Study of Ionization Form Factor: Insights from Argon and Xenon Targets

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DM-electron Scattering

A dark matter (DM) particle can scatter a bound electron from the (n, l) shell of an atom in a noble liquid and ionize it with an energy $E_{er} = k'^2/(2m_e)$. Considering that the coupling of the DM-electron interaction is model-independent [1], being parameterized by the dark matter form factor $F_{\rm DM}(q)$ and the non-relativistic cross-section for free scattering $\bar{\sigma}_e$ at a fixed momentum transfer $q = \alpha\, m_e$. The velocity-averaged differential ionization cross-section of this process is given by (1):

$$\frac{d\left\langle \sigma^{nl}_{ion} v\right\rangle}{d\ln E_{er}} = \frac{\bar{\sigma}_e}{8\mu^2_{\chi e}} \times \int dq\, q\, |f^{nl}_{ion}(k',\, q)|^2\, |F_{\rm DM}(q)|^2\, \eta(v_{min})\ , \qquad (1)$$

where $\eta(v_{min}) = \left\langle v^{-1}\,\theta(v - v_{min})\right\rangle$ is the inverse mean speed function of the minimum velocity, v_{min} , required for scattering. The standard halo model was assumed to describe the Maxwell-Boltzmann velocity distribution, with a circular velocity of the Sun equal to 220 km/s and a cutoff at 544 km/s. For the velocity of the Earth in the galactic rest frame, a velocity of 244 km/s was assumed [2].

Ionization Form Factor

The ionization form factor of a bound electron in the (n, l) shell with a final momentum k', after receiving a momentum q, is calculated numerically using equation (2) and shown in Figures 1 and 2.

$$|f^{nl}_{ion}(k', q)|^2 = \frac{4k'}{(2\pi)^3} \sum_{l'L} (2l+1)(2l'+1)(2L+1) \begin{bmatrix} l & l' & L \\ 0 & 0 & 0 \end{bmatrix}^2 \times \left| \int r^2 dr\, \tilde{R}_{k'l'}(r) R_{nl}(r)\, j_L(qr) \right|^2 . \qquad (2)$$

The radial part of the electron's wave function is described by Roothaan-Hartree-Fock wave functions, tabulated in [3] as linear combinations of Slater orbitals $R_{nl}(r)$. For the outgoing electron, the wave function of the final state, $\tilde{R}_{k'l'}(r)$, is approximated by a plane wave corrected by a Fermi factor [2].

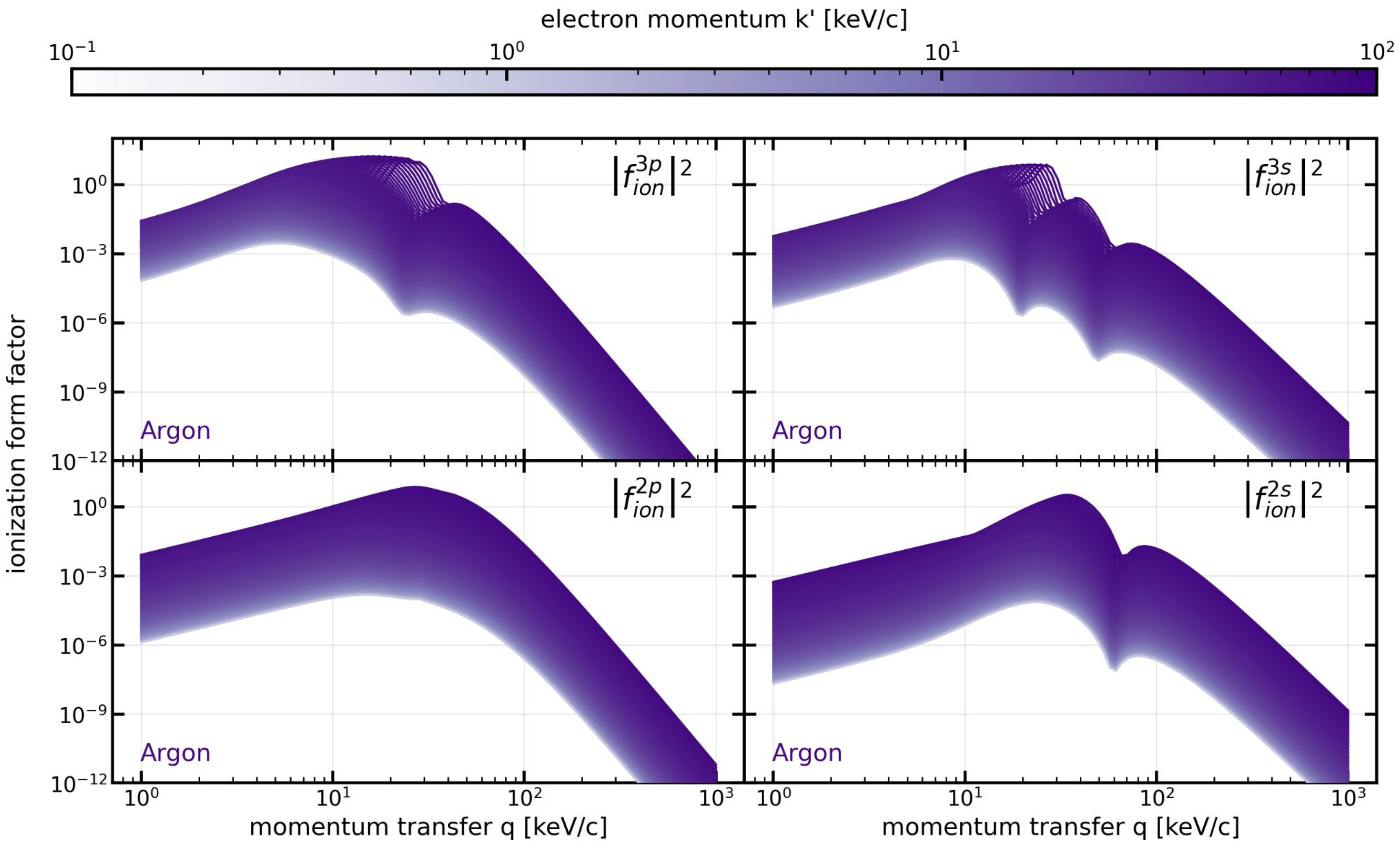

Fig 1. Ionization form factor for the four outermost shells of argon, calculated using DarkARC [4].

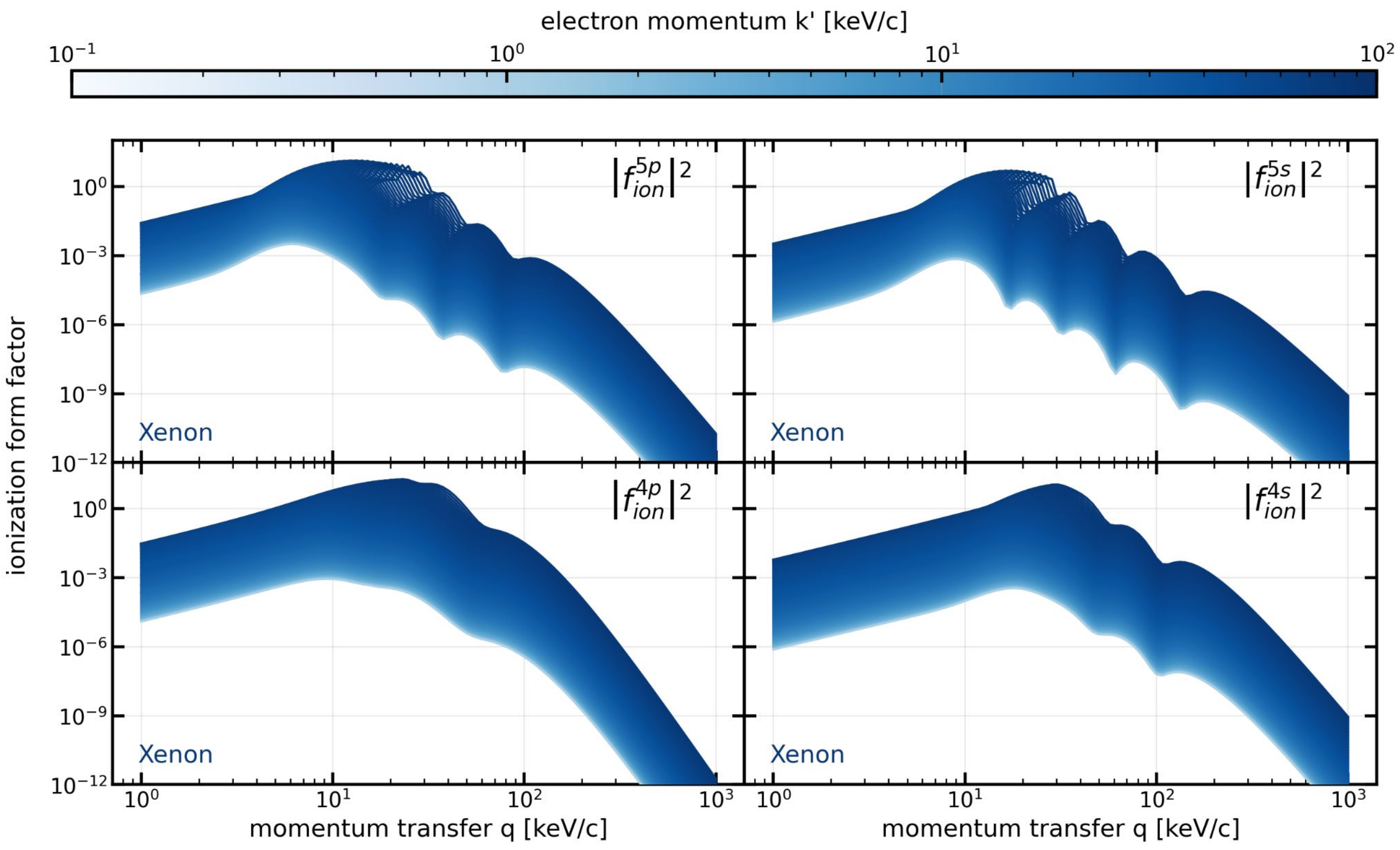

Fig 2. Ionization form factor for the four outermost shells of xenon, calculated using DarkARC [4].

[1] Essig R. et al, Phys. Rev. D 85, 076007 (2012).
[2] The DarkSide Collaboration, Phys. Rev. Lett. 121, 111303 (2018).
[3] BUNGE C. F. et al, Atomic Data and Nuclear Data Tables 53, 113 (1993).
[4] CATENA, R. et al, Phys. Rev. Research 2, 033195 (2020).

Acknowledgments

CAPES

Kinematic Limitations

From the kinematics of DM-electron scattering, it is possible to deduce that the minimum velocity required to ionize an electron from the (n, l) shell with equation (3):

$$v_{min}(E^{nl}_b,\, k',\, q) = \frac{E^{nl}_b + k'^2/(2m_e)}{q} + \frac{q}{2m_\chi}\ , \qquad (3)$$

where E^{nl}_b is the binding energy of the electron in question. The DM velocity distribution imposes a maximum velocity that is equal to the sum of the cutoff velocity and the Earth's velocity in the galactic rest frame [4]. Therefore, the accessible region of momentum space is equivalent to that allowed by relation (4), and its boundary is highlighted in Figure 3 for different values of DM mass.

$$k' < \sqrt{2m_e\left(v_{max}q - \frac{q^2}{2m_\chi} + E^{nl}_b\right)}\ . \qquad (4)$$

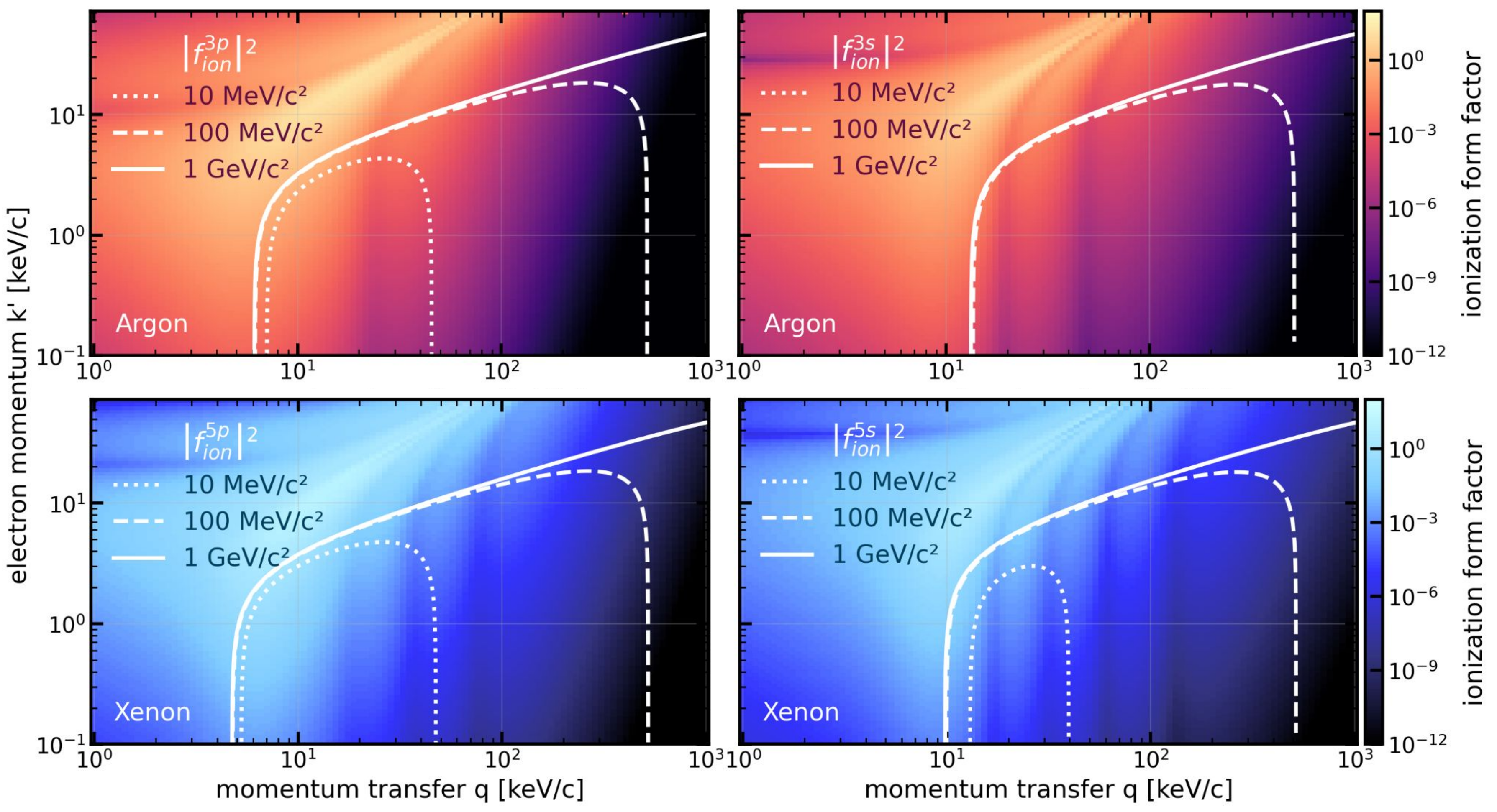

Fig 3. Boundaries of the allowed regions in the momentum space for DM-electron scattering, for the two outermost shells of argon (top) and xenon (bottom), considering different DM masses. The color bars indicate the form factor for these shells.

Theoretical Rates

The differential ionization rate as a function of the electron recoil energy, described in (5), depends on the the local dark matter density, $\rho_\chi = 0.4\,{\rm GeV/cm^3}$, number of target atoms per unit mass, N_T , as well as the cross-section (1).

$$\frac{dR^{nl}}{d\ln E_{er}} = N_T \frac{\rho_\chi}{m_\chi} \frac{d\left\langle \sigma^{nl}_{ion} v\right\rangle}{d\ln E_{er}}\ . \qquad (5)$$

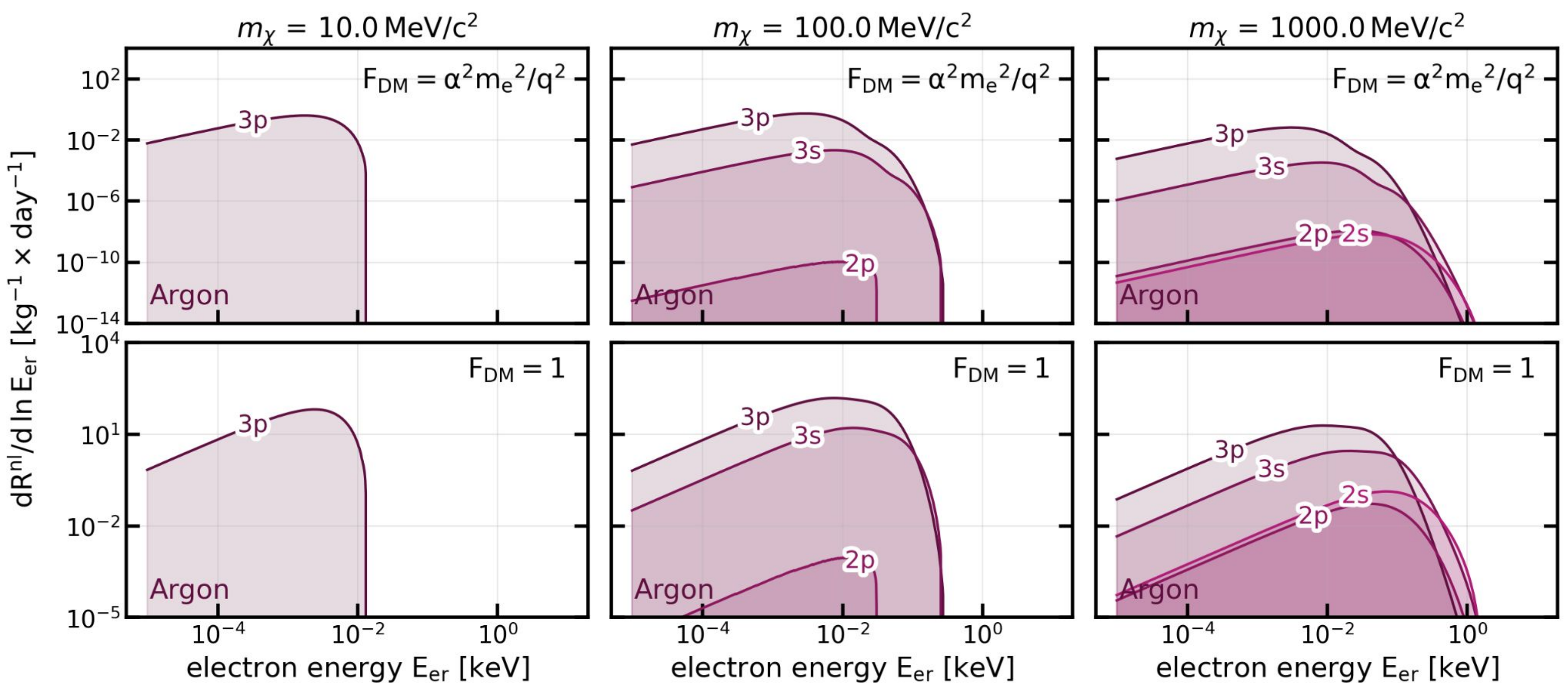

Fig 4. Theoretical differential ionization rate for each shell of argon, considering 3 DM masses and a cross-section of $\bar{\sigma}_e$=10^{-36} cm^2. Two models for the DM form factor are considered: light vector mediator (top) and heavy mediator (bottom).

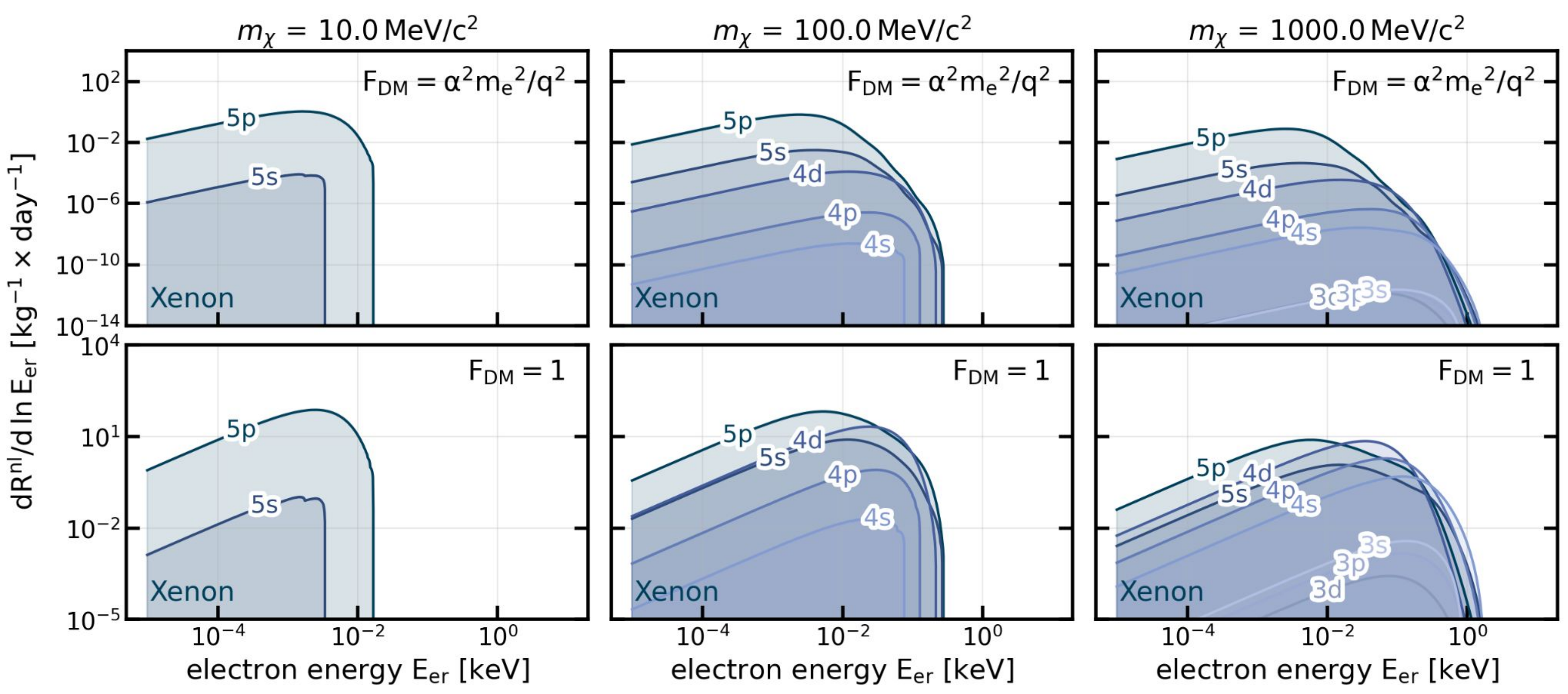

Fig 5. Theoretical differential ionization rate for each shell of xenon, considering 3 DM masses and a cross-section of $\bar{\sigma}_e$=10^{-36} cm^2. Two models for the DM form factor are considered: light vector mediator (top) and heavy mediator (bottom).

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