Scale Setting in Nf=2+1 QCD with Wilson fermions from RQCD

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RQCD Collaboration:arXiv:2211.03744

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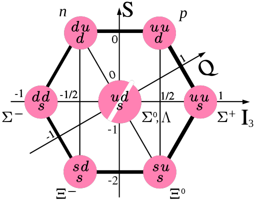
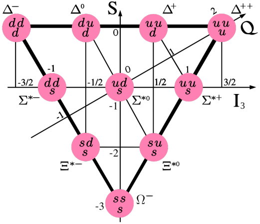
Baryon spectrum

Determine $t_{0,ph}$ using $\lim_{a\to 0}(\sqrt{t_0}m_\Xi)^{latt} = \sqrt{t_{0,ph}}m_\Xi^{ph}$.

Octet: $J^P = \frac{1}{2}^+$



decuplet $J^P = \frac{3}{2}^+$



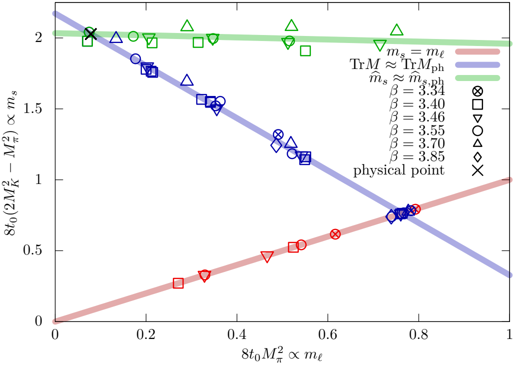
Fit: N, Σ, Λ, Ξ masses

Also fit together: N, Σ, Λ, Ξ and Δ, Σ^*, Ξ^*, Ω masses.

Unstable under strong decay: $\Delta \to N\pi$, $\Sigma^* \to \Lambda\pi, \Sigma\pi$ and $\Xi^* \to \Xi\pi$.

This work: for t_0 determination use the Wilson flow and the clover leaf definition of $E(t)$.

$N_f = 2+1$ CLS ensembles



44 ensembles: non-perturbatively $O(a)$ improved Wilson fermions on tree level Symanzik improved glue. Leading $O(a^2)$ errors in hadron masses.

- ⋆ **High statistics**: typically 6000-8000 MDUs, 1000-2000 configurations.
- ⋆ **Discretisation**: Six lattice spacings: $a = 0.1 - 0.04$ fm.
- ⋆ **Finite volume**: $Lm_\pi \gtrsim 4$ with additional smaller volumes.
- ⋆ **Quark mass**: $m_\pi = 410$ MeV down to m_π^{phys}.

Extrapolation of baryon multiplets

Fit form: $B \in \{N, \Lambda, \Sigma, \Xi\}$

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, L, \mathrm{a}) = \left[\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0) + \delta m_B^{FV}(\mathbb{M}_\pi, \mathbb{M}_K, L)\right] \times \left[1 + \mathrm{a}^2\left(c + \bar{c}\,\overline{\mathbb{M}}^2 + \delta c_B\,\delta\mathbb{M}^2\right)\right].$$

All correlations between m_B, M_π and M_K on each ensemble taken into account.

Discretisation coefficients: 6 parameters for the octet.

For $\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$ and $\delta m_B^{FV}(\mathbb{M}_\pi, \mathbb{M}_K, L)$ use

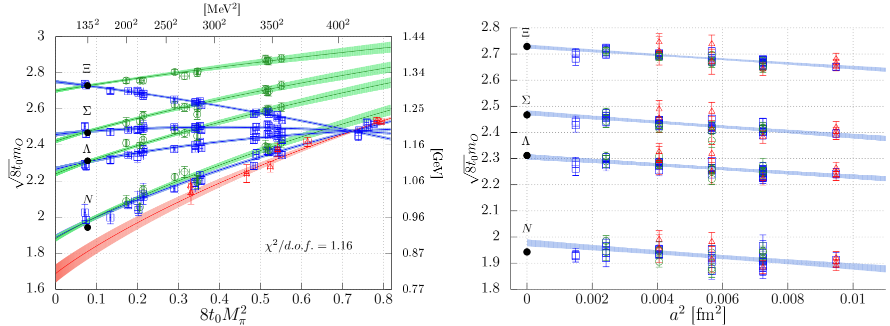
- $O(p^3)$ (NNLO) SU(3) baryon ChPT with EOMS regularisation [Ellis et al.,nucl-th/9904017]: 6 parameters for the low energy constants (LECs).

Also:
Heavy baryon NNLO SU(3) ChPT [Jenkins and Manohar,Phys. Lett. B 255 (1991) 558.]: 6 LECs.
Taylor expansion à la Gell-Mann-Okubo (GMO) about the symmetric point ($m_s = m_\ell$) [QCDSF,1102.5300]. NNLO leads to 11 parameters.

Baryon octet and decuplet fits using the small scale expansion, see e.g. [Martin Camalich et al.,1003.1929].

NNLO BChPT fit to the baryon octet

12 parameters to fit the 4 octet baryon masses, 125 d.o.f.

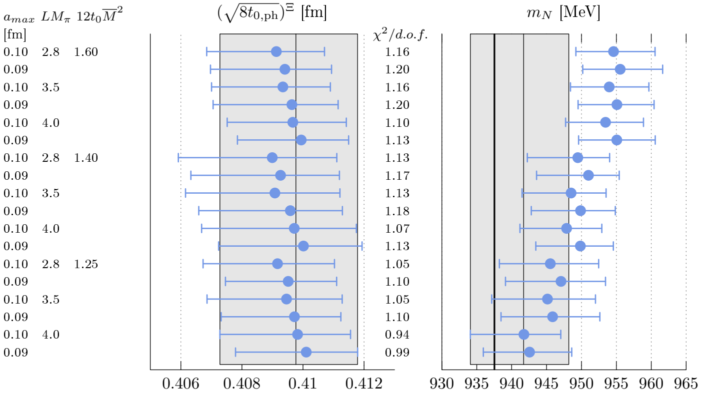


Discretisation effects are mild: around 3% from $a = 0.1$ fm to $a = 0$.

Finite volume effects are small, however, including FV terms in the fit (no extra parameters) improves the $\chi^2/d.o.f.$

$t_{0,ph}$, choice of hadronic scheme (isospin-corrected masses):
$M_\pi = 134.8(3)$ MeV, $M_K = 494.2(3)$ MeV from [FLAG 16,1607.00299].
$m_\Xi = \frac{1}{2}\left(m_{\Xi^0} + m_{\Xi^-} - \delta m^{\mathrm{QED}}\right) = 1316.9(3)$ MeV, $\delta m^{\mathrm{QED}} \approx 2.7$ MeV.

Variation with cuts on the data



$\chi^2/d.o.f$ improves with cuts on $\overline{M}^2 = (2M_K^2 + M_\pi^2)/3$.

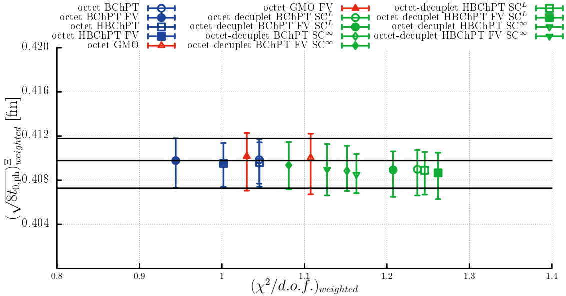
Values of $\sqrt{8t_{0,ph}}$ obtained are consistent. Agreement of m_N with corrected expt. value improves with cuts on $\overline{M}^2$.

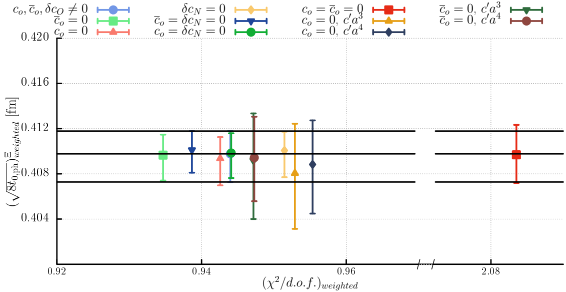
Grey bands indicate the weighted average of the results.

Variation with the fit form

Results for $\sqrt{8t_{0,ph}}$ are very stable (m_Ξ tightly constrained) w.r.t. variations in continuum fit form and discretisation terms.

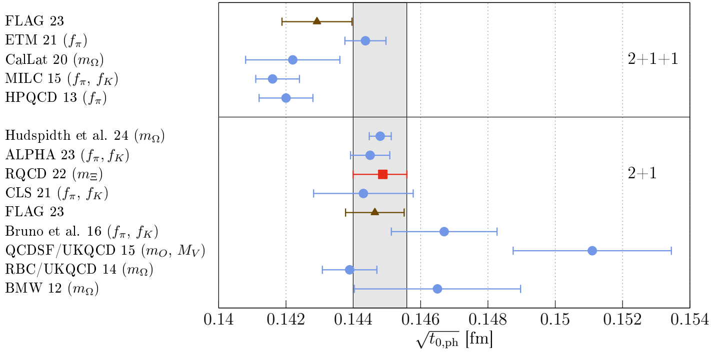
Poorer fit quality: HBChPT and GMO fit forms compared to EOMS BChPT, also when including the decuplet masses.





Comparison with other determinations of $\sqrt{t_{0,ph}}$

Final result: $\sqrt{t_{0,ph}} = 0.1449^{(7)}_{(9)}$ fm



$N_f = 2+1$ CLS ensembles: [Bruno et al. 16,1608.08900], [CLS 21,2112.06696] (proc.), [ALPHA 23,2309.14154,2401.11546], [Hudspith et al. 24,2404.02769].

Future: more ensembles and better statistics already available.