Photo diffusion in LArTPCs through a telegrapher equation

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The context

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- Lookup libraries can be too large for large detectors.
- The Radiative Transfer Equation (RTE) is high dimensional and might be slow.
- Simplifications of the RTE, like the diffusion equation, might not capture the physical reality.

A working approach

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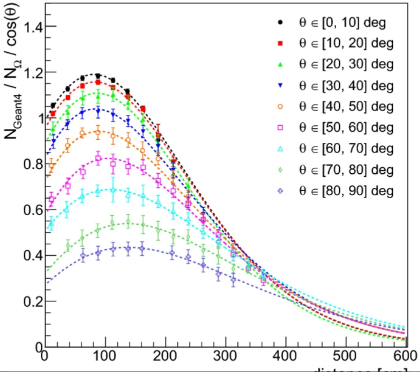
$$N_\Omega = e^{-d/\lambda_{abs}} \Delta E \cdot S_\gamma(\mathcal{E}) \frac{\Omega}{4\pi}$$



Relativistic diffusion

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- Their proposal is based on the explicit derivation of the equation for the 1d case and adjustments for the 3d case to have the correct limiting behavior.
- They found a Telegrapher's equation for the *photon density* φ:

$$\Delta\varphi = \frac{\partial^2\varphi}{v^2\partial t^2} + \left(\frac{2}{\lambda_{abs}} + \frac{3}{\lambda_{rs}^*}\right)\frac{\partial\varphi}{v\partial t} + \frac{1}{\lambda_{abs}}\left(\frac{1}{\lambda_{abs}} + \frac{3}{\lambda_{rs}^*}\right)\varphi$$

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$$\begin{aligned}\varphi(t,\vec{x}) =& \frac{3^{3/2} e^{-\frac{v}{2}\left(\frac{2}{\lambda_{ABS}}+\frac{3}{\lambda^*_{RS}}\right)t}}{4\pi (v\lambda^*_{RS})^{3/2}} \left[\frac{\delta(t-\|\vec{x}-\vec{x}_0\|/v)}{\|\vec{x}-\vec{x}_0\|} \sqrt{\frac{v\lambda^*_{RS}}{3}} I_0\left(\frac{3v}{2\lambda^*_{RS}}\sqrt{t^2-\frac{\|\vec{x}-\vec{x}_0\|^2}{v^2}}\right)\right. \\ &\left. + \frac{\sqrt{3v}}{2\sqrt{\lambda^*_{RS}}} \frac{H\left(\sqrt{3v/\lambda^*_{RS}}(t-\|\vec{x}-\vec{x}_0\|/v)\right)}{\sqrt{t^2-\|\vec{x}-\vec{x}_0\|^2/v^2}}\right] I_1\left(\frac{3v}{2\lambda^*_{RS}}\sqrt{t^2-\frac{\|\vec{x}-\vec{x}_0\|^2}{v^2}}\right) \\ =& \varphi_{wave}(t,\vec{x}) + \varphi_{dif}(t,\vec{x}),\end{aligned}$$

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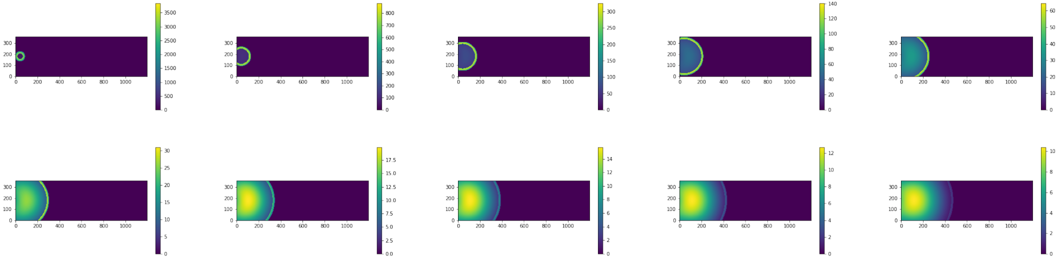
where I_0 and I_1 are modified Bessel functions of the first kind.

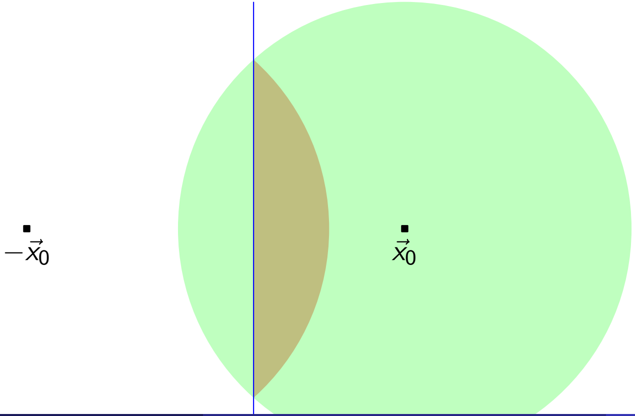
Propagation of radiation

Photon density distribution inside the detector for times 1.1, 2.1, 3.1, 4.1, 5.1, 6.1, 7.1, 8.1, 9.1 and 10.1 ns



Boundary effects and photon flow

The photon density does not give a direct way to calculate photon directions.
I consider flow on the boundary (where the detectors are) to be proportional to the fraction of the volume of the sphere that is inside the detector. Then reduce the photon density inside the detector using a mirror image of the emission point outside the detector.



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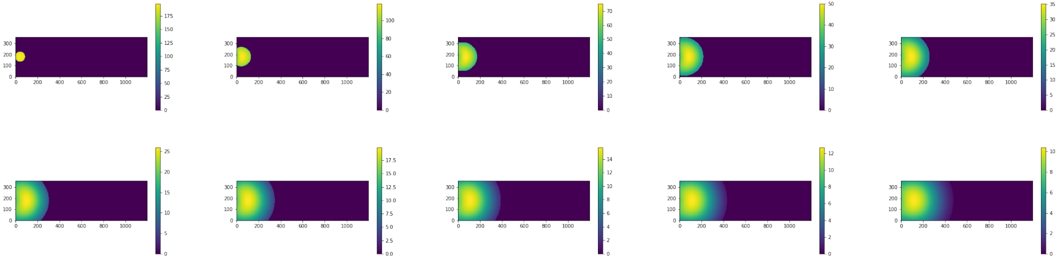
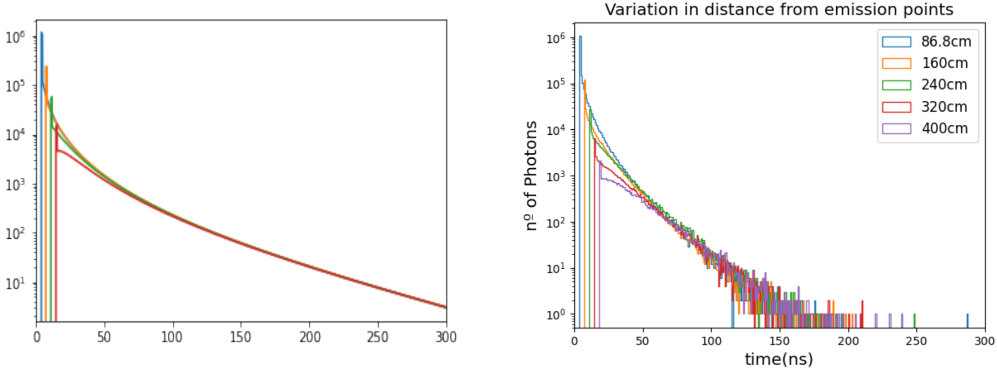
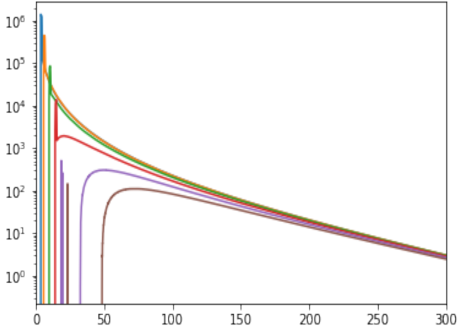


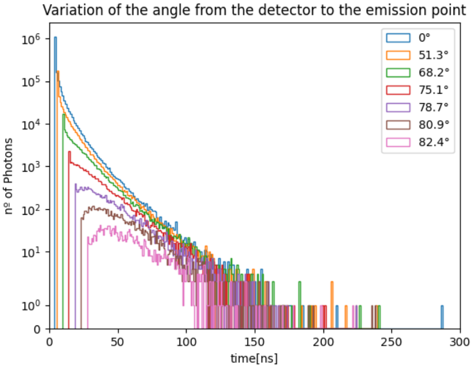
Total flow by distance

The number of photons through a detector (9.3×9.3 cm^2) in the middle of the LArTPC (absorbing box $365 \times 1200 \times 1400$ cm^3) with source in front of it at distances $d$cm (86.4; 160; 240; 320; 400). Comparison with MC (thanks to Vitor).



The number of photons through a detector (9.3×9.3 cm^2) from a source in the middle of one side 80cm away, with detector on the wall displaced by 0cm, 100cm, 200cm, ..., 600cm. Comparison with MC (thanks to Lorena).





Comparison of the total number of photons collected

Distance	Monte Carlo (MC)	Total Flow (TF)	MC / TF	Angle	Monte Carlo (MC)	Total Flow (TF)	MC / TF
86.4	1781035	1679980.2	1.06	51.3	480119	392953.6	1.22
160	417092	411190.4	1.01	68.2	107890	105181.6	1.03
240	157487	165655.9	0.95	75.1	32888	31557.4	1.04
320	70605	80408.1	0.88	78.7	11624	20003.2	0.58
400	34349	43188.8	0.8	80.9	4454	11758	0.38
				82.4	1678	6795.6	0.25

Thank you for your attention!