Near Threshold States in Coupled $DD^* - D^*D^*$ Scattering From Lattice QCD

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Based on arxiv:2405.15741 in collaboration with David J. Wilson [2] and Christopher E. Thomas [2]

Hadronic physics and heavy quarks on the lattice

Hamilton Mathematics Institute, TCD

The $T_{cc}^{+}(3875)$

- $T_{cc}^{+}(3875)$ was discovered by LHCb in the invariant mass spectrum of $D^0 D^0 \pi^+$

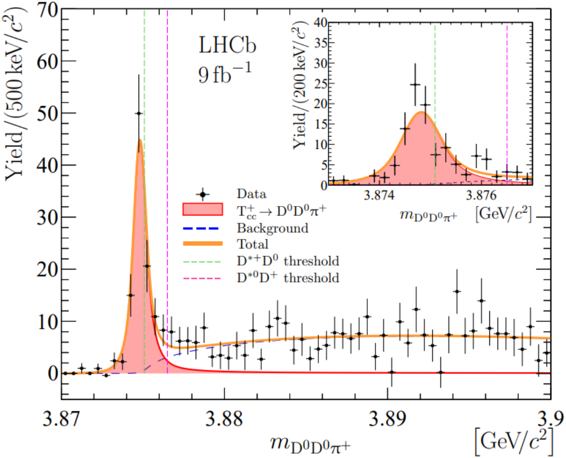
- The mass and width:

  $$\delta m = -360 \pm 40^{+4}_{-0} \ keV$$

  $$\Gamma = 48 \pm 2^{+0}_{-14} \ keV$$

- Consistent with $J^P = 1^+$, $I = 0$ and quark content $cc\bar{u}\bar{d}$

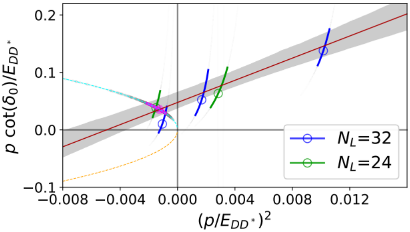
LHCb 2109.01038, 2109.01056



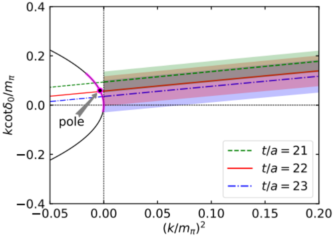
LHCb data ends at 3900 MeV. The $D^{*0}D^{*+}$ threshold opens at 4077.11 MeV, which may bring additional features

Lattice Studies of the $T_{cc}^{+}(3875)$

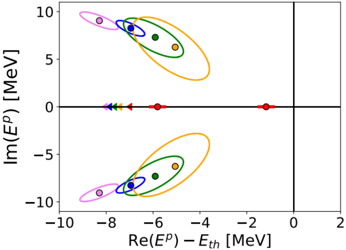
Padmanath and Prelovsek 2202.10110



Lyu et al 2302.04505

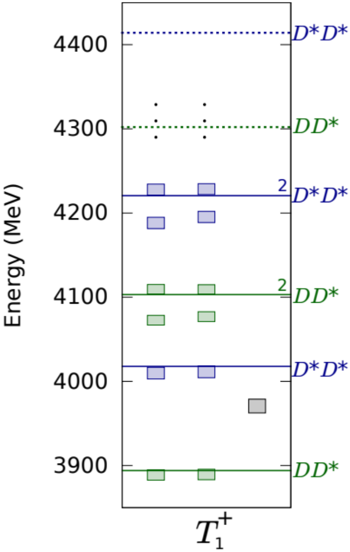


Collins, Nefediev, Padmanath, Prelovsek 2402.14715



Motivation for a Coupled Channel Calculation

- A variety of theoretical models have predicted a state below D^*D^* threshold in $J^P = 1^+$, $I = 0$
  - One gluon exchange Molina et al 1005.0335
  - Hadronic Molecular Models Liu et al 1902.03044
  - Heavy Quark Spin Symmetry M. Albaladejo 2110.02944
  - Chiral EFT Du et al 2110.13765
- Lattice calculation shows significant shifts of energy levels away from non interacting energies Cheung et al 1709.01417



Details of the Calculation

$(L/a_s)^3 \times (T/a_t)$	$N_{\rm cfgs}$	$N_{\rm tsrcs}$	$N_{\rm vecs}$
$16^3 \times 128$	478	8	64
$20^3 \times 256$	288	3-4	128
$24^3 \times 128$	553	3-4	160

	$a_t m$	m (MeV)
π	0.06906(13)	391(1)
D	0.33281(9)	1886(1)
D^*	0.35464(14)	2010(1)

	$a_t m$	m (Mev)
DD^*	0.68745(17)	3896(1)
$D^* D^*$	0.70928(20)	4020(1)
$DD\pi$	0.73468(18)	4163(1)

- Calculations performed on three anisotropic lattices of spatial extent $a_s \approx 0.12$ fm with $\xi = a_s/a_t \approx 3.444(50)$ Edwards et al 0803.3960, Lin et al 0810.3588
- Distillation used to smear the quark fields and efficiently calculate all necessary Wick contractions Peardon et al 0905.2160
- Ω baryon used to set the scale with $a_t^{-1} = m^{\Omega}_{phys}/a_t m_\Omega = 5667$ MeV Edwards et al 1212.5236

Details of the Calculation

- Finite volume energy levels are extracted by solving the GEVP Dudek et al 1004.4930

$$C_{ij}(t)v_j^{\mathfrak{n}}(t) = \lambda_{\mathfrak{n}}(t, t_0)C_{ij}(t_0)v_j^{\mathfrak{n}}(t) \tag{1}$$

where C is the matrix of correlation functions

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^{\dagger}(0)|0\rangle. \tag{2}$$

- The principal correlator, $\lambda_{\mathfrak{n}}(t, t_0)$, is used to obtain the energy $E_{\mathfrak{n}}$ of the $\mathfrak{n}^{th}$ eigenstate $|\mathfrak{n}\rangle$

$$\lambda_{\mathfrak{n}}(t, t_0) = (1 - A_{\mathfrak{n}})e^{-E_{\mathfrak{n}}(t-t_0)} + A_{\mathfrak{n}}e^{-E_{\mathfrak{n}}^{'}(t-t_0)} \tag{3}$$

and the eigenvectors of the GEVP are used to construct variationally optimized operators $\Omega_{\mathfrak{n}}^{\dagger} = \sum_i v_i^{\mathfrak{n}}\mathcal{O}_i^{\dagger}$ and define the operator overlaps

$$Z_i^{\mathfrak{n}} = \langle \mathfrak{n}|\mathcal{O}_i(0)|0\rangle \tag{4}$$

$$= \sqrt{2E_{\mathfrak{n}}}e^{(E_{\mathfrak{n}}t_0)/2}(v_j^{\mathfrak{n}})^{*}C_{ji}(t_0). \tag{5}$$

Interpolating Operators

- The finite volume of the lattice breaks the rotational symmetry of the infinite volume continuum
- Hadrons at rest with definite continuum momentum J are projected to the finite cubic irreducible representation (irrep), Λ, which is called *subduction* Dudek et al 1004.4930
- Utilize only meson-meson operators of the form Dudek et al 1212.0830, Dudek et al 1203.6041

$$\mathcal{O}^{\Lambda\,\dagger}_{D^{(*)}D^*}(\vec{P}) = \sum_{\vec{p_1},\vec{p_2}} \mathcal{C}(\vec{P}\Lambda; \vec{p_1}\Lambda_1; \vec{p_2}\Lambda_2) \times \Omega^{\Lambda_1\,\dagger}_{D^{(*)}}(\vec{p_1})\Omega^{\Lambda_2\,\dagger}_{D^*}(\vec{p_2}) \tag{6}$$

where the operators $\Omega_{D^{(*)}}$ are variationally optimized single $D^{(*)}$ meson operators subduced to Λ_1 and Λ_2

$$\Omega \sim \bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\Gamma\psi \tag{7}$$

D^*D^* Symmetry Considerations

- Bose symmetry dictates that the overall wavefunction of D^*D^* be symmetric

D^*D^*

ℓ	$^{2S+1}\ell_J$	J^P	Spin	Space	Flavor	Total
0	$^3S_1$	1^+	A	S	A	S
1	$^1P_1$ $^5P_{1,2,3}$	1^- $\{1,2,3\}^-$	S	A	A	S
2	$^3D_{1,2,3}$	$\{1,2,3\}^+$	A	S	A	S
3	$^1F_3$ $^5F_{1,\ldots,5}$	3^- $\{1,2,3,4,5\}^-$	S	A	A	S

- No such symmetry requirements for DD^*

DD^*

ℓ	$^{2S+1}\ell_J$	J^P
0	$^3S_1$	1^+
1	$^3P_{0,1,2}$	$\{0,1,2\}^-$
2	$^3D_{1,2,3}$	$\{1,2,3\}^+$
3	$^3F_{2,3,4}$	$\{2,3,4\}^-$

Subduction Tables for Mesons at Rest

Pseudoscalar-Vector Scattering for Mesons at Rest Woss et al 1802.05580

Λ^P	T_1^+	A_1^-	E^-	E^+	A_2^+
$J^P(^{2S+1}\ell_J)$		0^- $\begin{pmatrix} ^3P_0 \end{pmatrix}$			
	1^+ $\begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$				
			2^- $\begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	2^+ $\begin{pmatrix} ^3D_2 \end{pmatrix}$	
	3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$				3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
	4^+ $\begin{pmatrix} ^3G_4 \end{pmatrix}$	4^- $\begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	4^- $\begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	4^+ $\begin{pmatrix} ^3G_4 \end{pmatrix}$	

Identical Vector-Vector Scattering for Mesons at Rest

Λ^P	T_1^+	A_1^-	E^-	E^+	A_2^+
$J^P(^{2S+1}\ell_J)$	1^+ $\begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$				
			2^- $\begin{pmatrix} ^5P_2 \\ ^5F_2 \end{pmatrix}$	2^+ $\begin{pmatrix} ^3D_2 \end{pmatrix}$	
	3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$				3^+ $\begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
	4^+ $\begin{pmatrix} ^3G_4 \end{pmatrix}$	4^- $\begin{pmatrix} ^5F_4 \\ ^5H_4 \end{pmatrix}$	4^- $\begin{pmatrix} ^5F_4 \\ ^5H_4 \end{pmatrix}$	4^+ $\begin{pmatrix} ^3G_4 \end{pmatrix}$	

Pseudoscalar-Vector Scattering for Mesons at Overall Nonzero Momentum

Woss et al 1802.05580

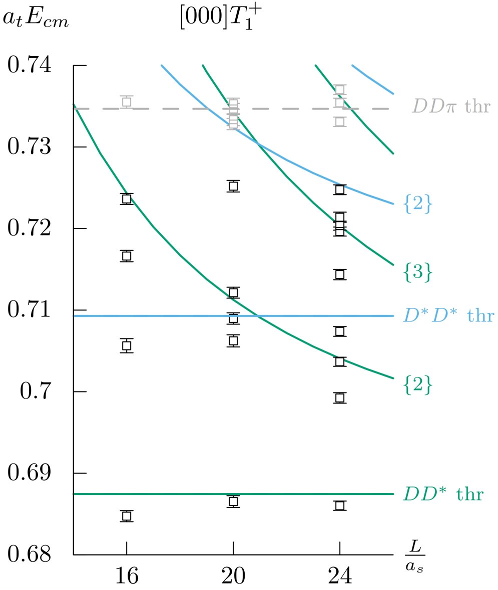
Λ^P	$[00n]A_2$	$[0nn]A_2$	$[nnn]A_2$
$J^P(^{2S+1}\ell_J)$	$0^- \, (^3P_0)$	$0^- \, (^3P_0)$	$0^- \, (^3P_0)$
	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
		$2^+ \, (^3D_2)$	
	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}_{[2]}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$
		$3^- \, (^3F_3)$	$3^- \, (^3F_3)$
	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[3]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$

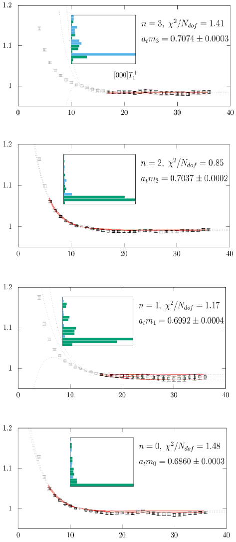
Subduction Tables for Mesons Inflight

Identical Vector-Vector Scattering for Mesons at Overall Nonzero Momentum

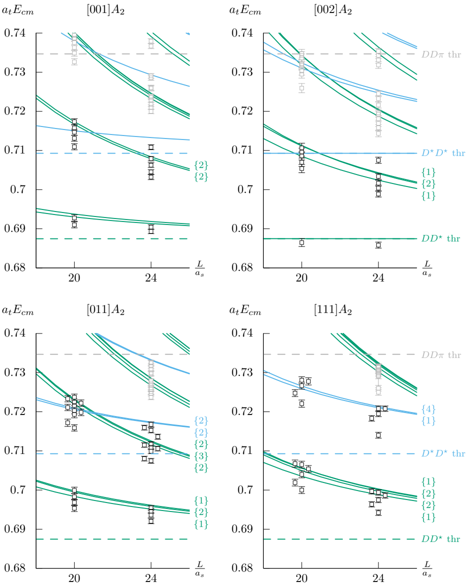
Λ^P	$[00n]A_2$	$[0nn]A_2$	$[nnn]A_2$
$J^P(^{2S+1}\ell_J)$	$1^+\begin{pmatrix}^3S_1\\ ^3D_1\end{pmatrix}$	$1^+\begin{pmatrix}^3S_1\\ ^3D_1\end{pmatrix}$	$1^+\begin{pmatrix}^3S_1\\ ^3D_1\end{pmatrix}$
		$2^+\,(^3D_2)$	
	$2^-\begin{pmatrix}^5P_2\\ ^5F_2\end{pmatrix}$	$2^-\begin{pmatrix}^5P_2\\ ^5F_2\end{pmatrix}_{[2]}$	$2^-\begin{pmatrix}^5P_2\\ ^5F_2\end{pmatrix}$
	$3^+\begin{pmatrix}^3D_3\\ ^3G_3\end{pmatrix}$	$3^+\begin{pmatrix}^3D_3\\ ^3G_3\end{pmatrix}_{[2]}$	$3^+\begin{pmatrix}^3D_3\\ ^3G_3\end{pmatrix}_{[2]}$
		$3^-\begin{pmatrix}^5P_3\\ ^5F_3\end{pmatrix}$	$3^-\begin{pmatrix}^5P_3\\ ^5F_3\end{pmatrix}$
	$4^-\begin{pmatrix}^5F_4\\ ^5H_4\end{pmatrix}_{[2]}$	$4^-\begin{pmatrix}^5F_4\\ ^5H_4\end{pmatrix}_{[3]}$	$4^-\begin{pmatrix}^5F_4\\ ^5H_4\end{pmatrix}_{[2]}$

Finite Volume Spectra

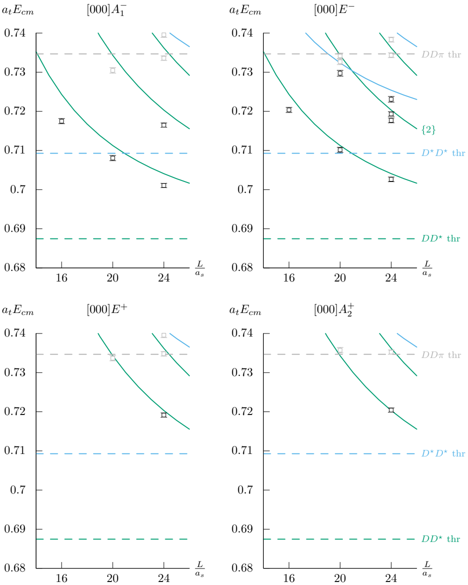




Finite Volume Energies



Finite Volume Energies



Scattering Analysis

- The Lüscher determinant equation for particles of arbitrary mass and spin M. Lüscher, Nucl. Phys. B354, Hansen and Sharpe 1204.0826, Briceño and Davoudi 1204.1110, Guo et al 1211.0929, and many others...

$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \mathbf{t} \cdot (\mathbf{1} + i\boldsymbol{\mathcal{M}})] = 0 \tag{8}$$

  where $\boldsymbol{t}(E_{cm})$ is the infinite volume scattering t-matrix.

- One choice to parameterize $\boldsymbol{t}$ is with a K-matrix parameterization

$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I} \tag{9}$$

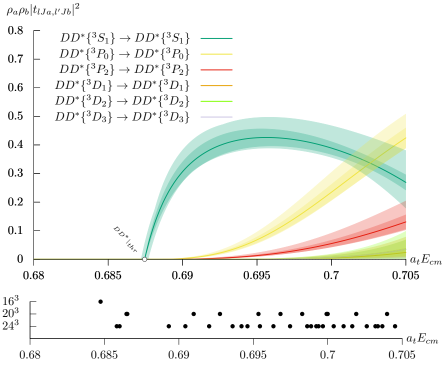
  with matrix elements

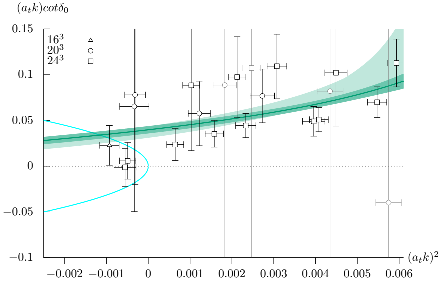
$$K(s)_{\ell SJa,\ell' S' Jb} = \sum_n \gamma^{(n)}_{\ell SJa,\ell' S' Jb} s^n \tag{10}$$

- We find a good description of the spectrum with a linear term in s for the S-wave amplitude

$$\chi^2/N_{dof} = 1.37 \qquad (11)$$

$$t = \frac{E_{cm}}{2}\frac{1}{k\cot\delta_0 - ik} \qquad (12)$$





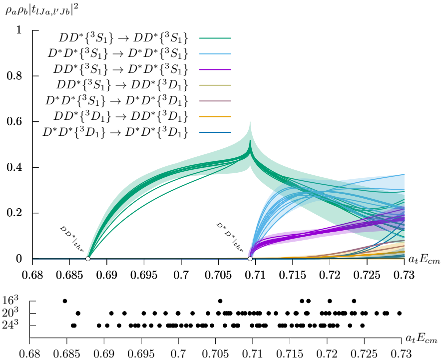
Coupled Channel DD^*-D^*D^* Scattering

		D^*D^*				
ℓ	$^{2S+1}\ell_J$	J^P	Spin	Space	Flavor	Total
0	$^3S_1$	1^+	A	S	A	S
1	$^1P_1$ $^5P_{1,2,3}$	1^- $\{1,2,3\}^-$	S	A	A	S
2	$^3D_{1,2,3}$	$\{1,2,3\}^+$	A	S	A	S
3	$^1F_3$ $^5F_{1,\dots,5}$	3^- $\{1,2,3,4,5\}^-$	S	A	A	S

- Utilizes 109 energy levels up to $a_t E_{cm} = 0.73$
- Starting from elastic DD^* analysis, we make use of all partial waves included there and assume $^3F_2$ is no longer zero
- Explicitly parameterize for D^*D^* $^3S_1$, $^3D_1$, $^5P_2$ and $^5F_2$

Coupled Channel DD^*-D^*D^* Scattering

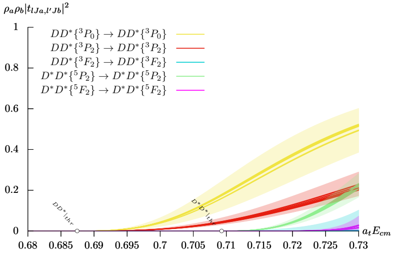
- To reduce bias, we use 14 parameterizations

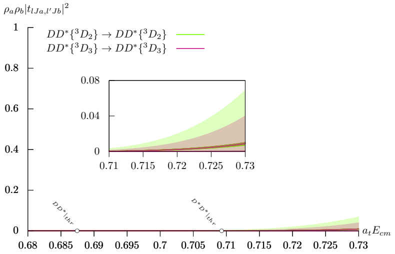


$$\chi^2/N_{dof} = 1.28$$

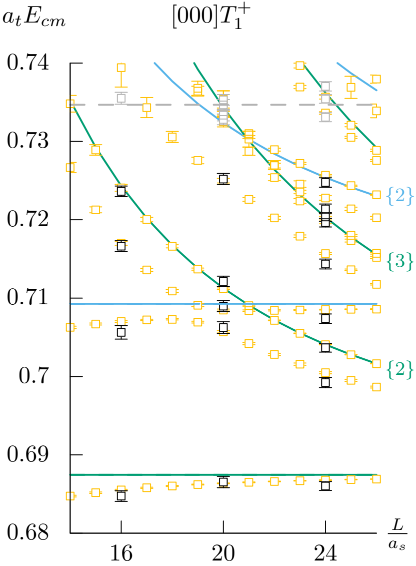
$$\gamma^{(0)}_{DD^*\{^3S_1\}\to D^*D^*\{^3S_1\}} = 4.11 \pm 0.51 \pm 0.94$$

- P-wave amplitudes significantly nonzero but well constrained





Parameterized Finite Volume Spectrum



Resonance Poles

- At energies close to the position of a pole, the infinite volume scattering t-matrix takes on the form

$$t_{ij} \sim \frac{c_i c_j}{s_{pole} - s} \tag{13}$$

which allows us to factorize the residue into the couplings c_i and c_j of the pole to each hadron channel

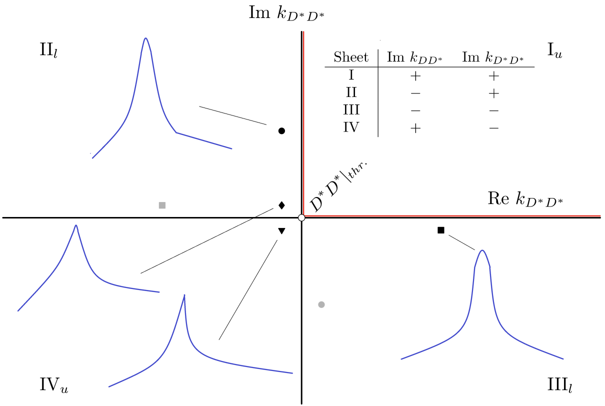
- Poles are characterized by their mass and width

$$\sqrt{s_{pole}} = m \pm \frac{i}{2}\Gamma \tag{14}$$

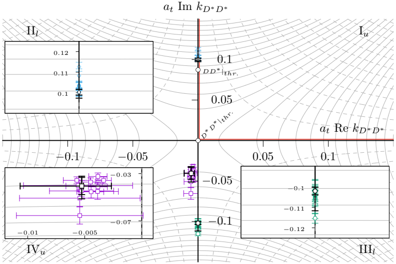
- A pole's influence on physical scattering determined by distance to physical scattering and can be observed in the complex momentum plane

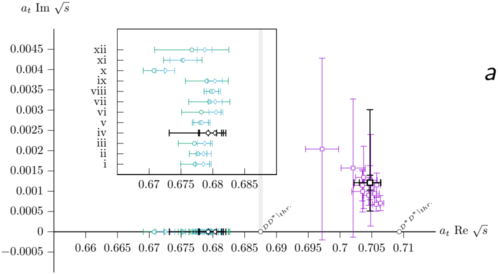
Resonance Poles

- In coupled channel scattering, there are 4 Riemann sheets
- The four sheets in the s-plane can be unfolded to a single sheet by using $k_{D^*D^*}$ as a "uniformizing variable" Newton, Journal of Mathematical Physics 2 188, Morgan and Pennington Phys. Rev. D48 1185



Poles in the $J^P = 1^+$ Scattering Amplitudes





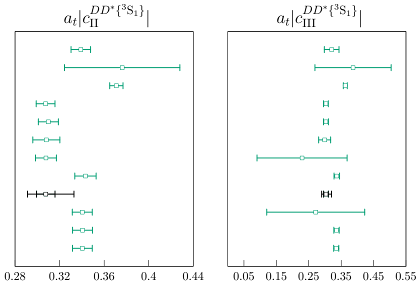
Three poles of interest on sheet II_ℓ, III_ℓ and IV_u:

$$a_t\sqrt{s_{\mathrm{II}}} = 0.6765(55)$$
$$= 3834(31)\ \mathrm{MeV}$$

$$a_t\sqrt{s_{\mathrm{III}}} = 0.6759(68)$$
$$= 3830(40)\ \mathrm{MeV}$$

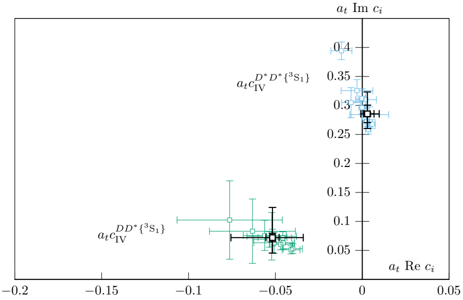
$$a_t\sqrt{s_{\mathrm{IV}}} = 0.7007(62) + \frac{i}{2}(0.0020(22))$$
$$= 3971(35) + \frac{i}{2}11(13)\ \mathrm{MeV}$$

Poles in the $J^P = 1^+$ Scattering Amplitudes



$$a_t c_{\mathrm{IV}}^{DD^*\{^3S_1\}} = -0.07(4) + i(0.10(7))$$
$$|c_{\mathrm{IV}}^{DD^*\{^3S_1\}}| = 692(195)\,\mathrm{MeV}$$
$$a_t c_{\mathrm{IV}}^{D^*D^*\{^3S_1\}} = 0.00(4) + i(0.33(8))$$
$$|c_{\mathrm{IV}}^{D^*D^*\{^3S_1\}}| = 1870(450)\,\mathrm{MeV}$$

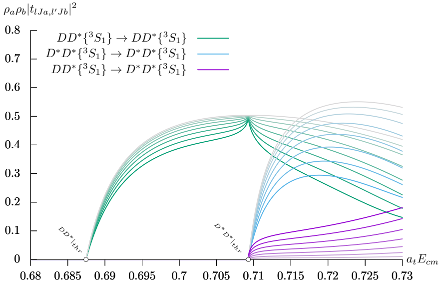
$$a_t c_{\mathrm{II}}^{DD^*\{^3S_1\}} = i(0.36(7))$$
$$|c_{\mathrm{II}}^{DD^*\{^3S_1\}}| = 2040(400)\,\mathrm{MeV}$$
$$a_t c_{\mathrm{III}}^{DD^*\{^3S_1\}} = i(0.30(21))$$
$$|c_{\mathrm{III}}^{DD^*\{^3S_1\}}| = 1700(1200)\,\mathrm{MeV}$$



Varying the $DD^* \to D^*D^*$ S-wave Amplitude

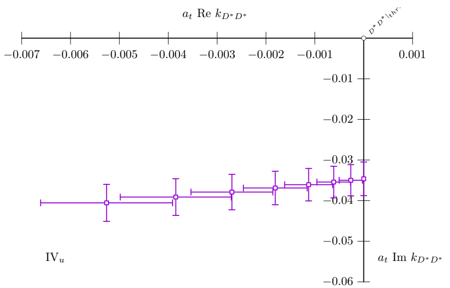
$\gamma^{(0)}_{DD^*\{^3S_1\}\to D^*D^*\{^3S_1\}}$	χ^2
4.11	**121.26**
3.60	127.47
3.08	140.04
2.57	156.40
2.06	174.20
1.54	191.53
1.02	206.91
0	223.26

- As the matrix element coupling the two channels in S-wave decreases, the χ^2 increases
- Suggests an incompatibility of the spectrum with zero coupling

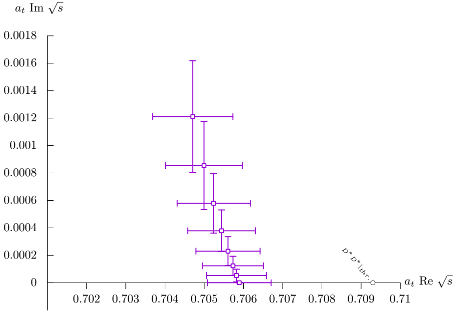


- As coupling is decreased, cusp at D^*D^* threshold disappears
- D^*D^* S-wave amplitude is correspondingly enhanced

Varying the $DD^* \to D^*D^*$ S-wave Amplitude



- As the coupling is decreased, the pole position moves from the complex plane on to the real axis in momentum and energy space



- Complex resonance becomes a virtual bound state in the limit that the channels are decoupled

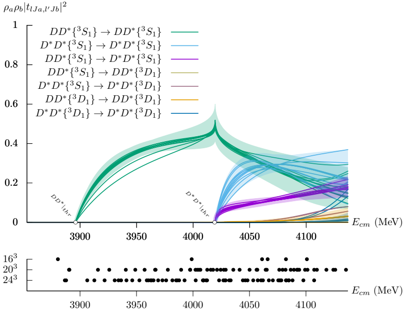
Varying the $DD^* \to D^*D^*$ S-wave Amplitude

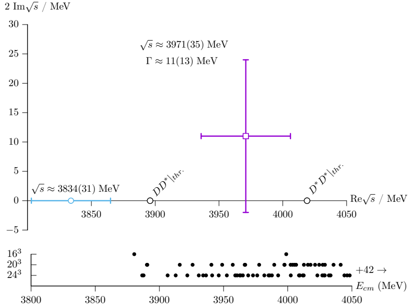
What about the sheet II and III poles?

$\gamma^{(0)}_{DD^*\{^3S_1\}\to D^*D^*\{^3S_1\}}$	Sheet II		Sheet III	
	$\text{Im}(k_{DD^*})$	$\text{Im}(k_{D^*D^*})$	$\text{Im}(k_{DD^*})$	$\text{Im}(k_{D^*D^*})$
4.11	**-0.0493(36)**	**0.1003(18)**	**-0.0529(43)**	**-0.1021(22)**
3.60	-0.0480(35)	0.0996(17)	-0.0507(39)	-0.1010(20)
3.08	-0.0469(34)	0.0991(16)	-0.0489(37)	-0.1001(18)
2.57	-0.0460(32)	0.0987(15)	-0.0473(33)	-0.0993(16)
2.06	-0.0453(31)	0.0984(14)	-0.0461(31)	-0.0988(15)
1.54	-0.0448(31)	0.0981(14)	-0.0452(31)	-0.0983(14)
1.02	-0.0444(30)	0.0980(14)	-0.0446(30)	-0.0980(14)
0	-0.0441(32)	0.0978(14)	-0.0441(32)	-0.0978(14)

- Differences between sheet II and III poles resolve as the channels are decoupled: the sheet III pole becomes an exact mirror pole of sheet II pole
- We prescribe it no physical significance due to its distance from the physical sheet

Summary and Outlook





- We observe two pole singularities in the DD^* and D^*D^* amplitudes: the virtual bound state $\sim$ 62 MeV below DD^* threshold which we identify as the T_{cc} and a resonance $\sim$ 49 MeV below the D^*D^* threshold, which we denote the T'_{cc}
- The T'_{cc} should be observable in the DD^* and D^*D^* final states of ongoing experiments investigating the $I = 0$, $J^P = 1^+$ doubly charmed sector
- Future work: examine the T'_{cc} with the added context of the left-hand cut $\to$ more machinery needed!

Acknowledgments

- Thank you to my collaborators for all of their help and guidance in this project
- This work funded through the Royal Society, UKRI STFC and Science Foundation Ireland
- Calculations performed on DiRAC and Jefferson Lab compute facilities
- Thank you for inviting me to give this talk!

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