Universität Münster

NRW-FAIR Netzwerk

Notes on Distillation Profiles

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Introduction

Topic:

- Lattice QCD

  $\hookrightarrow$ Hadron Spectroscopy

  $\hookrightarrow$ Distillation

  $\hookrightarrow$ Profiles

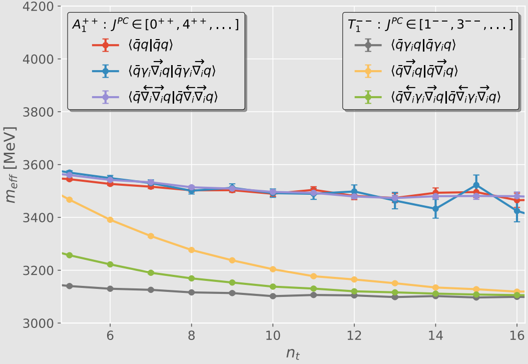
Who is involved:

- J. Heitger
- F. Knechtli
- M. Peardon
- J. Urrea-Niño
- T. Korzec
- R. Höllwieser
- J. Finkenrath

Outline:

1. Basics
2. Distillation
3. Distillation Profiles
4. Performing the contractions
5. Example1: Charmonium Spectroscopy
6. Example2: Comparing different particles
7. Other Applications and Outlook

Basics



$$\langle\langle \mathcal{O}_i(t)\mathcal{O}_j(0)\rangle\rangle = \sum_n Z_n \exp(-E_n t)$$

- **Isolate channel** with lattice group representation
- We want **good overlap** with physical states. (Typically the ground state)

Smearing

$$\langle\langle \mathcal{O}(t)\mathcal{O}(0)\rangle\rangle = \langle\langle \bar{q}_1 \bar{\Gamma} q_2 \bar{q}_1 \Gamma q_2 \rangle\rangle$$

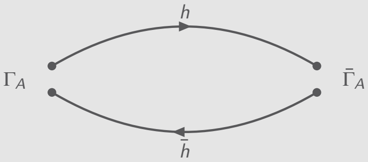


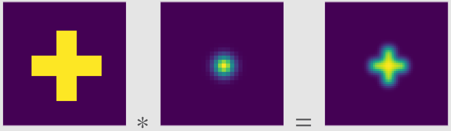
Figure: Sketch of the correlator. Time on x-axis.

- The correlator is the **trace** over this diagram.

- $\bar{q}\,\Gamma\,q$ might have spatial component.
- States are extended
  $\Longrightarrow$ operators should be extended

Smearing

- Interpolators must be **gauge invariant**.
- (Invariant Source)$\bigotimes$(Covariant Operation)=(New Invariant Source)



- This is a kind of **convolution**.
- We can start from a point-source.

Smearing

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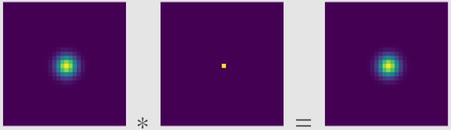
- (Invariant Source)$\bigotimes$(Covariant Operation)=(New Invariant Source)



- This is a kind of **convolution**.
- We can start from a point-source.

Smearing

$$q_{n+1} = c_1(\mathbb{1} + c_2 H) q_n, \quad \text{with} \quad H = \sum_i U_i(x)\delta_{x,y-i} + U_i^\dagger(x-i)\delta_{x,y+i}$$

- We act only on neighbors.
- This approaches convolution with a **Gaussian**.

$$q_{\text{final}} = f * f * f \dots f * q_{\text{initial}}$$

- Central limit theorem
- c_2 and number of iterations control the shape.
- It might not look like a Gaussian [G. Hippel et al. The Shape of Covariantly Smeared Sources in Lattice QCD]

Distillation

$$q_{n+1} = c_1(\mathbb{1} + c_2 H)q_n, \quad \text{with} \quad H = \sum_i U_i(x)\delta_{x,y-i} + U_i^\dagger(x-i)\delta_{x,y+i}$$

The Laplacian is

$$\Delta(x,y) = \frac{1}{6}H(x,y) - \delta_{x,y}.$$

We get

$$q_n = \left(\mathbb{1} + \frac{\sigma}{n}\Delta\right)^n q_0 \quad \Longrightarrow \quad \lim_{n\to\infty} q_n = e^{\sigma\Delta}q_0,$$

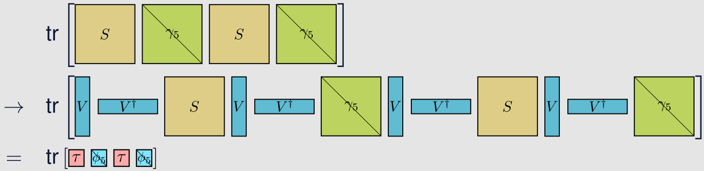
Resulting in suppression of higher **Laplacian eigenmodes**.

Standard Distillation

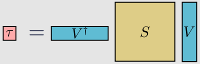
- Higher Laplacian eigenmodes are suppressed.
- Write quark fields in space of N_V **lowest eigenmodes**. [Michael Peardon et al., Physical Review]
- $q \longrightarrow VV^\dagger q$ with $\Delta V_i = \lambda_i V_i$
- Inversions can be **precomputed** and stored.
- Increasingly used for spectroscopy.

Standard Distillation

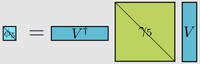
Example: One meson $\langle\langle PP \rangle\rangle$ correlator



With the **perambulator**:



And the the **elemental**:



Graphics by Tomasz Korzec

Distillation Profiles

- We are free to choose $q \to VJV^{\dagger} q$ instead [Francesco Knechtli et al., Physical Review D, 2022].
- J is diagonal with entries $g(\lambda_i)$, the **quark profile**
- **Gaussians** are used in practice.
- Degree of freedom analogous to smearing.
- Changes are **independent of inversion**.
- The **optimal profile** is determined by solving the GEVP.

Distillation Profiles

The perambulator:

$$\tau(t_1, t_2) = V^\dagger(t_1)\, D^{-1}\, V(t_2)$$

The elemental:

$$\Phi_{\substack{i,j \\ \alpha,\beta}}(t) = V_i^\dagger(t)\, \Gamma_{\alpha,\beta}(t)\, \mathbf{g}^*(\lambda_\mathbf{i}(\mathbf{t}))\, \mathbf{g}(\lambda_\mathbf{j}(\mathbf{t}))\, V_j(t)$$

The meson correlator:

$$-\langle \mathrm{tr}\left[\Phi_2(t)\, \tau_{q_a}(t,0)\, \bar{\Phi}_1(0)\, \tau_{q_b}(0,t)\right]\rangle_{\mathrm{gauge}}$$

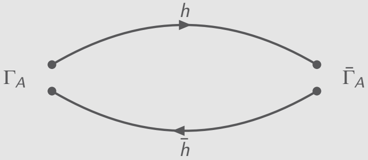
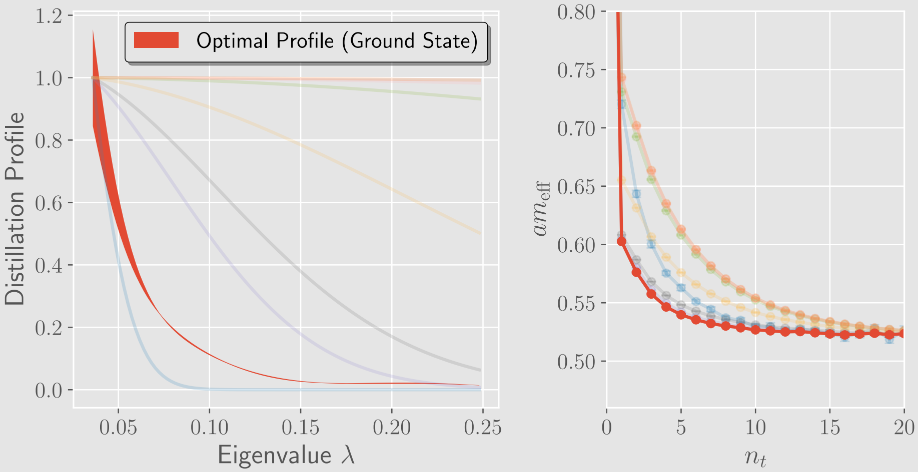


Figure: Sketch of the correlator. Time on x-axis.

Distillation Profiles



Performing the contractions

$$\langle\langle \mathrm{tr}\left[\Phi_0(0)\tau_{q_0}(t_0,t_1)\Phi_1(t_1)\tau_{q_1}(t_1,t_2)\ldots\Phi_{N-1}(t_{N-1})\tau_{q_{N-1}}(t_{N-1},t_0)\right]\rangle\rangle_{\text{gauge}}$$

- τ and Φ are $4N_V \times 4N_V$ matrices*
- Φ decomposes into $(4\times 4)\bigotimes(N_V\times N_V)$*
- Changing the profiles:
  - is volume independent
  - can be done independently for every t_n-combination

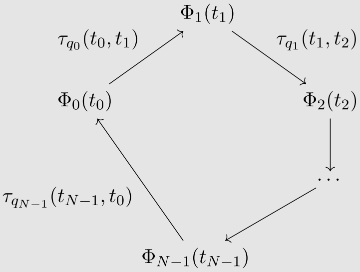


Figure: N-point diagram with distillation

Example1: Charmonium Spectroscopy

- We are interested in Ψ'' ($c\bar{c}$, 1^{--})
- 48×24^3 ($N_f = 2$)
- 8×8-GEVP with:
  - γ_i and $\gamma_4\gamma_i$
  - different smearings
  - covariant derivatives

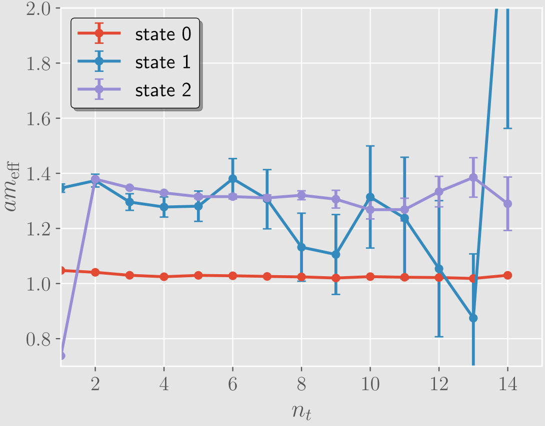


Figure: spectrum without distillation.

Example1: Charmonium Spectroscopy

With Distillation:

- 14×14-GEVP with:
  - γ_i and $\gamma_4\gamma_i$
  - different profiles
  - **No** covariant derivatives

- Similar dependence on γ_4 inclusion

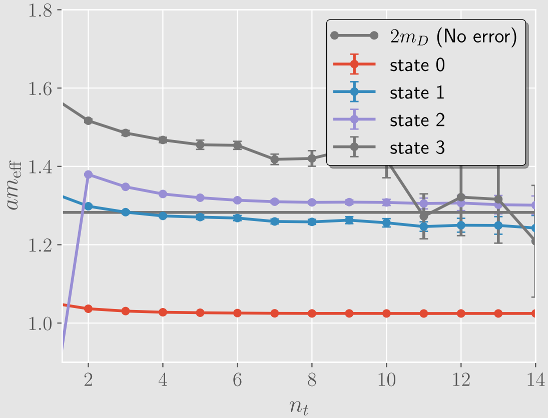


Figure: Spectrum with distillation.

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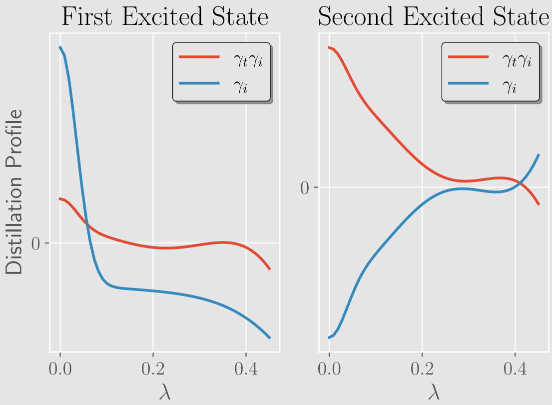

Figure: Distillation profiles

Example2: Comparing different particles

- Different particles show different optimal profiles
- Narrower profile $\leftrightarrow$ less localized contributions

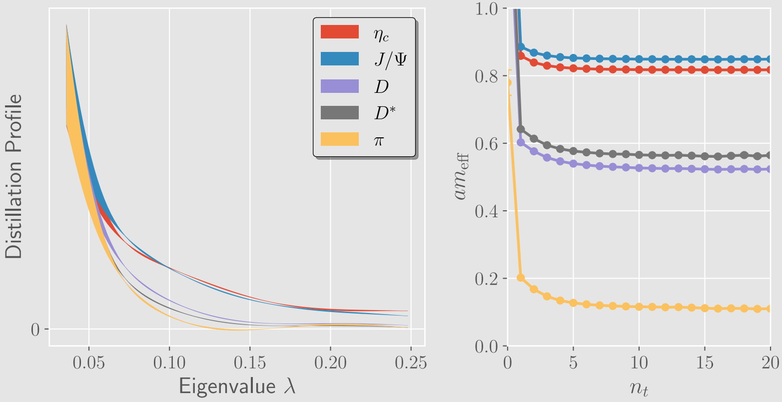

Figure: Different profiles and masses on $N_f = 3 + 1$ ensemble.

Example2: Comparing different particles

- Reconstructed operator **applied to point source**
- **Spatial slice** and average over configurations
- $\mathrm{tr}\,[\gamma_5\Gamma]$ and color average for **scalar value**
- [Francesco Knechtli et al., Physical Review D, 2022] see rings for excited states.

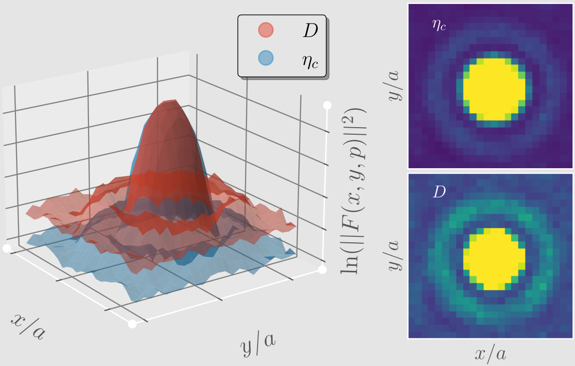

Figure: Profiles in real space.

Other Applications and Outlook

- Application of profiles in **two-particle operators**
- Profiles can be used with **lattice-momenta** and **twisted boundary conditions** [J.N. @ Lattice23]
- Goal is to study $\Psi'' \rightarrow D\bar{D}$ (and $\rho \rightarrow \pi\pi$).
- Charmonium spectroscopy on $N_f = 3 + 1$ [J. Urrea-Niño @ Lattice23]

Thank you for listening!

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