Exotic hadrons with heavy quarks - from exploratory calculations to reliable predictions

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Outline

What to call an exotic state in QCD ?

- Textbook: Quark-antiquark mesons and 3-quark baryons
- Historically, multiquark states and hybrids (made of quark and gluons) already suggested by Gell-Mann in addition
- We are now seeing some explicitly *exotic* states – in particular with heavy quarks
- Various possible structures: regular mesons/baryons; molecules; tetraquarks/pentaquarks; hybrid hadrons; glueballs; Di-Baryons
- For the purpose of this talk:
  I will also consider states with quantum numbers allowed by quark-antiquark states but unexpected properties as exotic

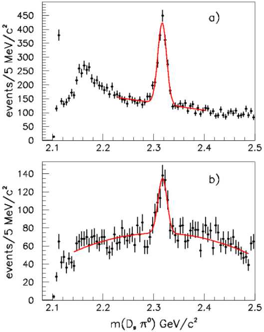
Exotic D_s and B_s candidates

Established s and p-wave hadrons:

D_s ($J^P = 0^-$) and D_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+),
$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)

B_s ($J^P = 0^-$) and B_s^* (1^-)
?
$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

$D_{s0}^*(2317)$:
PRL 90 242001 (2003)



- Corresponding $D_0^*(2400)$ and $D_1(2430)$ are broad resonances
- Perceived peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ (an old dispute; likely not the case)
- Additional exotic states are expected (in the sextet representation)

`See for example Kolomeitsev, Lutz, PLB 582, 39 (2004)`

- B_s cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ not (yet) seen in experiment

Tetraquarks - the T_{bb}

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark, T_{bb}, is the most concrete pure-tetraquark candidate phenomenologically and from the lattice in terms of being deeply-bound and strong-interaction-stable.
Cousin of the T_{cc} but likely has quite different physics,

T_{bb} **bound by** ≈ 100 **MeV**, T_{cc} by 360 KeV

T_{bb} often described by the diquark picture:

- "Good" (attractive) light diquark $(u^T C\gamma_5 d)$ - lighter diquark increases binding
- Color-Coulomb heavy antidiquark $(\bar{b}C\gamma_i\bar{b}^T)$ - deeper binding as heavy mass gets heavier

No Wick-contractions with annihilation $\rightarrow$ easy to compute on the lattice!

Determining the finite-volume spectra

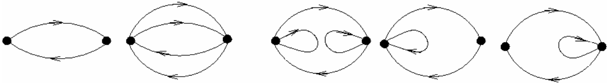
- In practical calculations $\bar{q}q$ and qqq interpolators couple very weakly to multi-hadron states

McNeile & Michael, Phys. Lett. B 556, 177 (2003); Engel et al. PRD 82, 034505 (2010); Bulava et al. PRD 82, 014507(2010); Dudek et al. PRD 82, 034508(2010);

- Similar observations in string breaking studies

Pennanen & Michael hep-lat/0001015;Bernard et al. PRD 64 074509 2001;

- This (often) necessitates the inclusion of hadron-hadron interpolators



- We also know: Energy levels $\neq$ resonance masses
  Naïve expectation: Correct up to $\mathcal{O}(\Gamma_R(m_\pi))$
- Was once upon a rime good enough for heavy pion masses where one would deal with bound states or very narrow resonances.

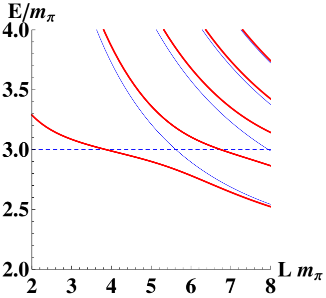
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite-volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \to \pi\gamma^*$)
- Significant progress for 3-particle scattering



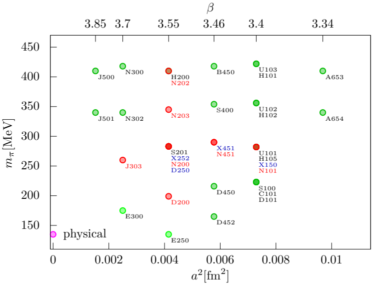
Challenges

- Hierarchy of difficulties
  - Meson systems are simpler than baryons
    (exponentially degrading signal to noise)
  - For deeply bound states Lüscher/scattering studies not strictly necessary
  - Cost of correlation functions much larger for systems with baryons
  - Complicated scattering amplitudes need more data (volumes, frames)
    single two-hadron channel; coupled two-hadron channels; three-hadron scattering
- Hierarchy of projects:
  - Proof of principle
  - Explore quark mass dependence
  - Full spectroscopy calculation including continuum limit
  - Structure observables (transitions, form factors, . . .)
- Two examples:
  - Low-lying positive-parity B_S mesons
    Most systematics can be addressed!
  - Doubly-heavy tetraquark states (see also Travis Whyte on Friday!)
    (illustrate different stages of progress/difficulties)

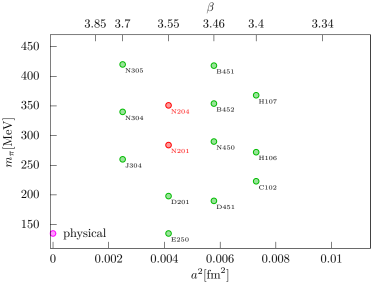
CLS gauge field ensembles

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



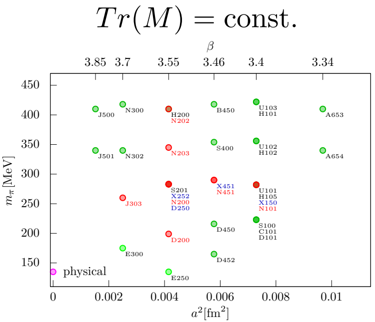
plot style by Jakob Simeth, RQCD

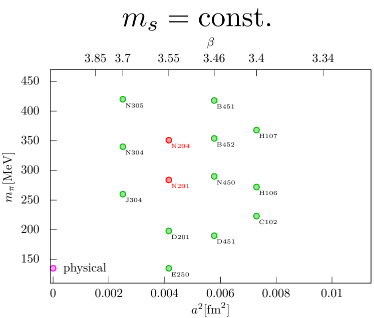
Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) physical quark masses

CLS gauge field ensembles

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)





plot style by Jakob Simeth, RQCD

Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Want to exploit (power law) finite volume effects (keeping exponential effects small)
- Calculation at (or extrapolation to) physical quark masses

NRQCD action

Typical tadpole-improved NRQCD action (here we will use n=4)

`Lepage et al., PRD 46, 4052-4067 (1992)`

$$
\begin{aligned}
H_0 &= -\frac{1}{2aM_0}\Delta^2, \\
H_I &= \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)(\Delta^2)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)} \\
H_D &= -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B} \\
\delta H &= H_I + H_D.
\end{aligned}
$$

Propagators generated through symmetric evolution equation

$$
G(x,t+1) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n \tilde{U}_t(x,t_0)^\dagger \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G(x,t).
$$

- We also tune a $\mathcal{O}(v^6)$ action with tree-level coefficients for the higher order terms

Neural net (RHQ and) NRQCD tuning and setup

R.J. Hudspith, DM, PRD 106, 034508 (2022)
R.J. Hudspith, DM, PRD 107, 114510 (2023)

- Calculate runs with a random distribution for the action parameters
- Let the neural network make parameter predictions
- Due to additive mass we must only consider splittings → we subtract the η_B from all states
- Perform tuning at $SU(3)_f$-symmetric point
- Gauge-fixed wall sources
- Tuning precision is about 1%

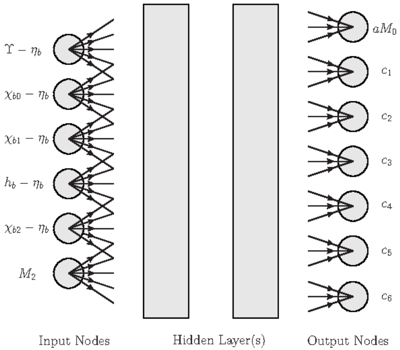


Figure: Schematic picture of our NRQCD setup

Input used for the tuning

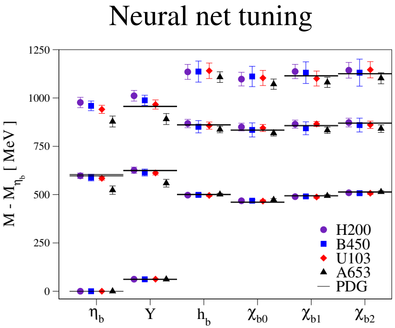
Consider only quark-line connected parts of simple meson operators

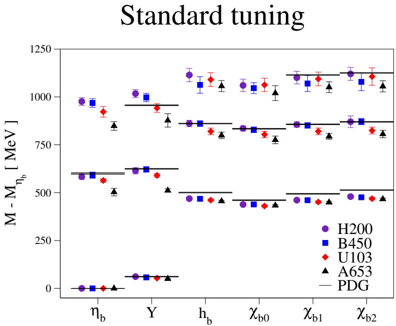
$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j\Delta_i - \sigma_i\Delta_j \ (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j\Delta_i + \sigma_i\Delta_j \ (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs.

NRQCD Neural Net Tuning: Stable s- and p-wave bottomonia





- Higher S- and P-wave states serve as a check whether our tuning leads to reasonable results
- Main results from the lattice spacing of U103; H200 used to estimate systematics

Outline

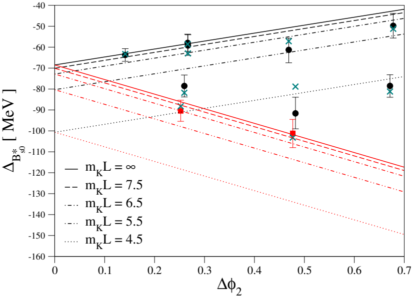
B_s: Chiral – infinite volume extrapolation

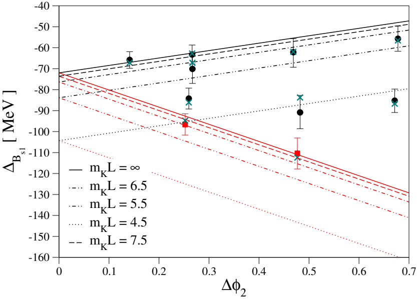
- We explore the previously predicted $J^P = 0^+$ and 1^+ bound states
- Mainly the CLS $\mathrm{Tr}M$ = const trajectory and $2\,m_S$ = const ensembles

Combined extrapolation:

$$\Delta_{B^*_{s0}/B_{s1}}(\Delta\phi_2, m_K L, a) = \Delta_{B^*_{s0}/B_{s1}}(0,\infty,a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

$$\Delta\phi_2 = \phi_2^{\mathrm{Lat}} - \phi_2^{\mathrm{Phys}} \qquad ; \qquad \phi_2 = 8t_0 m_\pi^2$$





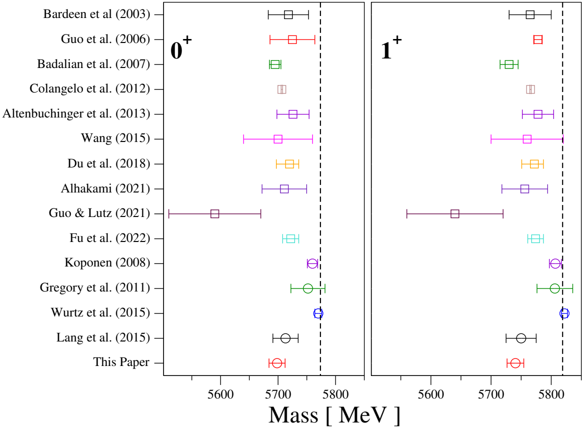
Systematic uncertainties and final result

Resulting binding energies:

$$\Delta_{B_{s0}^*}(0,\infty,0) = -75.4(3.0)_{\text{Stat.}}(13.7)_{\text{a}}\ [\text{MeV}],$$
$$\Delta_{B_{s1}}(0,\infty,0) = -78.7(3.7)_{\text{Stat.}}(13.4)_{\text{a}}\ [\text{MeV}].$$

- Small uncertainty from statistics + combined extrapolation
- Largest systematics from usage of NRQCD/discretization effects
- Central value shifted by applying half the mass difference between H200 and U103
- All other explored uncertainties (finite volume shapes, modified quark-mass dependence, etc.) small

Comparison to the literature



- Results agree well with models based on unitarized χPT
- Improved uncertainty estimate over older Lattice calculations

Outline

A high-statistics problem

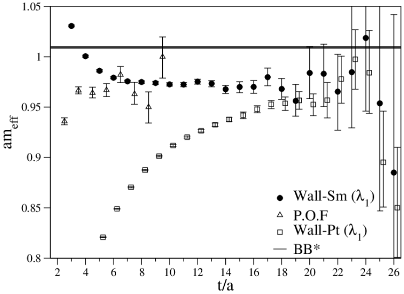
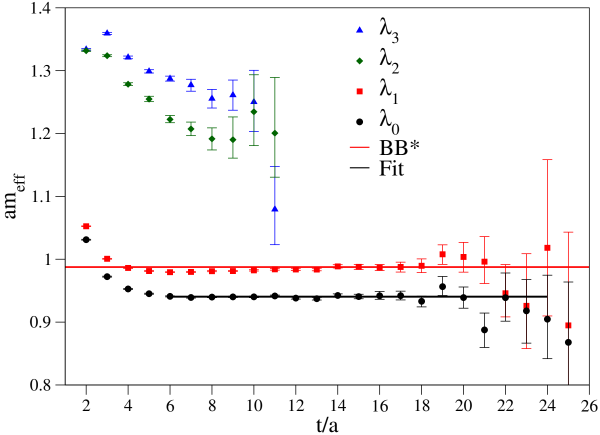


Figure: Effective mass of the T_{bb} correlator from ensemble U103 using 28,000 propagators.

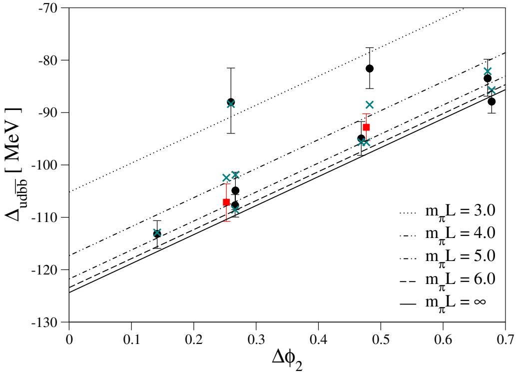
Wall-point data still not at plateau as it loses precision. Wall-sm shows stable plateau for long range of t. P.O.F plateaus fastest but is noisiest. Can easily fit a single exponential to Wall-point data and get too deep binding!

T_{bb} – Basis and effective masses (on N101)

$$D = ({u_a}^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T), \quad E = ({u_a}^T C\gamma_t\gamma_5 d_b)(\bar{b}_a C\gamma_i\gamma_t \bar{b}_b^T),$$
$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i d) - [u \leftrightarrow d], \quad N = (\bar{b} I u)(\bar{b}\gamma_5\gamma_i d) - [u \leftrightarrow d].$$



Combined mass and volume extrapolations



- Ansatz for a deeply-bound state:

$$\Delta_{ud\bar{b}\bar{b}}(\Delta\phi_2, m_\pi L, a) = \Delta_{ud\bar{b}\bar{b}}(0, \infty, a)(1 + A\Delta\phi_2 + Be^{-m_\pi L}).$$

- Strong $e^{-m_\pi L}$ volume effects and deeper binding at lighter pion mass.

Varying the NRQCD tuning

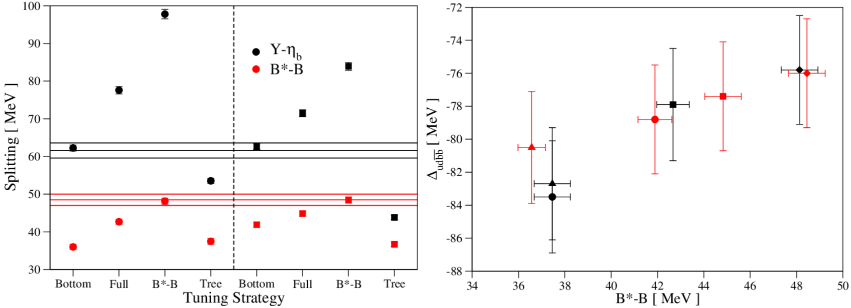
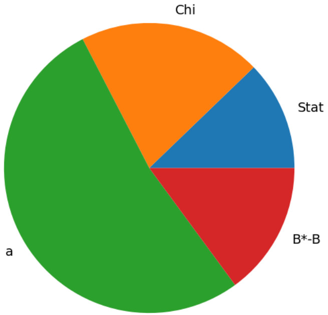


Figure: Alternative tuning strategies with/without B-mesons and higher-order terms (left). Clear correlation of the $B^* - B$ splitting with the T_{bb} bincing. (right)

- Simultaneously reproducing both hyperfine splittings seems impossible
- Tree-level performs poor; For our strategies higher order terms help.
- Shallower T_{bb} binding, with increased $B^* - B$ splitting.

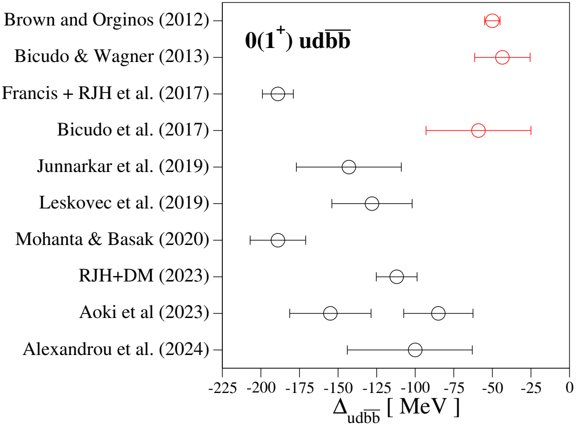
T_{bb} – quantifying systematics



$$\Delta_{ud\bar{b}\bar{b}}(0,\infty,0) = -112.0(2.7)_{\text{Stat.}}(4.5)_{\chi}(11.6)_{a}(3.3)_{B^*-B}$$

- $(..)_a$ uncertainty from comparison of the results for two lattice spacings (H200 vs. U103)
- Both leading systematic uncertainties come from discretization effects/ the use of Lattice NRQCD!

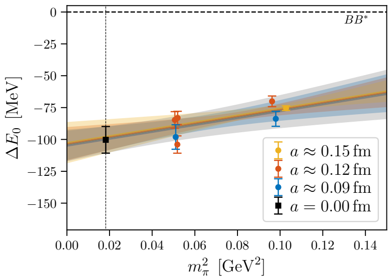
Overview of Lattice $I(J^P) = 0(1^+)$ T_{bb} determinations

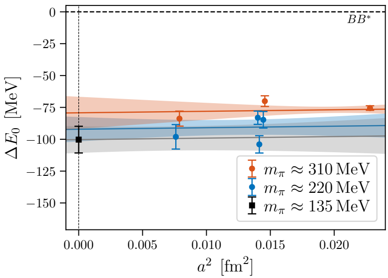


- Red: Static b-quarks; Black: Lattice NRQCD b quarks
- Interesting playground for understanding systematic uncertainties!

Recent study with local and scattering interpolators

Alexandrou *et al.* arXiv:2404.03588

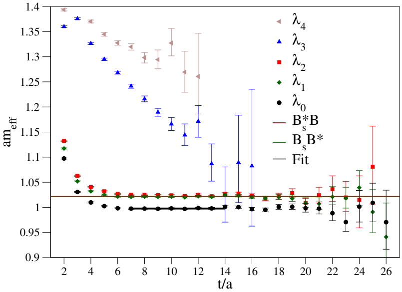




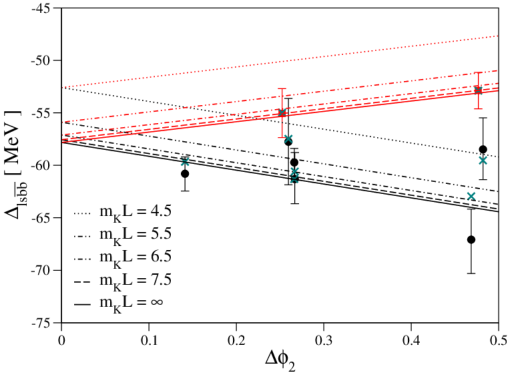
- NRQCD bottom; Wilson Clover with HYP-smearing for the valence quarks; HISQ 2+1+1 sea
- Authors assume/argue that finite-volume effects to be negligible compared to their statistical uncertainty

T_{bbs} – Basis and effective masses

$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i s), \quad N = (\bar{b}Iu)(\bar{b}\gamma_5\gamma_i s)$$
$$O = (\bar{b}\gamma_5 s)(\bar{b}\gamma_i u), \quad P = (\bar{b}Is)(\bar{b}\gamma_5\gamma_i u)$$
$$Q = \epsilon_{ijk}(\bar{b}\gamma_j u)(\bar{b}\gamma_k s).$$



T_{bbs} – chiral and infinite volume extrapolation

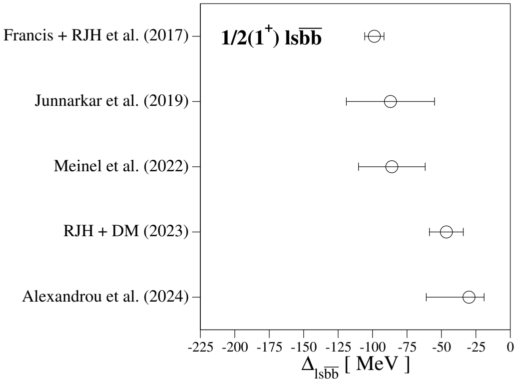


- Chiral/infinite-volume Ansatz:

$$\Delta_{\ell s\bar{b}\bar{b}}(\Delta\phi_2, m_K L, a) = \Delta_{\ell s\bar{b}\bar{b}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

- Large $e^{-m_K L}$ volume effects.
- Consistent with light-diquark picture.

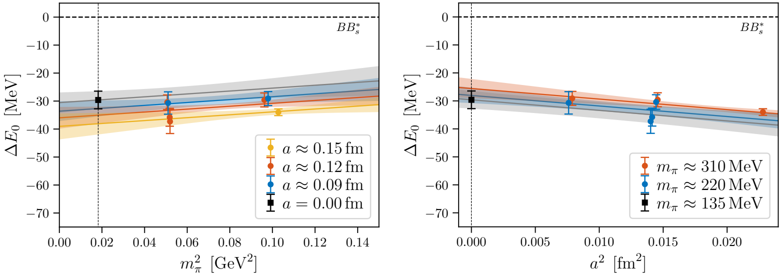
Overview of lattice T_{bbs} determinations



- Close/overlapping EM threshold $BB_s\gamma$, still possible that it is narrow and decays weakly

Recent study with local and scattering interpolators

Alexandrou *et al.* arXiv:2404.03588



- NRQCD bottom; Wilson Clover with HYP-smearing for the valence quarks; HISQ 2+1+1 sea
- Authors assume/argue that finite-volume effects to be negligible compared to their statistical uncertainty

Sad prospects for T_{bb}: Difficult to see at the LHC

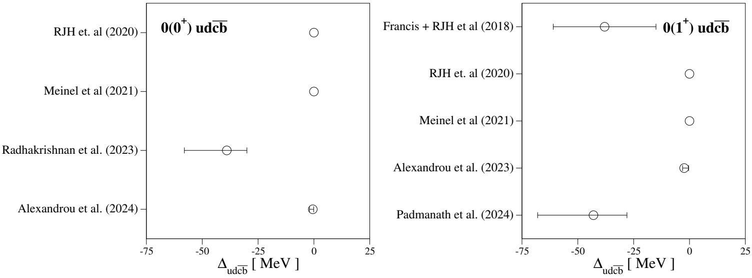
- T_{bb} is very heavy (≈ 10.5 GeV) and decays weakly
- A possible exemplary decay channel could be
  see Phys.Rev.Lett. 118 (2017) 14, 142001 - A. Francis, RJH et al.:

$$T_{bb} \to B^+ \bar{D}^0$$

- It is unlikely to be found anytime soon at the LHC

- Obvious next candidate 0^+ or 1^+ $ud\bar{c}\bar{b}$ "T_{cb}"
  potentially unbound or very weakly bound, due to the reduction of binding from the heavy antidiquark.
- Further exotic states $ud\bar{s}\bar{b}$ or $us\bar{c}\bar{b}$
  seem to be unlikely by diquark picture but worth investigating as some models predict these being deeply bound (mostly Chiral Quark models)

The $0^+/1^+$ T_{cb} - confusing results



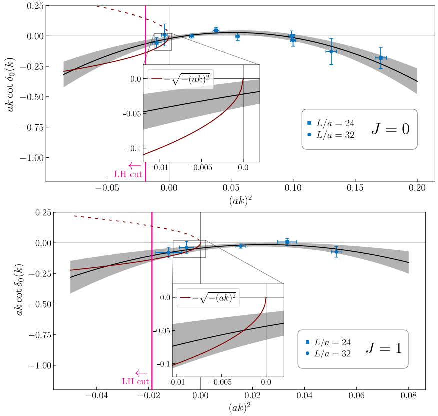
- New results:

Alexandrou *et al.*, PRL 132 151902 (2024)
Radhakrishnan *et al.*, arXiv:2404.08109
Padmanath *et al.*, PRL 132 201902 (2024)

- Close to threshold state could also be a virtual bound state
- Results are more or less incompatible

Shallow bound states and broad resonances in a scattering study

Alexandrou *et al.*, PRL 132 151902 (2024)



	Δm_{GBS} [MeV]	Δm_{R} [MeV]
$J=0$	$-0.5^{+0.4}_{-1.5}$	138(13)
$J=1$	$-2.4^{+2.0}_{-0.7}$	67(24)

- Obtained resonance poles just outside the radius of convergence of the ERE

Improving RHQ b-quarks

I think NRQCD for the T_{bb} and T_{bbs} is at an end. RHQ b-tuning using the "Tsukuba" action as we did for charm in .

- Again learn the dependence of states on parameters
- Absolute scales included
- Fixed $c^2 = 1$ to ensure relativistic nature
- 5-parameter tuning
- see large variations from 1 of r_s, ν, c_E, c_B

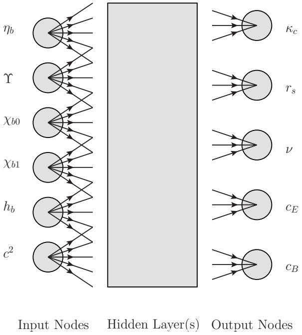
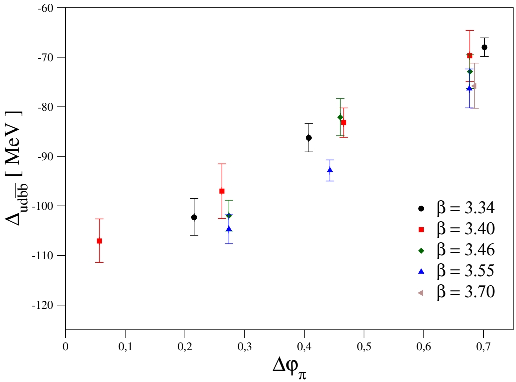
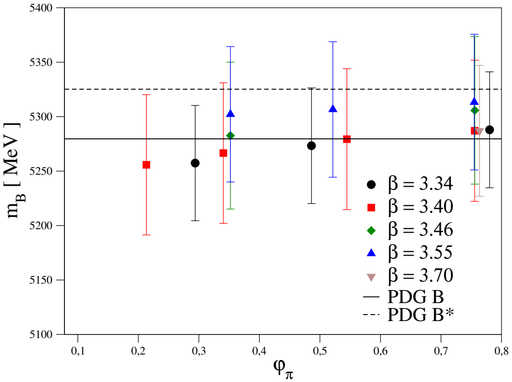


Figure: Schematic picture of our RHQ b-quark tuning

A glimpse at the future: T_{BB} with RHQ bottom



A glimpse at the future: Resulting B-meson masses



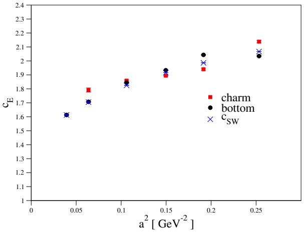
Outline

Conclusions and Outlook

- Positive-parity heavy-light mesons
  - NRQCD calculation with full uncertainty estimate for B_0^* and B_{s1} $\rightarrow$ refined predictions for LHCb, BelleII
  - Calculation could be further improved with RHQ action
  - Scattering amplitudes for the $D_{s0}^*(2317)$ and D_{s1} states using RHQ action planned
- Explicitly exotic heavy-quark tetraquarks
  - Lattice QCD is good at determining deeply-bound states and can rule out phenomenological models for states not yet observed in experiment
  - The calculations are systematically-improvable and we are seeing convergence for the easiest-to-compute quantities such as the T_{bb}
  - The smoking-gun tetraquark state T_{bb} is very difficult to see in current experiments; it is worth exploring weaker-bound candidates such as T_{bc}
  - More and more indications that the multi-quark exotic spectrum at heavy masses is diverse
  - Further insight can be gained from exploring the quark-mass dependence between charm and bottom.

Backup slides

Comparison of b and c parameters - c_E and c_B



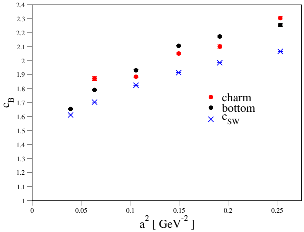


Figure: RHQ clover terms c_E and c_B for **bottom** and **charm**

As a rule of thumb $c_E \approx c_{SW}$, $c_B > c_E$. No big difference between bottom and charm!

Comparison of b and c parameters - κ, r_s, ν

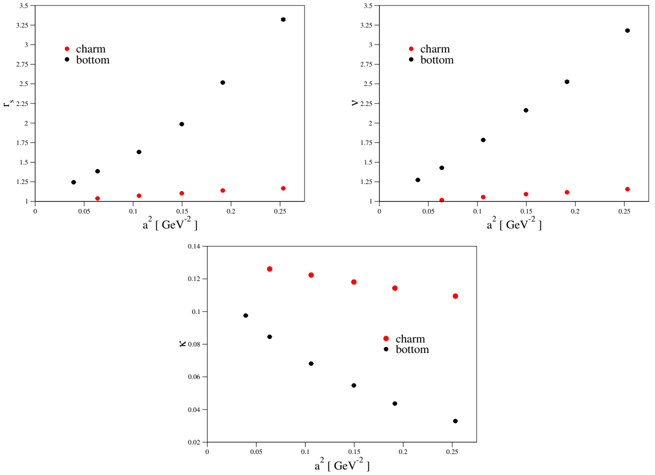


Figure: RHQ action terms r_s, ν, κ for **bottom** and **charm**

$D^*_{s0}(2317)$: D-meson – Kaon s-wave scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Charm-light hadrons

$D^*_{s0}(2317)^\pm$ $I(J^P) = 0(0^+)$

J, P need confirmation.

J^P is natural, low mass consistent with 0^+.

Mass $m = 2317.7 \pm 0.6$ MeV

$m_{D^*_{s0}(2317)^\pm} - m_{D_s^\pm} = 349.4 \pm 0.6$ MeV

Full width $\Gamma < 3.8$ MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L} Z_{00}\left(1; \left(\frac{L}{2\pi} p\right)^2\right)$$

$$\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

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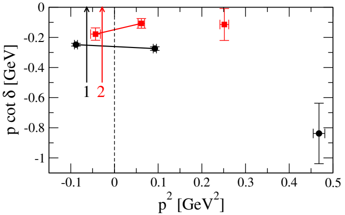
DM *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Results for ensembles (1) and (2)

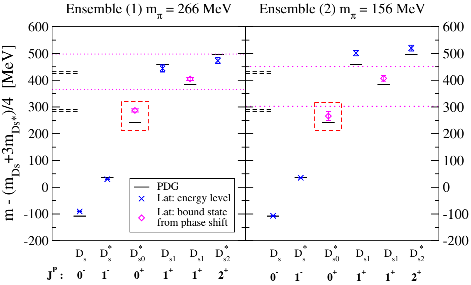


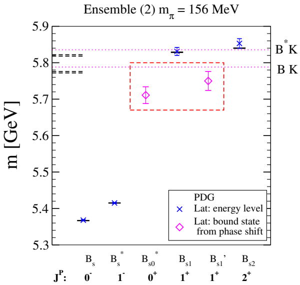
$$a_0 = -0.756 \pm 0.025\text{fm} \qquad (1)$$
$$r_0 = -0.056 \pm 0.031\text{fm}$$
$$a_0 = -1.33 \pm 0.20\text{fm} \qquad (2)$$
$$r_0 = 0.27 \pm 0.17\text{fm}$$

Positive-parity states in the D_s and B_s spectrum

DM *et al.* PRL 111 222001 (2013)
Lang, DM *et al.* PRD 90 034510 (2014)

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

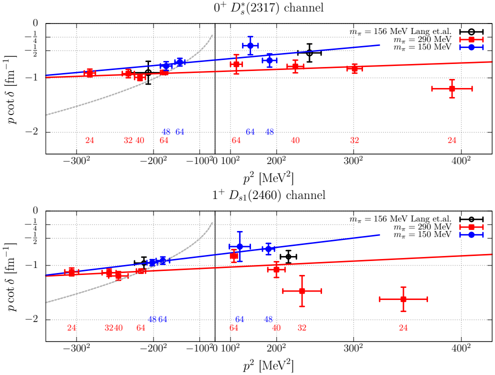




- Spectrum reliably extracted and agrees qualitatively with experiment
- Uncontrolled systematics sizable for the D_s states
- Full uncertainty estimate only for magenta B_s states
- Prediction of exotic states from Lattice QCD!

D_s results in multiple volumes from RQCD

Bali, Collins, Cox, Schäfer, PRD 96 074501 (2017)



- Study with different volumes at pion masses of 150, 290 MeV
- Results confirm basic behavior seen in a single volume
- Discretization effects remain unexplored

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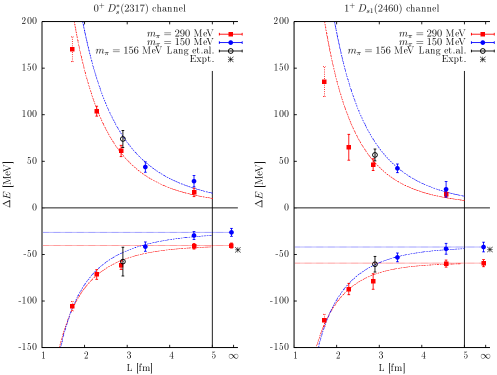

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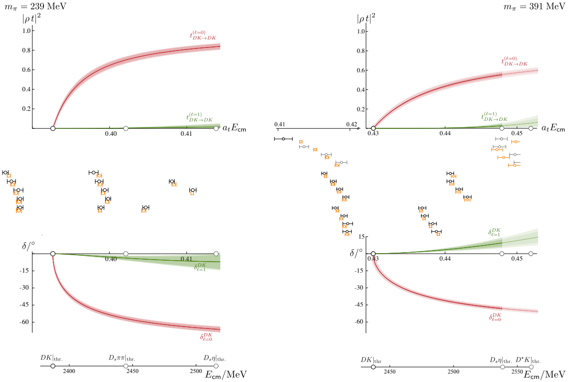
DK and $D\bar{K}$ scattering and the $D_{s0}^*(2317)$

Hadron spectrum collaboration, Cheung *et al.* JHEP 02 100 (2021)



- Study uses moving frames in addition
  results in large number of energy levels at $m_\pi = 238, 391$ MeV

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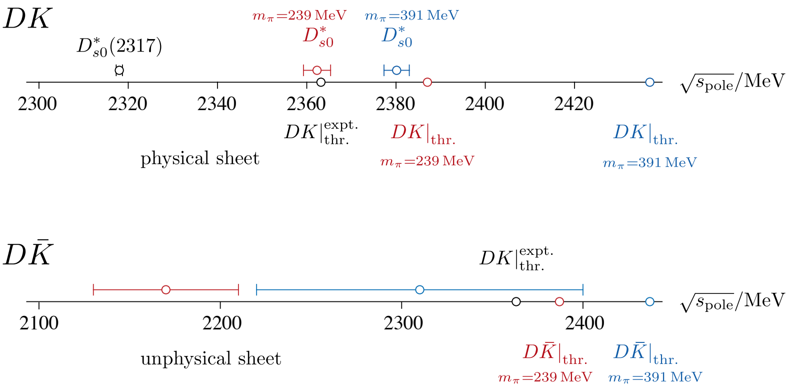

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