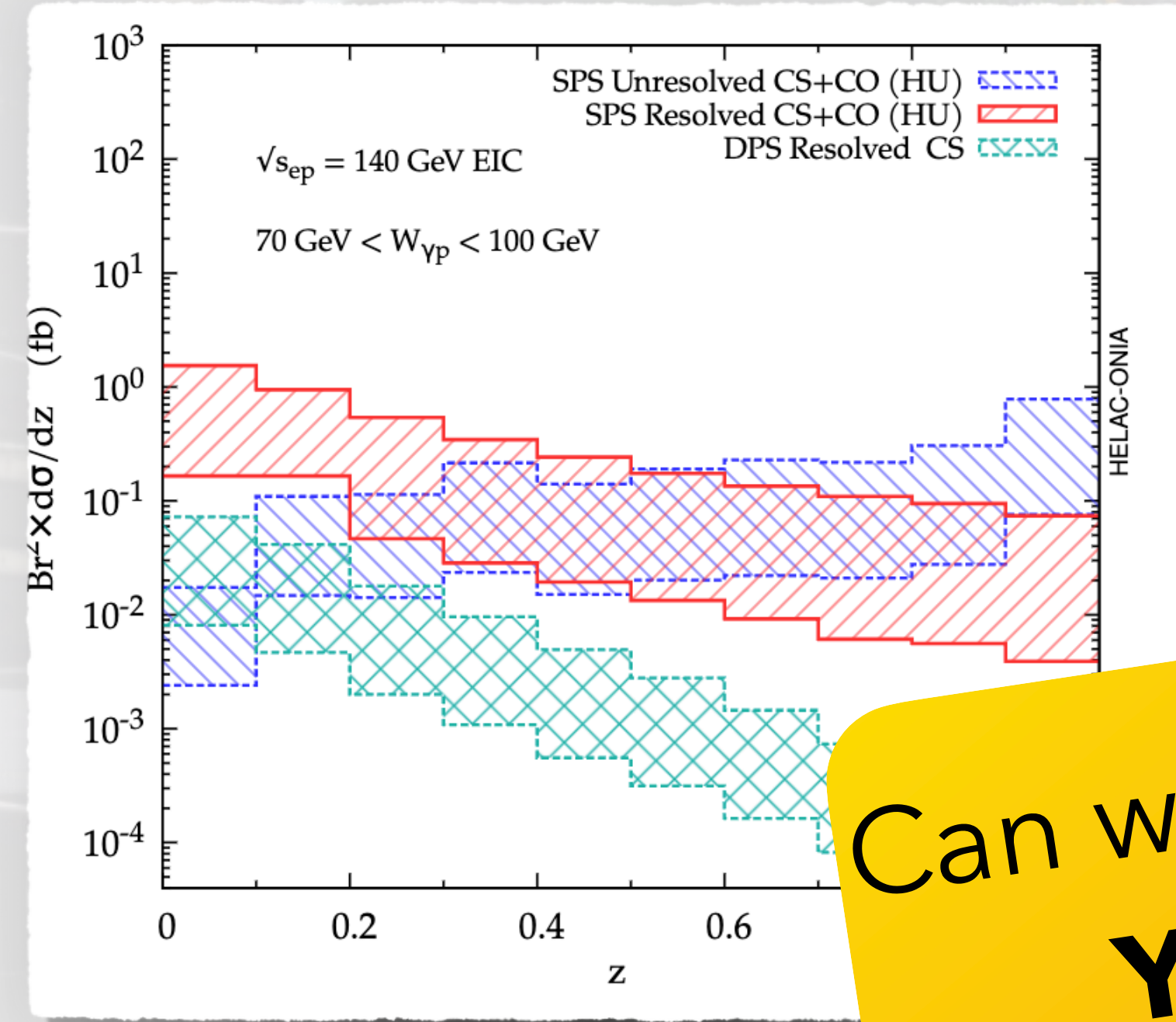


- * for $z < 0.1$, SPS resolved dominates → unique opportunity to investigate the PHOTON structure
- * for high z , the direct SPS contribution dominates → we test the quarkonia production via direct photoproduction
- * as for DPS studies @LHC, the cross-section dependence on the relative azimuthal angle is relevant to access the DPS contribution!

Numerical Results

PRELIMINARY

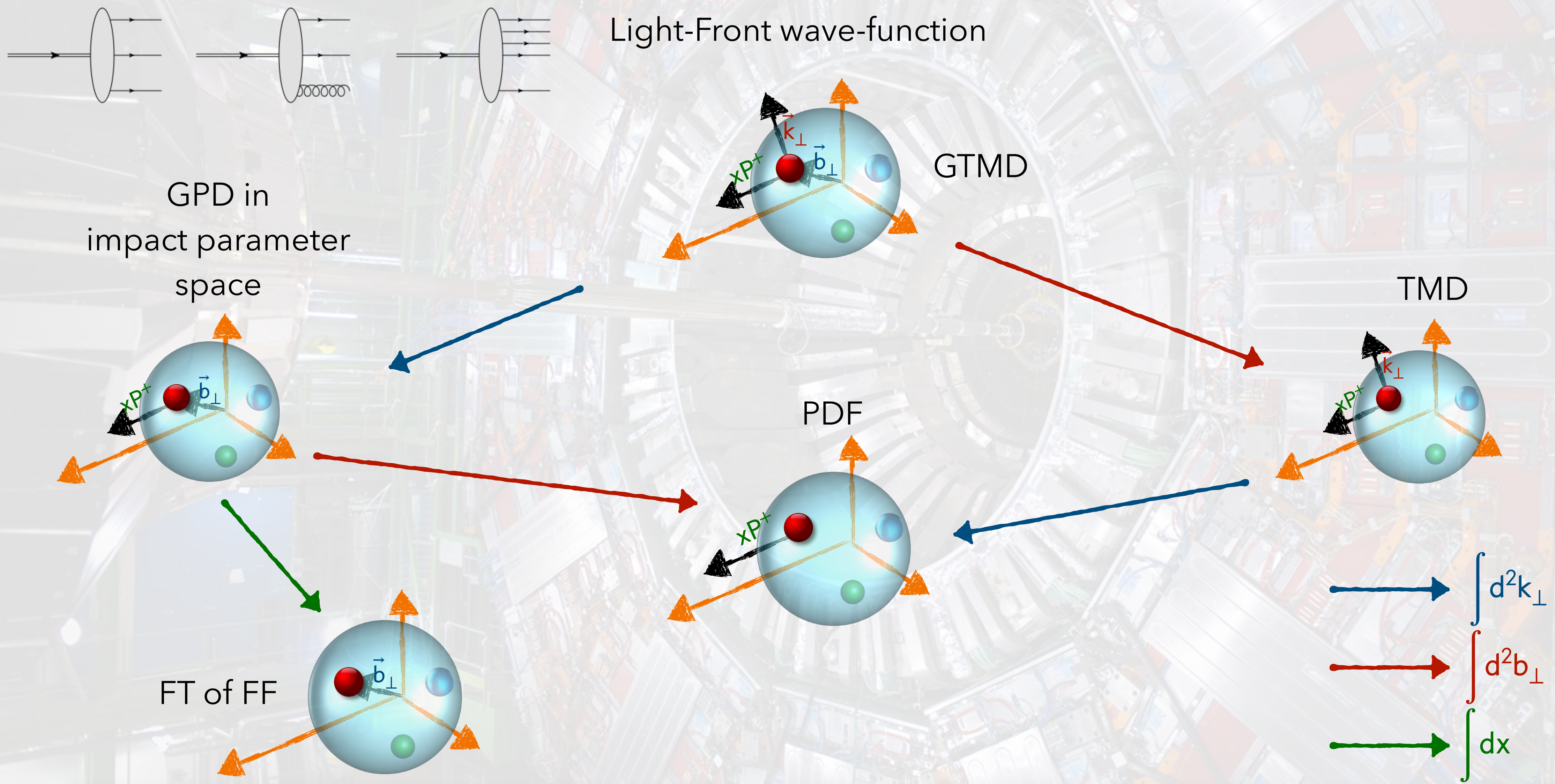
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



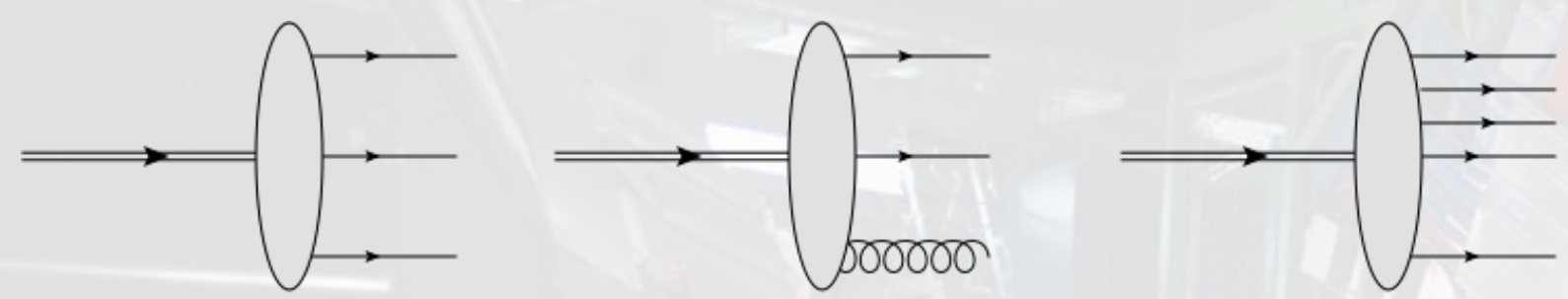
Can we increase the DPS signal?
YES! (Nuclear targets)

- * for $z < 0.1$, SPS resolved dominates → unique opportunity to investigate the PHOTON structure
- * for high z , the direct SPS contribution dominates → we test the quarkonia production via direct photoproduction
- * as for DPS studies @LHC, the cross-section dependence on the relative azimuthal angle is relevant to access the DPS contribution!

Multidimensional picture of hadrons

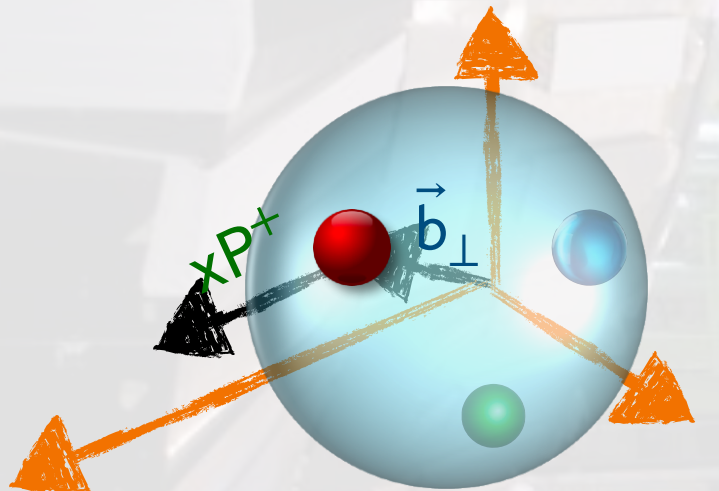


Multidimensional picture of hadrons

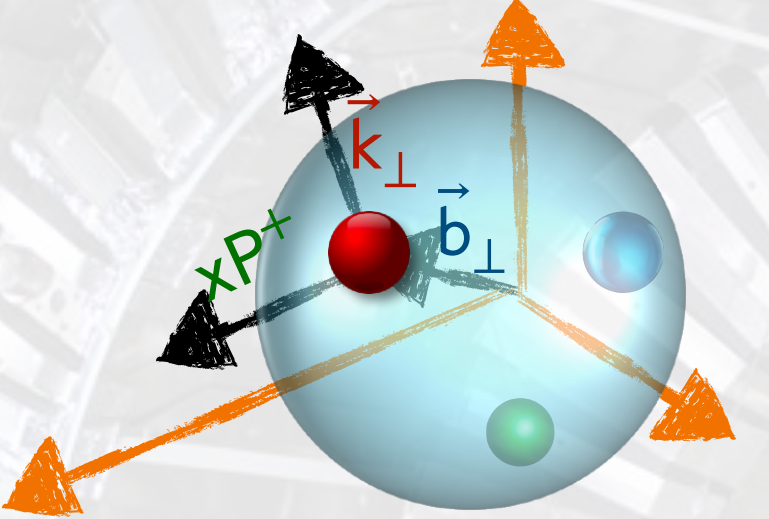


Light-Front wave-function

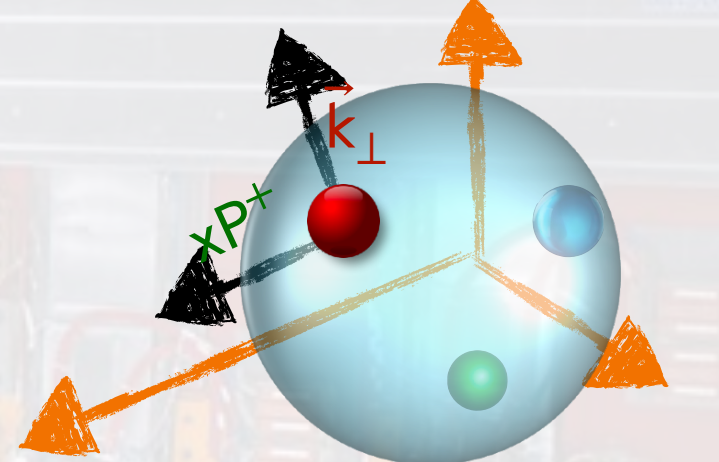
GPD in impact parameter space



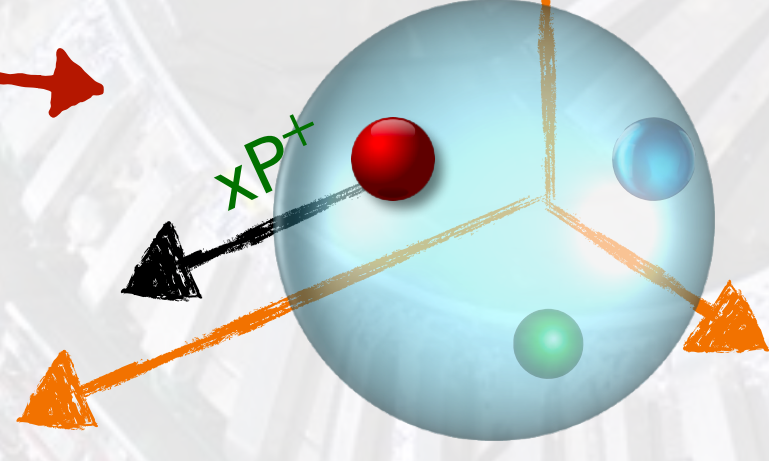
GTMD



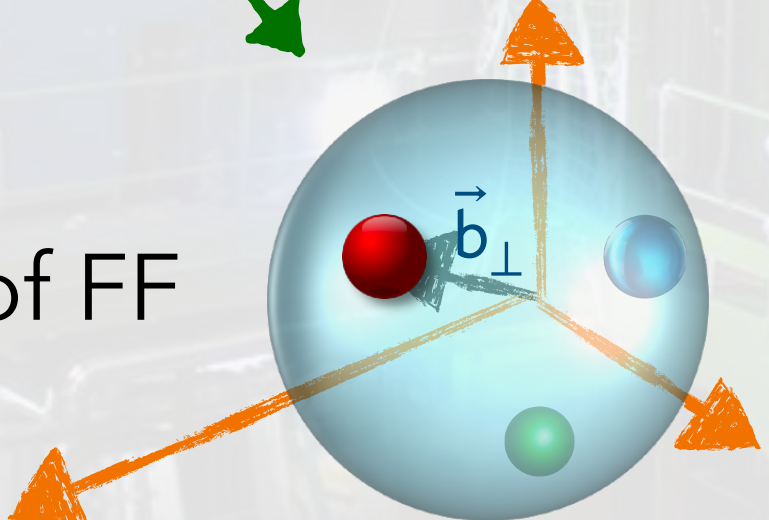
TMD



1-body Functions!



FT of FF



- $\int d^2k_{\perp}$
- $\int d^2b_{\perp}$
- $\int dx$