Charm-quark mass from massive non-perturbative renormalization scheme RI/mSMOM

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in collaboration with

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Hadronic physics and heavy quarks on the lattice meeting

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Overview

- Non-perturbative renormalization in Lattice QCD
- massive NPR scheme RI/mSMOM
- numerical implementation: charm-quark mass m_c

Almost all plots in this talk by Rajnandini Mukherjee (who was originally invited but unfortunately could not give this talk today!) and J Tobias Tsang

Non-perturbative renormalization in Lattice QCD

Lattice Renormalization

In lattice QCD we compute expectation values of bare operators

$$\langle \mathcal{O} \rangle_{\mathtt{lat}}$$

which we want to relate to physical observables like

- form factors
- decay constants
- mixing amplitudes

for which we need

$$\langle \mathcal{O} \rangle_{\mathtt{cont}}$$

- historically: renormalization constants from lattice perturbation theory
- coefficients often turned out to be large, other method preferable

Rome-Southampton method

- Completely avoids the use of lattice perturbation theory

⇒ Called ***Non-Perturbative Renormalization*** or ***NPR***

- First done for 2-fermion operators, which cannot be renormalized by solving the Ward identity [Martinelli et al., 1995]
- Idea is to fix renormalization conditions via tree-level matrix elements like

$$Z_\Gamma \langle p|O_\Gamma|p\rangle\Big|_{p^2=-\mu^2} = \langle p|O_\Gamma|p\rangle\Big|_{tree}$$

⇒ Renormalization constants can be computed on the lattice

- Other methods, like Schrödinger Functional, not part of this talk

Rome-Southampton method

First: convert to renormalization scheme S at finite lattice spacing a and mass scale μ

$$\langle \mathcal{O} \rangle_{lat}(am) \rightarrow \langle \mathcal{O} \rangle^{S}(am, a\mu)$$

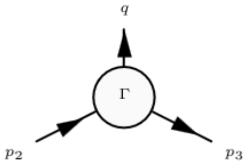
Second: take continuum limit

$$\langle \mathcal{O} \rangle^{S}(am, a\mu) \rightarrow \langle \mathcal{O} \rangle^{S}(m, \mu)$$

Third: *Continuum* perturbation theory to match to continuum scheme like e.g. $\overline{\text{MS}}$

$$\langle \mathcal{O} \rangle^{S}(m, \mu) \rightarrow \langle \mathcal{O} \rangle^{\overline{\text{MS}}}(\mu)$$

RI-SMOM



Kinematics for fermion bilinears:

- Original Rome-Southampton method **RI-MOM** [Martinelli et al., 1995]

$$p_1^2 = p_2^2 = -\mu^2, \quad p_1 = p_2 \Rightarrow q = 0$$

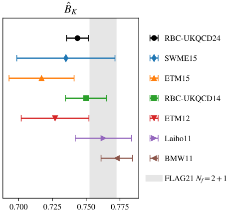
  which has *exceptional kinematics* $q^2 = 0 \ll \mu^2$, chiral symmetry breaking effects vanish with $1/p^2$
- *Non-exceptional kinematics* **RI-SMOM** [Sturm et al., 2009]

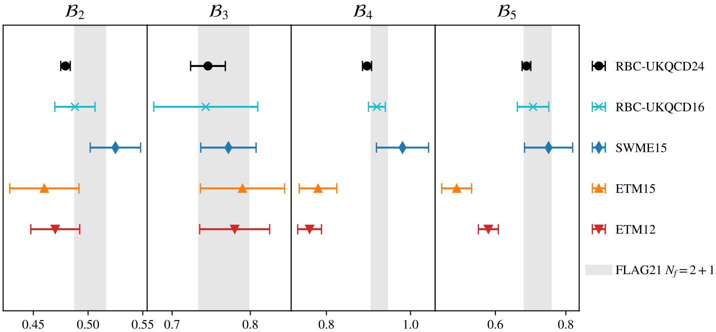
$$p_1^2 = p_2^2 = q^2 = -\mu^2, \quad q = p_1 - p_2$$

  chiral symmetry breaking and infrared effects vanish with $1/p^6$

TENSIONS IN BSM KAON MIXING [FE ET AL., ARXIV 2404.02297]

- Example $K - \bar{K}$ mixing (requires 4-quark vertices!)
- BSM bag parameters $\mathcal{B}_4$, $\mathcal{B}_5$ are in tension between results using RI-MOM (with manually removed pion poles) and RI-SMOM
- tension recently confirmed [FE et al., 2024]





massive NPR scheme RI-mSMOM

RI-mSMOM

- Both RI/MOM and RI/SMOM are defined in the chiral limit of QCD

$\Rightarrow$ these schemes are mass independent, i.e. all renormalization constants Z are independent of the fermion masses

$\Rightarrow$ using massless schemes for quark of mass m introduces discretization effects scaling with m/μ

- for c and b quarks these violations could be sizeable, when leaving the regime where $m \ll \mu \ll a^{-1}$, where the inverse lattice spacing a^{-1} is the UV cutoff

$\Rightarrow$ new renormalization conditions, imposed at finite value of renormalized mass, were suggested [Boyle et al., 2016], an extension of RI/SMOM

RI-mSMOM

The renormalization conditions are usually expressed in terms of amputated correlators

$$\Lambda^a_\Gamma(p_2,p_3) = S(p_3)^{-1} G^a_\Gamma(p_3,p_2) S(p_2)^{-1} \,,$$

where $S(p)$ is the fermion propagator

Renormalized quantities are defined as (subscript R denoting renormalized quantities)

$$\Psi_R = Z_q^{1/2}\Psi \,, \quad m_R = Z_m m \,, \quad M_R = Z_M M \,, \quad O_{\Gamma,R} = Z_\Gamma O_\Gamma$$

for fermion field Ψ, light-quark mass m, heavy-quark mass M and operator O_Γ

$$S_R(p) = Z_q S(p) \,, \quad \Lambda_{\Gamma,R}(p_1,p_2) = Z_\Gamma / Z_q \Lambda_\Gamma(p_1,p_2)$$

for propagator $S(p)$ and vertex function $\Lambda_\Gamma(p_1,p_2)$ (flavour index a suppressed)

RI-SMOM renormalization conditions

$$1 = \lim_{m_R \to 0} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1} \not{p}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12m_R} \left\{ \mathrm{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2} \mathrm{Tr}\left[(iq \cdot \Lambda_{A,R})\gamma_5\right] \right\},$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[(q \cdot \Lambda_{V,R}) \not{q}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[q \cdot \Lambda_{A,R} \gamma_5 \not{q}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12i} \mathrm{Tr}\left[\Lambda_{P,R} \gamma_5\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12} \mathrm{Tr}\left[\Lambda_{S,R}\right].$$

RENORMALIZATION CONDITIONS

RI-mSMOM renormalization conditions, evaluated at arbitrary mass scale $m_R = \overline{m}$

$$1 = \cancel{\lim_{m_R \to 0}} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1}\not{p}\right],$$

$$1 = \cancel{\lim_{m_R \to 0}} \frac{1}{12m_R} \left\{ \mathrm{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2}\mathrm{Tr}\left[(iq\cdot\Lambda_{A,R})\gamma_5\right] \right\},$$

$$1 = \cancel{\lim_{m_R \to 0}} \frac{1}{12q^2} \mathrm{Tr}\left[(q\cdot\Lambda_{V,R})\not{q}\right],$$

$$1 = \cancel{\lim_{m_R \to 0}} \frac{1}{12q^2} \mathrm{Tr}\left[(q\cdot\Lambda_{A,R} + 2m_R\Lambda_{P,R})\gamma_5\not{q}\right],$$

$$1 = \cancel{\lim_{m_R \to 0}} \frac{1}{12i} \mathrm{Tr}\left[\Lambda_{P,R}\gamma_5\right],$$

$$1 = \cancel{\lim_{m_R \to 0}} \left\{ \frac{1}{12}\mathrm{Tr}\left[\Lambda_{S,R}\right] + \frac{1}{6q^2}\mathrm{Tr}\left[2m_R\Lambda_{P,R}\gamma_5\not{q}\right] \right\}.$$

RI-mSMOM

- renormalization constants depend on $a\bar{m}$
- RI-mSMOM defines renormalization conditions for any value of the scale $\bar{m}$
- one can utilize $\bar{m}$ as a parameter, tuned to a value which leads to mild a dependence of observable

$\Rightarrow$ this could lead to a different $\bar{m}$ for different observables

numerical implementation: charm-quark mass m_c

RI-mSMOM - First numerical implementation

- As a first numerical implementation, we compute the renormalized charm-quark mass $m_c^R = Z_m m_c^{\text{bare}}$ using the RI-mSMOM scheme
- We use Domain-Wall Fermion ensembles at 3 lattice spacings (C, M, F) with the Möbius (M) and Shamir (S) kernels

name	L/a	T/a	$a^{-1}[\text{GeV}]$	$m_\pi[\text{MeV}]$	am_l	am_s
C1M	24	64	1.7295(38)	276	0.005	0.0362
C1S	24	64	1.7848(50)	340	0.005	0.04
M1M	32	64	2.3586(70)	286	0.004	0.02661
M1S	32	64	2.3833(86)	304	0.004	0.03
F1M	48	96	2.708(10)	232	0.002144	0.02144
F1S	48	96	2.785(11)	267	0.002144	0.02144

CHARM-QUARK MASS

- goal: renormalized charm-quark mass

$$m_R^{\text{mSMOM}} = Z_m^{\text{mSMOM}}(a\mu, a\bar{m})(am_q + am_{\text{res}})a^{-1}$$

- Z_m at mass scale $a\mu$ and mSMOM scale $a\bar{m}$
- quark mass am_q and residual mass am_{res}
- Good testing ground:
  - only 2pt-functions and NPR bilinears needed
  - charm-quark can be computed fully relativistically

  ⇒ avoid difficult b-quark, issues covered in [Simon Kuberski, Tue 9:30]

COMPUTATION DETAILS

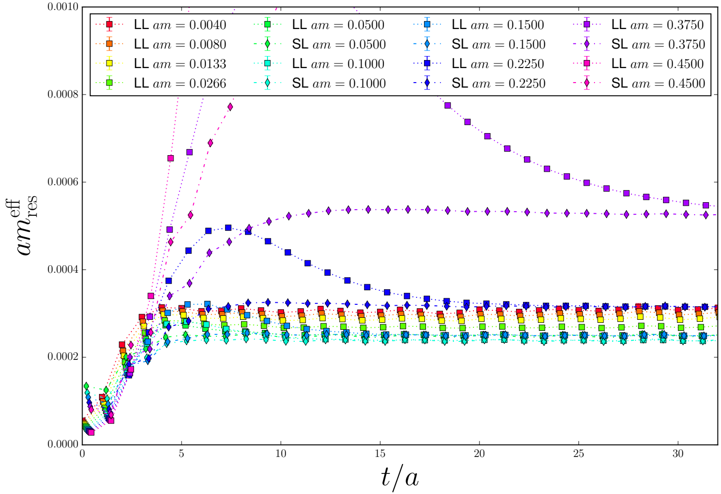
- Compute "η_c" 2pt-functions and NPR vertices at am_l, $2am_l$, $am_s/2$ and am_s and heavy-quark masses

ens	am_q
C1M	0.05, 0.1, 0.15, 0.2, 0.3
C1S	0.05, 0.1, 0.15, 0.2, 0.3
M1M	0.05, 0.1, 0.15, 0.225, 0.3, 0.375
M1S	0.05, 0.1, 0.15, 0.225, 0.3, 0.375
F1M	0.033, 0.066, 0.099, 0.132, 0.198, 0.264, 0.33, 0.36, 0.396
F1S	0.033, 0.066, 0.099, 0.132, 0.198, 0.264, 0.33, 0.36, 0.396

- at each simulated am_q we determine $M(am_q)$, $Z_A(M(am_q))$, $Z_m(\mu, M(am_q))$, and $am_{\text{res}}(am_q)$

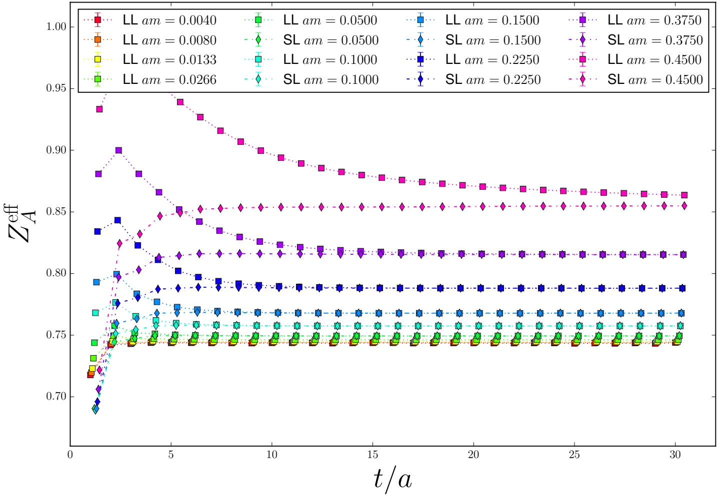
- $am_{res}(am_q)$ for local and Jacobi-smeared quarks
- DWF residual mass
- from pseudoscalar density

$$am_{res}^{eff}(t) = \frac{\langle PJ_{5q}\rangle(t)}{\langle PP\rangle(t)}$$

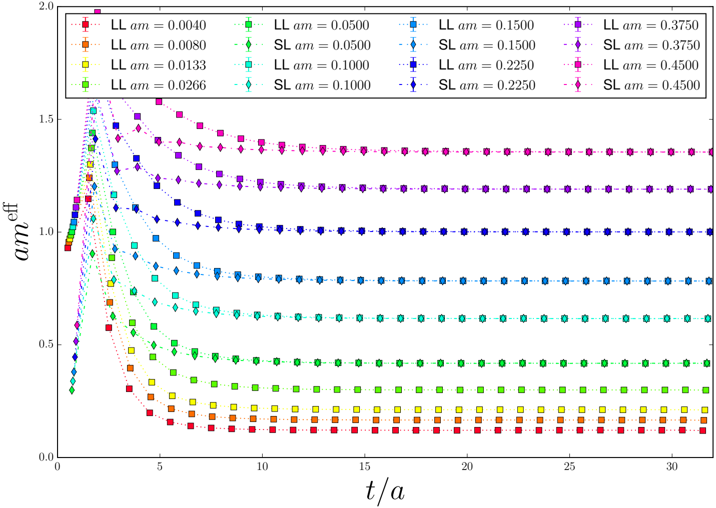


- $Z_A(am_q)$ for local and Jacobi-smeared quarks
- from local and conserved currents
- statistically cleaner than from mSMOM condition

$$Z_A^{\text{eff}}(t) = \frac{1}{2}\left[\frac{C(t+\frac{1}{2}) + C(t-\frac{1}{2})}{2L(t)} + \frac{2C(t+\frac{1}{2})}{L(t)+L(t+1)}\right]$$

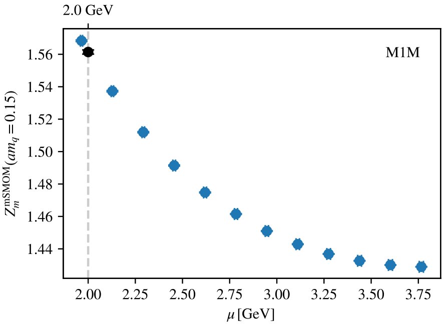


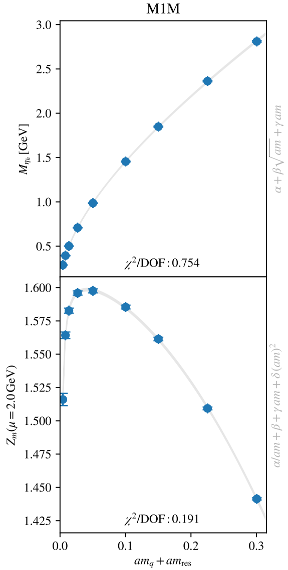
- $M(am_q)$ for local and Jacobi-smeared quarks
- effective heavy-heavy meson mass



Z_A AND MESON MASS

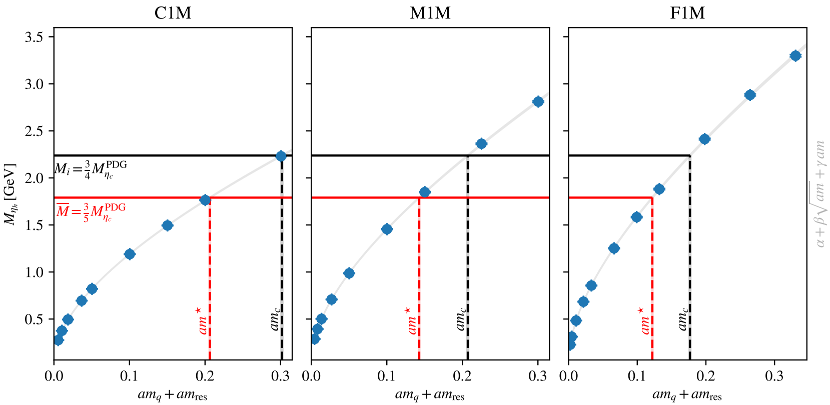
- top: heavy-heavy meson mass at simulated am_q values, and interpolated
- bottom: $Z_M(a\mu, am_q)$ interpolated to momentum scale (here $\mu = 2\text{GeV}$ – plot below)





BARE QUARK MASS

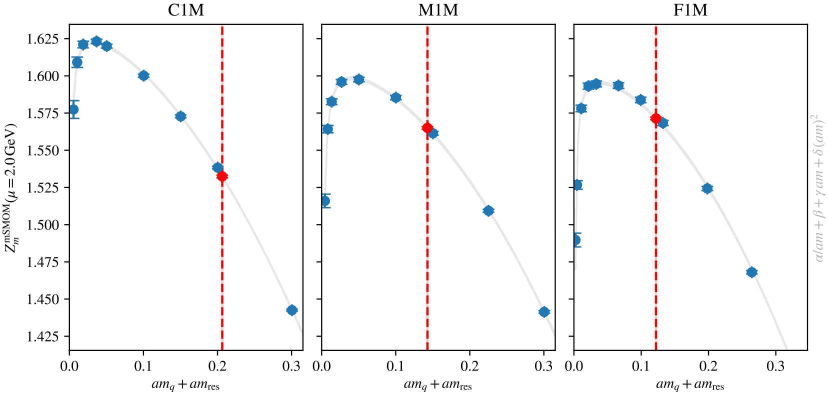
choose mass scale M_i at which we renormalize, and identify corresponding quark mass am_i^{bare}.



repeat for $M_i / M_{\eta_c}^{\text{PDG}} \in \{0.4, 0.5, 0.6, 0.75, 0.9, 1\}$

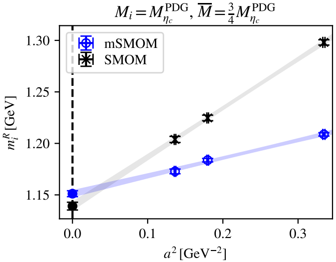
Z_m

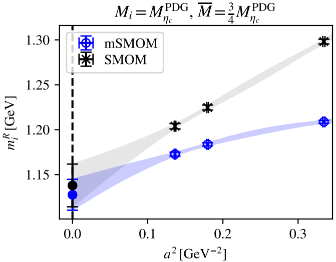
At determined am_i^{bare}, evaluate Z_m



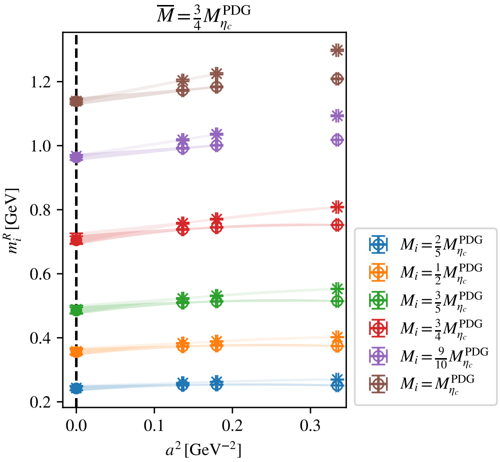
CONTINUUM EXTRAPOLATION

- comparison of bare quark mass RI-SMOM and RI-mSMOM at $M_i/M_{\eta_c}^{PDG} = 1$ and $\bar{M}/M_{\eta_c}^{PDG} = 1$
- for this particular example, RI-mSMOM data has a flatter approach to continuum than RI-SMOM data
- comparison of fits linear and quadratic in a^2



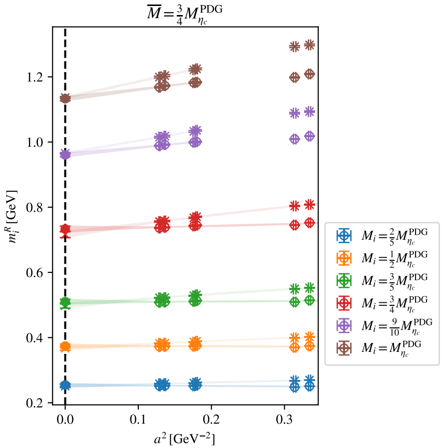


- Take the continuum limit of the quark mass m_i renormalised in the RI-mSMOM (at $\overline{m}$) $\Rightarrow m_i^R(\overline{m})$.
- for largest m_i, continuum limit only from M,F ensembles.
- crosses: RI-SMOM, circles: RI-mSMOM
- we repeat this for all values of M_i

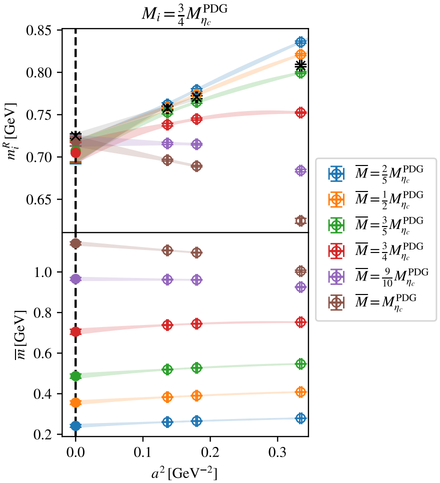


- Möbius and Shamir lattice data compatible for medium and fine lattice spacing
- Discrepant for coarse lattice spacing
- Currently investigating various fits to describe this

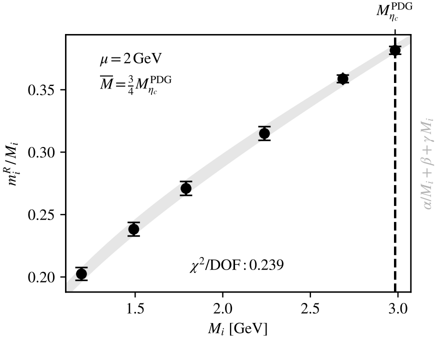
⇒ am_{res} effect?



- m_i^R and $\bar{m}$ for various values of $\bar{M}/M_\eta$ at fixed M_i
- NB: continuum extrapolations do not need to agree with each other, as defined at different $\bar{m}$
- black data : RI-SMOM
- flatter slope in discretization effects for some values of $\bar{M}$



- Having obtained the continuum limit extrapolated values $m_i^R(\overline{m}_j)$, perform a fit of these values against M_i to obtain $m_i^R(\overline{m}_j)$.
- This is a PRELIMINARY fit to preliminary data



Outlook

Perturbative matching to continuum scheme:

- all our results are in RI-SMOM or RI-mSMOM
- matching coefficients known

Full estimation of uncertainties:

- some underlying data points have not a rigorously estimated systematic error budget yet
- full error budget on final result m_c^R will have to be assembled step by step
- consistency checks:
  - converting RI-mSMOM to RI-SMOM for various choices of $\bar{m}$
  - converting to $\overline{MS}$ for various choices of $\bar{m}$
  - full calculation at different scales μ

Conclusions

- Non-perturbative renormalization is increasingly important in lattice QCD computations
- The Rome-Southampton method provides a rigorous approach to achieve this
- Some observables still have large cut-off effects
- massive schemes are a promising avenue to moderate those
- We are working on a first numerical implementation of the RI-mSMOM scheme, to compute the fully renormalized charm-quark mass, comparing with RI-SMOM
- systematic error analysis and final fits still pending

$\Rightarrow$ Stay tuned, aiming for arxiv submission within 2 months

- if this turns out to be successful, we will think about 4-quark vertices in RI-mSMOM

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Backup

Results from calculations of BSM bag parameters in $\overline{\text{MS}}(\mu = 3\,\text{GeV})$ from RBC-UKQCD, SWME and ETM show tensions for $\mathcal{B}_4$ and $\mathcal{B}_5$. The results obtained by ETM, which were renormalised via RI-MOM, agree with RBC-UKQCD's results obtained via RI-MOM. The SWME results, obtained via a 1 loop intermediate scheme agree with RBC-UKQCD's results obtained via RI-SMOM, for both γ_μ and $\not{q}$. This suggests tensions arise from the implementation of intermediate schemes, in particular caused by RI-MOM exhibiting exceptional infrared behaviour which is absent in RI-SMOM. All results are shown in the SUSY basis.

	ETM12	ETM15	RBC-UKQCD12	SWME15	RBC-UKQCD16		THIS WORK
N_f	2	2+1+1	2+1	2+1	2+1	2+1	2+1
scheme	RI-MOM	RI-MOM	RI-MOM	1 loop	RI-SMOM	RI-MOM	RI-SMOM
$\mathcal{B}_2$	0.47(2)	0.46(3)(1)	0.43(5)	0.525(1)(23)	0.488(7)(17)	0.417(6)(2)	0.4794(25)(35)
$\mathcal{B}_3$	0.78(4)	0.79(5)(1)	0.75(9)	0.773(6)(35)	0.743(14)(65)	0.655(12)(44)	0.746(13)(17)
$\mathcal{B}_4$	0.76(3)	0.78(4)(3)	0.69(7)	0.981(3)(62)	0.920(12)(16)	0.745(9)(28)	0.897(02)(10)
$\mathcal{B}_5$	0.58(3)	0.49(4)(1)	0.47(6)	0.751(7)(68)	0.707(8)(44)	0.555(6)(53)	0.6882(78)(94)