Heavy quark physics with partially-quenched twisted quarks

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ALPHA Collaboration

Hadronic physics and heavy quarks on the lattice, Dublin, June 4, 2024

lattice QCD mixed action

setup : mixed action with Wilson twisted mass (Wtm) valence quarks
on CLS $N_\mathrm{f} = 2 + 1$ ensembles

motivation :

- alternative/complementary way to control lattice artefacts
  $\rightsquigarrow$ universality

- steps :

  - light-quark sector : sea/valence matching, scale setting
    light-quark masses

  - heavy-quark sector : [2309.14154]
    charm quark mass
    D-mesons leptonic and semi-leptonic decays

sea sector : $N_f = 2+1$ CLS [1411.3982, 1608.08900, 1712.04884]

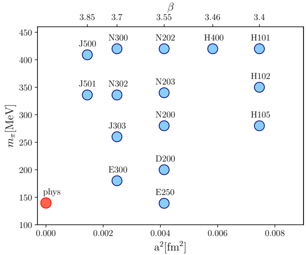
- lattice action:
  - gauge action: Lüscher-Weisz gauge action (tlSym)
  - fermion action: $N_f = 2+1$ Wilson fermions with non-perturbative $c_{\rm SW}$
- open boundary conditions in time : relevant for heavy-quark physics
- chiral trajectory $M_q = \mathrm{diag}\,(m_{q_u}, m_{q_d}, m_{q_s})$

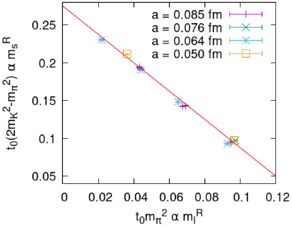
$$\mathrm{tr}\, M_q \;=\; m_{q_u} + m_{q_d} + m_{q_s} \;=\; \mathrm{const.}$$

renormalised chiral trajectory [Bruno, Korzec, Schaefer, 1608.08900]

$$\phi_4 \;\equiv\; 8t_0\left(\tfrac{1}{2}M_\pi^2 + M_K^2\right) \;=\; \tfrac{1}{2}\phi_2 + \phi_K \;=\; \mathrm{const} \;=\; \phi_4^{\rm phys}$$

- lattice spacings: $a \approx$ 0.085, 0.075, 0.063, 0.050, 0.038 fm $M_\pi L \geq 3.9$





valence quarks: Wilson twisted mass

[ALPHA, hep-lat/0101001; Frezzotti and Rossi hep-lat/0306014, Pena et al., hep-lat/0405028]

- valence action

$$D_{\mathrm{Wtm}} = D_{\mathrm{W}}^{\mathrm{SW}} + m \pm i\gamma_5\mu$$

- maximal twist $\alpha = \frac{\pi}{2}$:

$$m = \tilde{m}_{cr} \quad \rightsquigarrow \quad m_{12}^{\mathrm{val}} = 0$$
$$\mu = \{\mu_{ud}, \mu_s, \mu_c\}$$

flavours: i=1, 2 $\rightarrow (u, d)$; i=3, 4 $\rightarrow (s, s')$

- properties:
  - absence of $\mathcal{O}(a\mu)$ lattice artefacts in physical quantities at maximal twist
  - SW term: same renormalization in sea and valence, valence flavour breaking cutoff effects
  - μ acts as an infrared cutoff

- mixed action: match sea & valence quark masses (at maximal twist)

mixed action : lattice artefacts

[A. Bussone et al., 1812.01474]

extension of [Bhattacharya et al., hep-lat/0511014] to Wtm

- singlet and non-singlet bilinears and masses
- improvement of the twisted mass μ_j : mixed action with valence Wtm at maximal twist

$$\widehat{\mu}_j = \frac{1}{Z_\mathrm{P}} \mu_j \left[1 + a\overline{b}_\mu \,\mathrm{tr}\, M_\mathrm{sea}\right] + \mathrm{O}(a^2)$$

$\overline{b}_\mu = O(g_0^4)$

- Wilson fermions : current quark mass from PCAC relation

$$\widehat{m}_\mathrm{ij} = \frac{Z_\mathrm{A}}{Z_\mathrm{P}} m_\mathrm{ij} \left[1 + a(\bar{b}_\mathrm{A} - \bar{b}_\mathrm{P}) \,\mathrm{tr}\, M_\mathrm{sea} + a(\tilde{b}_\mathrm{A} - \tilde{b}_\mathrm{P}) m_\mathrm{ij}\right] + \mathrm{O}(a^2)$$

Z_P : non-perturbative [Schrödinger Functional (SF)]

[ALPHA, 1802.05243]

Z_A : non-perturbative [chirally rotated SF]

[ALPHA, 1808.09236]

m_ij includes non-perturbative c_A

[ALPHA, 1502.04999]

$\tilde{b}_\mathrm{A} - \tilde{b}_\mathrm{P}$: non-perturbative

[ALPHA, 1906.03445]

$\bar{b}_\mathrm{A}$ & $\bar{b}_\mathrm{P} = O(g_0^4)$

mixed action : lattice artefacts   twist angle

- flavours : i=1,2 $\to (u, d)$; i=3,4 $\to (s, s')$

$$\widehat{\mu}_{\rm i} = = \frac{1}{Z_{\rm P}}\, \mu_{\rm i}\, \left[1 + a\bar{b}_\mu\, \mathrm{tr}\, M_{\rm sea}\right] + \mathrm{O}(a^2)$$

$$\bar{b}_\mu = O(g_0^4)$$

- Wilson twisted mass fermions : current quark mass

$$\widehat{m}_{\rm ij}^{\rm val} = \frac{Z_{\rm A}}{Z_{\rm P}}\, m_{\rm ij}^{\rm val}\, [1 + a(\bar{b}_{\rm A} - \bar{b}_{\rm P})\, \mathrm{tr}\, M_{\rm sea} + a(\tilde{b}_{\rm A} - \tilde{b}_{\rm P}) m_{\rm ij}^{\rm val}] + \mathrm{O}(a\mu_{\rm i}^2) + \mathrm{O}(a^2)$$

$m_{\rm ij}^{\rm val}$ includes non-perturbative c_A [ALPHA, 1502.04999]

$\tilde{b}_{\rm A} - \tilde{b}_{\rm P}$: non-perturbative [ALPHA, 1906.03445] ; $\bar{b}_{\rm A}$ & $\bar{b}_{\rm P} = O(g_0^4)$

- deviation from maximal twist : $\theta_{\rm i}$

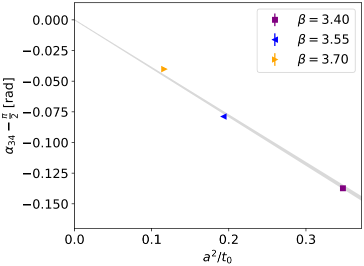
$$\tan\theta_{\rm ij} = \tan\left(\alpha_{\rm ij} - \frac{\pi}{2}\right) = \frac{\widehat{m}_{\rm ij}^{\rm val}}{\widehat{\mu}_{\rm i}} = \frac{Z_{\rm A} m_{\rm ij}^{\rm val}}{\mu_{\rm i}}\, [1 + a(\tilde{b}_{\rm A} - \tilde{b}_{\rm P}) m_{\rm ij}^{\rm val}] + O(a\mu_{\rm i}) + O(a^2)$$

tuning to maximal twist : $\theta_{12} = 0$

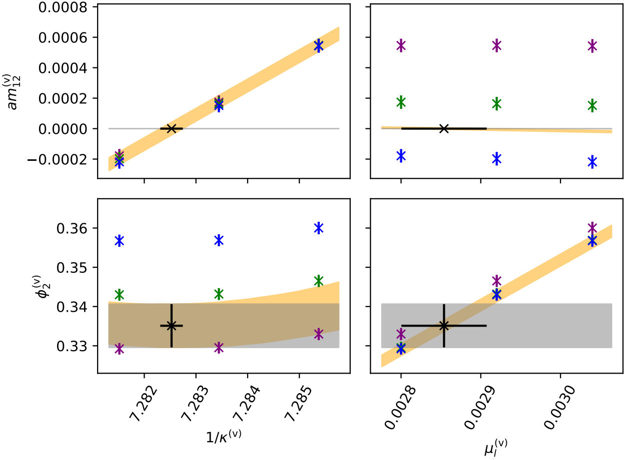
$\rightsquigarrow$ what are the deviations θ_{34}

in the strange quark sector?

$$\phi_2 = 0.284(2)$$
$$\phi_4 = 1.101(9)$$



matching of mixed action



$\phi_2^{\mathrm{val}} \hat{=} \phi_2^{\mathrm{sea}}$

$\phi_4^{\mathrm{val}} \hat{=} \phi_4^{\mathrm{sea}}$

$m_{12}^{\mathrm{val}} \hat{=} 0$

$\phi_2 \equiv 8t_0 M_\pi^2$

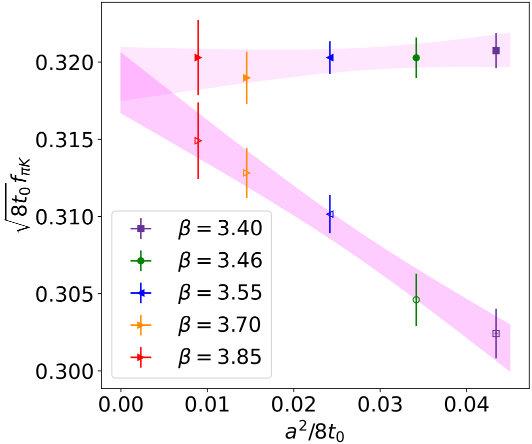
$\phi_4 \equiv 8t_0 \left(\frac{1}{2} M_\pi^2 + M_K^2\right) = \phi_4^{\mathrm{phys}}$ [1.3%]

$\left(M_\pi^{\mathrm{val}}\right)^2 \propto \sqrt{\left(m_\ell^{\mathrm{R}}\big|_v\right)^2 + \left(\mu_\ell^{\mathrm{R}}\right)^2}$

$a = 0.085$ fm
$M_\pi = 280$ MeV
$M_\pi L = 3.9$

continuum-limit scaling

$$f_{\pi K} \equiv \frac{2}{3}\left(\frac{1}{2} f_\pi + f_K\right)$$



Wilson twisted mass

Wilson

symmetric point: $m_\ell = m_s$, $M_\pi = M_K = 420\,\mathrm{MeV}$

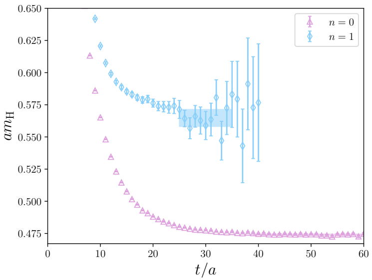
charm sector : ground-state extraction

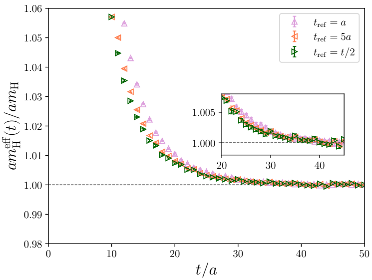
- setup : mixed action with Wilson twisted mass (Wtm) valence quarks
  on CLS $N_\mathrm{f} = 2 + 1$ ensembles
  Wtm charm quark is partially quenched : three masses around physical value
- meson masses and matrix elements extracted from a GEVP

$$C(t) = \begin{bmatrix} f_\mathrm{P}(t) & f_\mathrm{P}(t+\tau) \\ f_\mathrm{P}(t+\tau) & f_\mathrm{P}(t+2\tau) \end{bmatrix}$$

$\tau = 3a$

$$C(t) v_n(t, t_\mathrm{ref}) = \lambda_n(t, t_\mathrm{ref}) C(t_\mathrm{ref}) v_n(t, t_\mathrm{ref})$$

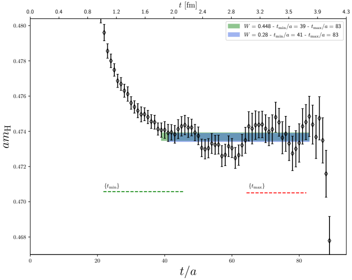


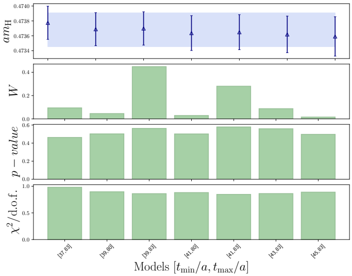


$a = 0.050$ fm ; $M_\pi = 190$ MeV ; $M_\pi L = 4.1$

charm sector : ground-state extraction

meson masses and matrix elements extracted from a GEVP



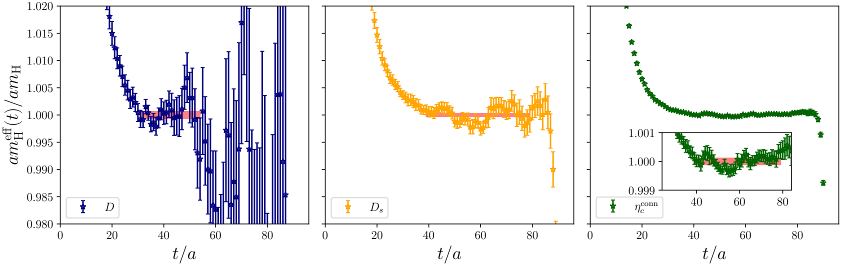


$a = 0.050$ fm ; $M_\pi = 190$ MeV ; $M_\pi L = 4.1$

1100 configurations , $N_{\mathrm{noise}} = 6$

charm sector : ground-state extraction

meson masses : m_D, m_{D_s}, $m_{\eta_c}^{(\text{conn.})}$



signal/noise :

$(m_{\eta_C} + m_\pi) - 2m_D \approx -610\,\text{MeV}$

$(m_{\eta_C} + m_{\bar{s}s}) - 2m_{D_s} \approx -260\,\text{MeV}$

$(m_{\eta_C} + m_{\eta_C}) - 2m_{\eta_C} = 0$

$a = 0.050$ fm ; $M_\pi = 190$ MeV ; $M_\pi L = 4.1$
1100 configurations , $N_{\text{noise}} = 6$

matching charm quark mass

- consider two matching conditions

$$\phi_H^{(i)} \equiv \sqrt{8t_0}m_H^{(i)} \triangleq \phi_H^{(i),\mathrm{isoQCD}}$$

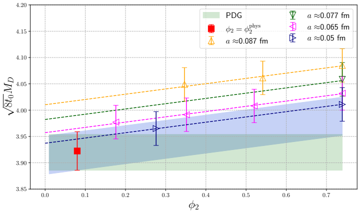
$$m_H^{(1)} \triangleq m_{\overline{D}}^{\mathrm{isoQCD}} \equiv \frac{2}{3}m_D^{\mathrm{isoQCD}} + \frac{1}{3}m_{D_s}^{\mathrm{isoQCD}}$$

$$m_H^{(2)} \triangleq m_{\eta_c}^{(\mathrm{conn.,isoQCD})}$$

[includes 100% uncertainty from absence of quark-disconnected contributions]

- chiral-continuum behaviour of $D_{(s)}$ meson masses

$$\sqrt{8t_0}m_{D_{(s)}}(a, \phi_2, \phi_H^{(i)}) = p_0 + p_1\phi_2 + p_2\phi_H^{(i)} + c_1\frac{a^2}{8t_0}$$



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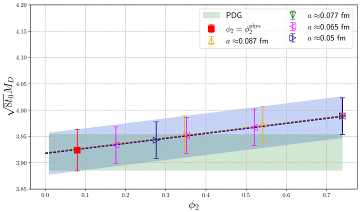

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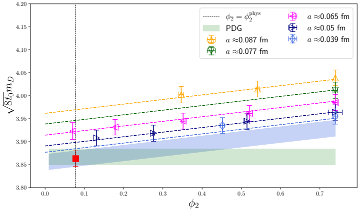
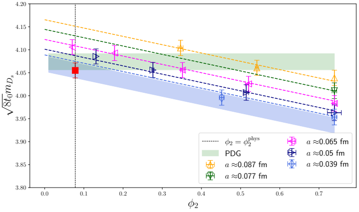




$$m_H^{(2)} \hat{=} m_{\eta_c}^{(\mathrm{conn.,isoQCD})}$$

charm quark mass

RGI quark mass

$$\mu_c^{\mathrm{RGI}} = \frac{M}{\widehat{m}(\mu_{\mathrm{had}})} \; Z_{\mathrm{P}}^{-1}(g_0^2, \mu_{\mathrm{had}})\, \mu_c$$

non-perturbative running [Schrödinger Functional] [ALPHA, 1802.05243]

$$\frac{M}{\widehat{m}(\mu_{\mathrm{had}})} = 0.9148(88) \qquad [1\%]$$

from $\mu_{\mathrm{had}} = 233(8)\ \mathrm{MeV}$ to $\mu_{\mathrm{pt}} \sim \mathrm{O}(M_{\mathrm{W}})$

continuum factor: applies to Wilson and Wtm regularizations

- mass dependence $\phi_2 = 8t_0 M_\pi\,, \quad \phi_H^{(i)} = \sqrt{8t_0} M_H^{(i)}$

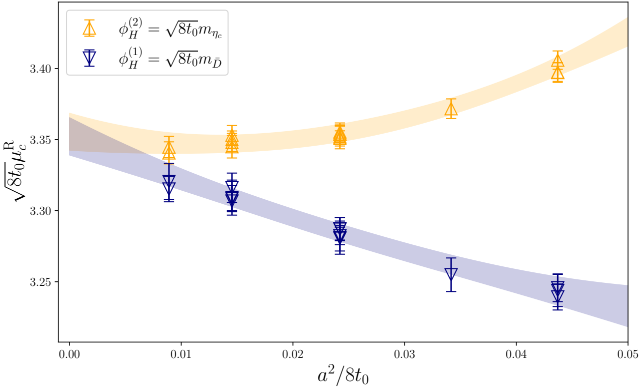
$$\sqrt{8t_0}\, m_c^{\mathrm{RGI}}(0, \phi_2, \phi_H^{(i)}) = p_0 + p_1 \phi_2 + p_3 \phi_H^{(i)}$$

- discretization effects

$$c_M(a, \phi_2, \phi_H) = \frac{a^2}{8t_0}\left(c_1 + c_2 \phi_2 + c_3 \phi_H^2\right) + \frac{a^4}{(8t_0)^2}\left(c_4 + c_5 \phi_H^2 + c_6 \phi_H^4\right)$$

charm quark mass

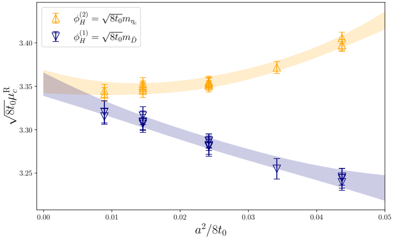
lattice spacing dependence

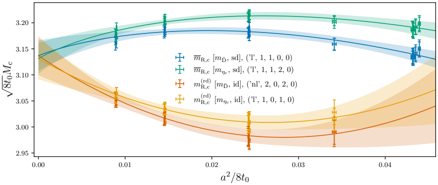


$$m_H^{(2)} \hat{=} m_{\eta_c}^{(\mathrm{conn.,isoQCD})}$$
$$m_H^{(1)} \hat{=} m_{\overline{D}}^{\mathrm{isoQCD}}$$

charm quark mass

lattice spacing dependence





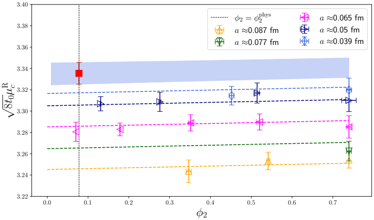
Wilson twisted mass ; SF scheme

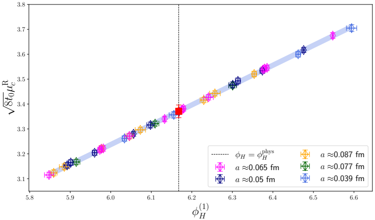
Wilson ; RGI mass

[J. Heitger, F. Jorwig & S. Kuberski, 2101.02694]

charm quark mass

mass dependence





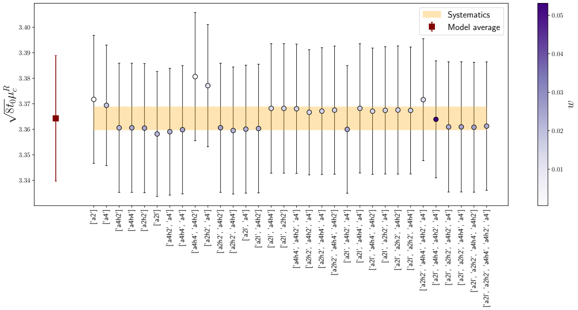
$m_H^{(1)} \hat{=} m_{\overline{D}}^{\mathrm{isoQCD}}$

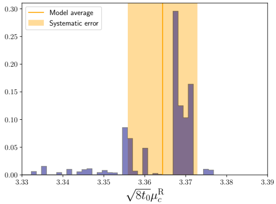
$m_H^{(2)} \hat{=} m_{\eta_c}^{(\mathrm{conn.,isoQCD})}$

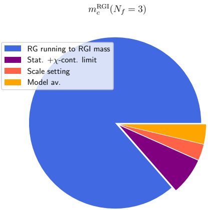
charm quark mass [preliminary]

lattice spacing dependence :

$$c_{\mu_c}(a, \phi_2, \phi_H) = \frac{a^2}{8t_0}(c_1 + c_2\phi_2 + c_3\phi_H^2) + \frac{a^4}{(8t_0)^2}(c_4 + c_5\phi_H^2 + c_6\phi_H^4)$$







$$m_H^{(2)} \hat{=}\, m_{\eta_c}^{(\mathrm{conn.,isoQCD})}$$

$$M_c^{\mathrm{RGI}}(N_f = 3) = 1.486(5)(3)(15)\ \mathrm{GeV}\ \ [1\%]$$

systematic effects: model averaging

- generalised least square

$$\chi^2_K(a) = [y - f(a)]^T K^{-1} [y - f(a)]$$

$K = C$ covariance matrix; $K = K_{\rm syst}$

- Akaike Information Criterion

$$\mathrm{AIC} = \chi^2_C + 2N_{\rm par}$$

- Takeuchi Information Criterion

[Frison, 2302.06550]

$$\mathrm{TIC} = \chi^2_K - 2\langle\chi^2_K\rangle$$

$\langle\chi^2_K\rangle$: [Bruno & Sommer, 2209.14188]

- model m: apply weight W_m

$$W_m \propto \exp\left(-\frac{1}{2}\mathrm{TIC}_m\right) \quad \text{with} \quad \sum_{m=1}^{M} W_m = 1$$

- if $K = C$ $\rightarrow$ $\langle\chi^2_C\rangle = N_{\rm dof} = N^{\rm tot}_{\rm dat} - N_{\rm par}$

  if data cuts: $N^{\rm cut}_{\rm dat} = N^{\rm tot}_{\rm dat} - N_{\rm cut}$ $\rightarrow$ $\langle\chi^2_C\rangle = N^{\rm cut}_{\rm dat} - N_{\rm par}$

$$W_m \propto \exp\left(-\frac{1}{2}\left[\chi^2_C + 2N_{\rm par} + 2N_{\rm cut}\right]_m\right)$$

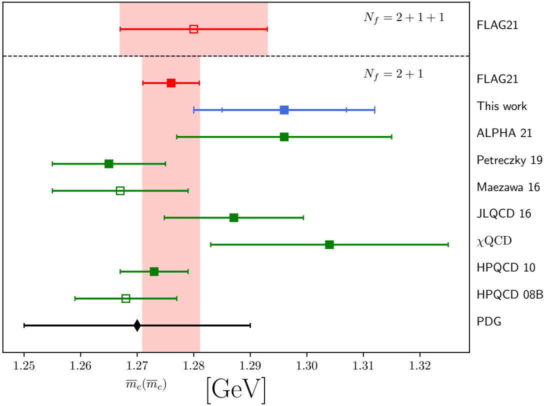
[Jay & Neil, 2008.01069]

- model average (MA)

$$\langle O\rangle_{\rm MA} = \sum_{m=1}^{M} \langle O_m\rangle W_m$$

$$\sigma^2_{\rm syst} = \langle O^2\rangle_{\rm MA} - \langle O\rangle^2_{\rm MA}$$

charm quark mass [preliminary]



$$\overline{m}_c^{\overline{\mathrm{MS}}}(\mu = \overline{m}_c, N_f = 4) = 1.295(11)(13)_\Lambda(5)_{\mathrm{trunc.}}\ \mathrm{GeV}$$

$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 341(12)$ MeV [ALPHA, 1706.03821]

decay constants : $f_{D_{(s)}}$

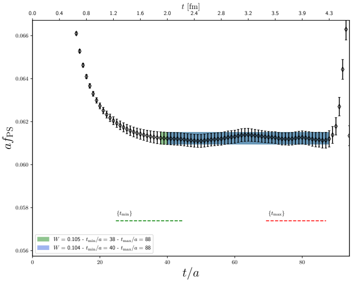
$$f_{\rm PS}^{qr} = \sqrt{\frac{2L^3}{m_{\rm PS}^3}}(|\mu_q| + |\mu_r|)\ |\langle 0|P^{qr}|P^{qr}(\mathbf{p}=\mathbf{0})\rangle|$$

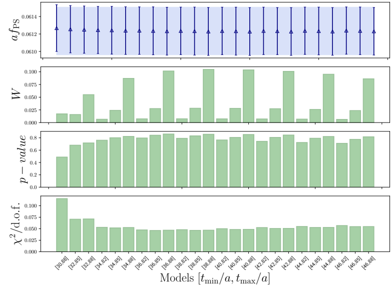
matrix element extracted from GEVP

combined study of mass and lattice spacing dependence of ϕ_D & ϕ_{D_s}

$$\phi_{D_{(s)}} = (8t_0)^{3/4}\, f_{D_{(s)}} \sqrt{m_{D_{(s)}}}$$

matrix elements extracted from a GEVP





$a = 0.050$ fm ; $M_\pi = 190$ MeV ; $M_\pi L = 4.1$

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matrix element extracted from GEVP

combined study of mass and lattice spacing dependence of ϕ_D & ϕ_{D_s}

$$\phi_{D_{(s)}} = (8t_0)^{3/4}\, f_{D_{(s)}} \sqrt{m_{D_{(s)}}}$$

- global fit of Φ_D and Φ_{D_s}
- generic functional form

$$\Phi_{D_s} = \Phi_\chi[1 + \delta_{\chi\rm PT}^{D_{(s)}}][1 + \delta_a^{D_{(s)}}]$$

decay constants : $f_{D_{(s)}}$

- continuum heavy-quark mass dependence

$$\Phi_\chi = \Phi_0 \left[1 + p_h^{(1)} \frac{1}{\Phi_H} + p_h^{(2)} \frac{1}{\Phi_H^2} + \ldots \right], \qquad \Phi_H = \sqrt{8t_0} m_H$$

decay constants : $f_{D_{(s)}}$

- continuum light-quark mass dependence

$$\delta^{D}_{\chi\mathrm{PT}} = -\frac{1+3g^2}{64\pi^2\phi_f^2}\left[3\mathcal{L}_\pi + 2\mathcal{L}_K + \frac{1}{3}\mathcal{L}_\eta\right] + \frac{4\phi_2}{\phi_f^2}\left(p_\chi^{(0)} + p_\chi^{(2)}\frac{\phi_2}{\phi_f^2} + \frac{p_\chi^{(4)}}{\phi_H}\right)$$

$$\delta^{D_s}_{\chi\mathrm{PT}} = -\frac{1+3g^2}{64\pi^2\phi_f^2}\left[4\mathcal{L}_K + \frac{4}{3}\mathcal{L}_\eta\right] + \frac{8\,(\phi_4-\phi_2)}{\phi_f^2}\left(p_\chi^{(0)} + p_\chi^{(2)}\frac{\phi_2}{\phi_f^2} + \frac{p_\chi^{(4)}}{\phi_H}\right)$$

where

$$\mathcal{L}_\pi = \phi_2\log(\phi_2), \qquad \mathcal{L}_K = \left(\phi_4 - \frac{1}{2}\phi_2\right)\log\left(\phi_4 - \frac{1}{2}\phi_2\right), \qquad \mathcal{L}_\eta = \left(\frac{4}{3}\phi_4 - \phi_2\right)\log\left(\frac{4}{3}\phi_4 - \phi_2\right)$$

decay constants : $f_{D_{(s)}}$

- lattice spacing dependence

$$\delta_a^D = \frac{a^2}{8t_0}\left[p_a^{(0)} + \phi_2\left(p_a^{(1)} + p_a^{(3)}\phi_H^2\right) + p_a^{(2)}\phi_H^2\right]$$

$$\delta_a^{D_s} = \frac{a^2}{8t_0}\left[p_a^{(0)} + 2\left(\phi_4 - \phi_2\right)\left(p_a^{(1)} + p_a^{(3)}\phi_H^2\right) + p_a^{(2)}\phi_H^2\right]$$

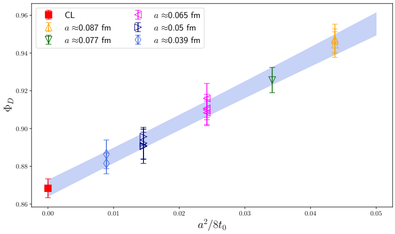
decay constants: $f_{D_{(s)}}$ [preliminary]

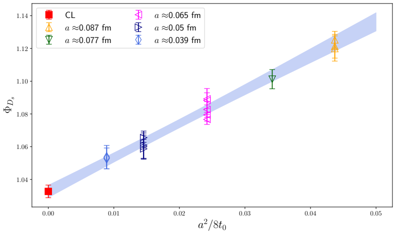
$$f_{\rm PS}^{rq} = \sqrt{\frac{2L^3}{m_{\rm PS}^3}}(|\mu_q| + |\mu_r|)\ |\langle 0|P^{qr}|P^{qr}(\mathbf{p}=\mathbf{0})\rangle|$$

matrix element extracted from GEVP

combined study of mass and lattice spacing dependence of ϕ_D & ϕ_{D_s}

$$\phi_{D_{(s)}} = (8t_0)^{3/4}\, f_{D_{(s)}} \sqrt{m_{D_{(s)}}}$$



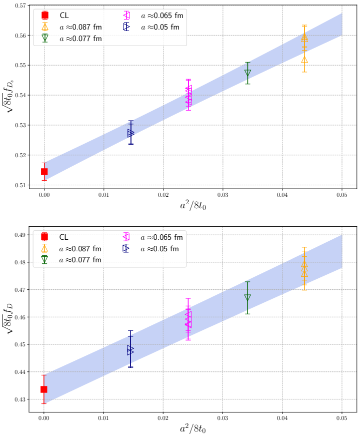


fits based on $O(a^2)$ lattice artefacts + NLO HMχPT

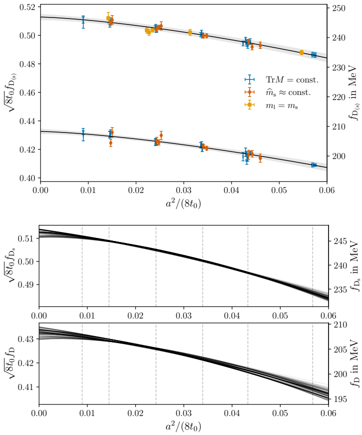
$m_H^{(1)} \hat{=} m_{\overline{D}}^{\rm isoQCD}$

decay constants : $f_{D_{(s)}}$

lattice spacing dependence



Wilson twisted mass



Wilson

[RQCD/ALPHA, S. Kuberski et al., 2405.04506]

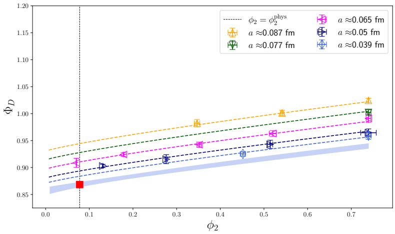
decay constants : $f_{D_{(s)}}$ [preliminary]

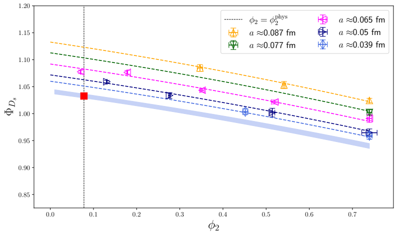
$$f^{rq}_{\mathrm{PS}} = \sqrt{\frac{2L^3}{m^3_{\mathrm{PS}}}}(|\mu_q| + |\mu_r|)\ |\langle 0|P^{qr}|P^{qr}(\mathbf{p}=\mathbf{0})\rangle|$$

matrix element extracted from GEVP

combined study of mass and lattice spacing dependence of ϕ_D & ϕ_{D_s}

$$\phi_{D_{(s)}} = (8t_0)^{3/4}\, f_{D_{(s)}} \sqrt{m_{D_{(s)}}}$$





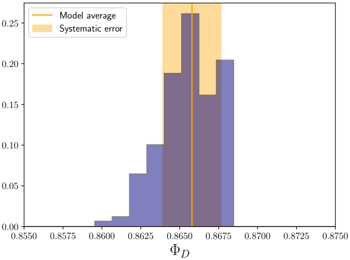
fits based on O(a^2) lattice artefacts + NLO HMχPT

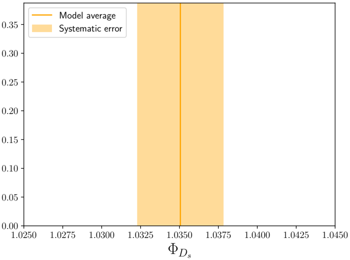
$m_H^{(1)} \hat{=}\, m_{\overline{D}}^{\mathrm{isoQCD}}$

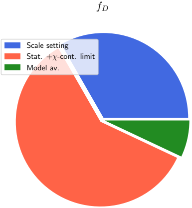
decay constants: $f_{D_{(s)}}$ [preliminary]

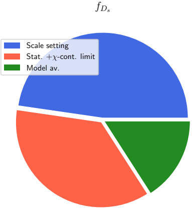
$$\Phi_{D_{(s)}} = (8t_0)^{3/4} f_{D_{(s)}} \sqrt{m_{D_{(s)}}}$$

- global fit between Φ_D and Φ_{D_s}
- 57 models $\times$ 2 matching prescriptions
- model average



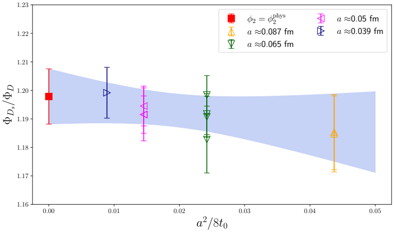


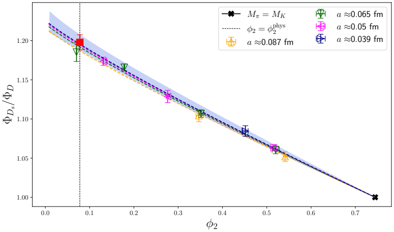




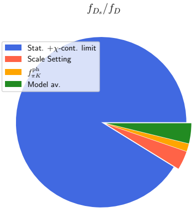
decay constants: f_{D_s}/f_D [preliminary]

$$\frac{\Phi_{D_s}}{\Phi_D} = \left[1 - \frac{1+3g^2}{64\pi^2\phi_f^2}\left[2\mathcal{L}_K + \mathcal{L}_\eta - 3\mathcal{L}_\pi\right] + \frac{4\,(2\phi_4 - 3\phi_2)}{\phi_f^2}\left(p_\chi^{(0)} + p_\chi^{(2)}\frac{\phi_2}{\phi_f^2} + \frac{p_\chi^{(4)}}{\phi_H}\right)\right] \times \left[1 + \frac{a^2}{8t_0}\,(2\phi_4 - 3\phi_2)\,\left(p_a^{(1)} + p_a^{(3)}\phi_H^2\right)\right].$$

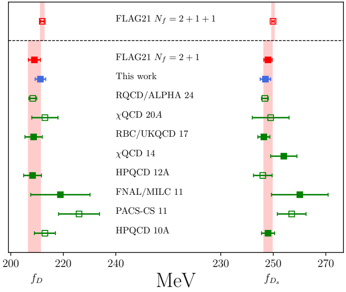


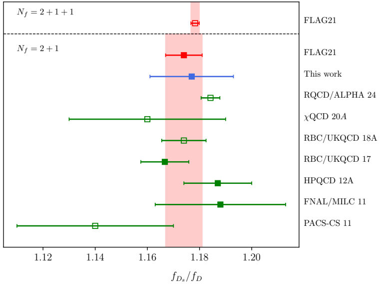


fits based on O(a^2) lattice artefacts + NLO HMχPT or Taylor expansion; $m_H^{(1)} \hat{=} m_{\overline{D}}^{\rm exp}$



decay constants : $f_{D_{(s)}}$





[2309.14154]

"RQCD/ALPHA 24" : [S. Kuberski, F. Joswig, S. Collins, J. Heitger, W. Söldner, 2405.04506]

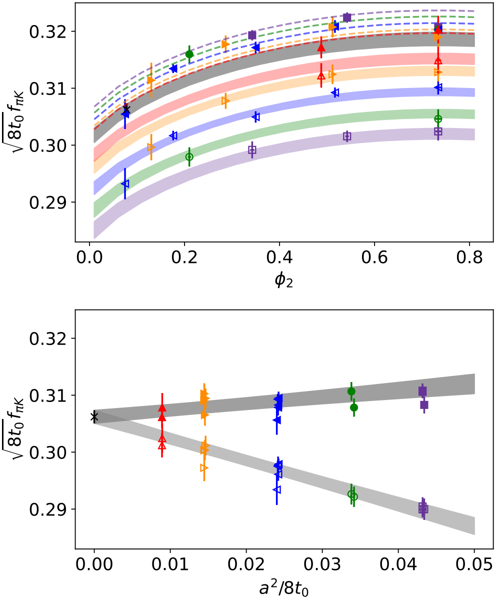
relative precision on f_{D_s} [0.9%]

continuum-chiral extrapolation of $f_{\pi K}$

$f_{\pi K} \equiv \frac{2}{3}\left(\frac{1}{2} f_\pi + f_K\right)$

systematic effects:

- additional term in χ^2: systematic effects at largest a & m_π
- SU(3) & SU(2) NLO χPT + Taylor series
- discretization effects: $O(a^2)$, $O(\phi_2 a^2)$, $O(a^2 \alpha_s^\Gamma)$ [Husung, 2206.03536]
- cuts in M_π, a, $M_\pi L$
- excited states contamination



systematic effects : t_0 [preliminary]

physical input [FLAG '21 , PDG '22]

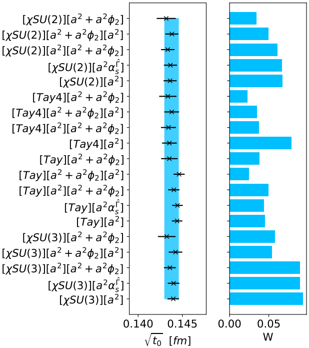
$$m_\pi^{\mathrm{isoQCD}} = m_{\pi^0}^{\mathrm{exp}} = 134.9768(5)\ \mathrm{MeV}$$

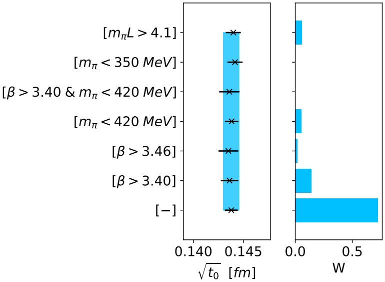
$$m_K^{\mathrm{isoQCD}} = m_{K^0}^{\mathrm{exp}} = 497.611(13)\ \mathrm{MeV}$$

$$f_\pi^{\mathrm{isoQCD}} = 130.56(2)_{\mathrm{exp}}(13)_{\mathrm{QED}}(2)_{V_{ud}}\ \mathrm{MeV}\ \ [0.1\%]$$

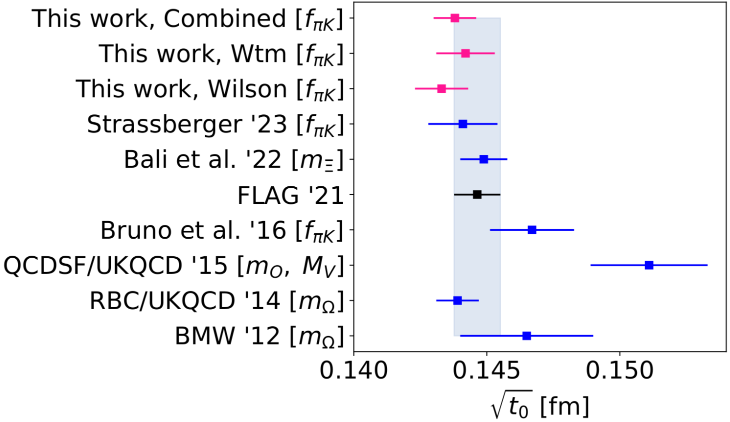
$$f_K^{\mathrm{isoQCD}} = 157.2(2)_{\mathrm{exp}}(2)_{\mathrm{QED}}(4)_{V_{us}}\ \mathrm{MeV}\ \ [0.3\%]$$

combined fit of Wilson and twisted mass data





scale setting: t_0 $N_f = 2 + 1$ [preliminary]



$\sim$ 12% of the error (squared) is due to $|V_{us}|$

conclusions

- mixed action: Wilson twisted mass on Wilson fermions
- determination of t_0 , m_c , $f_{D_{(s)}}$

ongoing:

- analysis of systematic effects
- D-meson semileptonic decays